A Review of Pool-Boiling Processes Based on Bubble-Dynamics Parameters

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Abstract: Immersion cooling is widely used for thermal management of servers. The two-phase immersion cooling, which transfers heat by boiling, possesses efficient temperature control ability under intensive heat generation. In the process of temperature control through boiling, the generation and transportation of bubbles play a crucial role in calculating the heat-transfer capacity. Therefore, it holds immense significance to obtain a profound understanding of the mechanisms underlying bubble formation and detachment. Currently, numerous mechanistic explanations and empirical correlations have been proposed to elucidate the various parameters of bubbles during the boiling process. These findings were considered to be valuable references when selecting appropriate boiling media and designing efficient heating surfaces. To comprehensively present the progress of bubble formation and heat transfer in the boiling system, the forces exerted on the bubbles are highlighted in this article. A meticulous review of bubble-force analysis and correlation formulae pertaining to various relevant parameters (e.g., nucleation sites density, bubble growth rate, bubble growth period, and detachment frequency) was conducted. This review article was also expected to provide a novel foundation for further exploration of enhanced boiling heat transfer.

Keywords: nucleate boiling; force analysis; bubble-dynamics parameters; heat-transfer enhancement

1. Introduction

In the context of ongoing industrialization and informatization, electronic devices have become increasingly miniaturized and highly integrated, especially in sectors such as automotive engineering, artificial intelligence, and data centers [1,2]. As a consequence, the heat-dissipation challenges from these advancements have become severe. The heat flux of individual chips could reach 100–1000 W/cm², which exceeds the cooling performance of traditional air-cooling techniques [3–5]. Therefore, the boiling cooling, which provides excellent heat-transfer efficiency and uniform temperature distribution, has attracted considerable attention. Boiling could transfer heat rapidly and control the wall temperature effectively. Currently, the heat-dissipation capacity of boiling cooling achieves as high as 202 W/cm², which is about six times higher than that of conventional air-cooling methods [6]. Therefore, boiling-cooling technology is widely used in various fields, such as nuclear power plants [7], air-conditioning units [8], heat pipes [9], rocket engines [10], battery cooling [11], and electronics cooling [12,13].

Boiling can be divided into two main categories: pool boiling (e.g., two-phase immersion cooling) and flow boiling (e.g., two-phase minichannel cold plate). Pool boiling, as a passive method of heat transfer through phase change, relies solely on the buoyancy to facilitate the cooling process. This characteristic not only gives pool boiling a high heat-transfer efficiency, but also provides it with several advantageous features. First,
it requires minimal maintenance due to its self-sustaining nature. Second, pool boiling eliminates the need for pump induction, resulting in simplified system design and lower energy consumption. Third, the packaging and implementation of pool-boiling systems are relatively easy, making the deployment suitable for various applications. These features allow for a remarkable reduction in energy consumption, making it widely adopted in various industries [14]. In the process of realizing temperature control with pool boiling, the heat-transfer capacity is largely determined by the amount of bubbles generated and transferred. Thus, understanding the mechanism of bubbles formation, growth and detachment is crucial for enhancing heat transfer.

In the process of pool boiling, the bubbles will follow a cycle of nucleation, growth, detachment, and nucleation again, as shown in Figure 1. After the bubbles detach from the heated surface during pool boiling, the region will be temporarily devoid of areas for new bubbles. During the early stage of boiling, when the heated wall surface meets the superheating criteria, nucleation sites are formed at the concave pits and the cracks on the wall, inducing bubble nucleation. Then bubbles grow through the combined effects of the thermal boundary layer, micro-convection, and the micro-liquid layer. In this process, the bubbles are mainly subject to the surface tension $F_s$, the liquid static pressure $F_p$, and the excess vapor pressure $F_m$, and the role of gravitational force is usually negligible. When the bubble detachment force exceeds the force dragging the bubble, the bubble detaches from heated surfaces. The superheated wall continues to undergo convective heat transfer with the liquid until the nucleation conditions are met again and the bubble cycle is repeated.

![Figure 1](image_url). Various stages of bubbles in the pool-boiling process.

Evaluation of the correlations of pool boiling showed that the key factors determining the boiling-heat-transfer process include the nucleation site’s density $N_s$, growth rate, detachment diameter $D_d$ and detachment frequency $f$. Among them, $N_s$ is related to the surface conditions and temperature field, the bubble growth rate is closely related to the force-balance model, and the bubble detachment frequency $f$ is determined by the growth time $t_g$ and the waiting time $t_w$. The complex relationship of the above parameters can be represented by the relationship diagram shown in Figure 2.
Many researchers have proposed empirical correlations for various bubble-related parameters, force-balance models, and heat-transfer coefficients. However, there are strict constraints on the use of these empirical correlations, and it is not easy to achieve ideal results outside of their own experimental conditions [15]. The prediction of various parameters of bubble dynamics always relies on accurate boiling-heat-transfer mechanisms, and these theoretically derived parameters of bubble dynamics will, in turn, affect subsequent heat-transfer calculations, making accurate prediction of the boiling process significantly more difficult [16]. Therefore, the goal of this paper is that mathematical descriptions of the various stages of bubble behavior and force analysis are reviewed and summarized, and the accuracy and applicability of these models are further analyzed to provide valuable references for subsequent boiling-heat-transfer calculations and heat-transfer enhancement.

2. Bubble Nucleation

The formation of bubbles in boiling is now generally categorized into two types: homogeneous nucleation and heterogeneous nucleation. The conditions for homogeneous nucleation are more demanding and require the liquid to be superheated to a sub-stable state [17], such as heating water in a smooth cup using microwaves. It needs to be noted that the superheated liquid is very unstable and may induce violent boiling by slight perturbation, causing liquid to be splashed out of the container, which is also known as the burst-boiling phenomenon. Heterogeneous nucleation is the formation of bubbles on heated surfaces with the help of nucleation sites, and is the main form of boiling [18]. Compared with heterogeneous nucleation, the homogeneous nucleation symmetric uniform model is easier to be described by mathematics, thus many researchers have taken advantage of the relative simplicity of the modeling conditions for homogeneous nucleation to assist in the calculation of heterogeneous nucleation mechanisms.

2.1. Homogeneous Nucleation

Homogeneous nucleation tends to happen in superheated water when liquids are heated uniformly on smooth surfaces [17], such as heating smooth test tubes, or microwaving water in a clean cup and releasing the pressure rapidly at high temperature. Superheated water is so unstable that the slightest disturbance may induce violent boiling. Operators should take care of safety in related experiments.

In the case of low-boiling-point liquids stored under high pressure, if the integrity of the container is compromised, the pressure will rapidly drop below the saturation vapor pressure, which could lead to the rapid expansion of liquid or even cause explosions,
resulting in safety hazards. This phenomenon is also known as boiling liquid expanding vapor explosion (BLEVE) [19]. BLEVEs are also found in thermal micro-electric mechanical systems [20], laser-assisted phase change systems [21] and in the transient cooling of hot surfaces in the metal processing industry [22].

2.2. Heterogeneous Nucleation

Rough surfaces with pits, crevices, and cracks are more likely to retain gas and induce bubble nucleation. The heating effect is also stronger than for flat regions, as shown in Figure 3 [23]. The superheating around the bubbles is not uniform because of the thermal boundary layer and micro-liquid layer, making the growth of bubbles in non-homogeneous boiling more complex than in homogeneous boiling.

\[\Delta T = \frac{p_v - p_l}{\rho_v - \rho_l} \approx \frac{1}{\rho_v} \cdot 2\sigma\]

(1)

Figure 3. Pits and crevices are more likely to entrap gas [23].

It is widely recognized that bubble formation requires cavities capable of trapping gas. S.G. Bankoff [24] simplified the cavity to a conical cavity, where the condition for trapping the gas requires the contact angle to be greater than the cone angle of the cavity, i.e., \(\theta > \pi - 2\alpha\), as shown in Figure 4. Cornwell [25] synthesized the influence of microstructure in cavities and extended the bubble-trapping criterion to various shapes, proposing that the gas-trapping surface should satisfy \(\theta > \pi - \alpha - \cos^{-1}(1/S)\), where \(S\) is the ratio of the wetted area to the projected area of the cavity surface, i.e., \(S = S_{\text{actual}}/S_{\text{projected}}\). The Cornwell criterion is in agreement with the Bankoff criterion when the cavity is a tapered cavity. Both equations indicate that hydrophobic surfaces (large contact angles) are more likely to trap gas and form nucleation sites than hydrophilic surfaces (small contact angles).

Figure 4. Schematic of the gas entrapment model proposed by Bankoff [24].

The formation of nucleation sites requires not only pre-existing gas, but also a certain degree of superheat to induce nucleation growth of the bubbles. A combination of force equilibrium and thermodynamic equilibrium is currently commonly used to calculate the required superheat of the bubble embryo, i.e., it is predicted by Equation (3) obtained by combining the Young–Laplace equation and the Clausius–Clapeyron equation, as shown in Equations (1) and (2), where \(\Delta v = \frac{1}{\rho_v} - \frac{1}{\rho_l} \approx \frac{1}{\rho_v}\).

\[R(p_v - p_l) = 2\sigma\]
Hsu [26] argued that the size of the cavity that satisfies the nucleation condition should be an interval rather than the specific size predicted by Equation (1). Therefore, a criterion related to the liquid properties and to the wall temperature was proposed. As shown in Figure 5, the bubble embryo will nucleate successfully when the ambient temperature reaches the bubble temperature, i.e., $T_i \geq T_v$ when the relative height to the wall surface $h = h_b$. The liquid temperature Equation (4) was obtained by solving the transient one-dimensional heat-transfer equation, and the bubble temperature was calculated by Equation (3). Thus, Hsu arrived at a quadratic equation whose solution bounds the size interval of the nucleation cavity, as shown in Equation (5).

$$T_i(y) = (T_w - T_\infty) \left(1 - \frac{y}{\delta_l}\right) + T_\infty$$

$$\{r_{c,\text{min}}, r_{c,\text{max}}\} = \frac{\delta_l \sin \varphi}{2(1 + \cos \varphi)} \frac{\Delta T_w}{\Delta T_{\text{sub}}} \times \left[1 \mp \sqrt{\frac{8\sigma T_5 (1 + \cos \varphi)(\Delta T_{\text{sub}})}{h_{lv} \rho_v \delta_l \Delta T_{\text{sub}}^2}}\right]$$

![Figure 5. Schematic diagram of the thermal boundary layer nucleation model proposed by Hsu [26].](image)

In Equation (5), the angle $\varphi$ is the shape angle formed by the embryonic bubble and the surface measured from the liquid side when the bubble temperature is equal to the liquid temperature, as shown in Figure 5. The shape angle is related to surface tension and hydrophilicity, and the more hydrophilic the smaller the angle $\varphi$. It is important to note that during boiling, cavities may still settle after activation to produce bubbles, resulting in a rapid rise in local surface temperature. Qi and Klausner [27] studied the heterogeneous nucleation of artificial cylindrical cavities fabricated on silicon substrates, with diameters ranging from 8 to 60 $\mu$m and depths of 45 $\mu$m, and observed that bubble nucleation during the boiling process showed an intermittent pattern, with the cavities experiencing a few minutes of silence before next nucleating. This phenomenon may be due to the enhancement of convection induced by boiling, which leads to the thinning of the boundary layer and inhibits nucleation. Therefore, the stability of nucleation sites during boiling is also an important factor that should be considered in surface design.
Based on the nucleation radius model proposed by Hsu [26], water and Novec-649 fluoride liquid were selected as the research objects in this paper in order to investigate the effects of different shape angles on their nucleation radius. The thickness of the thermal boundary layer $\delta_l = 300 \, \mu m$, recommended by Liu et al. [28], is used for calculations, and the boiling is assumed to be saturated boiling, i.e., $\Delta T = 0$. The thermophysical properties of water and Novec-649 fluoride ($p_0 = 1 \, \text{bar}$) are shown in Table 1. Based on the data in the table and Equation (5), the relationship between the range of the nucleation radius and the wall superheat $\Delta T$ for water and Novec-649 at different shape angles is obtained in Figure 6, where the middle part of the curves indicates the range of the nucleation radius. It can be seen from the figure that, according to the nucleation radius model proposed by Hsu [26], the nucleation radius range of boiling increases with increasing shape angle, which also verifies that the stronger the hydrophobicity is, the easier the nucleation is, and the superheat degree does not have much effect on the nucleation radius after reaching a certain standard.

**Table 1.** Physical properties of water and Novec-649 fluoride liquid.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$\rho_l$ (kg/m$^3$)</th>
<th>$\rho_v$ (kg/m$^3$)</th>
<th>$\sigma$ (mN/m)</th>
<th>$h_{lv}$ (kJ/kg)</th>
<th>$T_{sat}$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water [29]</td>
<td>958.35</td>
<td>0.597</td>
<td>59</td>
<td>2256.5</td>
<td>100</td>
</tr>
</tbody>
</table>

**Figure 6.** Range of nucleation radius at different shape angles for (a) water, and (b) Novec-649.

For heterogeneous nucleation, the nucleation site’s density $N_s$ refers to the number of active sites per unit of heated area [31]. The active nucleation site’s density is essential for the determination of heat flux and heat transfer coefficient.

Sakashita and Kumada [32] calculate the density of nucleation sites by Equation (7), which is the function of cavity radius $R_c$ and Jakob number $Ja$, where $Ja$ is a dimensionless measure of wall superheat, as shown in Equation (6). In the following equation, the constant $C_2$ was determined from experimental data, and cavity size $R_c$ was calculated according to the Hsu and Graham model [33].

$$Ja = \frac{\rho_l c_p \Delta T_{sl}}{\rho_v h_{lv}} \quad (6)$$

$$N_s = C_2 \left( \frac{Ja \times R_c^{0.3}}{R_c} \right)^{10} \quad (7)$$
Xiao et al. [34] suggested a predictive correlation for nucleation sites’ density according to the statistical properties of the boiling process, and the cumulative size distribution of the value of \( N_s \) was obtained based on the correlation given by Wang and Dhir [35]. The value of nucleation cavities’ density was expressed by calculating the minimum nucleation cavity density and the maximum nucleation cavity size:

\[
N_s = 7.8125 \times 10^{-29} \times (1 - \cos \theta) R_{c,\text{min}}^{-6} \left[ 1 - \left( \frac{R_{c,\text{min}}}{R_{c,\text{max}}} \right)^6 \right]
\]  

(S8)

Suszko and El-Genk [36] synthesized the conservation of energy and presented the nucleation site density calculation equation involving the bubble detachment diameter, bubble departure frequency, and Jacob number as follows, where the Archimedean number

\[
Ar = \left( \frac{g}{\nu L^2} \right) \left( \frac{\sigma}{\rho L g} \right)^{3/2}.
\]

\[
N_s = \frac{6q}{\pi D^3 \rho_l h_{l,t} f (1 + Ar)}
\]

(S9)

From the perspective of enhanced boiling, boiling surfaces are often expected to have lower nucleation onset (ONB) and denser nucleation sites, thus generating more phase transitions and transferring more heat at low superheat. Cornwell’s [25] criterion can be applied to a wide range of irregular surfaces and provides a valuable reference for surface cavity design. According to this criterion, the wall-contact angle of the liquid should be as large as possible, i.e., more hydrophobic. Bourdon et al. [37] also noted that hydrophobic surfaces have lower boiling-onset temperatures and higher heat-transfer coefficients than hydrophilic surfaces under low superheat conditions. It should be noted, however, that while hydrophobic surfaces are more likely to nucleate, bubbles on hydrophilic surfaces are more likely to detach. Jo et al. [38] found that hydrophobic surfaces have lower boiling-onset superheat and higher boiling heat-transfer coefficients than hydrophilic surfaces, but their critical heat flux (CHF) is lower, which may be caused by the excessive bubble size on hydrophobic surfaces, which results in no liquid wetting underneath the bubbles [39]. Therefore, a combination of hydrophobic and hydrophilic surfaces is considered a viable approach to surface design [40,41].

It needs to be explained that the above criteria (Figures 4–6) can only be applied to ordinary rough surfaces. However, as shown in Figure 7, more and more complex surface structures are being used to enhance boiling heat transfer, such as metal foams [42], nanostructures [43], and porous media [44]. How to describe the disordered microstructure of these complex surfaces by mathematical logic is the urgent problem that still needs to be solved. Moreover, the force analysis of bubbles has an important effect on the disordered microstructure, so the research on force analysis of bubbles is reviewed in order to research the mechanisms of growth and detachment of bubbles after nucleation.

![Figure 7. Surfaces of three complex structures: (a) nanostructures [43], (b) porous media structure [44], and (c) copper metal foam structure [45].](image-url)
3. Mechanical Analysis of the Bubbles

The forces on the bubbles have a great influence on the state of motion and the shape of the bubbles in the processes of growth and separation after nucleation, and it is very important to analyze and model the forces acting on the bubbles. The growth and separation of bubbles on solid surfaces are affected by various forces such as surface tension, buoyancy, and drag force. These forces can be divided into the lifting forces that separate the bubbles (e.g., buoyancy and internal pressure), as well as the resistance forces that make the bubbles attach to the heated surfaces (e.g., surface tension and drag force).

In the early boiling stage, the impact of lift is less than that of resistance, so the bubbles stay attached to the solid wall. When the bubbles are large enough, the influence of lift force begins to exceed that of resistance, so the bubbles detach from the heated surface, rise rapidly to the liquid surface and break up.

In order to research the growth and detachment of bubbles during boiling, many researchers have mathematically modeled the forces on bubbles. Klausner et al. [46] theoretically analyzed the forces on bubbles in flow-boiling, and provided expressions for various forces. Their theory suggests that the growth and detachment of bubbles on a heated wall perpendicular to the direction of gravity, are subjected to a combination of surface tension \( F_s \), unsteady drag force \( F_{du} \), buoyancy \( F_b \), hydrodynamic pressure force \( F_h \), and contact pressure force \( F_{cp} \). Among these forces, the surface tension \( F_s \) originates from the area between the bubbles and the wall and acts along the bubble interface, which prevents the bubbles from detaching; the unsteady drag force \( F_{du} \) originates from the bubble growth process, also known as the bubble growth force \( F_g \), which includes the resistance caused by the asymmetric bubble growth \( F_d \) and the dynamic effects induced by the unsteady flow, such as the additional mass force \( F_i \); and the contact pressure \( F_{cp} \) is induced by the difference in pressure between the inside and outside of the bubble, which acts in the direction perpendicular to the heated surface.

For flow boiling, there will also be a shear lift force \( F_{sl} \) due to the flow of the liquid as well as a liquid flow force \( F_h \), where the shear lift force \( F_{sl} \) acts perpendicular to the direction of the flow and tries to lift the bubbles off the walls. Van Helden et al. [47] believe that this force is influenced by Bernoulli suction and vortex volume. To calculate it, Mei and Klausner [48] proposed a relation based on the assumption of spherical bubbles in an infinite flow field at a low Reynolds number, and related it to Auton’s relation [49] for the shear lift of bubbles in viscous flow at low tension rates, giving an equation for shear lift in a wide range of Reynolds numbers:

\[
F_{sL} = \frac{1}{2} C_L \rho_l \pi u^2 R^2_b 
\]  

(10)

where \( G_s = \left| \frac{du}{dx} \right| \frac{R_b}{\pi} \) and \( C_L \) is the lift-off coefficient, expressed as:

\[
C_L = 3.877 G_s^{1/2} \left[R e_p^{-m/2} + \left(0.344 G_s^{1/2}\right)^{n} \right]^{1/m}, m = 4
\]  

(11)

The liquid flow force \( F_h \) is derived from the differential pressure generated by the liquid flow, which is given by:

\[
F_h = \frac{9}{8} \rho_l \pi u^2 R^2_b 
\]  

(12)

Thus, its mechanics are described as:

\[
\Sigma F = F_s + F_{sL} + F_{du} + F_b + F_{cp} + F_h = 0
\]  

(13)

The flow velocity \( u \) of the liquid can be approximated as zero in pool boiling, so for pool boiling \( F_h \) and \( F_{sl} \) can be neglected, i.e.,

\[
\Sigma F = F_s + F_{du} + F_b + F_{cp} = 0
\]  

(14)
Klausner’s mechanical model [46] has been applied and improved by many researchers. Thorncroft et al. [30] proposed that the static force wall-contact pressure \( F_{cp} \) as well as the first term of the unsteady resistance \( F_{du} \) should be neglected. Therefore, the second coefficient of \( F_{du} \) was changed from 1.5 to 2, and the wall-contact pressure \( F_{cp} \) was modified to obtain the mechanical equilibrium on the bubble detachment in the pool-boiling, as shown in Equation (15):

\[
\frac{4}{3}(\rho_l - \rho_v)\pi R_b^3 g - 2\pi \rho_b R_b^2 \left( \frac{d^2 R_b}{dt^2} \right)^2 - 2\pi R_b \sigma \sin^2 \varphi = 0
\]  

(15)

Jiang et al. [51] modified Klausner and Thorncroft’s model by proposing a different growth-force equation and considering the drag force caused by the wake of the bubble after it leaves. They proposed that as the bubble grows, the vapor in the bubble and the liquid expelled around the bubble will exert a growth or inertial force on the bubble. They modeled this inertial force on the growth rate of the bubbles and developed a dynamic-boiling model that takes into account the effects of evaporation action at the contact line and heat transfer from the thermal boundary layer, and which also takes into account the change in the contact angle during the growth of the bubbles. Wang et al. [52] made some modifications to the coefficients and considered the Marangoni stress acting on the bubbles, which is a stress gradient caused by the temperature difference, with the greater the liquid subcooling, the greater the effect of Marangoni force. Bhati et al. [53] and Paruya et al. [54] on the other hand, proposed that for bubbles in boiling liquid the effect of contact pressure can be neglected.

Siddharth [55] performed further calculations on the volume of the truncated sphere and modified the drag coefficient. Among these models, Klausner, Wang and Siddharth’s models are most widely used for reference. The specific relational equations they proposed are listed in the Table 2, where \( R_{cur} = 5 \), \( R_b \) is the radius of curvature at the bottom of the bubble, and \( C_d \) is the drag coefficient, which is obtained by the rate of growth of the radius of the bubble [56], whose expression is \( C_d = 5360 \mu_1^{-0.79} / [\rho_l (dR/dt)]^{0.79} \).

<table>
<thead>
<tr>
<th>( F_s )</th>
<th>( F_b )</th>
<th>( F_{cp} )</th>
<th>( F_d )</th>
<th>( F_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{s} = -2\pi R_b \sigma \sin^2 \varphi )</td>
<td>( F_{b} = \frac{2}{3}\pi R_b^2 (\rho_l - \rho_v) )</td>
<td>( F_{cp} = \frac{2\pi}{R_{cur}} \pi R_b^2 \sin^2 \varphi )</td>
<td>( F_{d} = -\frac{2}{3} \rho_l \pi R_b^2 \left( \frac{dR_b}{dt} \right)^2 )</td>
<td>( F_i = -\rho_l \pi R_b^2 \frac{d^2 R_b}{dt^2} )</td>
</tr>
</tbody>
</table>

Bucci et al. [57] took high-speed video and infrared thermometry to modify these formulae by quantifying the forces. They conducted a series of experimental studies on the growth and detachment of bubbles under boiling conditions on the horizontal pool. By contouring the liquid–gas interface, they calculated the external forces acting on the bubbles. These forces versus time during bubble growth and detachment are shown in Figure 8. Bucci et al. argue that the force-balance approach relies on predicting, for example, the rate of change of bubble momentum, which is very small, to give a basis for the viscous drag \( F_d \) and inertial force \( F_i \) of the bubble, leading to experiments with very high demands on the precision of the measurements. This relationship makes it difficult to obtain the desired results for bubble-detachment-radius prediction by force-balance methods.
Liu et al. [59] calculated the pressure on the bubble surface by spherical coordinate integration, and considered the effect of the micro-liquid layer at the bottom of the bubble on the overall pressure. The receding contact angle \( \alpha \) was taken in their calculations, which was obtained by photographing the state of the bubble before it detached. The forces on the bubble in their mechanical model include the surface tension \( F_s \), the liquid pressure \( F_p \), the gravity \( F_g \), the momentum change force \( F_d \), the internal pressure of the bubble \( F_m \), the instantaneous expansion force acting on the boundary of the bubble \( F_a \), which is oriented perpendicular to the boundary of the bubble, and the combined force acting on the bubble, which was denoted as \( F_Z \). The comparison between Wang et al.’s [52] proposed force model and that of Liu et al. [59] is shown in Figure 9, and the computational equations of various forces are shown in Table 3, where \( H \) denotes the height from the wall to the liquid surface, \( \beta \) is the ratio of the area of the micro-fluid layer to the area of the bottom of the bubble, \( q_{\text{fl}} \) denotes the heat flux at the gas–liquid interface around the bubble, and \( S_b \) denotes the area of the gas–liquid interface, with the negative sign indicating that the direction is along the direction of gravity. In its assumptions, the radius of the micro-fluid layer is approximately twice the radius of the dry portion.

In all of the above models, the researchers used Archimedes’ principle for the calculation of buoyancy, i.e., \( F_b = \rho_g V \). It should be noted that Archimedes’ law can only be applied to the bubbles after they have detached from the wall, and for the bubbles that have not yet been detached Archimedes’ law does not apply because the part of the bubble that is in contact with the wall is not subjected to the pressure of the fluid [58].

Liu et al. [59] calculated the pressure on the bubble surface by spherical coordinate integration, and considered the effect of the micro-liquid layer at the bottom of the bubble.
Table 3. Liu et al.’s [59] proposed bubble-force equations.

<table>
<thead>
<tr>
<th>Types of Forces</th>
<th>Mathematical Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_s$</td>
<td>$F_s = -\pi R_b \sigma \sin^2 \alpha$</td>
</tr>
<tr>
<td>$F_p$</td>
<td>$F_p = F_{p1} + F_{p2}$</td>
</tr>
<tr>
<td>$F_{p1}$</td>
<td>$F_{p1} = 2\pi \rho_l g R^2 \left[ \frac{1}{4} (H - R \cos \alpha)(\cos 2\alpha - 1) + \frac{1}{3} R (\cos^3 \alpha + 1) \right]$</td>
</tr>
<tr>
<td>$F_{p2}$</td>
<td>$F_{p2} = \beta \rho_l g H \pi (R \sin \alpha)^2$</td>
</tr>
<tr>
<td>$F_G$</td>
<td>$F_G = -\rho_l g \frac{2R}{R} (2 + 3 \cos \alpha - \cos^3 \alpha)$</td>
</tr>
<tr>
<td>$F_e$</td>
<td>$F_e = -\frac{\pi^2 \rho_l q''}{2} \left[ \frac{5}{4} R \cos \alpha + 1 \right]$</td>
</tr>
<tr>
<td>$F_m$</td>
<td>$F_m = \Delta p \pi \left( \frac{1}{2} R \sin \alpha \right)^2$</td>
</tr>
</tbody>
</table>

It can be seen from the equation that the bubble force is also affected by the liquid level height $H$ in Liu et al.’s model, which provides a new insight for the design of the boiling system. Kopchikov et al. [60] also pointed out that the boiling heat transfer coefficient increases significantly with the reduction in liquid height when the liquid height is reduced to a certain value.

Liu et al. [59] proposed the formula $p = \rho_l g h$ for liquid pressure, however, the effect of external air pressure should also be taken into account due to the adhesion of the bubbles to the wall, i.e., $p = \rho_l g h + p_0$. The integration of the actual liquid pressure on the bubbles in combination with the external pressure results in:

$$F_p = \frac{1}{3} \rho_l g \pi R_b^2 (2 + 3 \cos \varphi - \cos^3 \varphi) - (\rho_l g h + p_0) \pi R_b^2 \sin^2 \varphi \quad (16)$$

The relationship between bubble growth and systemic pressure has also been confirmed by many researchers. Labuntsov et al. [61] and Akiyama et al. [62] pointed out that the bubbles grow more slowly with increasing pressure. Moreover, Miglani et al. [63] and Gao et al. [64] observed that the higher the pressure, the smaller the diameter of the bubble detachment.

A review of these forces shows that the bubble forces are related to the density of the liquid, the surface tension, wall wettability, and the growth rate of the bubbles, so all these physical properties of the liquid should be taken into account when selecting a boiling medium. The equilibrium of forces on the bubbles is critical for the prediction of the bubble detachment diameter, which is discussed in Section 4.3. The analytical difficulties among these forces are that the first- and second-order derivatives of bubble size, with respect to time, contained in unsteady resistance $F_{du}$, are difficult to predict accurately. The ever-changing mass transfer process also keeps the center of mass of the bubbles in an unsteady state, and an accurate bubble growth and detachment model will help in the calculation of these variable quantities.

4. Bubble Growth and Detachment Model

Mahmoud et al. [65] conducted an experimental study on the growth of bubbles in the saturated pool-boiling of deionized water on the surface of pure copper under atmospheric and sub-ambient pressures. The measurements were conducted using a high-speed, high-resolution camera, and the complete cycle of bubbles from nucleation, growth, detachment to nucleation again is shown in Figure 10. They also evaluated existing uniform and non-uniform bubble growth models based on experimental data [66]. They found that the existing models are difficult to unify after summarizing these studies, most of which are only applicable under the experimental conditions employed by the proposers themselves, and suggested that the phenomenon may be caused by the excessive differences in the mechanisms affecting bubble kinetics (bubble growth rate, bubble detachment diameter, bubble growth period, etc.), liquid properties and surface structure. They also point out that modeling uniform boiling can provide a reference for calculations of non-uniform
boiling. Modeling of bubble growth during uniform and non-uniform boiling is presented in detail next.

\[
\frac{p_v - p_{\text{Sat}}}{(1 - \rho_v/\rho_l)\rho_l} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{2\sigma}{(1 - \rho_v/\rho_l)\rho_l R} + \frac{4\mu_l R}{(1 - \rho_v/\rho_l)\rho_l} \frac{dR}{dt} \quad (17)
\]

The homogeneous nucleation of bubbles was categorized into three distinct phases by Mahmoud [66]: surface tension growth, inertial growth, and thermal diffusion growth, as shown in Figure 11.

Figure 10. One complete bubble cycle during boiling at \( p = 1 \text{ bar}, \Delta T = 9.4 \text{ K} \) [65].

4.1. Bubble Growth in Uniform Boiling

The second-order non-linear type of ordinary differential equation, as shown in Equation (17), for the bubble when uniformly heated can be obtained from the mass conservation equation and momentum conservation equation of the bubble. These equations are grounded in the following assumptions.

(i). Bubbles grow in an infinite medium and that their growth has spherical symmetry;
(ii). The surrounding fluid is Newtonian fluid;
(iii). The bubbles are assumed to be unaffected by external forces;
(iv). The temperature and pressure inside the bubble are uniform;
(v). Due to mass transfer at the interface, the liquid around the bubble is assumed to flow at a velocity of \( u_l = \left(1 - \rho_v/\rho_l\right) \frac{dR}{dt} \);
(vi). There is no translational or rotational motion in bubble growth.
The liquid temperature at the bubble interface is equal to the vapor temperature in thermodynamic equilibrium, which is provided by the transient energy equation without an internal heat source (Equation (20)), and the boundary conditions at the bubble interface take the second type of boundary conditions (heat flux is known). Thus, the solutions of different growth stages of homogeneous nucleation [66].

![Figure 11](image)

In the surface tension growth phase, the initial equilibrium radius of the bubble is \( R_0 \), which is obtained from the pressure difference between inside and outside the bubble, and the surface tension equilibrium is \( p_v - p_{\infty} = 2\sigma/R \), i.e., the Young–Laplace equation. The temperature of the bubble is equal to the temperature of the surrounding liquid, i.e., \( T_v = T_{L\infty} \). At this point, the velocity and acceleration are so tiny that its inertial terms are negligible, and the dynamic growth of the bubble is driven by the pressure difference.

In the inertial growth phase of the bubbles (\( R \approx 10\ R_0 \)), the bubble radius is significantly larger than the initial radius \( R_0 \). Therefore, the third term on the right-hand side of surface tension Equation (17) becomes negligible and it is the inertia of the liquid that dominates the growth of the bubbles. It needs to be noted that the time scales of the first and second phases are in the order of microseconds. Sernas et al. [67] and Forster et al. [68] confirmed the weakening of the dynamic effect after 50 \( \mu \)s (from experimental measurements) and 100 \( \mu \)s (from numerical analysis), respectively.

During the thermal-diffusion-growth phase, phase change appears on the bubble interface and its thermal boundary layer, called the “cooling effect” [69]. As a result, the vapor temperature reduces from the initial superheat to saturation temperature, so that the vapor pressure \( p_v \) equals the system pressure \( p_{\infty} \). This makes the left term of Equation (17) zero, the dynamic effect no longer drives bubble growth, and it is the temperature difference \( T_{\infty} - T_v \) that drives bubble growth.

Although the above differential equation does not have an analytical solution, each of the individual growth stages in it has an approximate solution or a numerical solution. Rayleigh [66] neglected the surface tension phase as well as the vapor–liquid density ratio \( \rho_v/\rho_l \), and assumes that the bubble grows isothermally to obtain a Rayleigh solution (Equation (18)) to inertia-controlled growth, which shows that radius is proportional to time.

\[
R = \sqrt[3]{\frac{2}{3} \frac{p_v - p_{\infty}}{\rho_l}}
\]

The energy equation needs be coupled with the kinetic Equation (17) in order to combine the heat transfer with the bubble growth problem. The association between Equations (17) and (18) is realized using the pressure difference \( p_v - p_{\infty} \) in Equation (17), e.g., the well-known Clausius–Clapeyron equation (Equation (19)), which allows us to relate the pressure difference term to the liquid superheat term \( T_{L\infty} - T_v \). The liquid temperature at the bubble interface is equal to the vapor temperature in thermodynamic equilibrium, which is provided by the transient energy equation without an internal heat source (Equation (20)), and the boundary conditions at the bubble interface take the second type of boundary conditions (heat flux is known). Thus, the solutions of
Equations (17) and (19), with the proper initial assumptions, are able to model the growth of bubbles, and these equations can be solved by the numerical calculations

$$\Delta P = P_0 - P_{\infty} = \frac{\rho_0 h_{10}}{T_{\text{sat}}} (T_{\infty} - T)$$  \hspace{1cm} (19)

$$\frac{\partial T}{\partial t} + \frac{(1 - \rho_0 / \rho_1)R^2}{r^2} \frac{d r}{d t} \frac{\partial T}{\partial r} = \frac{1}{r^2 a_1} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right)$$  \hspace{1cm} (20)

$$k_1 \left( \frac{\partial T}{\partial r} \right)_{r=R} = \rho_0 h_{10} \frac{d R}{d t}$$  \hspace{1cm} (21)

Since the numerical solution does not provide an explicit expression for the bubble radius and the kinetic effect of the bubble is only important for a tiny time interval, the complicated inception phase (i.e., surface tension growth and inertial growth) has been ignored and an approximate analytical solution is provided for the asymptotic phase in which the radius of the bubble is proportional to the square root of time (i.e., $$R \propto \sqrt{t}$$). Moreover, the asymptotic solution could be calculated based on Equation (21) if the temperature gradient $$\left( \frac{\partial T}{\partial r} \right)_{r=R}$$ is known.

A comparison of correlations for uniform boiling [70–75] was conducted and shown in Table 4. In the following correlations, $$Ja$$ is the Jacob number and the expression is $$Ja = \rho c p_0 \Delta T_w / \rho_0 h_{10}$$, $$a_1$$ is the thermal diffusivity and the expression is $$a_1 = L / \rho c p_0$$.

These correlations were compared with pool-boiling experiments by Mahmoud et al. [65]. It was found that for bubble growth at atmospheric pressure, the prediction curve of the formula proposed by Fritz et al. [70] is most consistent with the experimental curves, with relative errors between 18.5% and 31.6%; for the conditions at 0.5 bar and 0.15 bar, these homogeneous models are difficult to coincide with the experimental curves.

Table 4. Growth curves of bubbles in uniform boiling.

<table>
<thead>
<tr>
<th>Author</th>
<th>Mathematical Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fritz and Ende [70]</td>
<td>( R = \sqrt{4 / \pi}</td>
</tr>
<tr>
<td>Plesset and Zwick [69]</td>
<td>( R = \sqrt{12 / \pi}</td>
</tr>
<tr>
<td>Forster and Zuber [68]</td>
<td>( R = \sqrt{12 / \pi}</td>
</tr>
<tr>
<td>Scriven [71]</td>
<td>( R = \sqrt{12 / \pi}</td>
</tr>
<tr>
<td>Avdeev and Zudin [72]</td>
<td>( r_+ = \frac{r_1}{(1 + F)^{1/3}}, r_- = \frac{r_1}{1 + \frac{2}{\sqrt{7}}}, t_{ch} = \frac{2 a_1}{\tau}, U = \sqrt{\frac{2 \Delta T}{3p_0}}, F = \frac{2 a_1}{\sqrt{7}}, \tau = \frac{1}{c_p}, k_0 = \frac{2 a_1}{\tau}, r_1 = \frac{1}{2} \left( 1 + \frac{2}{\sqrt{7}} \right)^{3/2} - 3 \left( 1 + \frac{2}{\sqrt{7}} \right)^{1/2} + 2 \right] ), ( \psi = \left[ 1 + \sqrt{2} \left( \frac{1}{\sqrt{1-N}} - 1 \right) \right], N = \frac{c_p \Delta T}{h_{10}} )</td>
</tr>
<tr>
<td>Mikic [73]</td>
<td>( r_+ = \frac{3}{2} \left( (r_1 + 1)^{3/2} - (r_1)^{1/2} \right) - 1 ), ( r_- = \frac{\Delta T}{(B-1)^{1/2}} ), ( B_+ = \left( \frac{12}{\pi} a_1 \right)^{1/2} /</td>
</tr>
</tbody>
</table>

The growth rate of bubbles is mainly determined by the $$Ja$$ number and the thermal diffusivity of the liquid in uniform growth models from Table 4. Early studies considered the bubble growth rate to grow linearly with the square root of time, while later studies mostly took the form of dimensionless numbers to describe the bubble growth rate in uniform growth models.
4.2. Bubble Growth in Non-Uniform Boiling

The growth of bubbles in non-uniform boiling is more complex and unpredictable than the situation in uniform boiling. The radius $R$ is mostly proportional to $t^{0.5}$ in most of the above-discussed correlations, while the time exponent can be less than 0.5 in non-uniform boiling, e.g., Strenge et al. [74] measured the growth of bubbles during a saturated-boiling experiment with n-pentane and ether at atmospheric pressure and noticed that the radius of the bubbles conforms to the relation $R \propto t^n$, where $n$ takes values in the range of 0.19~0.475. The main difference in the modeling of bubble growth during the asymptotic stage of non-uniform boiling lies in the assumptions made on the heat-transfer mechanism of the bubbles. The researchers assumed two main mechanisms, one in which the increase in bubble mass originates mainly from the evaporation of liquid in the micro-fluid layer below the bubble, and the other attributed to the evaporation of the thermal boundary layer around the bubble surface.

It is now generally accepted that bubble growth is attributed to the action of the micro-fluid or thermal boundary layer, as shown in schematic Figure 12. In Figure 12a, bubble growth is driven by evaporation from the boundary layer carried by the bubbles from the heated surface. In Figure 12b, the bubbles protrude outside the wall’s thermal boundary layer (WTBL) and grow from microlayer evaporation. The mechanism in Figure 12c is similar to that in Figure 12a, except that the thermal boundary covers only a part of the bubble. In Figure 12d, the growth of the bubble comes from the combined effect of the micro-fluid layer and the thermal boundary layer around the bubble.

![Figure 12](image)

Different growth modes of heterogeneous nucleating bubbles [66]. (a,c) boundary layer evaporation, (b) microlayer evaporation, (d) combined microlayer and superheated liquid evaporation.

Mahmoud et al. [66] concluded that models based only on micro-fluid layer evaporation are insufficient to explain bubble growth, and can only reach good agreement with the results under certain specific experimental conditions after a detailed evaluation of several mathematical models of the micro-fluid layer. Bubble growth is mainly promoted by evaporation from the thermal boundary layer around the bubble, and the effect of the micro-liquid layer is relatively small for some fluids such as water.

4.2.1. Thermal Boundary Layer Bubble Growth Modeling

Bubbles in non-uniform boiling are not homogeneous spheres, which are usually calculated as truncated spheres, and the heat in a thermal system usually comes from the superheated wall at the bottom of the bubble. In this regard, Zuber [75] modified the uniform-growth model proposed by Fritz and Ende [70] to incorporate the effects of superheat. This model takes a uniform thickness $\sqrt{\pi a t}$ based on a “thin boundary layer” approximation of the interfacial heat balance. At uniform boiling, each heat-flux vector points to the internal bubble interface. On the contrary, in non-uniform boiling, partial heat is transferred to the bubble interface as a temperature potential ($T_{\text{max}} - T_{\text{sat}}$) and the other part of the heat is transferred to the liquid as a temperature difference ($T_{\text{max}} - T_{\text{Lk}}$). In other words, there is a temperature peak $T_{\text{max}}$ which exists in the boundary layer, which
decreases towards the bubble interface and the liquid. $T_{\text{max}}$ is considered to be equal to the superheated surface temperature $T_w$ and the heat flux $q_{Lb}$ to the liquid body is assumed to be equal to the wall heat flux. The factor $b$ is proposed to modify the interference of the interface curvature on the temperature gradient, usually between one and $\sqrt{3}$, and the recommended value $b = \pi/2$ is obtained by comparing with the experimental data. The energy equation in non-uniform boiling was corrected to Equation (22) to obtain the bubble radius in Equation (23).

$$
\rho_v h_v \frac{dR}{dt} = b \left[ k_L \frac{T_w - T_{\text{sat}}}{\sqrt{\pi} \alpha_L t} - q_{Lb} \right]
$$

(22)

$$
R = b \frac{2}{\sqrt{\pi}} Ja \sqrt{\alpha_L t} \left[ 1 - \frac{q_{Lb} \sqrt{\pi} \alpha_L t}{2k_L (T_w - T_{\text{sat}})} \right]
$$

(23)

A relaxation phenomenon exists in all fields of physics, whereby when the system is out of equilibrium, it requires a delay time to return to its initial state, and this process shows exponential regression. Van Stralen [76] launched a study on the growth model of bubbles with the help of this concept. As the bubble expands radially, superheated fluids accumulate around the bubble up to a certain height $y$ measured from the heated surface. The boundary layer around the bubble up to this height $y$ is assumed to have a uniform thickness and is referred to as the “relaxation layer”. Van Stralen also assumed that the liquid superheat in the “relaxation layer” drops exponentially with time from the initial maximum ($T_w$), as in Equation (24). Van Stralen used the bubble detachment time $t_d$ as the characteristic time in Equation (24) based on the assumption that bubble growth followed the relaxation law. This correlation performed an energy balance for partially heated bubbles and incorporated the time-dependent superheating term into the relationship. Its calculation result (Equation (25)) is similar to that of the Plesset and Zwick [69] model of uniform boiling, and his model was also multiplied by the factor $b^*$ defined by Equation (26), which indicates the proportion of the bubble surface area covered by the superheated liquid layer.

$$
\Delta T = \Delta T_w \exp \left( -\sqrt{\frac{t}{t_d}} \right)
$$

(24)

$$
R = b^* \sqrt{\frac{12}{\pi}} Ja \sqrt{\alpha_L t} \left[ -\frac{t}{t_d} \right] \sqrt{\alpha_L t}
$$

(25)

$$
\frac{b^*}{\sqrt{12/\pi}} \frac{2.7183 R d \rho_v h_v}{\sqrt{12/\pi} \sqrt{\rho_\ell c_\ell} k_L \Delta T_w \sqrt{t_d}}
$$

(26)

The “relaxed boundary layer” model proposed in Equation (25) predicted all the data well in all types of experiments, with deviation errors within 15% for a wide range of superheat and low- and medium-pressure conditions [66]. The limitation of this model is that the bubble detachment period $t_d$ and the radius of detachment $R_d$ need to be provided. The departure time, as well as the bubble detachment diameter, were taken directly from the previous experimental data in the above calculation. Therefore, accurate bubble detachment diameter and bubble detachment period need to be calculated first for exact calculation.

4.2.2. Empirical Modeling of Bubble Growth

Many models of bubble growth in non-uniform boiling have been proposed based on experimental results. Cole and Shulman [77] observed bubble growth in the saturated boiling of various liquids on a smooth metallic zirconium belt at different wall-superheat levels. They evaluated a number of homogeneous growth models based on their data and proposed that the empirical growth coefficient $\beta$, which is usually a function about $Ja$, would be smaller than the homogeneous model due to non-uniform superheating in non-uniform boiling. These data were correlated in the form given by Equation (27), and
the empirical coefficient 2.5 and the exponent 0.75 to the Jacob number were taken (mostly one in uniform boiling).

\[ R = 2.5 J a^{0.75} \sqrt{\alpha_L t} \]  

(27)

Du et al. [78] synthesized experimental data from various studies on saturation boiling with water on different metal surfaces at different pressures and wall superheat, and proposed fitting these data with the formula by Equation (28), where the exponent of time, \( n \), depends on the ambient pressure in Mpa. Factor \( n \) varies equally as the main factor controlling the growth rate varies. This is because growth is much more influenced by the inertial growth stage at low pressures, whereas growth is mainly controlled by heat transfer under high-pressure ambient conditions.

\[ R = \beta a^{1/2} t^n, \beta = 2.1077 J a^{0.7902} \]  

(28)

\[ n = 1.0012 e^{-p/0.3257} - 0.9624 e^{-p/0.6161} + 0.5 \]  

(29)

Benjamin and Balakrishnan [79] collected saturated pool boiling data for a variety of liquids (including water, CCl\(_4\), etc.) from the literature, on metal surfaces in the horizontal direction, and they proposed Equation (30) to fit these data based on their analysis, in which the correction factor \( B = 1.55 \) for water, CCl\(_4\), and n-hexane, and \( B = 0.645 \) for n-pentane and acetone.

\[ R = \frac{1}{2} B A r^{0.135} J a^{0.5} \sqrt{\alpha_L t} \]  

(30)

The accuracy of the above relation was evaluated by Mahmoud et al. [66] based on experimental data from non-uniform boiling and it was found that the Du et al. [78] model has better accuracy at atmospheric pressure, while Benjamin et al.’s [79] model showed better accuracy at low and medium pressures. This phenomenon may be attributed to the incorporation of the Archimedes number, which gravity into account, which is neglected in other studies.

4.3. Bubble Detachment Diameter

The bubble detachment diameter is the bubble diameter when it leaves the boiling surface, which is an important parameter in calculating heat flux during the boiling process. The bubble detachment radius ranges from micrometers to centimeters under different working conditions [80]. Bubble detachment diameter is usually determined by mathematical calculations with the aid of force balances or experimental measurements using a high-speed camera to capture the boiling process of a single bubble image. Such measurements are limited to low heat flux, i.e., in the early stage of boiling. The existing models are insufficient to analyze the heat mechanism of bubble merging and still need to be improved.

It is generally accepted that bubble detachment diameter prediction requires a combination of various factors, such as surface tension, wall wettability, and wall superheat. Many correlation equations for bubble departure diameter have been established by considering various factors, e.g., pressure, roughness, and cavity radius, in conjunction with the thermophysical properties. Fazel et al. [81] performed nucleation boiling experiments with three electrolyte solutions and observed the characteristics of bubbles. It was noted from the experimental results that the detachment diameter increased with higher heat flux and electrolyte concentration. Thus, they obtained the bubble detachment diameter prediction model by combining heat flux and dynamic viscosity, as shown in Equation (31). The model predicts the bubble detachment diameters from experimental data with errors of 2%, 10% and 5% for NaCl solution, KNO\(_3\) solution and Na\(_2\)SO\(_4\) solution, respectively.

\[ D_d = 40 \left[ \mu_v \left( \frac{q}{h_v P_v} \right) / \sigma \cos \theta \right]^{1/3} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2} \]  

(31)
Phan et al. [82] modified Fritz’s relational equation to propose the bubble detachment diameter prediction correlation about several organic fluids on the basis of the concepts of macroscopic and microscopic contact angles. The following assumptions were made: (i) maximum bubble size is determined by force balance; (ii) bubble growth comes from phase change of the microlayer, and the bubble detachment size is related to the conservation in mass; (iii) mass transfer during the rewetting stage of the liquid is assumed to be negligible. Then, the bubble detachment diameter was related to the liquid properties and surface structure, as shown in Equation (32). The model predicted the experimental results for bubble detachment diameters at low superheat and low subcooling within ±30%, but the proposed correlations show large deviations in comparison with the experimental results at high concentrations.

\[
D_d = 54^{1/6} \left( \frac{\rho_l}{\rho_v} \right)^{-1/2} \left( \frac{\rho_l}{\rho_v} - 1 \right)^{1/3} \tan^{-1/6} \theta \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2}
\]  

Equation (32)

Nam et al. [31] conducted experiments by fabricating separated microcavities from CuO nanostructures on silicon substrates with superhydrophobic surfaces. The model of detachment-diameter prediction was proposed by equating uplift force and surface-tension force acting on bubbles based on Fritz’s [83] model, ignoring other factors, e.g., contact pressure and inertial force. It was found that the effect of wettability on bubble dynamics had been demonstrated experimentally on superhydrophilic surfaces and the root term \(\sqrt{24 \sin^2 \theta / [2 + 3 \cos \theta - \cos^3 \theta]}\) was used to express the effect of contact angle.

\[
D_d = \sqrt{24 \sin^2 \theta / [2 + 3 \cos \theta - \cos^3 \theta]} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2}
\]  

Equation (33)

Suszko and El Genk [36] carried out experiments on smooth and rough copper surfaces under saturated-boiling conditions with PF-5060 liquid. The roughness of the surfaces varied about 0.21–1.79 \(\mu\)m, and the roughness of these surfaces varied isometrically at intervals of 0.039 \(\mu\)m. They derived correlations between bubble detachment diameter and bubble growth period for two surfaces. For the smooth surface they used Equation (34), and for the rough surface Equation (35) was used. The relative errors between the prediction results and the experimental results were 15% and 8%, respectively.

\[
D_d = 234 + 81 \sqrt{T_g}
\]  

Equation (34)

\[
D_d = 206 + 48 \sqrt{T_g}
\]  

Equation (35)

Bovard et al. [84] conducted an experimental study of boiling with different liquids with heat fluxes ranging from 1–10 W/cm\(^2\). Different surface materials and roughness were selected for the experiment. The Buckingham’s \(\pi\)-theorem was taken to derive the dimensionless correlation Equation (36) for the detachment diameter, taking into account the forces acting on bubbles according to the dimensionless parameters (e.g., Capillary number, Bond number, Jacob number, and heat-diffusion coefficient), and the experimental data-predicted error bands were less than ±15%. In the relation, the Capillary number \(Ca = \mu_b \nu_b / \sigma \cos \phi\), where \(\mu_b\) denotes the kinetic viscosity, \(\nu_b\) denotes the velocity of bubble motion, \(c_0=17.952177, c_1=0.0172742, c_2=1.285607, c_3=0.661205, c_4=0.025346\).

\[
D_d = c_0 \left[ c_1 + J a^2 C a \frac{\epsilon_1}{\epsilon_3} \right]^{c_4} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2}
\]  

Equation (36)

Kumar et al. [85] considered the effect of wall superheat and liquid properties through the Jacob, proposing that the gas density and latent heat in the Jacob partially incorporate the effect of system pressure, and expressing the effect of solid–liquid–gas interfacial
interaction on bubble detachment through the $\sin \varphi$ term. Therefore, Kumar et al. [85] synthesized the role of the Jacob number index in the Cole [86] model and Du et al. [78] model, and the effect of contact angle in the Fritz [83] model and Phan et al. [82] model. A prediction equation applicable to nucleate boiling individual bubbles was proposed with the following constant $C = (\sin \theta)^{1.25}$, which ignored the effects of neighboring bubble interactions and lateral motion of the liquid. The model was able to obtain errors within 25% for 90% of the data in comparison with other experimental data.

$$D_d = 0.04(\sin \varphi)^{1.5} Ja \left( \frac{3\sigma \sin \varphi}{g(\rho_1 - \rho_v)} \right)^{\frac{1}{2}}$$  \hspace{1cm} (37)

Most of the above correlation equations consider the bubble-force equilibrium, and the refinement of the force analysis will help to provide more accurate detachment-size predictions. Researchers usually enhance heat transfer by reducing the detachment radius in practice. Dong et al. [87] conducted experiments on microstructured and nanostructured surfaces and proposed that reducing the detachment diameter and improving the departure frequency can accelerate the bubble departure, since the nanostructures essentially retard bubble merging and prevent the formation of a vapor film on the surface. Wang et al. [88] fabricated a kind of thin liquid film with the help of nanoporous membranes, utilized the capillary force provided by the pore size to maintain the liquid film height, controlled the bubble detachment size below the millimeter level with the very low liquid-film height, and obtained an ultra-high critical heat flux of 1.85 kW/cm$^2$. Lim et al. [89], on the other hand, obtained better heat transfer with the help of a matrix arrangement of hydrophobic patterns and hydrophilic surfaces, the bubble detachment diameters and location were controlled by the size and spacing of the patterns.

5. Bubble Departure Frequency

The bubble-cycle time and departure frequency are inverses of each other, and both of them are important parameters affecting the boiling-heat flux. The bubble period is the time interval between the first nucleation and the second nucleation in the boiling process, which is usually divided into two parts: the bubble waiting time and the bubble growth time [90]. The bubble cycle is usually divided into two parts: the bubble waiting cycle and the bubble growth cycle. The frequency is usually measured by recording the total number of bubbles emerging from the cavity per unit time. The single-cavity bubble frequency during boiling can be expressed as:

$$f = \frac{1}{t_w + t_g}$$  \hspace{1cm} (38)

5.1. Bubble Growth Time $t_g$

The bubble growth time has a direct effect on the detachment frequency of bubbles and therefore has a significant impact on the boiling heat transfer, which is calculated according to the time interval from the start of bubble growth to its departure.

Zuber [75] presented a correlation for the bubble growth time by considering the detachment diameter, the liquid-thermal-diffusion coefficient and the dimensionless Jacob number. It is considered that the bubble growth time is proportional to the square of the bubble detachment diameter, as shown in Equation (39), where $b$ is an empirical constant, whose size varies from one to $\sqrt{3}$.

$$t_g = \frac{D_d^2}{16b^2(\mu)^{2}a_l}$$  \hspace{1cm} (39)

Hatton and Hall [91] analyzed the Plesset and Zwick [69] expression for the bubble growth rate and considered Zuber’s [75] parameters. A new bubble growth period relation Equation (40) is proposed by eliminating the Jacob number and considering the cavity
radius, which suggests that the bubble growth time is positively proportional to the square of the bubble detachment diameter and the nucleation-cavity radius:

\[ t_g = \frac{\pi a_1}{3} \left\{ \frac{(\rho_v h_{lv})^2 D_d R_c}{8 k_c \sigma T_{sat}} \right\}^2 \]  

(40)

Lee et al. [92] conducted an experimental study of saturated pool boiling for R11 and R113, and the bubble-asymptotic-growth behavior was described with the help of two dimensionless parameters related to time and bubble radius. By utilizing the values of the dimensionless bubble detachment radius and time, the bubble growth period can be obtained as follows:

\[ t_g = 67.5 J_a \alpha \rho_l D_d \sigma \]  

(41)

5.2. Bubble Waiting Time \( t_w \)

The bubble waiting time is the time interval from the detachment of the bubble to the start of next nucleation from the same cavity, representing the process by which the superheated wall activates the nucleation site by natural convection. Han et al. [93] suggested a waiting-time correlation according to the guidelines of bubble nucleation and flow theory, and Equation (42) was proposed for the waiting period by simplifying the temperature distribution equation. The irregularity in the bubble waiting periods was attributed to random differences in the temperature of the liquid that flows in after the bubbles have left.

\[ t_w = \frac{9}{4 \pi a_1} \left\{ \frac{(T_w - T_l) R_c}{T_w - T_{sat}[1 + (2 \sigma / R_c \rho_v h_{lv})]} \right\}^2 \]  

(42)

Hsu et al. [94] developed a detailed analysis of the relationship between cavity size and bubble waiting time for a given wall temperature. The exact radius value of the nucleation cavity in the experiment is difficult to calculate. Thus, they analyzed the liquid-temperature line and the system-equilibrium-temperature profile to obtain the minimum bubble waiting time, and their model is shown in Equation (43).

\[ t_{w,\text{min}} = \frac{144(T_w - T_l)^2 T_{sat}^2 \sigma^2}{\pi a_1 \rho^2_v \rho h_{lv} (T_w - T_{sat})^4} \]  

(43)

Van Stralen et al. [95] proposed the relationship between the bubble waiting time and the growth time about monolithic fluids, and the bubble waiting period can be simply estimated by tripling the bubble growth period, i.e.,:

\[ t_w = 3 t_g \]  

(44)

5.3. Bubble Departure Frequency

The equations mentioned above for bubble growth time and waiting time show that the detachment frequency of bubbles largely depends on the wall superheat, the fluid thermophysical properties, the angle of contact, the size of the cavity, and the interaction between neighboring bubbles. Jakob and Fritz [96] found an inverse relationship between the detachment frequency and the detachment diameter in boiling experiments with water, and LN. Jakob [97] assumed that the bubble growth period and the bubble waiting period satisfy a certain relationship and corrected it by adding factors such as the thermophysical properties and surface tension, thus the correlation Equation (45) was obtained.

\[ f D_d = \left[ \frac{\sigma g (\rho_l - \rho_v)}{\rho_l^2} \right]^{1/4} \]  

(45)

Sakashita et al. [98] suggested a bubble-departure-frequency prediction model of high-pressure water on a rectangular plate at high heat flux. Their experiments showed that heat
flux and pressure no longer have a significant effect on the detachment frequency at high heat flux, and gave a relation Equation (46) for the detachment frequency at this time. This expression includes the density property, kinematic viscosity property and surface tension, and the effect of the detachment radius on it was ignored.

\[
f = 0.6 \left[ \frac{g(\rho_l - \rho_v)}{\rho_1} \right]^{2/3} \left\{ \frac{g(\rho_l - \rho_v)}{\sigma} \right\}^{0.25} (46)
\]

Migliani et al. [63] conducted experiments on the surface of an inconel alloy during high-pressure pool boiling of the refrigerant R134a, with heat fluxes ranging from 0.25 W/cm² to 1.2 W/cm². A dimensionless correlation between the bubble departure frequency and the modified boiling point was proposed according to their experimental results and the regression relation was derived, as shown in Equation (47), where \( Bo_m = \frac{d}{\nu h_{sv}} \) and \( G = \frac{2}{3} D_d^3 \rho_v N_s f \).

\[
f^* = 28.89 Bo_m + 116.8 (47)
\]

Hamzekhani et al. [99] established the prediction equation about the bubble detachment frequency from experimental boiling data with water and NaCl solutions at low and medium heat fluxes at standard atmospheric pressure. The Buckingham’s \( \pi \)-theory was used to perform a dimensionless analysis for the influential factors, e.g., the detachment diameter, detachment frequency, gravitational acceleration, gas–liquid density difference, and so on. Three \( \pi \)-terms were summarized as shown in Equation (48).

\[
f = 0.015 \left( \frac{\Delta \rho^{0.25} \sigma^{0.75}}{\sigma^{0.25}} \right) \left( \frac{q}{\Delta \rho^{0.25} \sigma^{0.75} D_d} \right)^{0.44} \left( \frac{\Delta \rho^{0.25} \sigma^{0.75} D_d}{\sigma^{0.5}} \right)^{0.88} (48)
\]

Several researchers [100–102] proposed to use the form of \( D^{0.5} f = C \sigma^{0.5} \) to describe the relationship between the detachment frequency and the detachment radius of bubbles. Cole [100] analyzed the force balance between the buoyancy force and the drag force on the bubbles and derived the correlation equation of \( D^{0.5} f = 1.15 \sigma^{0.5} \) according to the experimental data. McFadden and Grassmann [101] supplemented the magnitude analysis for all dynamic variables, adopting 0.56 as an empirical constant in their experiments. Ivey [102] analyzed experimental observations on bubbles with different bubble detachment sizes and concluded that the value of the constant \( C \) should be taken as 0.90 when \( D_d < 3 \) mm. Zhang et al. [80] compared the three above-mentioned correlations with boiling-bubble data for different liquids and surface materials, which included R113 on a glass surface [103] and water on the surface of 316SS stainless steel [104], indium tin oxide (ITO) surfaces [105], copper metal surfaces [106,107] and metallic zirconium (Zr) surfaces [100]. The comparison results are shown in Figure 13a.

![Figure 13. Comparison between different theories and experimental results [80]: (a) comparison of the three models of Cole et al. with experimental results; and (b) comparison of Zhang’s model with experimental results.](image-url)
Zhang et al. [80] analyzed the heat-transfer process in the bottom region of the boiling cycle, and proposed that the heat transfer leading to bubble re-growth consists of two parts, i.e., rewetting of the surrounding superheated liquid and heat conduction between the solid and the gas, which they model as transient heat conduction at a semi-infinite wall with convective boundary conditions, and they obtained the correlation Equation (49), where \( C_w = Nu \times Ra^{-0.25} \), and \( Ra = \frac{g\Delta\rho D^3}{\mu l} \). Figure 13b shows the comparison between the predicted straight line of Equation (49) and the experimental value. The straight line is obtained by different physical properties of the wall and the liquid, and the experimental and predicted straight lines for the same conditions have the same color.

\[
D^{0.5} f = \left( C_w \left( \frac{k_l}{k_s} \right)^2 \left( \frac{\nu l}{\alpha_s} \right)^{-\frac{1}{2}} \alpha_s \right) g^{0.5}
\]  

The above relational equations show that there are different types of relationships between bubble departure frequency and bubble detachment diameter, and the accurate prediction of the detachment diameter makes the prediction of the bubble departure frequency more difficult to obtain. In addition to the bubble detachment diameter, the bubble departure frequency is also affected by various surface properties and liquid properties. A better correlation formula may be obtained by a dimensionless analysis of parameters such as wall roughness, cavity dimensions, pressure, and wall-superheating conditions.

6. Boiling-Heat-Transfer Coefficient \( h \) and Heat Flux \( q \)

Boiling-heat-transfer coefficient and boiling-heat flux are the most important two parameters to measure the heat-transfer effect of the boiling process. The former determines the wall superheat at equilibrium, the latter determines the heat taken away per unit time. The traditional methods of measuring the heat flux of pool boiling mainly include the Joule effect method, and temperature gradient method and thermoelectric effect method, etc. [108] The Joule effect method utilizes the voltage and current applied to the heating element to directly calculate the heat flux, and is suitable for systems where the boiling surface area is larger [109]. The gradient method determines the boiling surface temperature gradient by measuring the temperature difference between solid layers \( \nabla T \) and obtains a linear temperature distribution under steady-state conditions, and the heat flux is calculated by Fourier’s law \( q = -k \nabla T \) [110–112]. The principle of the thermoelectric effect method, on the other hand, is that materials with anisotropic thermal conductivity generate an electric field with a transverse component in the main axis of the material when heat passes through it due to the Seebeck effect, thus enabling the heat flux to be obtained by detecting the electrical signal, which allows for the ultra-fast response and is suitable for transient heat flux measurements [113]. With the continuous development of Machine Learning, the image [114,115] and acoustic signals [116] of boiling are detected in order to develop a boiling heat flux measurement system with the aid of the Convolutional Neural Networks (CNNs) [117] and Multilayer Perceptron Neural Networks (MLPNNs) [118].

Accurate prediction equations for boiling heat transfer coefficient and heat flux can help us to predict the experimental heat transfer intensity in advance, as well as to verify the experimental results. Kim [119] reviewed bubble heat transfer models of pool boiling according to transient conduction, microlayer evaporation, enhanced convection, and three-phase line mechanisms, and a large number of correlation formulae for determining boiling heat transfer coefficients according to experimental results and theoretical analyses were compiled. Kim found that most of the heat transfer models were developed based on a combination of factors such as surface structure, heat flux, liquid properties, pressure, and wall temperature. Mohanty et al. [120] also suggested that the accuracy of the equations was highly dependent on a proper and accurate mathematical description of the bubble behavior in reviewing the prediction equations for boiling heat transfer coefficients and heat flux.
6.1. Boiling Heat Transfer Coefficient $h$

It has been widely recognized that it is possible to predict the boiling heat transfer coefficient through modeling theoretical or empirical relational equations from experimental data. These correlations also reduce time and economic cost compared to measuring experimentally. Therefore, it is necessary to explore the boiling heat transfer coefficients (HTC) for various combinations of surfaces and liquids.

Stephan and Preusser [121] analyzed the bubble detachment diameters and liquid properties during pool boiling with pure liquids. An experiential model was obtained as shown in Equation (50). Based on these correlations, the factors affecting the boiling heat transfer coefficient include liquid thermal conductivity, bubble detachment diameter, heat flux, saturation temperature, and so on.

$$h = 0.1 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^{0.67} \left( \frac{\rho_v}{\rho_l} \right)^{0.156} \left( \frac{h_{liq}D_d^2}{\alpha_l^2} \right)^{0.371} \left( \frac{\alpha_l^2\rho_l}{\sigma D_d} \right)^{0.35} \left( \frac{\mu_{C_{pl}}}{k_l} \right)^{-0.16}$$ (50)

Stephan and Abdelsalam [122] conducted pool boiling experiments on a wide range of liquids. Based on a large amount of experimental data, semi-empirical correlations of boiling heat transfer coefficients were established. The predicted correlations for hydrocarbons, refrigerants, and other liquids are shown in Equations (51)–(53), respectively.

For hydrocarbons

$$h = 0.0546 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^{0.5} \left( \frac{\rho_v}{\rho_l} \right)^{0.581} \left( \frac{\rho_v - \rho_l}{\rho_l} \right)^{-4.33} \left( \frac{h_{liq}D_d^2}{\alpha_l^2} \right)^{0.248}$$ (51)

For refrigerants

$$h = 207 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^{0.745} \left( \frac{\rho_v}{\rho_l} \right)^{0.53}$$ (52)

For other liquids

$$h = 0.23 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^{0.674} \left( \frac{\rho_v}{\rho_l} \right)^{0.297} \left( \frac{h_{liq}D_d^2}{\alpha_l^2} \right)^{0.371} \left( \frac{\alpha_l^2\rho_l}{\sigma D_d} \right)^{0.35} \left( \frac{\rho_l - \rho_v}{\rho_l} \right)^{-1.73}$$ (53)

Jung et al. [123] performed a boiling experiment on a horizontal smooth tube, with an outer diameter of 19 mm, with various refrigerants. They reduced the heat flux from 8 W/cm² isotropic to 1 W/cm². The Equation (54) was proposed based on their experimental data, which described the relationship between the boiling heat transfer coefficient and the detachment diameter, the heat flux, the thermal diffusivity, the liquid saturation temperature and the thermophysical properties, where $c = 0.855 \left( \frac{\rho_v}{\rho_l} \right)^{0.309} \left( \frac{\rho_v}{\rho_l} \right)^{-0.437}$, $p_r$ and $T_r$, respectively, are the reduced pressure and the reduced temperature, i.e., the ratios of the measured values to the critical values.

$$h = 10 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^c p_r^{0.1} (1 - T_r)^{-1.4} \left( \frac{\rho_l - \rho_v}{\rho_l} \right)^{0.33} \left( \frac{\beta_1}{\beta_1} \right)^{-0.25}$$ (54)

Jung et al. [124] investigated the experimental data of a nucleate boiling pool with several flammable coolants, refined the Equation (54) and obtained Equation (55), which was found to agree with the experimental results within an error of ±5.3%.

$$h = 41.4 \left( \frac{k_1}{D_d} \right) \left[ \frac{qD_d}{k_1T_{sat}} \right]^{0.685(p_l^{0.33})} \left[ -\log_{10}(p_r) \right]^{-1.52} \left( \frac{\rho_l - \rho_v}{\rho_l} \right)^{0.33}$$ (55)
Rao and Balakrishnan [125] carried out pool boiling experiments on a 28.9 mm diameter aluminum block for acetone–isopropanol–water and acetone–MEK (Methyl Ethyl Ketone)–water ternary systems, and proposed a new correlation, i.e., Equation (56), to estimate mixture-heat-transfer coefficients, which is within a 16% error compared to the experimental results.

\[
h = 0.74 \frac{k_w}{D_d} \left( \frac{qD_d}{k_{\text{sat}}} \right)^{0.674} \left( \frac{\rho_v}{\rho_l} \right)^{0.297} \left( \frac{h_{\text{sat}}D_d^2}{\sigma} \right)^{0.371} \left( \frac{\sigma_{\text{sat}}}{\sigma} \right)^{0.350} (56)
\]

where \( \gamma = \sqrt{\left( \frac{k_w \rho_w C_{pw}}{k_l \rho_l C_{pl}} \right)} \), indicates the effect of wall physical properties and liquid physical properties on boiling.

Judd and Hwang [126] et al. proposed to divide the heated surface into bubble zones and natural convection zones and performed the calculation of the convective heat transfer coefficient. Fazel and Mahboobbour [127] carried out further calculations on this theory and experimentally verified the saturated pool boiling of aqueous ethylene glycol solutions with different concentrations at atmospheric pressure. They assumed that the boiling surface maintains a uniform temperature and used Newton’s law of cooling to measure the boiling heat transfer to obtain Equation (57), where \( h_{nc} \) is the natural convective heat transfer coefficient proposed by Mikic and Rohsenow [128]. This predicted equation is within a 12% relative error compared to the experimental results.

\[
h = h_{nc} + \frac{\pi}{4} \left( \sqrt{\pi k_l \rho_l f} - h_{nc} \right) N_s D_d^2 (57)
\]

6.2. Heat Flux \( q \)

Models for the calculation of heat flux have also been developed by many researchers. Many researchers attempting to describe the boiling have proposed using mechanistic prediction and these descriptions are highly relevant to the various parameters of the bubble. Due to the perturbations caused by bubble generation and detachment, the boiling system can usually be divided into different zones for the calculation of heat transfer: the natural convection zone without bubble generation, the bubble zone, and the peripheral zone affected by the bubbles.

Based on above assumption, Han and Griffith [93] proposed a total heat flux consisting of the natural convective and transient heat transfer due to thermal boundary reconstruction. Mikic and Rohsenow [128] modified the Han and Griffith [93] model and the effect of heating characteristics was given extra thought. They developed a total heat flux that includes transient conduction and natural convection. Then Judd and Hwang [126] refined the Mikic and Rohsenow [128] thermal mechanism by summarizing the heat flux of evaporation from the micro-fluid layer at the bottom of the bubble and expressing it in terms of various bubble-dynamics parameters and liquid properties. Paul and Abdel-Khalik [129], on the other hand, expressed the total heat flux in three parts, i.e., natural convection, forced convection, and vaporization.

Yu and Cheng [130] modeled heat transfer in pool boiling according to the fractal distribution of nucleation locations on the boiling surface. In the computational model proposed by Mikic and Rohsenow [128], they divided the boiling heat flux density \( q_{\text{total}} \) into phase-change heat flux \( q_p \), micro-fluid layer evaporation heat flux \( q_{\text{me}} \) and heat flux density \( q_{nc} \), and validated the model with the help of experimental data from Wang and Dhir at different contact angles [35]. The experimental data at different contact angles were verified and good agreement was obtained, and the value of the total nucleation boiling heat flux is expressed in Equation (58).

\[
q_{\text{total}} = q_p + q_{\text{me}} + q_{nc} (58)
\]
Chu et al. [131] further analyzed the theoretical research proposed by Han and Griffith [93]. Then the heat transfer was further described, combining the transient conduction heat flow \( q_{tc} \) due to bubble-thermal-boundary reconstruction and the evaporation heat flux \( q_{me} \) of the liquid micro-fluid layer. The weighted average distribution of bubble growth time and bubble waiting time was adopted for the two kinds of heat flux, and their new descriptive model is shown in Equation (59).

\[
q_{\text{total}} = (q_{me} \times t_g + q_{tc} \times t_w) f + q_{nc}
\]  

(59)

Sateesh et al. [132] recommended the refined model for pool boiling heat transfer by considering the effect of sliding bubbles on the vertical heated wall. Four different heat transfer mechanisms were proposed for the calculation of heat flux, i.e., latent heat of microlayer evaporation, transient conduction of thermal boundary reconstruction, natural convection, and vertical heated surface sliding-bubble heat transfer, and the total heat flux was calculated as Equation (60), where the suffix of subscript \( s \) denotes the sliding bubbles and the superscript \( m \) denotes the modified model.

\[
q_{\text{total}} = (q_{me}^m + q_{tc}^m) + (q_{mes}^m + q_{tcs}^m) + q_{nc}
\]  

(60)

Kaniowski et al. [30] conducted pool boiling experiments on microchannels of different heights and arrangements with Novec-649 at atmospheric pressure, visualization images of vapor bubbles were made, and the diameters of bubble departure and the frequency of departure from the surface were measured. Based on the model of transient conduction proposed by Mikic and Rohsenow [128] and the model of natural convection proposed by Han and Griffith [93], the total heat flux can be determined ultimately from the relationship from Equations (61)–(64), where \( \beta \) is the thermal expansion coefficient.

\[
q = q_{\text{con}} + q_{me} + q_{nc}
\]  

(61)

\[
q_{\text{con}} = 2\sqrt{\pi \lambda_l \rho_l c_{pl} f D_b^2 N_s}
\]  

(62)

\[
q_{me} = \rho_v h_{ic} N_s f \frac{\pi D_b^3}{6}
\]  

(63)

\[
q_{nc} = 0.14 \rho_l c_{pl} \left[ \frac{\beta g \Delta T^4 (\lambda_l / \rho_l c_{pl})}{\rho_l c_{pl}} \right]^{0.33} \nu^{-0.33}
\]  

(64)

The proposed prediction models of Yu et al. [130], Sateesh et al. [132] and Chu et al. [131] were compared with experimental data measured by Wang et al. [35], and the result is shown in Figure 14a by Mohanty et al. [120]. The comparison shows that about 80% of the experimental data obtain within 25% of the error with the predictive models proposed by Yu et al. [130] and Sateesh et al. [132], while the Chu and Yu [131] models exhibit a low degree of agreement with the experimental data. The Sateesh et al. [132] model is the best fit for the working conditions at high heat flux, and further validation was carried out by Mohanty et al. [120]. Figure 14b shows the results of the predictive correlation proposed by Sateesh et al. in comparison with three different pool boiling heat flux experiments, i.e., the pool boiling of water on the vertical surface which was conducted by Wang and Dhir [35], the pool boiling of R134a on the horizontal tube which was conducted by Barthu and Hahne [133], and the pool boiling of propane on the horizontal tube surface which was conducted by Luke and Gorenflo [134]. As can be seen from the Figure 14b, the Sateesh et al. [132] model has a prediction error within ±25% from the results of the three sets of experimental data.
In this work, the mechanisms and mathematical correlations of each period of bubbles were reviewed and analyzed. The conclusions were made as follows:

(1) Whether bubbles can nucleate or not depends upon solid surface conditions and the wall superheat. Hydrophilic walls have higher critical heat flux, while hydrophobic surfaces have lower initial nucleation criteria. The density of nucleation sites is affected by a variety of factors (e.g., cavity shape and size, wall wettability, heat flux, wall temperature, and system pressure). There is no correlation formula yet that can predict all surfaces, which is important as more and more complex surfaces emerge.

(2) Bubbles in the growth period and detachment period are mainly affected by the surface tension and the differential pressure between the inside and outside of the bubble. The bubble force analysis is of importance for the prediction of bubble

Figure 14. Comparison of multiple heat flux prediction models with experimental results [120]: (a) comparison of heat flux prediction models from Sateesh et al. [132], Yu and Cheng’s [130], and Chu and Yu’s. [131] with Wang and Dhir’s [35] experimental boiling heat flux; and (b) comparison of the Sateesh [132] boiling heat flux density prediction model with experimental boiling heat flux density results given by Wang and Dhir [35], Barthu and Hahne [133], and Luke and Gorenflo [134].

It is well known that the relationship between the heat flux $q$ and the heat transfer coefficient $h$ is $q = h\Delta T$ from Newton’s law of cooling. In the prediction about the heat transfer coefficient $h$ or the heat flux $q$, if the wall superheat is known, it can be quickly achieved through one quantity to obtain the value of the other, and the appropriate choice should be made according to the demands in relevant experiments.

Mechanistic correlations are insufficient to capture the interactions among parameters, and this can be accomplished with computer assistance [135]. In order to solve the heat transfer problem during boiling, many researchers [136–138] have attempted to solve the complex flow and heat transfer problems during boiling by means of Computational Fluid Dynamics (CFD). For the simulation of boiling processes by CFD, various bubble correlations can be used to improve the accuracy of the simulation. Although CFD simulations have been very successful and widely used in predicting single-phase flows, their effectiveness for two-phase flows has not been fully realized [139], and CFD also has difficulty in accurately describing a variety of complex structural surfaces. For unstudied boiling cases, the difficulty of CFD modeling and the degree of similarity to other conditions should be weighed to decide whether to adopt mathematical formulation models or CFD models.

By reviewing various behaviors of bubbles in the boiling process, it can be found that the complexity and unpredictability of boiling mainly comes from the complex coupling relationship between the mass transfer brought by bubble nucleation as well as growth, and the energy transfer brought by thermal convection as well as phase change. Moreover, the parameters in the boiling process are changing from moment to moment, which affects and is affected by other bubble characteristic qualities.

7. Conclusions

In this work, the mechanisms and mathematical correlations of each period of bubbles were reviewed and analyzed. The conclusions were made as follows:

(1) Whether bubbles can nucleate or not depends upon solid surface conditions and the wall superheat. Hydrophilic walls have higher critical heat flux, while hydrophobic surfaces have lower initial nucleation criteria. The density of nucleation sites is affected by a variety of factors (e.g., cavity shape and size, wall wettability, heat flux, wall temperature, and system pressure). There is no correlation formula yet that can predict all surfaces, which is important as more and more complex surfaces emerge.

(2) Bubbles in the growth period and detachment period are mainly affected by the surface tension and the differential pressure between the inside and outside of the bubble. The bubble force analysis is of importance for the prediction of bubble
detachment diameters. The influence of ambient pressure is proposed to be considered in the force analysis. Accurate mathematical descriptions of the drag force $F_d$ and the inertial force $F_i$ are still missing, and how to capture the center-of-mass trajectory of the bubble remains to be solved as well.

(3) The growth rate of bubbles is mainly determined by the various properties of liquids and walls, and the growth of bubbles mainly comes from the evaporation of the thermal boundary layer. The difficulty in accurately predicting growth rates stems from the need for correct mechanistic analyses at different time periods. There is a lack of research on the flow physics of bubbles in existing studies, which we believe plays an important role in the refinement of bubble-growth models.

(4) Most of the studies on the relational equations for the bubble detachment frequency $f$ need to be calculated by the boiling heat transfer coefficient $h$ and the heat flux $q$, which is contrary to the commonly expected need to calculate the above two parameters with the help of $f$. So, it is worthwhile to exclude the above two parameters from the correlation equations and replace them with other factors such as the wall superheat.

In the current study, the bubble-dynamics correlations are always highly specific to the experimental data observed by themselves, and the agreement always reduces when these expressions are used for other boiling conditions. This is attributed to the fact that these correlations depend on a variety of influential parameters. However, usually only some of the highly correlated factors were selected to make the correlation guess based on their own experimental data, and other factors were ignored. Therefore, it is necessary to conduct a full range of experiments and to take all factors into account in the correlations, which requires a very sophisticated experimental design.

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Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

**Abbreviations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$Ar$</td>
<td>Archimedes number $\left(\frac{g}{\nu^2}\right)\left(\frac{\sigma}{\rho L g}\right)^{3/2}$</td>
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<tr>
<td>$Ca$</td>
<td>Capillary number $\left(\frac{\mu_b v_b}{\sigma \cos \phi}\right)$</td>
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<tr>
<td>$C_L$</td>
<td>Lift-off coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure $(J \text{ kg}^{-1} \text{K}^{-1})$</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter (m)</td>
</tr>
<tr>
<td>$f$</td>
<td>Detachment frequency $(s^{-1})$</td>
</tr>
<tr>
<td>$F_b$</td>
<td>Buoyancy force (N)</td>
</tr>
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<td>$F_{du}$</td>
<td>Unsteady drag force (N)</td>
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<td>$F_m$</td>
<td>Excess vapor pressure (N)</td>
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<tr>
<td>$F_p$</td>
<td>Liquid static pressure (N)</td>
</tr>
<tr>
<td>$F_s$</td>
<td>Surface tension (N)</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity $(m \text{ s}^{-2})$</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient $(W \text{ m}^{-2} \text{K}^{-1})$</td>
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<tr>
<td>$H$</td>
<td>Height of liquid (m)</td>
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**Greek symbols**

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<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity $(m^2 s^{-1})$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density $(kg \text{ m}^{-3})$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Difference</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity $(kg \text{ m}^{-1} \text{s}^{-1})$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity $(m^2 \text{ s}^{-1})$</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>Thermal boundary layer thickness $(\mu m)$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Surface tension $(N \text{ m}^{-1})$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Contact angle $(^\circ)$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Shape angle $(^\circ)$</td>
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**Superscripts**

<table>
<thead>
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<tr>
<td>$+$</td>
<td>Dimensionless</td>
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**Subscripts**

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\( h_v \) heat of vaporization (J kg\(^{-1}\))

\( Ja \) Jakob number (\( \rho c_p \Delta T_{w}/\rho_0 h_v \))

\( k \) thermal conductivity (W m\(^{-1}\) K\(^{-1}\))

\( Nu \) Nusselt number

\( N_s \) nucleation sites density (m\(^{-2}\))

\( p \) pressure (bar)

\( q \) heat flux (W m\(^{-2}\))

\( R_s \) surface roughness (µm)

\( R_b \) bubble radius (m)

\( R_c \) cavity radius (m)

\( S \) dimensionless surface factor

\( T \) temperature (K)

\( t \) time (s)

\( t_w \) waiting time (s)

\( t_g \) growth time (s)

\( v \) specific volume (m\(^3\) kg\(^{-1}\))

\( \varphi \) cavity

\( \delta \) bubble departure

\( l \) liquid

\( \mu \) microlayer evaporation with sliding

\( \rho_0 \) density

\( \rho \) reduced variable

\( \Delta \) saturation state

\( \tau \) transient conduction with sliding

\( \sigma \) heated surface

\( \infty \) bulk

\( \Delta \) liquid

\( \Delta \) cavity

\( \Delta \) transient

\( \Delta \) bulk

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