Operation Optimization of the Sea Container Fleet Based on the Double-Level Planning Model

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Abstract: In response to the optimal operation of ocean container ships, this paper presents a two-level planning model that takes into account carbon tax policies. This model translates the CO2 emissions of ships into carbon tax costs and aims to minimize the overall operation costs of the ships. In top-level planning, the model considers factors such as speed, cargo load, and energy consumption to establish an objective function and optimization strategy. In bottom-level planning, the model involves ship stability and imposes corresponding constraints. By integrating the two levels of planning, a ship operation optimization model that considers multiple factors is obtained. With practical ocean container ships as cases, through numerical examples and sensitivity analysis, the constraint, stability, and structural feasibility of the constructed model are confirmed. The research results of this paper provide a decision-making basis for optimizing the operation of oceanic container ships.

Keywords: two-level planning model; carbon tax policies; overall operation costs of ships; sensitivity analysis

1. Introduction

Ocean container transportation has gained increasing significance in contemporary sea transportation, mainly due to its convenient and efficient nature [1]. In addition, with the continuous progress of green ship technology and the continuous instability of the global shipping market [2], shipping companies must adopt an optimal route configuration, voyage, and speed design to improve the capacity of container ships and achieve the maximum output or efficiency with the least input or consumption [3].

In order to achieve the reasonable allocation of container ship operation planning on multiple routes, Wang et al. [4] used the self-correlation feature scheduling method based on correlation rule information fusion to improve the automatic dispatching ability of container route dynamic loading. Ma [5] established an optimization model to allocate time windows and appointment quotas for each ship at sea freight container terminals. Yang et al. [6] proposed a fixed route speed optimization model that distinguished water speed from land speed to minimize the total fuel consumption of the voyage. Chen et al. [7] used the fuzzy quality function expansion (QFD) method to construct an evaluation index system for the collaborative operation level of container shipping alliances and improved shipping efficiency by improving the management operation level. Febriyanti et al. [8] used the saving matrix method to determine the distribution route to minimize transportation costs. Tian et al. [9] used a machine learning paradigm to solve the prediction difficulties of ship detention. Xin et al. [10] established a joint optimization model for shipping network design–infrastructure investment and minimized the generalized transportation costs in the study area by jointly optimizing the container port’s throughput and shipping network operation plans. Chen et al. [11] proposed an ellipsoid uncertainty set to study the impact of low sulfur fuel costs on container liner scheduling. Du et al. [12] proposed an improved
three-dimensional dynamic programming algorithm for ship route optimization with weather and other constraints to obtain the optimal ship route.

The scattered nature of research on various aspects of ports, ships, and fuel, as mentioned above and as observed by Song et al. [13], is a primary cause of the inefficiency of shipping optimization models. Gülmez et al. [14] and Paridaens et al. [15] separately examined and studied the classification of maritime logistics and the integration of shipping logistics, and they concurred that integrating each logistics link in shipping would be a key research direction in the future. Simultaneously, ocean transportation-induced environmental pollution has sparked concern, leading to heightened international expectations regarding carbon emission issues [16]. Currently, there is a lack of sufficient quantitative analysis on factors like carbon tax [17].

Based on this, considering the carbon tax cost, meeting freight demand, and ensuring transportation income, the scientific novelty of this paper is to build a comprehensive optimization model for container ship configuration, route, and speed based on nonlinear programming aimed at reducing the overall operation costs of ships. In view of the complexity of the model, this paper formulates the problem as a multi-objective optimization problem [18], uses the discrete algorithm, and analyzes the effectiveness and sensitivity of the model based on basic data of ship speed on specific routes to prove that this model can provide practical support for optimizing container fleet operation strategies.

2. The Establishment of the Model
2.1. The Assumption and Description of the Problem

The optimization of multi-routing and multi-vessel types is a complex task that involves numerous parameters and variables. Table 1 summarizes the key parameters and variables considered in this paper. To facilitate the analysis and optimization of ship scheduling and vessel allocation, this paper outlines a hypothetical framework as follows:

1. Vessel Diversity: each vessel type possesses a unique cargo-carrying capacity and operating costs. The selection of vessel types is tailored to the characteristics of distinct routes.
2. Intercontinental Trunk Shipping: the order of berthing ports on intercontinental trunk routes is determined, and the container demand between intercontinental endpoints is anticipated and confirmed.
3. Fixed Departure Frequency: the frequency of ship departures is standardized to once per week for all flights within the same week.
4. Vessel Type Consistency: each flight adheres to a predefined vessel type, ensuring only vessels of the same type are utilized within the same flight.
5. Uniform Speed and Fuel Consumption: A consistent speed is maintained by vessels of the same type across different sections, and their fuel consumption function remains unchanged across different sections.

Drawing on these parameters and assumptions, this paper aims to explore methods for maximizing operating efficiency across various vessel types, routes, and operating costs. The principle is shown in Figure 1.

2.2. The Establishment of Comprehensive Optimization Model
2.2.1. Variable Relationships

1. The relationship between air cargo volume and port container demand

On route r, if the ships can only call at each port once, its route system is that shown in Figure 2.
Total weekly operating costs for ships on each route

Table 1. Various parameters involved in ocean shipping.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Route, $r \in {1, 2, \ldots, R}$</td>
</tr>
<tr>
<td>$v$</td>
<td>Type of vessel, $v \in {1, 2, \ldots, V}$</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Total number of ports of call on route</td>
</tr>
<tr>
<td>$k$</td>
<td>Segment, $k = 1, 2, \ldots, m_r$</td>
</tr>
<tr>
<td>$Q_k^r$</td>
<td>Cargo volume on sector $k$ of route</td>
</tr>
<tr>
<td>$d_{ij}^r$</td>
<td>Port on route $i$ to port $j$ weekly container demand (TEU)</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Distance of route (miles)</td>
</tr>
<tr>
<td>$C_{ap,v}$</td>
<td>Rated capacity of $v$ type of container (TEU)</td>
</tr>
<tr>
<td>$S_{min}$</td>
<td>Minimum speed of type ship (kn)</td>
</tr>
<tr>
<td>$S_{max}$</td>
<td>Maximum speed of type ship (kn)</td>
</tr>
<tr>
<td>$t_r$</td>
<td>Time (in days) required for a ship to complete a voyage on route $r$</td>
</tr>
<tr>
<td>$T_{port}$</td>
<td>Ship’s port call and berthing time on route $r$ (days)</td>
</tr>
<tr>
<td>$C_{av}$</td>
<td>Daily operating costs for type of vessel</td>
</tr>
<tr>
<td>$K_v$</td>
<td>Fuel consumption constants for ship types</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Heavy oil price (USD/t)</td>
</tr>
<tr>
<td>$P_l$</td>
<td>Diesel price (USD/t)</td>
</tr>
<tr>
<td>$F_{1,v}$</td>
<td>Weekly by-engine fuel consumption per vessel $v$</td>
</tr>
<tr>
<td>$F_{f,v}$</td>
<td>Main engine fuel consumption per day per vessel $v$</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Average daily total oil consumption of individual main engines</td>
</tr>
<tr>
<td>$Z$</td>
<td>Total weekly operating costs for ships on each route</td>
</tr>
<tr>
<td>$x_{v,r}$</td>
<td>When the $r$ route is equipped with the $V$-type ship, it is defined as 1; otherwise, it is set to 0.</td>
</tr>
<tr>
<td>$n_{V,r}$</td>
<td>Number of $V$-type ships on route $r$</td>
</tr>
<tr>
<td>$S_v$</td>
<td>The speed of a $V$-shaped ship</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Carbon tax costs</td>
</tr>
</tbody>
</table>
Here, the seaport inside the dashed circle is situated on the opposite side of the intercontinental route. The ports linked by the dashed line do not engage in goods exchange. On the route, assuming there are \( n_1 \) ports at the starting point A and \( n_2 \) ports at the ending point B, there are \((n_1 + n_2)\) segments in total on the route \( r \). The relationship between freight volume and container transportation demand on each section of the route \( r \) can be represented as follows:

On the route, assuming that there is 1 port at the starting point A and 2 ports at the ending point B.

\[
Q_r^k = \begin{cases}
\sum_{i=1}^{k} \sum_{j=i+1}^{n_1+n_2} d_{ij}^r + \sum_{i=n_1+1}^{n_1+n_2} \sum_{j=k+1}^{n_2} d_{ij}^r & k = 1, 2, \ldots, n_1 - 1 \\
\sum_{i=1}^{n_1} \sum_{j=k+1}^{n_1+n_2} d_{ij}^r, k = n_1r & k = n_1r + 1, \ldots, n_1 + n_2 - 1 \\
\sum_{i=n_1+1}^{n_1+n_2} \sum_{j=1}^{n_1} d_{ij}^r, k = n_1 + n_2r & k = n_1 + n_2r
\end{cases}
\]

2. The relationship between speed, roundtrip time, and the number of ships

The roundtrip time of each voyage of the ship on each route is mainly determined by the water travel time and the berthing time in the port. The specific expression is as follows:

\[
t_r = T_{port} + L_r / 24 \sum_{v=1}^{V} x_{vr} s_v
\]

In the ocean liner route, the frequency of the service is usually arranged on a weekly basis. Therefore, the number of \( v \)-shaped ships that are equipped for route \( r \) can be represented as follows:

\[
n_{vr} = x_{vr} \lceil t_r / 7 \rceil
\]

3. The relationship between the speed and fuel consumption of ships

The fuel cost encompasses the diesel consumption of the auxiliary engine and the heavy oil consumption of the main engine. When comparing ships of the same type, the weekly fuel consumption of the auxiliary engine remains constant, whereas the power of the main engine is roughly proportional to the cube of its speed.

\[
P_M = 0.7355 D^{2/3} s^3 / C
\]

where \( P_M \)(kw) is the actual engine power, \( D(t) \) is the ship’s displacement, and \( C \) is a naval constant.

The daily total oil consumption of a single ship’s main engine is directly proportional to the power of the main engine as follows:

\[
F_f = 24 \times 10^{-3} P_M S = 17.625 \times 10^{-3} gD^{2/3} s^3 / C
\]

where \( g(\text{kg/(kw\cdot h)}) \) refers to the heavy fuel oil consumption rate of the main engine. As the fuel consumption of the main engine is proportional to the cube of speed, let \( k_0 \) be the proportionality coefficient for \( V \)-type ships, then \( F_{f,v} = K_0 s_v^3 \). In this way, the weekly heavy fuel oil consumption of ships can be expressed as \((F_{1,v} + 7K_0 s_v^3)\).

4. The composition of the cost of the fleet
The cost of the liner company’s fleet mainly considers the variable cost, which primarily consists of fuel and operating expenses. The fuel cost encompasses auxiliary diesel oil and main engine heavy oil expenses. In practical operations, if the average weekly auxiliary fuel consumption of various ship types is $F_{1,v}$, the total fuel cost per ship type can be expressed as $(P_1F_{1,v} + 7P_7F_{F_v})$.

The operating expenses of the ships cover crew salaries, daily ship maintenance, and upkeep, management fees, etc. Depending on the number of ships deployed on a route, there can be variations in operating costs. Assuming that the daily operating cost of each type of $V$ ship is $C_v$, the total weekly operating cost of ships on route $r$ is as follows:

$$c_r = \sum_{v=1}^{V} \left( P_1F_{1,v} + 7K_vS_v^3 \right) x_{vr} + \sum_{v=1}^{V} x_{vr}C_v/7$$

(6)

5. The composition of the carbon emissions cost

The ships emit carbon emissions during the process of fuel consumption. In view of the coefficient of conversion from ship fuel to carbon dioxide, this paper transforms the cost of carbon emissions into the cost of carbon tax, as follows:

$$t_s = 56 \times 3.17 \times (F_{1,v} + 7K_vS_v^3)$$

(7)

where 56 is the national tax per ton of carbon dioxide emissions, and 3.17 is the IPCC coefficient, which means that 1 t of marine fuel produces 3.17 t of carbon dioxide.

2.2.2. Model Construction

Based on the above analysis, we established an optimized model of ship allocation on the basis of speed and route. The objective function is as follows:

$$\text{min} W = \sum_{r=1}^{R} \sum_{v=1}^{V} \sum_{r=1}^{V} \left( P_1F_{1,v} + 7K_vS_v^3 \right) x_{vr}n_v + \sum_{r=1}^{R} \sum_{v=1}^{V} x_{vr}C_v/7 + ts$$

(8)

The constraints of the objective function are as follows:

$$\begin{align*}
\sum_{v=1}^{V} x_{vr}C_{ap,v} & \geq Q_k^p, k = 1, 2, \ldots, m_r, \forall r \in \{1, 2, \ldots, R\} \\
t_r & = T_{port} + L_r/24 \sum_{v=1}^{V} x_{vr}s_v, \forall v \in \{1, 2, \ldots, V\}, r \in \{1, 2, \ldots, R\} \\
n_{vr} & = x_{vr} \left[ \frac{V}{7} \right], \forall v \in \{1, 2, \ldots, V\}, r \in \{1, 2, \ldots, R\} \\
S_{min,v} & \leq S_v \leq S_{max,v}, \forall v \in \{1, 2, \ldots, V\}, r \in \{1, 2, \ldots, R\} \\
\sum_{v=1}^{V} x_{vr} & = 1, \forall r \in \{1, 2, \ldots, R\} \\
x_{vr} & \in \{0, 1\}, \forall v \in \{1, 2, \ldots, V\}, r \in \{1, 2, \ldots, R\}
\end{align*}$$

(9)

The specific constraints include the following:

Constraint 1 (cargo-carrying capacity): Ensure that the ship’s carrying capacity on the route is adequate to meet the freight demand of each segment. This prevents cargo backlog or transportation delays by ensuring that the ship can meet the demand for cargo transportation.

Constraint 2 (speed and voyage time): Consider the relationship between the ship’s speed and the time required to complete a voyage. This includes both the port berthing time and sea navigation time to ensure timely arrival at the destination and successful completion of the cargo transportation mission.

Constraint 3 (ship allocation number on route): Ensure that the number of ships deployed on the route matches the desired frequency of departure per week, maintaining the continuity and reliability of transportation services.
Constraint 4 (speed limits): set minimum and maximum speeds for ships to ensure their safe operation and technical compliance.

Constraint 5 (uniform ship types): there is a requirement for consistency in the type of ships used on each route for improved operational efficiency, facilitated management, and consistency in ships.

Constraint 6 (flexible ship type configurations): allows for adjustments to the ship’s configuration based on specific route needs and regulations.

By comprehensively considering these constraints, it is possible to develop a rational and feasible solution process for ship speed and route allocation problems. By balancing these constraints appropriately, an ideal route allocation plan can be developed that minimizes operating costs while meeting practical business and technical constraints.

2.3. Model Solution

The model introduced in this paper exhibits nonlinear mixed integer programming features, encompassing both integer variables (the allocation of ships to routes) and continuous variables (navigation speed). Notably, Equation (8) and constraints 3 and 4 are nonlinear expressions, with constraint 3 requiring rounding up. In practical applications, ship enterprises often face constraints on their number of intercontinental mainline routes and ship type selection. Based on this, this paper suggests a step-by-step approximation algorithm [19] combined with the enumeration strategy to address the challenges encountered during model construction.

The specific process includes the following:

1. The routes and vessel types should be numbered sequentially, and the enumerative method should be used to find all the permutations of R routes based on their serial numbers \((R! = R\cdot (R-1)\cdot \ldots \cdot 1))\). Beginning with permutation 1, for each permutation \(j (j \in R!))\), all the routes should be equipped with vessels in the order of route numbers and in the order of vessel type numbers.

2. In the selected sequential order of arrangement \(a\), the sequentially selected \(V\)-shaped ships are configured based on the sequentially selected \(r\) route. Starting from the maximum speed \(S_{\text{max},v}\), let \(S_v = S_{\text{max}}\) and use constraint 3 to calculate the number of \(V\)-shaped ships required on the route, \(n_{m,pr}\). According to constraint 1, it is judged whether the configuration of \(V\)-shaped ships on this route can meet the weekly freight capacity. If it is not satisfied, proceed to step 5; if it is satisfied, use \((P_1F_{1,r} + 7P_jF_{j,r})\) to obtain the total cost of this route \(c_{m,pr}\), and note \(c_{a^s,pr} = c_{m,pr}, s_{a^s,pr} = s_v, n_{a^s,pr} = n_{m,pr}\).

3. Let \(s_0 = s_v - 0.1\); use constraint condition 3 to calculate the quantity \(n_{m,v}\) of this type of ship required on the route, and use Equation (6) to calculate the total cost \(C_{s,r}\) of the route. If \(c_{s,r} < c_{j,s,r}\), then let \(c_{a^s,r} = c_{s,r}, s_{a^s,r} = s_v, n_{a^s,r} = n_{m,pr}\). Repeat this step until \(s_0 = S_{\text{min,v}}\).

4. According to step 2, the ships of the following route in the arrangement are allocated. If the preceding route has been allocated with type \((v+1)\) ships, the allocation continues on this route until all routes in the arrangement are completed. At this time, the total operation cost of the ships on each route \(r\) in arrangement \(a\) is \(c_{a^s,r}\), and the speed of ship type \(v\) is \(s_{a^s,pr}\). Let \(Z_{a^s} = \sum_{r \in K} c_{a^s,r}\), and proceed to step 6.

5. Renumber ship types \(v+1, \ldots, V\) as \(v, \ldots, V-1\), Re-number \(v\) as \(V\) and move to step 2.

6. Let \(a = a + 1\). If \(a \leq R!\), then proceed to step 2; otherwise, record \(a^* = a_0|a_0 = \min\{Z_{a,pr} | \in R!\}\); \(a^*\) is the optimal permutation, and its corresponding ship speed is the optimal speed. The route ship allocation scheme is the optimal scheme. Thus, the algorithm ends.

3. Simulation Analysis and Verification

3.1. Example Analysis

Based on reference [20], the mathematical model is simulated using Matlab 2023 software. This paper selects three Asia–Europe routes operated by a certain company to find the optimal route allocation and speed plan. The specific port call sequence is shown...
in Table 2. The corresponding freight demand and relevant parameters of ships for each segment of the three routes are shown in Tables 3 and 4.

Table 2. Distance of routes in order of port calls.

<table>
<thead>
<tr>
<th>Shipping Route</th>
<th>Total Torque</th>
<th>Port of Call</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21,036</td>
<td>Hong Kong → Xiamen → Kaohsiung → Yantian → Singapore → Rotterdam → Hamburg → Felixstowe → Antwerp → Singapore → Hong Kong</td>
</tr>
<tr>
<td>2</td>
<td>23,281</td>
<td>Yantian → Hong Kong → Tianjin → Dalian → Qingdao → Ningbo → Singapore → Rotterdam → Felixstowe → Hamburg → Antwerp → Yantian</td>
</tr>
<tr>
<td>3</td>
<td>21,870</td>
<td>Yantian → Kaohsiung → Ningbo → Shanghai → Hong Kong → Singapore → Rotterdam → Hamburg → Antwerp → Singapore → Yantian</td>
</tr>
</tbody>
</table>

Table 3. The freight volume of each route segment.

<table>
<thead>
<tr>
<th>Shipping Route</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 1</td>
<td>3553</td>
<td>4401</td>
<td>4635</td>
<td>5369</td>
<td>6313</td>
<td>5984</td>
<td>5759</td>
<td>4989</td>
<td>3650</td>
<td>2990</td>
<td>0</td>
</tr>
<tr>
<td>Route 2</td>
<td>7415</td>
<td>6863</td>
<td>6240</td>
<td>5563</td>
<td>5099</td>
<td>4516</td>
<td>4555</td>
<td>5541</td>
<td>6373</td>
<td>7056</td>
<td>7957</td>
</tr>
<tr>
<td>Route 3</td>
<td>3947</td>
<td>4352</td>
<td>4648</td>
<td>5123</td>
<td>5332</td>
<td>6293</td>
<td>6119</td>
<td>5380</td>
<td>4105</td>
<td>3308</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Vessel parameters.

<table>
<thead>
<tr>
<th>Ship Type</th>
<th>Container Capacity THU</th>
<th>Weekly Operating Costs</th>
<th>Fuel Consumption Constant</th>
<th>Weekly Diesel Fuel Consumption/t</th>
<th>Minimum Speed/kn</th>
<th>Maximum Speed/kn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8000</td>
<td>332,000</td>
<td>0.0068</td>
<td>82</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>9500</td>
<td>345,000</td>
<td>0.0072</td>
<td>85</td>
<td>14</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>7000</td>
<td>310,000</td>
<td>0.0065</td>
<td>80</td>
<td>16</td>
<td>30</td>
</tr>
</tbody>
</table>

Given that the berthing duration of all ports is two days, and the prices of heavy fuel oil and gasoline are USD 350/ton and USD 650/ton, respectively, the ship allocation plans for the routes shown in Table 5 can be obtained under the assumption of ignoring the actual restrictions on the freight volume of each route segment.

From Table 5, it is evident that while scheme d can minimize the overall operation cost of the ships, the third ship type with a capacity of 6500 TEU, as mentioned in Table 3, falls short of meeting the maximum freight demand of 7957 TEU on route 2. Hence, this scheme is not feasible. Similarly, in scheme a, the second ship type with a capacity of 7000 TEU cannot cater to the transportation requirements on route 2. Moreover, both scheme c and scheme e encounter the same problem. Among the feasible options, scheme b totals an operation cost of USD 13.857 million, while scheme f amounts to US 13.818 million. Both schemes incur a carbon tax of USD 179,000, indicating identical carbon emissions. Consequently, scheme f is identified as the optimal ship allocation scheme.

In scheme f, route 1 is equipped with 11 type 2 ships with a container capacity of 7000 TEU and the best speed of 17.7 knots; route 2 is equipped with 10 type 1 ships with a container capacity of 8000 TEU and the best-expected speed of 17.6 knots; route 3 is equipped with 10 type 3 ships with a container capacity of no less than 6500 TEU and the best speed of about 18.3 knots. Compared with scheme b, scheme f saves USD 39,000 in total weekly operating costs, reflecting the economic benefits of speed and route allocation.
Table 5. Allocation of route vessels.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Shipping Route</th>
<th>Ship Type</th>
<th>Speed of Ship</th>
<th>Quantities</th>
<th>Carbon Tax/USD 10,000</th>
<th>Fuel Cost/USD 10,000</th>
<th>Airline Operating Costs/USD Million</th>
<th>Total Operating Costs/USD Million</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2</td>
<td>2</td>
<td>17.7</td>
<td>11</td>
<td>5.8</td>
<td>150.1</td>
<td>493.9</td>
<td>1382.7</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>1</td>
<td>18.3</td>
<td>10</td>
<td>6.6</td>
<td>155.4</td>
<td>493.1</td>
<td>1385.7</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>2</td>
<td>17.6</td>
<td>10</td>
<td>5.7</td>
<td>135.0</td>
<td>447.8</td>
<td>1377.4</td>
</tr>
<tr>
<td>d</td>
<td>1</td>
<td>1</td>
<td>17.6</td>
<td>10</td>
<td>6.6</td>
<td>155.4</td>
<td>493.1</td>
<td>1377.1</td>
</tr>
<tr>
<td>e</td>
<td>2</td>
<td>2</td>
<td>17.6</td>
<td>10</td>
<td>5.7</td>
<td>155.4</td>
<td>493.1</td>
<td>1385.2</td>
</tr>
<tr>
<td>f</td>
<td>1</td>
<td>2</td>
<td>17.7</td>
<td>11</td>
<td>5.8</td>
<td>150.1</td>
<td>493.9</td>
<td>1381.8</td>
</tr>
</tbody>
</table>

Through the above case, it can be seen that the ship speed and route allocation model constructed in this paper accurately reveals the internal relationships between ship speed, fuel cost, ship speed changes, and route allocation effectiveness. At the same time, it highlights the significant impact of ship speed changes on route allocation outcomes. The adopted solution method is simple and efficient and can meet the practical needs of liner companies for ship speed and route allocation management on ocean routes.

3.2. Fuel Sensitivity Analysis

Based on the ship allocation results of the route, this paper analyzes fuel sensitivity. Taking the second type of ship configured in route 1 as the research object, under the assumption that the fluctuation range of diesel and heavy oil prices is both 10%, the fuel sensitivity change is shown in Figure 3. The influence of fuel price on shipping factors is shown in Figure 4.

Figure 3. Fuel sensitivity changes.
Figure 4a,b clearly indicate that as fuel prices rise, the optimal speed of liner shipping remains consistent, though there is a noticeable slowdown trend. Additionally, while the optimal number of ships remains stable, there is a general upward trend. Furthermore, when the optimal speed and number of ships are held constant, the fuel cost increases correspondingly with the rise in fuel prices. It is noteworthy that, at the point where the optimal speed begins to decrease while the optimal number of ships starts to rise, the increase in fuel prices actually results in a decrease in fuel costs.

Figure 4c clearly illustrates the changes in carbon tax, total weekly operating expenses for ships, and speed under various fuel prices. As fuel prices increase, the optimal speed of liner ships remains relatively consistent within a specific range, but there is a general trend towards a gradual decrease in speed. Furthermore, the cost of carbon tax decreases gradually as the speed decreases.

Similarly, Figure 4d shows the changes in carbon tax, the total weekly operating cost of ships, and the number of ships under different fuel prices. The optimal fleet size remains stable within a certain range, and the general trend is to increase with the rise in fuel prices. Conversely, the carbon tax cost gradually decreases. At the inflection point where the optimal fleet size increases, there is a situation where fuel prices rise while carbon tax costs decrease.

4. Conclusions

This paper focuses on the optimization of the operation of the ocean container fleet, considering multiple constraints such as cargo capacity, speed, journey time, and the number of ships and speed. A nonlinear mixed integer model based on double-level programming is constructed. This model not only reveals the inherent connection between speed, fuel cost, and route allocation results but also highlights the significant impact of speed changes on route allocation results. To demonstrate the model’s advantages, this paper takes three Asia–European routes as examples and obtains the optimal route
allocation plan and optimal speed. For example, in speed and route allocation, scheme 6 saves USD 39,000 in total weekly operating costs.

Through fuel sensitivity analysis, it can be seen that as fuel prices rise, the optimal speed of liner ships gradually slows down, and the number of ships gradually increases. Carbon tax costs should be adjusted accordingly. This provides an important reference for decision makers to adjust route allocation and speed strategies under different market conditions.

In view of the importance of green, low-carbon shipping in future development, this paper only considers carbon tax as a variable. Therefore, in the process of building an optimized model of container ship operating costs, it should be combined with environmental sustainability (low carbon), development, and other factors for further research.

**Author Contributions:** Conceptualization, P.X. and H.W.; Methodology, P.X. and H.W.; Software, P.X. and H.W.; Validation, P.X. and H.W.; Formal analysis, P.X. and H.W.; Investigation, P.X. and H.W.; Resources, P.X. and H.W.; Data curation, P.X. and H.W.; Writing—original draft, P.X.; Writing—Review and editing, H.W.; Visualization, P.X. and H.W.; Supervision, P.X. and H.W.; Project administration, H.W. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Data are contained within the article.

**Conflicts of Interest:** The authors declare no conflicts of interest.

**References**


5. Ma, M.; Fan, H.; Jiang, X.; Guo, Z. Truck arrivals scheduling with vessel dependent time windows to reduce carbon emissions. *Sustainability* 2019, 11, 6410. [CrossRef]

6. Yang, L.; Chen, G.; Zhao, J.; Rytter, N.G.M. Ship speed optimization considering ocean currents to enhance environmental sustainability in maritime shipping. *Sustainability* 2020, 12, 3649. [CrossRef]


13. Song, D. A literature review, container shipping supply chain: Planning problems and research opportunities. *Logistics* 2021, 5, 41. [CrossRef]


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