Collaborative Allocation Method of En-Route Network Resources Based on Stackelberg Game Model

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Abstract: To further enhance fairness in the allocation of en-route space–time resources in the collaborative trajectory selection program, a study on the plan preferences between air traffic control (ATC) and airlines in the selection process of the final plan is conducted based on the initial resource allocation plans, considering the roles of airlines in resources allocation decisions. By using Stackelberg game theory, a game model is established for the role played by ATC and airlines in the process of selecting plans. Then, combining the overall consideration of ATC for all affected flights, the preferences of airlines for initial allocation plans are obtained, and the option range of selectable plans is narrowed down to determine the optimal allocation plan. The results of the example analysis show that the proposed model and method can effectively select the optimal allocation plan from the six initial allocation plans, select the trajectories and entry slots in the congestion areas for airlines that better meet the operation demand, and provide the decision basis with more preferences for ATC to select the final allocation plan. When ATC prefers the lowest overall delay cost, the delay cost of the selected optimal allocation plan is 267.7 min, which is 23.84% lower than the traditional RBS algorithm; when considering the preferences of the main base airline in East China, the delay cost of the selected optimal allocation plan is 287.7 min, which is 18.15% lower than the traditional RBS algorithm.

Keywords: air traffic management; en-route network; Stackelberg game model; resource allocation; selection preference; fairness

1. Introduction

The traditional ground waiting strategies, diversion strategies, air traffic management procedures, and the latest Collaborative Trajectory Options Program (CTOP) are the air traffic flow management strategies which are mainly used, the core of which is the rational allocation of limited slot resources and trajectory resources to achieve the efficient utilization of airspace [1,2].

Most of the early studies focused on the perspective of ATC and study the process of resource allocation aiming at maximizing the utilization of airspace. Subsequently, the research gradually turned to the study of fairness issues. Various theories, models, and algorithms have been applied to the research of fair resource allocation, including the Collaborative Decision Making (CDM) mechanism [3], the 0–1 integer programming model for collaborative multi-route resource allocation integrating diversion strategies and waiting strategies [4], the flight slot assignment model for airlines under the collaborative decision-making mechanism [5], and the slot exchange model based on cooperative game...
theory with the goal of minimizing passenger delays for all parties involved in the exchange [6]. In the above results, the focus considers delay fairness from the perspective of ATC based on the number of passengers to allocate the flight slot resources or/and trajectory resources. The fairness of airlines is mainly reflected through the allocation of passenger delays, and the different preferences of different airlines are not taken into account, so the fairness achieved by this still remains under the perspective of a single perspective of ATC.

With the widespread implementation of air traffic flow management strategies, especially CTOP strategies, the airlines, as important participants and major beneficiaries of resource allocation, have become increasingly involved in flow management decisions. Therefore, some studies have included airlines in the resource allocation process, changing the original resource allocation problem under the single perspective of ATC to a resource allocation problem under the domination of ATC that considers the preferences of airlines. Some studies have shown that the idea of collaborative route resource allocation considering the competitive behavior of airlines is the main research direction of future traffic management technology [7,8]. For the implementation process of CTOP, considering the gaming behavior between the two airlines participating in the collaborative en-route technology can effectively reduce flight delays [9]. In order to implement and analyze the CTOP program more effectively, a variety of models and methods are proposed to explore the trajectory preference of airlines, study the route resource allocation issues, and improve airspace operation efficiency and resource allocation fairness, as well as reducing the total delay costs. These include an improved Flow Constrained Area (FCA), which is a Potentially Constrained Area (PCA) model combined with heuristic algorithms [10], the stochastic optimization models to set flow in the new traffic management initiatives of CTOP based on linear optimization-based alternative flight scheduling methods [11], the centralized en-route resource allocation mechanism considering fairness [12], the new calculation method of relative en-route flying costs for the difficulty to calculate the relative en-route flying costs for airlines [13], using fuzzy comprehensive evaluation method to evaluate flight delay time, delay costs and the number of turning points [14], using Gini coefficient to define the fairness performance indicators of en-route collaborative allocation and constructing a dual-objective nonlinear integer programming model that considers both efficiency and fairness [15], the multi-objective nonlinear 0–1 integer programming model with the goal of minimizing the total flight delay costs and the deviation coefficient of delay fairness loss for airlines [16], etc. Some researchers have studied the game process among airlines, ATC, or other participants in the process of resource allocation. The relevant results mainly focus on solving the problem of combining the CDM framework that the CTOP relies on and the game theory [17–20]. Among them, the CDM model based on satisfaction game theory [21], the CDM model based on the deferred acceptance mechanism by using the bilateral market matching method of game theory [22], a Game Theory Model Applied to Resource Management of Large scale UAV Communication Networks [23], and the Bayesian game model for the game process of competing for advantageous resources of airlines around the three key issues in the CTOP implementation process [24] have been proposed.

In summary, most of existing research tends to use multi-objective decision-making techniques to select the global allocation plan. The final alternative plans are numerous, and the uncertainty is larger, and the research on resolving the conflicts of interest among airlines during the implementation of the CTOP is still not fully developed, which is a loss of fairness to some extent. But the maturity of CTOP strategies is closely related to fairness, as a mature CTOP system should be capable of balancing fairness among different airlines, flights, and flight plans. It ensures the reasonable utilization of resources while maintaining efficient air traffic. Therefore, on the issue of balancing the global goals of CDM and the individual goals of airlines, game theory can be used to fully consider the impact of airline decisions and enhance airline participation. At the same time, calculating the preferred en-route resource allocation plans for each airline from the perspective of
game theory can provide stronger quantitative indicators for ATC to choose the final plan, finally selecting an allocation plan that is acceptable to multiple parties. This paper introduces game theory into collaborative en-route network resource management. Combining with the CTOP framework and fully considering the role of airlines in the resource allocation process, it analyzes the roles played by ATC and airlines in the plan selection process and the game process and establishes the Stackelberg game model. Finally, the option range of selectable plans is narrowed down according to the obtained mixed strategy results, and then, an optimal allocation plan is obtained, which enhances the fairness of multi-party participation in the CTOP decision-making process and the allocation plans.

2. Analysis of the Game Process in Collaborative Allocation of En-Route Network Resources

2.1. Game Construction and Principle Analysis

Collaborative allocation of en-route network resources involves both ATC, which makes overall decisions, and participating airlines, which have an impact on the decision-making process. In this game process, ATC, as the ultimate decision-maker, has the right to take priority action and determine the final allocation plan and acts as the leader in the game process, while airlines, as participants, need to choose the plan according to the initial decisions made by ATC, playing the role of followers. This constitutes a Stackelberg game. The Stackelberg game is a staged decision problem involving a leader and one or more followers, in which the leader announces their strategy, and the followers make their own decisions or improvements based on the leader’s strategy. The main idea of the Stackelberg game is that both the leader and the followers make decisions based on the possible strategies the other party may adopt to maximize their own interests or minimize their losses, aiming to achieve a stable state.

The game process of collaborative allocation of en-route network resources mainly involves the following principles:

(1) Principle of individual rationality: In this game process, both ATC and the airlines, as individual entities participating in the game, are absolutely rational. ATC pursues the overall system optimum, including efficiency and fairness. Efficiency refers to the overall optimality of the system, encompassing both the entire air traffic management system and the objectives pursued by individual airlines during the game process. Fairness refers to the equitable treatment of all participating parties during the game process. It can also be understood as ensuring that there is no undue favoritism towards any specific airline throughout the gaming process. The initial allocation plan has already taken this condition into account, so this constraint does not need to be considered in the subsequent game. Each airline aims to maximize its own interests, specifically minimizing the total delay time of all flights belonging to the airline. (For airlines, there are numerous factors influencing total costs. However, for the purpose of simplifying calculations and ensuring a more intuitive presentation of results, we assume that maximizing the airline’s interests is equivalent to minimizing the total delay time). There are no alliances between airlines, and the final allocation plan obtained through the game process should result in less delay time than the random selection, so that airlines will actively participate in the collaborative decision-making process.

(2) Principle of relatively optimal strategy: In the case that no matter which plan other airlines choose, the plan chosen by the airline can ensure that its delay time is no less than other plans, then this plan is called the optimal strategy of the airline. In the game process, each airline pursues its own optimal strategy. When all airlines implement the optimal strategy, the system also reaches an optimal state. However, in the actual game process, due to the conflicts of interest, it is impossible to achieve the optimal state of each airline. Therefore, the final equilibrium is determined by ATC to achieve a relatively optimal state for the entire system.
To ensure the smooth progress of the game process and the effectiveness of the model, the following assumptions need to be made before establishing the model:

1. There is no alliance among airlines, and the airlines have a purely competitive relationship. Each airline aims to minimize its own delay time and actively secure advantageous trajectories and time slots. This game is a non-cooperative game.
2. When airlines engage in the game, all airlines make strategy selection among the feasible plans provided by ATC. Other trajectory time slot allocation plans are not considered, so as to ensure that each airline can still meet the overall relatively optimal state of the system after strategy selection, which means satisfying the requirements of ATC.
3. The initial allocation plan derived by ATC is made public to all airlines, and all airlines simultaneously make their choices after ATC issues the guidance information. All airlines are aware of the available trajectory and slot resources and the payment function (the resulting delay situation) of other airlines when choosing plans. This game is a complete information static game based on this assumption.

2.2. Game Process and Elements Analysis

Based on the basic framework of the Stackelberg game model, the game process of collaborative allocation of en-route network resources mainly involves three stages, as shown in Figure 1:

Stage 1: ATC acts as the leader and calculates the initial allocation plan based on the available trajectory and slot information, as well as the set of trajectory options (TOS) submitted by airlines, with the goal of optimizing the overall system state (maximizing efficiency and fairness).

Stage 2: Each airline engages in the game under the initial allocation plan provided by ATC and pursues to minimize its total delay time while considering the impact of the plan chosen by other airlines on itself, so as to measure the preference for different plans and finally choose the optimal plan for itself.

Stage 3: The airlines feedback the selection situation of plans obtained from the game to ATC, then ATC finally selects the allocation plan that maximizes the benefits of each airline and maximizes the overall operational efficiency from the obtained initial allocation plans, which is the optimal resource allocation plan.

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**Figure 1.** Flow chart of the game process.

In the second stage, the game between airlines is a non-cooperative game. To simplify the game process and reduce the calculation complexity of the game model, the game is transformed into a two-player game process, then each airline is treated as an individual player to play a multi-round game. This game problem mainly needs to clarify the meaning of the following three elements:
(1) Set of participants: This is the set of all airlines \( A = \{a_1, a_2, \cdots, a_N\} \), in each round of the game, and the airlines are divided into two opposing entities. One entity represents an individual airline \( A_1 \) and the other entity represents the set of airlines \( A_2 \).

(2) Set of strategies for each participant: this is the set of allocation plans \( S = \{s_1, s_2, s_3, \cdots, s_m\} \) that each airline can select, which includes the reasonable and viable plans obtained from the initial allocation plan.

(3) Payment function (utility): When one airline chooses a particular plan \( s_i \) and the other set of airlines participating in the game chooses the plan \( s_j \), the delay time of the airline can be obtained. This delay time is the payment function \( F(s_i, s_j) \) of the corresponding airline.

3. Initial Resource Allocation Multi-Objective Optimization Model

The first stage of the collaborative resource allocation game process for the en-route network is to establish a multi-objective allocation model with the objective of global optimality and calculate the initial resource allocation scheme.

The problem of allocating route resources to affected flights is a large-scale, multi-objective, and multi-constrained resource allocation problem. Suppose the set of affected flights of airlines is \( f \), the sets of route options and time slots are \( C \) and \( T \), respectively, and there are \( K \) FCAs in each route. Let \( f, c, k, \) and \( t \) denote the flight \( f \), the FCA \( k \) in the route \( c \), and the time slot \((f \in F; c \in C; k \in K; t \in T)\), then the decision variables for the resource allocation problem are \( x_{ij}^{fc} \) [16]:

\[
x_{ij}^{fc} = \begin{cases} 1, & \text{flight } f \text{ is assigned to the route option } c \text{ and} \\ 0, & \text{departs from the FCA } k \text{ on this route before time } t \\
0, & \text{else} \\ \end{cases}
\]

Introduce the following auxiliary variables:

\[
\lambda_{ij}^{fc} = \begin{cases} 1, & \text{flight } f \text{ submitted route option } c \\ 0, & \text{flight } f \text{ has not submitted route option } c \\ \end{cases}
\]

The definitions of other parameters involved in the model are as follows:

\( \delta_{k+1} \): the time required for the affected flight to move from the FCA \( k \) to the FCA \( k + 1 \) adjacent to it on the same route.

\( \delta_{k}^{fc} \): the time required for the affected flight \( f \) to pass through the FCA \( k \) on route \( c \).

\( t_{i}^{fc} \): the time assigned for the affected flight \( f \) to enter the FCA \( k \) on route \( c \).

\( t_{i}^{fc} \): the time submitted by the airline for the affected flight \( f \) to enter the FCA \( k \) on route \( c \), i.e., the ETA (in actual implementation, airlines only need to submit the time for the affected flight \( f \) to enter the first FCA on route \( c \), and the time for entering subsequent FCAs can be calculated based on the distance and flight speed).

\( \omega_{i}^{fc} \): a binary variable \((0–1)\) indicating whether the affected flight \( f \) is assigned to route \( c \).

\( T_{i}^{k} \): The set of available time windows for the FCA \( k \) in the route \( c \). Among them, the earliest available time window is \( T_{i}^{k} \), while the latest available time window is \( T_{i}^{k} \).

\( M_{i}^{fc} \): the capacity of the FCA \( k \) on the route \( c \) within a specified time frame.
The optimization objectives of this study need to simultaneously consider efficiency and fairness. It is not only aimed at minimizing the total delay costs for all affected flights to maximize the utilization of restricted airspace but also involves ensuring that the allocated delay times for individual airlines are as close as possible. Objective functions and constraint conditions are shown below.

\[
\min E = \sum_{j \in \mathcal{F} \setminus \mathcal{K}} \sum_{c \in \mathcal{C}} x_{j,c} - t_{j,c} \omega_{j,c}^{f} + \beta \sum_{j \in \mathcal{F} \setminus \mathcal{K}} \sum_{c \in \mathcal{C}} x_{j,c} - t_{j,c} \omega_{j,c}^{f}
\]

\[
\min T = \sum_{j \in \mathcal{F} \setminus \mathcal{K}} d_{j} \cdot \text{lg}(n \cdot \frac{d_{j}}{q_{j}})
\]

\[
\sum_{i \in T_{k}} \sum_{k} x_{j,c} = \lambda_{c}^{f}, \forall f \in \mathcal{F}, c \in \mathcal{C}
\]

\[
t_{j,c} = t - \delta_{j,c}, x_{j,c} - x_{j,c-1} = 1, \forall f \in \mathcal{F}, c \in \mathcal{C}
\]

\[
t_{j,c} \geq t_{j,c} \cdot \lambda_{c}^{f}, \forall f \in \mathcal{F}, c \in \mathcal{C}, t \in T_{k}
\]

\[
\omega_{j,c}^{f} = \begin{cases} 1, & \sum_{i \in T_{k}} \sum_{k} x_{j,c} \geq 1 \\ 0, & \sum_{i \in T_{k}} \sum_{k} x_{j,c} = 0 \end{cases}
\]

\[
\sum_{i \in T_{k}} \sum_{k} x_{j,c} = 1, \forall f \in \mathcal{F}
\]

\[
\sum_{f \in \mathcal{F}} x_{j,c} \leq 1, \forall c \in \mathcal{C}, t \in T_{k}
\]

\[
x_{j,c} - x_{j,c-1} \geq 0, \forall f \in \mathcal{F}, c \in \mathcal{C}, t \in T_{k}
\]

\[
x_{j,c} - x_{j,c-1} \leq 0, \forall f \in \mathcal{F}, c \in \mathcal{C}, t \in T_{k}
\]

\[
\sum_{i \in T_{k}} \sum_{j \in \mathcal{F}} (x_{j,c} - x_{j,c+1}) \leq M_{c}, \forall c \in \mathcal{C}
\]

Equations (3) and (4) serve as objective functions, representing the minimization of total delay costs (including ground and airborne delay costs) and the minimization of the disparity in delay allocation among different airlines (minimizing fairness loss), respectively. Equations (5)–(7) serve as allocation feasibility constraints, indicating that for any given flight \( f \), a specific route can only be assigned to the corresponding flight if and only if the airline has submitted the affected flight and has the option to choose that particular route. Furthermore, the assignment of a route to flight \( f \) must occur no earlier than the
time it was submitted. Equations (8)–(10) represent uniqueness constraints for allocation. Specifically, they indicate whether flight \( f \) is assigned to the route \( c \). Each flight can only be assigned to one slot on one route, and it must be assigned to a slot, ensuring that each slot can be assigned to at most one flight. Equation (11) represents the time continuity constraint, ensuring the temporal continuity of flights through various FCAs. Equation (12) serves as the resource continuity constraint, guaranteeing the continuity of resources between FCAs. Equation (13) represents the capacity allocation constraint, ensuring that the number of flights assigned to each route does not exceed the capacity of the corresponding airway.

4. Resource Collaborative Allocation Model Based on Stackelberg Game

According to the utility theory, the utility of airlines participating in the game is the benefits obtained by airlines when choosing different plans. In this case, the negative utility is represented by delay costs, that is the utility function of the airline \( A_n \) for choosing plan \( s_i \) is defined as:

\[
U_{A_n} = \delta f(s_i, s_j)
\]

where \( \delta \) is the utility conversion parameter, which is mainly used to adjust the utility values of each plan, so that the utility value of the plan with the minimum delay cost is set to 1, and the utility value of the plan with the maximum delay cost is equal to 0, ensuring that the utility values of the chosen plans are within the range of \([0,1]\). Since the number of flights included in each involved airline is generally larger than 1, the utility value of the airline is the sum of the utility values of its flights:

\[
U_{A_n} = \sum t_{ij}(s_i, s_j)
\]

where \( t_{ij}(s_i, s_j) \) is the delay cost of flight \( f \) belonging to airline \( A_n \) when choosing plan \( s_i \), which is positively correlated to the time difference between the allocation time of the chosen plan and the time specified in the original submitted TOS. This delay cost includes both the ground delay cost and the air delay cost. The relative coefficient of air delay cost to the ground delay cost is assigned as \( \beta \). Therefore, the delay cost of each flight under plan \( s_i \) is expressed as follows:

\[
t_{ij}(s_i, s_j) = (t_{ij}^{g} - t_{ij}^{k}) + \beta(t_{ij}^{a} - t_{ij}^{k})
\]

The airline can compare each initial allocation plan based on the utility function. The probability of the airline selecting the plan \( s_i \) is as follows:

\[
p_{A_n}^{s_i} = \text{prob}[U_{A_n}^{s_i} \geq U_{A_n}^{s_j}], \forall A_n \in A, s_i, s_j \in S, i \neq j
\]

According to the LOGIT model, it can be further calculated as follows:

\[
p_{A_n}^{s_i} = \frac{\exp[-\varepsilon f(s_i, s_j)]}{\sum_{s_i} \exp[-\varepsilon f(s_i, s_j)]}
\]

where \( \varepsilon \) is the parameter that represents the degree of preference of the airline for each plan, determining the importance of the utility value of the corresponding plan to the airline. This parameter can be derived from the corresponding delay cost.

The following constraints are simultaneously satisfied:
(1) The sum of the probabilities of each airline selecting all plans is 1:
\[
\sum_{s} p_{s}^{i} = 1, \quad \forall A_{n} \in A, \ s_{i} \in S
\]  
(19)

(2) When each airline chooses different plans, there is no conflict between the occupied slots, that is, the slots cannot be repeatedly occupied:
\[
\sum_{A_{n}} \sum_{s_{i}} p_{s}^{i} \times \sum_{j < A_{n}} D_{j}^{y,j} \leq 1
\]  
(20)

where \( D_{j}^{y,j} \) represents the slot situation occupied by the flight \( j \) when the airline \( A_{n} \) chooses the plan \( s_{j} \).

5. Experiment Analysis

The hardware and software configuration of the computer used for the experiment analysis is as follows: the operating system is Windows 11 (64-bit); the processor is Intel (R) Core (TM) i7-1165G7; the CPU frequency is 2.80 GHz; the onboard memory is 16.0 GB; and the programming language is Python 3.11.

The actual operational and simulated data of an en-route sector in Shanghai Flight Information Region (FIR) in the eastern region of China in April 2019 are used for analysis. Congestion is more likely to occur on the A599 and H24 high-altitude en-routes in the Shanghai sector. ATC initiated the CTOP in this area, and the FCA is set according to the actual operational data and simulated data as shown in Figure 2. The relevant available slot information and flight information are shown in Tables 1 and 2.

![Figure 2. FCA layout of sector.](image)

<table>
<thead>
<tr>
<th>Table 1. Available time slot information.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCA001</td>
</tr>
<tr>
<td>17:08:00</td>
</tr>
<tr>
<td>17:14:00</td>
</tr>
<tr>
<td>17:21:00</td>
</tr>
<tr>
<td>17:26:00</td>
</tr>
<tr>
<td>17:32:00</td>
</tr>
<tr>
<td>FCA001</td>
</tr>
<tr>
<td>17:38:00</td>
</tr>
<tr>
<td>17:45:00</td>
</tr>
</tbody>
</table>
Table 2. Flight information.

<table>
<thead>
<tr>
<th>Flight Number</th>
<th>Airlines</th>
<th>Estimated Time to Enter FCA001</th>
<th>Estimated Time to Enter FCA002</th>
<th>Estimated Time to Enter FCA003</th>
<th>Estimated Time to Enter FCA004</th>
<th>Earliest Entry Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>fi</td>
<td>A</td>
<td>17:15:18</td>
<td>17:00:18</td>
<td>17:20:18</td>
<td>17:05:18</td>
<td>17:00:18</td>
</tr>
<tr>
<td>f2</td>
<td>B</td>
<td>17:03:46</td>
<td>17:28:46</td>
<td>17:08:46</td>
<td>17:23:46</td>
<td>17:03:46</td>
</tr>
<tr>
<td>f5</td>
<td>B</td>
<td>17:10:16</td>
<td>17:25:16</td>
<td>17:15:16</td>
<td>17:30:16</td>
<td>17:10:16</td>
</tr>
<tr>
<td>f6</td>
<td>C</td>
<td>17:27:03</td>
<td>17:12:03</td>
<td>17:32:03</td>
<td>17:17:03</td>
<td>17:12:03</td>
</tr>
<tr>
<td>f9</td>
<td>A</td>
<td>17:21:05</td>
<td>17:36:05</td>
<td>17:26:05</td>
<td>17:41:05</td>
<td>17:21:05</td>
</tr>
<tr>
<td>f11</td>
<td>C</td>
<td>17:45:34</td>
<td>17:30:34</td>
<td>17:40:34</td>
<td>17:25:34</td>
<td>17:25:34</td>
</tr>
<tr>
<td>f14</td>
<td>A</td>
<td>17:32:01</td>
<td>17:47:01</td>
<td>17:37:01</td>
<td>17:52:01</td>
<td>17:32:01</td>
</tr>
<tr>
<td>f16</td>
<td>C</td>
<td>17:35:50</td>
<td>17:50:50</td>
<td>17:40:50</td>
<td>17:55:50</td>
<td>17:35:50</td>
</tr>
<tr>
<td>f18</td>
<td>C</td>
<td>18:00:33</td>
<td>17:45:33</td>
<td>18:15:33</td>
<td>17:50:33</td>
<td>17:45:33</td>
</tr>
</tbody>
</table>

Note: Airlines A, B, and C, respectively, correspond to Juneyao Airlines, Shenzhen Airlines, and Eastern Airlines. Based on the above situational information, the initial allocation of en-route resources is conducted with the objectives of efficiency and fairness. A multi-objective optimization algorithm is used to allocate 20 flights from different airlines to the available time slots to obtain feasible allocation plans. Then, the Pareto front is calculated based on the obtained allocation plans, and the objective function values of these plans are compared with the results of the allocation plans obtained by the commonly used Ration-by-Schedule (RBS) algorithm [25] to obtain six superior allocation plans that completely dominate the RBS method in the Pareto optimal solution set. The specific allocation details of these six plans are shown in Figure 3, while the efficiency and fairness indicator values of each plan, that is the total delay cost and fairness loss degree, are shown in Figure 4, which are calculated for each scenario based on the model constructed in this paper. In Figure 3, the allocated trajectory of each flight and the time order of entering the trajectory is used to represent the specific situation of each allocation, the specific position of each flight point on the timeline do not directly correspond to the actual time slot situation, and the relative interval size of each flight within the same trajectory option represents the length of the interval time between the allocated time slots.
Figure 3. Allocation situations of each flight for the initial allocation plan.

Figure 4. Efficiency and fairness statistics of the initial allocation plan.
According to the specific allocation information of the obtained plans, the delay costs of each airline under each plan can be calculated. The calculation results are shown in Figure 5, where Juneyao Airlines, Shenzhen Airlines, and Eastern Airlines, respectively, correspond to the airlines A, B, and C.

![Figure 5](image.png)

**Figure 5.** Delay costs of each airline.

After obtaining the above initial allocation plan information, the first stage of the Stackelberg game is completed. The preliminary allocation plan is used as the guiding information by the ATC to pass to the airlines, and then, the game between the airlines is the second stage of the Stackelberg game.

Based on the flight information in Table 2 and the preliminary allocation plan information in Table 3, it can be seen that there are three airlines participating in the game, and each airline has six different options. According to the game model between airlines established earlier, the probability of Juneyao Airlines choosing each option is calculated first. The calculation results are shown in Figure 6. From the game results of Juneyao Airlines shown in Figure 6, it can be seen that when Juneyao Airlines chooses plan 4, the probability of other airlines (that is the set of airlines participating in the game excluding Juneyao Airlines) choosing a plan other than plan 4 is relatively small. Combining specific allocation information, it can be known that this is because when all flights of Juneyao Airlines choose plan 4, the other airlines will be more likely to conflict with the flight time of Juneyao Airlines when making their choices among other plans, so the probability of choosing other plans in this case is very small, which is in line with the actual situation. At the same time, this also has a reverse influence on the choosing inclination towards plan 4 of Juneyao Airlines, resulting in a lower probability of choosing option 3 for Juneyao Airlines. Similarly, it can be seen that when Juneyao Airlines chooses option 1, it has the least influence on the other airlines’ choices of plans. Other airlines have a higher probability of choosing each plan, which in turn makes Juneyao Airlines more inclined to choose option 1. Furthermore, when analyzing the game results combined with the delay costs for Juneyao Airlines in each option shown in Table 3, it is found that the probabilities of airlines choosing each plan do not necessarily have a linear relationship with their own delay situations. This is because when the airline chooses the plan with the least delay for itself, it is equivalent to taking the first advantageous time, and then opportunities for other airlines to choose will be reduced. As a result, in such a situation, the opportunities for other airlines to make their choices are reduced, which in turn affects the airline’s own choice of the plan.
Figure 6. Probability of Juneyao Airlines choosing each plan.

Similar to the game process of Juneyao Airlines, the game process between Shenzhen Airlines and Eastern Airlines is conducted in turn. The probabilities of each airline choosing each plan are calculated, and the results are shown in Figures 7 and 8. From the probability information shown in the figure, it can be seen that Shenzhen Airlines prefers choosing option 4, while Eastern Airlines prefers choosing option 3.

Figure 7. Probability of Shenzhen Airlines choosing each plan.
After the above game process, the selection tendency of each airline is obtained, and the specific choosing situation is shown in Table 3. The maximum probability is the average of the probabilities of the other airlines choosing each option while one airline chooses its optimal option. From the results, it can be seen that the optimal plan for each airline is different. Then, the next step of the game will be played, that is, all airlines will feedback their selection tendency of the plans to the ATC side. Based on the choice preference of each airline, ATC will make the decision on the final allocation plan.

Table 3. Optimal options for each airline.

<table>
<thead>
<tr>
<th>Airlines</th>
<th>Maximum Probability</th>
<th>Selected Optimal Plan</th>
<th>Delay Cost under Optimal Plan</th>
<th>Selected Suboptimal Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juneyao Airlines</td>
<td>0.346</td>
<td>Plan 1</td>
<td>80.3</td>
<td>Plan 2</td>
</tr>
<tr>
<td>Shenzhen Airlines</td>
<td>0.363</td>
<td>Plan 4</td>
<td>104.3</td>
<td>Plan 1</td>
</tr>
<tr>
<td>Eastern Airlines</td>
<td>0.358</td>
<td>Plan 3</td>
<td>90.2</td>
<td>Plan 1</td>
</tr>
</tbody>
</table>

The third stage of the game is an information exchange between ATC and the airlines. As the overall controlling party, ATC considers the overall situation at this time, but it also takes into account the choosing preferences of the airlines, reflecting the collaborative decision-making process. From the optimal option of each airline shown in Table 3, it can be seen that the preference of each airline is different. At this time, ATC needs to make a decision while adequately considering the demands of the airlines and combing the overall situation. From the perspective of ATC, the comparison situation of the three plans is shown in Figure 9. It can be seen from Figure 9 that plan 1 has the lowest total delay cost and is the optimal strategy overall, which can fully satisfy the demand of Juneyao Airlines. At the same time, combing the probability results of selecting each plan in the game, it can be seen that plan 1 is the suboptimal option for Shenzhen Airlines and Eastern Airlines, which can also meet their preferences. Therefore, the final optimal solution is plan 1.
Through the game analysis on the six initial allocation plans, the preferences of each airline are obtained, and ATC makes the final decision on this basis. The total delay costs for plan 1, plan 3, and plan 4 are 267.7 min, 287.7 min, and 321.7 min, respectively. When the ATC prefers minimizing the overall delay cost, the optimal allocation plan is plan 1. However, if ATC intends to prioritize the flights of the airline because it is the main base airline, it will choose the optimal plan which this airline prefers. For example, Eastern Airlines has a comparatively large operating scale in the East China region, and therefore, ATC may select the plan which is preferred by Eastern Airlines as the final allocation plan. The delay costs of Eastern Airlines choosing plan 1, plan 3, and plan 4 are 94.8 min, 90.2 min, and 104.2 min, respectively. And the delay cost of choosing plan 3 is the lowest, that is, Eastern Airlines prefers plan 3. Therefore plan 3 is taken as the optimal allocation plan.

In order to test the effectiveness of the final allocation plan, the total delay cost of the allocation plan obtained from the game is compared with the results of the allocation plan obtained from the commonly used RBS algorithm. The total delay cost of the plan allocated according to the results of the Ration-by-Schedule (RBS) algorithm is 351.5 min. Compared to the results of the RBS algorithm, the total delay cost of plan 1, plan 3, and plan 4 decreased by 23.84%, 18.15%, and 8.47%, respectively, which are all better than the traditional RBS algorithm. So, this method can provide the ATC department with an alternative and better allocation plan.

6. Conclusions

Based on the principle of the Stackelberg game, fully considering the decision preferences of airlines in the “collaborative” allocation process and aiming at further improving the fairness of resource allocation, a Stackelberg game model between ATC and airlines is established, and the study of the selection preferences between ATC and airlines in the process of selecting the final allocation plan is conducted. Based on the data of en-route network in East China, an analysis of selection preferences of airlines is conducted on the initial allocation plans, narrowing down the range of optimal allocation plans based on the game results of airlines. Finally, combined with the overall optimal principle of air traffic control, the optimal allocation plan was locked in. The results showed that when considering the main base airline’s time and space management, they tend to prioritize the airline’s flights. Different airlines have different preferences, resulting in different optimal allocation plans. This provides a decision-making reference for further improving
airspace operation efficiency and improving resource allocation fairness, thereby enhancing the airline’s participation in the implementation process of the CTOP.

In the complete implementation process of the CTOP, after air traffic control formulates the allocation plan, airlines can also choose whether to modify the TOS of the corresponding flight based on whether the plan provided by air traffic control meets their own preferences. If some airlines choose to modify it, they need to consider the new flight information and reallocate the trajectory and time slot resources. Airlines can choose to modify it at any time, and for flights that have not yet been executed at the time of modification, new allocation situations need to be considered. This process has a significant impact, so in subsequent research, it is necessary to consider the impact of TOS updates on resource allocation and the development of new allocation plans.

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**References**


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