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Article

Analysis of Lateral Forces for Assessment of Safety against Derailment of the Specialized Train Composition for the Transportation of Long Rails

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Abstract: This study proposes a theoretical method for evaluating the "safety against derailment" indicator of a specialized train composition for the transportation of very long rails. A composition of nine wagons, suitable for the transportation of rails with a length of 120 m in three layers, is considered. For the remaining recommended rail lengths, the number of wagons is reduced or increased, with the calculation model being modified depending on the required configuration. When the composition is in a curve with the minimum radius (R = 150 m), the rails bend, and some of them come into contact with the vertical stanchions of the wagon and cause additional lateral forces. These forces are then transferred through the wagon body, central pivot, bogie frame, and wheels and act on the wheel–rail contact points. They could potentially lead to derailment of the train composition. The goal of this study is to determine the additional lateral forces that arise because of the bent rails. For the purposes of this study, the finite element method was used. Based on the displacements of the support points of the rails (caused by the geometry of the curve), the bending line of the elastic load is determined and the forces in the supports are calculated. The resulting forces are considered when determining the derailment safety criterion. The analysis of the results shows that the wagon with fixing blocks is the most at risk of derailment. The front and intermediate wagons have criterion values very close to that of the empty wagon. This shows that the emerging horizontal elastic forces do not significantly influence the derailment process. The obtained results show that the transportation of long rails with specialized train composition can be realized on four layers. This will significantly increase the efficiency of delivering new long rails.

Keywords: railway; derailment; long rails transportation; FEM

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1. Introduction

The European Commission's road map [1] (with a time horizon of 2050) identifies 10 goals in defining transport policy and offers a list of 40 concrete initiatives to achieve the goals. One of them concerns passenger and freight transport, with the following aims:

- Reducing the share of road transport, with 50% of all medium-distance transport being carried out through rail and water transport;
- By 2030, 30% of freight transport over 300 km should be carried out through rail or water transport, and by 2050, over 50%;
- By 2050, most of the passenger transport over more than 300 km should be carried out through rail transport.

The main reasons for the strategic plans formulated in this way are dictated by the huge advantages that rail and water transport have over road and air transport: developed infrastructure, high speed of movement, the possibility of transporting loads of large volume and mass, low cost per unit of work, minimal staff per unit of work, environmental friendliness, high level of safety, and others.

In terms of safety (as well as personnel and environmental indicators), rail transport is the undisputed leader among other modes of transport. This stems from the strict regulatory documents for the admission of new vehicles into operation and the regulation of a strict system for operation and maintenance.

A special place for guaranteeing accident-free transport is occupied by the wheel–rail interaction. This is the reason why these elements are the subject of a few current studies [2–5], in which analytical, numerical, and experimental methods related to wheel and rail wear and the derailment phenomenon have been proposed and analyzed. A significant concern in modern publications is the elucidation of the derailment mechanism [6–13], in which new factors related to the process are introduced. Their reasons are logical since the phenomenon of derailment is a not infrequent event in the modern reality of railway transport and is associated with serious material damage and casualties. Derailments can occur due to the infrastructure or to rolling stock failures as stated in [14]. Some of the publications [8,9,11,15–17] investigate specific elements (turnouts, guardrails, switches, track defects, lubricators, crossings) or operational indicators (speed, train length, location of wagons carrying dangerous loads) affecting the wheel–rail system and contributing to the derailment of rolling stock. To better investigate the complex phenomenon of derailment, some research activities were devoted to studying the problem from an experimental point of view [18]. Many authors [8,9,19] propose the use of numerical methods to theoretically determine the "safety against derailment" indicator. In [20], a new type of finite element for FEM analysis is proposed, which more accurately reflects the interaction between the wheel and the rail.

The analysis made above clearly shows that there are many theoretically unexplained problems for the occurrence of the derailment process. This is clearly stated in [21], where a critical reading of the currently effective regulatory documents for the admission of new railway vehicles into operation of the European Union, CIS, Great Britain, the USA, China, South Korea, and Japan was made. The authors provide specific explanations of the most common trade-offs, point out the advantages and disadvantages of the methods from the relevant papers, and provide useful recommendations on using simulation methods to assess safety against derailment.

The extensive analysis of available publications allows for the determination of the following conclusions:

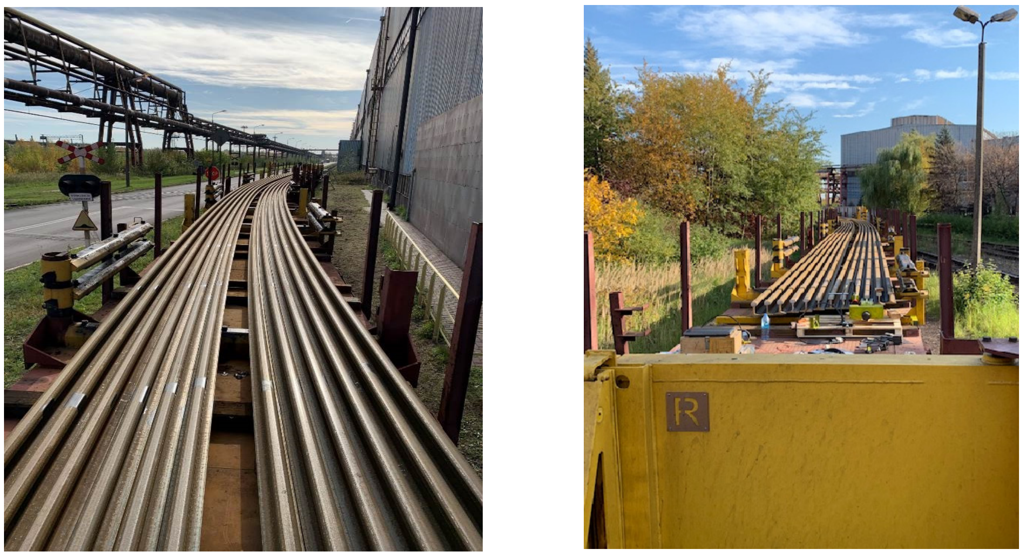
- The derailment phenomenon has a distinctly stochastic character. It depends on many factors, which at a certain moment in a complex combination can lead to its occurrence. Current research addresses the issue of derailment from the point of view of theory enrichment by accounting for one or more additional factors that could influence the occurrence of the adverse event. This effort to clarify the mechanism of the process is undeniably positive for two main reasons: first, although the wagons commissioned into operation meet the requirements of the regulatory documents, in practice, railway accidents caused by derailments often occur; and second, it could lead to an adjustment of national or international standards, which would reduce the likelihood of derailment.
- Although the proposed criteria are generally valid for the cases of examination of railway vehicles in a loaded or empty state, the safety assessment against derailment is carried out on an unloaded wagon. The reasons for this are logical—with a full wagon, the vertical force Q increases significantly (especially for freight wagons up to 3–4 times), while the lateral force Y increases less or in the same order under the influence of centrifugal forces.
- Currently, there are regulatory documents that, regardless of their shortcomings, should be respected to reduce the likelihood of derailment.
- In most analyzed publications and in all normative documents related to the derailment process, the evaluation is reduced to the application of Nadal's criterion [21]. The

limit value of the ratio (Y/Q) at a coefficient of friction equal to 0.36 was determined via Nadal's criterion using Equation (1):

$$\left(\frac{Y}{Q}\right)_{lim} = \frac{\tan\gamma - 0.36}{1 + 0.36\ \tan\ \gamma} \tag{1}$$

where γ is the angle of inclination of the wheel flange.

Based on the third conclusion, a theoretical method for evaluating the safety against derailment of a specialized rolling stock for the transport of long rails is proposed in the present study. In this regard, the second conclusion has relevance to the problem: when transporting long rails, the load is placed not on one wagon, but on a whole specialized train composition (Figure 1). When passing through a curved section of the track, the elastic rails are deformed and describe a complex curved line, the shape of which depends on the radius of the curve, the number of rails, the type of rails, the method of fixing the rails, the number of vertical columns limiting the lateral displacements inside the gauge of the wagon, etc.



(**a**) (**b**)

Figure 1. Train composition for the transport of long rails in a curve: (**a**) middle view; (**b**) front view (source: personal archive).

As a result of the interaction of the elastic rails with the vertical columns of the wagon, enormous lateral forces arise between them, which are transmitted through the vehicle structure and balanced on the rail track. This leads to an increase in the lateral forces Y. The mechanism described above necessitates an assessment of safety against derailment not only in a single empty wagon (the case when a new wagon is commissioned into service), but also in the whole train composition, which is loaded to the permissible values. There is a very small number of studies dealing with this issue. One of them is [22], which uses a multibody dynamics simulation for the assessment of safety against derailment when transporting long rails. In this case, rails are "only" 50 m long, loaded in one layer, using four wagons. This is not a common case according to European standards, where longer rails (mostly 120 m long) in three layers are transported in one train composition, which dramatically increases the efficiency of transport work.

Based on the above considerations, the study of the safety criterion against derailment should be carried out through two distinct stages:

- Determination of the lateral forces acting on the central bearings of the bogies, obtained because of the elastic deformations of the long rails transported.

- Application of the method for theoretical assessment of safety against derailment, considering the permissible vertical load and the total values of lateral forces resulting from the movement of the train composition in a curved section of the track and from the elastic deformation of the transported rails.

The first stage takes place in the following sequence:

1. Development of a computational model for the research object;
2. Numerical solution for determining the interaction forces between the deformed rails and the vertical columns of the wagon;
3. Determination of the additional forces acting on the central bearings of the bogies.

2. Computational Model

The task is complex and complicated since the dimensions of the studied objects are large, and the deformations are significant. In this case, non-linear effects occur that do not allow us to obtain an accurate solution even when using specialized software SolidWorks 2014 SP5. In the literature, there are studies investigating beams where the deformations are large [23–27]. In them, the following task is solved: at a given load, the bending line of the beam is sought. In the present work, the reverse task should be solved: determining the forces that arise at the support points at a given deformation of the investigated beam. For this purpose, a linear analysis method was used, and the research methods from [23–27] can be used to assess the accuracy of the solution.

The transportation of long rails can be carried out with wagons that have different parameters (length, base, width, own weight, load capacity, etc.). After analyzing the mentioned parameters, the authors propose as the most efficient version of the specialized train composition, one that should consist of nine wagons with precisely defined parameters. The idea is to obtain maximum use of both the wagons' carrying capacity and their length, and to have the necessary safety factor against derailment. Therefore, the specific technical parameters of the wagon, which in our opinion are the most efficient, are described. When using another modification in the wagons, the mentioned efficiency cannot be guaranteed.

Basic data and prerequisites for solving the task at these conditions are:

- In accordance with [28], the train composition is in a curve with radius R = 150 m (Figure 2a). The locomotive is not shown in the figure.
- The composition consists of nine wagons; the rails are located on support frames (Figure 2b) and are grouped in separate sections of five rails each.
- The middle wagon is equipped with a system to guarantee the immobility of the rails (so called "fixing block") (Figure 2b), and the load/pressure forces are shown in Figure 2c.
- On the remaining wagons (not equipped with fixing blocks), three or two (for the end wagons) support frames [29] (Figure 2b) are installed, depending on the length of the transported rails and the number of wagons.
- In the support frames, the rails are placed freely, and unlimited longitudinal displacement is allowed as well as limited lateral displacement within Δh, (Figure 2d). Movements along the vertical direction are limited by the supporting beams of the frames and the clearances Δv (Figure 2d).
- It is assumed that the first four wagons have entered the curve.

The last premise allows for only one half of the train composition to be considered (Figure 3). Element A_0 defines the position of the fixing block in the middle wagon, and the remaining elements (A_1 to A_{12}) define the position of the support frames. The front wagon is equipped with two support frames (A_{11} and A_{12}) in accordance with the requirements of [29]. The locomotive is not illustrated in Figure 3.

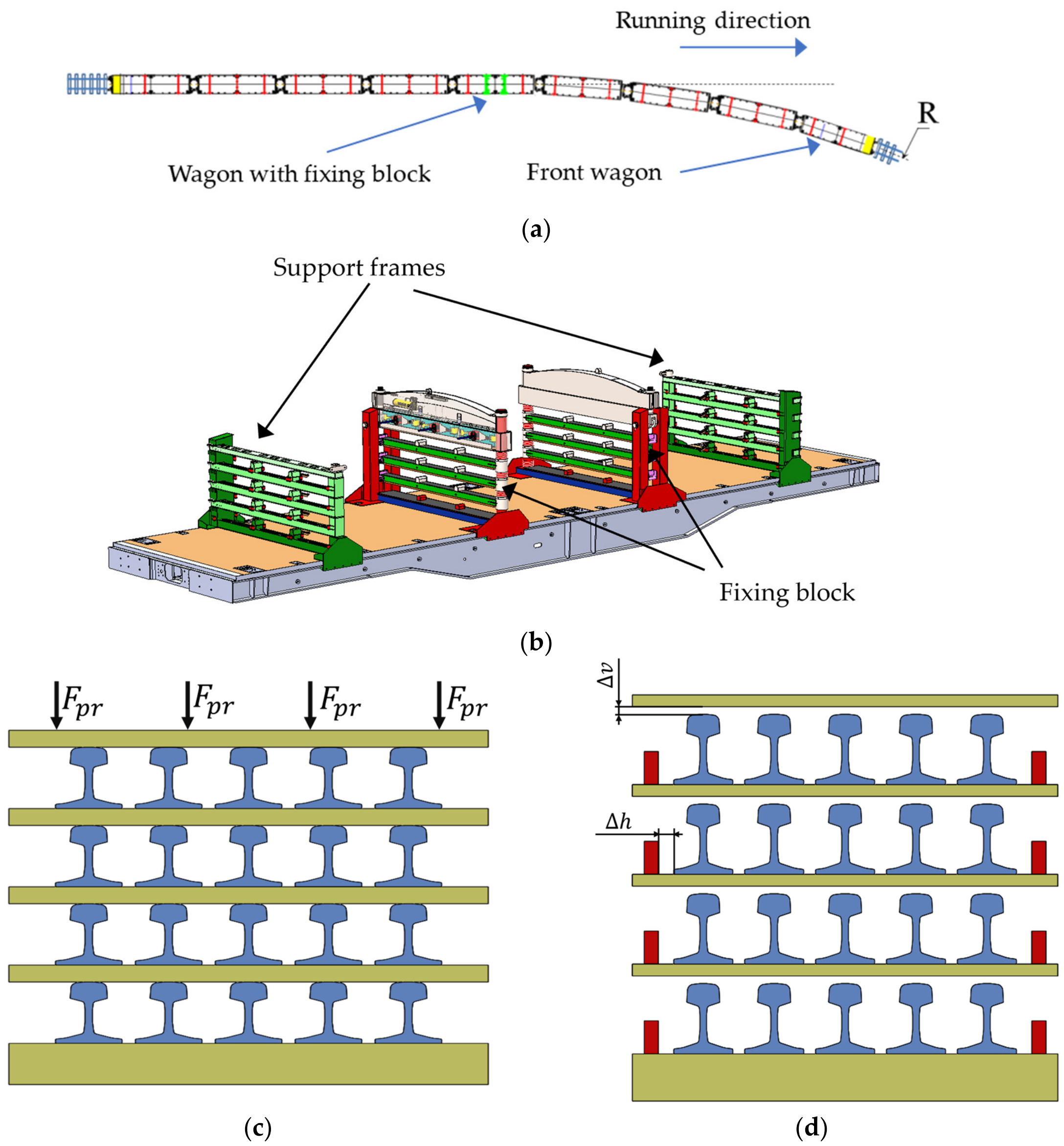


Figure 2. Train composition for transportation of long rails: (**a**) composition in the curve; (**b**) middle wagon with frames; (**c**) load/pressure forces in the fixing block; (**d**) movement limitations in support frames.

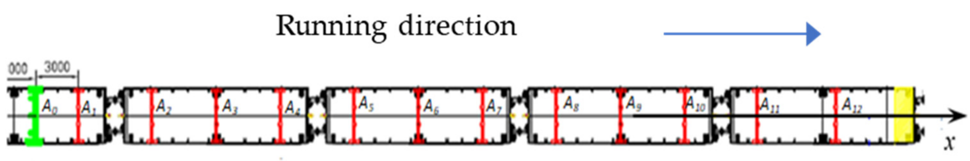


Figure 3. Half of the train composition with elements needed for calculation.

The following additional assumptions were made during the compilation of the computational model:

- The bending of a group of five rails (corresponding to one separate section) in the plane *Oxy*, referred to as "group of rails" or "beam" for brevity (Figures 4 and 5);
- Each rail group has a moment of inertia I_{zi} = 0.00000486 m^4 [30,31]. The total moment of inertia for five rails is $I_z = 5.\ I_{zi}$;

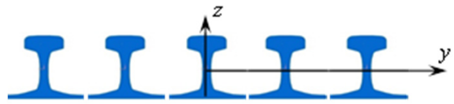


Figure 4. Cross section of one rail group.

- The friction between the rails is neglected due to the peculiarities of the investigated structure.
- In the wagon with the fixing blocks, it is assumed that the rails are immovably fixed, i.e., in the fixing block, it is assumed that we have a fixed beam (Figure 5a);
- At the locations of the supports A_i, it is assumed that unknown forces caused by the deformation of the beam act on the lateral direction (y axis) as shown in Figure 5b. The forces are directed perpendicular to the beam axis (x axis);
- Because of the large dimensions and deformations of the system and considering that the task is statically indeterminate, it is solved with a linear analysis method for the approximate determination of the forces in the supports. The obtained results contain some inaccuracies, which can be estimated by the studies presented in [25,27];
- When moving in a curve with radius R = 150 m, the speed of the train composition is small; therefore, the inertial forces are neglected.

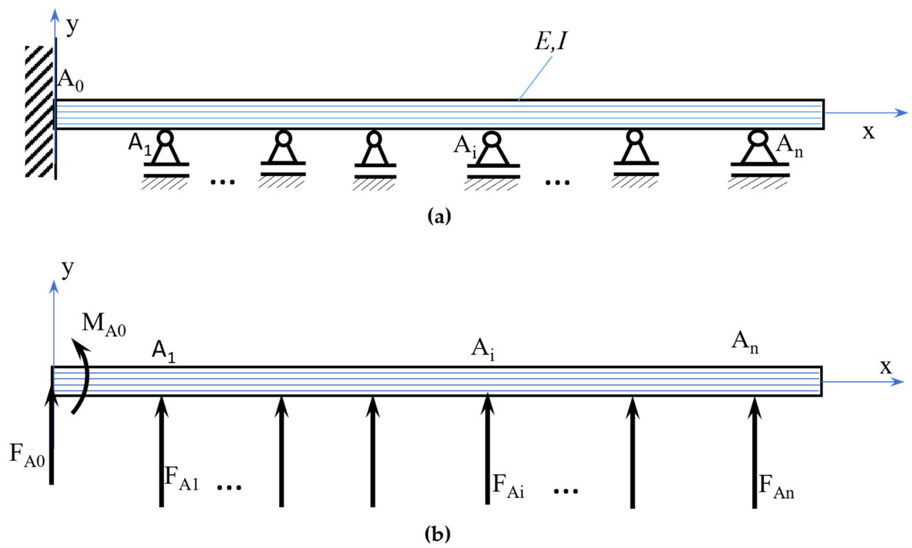


Figure 5. Calculation model of the fixed beam and lateral forces acting on it: (**a**) fixed beam in fixing block; (**b**) unknown forces acting on the lateral direction.

The classical form of the differential equation of the bending line in the given coordinate system and for each section of the beam is given in Equation (2) [32]:

$$\frac{d^2}{dx^2}\left(EI_z\frac{d^2u_y}{dx^2}\right) = q_y \tag{2}$$

For the analytical solution, it is necessary to integrate Equation (2), where a significant number of integration constants appear, depending on the number of sections, which must subsequently be determined by the boundary conditions. For this purpose, the finite element method was applied [32,33]. The large dimensions of the beams make it difficult to use precise models like those shown in [2,34,35]. The beam is divided into n-number of finite elements, and each of them is located between two adjacent supports A_i (Figure 5). The type of finite element is shown in Figure 6 [32].

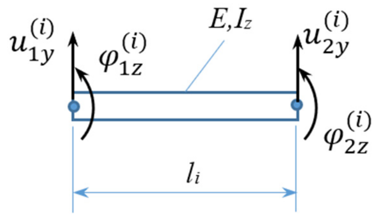


Figure 6. Finite beam element [28].

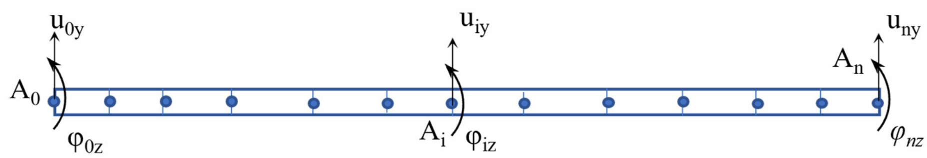
The position of each element is described by four local coordinates—two displacements and two rotations ($u_{1y}^{(i)}, \varphi_{1z}^{(i)}, u_{2y}^{(i)}, \varphi_{2z}^{(i)}$) of each of the nodes [32].

The stiffness matrix [32,33] of the element i is given in Equation (3):

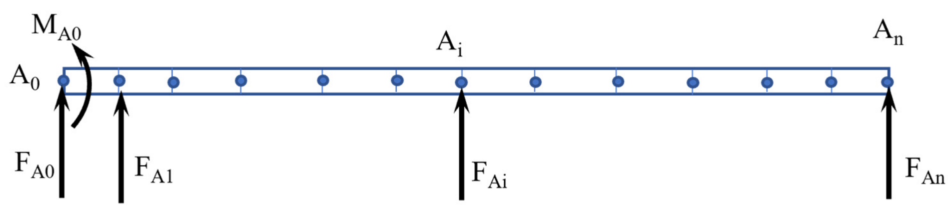
$$\left[K^{(e)}\right] = \frac{2EI_z}{l_i^3}\begin{bmatrix} 6 & 3l_i & -6 & 3l_i \\ 3l_i & 2l_i^2 & -3l_i & l_i^2 \\ -6 & -3l_i & 6 & -3l_i \\ 3l_i & l_i^2 & -3l_i & 2l_i^2 \end{bmatrix} \tag{3}$$

where l_i is the length of the element and E is the modulus of elasticity.

The discretized model has n number of elements (n = 12 in Figure 7) with different lengths l_i and correspondingly different stiffness matrices. Its position is described by $2n + 2$ global coordinates (Figure 7a). The forces act on the nodes, and the moment acts in node A_0 (Figure 7b).



(**a**)



(**b**)

Figure 7. Discretized model of the beam: (**a**) with defined nodes coordinates; (**b**) with defined loads in nodes.

For the beam in Figure 7, Equation (4) is valid:

$$[K_0]\{U_0\} = \{F_0\} \tag{4}$$

where $\{U_0\} = \{u_{0y}, \varphi_{0z}, u_{1y}, \varphi_{1z}\ldots\ldots, u_{iy}, \varphi_{iz}\ldots\ldots u_{ny}, \varphi_{nz},\}^T$ is a vector of global coordinates with dimension (2n + 2) × 1 and $\{F_0\} = \{F_{A0}, M_{A0}, F_{A1}, 0, F_{A2}, 0\ldots.F_{Ai}, 0..F_{An}, 0\}^T$ is a vector of node loads with dimension (2n + 2) × 1. $\{F_0\}$ contains forces and moments applied at the nodes. Except for A0, there are no applied moments in all other nodes. $[K_0]$

is the global stiffness matrix, with dimension (2n + 2) × (2n + 2). Considering that $u_{0y} = 0$ and $\phi_{0z} = 0$, Equation (4) is reduced to Equation (5):

$$[K]\{U\} = \{F\} \tag{5}$$

where $[K] = [K_0\ (3{:}2n + 2, 3{:}2n + 2)]$ is the stiffness matrix after considering the boundary conditions and has a dimension of $2n \times 2n$ (the first two rows and columns of the matrix $[K_0]$ are removed), $\{U\} = \{u_{1y}, \varphi_{1z}......, u_{iy}, \varphi_{iz}, ..., u_{ny}, \varphi_{nz},\}^T$ is a vector of global coordinates after considering the boundary conditions and has a dimension of $2n \times 1$, and $\{F\} = \{F_{A1}, 0, F_{A2}, 0....F_{Ai}, 0..F_{An}, 0\}^T$ is a vector of node loads after considering the boundary conditions and has a dimension of $2n \times 1$. F_{A0} and M_{A0} are determined using Equation (6):

$$[K_1]\{U\} = \begin{Bmatrix} F_{A0} \\ M_{A0} \end{Bmatrix} \tag{6}$$

where $[K_1] = [K_0\ (1{:}2, 3{:}2n + 2)]$ is a $2 \times 2n$ matrix formed by the first two rows and columns from 3 to $2n + 2$ of the matrix $[K]$. Classical FEM analysis was used up to this step. In Equation (5) (unlike the classical problem), the displacements u_{iy} are known, but the forces F_{Ai} and rotations φ_{iz} of the nodes are not known. To make the equations easier to solve (with MATLAB R2020b software), it is necessary to discretize this continuous mechanical system. To determine the unknowns, the system is transformed into the form given in Equation (7):

$$[X]\{p\} = \{Z\} \Rightarrow \{p\} = [X]^{-1}\{Z\} \tag{7}$$

In Equation (7), the term $\{p\}$ has a dimension of $2n \times 1$ and contains the unknowns F_{Ai} and φ_{iz}. It is determined using Equation (8):

$$\{p\} = \{F_{A1}, F_{A2}, ...F_{An}, \varphi_{1z}, \varphi_{2z}......\varphi_{nz}\}^T \tag{8}$$

The term $\{Z\}$ in Equation (7) has a dimension of $2n \times 1$ and contains the predetermined displacements u_{iy} of the nodes (coordinates of $\mathrm{A_i}$ along the *y*-axis direction) and elements of the matrix $[K]$. It is determined using Equation (9):

$$\{Z\} = \begin{Bmatrix} K_{1,1}u_{1y} + K_{1,3}u_{2y}....... + K_{1,2n-1}u_{ny} \\ K_{3,1}u_{1y} + K_{3,3}u_{2y}....... + K_{3,2n-1}u_{ny} \\ ... \\ .... \\ K_{2n-1,1}u_{1y} + K_{2n-1,3}u_{2y}....... + K_{2n-1,2n-1}u_{ny} \\ K_{2,1}u_{1y} + K_{2,3}u_{2y}....... + K_{2,2n-1}u_{ny} \\ K_{4,1}u_{1y} + K_{4,3}u_{2y}....... + K_{4,2n-1}u_{ny} \\ ... \\ .... \\ K_{2n,1}u_{1y} + K_{2n,3}u_{2y}....... + K_{2n,2n-1}u_{ny} \end{Bmatrix} \tag{9}$$

The matrix $[X]$ has a dimension of $2n \times 2n$, and contains the null matrices and the elements of matrix $[K]$. It is determined using Equation (10):

$$[X] = \begin{bmatrix} 1, O_{1,n-1}, -K_{1,2}, -K_{1,4}, -K_{1,6}\ldots\ldots\ldots, -K_{1,2n} \\ 0, 1, O_{1,n-2}, -K_{3,2}, -K_{3,4}, -K_{3,6}\ldots\ldots\ldots, -K_{3,2n} \\ \ldots. \\ \ldots \\ O_{1,n-1}, 1, -K_{2n-1,2}, -K_{2n-1,4}, -K_{2n-1,6}\ldots\ldots\ldots, -K_{2n-1,2n} \\ O_{1,n}, -K_{2,2}, -K_{2,4}, -K_{2,6}\ldots\ldots\ldots, -K_{2,2n} \\ O_{1,n}, -K_{4,2}, -K_{4,4}, -K_{4,6}\ldots\ldots\ldots, -K_{4,2n} \\ \ldots \\ \ldots.. \\ O_{1,n}, -K_{2n,2}, -K_{2n,4}, -K_{2n,6}\ldots\ldots\ldots, -K_{2n,2n} \end{bmatrix} \tag{10}$$

In this way, the unknowns from Equation (8) can easily be solved with MATLAB software. After determining the unknowns $F_{A1}, F_{A2}, \ldots F_{An}, \phi_{1z}, \phi_{2z}\ldots\ldots\phi_{nz}$ (contained in $\{p\}$), M_{A0} and F_{A0} can be determined from Equation (6) or from the two equilibrium conditions of the entire beam.

3. Results from the Numerical Calculations

As mentioned above, in numerical calculations, the first step is to determine the interaction forces between the deformed rails and the vertical columns of the wagon. The second step is the determination of the additional forces acting on the central bearings of the bogies from the forces calculated in the first step. These lateral forces are transferred via central bearings to the bogie frame and then on to the wheels, acting on wheel–rail contact and causing additional lateral force. This force is used for the assessment of safety against derailment according to Equation (1).

3.1. Results from Calculation of Lateral Forces between Rails and Vertical Columns of the Wagon

To determine the coordinates of the points A_i (and the displacements u_{iy}, respectively) along the y-axis, it is assumed that the composition is in a curve with radius R (Figure 2a). It is assumed that the points A_1, A_2, A_4, A_5, A_7, A_8, A_{10} and A_{11} lie on the theoretical arc of a circle because that is where the central bearings of the wagons are positioned. The remaining points A_3, A_6, A_9 and A_{12} are located on the chords between the above points (Figure 3). Their position is determined by the line connecting the central bearings of the wagon. Point A_1 is located at the beginning of the curve.

The theoretical curve of the bending line, described with the coordinates from Table 1, is shown in Figure 8.

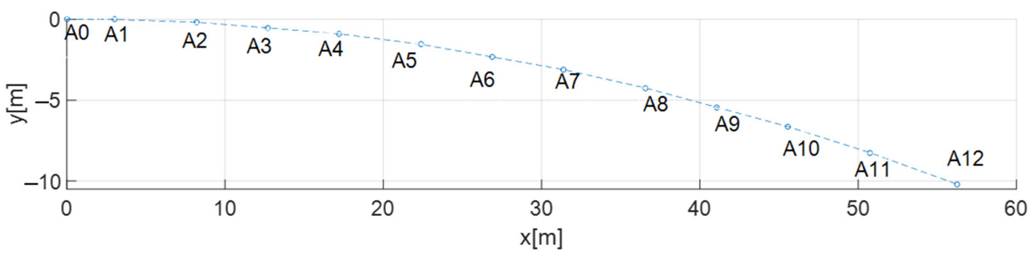


Figure 8. Bending line with coordinates of points A_i.

The coordinate system $A0xy$ is introduced so that the x-axis is in the direction of line A_0–A_1, and its direction relative to the curve is determined by the two central bearings of the wagon with the fixing blocks (Figure 2a).

The determination of the unknown forces F_{Ai} and the moment M_{A0} is carried out in the MATLAB program environment using the displacements from Table 1 (third column). Table 1 (column 5) also contains the obtained values for force reactions in points A_i. It should be noted that the values are obtained under the condition that there is no horizontal clearance between the rails and the support stanchions, i.e., $\Delta h = 0$ (Figure 2d).

Table 1. Coordinates of points A_i along the bending line and values of reaction forces (and moment M_{A0}) in points A_i without horizontal clearance between the rails and the support stanchions ($\Delta h = 0$).

Point	Coordinate x (m)	Coordinate $y = u_{iy}$ (m)	Reaction Identifier (Unit)	Value
A_0	0	0	M_{A0} (kN·m)	−20.9627
A_0	0	0	F_{A0} (kN)	−20.9627
A_1	3.000	0	F_{A1} (kN)	15.4067
A_2	8.190	−0.168	F_{A2} (kN)	28.7388
A_3	12.690	−0.525	F_{A3} (kN)	−45.0839
A_4	17.190	−0.883	F_{A4} (kN)	22.1165
A_5	22.380	−1.540	F_{A5} (kN)	21.0936
A_6	26.880	−2.320	F_{A6} (kN)	−42.6837
A_7	31.380	−3.100	F_{A7} (kN)	21.6934
A_8	36.570	−4.241	F_{A8} (kN)	20.7446
A_9	41.070	−5.436	F_{A9} (kN)	−43.0170
A_{10}	45.570	−6.631	F_{A10} (kN)	25.6134
A_{11}	50.760	−8.245	F_{A11} (kN)	4.9837
A_{12}	56.260	−10.200	F_{A12} (kN)	−8.6432

During the calculations, the presence of large forces was found (Table 1, last column), especially in the points that do not lie on the theoretical arc of a circle (A_3, A_6 and A_9). This is logical and is dictated by the fact that the clearance Δh is not provided in the model, i.e., $\Delta h = 0$ (Figure 2d). To increase the adequacy of the model, a limited horizontal displacement of the points was allowed due to the available constructively provided clearances between the rails and the support frames. With successive iterations based on a possible displacement of the bending line in the support points, new results for the y-coordinates of points A_i and values for the force reactions in points A_i were obtained and are presented in Table 2.

Some of them have been adjusted relative to their original position up to 90 mm (constructively provided clearance between the rails and the support frames) in the lateral direction. In this case, the group of rails does not necessarily rest against each of the side supports, but settles freely, hindered by the minimal frictional forces at the base of the rail. This fact was established during real measurements carried out on another specialized train composition for the transport of long rails. The clearance and possible settlement of the beam at a given point was obtained by iterations through a certain step with a numerical experiment until the actual measured values were obtained.

The iterations of horizontal clearance between the rails and limiting pins Δh (Figure 2d) show that the reaction forces decrease because the bending line changes.

With new coordinates from Table 2 (third column), new values of reaction forces are determined and are presented in the last column of Table 2.

An analysis of the data from Table 2 shows that the maximum lateral forces are at points A_0 and A_1 of the wagon with fixing block and at points A_{11} and A_{12} of the front (closest to the locomotive) wagon.

These lateral forces are transferred via central bearings to the bogie frame and then on the wheels, acting on wheel–rail contact and causing additional lateral force. The results from the calculation of these additional forces for different wagons of a specialized train composition are presented in the next subsection.

Table 2. Adjusted coordinates of points A_i along the bending line and values of reaction forces (and moment M_{A0}) in points A_i with horizontal clearance between the rails and the support stanchions (Δh = 90 mm).

Point	Coordinate x (m)	Coordinate $y = u_{iy}$ (m)	Reaction Identifier (Unit)	Value
A_0	0	0	M_{A0} (kN·m)	−21.8257
A_0	0	0	F_{A0} (kN)	−21.8257
A_1	3.000	0	F_{A1} (kN)	22.8997
A_2	8.190	−0.1436	F_{A2} (kN)	−1.0583
A_3	12.690	−0.4350	F_{A3} (kN)	1.3180
A_4	17.190	−0.8733	F_{A4} (kN)	−1.0934
A_5	22.380	−1.5400	F_{A5} (kN)	−1.1026
A_6	26.880	−2.2530	F_{A6} (kN)	1.1627
A_7	31.380	−3.1000	F_{A7} (kN)	−0.0952
A_8	36.570	−4.2410	F_{A8} (kN)	−0.2744
A_9	41.070	−5.3690	F_{A9} (kN)	−0.4341
A_{10}	45.570	−6.6274	F_{A10} (kN)	1.2717
A_{11}	50.760	−8.2450	F_{A11} (kN)	4.8339
A_{12}	56.260	−10.1100	F_{A12} (kN)	−5.6022

3.2. Results from Calculation of Additional Lateral Forces in Wheel–Rail Contact

The next step is to determine the additional forces acting on the central bearings of the bogies from forces calculated in the previous subsection. They are used to assess the safety against derailment of the entire specialized train composition. The calculation is carried out only for the wagon with the fixing blocks and the front wagon.

The forces at points A_0 and A_1 of the wagon with fixing block with their respective values from Table 2 are applied to the wagon, as shown in Figure 9.

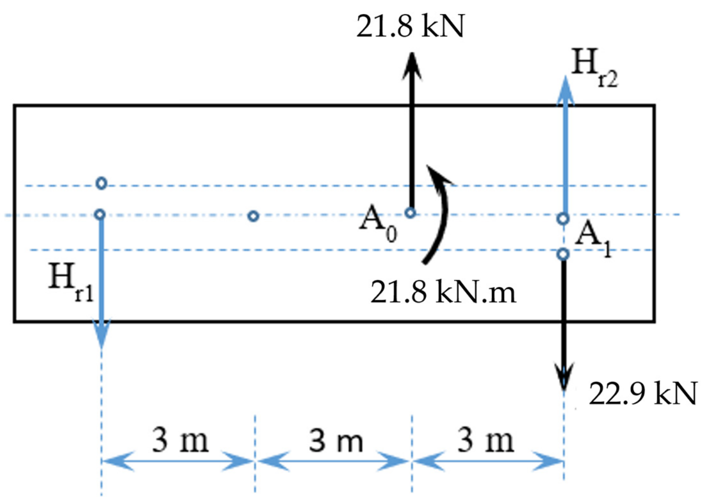


Figure 9. Forces and moment acting on a wagon with fixing block when entering the curve.

The reason for analyzing the front wagon is that it is at risk of derailment due to the incomplete vertical load from the rails' weight (leading to a reduction in the vertical force Q from Equation (1)) and due to the asymmetric distribution of the load in the structure resulting from the placement of the support frames on the wagon (Figure 3). The forces at points A_{11} and A_{12} of the front wagon with their respective values from Table 2 are applied to the wagon, as shown in Figure 10.

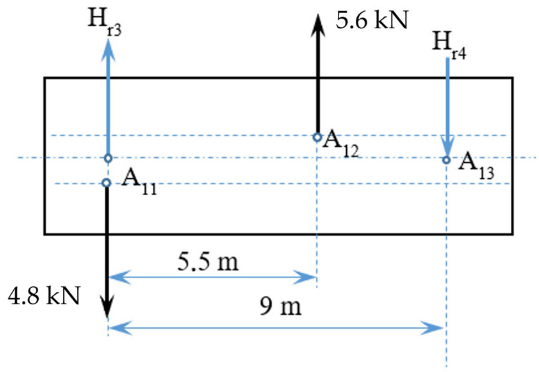


Figure 10. Forces acting on a front wagon.

The determination of the additional horizontal forces H_{ri} is trivial, and their values are given in Table 3.

Table 3. Values of additional lateral forces H_{ri}.

Wagon with Fixing Block	Front Wagon	Unit	Value
H_{r1}	-	(kN)	4.84
H_{r2}	-	(kN)	5.94
-	H_{r3}	(kN)	2.62
-	H_{r4}	(kN)	3.42

4. Assessment of Safety against Derailment

The assessment of safety against derailment of the specialized train composition for transportation of long rails was performed theoretically in full accordance with the method proposed in [36]. Briefly, this is an analytical method developed according to EN 14363:2019 [28], Section 6.1, Method 2 (paragraph 6.1.5.2). The method proposed in [36] has been verified by comparing the results of theoretical and experimental studies and shows a very good compliance.

For assessment purposes, the following corrections were made in the present method compared to that in [36]:

- The vertical load from the weight of 60 (4 rows × 15 rails per row) rails of type 60E1 with a length of 120 m (on the entire train composition) was applied;
- The horizontal forces from Table 3 are applied in the central bearings of the wagon.

It is assumed that the specialized train composition for transportation of long rails is composed of nine flat wagons of the Smmns series with the following parameters:

- The tare weight of each wagon is 21 tons;
- The distance between pivots is 9000 mm;
- The maximum load capacity is 69 tons;
- The maximum permissible load capacity when transporting rails is 0.8 × 69 tons = 49.6 tons according to [29];
- The wagon gauge is G1;
- The weight of two fixing blocks at the middle wagon is 2 × 2.5 tons = 5 tons;
- The weight of one support frame is 1 ton.

The calculations were performed for a horizontal track without overhang at a friction coefficient $\mu = 0.36$.

Three types of wagons from the loaded train composition were studied:

- The wagon equipped with fixing block to limit the longitudinal movements of the rails during transportation—the reason for this is that this is the wagon with the maximum tare weight (taking into account the weight of the installed frames) and in it, the lateral forces F_{A0} and F_{A1} (Table 2), respectively, and the lateral force H_{ri} have a maximum values;

- Front wagon with protective walls. It has a reduced weight of the payload, which is placed asymmetrically on the wagon;
- Intermediate wagon (heaviest loaded wagon of all the wagons discussed above).

Additionally, an assessment was also performed for the empty Smmns series wagon in accordance with the requirements of [28], to assess the influence of the load on the derailment process. Detailed data from the simulation are presented in Table 4 only for the first type of wagon—wagon with fixing block—and for the other types of wagons only the final assessments are given separately in Table 5.

Table 4. Results from calculation of parameters used for assessment of safety against derailment of wagon with fixing block.

Parameter Description	Symbol/Equation	Value
Total reaction force of the attacking wheel of wheelset [1]	Y_{1a} [1]	122.814 kN
Total reaction force of the non-attacking wheel of wheelset [1]	Y_{1i} [1]	−70.732 kN
Minimum deflection of the bogie frame to be reached during tests of the wagon [28]	$\Delta z^{+}_{jk} = g^{+}_{lim}.2a^{+}$	0.0076 m
Twist of the bogie frame during test (2a$^+$ is distance between wheelsets) [28]	$g^{+}_{lim} = 7 - 5/2a^{+}$	4.2222‰
Minimum deflection of the wagon frame to be reached during tests of the wagon [28]	$\Delta z* = g^{*}_{lim}.2a$	33.2 mm
Twist of the wagon frame during test (2a is the pivot distance) [28]	$g^{*}_{lim} = \frac{15}{2a}+2$	3.6483‰
Load force during torsion test	ΔF_p	10 kN
Deflection of the wagon frame under ΔF_p	Δz_p	88.22 mm
Force needed to achieve Δz*	$\Delta F_{z*} = \frac{\Delta F_p . \Delta z^*}{\Delta z_p}$	3.763 kN
Maximum force additionally loading bogie frame under ΔF_{z^*}	$\Delta F'_{z*,max} = \frac{\Delta F_{z*}.(b_{1F} + b_s)}{2b_{1F}}$	3.481 kN
Minimum force additionally loading bogie frame under ΔF_{z^*}	$\Delta F'_{z*,min} = \Delta F_{z*} - \Delta F'_{z*,max}$	0.282 kN
Maximum force overloading first wheel	$\Delta F'_{1z*,max} = \frac{\Delta F'_{z*,max}}{2}$	1.7405 kN
Minimum force overloading first wheel	$\Delta F'_{1z*,min} = \frac{\Delta F'_{z*,min}}{2}$	0.141 kN
Additional maximum vertical reaction force in wheels (Equation (25) of [36])	$\Delta Q_{1,max}$	2.007 kN
Additional minimum vertical reaction force in wheels (Equation (25) of [36])	$\Delta Q_{1,min}$	−0.125 kN
Nominal vertical force loading the wheels	$Q_{nom} = Q/N$	93.401 kN
Minimum vertical wheel reaction force	$Q_{jk,min} = Q_{nom} + \Delta Q_{1,min}$	93.276 kN
Minimum vertical wheel reaction force	$Q_{jk,max} = Q_{nom} + \Delta Q_{1,max}$	95.408 kN

[1] Calculated according to methodology described in [36].

With the data from Table 5, the final assessment of safety against derailment can be conducted. This is performed using Equation (11) and the calculated value is equal to 1.0698. According to [28], when using the theoretical assessment methods, the limit value of 1.2 is reduced by 10%, which means that the limit value of Nadal's criterion should be set to 1.08 and compared with the calculated value, as shown in Equation (12).

$$\left(\frac{Y}{Q}\right)_{ja} = \frac{Y_{ja}}{Q_{jk,\min} + \Delta Q_j}; \qquad \frac{Y_{ja}}{Q_{jk,\min} + \Delta Q_j} \leq \left(\frac{Y}{Q}\right)_{\lim} \tag{11}$$

$$\left(\frac{Y}{Q}\right)_{ja} \leq \left(\frac{Y}{Q}\right)_{lim}; \qquad 1.0698 \leq 1.08 \tag{12}$$

Data for the safety against derailment evaluation criterion for all wagon types considered are given in Table 5.

Table 5. Safety criterion for different wagon types.

Criterion	Wagon with Fixing Block	Front Wagon	Intermediate Wagon	Empty Wagon
$\left(\frac{Y}{Q}\right)_{ja} \leq 1.08$	1.0698	0.8024	0.8295	0.8268

The obtained values of the safety criterion for all wagon types considered are lower than the limit value of 1.08, which means that for specialized train composition for transportation of long rails, the requirement for safety against derailment is fulfilled.

The developed model allows us to evaluate the influence of individual factors like pivot distance, own weight, number of rails, movement speed, overhang of the outer rail of the track, friction coefficient, etc., on the criterion. It was found that the friction coefficient has the most serious influence. By increasing its value by only one hundredth from 0.36 to 0.37, the criterion increases from 1.0698 to 1.0874 and exceeds the permissible limits for theoretical evaluation.

The remaining parameters have relatively less influence on the change in intensity of the criterion. For example, when reducing the number of rails from 60 (four rows with 15 rails each) to 45 (three rows with 15 rails each) as recommended in [29] (the reduction by 25%), the criterion decreases from 1.0698 to 1.0450, i.e., by only 2.37%, which is within the computational error. This makes it inexpedient to use an approach from [29] with only three layers of long rails in the composition.

5. Conclusions

The present study proposes a theoretical method for assessment of safety against derailment of a specialized train composition for the transportation of long rails. For this purpose, a method was developed to determine the horizontal forces in the vertical stanchions of the wagon, occurring when they contact the transported rails in a curved section of the track. The forces are used in the computational model for the study of safety against derailment and an evaluation is made according to Nadal's criterion.

- Four different types of train composition wagons were considered: a wagon with fixing blocks, front wagon, intermediate wagon, and an empty wagon. The analysis of the results shows that the wagon with fixing blocks is most at risk of derailment. The obtained value (Table 4) fully corresponds to the requirements of the normative documents not only for real tests (<1.2), but also for theoretical studies (<1.08).
- The front and intermediate wagons have criterion values very close to that of the empty wagon. This shows that the emerging horizontal elastic forces do not significantly influence the derailment process.
- The developed model allows us to evaluate the influence of individual factors like pivot distance, own weight, number of rails, movement speed, overhang of the outer rail of the track, friction coefficient, etc., on the safety against derailment criterion.
- The transportation of long rails can be carried out with wagons that have different parameters (length, base, width, own weight, load capacity, etc.). After analyzing the mentioned parameters, the authors propose the most efficient version of the specialized train composition for 120 m long rails. When using another modification in the wagons or another rail length, the mentioned efficiency cannot be guaranteed.
- The obtained results show that the transportation of long rails with specialized train composition can be realized on four levels. This will significantly increase the efficiency of operators when delivering long new rails.

The transportation of rails on more than three levels is not new in global practice and is widely used in India (a record was set there for the transportation of rails with a length of 260 m, in five rows with 12 rails each), the USA, Russia, etc. (in four rows with 12–15 rails each).

For future research, it would be necessary to conduct a physical validation of the model. This will certainly be carried out if there is a proposal from companies carrying out the transportation of long rails.

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