

Article

Numerical Investigation of the R-Curve Effect in Delamination of Composite Materials Using Cohesive Elements

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Abstract: This paper presents a numerical investigation of the R-curve effect in delamination propagation in composite materials. The R-curve effect refers to the phenomenon whereby resistance to crack propagation increases with the advancement of the delamination, due to toughening mechanisms, such as fiber bridging. Numerical models often neglect this effect assuming a constant value of the fracture toughness. A numerical approach based on cohesive elements and on the superposition of two bilinear traction-separation laws is adopted here to accurately predict the R-curve effect in skin-doubler composite specimens subjected to three-point bending tests. The carbon-epoxy material presents two different sensitivities to the fiber bridging phenomenon resulting in two different R-curves. Comparisons with literature experimental data, in terms of load and delaminated area vs. applied displacement, and ultrasonic C-scan images show the effectiveness of the adopted approach in simulating the R-curve effect. The predicted numerical stiffness aligns with the experimental scatter, although the maximum load is slightly underestimated by approximately 15% compared with the average experimental results. The numerical model accurately predict the R-curve effect observed in the experimental data, demonstrating a 31% increase in the maximum load for the material configuration exhibiting greater sensitivity to fiber bridging.

Keywords: composite materials; crack propagation; finite element analysis; cohesive elements; R-curve



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1. Introduction

Composite materials play a crucial role in aerospace applications due to their exceptional strength-to-weight ratio, corrosion resistance, and design flexibility. They are extensively used in aircraft structures, such as wings and fuselages, to reduce weight, improve fuel efficiency, and enhance performance. However, one of the significant challenges associated with composite materials in aerospace is the occurrence of delamination. Delamination refers to the separation of layers within a composite laminate, which can weaken the structure and compromise its integrity. Investigating the problem of delamination is essential because often this type of damage is not visible and can propagate under service loads leading to catastrophic failures, especially in critical components subjected to high mechanical loads and environmental stresses [1]. Understanding the mechanisms and factors influencing delamination is crucial for developing effective preventive measures and design guidelines to ensure the safety and reliability of composite structures in aerospace applications. By addressing the issue of delamination, aerospace engineers can optimize composite material performance, extend component lifespan, and enhance overall aircraft safety [2,3].

Unlike traditional homogeneous materials, composite materials exhibit non-linear fracture behavior due to their heterogeneous microstructure and complex damage mechanisms. Delamination propagation is influenced by mechanisms such as fiber bridging, matrix toughening, and fiber pull-out, which contribute to energy dissipation and crack deviation, thus enhancing the material's resistance to fracture and playing a crucial role

in determining the fracture behavior and overall durability of composite structures [4]. Fiber bridging represents the main toughening mechanism, it occurs when reinforcing fibers bridge across the delaminated interface, effectively resisting crack propagation, and enhancing the material's fracture toughness, the energy needed to advance a pre-existing delamination within a composite laminate [5–7]. The R-curve refers to the resistance curve, representing the relationship between crack extension and fracture toughness. The R-curve effect indicates that as the crack propagates, the fracture toughness increases until it reaches a steady-state value, reflecting the material's ability to resist crack growth [8]. Understanding and characterizing the R-curve behavior is essential for predicting the structural integrity and failure behavior of composite materials under various loading conditions, facilitating the design of robust and reliable composite structures for aerospace applications.

The standard experimental tests used to determine the fracture toughness for composite materials are intended for delamination propagation in unidirectional specimens, where the fiber bridging and the R-curve effect is often negligible [9,10]. However, in real structural applications, delamination often occurs between plies oriented at different angles, resulting in a range of damage mechanisms, mainly fiber bridging, that contribute to an increase in fracture toughness as the delamination advances [11,12]. Several works can be found in the literature that address the R-curve effect on the delamination propagation both numerically and experimentally. In [13–15] researchers experimentally studied the effect of the fiber orientation at the interface on the delamination propagation in standard Double cantilever Beam (DCB), End-Notched Flexure (ENF), and Mixed-Mode Bending (MMB) specimens, achieving consistent values of the fracture toughness at the onset, while significantly different R-curve behavior, with large increases in the fracture toughness as the delamination advances.

The numerical approaches adopted to simulate delamination propagation in Finite Element (FE) analysis typically use the value of the fracture toughness measured at the onset of the delamination, underestimating the load-carrying capability of the structure and resulting in conservative designs, as the increasing resistance of the delamination, the R-curve effect, is ignored [16].

Delamination propagation can be numerically simulated in FE analysis using approaches based on fracture mechanics, such as the Virtual Crack Closure Technique (VCCT) [17,18], or damage mechanics, such as the Cohesive Zone Modelling (CZM) [19–21]. In the last decades, both these approaches have been modified to take into account the R-curve effects. Several researchers have demonstrated the possibility of using VCCT-based methodologies by modifying the fracture toughness value used in the propagation criteria following the experimentally determined curves of the fracture toughness as function of the crack opening [22–24]. These approaches can be easily incorporated in a FE analysis, however their applicability is limited to scenarios where the location of the initial delamination front and the direction of propagation are known a priori.

On the other hand, conventional cohesive zone model (CZM) using a bilinear traction-separation law has been developed and extensively utilized in numerical simulations to study delamination initiation and growth, without considering fiber bridging. Subsequently, various modifications to the traction-separation law have been proposed to consider the additional fiber bridging effect based on the R curve obtained experimentally. Dávila et al. [25–27] proposed a tri-linear law, obtained from the combination of two bilinear cohesive laws to simulate the toughening mechanisms during the propagation of the delamination. This approach has been adopted with different modifications by other researchers [28,29] demonstrating its effectiveness in considering the effect of the R-curve in standard specimens. Although the use of cohesive elements is more computationally expensive compared to VCCT, they enable the simulation of delamination initiation without the need to define an initial crack tip.

Approaches based on the cohesive zone model to represent the R-curve effect have been mainly applied to standard specimens, such as the Double Cantilever Beam (DCB). The objective of this research is to demonstrate the applicability of this numerical technique

to a more complex specimen. In this work, the effect of R-curve on the delamination propagation in skin-doubler specimens, representative of the stringer flange of stiffened composite panel, subjected to three-point bending test, is numerically investigated using a superposition of two bilinear cohesive law. The specimens were manufactured with the same material exhibiting two different fracture behavior: one with limited R-curve effect, indicated as “Low Bridging”, and the other with a significant increase in the fracture toughness as the delamination propagate, indicated as “High Bridging”. The numerical results are compared with the experimental data in terms of ultrasonic C-scan images and load vs. applied displacement curves obtained in [23]. The comparisons validate the effectiveness of the adopted approach in replicating the increase in strength due to the R-curve effect observed experimentally and in predicting the evolution of the delaminated area as a function of the applied displacement.

In Section 2, the theoretical background of the cohesive zone model employed to represent the R-curve effect is introduced. In Section 3, the geometrical characteristics of the skin-doubler specimens are described together with the details of the FE model, while in Section 4, the calibration of the cohesive zone model parameters is performed for the materials under consideration. Finally, in Section 5, the numerical results are presented and compared with the experimental data.

2. Cohesive Zone Model

Cohesive elements are widely employed in the simulation of delamination propagation in composite materials. Cohesive elements model the interaction between adjacent layers by defining the traction-separation relationship along the delaminated interface. These elements capture the nonlinear behavior of the interface, allowing for accurate simulation of crack initiation and propagation.

The behavior of cohesive elements is governed by the cohesive law [30], which defines the traction-separation relationship along the interface or crack. The cohesive law describes how the traction (force per unit area) between the surfaces changes as the separation (relative displacement) between them increases. Different models can be found in literature, but the simplest and most widely used employs a bilinear traction-separation law, as shown in Figure 1.

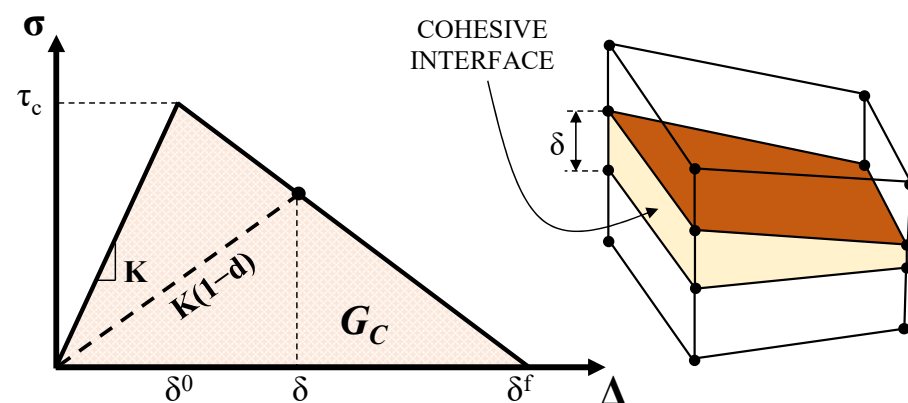


Figure 1. Constitutive response for traction-separation cohesive elements.

At low levels of separation (δ), the cohesive traction rapidly grows following the penalty stiffness (K), which represents the strong bonding between the laminae. As the separation increases, the stress reaches the interface strength of the material (τ_c) at the damage initiation displacement (δ^0) and the cohesive traction starts to decrease, indicating the progressive degradation of the interface and the opening of the crack. When the traction becomes zero, at the final displacement (δ^f), the element completely fails to represent the

propagation of the crack. The degradation of the interface stress is driven by the damage variable, d , which can be calculated using Equation (1):

$$d = \frac{\delta f (\delta - \delta^0)}{\delta (\delta f - \delta^0)} \quad (1)$$

The integral of the traction-separation curve represents the fracture toughness of the material (G_C).

In a delamination propagation analysis, the length of the process zone (LPZ) [26], l_{pz} , is defined as the distance between the crack tip and the location where the maximum cohesive stress occurs. The LPZ is a material property that depends on the response of the different material to the presence of a crack. The value of l_{pz} can be estimated for mode I loading conditions as follow:

$$l_{pz} = \gamma \frac{EG_C}{(\tau_C)^2} \quad (2)$$

where, E is the transversal elastic modulus and γ is a nondimensional parameter that depends on the elasto-plastic response of the material. Different models are available in literature to estimate the value of the parameter γ , in this work a value of $\gamma = 0.884$ has been adopted [30].

The conventional assumption is that the fracture toughness of a composite material remains constant and independent of the crack length, and mechanisms like fiber bridging, which can modify fracture propagation characteristics, are frequently neglected. However, this assumption is only valid for brittle crack propagation where the size of the process zone (l_{pz}) is negligible compared to the others characteristic dimensions, like the crack length. In practice, as the crack develops, due to the already mentioned toughening mechanisms, the formation of the process zone behind the crack tip is gradual, resulting in an increase of the fracture toughness until its value stabilize (Figure 2).

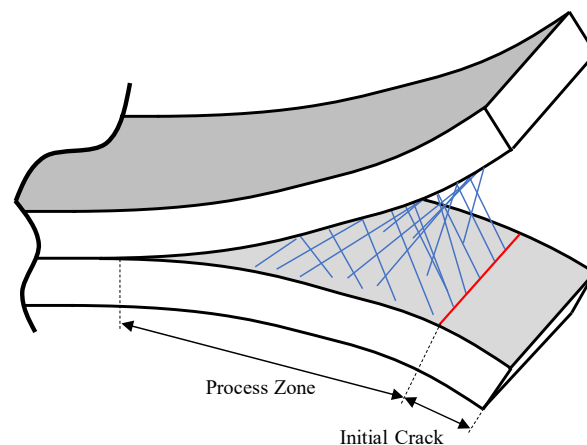


Figure 2. Fiber-Bridging representation.

Under the assumption that the cohesive stresses are independent from the profile of the crack opening and are a function of only the distance from the crack tip, an approximate expression for the resistance curve is developed in [5] using a linear idealization:

$$G_R = \begin{cases} G_T + G_b \frac{\Delta a}{l_{pz}} \left(2 - \frac{\Delta a}{l_{pz}} \right) & \text{for } \Delta a < l_{pz} \\ G_T + G_b & \text{for } \Delta a \geq l_{pz} \end{cases} \quad (3)$$

where, G_T , is the fracture toughness related with the crack tip, G_B , is the fracture toughness related to the toughening mechanisms happening during the development of the crack, G_R , represents the steady-state value of the fracture toughness, and Δa is the extension of the crack. The approximated R-curve is represented in Figure 3.

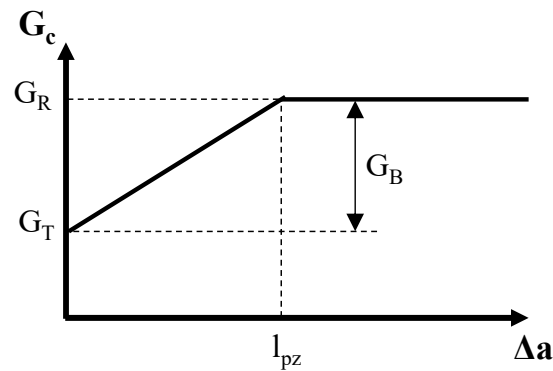


Figure 3. Linear idealization of the R-curve.

Considering Equation (3) and Figure 3, it is evident that if $l_{pz} \ll \Delta a$ the R-curve effect can be ignored with the assumption that all the toughening mechanisms are happening at the crack tip.

In order to represent this behavior using cohesive elements, Dávila et al. [25–27] proposed to use of a superposition of two bilinear softening laws, resulting in a trilinear traction-separation curve, as shown in Figure 4.

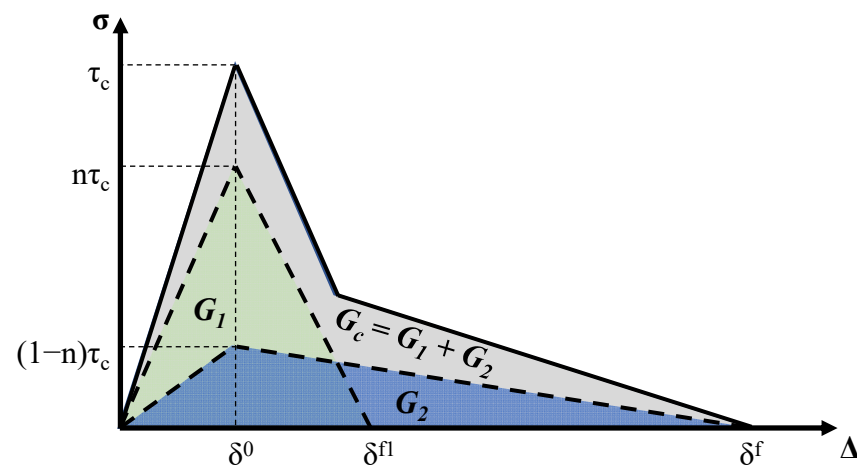


Figure 4. Superposition of two bilinear cohesive laws.

The two cohesive laws represent the two phenomena happening at the crack tip during the propagation of a delamination: the first is characterized by high strength and lower maximum displacement indicating the brittle fracture of the crack, while the other, with low strength and high final displacement is representative of the toughening mechanisms, such as fiber bridging.

The strength and critical energy release rate of the two cohesive laws are defined as follow:

$$\begin{aligned} \tau_{c1} &= n\tau_c, & \tau_{c2} &= (1-n)\tau_c \\ G_1 &= mG_c, & G_2 &= (1-m)G_c \end{aligned} \quad (4)$$

where, the coefficient m is obtained considering the ratio of the fracture toughness (G_1/G_c), while the coefficient n is calculated considering the relationship between the characteristic length of the process zone for the trilinear cohesive law and the two bilinear cohesive laws, as shown in Equation (5) [26].

$$n = 1 - \frac{2}{3}\gamma \frac{1-m}{l_{pz}} \frac{EG_c}{(\tau_c)^2} \quad (5)$$

3. Specimen Description and FE Model

The numerical model described in the previous section is applied here to replicate the experimental results of skin-doubler specimens subjected to a three-points bending test [23]. The specimens are made of a 200 mm $\times$ 100 mm skin with a 70 mm long doubler on top representing the stringer flange (Figure 5). The skin and the doubler are made of 8 plies of carbon-epoxy with the stacking sequence $[45/0/-45/90]_s$, and a ply thickness of 0.1875 mm. The total thickness of the skin and the doubler is 1.5 mm. The material properties adopted in the numerical analysis are reported in Table 1.

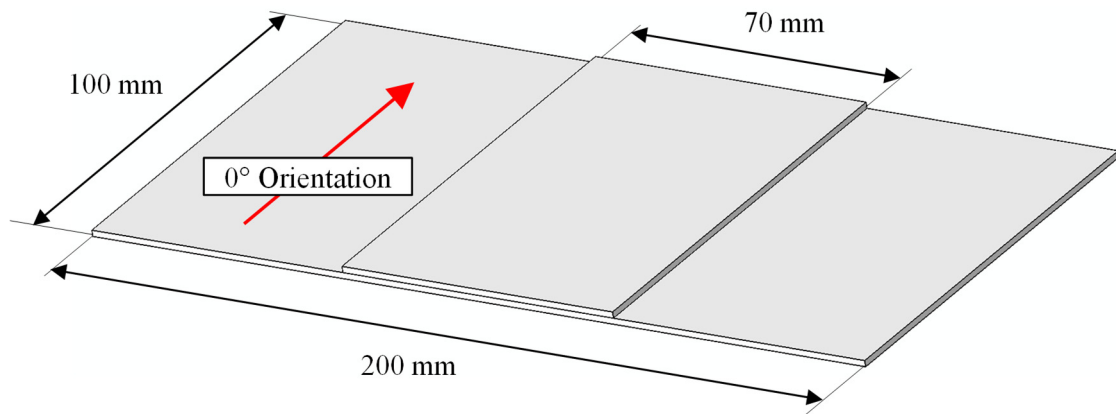


Figure 5. Specimen geometry.

Table 1. Material properties [23].

Lamina Properties		
E_1	[MPa]	147,000
$E_2 = E_3$	[MPa]	8500
$G_{12} = G_{13}$	[MPa]	4500
G_{23}	[MPa]	4000
$\nu_{12} = \nu_{13}$		0.36
ν_{23}		0.45
τ_C	[MPa]	40
G_{2C}	[J/m ²]	514

The model is discretized in the Finite Element (FE) code ABAQUS [31] using a combination of continuum shell elements (SC8R) for the composite skin and the doubler, and zero-thickness cohesive elements (COH3D8) at the interface to simulate the initiation and growth of the delamination. The properties of the composite laminate are assigned to both the skin and the doubler using the Composite Layup functionality of ABAQUS, which allow to define the thickness and orientation of each lamina within the laminate. A global element size of 1 mm is adopted for the entire model, while a more detailed discretization is implemented in the region potentially interested by the propagation in order to have an accurate representation of the cohesive process zone. According to [30], a minimum of 5 elements is required to accurately represent the stress variation within the cohesive process zone. Using Equation (2) and the properties of the material under investigation, the length of the process zone is estimate to be approximately 1 mm. Therefore, to maintain a conservative approach, an element length of 0.1 mm is selected for the cohesive elements. The total number of elements in the FE model is approximately 220,000. The skin and the doubler are connected through shared nodes between the cohesive layer and the two plates. The nodes on top of the cohesive elements are shared with the elements of the doubler, while the nodes on the bottom are shared with the nodes of the skin.

To model the trilinear traction-separation law, the zero-thickness cohesive elements at the interface are doubled, resulting in two bilinear cohesive elements connected to the same nodes, as graphically represented in Figure 6.

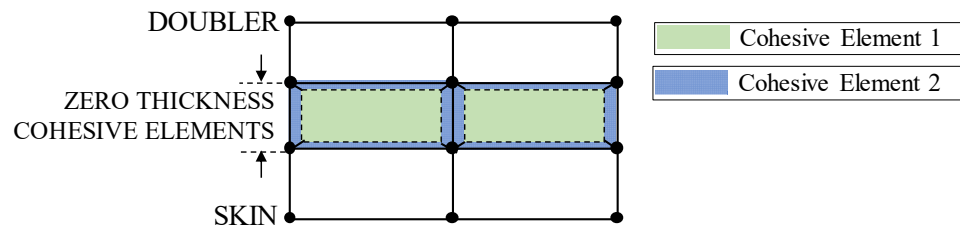


Figure 6. Superposition of two bilinear cohesive elements.

Considering the symmetry of the specimen and of the loading and boundary conditions, in the numerical analysis the delamination will start from both the edges of the doubler at the same time. However, during the experimental tests it can be observed that the damage always initiates from one of the corners, due to microstructural defects, and propagates toward the center of the specimen until the global collapse. To numerically represent this phenomenon, the delamination is allowed to propagate from one side of the specimen, while on the other side, only the elastic properties are assigned to the cohesive elements. The FE model and the stacking sequence are shown in Figure 7.

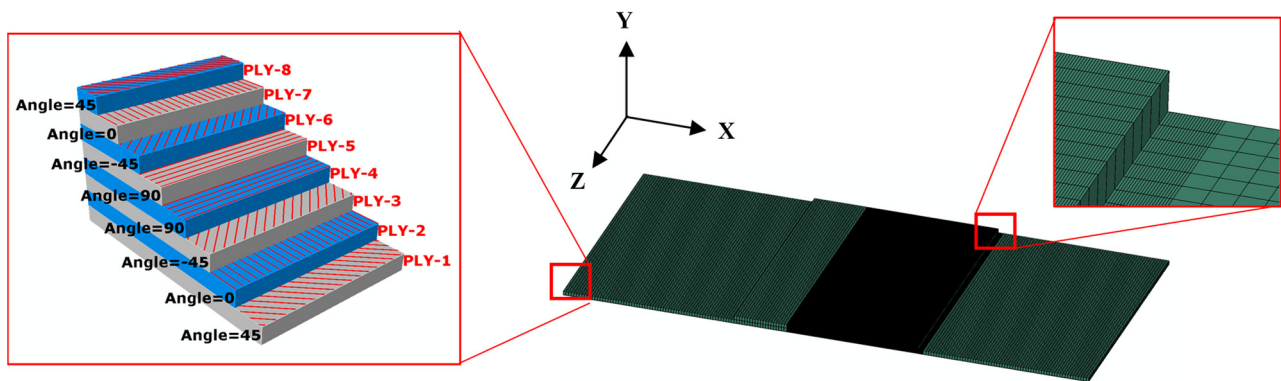


Figure 7. Details of the FE Model.

In the experimental tests, the specimens are loaded and fixed using cylindrical supports located at a distance of 100 mm. In the numerical model, to reduce the computational times, the supports are not explicitly modelled, but appropriate boundary conditions are introduced to represent them. In particular, the vertical displacements of the nodes under the specimen located on the supports are blocked, while, for the nodes in the centerline of the specimen, the lateral displacements are fixed, and the load is applied imposing a vertical displacement of 10 mm. The boundary conditions are shown in Figure 8.

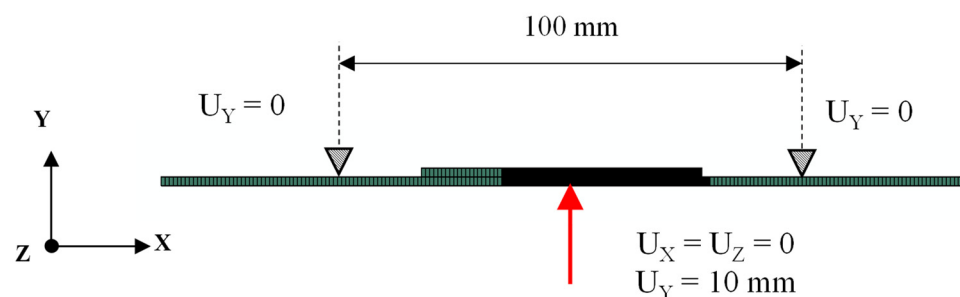


Figure 8. Boundary conditions.

4. Model Parameters Calibration

The carbon-epoxy reinforced composite under investigation exhibits two different toughening behavior under mode I loading conditions, according to the curing process, as reported in Figure 9, where the two experimentally determinate R-curves are reported.

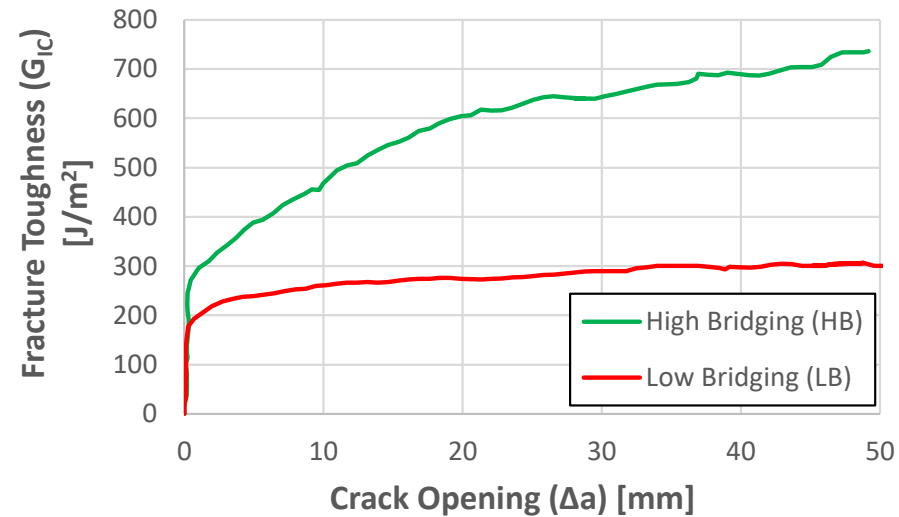


Figure 9. Fracture toughness vs. crack opening.

From Figure 9, it is possible to distinguish two different trends: for the material identified as “Low Bridging (LB)”, the fracture toughness experiences a limited growth with the delamination size, showing a reduced sensitivity to the fiber bridging. On the other hand, the “High Bridging (HB)” material shows a strong increase in the fracture toughness as the crack propagates, representative of a high sensitivity to the fiber bridging, stabilizing at a final value more than double the initial fracture toughness.

Using the approach described in Section 2, the two R-curves have been linearly idealized, as shown in Figure 10.

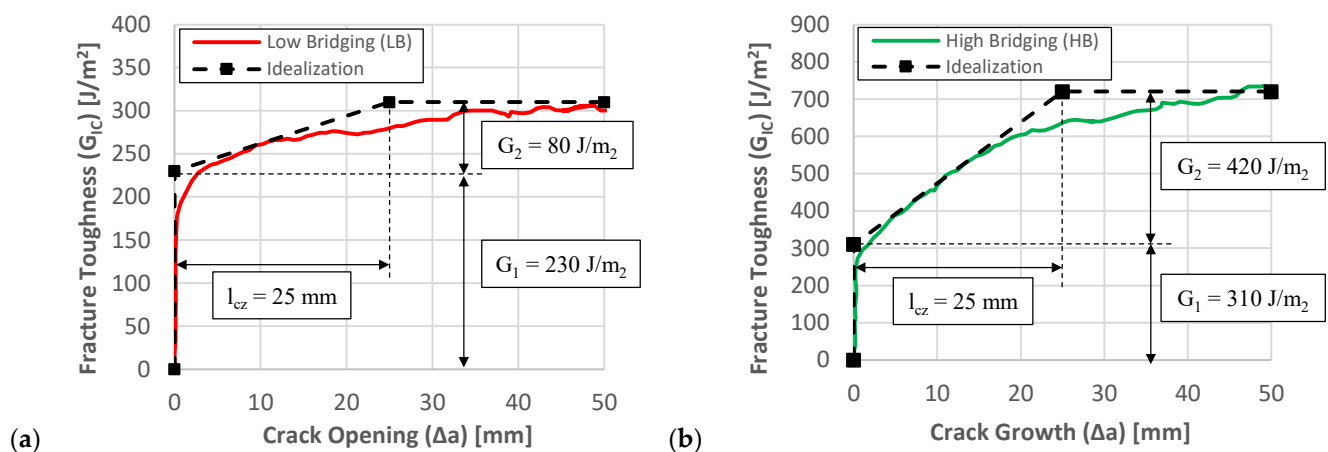


Figure 10. Linear idealization of the R-curve for: (a) “Low Bridging (LB)”; (b) “High Bridging (HB)”.

From the idealization performed in Figure 10, it is possible to derive the parameters of the two bilinear cohesive laws for both the “Low Bridging (LB)” and the “High Bridging (LB)” material configurations, using Equations (4) and (5). The values obtained are reported in Table 2.

Table 2. Cohesive laws parameters.

Low Bridging (LB)				High Bridging (HB)			
n		0.9899		n		0.9479	
m		0.7419		m		0.4305	
G_1	[J/m ²]	230		G_1	[J/m ²]	310	
τ_{1c}	[MPa]	39.59		τ_{1c}	[MPa]	37.91	
G_2	[J/m ²]	80		G_2	[J/m ²]	420	
τ_{2c}	[MPa]	0.40		τ_{2c}	[MPa]	2.08	

5. Numerical Results and Experimental Comparisons

Non-linear static analysis have been performed for the two material configurations under displacement-controlled loading conditions to replicate the experimental tests, using the cohesive parameters obtained in the previous section. The deformed shape with the out-of-plane displacements contour plot is reported in Figure 11 at different values of the applied displacement for the model with the “Low Bridging (LB)” material configuration.

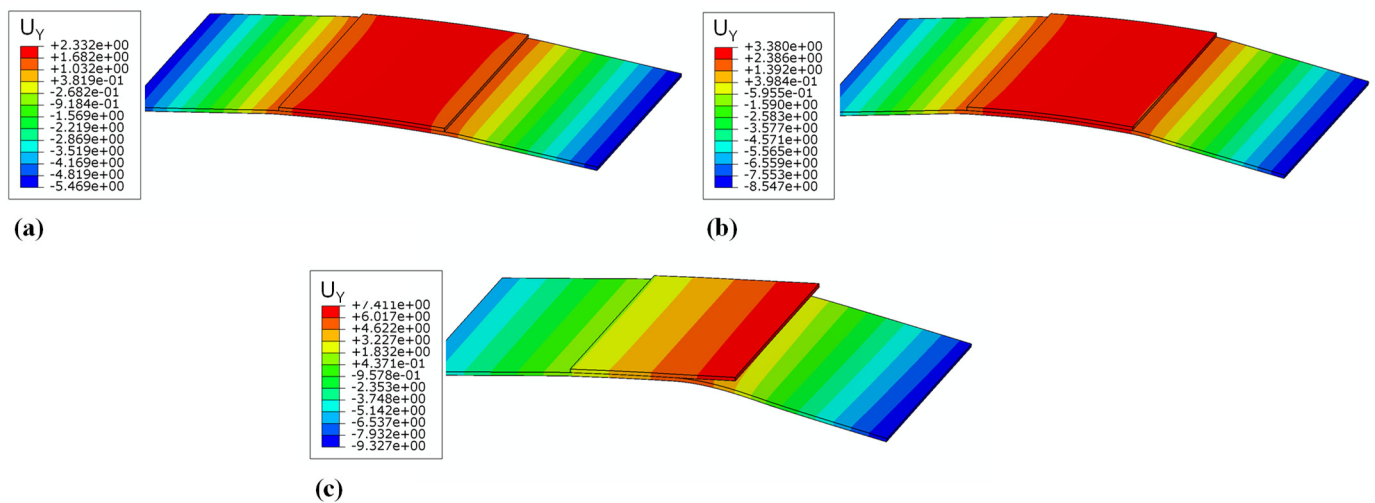


Figure 11. Deformed shape with out-of-plane displacements contour plot at an applied displacement of: (a) 2.31 mm; (b) 3.33 mm; (c) 4.68 mm (“Low Bridging”).

The delamination initiates from the right edge of the doubler at an applied displacement of 2.31 mm (Figure 11a). As the damage propagates, the displacement distribution loses its symmetry, as shown in Figure 11b where the deformed shape at the maximum load is presented. When the doubler is almost completely separated from the skin (Figure 11c), the specimen fails. An identical sequence of events occurs for the “High Bridging (HB)” material configuration, with the delamination propagation delayed due to the higher fracture toughness.

The numerical results in terms of load vs. applied displacement for the “Low Bridging (LB)” and “High Bridging (HB)” material configurations are compared with the data of the experimental tests [23] in Figure 12a,b, respectively. For each material configuration, three specimens were tested, and the displacements were measured using two laser sensors (L1 and L2) positioned close to the center of the specimen.

For both the models, the numerical stiffness is slightly overestimated, although it still falls within the range of the experimental data. The reason is probably related to the numerical boundary conditions, where the displacement in the vertical direction of the nodes located on the supports is blocked. During the experimental tests these nodes can slide on the supports, resulting in a higher compliance of the structure.

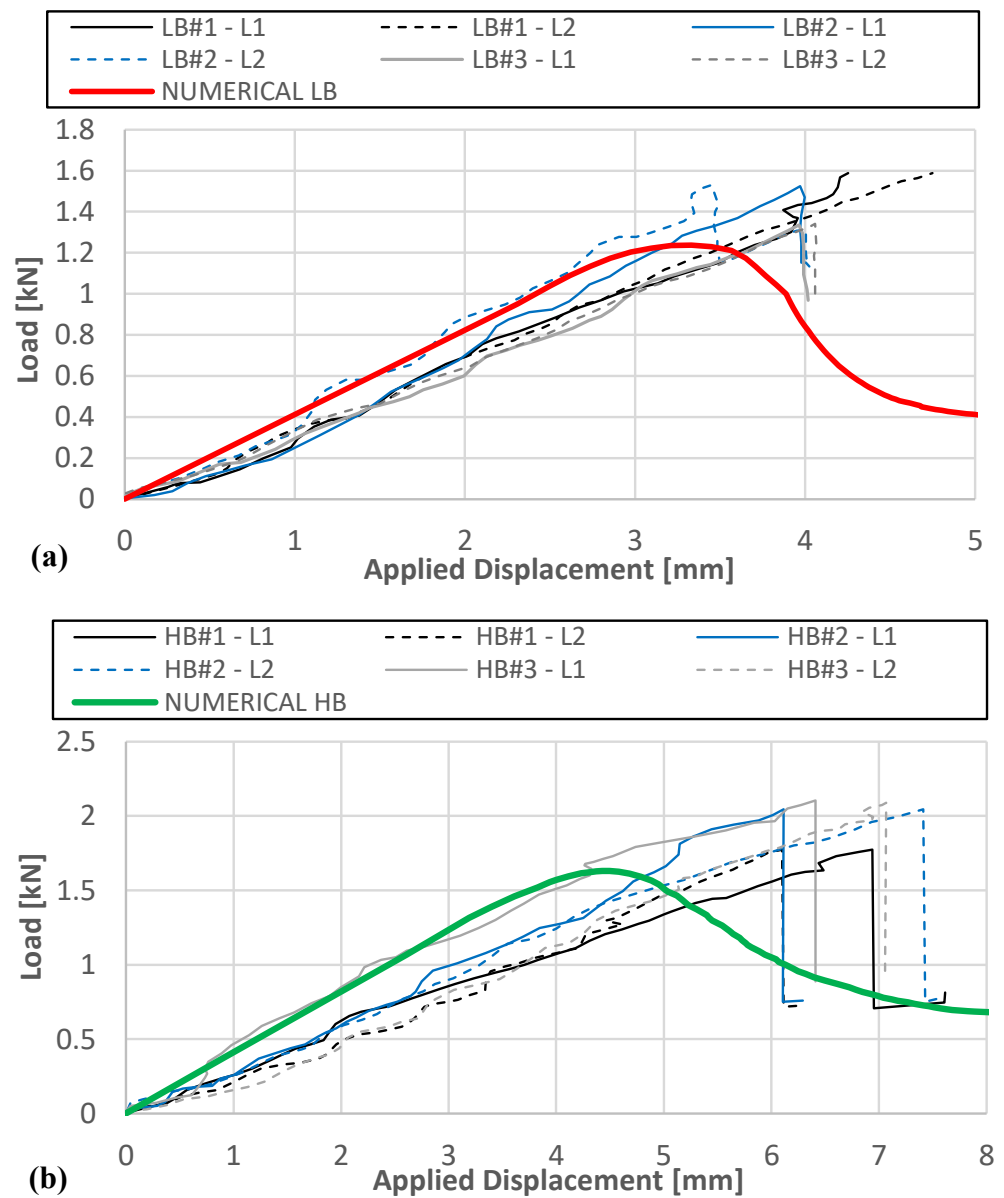


Figure 12. Numerical-experimental Comparisons of the load-displacement curves for: (a) “Low Bridging”; (b) “High Bridging”.

The effect of the different R-curves of the two material configurations is clearly visible observing the comparisons of the maximum load between the experimental tests and the numerical models, reported in Table 3.

Table 3. Experimental and Numerical maximum loads.

Low Bridging (LB)			High Bridging (HB)		
EXP	LB#1	1.58 kN	EXP	HB#1	1.77 kN
	LB#2	1.52 kN		HB#2	2.04 kN
	LB#3	1.30 kN		HB#3	2.08 kN
NUM		1.25 kN	NUM		1.64 kN

The higher fracture toughness results in an increase in the maximum load in the experimental tests of 34% on average. Although both the numerical models slightly underestimate the maximum load, they exhibit the same trend of the experimental results with a 31% increase in the maximum load for the “High Bridging (HB)” material configuration. The lower maximum load displayed by the numerical models can be explained by the fact that the R-curve effect for the opening mode II has been neglected, since no experimental data were available.

The differences in the damage propagation rates between the two numerical models can be appreciated in Figure 13, where the evolution of the cohesive damage state is presented for the two configurations at the same values of the applied displacement. In Figure 13, the fully damaged cohesive elements are represented in red.

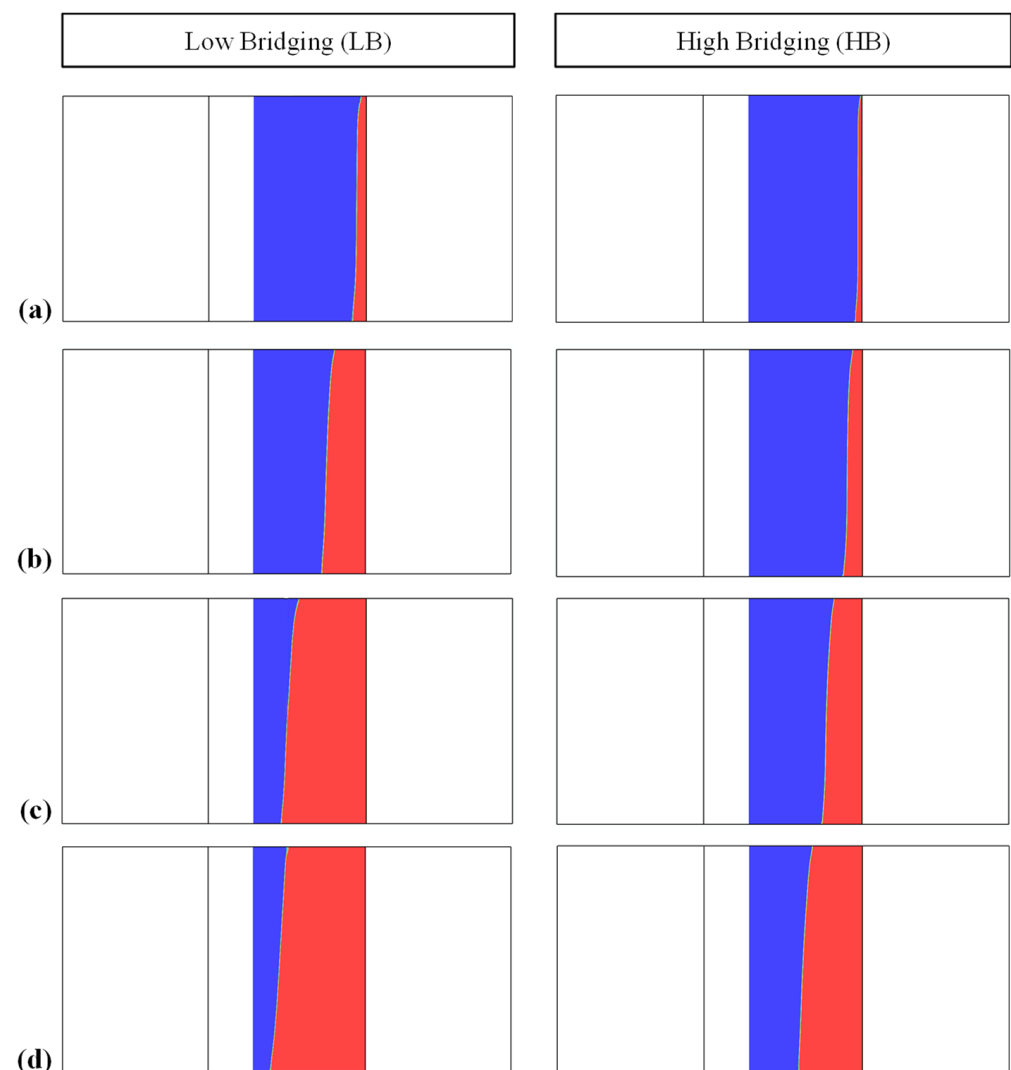


Figure 13. Damage propagation at an applied displacement of: (a) 3.3 mm; (b) 4.0 mm; (c) 5.0 mm; (d) 6.0 mm.

As shown in Figure 13, during the propagation the delamination front is not straight due to the different orientation of the laminae at the interface (+45/−45). As expected, the growth rate is higher for the “Low Bridging (LB)” configuration indicating a lower load-carrying capability. In Figure 14, the damage extensions predicted by the numerical models are compared with the images of ultrasonic C-scans captured at an applied displacement of 4 mm.

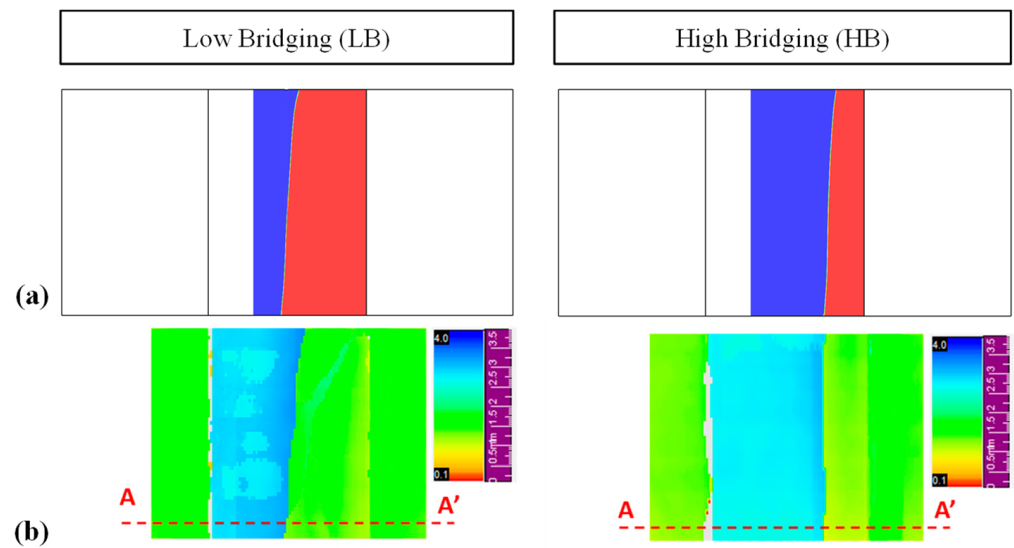


Figure 14. Comparison between numerical and experimental damage propagation: (a) Numerical; (b) C-Scan [23].

The comparison in Figure 14 shows an excellent agreement between the numerical results and the experimental data both in terms of delamination size and delamination front shape.

The effectiveness of the adopted numerical approach in simulating the R-curve effect using a trilinear traction-separation law is demonstrated in Figure 15, where the delaminated area vs. applied displacement is compared with the experimental data and with the results of analysis performed using a single bilinear traction-separation law, where only the steady-state value of the fracture toughness has been considered.

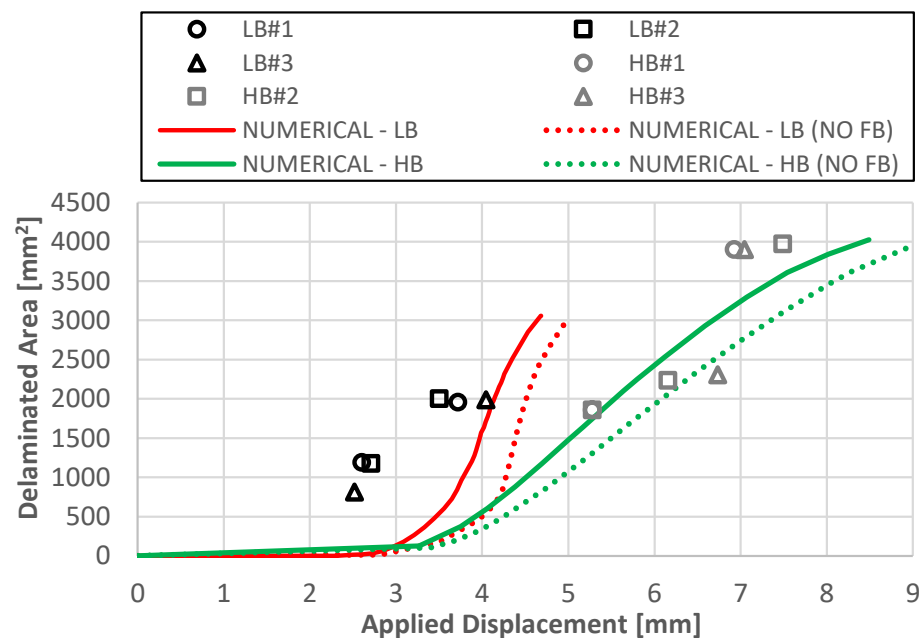


Figure 15. Comparison between numerical and experimental delaminated area vs. applied displacement.

As can be seen from Figure 15, the numerical results obtained using a trilinear softening law are much closer to the experimental data at the same value of the applied displacement. For both material configurations, delamination initiation occurs earlier for the models with a trilinear traction-separation law, due to the lower initial values of the fracture toughness.

Then, when the fracture energies reach their steady-state values, the propagation rates obtained from the two different traction-separation laws becomes almost identical.

6. Conclusions

In this paper, the R-curve effect in delamination propagation analysis for composite materials has been numerically investigated. An approach based on cohesive elements and on the combination of two bilinear traction-separation laws has been adopted to simulate the increase in the fracture toughness during the propagation of the delamination due to fiber bridging. The methodology has been applied to analyze the structural response of skin-doubler specimens subjected to three-points bending test using literature experimental data as a reference.

The carbon-epoxy composite material considered in this work exhibits two different sensitivities to the fiber-bridging phenomenon, resulting in two different R-curves. The R-curves determined experimentally have been linearly idealized and used to calibrate the parameters of the two bilinear traction-separation laws. Comparisons between the numerical results and the experimental data both in terms of load and delaminated area vs. applied displacement have demonstrated the effectiveness of the employed approach in accounting for the R-curve effect in delamination propagation analysis in composite materials, although some differences have been found in the maximum load predicted by the numerical model. From the numerical-experimental comparison, it has been observed a good agreement in terms of stiffness, while an underestimation of the maximum load of about 15% has been noted in both numerical models. However, the R-curve sensitivity has been captured by the numerical analysis with an increase in the maximum load of around 31% for the material configuration with higher fiber bridging sensitivity, perfectly matching the experimental data.

The results of the present work have highlighted the importance of taking into account the effect of the R curve in the analysis of delamination propagation in composite structures. Further studies are needed to include the R-curve effect for pure mode II and different mode-mixities to improve the results and extend the approach to more complex structural problems. The methodology could be applied to fatigue loading conditions and to specimens located at the component or sub-component level of the pyramid of test certification, such as stiffened panel subjected to post-buckling loads.

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Conflicts of Interest: The author declares no conflict of interest.

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