Analysis of Multi-Modal Public Transit Competing Relationships and Evolutionary Mechanisms in Cities in Cold Regions

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Abstract: In order to reveal the characteristics of the competition and cooperation of multi-modal public transportation in the cities of cold areas and to study the competitive relationship of multi-modal public transportation in cold areas, the urban public transportation system in cold areas is divided into three sub-systems, namely, conventional buses, urban mass transit, and cabs, to analyze the competitive relationship and evolution mechanism of multi-modal public transportation in cities of cold areas. We construct a multi-modal public transit Lotka–Volterra competitiveness model, considering that conventional buses and urban mass transit are mainly dominant in urban public transit systems in cold regions, and introduce the competitive influence effect coefficients and cooperative influence effect coefficients to analyze the mechanism of the evolution of the competitiveness between conventional buses and urban mass transit. The data on traffic supply and demand and travel of the Harbin bus system, a typical city in China, are collected, and the simulation analysis of multi-modal competing relationship and evolution mechanism of conventional bus and urban mass transit competition is carried out, respectively, with Harbin as the research object. The results show that the constructed model can better describe the multi-modal bus competition relationship and evolution mechanism.

Keywords: cities in cold regions; evolutionary mechanism model; competing relationship model; multi-modal transit system

1. Introduction

In cold climates, the design and operation of urban transportation systems become more complex, and the need for multi-modal transit becomes more urgent. With its flexibility and adaptability, multi-modal transit systems have become an important means to solve urban transportation problems. However, in multi-modal public transportation systems, the competing relationship and evolution mechanism between different transportation modes is complex and worthy of in-depth study. How to deal with the relationship between them, so that they operate in synergy, jointly improve efficiency, and reasonably allocate the share of passenger sources, so as to improve the efficiency of transportation has become a research hotspot for scholars at home and abroad, and its competition mechanism is one of the most important research objects.

With regard to the competitive relationship in the field of transportation, domestic and foreign studies are mainly divided into the competition between passenger transport modes and the competition between freight transport modes, passenger transport, including civil aviation and high-speed railways, railroads and long-distance passenger transport, and the analysis of the competitive relationship between urban conventional buses and rail transit, and freight transport, mainly including highways, railroads,
waterways and airways, four modes of freight transport, and the study of the competitive relationship between the ports. Roman C et al. [1] (2007) analyzed the competition between aviation and railroads with the example of the passenger transport corridor between Madrid and Barcelona, and the results showed that in medium- and long-distance transportation, high-speed rail can make itself more competitive than aviation by improving the punctuality rate. Juan C M et al. [2] (2014), based on the spatial data of the transportation corridor from Madrid to Barcelona, analyzed the impact of the arrival and departure time of the terminal on the competitiveness of the corridor between the aviation and high-speed rail competitiveness. Albalate D et al. [3] (2015) and others studied the competition and cooperation relationship between high-speed rail and air transportation in Europe and found that there is a direct competition between high-speed rail and aviation, and at the same time, it can also provide pick-up and drop-off services for hub airports, i.e., there is a cooperative relationship. Xia W et al. [4] (2016) took air transportation and high-speed rail in different travel distance range vertical differences within the range of travel distances and analyzed the impacts of air and HSR competition on fares, traffic volume, and welfare. Hun-Koo et al. [5] (2016) analyzed the impacts caused by HSR competition on aviation, based on the dataset of Japan and South Korea for the period of 1994–2012. The results showed that HSR services with a maximum speed of 300 km/h have a small additional impact on medium- and long-distance routes but a negative impact. Matin D et al. [6] (2018) constructed a binomial logit model to assess the competition between high-speed rail and aviation by selecting variables such as fare, travel time, ticket price, and convenience in the case of the Tehran–Isfahan route. Oscar A et al. [7] (2021) analyzed the impact of mutual cooperation between railroads and aviation infrastructures on the transportation network in terms of the aviation and the high-speed railroads caused by the impact. The results showed that with a sufficiently low degree of substitution, cooperation between modes of transportation reduces the operating costs of high-speed rail while increasing the number of high-speed rail trips compared to competition. Yuen [8] (2012) used hierarchical analysis to screen the factors affecting container ports from the perspective of port users and found that in-port costs and port location were the most important factors affecting carriers and shippers.

Previous studies on the evolution of the competitive relationship between transportation modes are often carried out from the perspective of urban economics, reflecting the individual traffic impacts of travel choices with individual traveler’s psycho-behavioral characteristics, and analyzing them by using competition models based on stochastic utility functions or generalized cost functions [9–11] or competition models based on game theory [12]. In recent years, scholars have begun to explore the competitive relationship between transportation modes and their development and evolution from the intrinsic dynamics of the relationship between transportation modes, and transportation modes and transportation modes constitute a system that has a very similar ecological mechanism with the biological and natural environments, and this similarity will analyze the dynamics of the ecosystem evolution of the dynamics of the approach to the introduction of the study of the development of transportation modes to provide the theoretical basis. Among them, the Lotka–Volterra model, which reflects interspecific competitive cooperation, is more common.

Wei J et al. [13] (2020) applied competition and cooperation indices to define the cooperative relationship between bus and subway lines and constructed a bus line optimization model based on the cooperation coefficient. Zhou Zhixiang [14] (2015) introduced the Lotka–Volterra model to analyze the study of competitive relationship between container ports. Xiang Jun et al. [15] (2016) constructed a regional network freight transportation mode advantageous distance model based on the current situation of competition among four modes of freight transportation, road, railroad, waterway, and air, and determined the advantageous distance of the four modes of freight transportation. Ci Yusheng et al. [16] (2017) divided the resident travel groups within the urban passenger transportation corridor into three different types, and based on this, they constructed a macro-
competition model of private transportation and public transportation based on the Lotka–Volterra model. Zhao Xueyu et al. [17] (2017) constructed a model of the competition mechanism between urban rail transit and surface buses and analyzed the competition effect generated when the two modes coexist. Luo Jiaqi et al. [18] (2019) constructed a model of the competing relationship between high-speed rail and civil aviation based on a two-stage game model and delineated the distance intervals of the competition between high-speed rail and civil aviation. Li Zhaolei et al. [19] (2021) established the Lotka–Volterra evolutionary model of net cabs and cabs in competitive and cooperative environments based on the theory of collaborative evolution and determined the appropriate size of urban net cabs and cabs when they reach equilibrium under different environments. Chen Qixiang et al. [20] (2022) analyzed the competitive relationship between cabs and metro in combination with the spatial heterogeneity of the built environment and characterized it as competing with, extending, and complementing the metro. Han Baorui et al. [21] (2023) combined the ecological characteristics of the development process of the two modes of transportation and established an evolutionary model of competition between modes of transportation with adaptive characteristics based on the Lotka–Volterra model of interspecific competition. Jiang Huifeng et al. [22] (2010) developed the Lotka–Volterra model to analyze the stability and evolutionary dynamics of road and rail transportation from a biomechanical perspective. Tianjun Feng et al. [23] (2019) applied a two-tier planning model to describe the competition problem among private cars, cabs, buses, and rail transit in terms of modal choices, and the model considered the dual benefits for both city managers and transit travelers.

At this stage, scholars at home and abroad mainly analyze the competitive relationship between passenger transport modes and cargo transport modes, and the existing studies are mostly from the perspective of passengers or cargo shippers, screening the relevant factors affecting the choice of transport modes and constructing the sharing rate model to analyze the competition of various modes of transport, and there is also a study based on the Lotka–Volterra model of ecology to construct the mode of transport competition analysis model and to analyze the current situation of the competition between various modes of transport, as well as the evolution of the competition relationship. Few studies have analyzed the competing relationships among conventional bus, urban mass transit, and cab subsystems from the perspective of urban public transportation systems, while fewer or insufficiently in-depth studies have been conducted on the advantageous spacing of each public travel mode in cities in cold regions.

The urban climate in cold regions during the snow and ice period is characterized by two main aspects: first, low temperatures, long cold periods, and large temperature differences. The second is the frequency of snowfall during the snow and ice period and the regularity of snow formation in winter. At the same time, cold temperatures reduce residents’ willingness to travel by transportation, making them prefer to stay warm and indoors, not to wait for long periods of time at bus stops, and not to have excessive walking distances and too many transfers. Based on this, this paper considers the level of economic development in the cold region in a certain period of time, combines the climate characteristics of the cities in the cold region during the ice and snow period and the travel characteristics of the residents, and constructs a multi-modal public transportation Lotka–Volterra competitive relationship model, with the help of the safety, economy, efficiency, and comfort of each mode of travel to fully reflect the competition coefficients and role coefficients of each mode of travel in the model. It takes the average competition coefficients of the cities in the cold region, and the role coefficients are calibrated. This paper introduces the competition effect coefficients and the cooperation effect coefficients to analyze the evolutionary mechanism of the competition between conventional buses and urban mass transit. Finally, the simulation analysis of the multi-modal competing relationship and the evolution mechanism of the competition between conventional buses and urban mass transit is carried out with the background of the overall passenger flow development of the bus passenger transportation system in Harbin City as an example.
2. Lotka–Volterra Based Modeling of Competing Relationships

2.1. Lotka–Volterra Model

The Lotka–Volterra model is an important model used to describe the relationship between predators and prey in an ecosystem, which is an extension and expansion of the Logistic model and has been widely used in the analysis of competitive and cooperative relationships within an ecosystem or between enterprises. Because the model can describe the interactions of different variables under the same environmental capacity and can reflect the relationship between the variables more intuitively, it has been applied to many fields such as economics and epidemiology and also has some applications in the study of transportation systems [13]. The basic form of this model is that the two populations are represented by \( X \) and \( Y \), and the growth rates of natural growth of population 1 and population 2 are represented. The predation of population 2 on population 1 is considered, and the chance of an encounter between population 1 and population 2 and the growth rate of population 2 after predation are represented, so that the equations for the change in the number of populations 1 and 2 can be shown in Equations (1) and (2):

\[
\frac{dx(t)}{dt} = a_1 x(t) - b_1 x(t) y(t) \tag{1}
\]

\[
\frac{dy(t)}{dt} = -a_2 y(t) + b_2 x(t) y(t) \tag{2}
\]

The combination of the above equations is the initial form of the Lotka–Volterra model, taking into account the growth-retarding effect of each population itself and the mutualistic symbiosis, mutualistic bias, and many other relationships between populations. The general form of the Lotka–Volterra model can be obtained as Equation (3):

\[
\frac{dx(t)}{dt} = x(t) (a_1 + b_1 x(t) + c_1 y(t))
\]

\[
\frac{dy(t)}{dt} = y(t) (a_2 + b_2 y(t) + c_2 x(t))
\]

where \( x(t) \) is the size of population 1; \( y(t) \) is the size of population 2; \( a_1 \) and \( a_2 \) are the endowment growth rates of population 1 and population 2; \( b_1 \) and \( b_2 \) are the coefficients of intraspecific interactions of population 1 and population 2 affected by their own blockage; and \( c_1 \) and \( c_2 \) are the coefficients of interactions of population 1 and population 2 affected by the interaction of the other population.

From the above relationship, it can be seen that when \( \frac{dx(t)}{dt} = \frac{dy(t)}{dt} = 0 \), the competition between population 1 and population 2 reaches equilibrium. The positive and negative coefficients of the interaction coefficients reflect the various relationships between the populations, as shown in Table 1.

<table>
<thead>
<tr>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>Interaction between Populations</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0</td>
<td>&gt;0</td>
<td>mutually beneficial symbiosis</td>
</tr>
<tr>
<td>&gt;0</td>
<td>=0</td>
<td>commensalism</td>
</tr>
<tr>
<td>&gt;0</td>
<td>&lt;0</td>
<td>population 1 feeds on population 2</td>
</tr>
<tr>
<td>&lt;0</td>
<td>&gt;0</td>
<td>population 2 feeds on population 1</td>
</tr>
<tr>
<td>&lt;0</td>
<td>&lt;0</td>
<td>prejudice</td>
</tr>
<tr>
<td>&lt;0</td>
<td>=0</td>
<td>compete with each other</td>
</tr>
</tbody>
</table>
2.2. Analysis of Multi-Mode Bus Competition

The urban public transportation system in the cold region defined in this paper consists of three subsystems, conventional public transportation, rail transportation, and cabs, and the three modes of transportation are interconnected, interacting, and influencing each other in an integrated transportation system, so it is feasible to apply the Lotka–Volterra model to analyze the competitiveness among the three subsystems of urban public transportation.

(1) Competitive Relationship Model Assumptions

Aiming at the interdependence and mutual constraints of each transport mode in the urban public transport network, the Lotka–Volterra model is used to study the competing relationships among three travel modes: conventional bus, rail transit, and taxi. In urban public transportation systems, the factors affecting the passenger volume of conventional buses, rail transit, and cabs and the competition among the three are quite complex in both the internal and external environments, so reasonable assumptions need to be made to account for the real-world environment.

Assumption 1. The passenger traffic of each of the three subsystems in a region with three modes of transport in the absence of competition is consistent with the logistic model.

Assumption 2. The external environment is stable. As the economic level of the region increases, the passenger traffic of the three modes will gradually increase, but the increase will gradually decrease and eventually reach saturation, maintained at a certain amount, and the various modes ultimately assume a relatively stable passenger traffic.

Assumption 3. Market demand is stable, i.e., the patronage of the region maintains a steady growth, considering only its own deterrent growth effect.

(2) Multi-modal Public Transportation Competitive Relationship Modeling

The basic form of the Lotka–Volterra model mainly describes the relationship model between the two clusters, and we extend the basic form of Lotka–Volterra model in order to study the relationship of multi-modal transit competition in the cold region. The transportation development of cities in cold regions is affected by their economic level, and cities in cold regions need to invest a lot of money and labor to maintain houses and roads in winter and need to spend more money and energy on city construction. Considering the level of economic development of cities in cold regions, the model is as follows:

\[
\frac{dx_j}{dt} = r_j x_j \left(1 - \frac{x_j + \sum_{i=1}^{n} a_{ij} \cdot x_i}{K_j}\right)
\]

where \(x_j\) is the passenger traffic volume of transportation mode \(j\) at moment \(i\); \(r_j\) is the endowment growth rate of transportation mode \(j\); \(K_j\) is the threshold value (passenger saturation capacity) of transportation mode \(j\) when it develops alone under the level of economic development of the cold region for a certain period of time; and \(a_{ij}\) and \(\frac{a_{ij}}{K_j}\) are the regional average competition coefficient and average role coefficient of transportation mode \(i\) to transportation mode \(j\) under the level of economic development of the cold region for a certain period of time.

This leads to the competitive relationship between conventional buses, rail transit, and cabs in cities in cold regions as in Equation (5):
where $\frac{\alpha_{ij}}{K_i}$ is the competition coefficient of system $i$ to system $j$, when $i=j$ represents the mode’s own blocking coefficient; $\frac{dx_i}{dt}, \frac{dx_2}{dt}, \frac{dx_3}{dt}, K_1, K_2, K_3$ are the growth rate of the travel volume of the three travel modes of conventional buses, rail transit, and cabs collectively; and $K_1, K_2, K_3$ are the scale threshold of the conventional buses, rail transit, and cabs’ traffic volume.

For solving the parameters of the Lotka–Volterra model, the related literature uses the nonlinear least squares method to estimate the model parameters, but this method is sensitive to the initial values of the parameters. Considering the first-order linear differential equation properties and data availability of the Lotka–Volterra competing model constructed in this paper which considers the urban transit subsystems in three cold regions, this thesis introduces the idea of gray direct modeling in the gray system model and discretizes the model, on the basis of which the model parameters are solved.

The model is first simplified by applying the following transformations to the model parameters:

$$
\begin{align*}
    a_1 &= r_1, b_1 = \frac{\alpha_{11}}{K_1}, c_1 = \frac{\alpha_{21}}{K_2}, d_1 = \frac{\alpha_{31}}{K_3} \\
    a_2 &= r_2, b_2 = \frac{\alpha_{12}}{K_1}, c_2 = \frac{\alpha_{22}}{K_2}, d_2 = \frac{\alpha_{32}}{K_3} \\
    a_3 &= r_3, b_3 = \frac{\alpha_{13}}{K_1}, c_3 = \frac{\alpha_{23}}{K_2}, d_3 = \frac{\alpha_{33}}{K_3}
\end{align*}
$$

After parametric transformations, the Lotka–Volterra model constructed above is transformed as follows:

$$
\begin{align*}
    \frac{dx_1}{dt} &= x_1\left(1 - \frac{\alpha_{11}x_1}{K_1} - \frac{\alpha_{21}x_2}{K_2} - \frac{\alpha_{31}x_3}{K_3}\right) \\
    \frac{dx_2}{dt} &= x_2\left(1 - \frac{\alpha_{12}x_1}{K_1} - \frac{\alpha_{22}x_2}{K_2} - \frac{\alpha_{32}x_3}{K_3}\right) \\
    \frac{dx_3}{dt} &= x_3\left(1 - \frac{\alpha_{13}x_1}{K_1} - \frac{\alpha_{23}x_2}{K_2} - \frac{\alpha_{33}x_3}{K_3}\right)
\end{align*}
$$

As a result, the previous solution for $\frac{dx_i}{dt}$ is simplified to the solution for $a_i$, $b_i$, $c_i$, and $d_i$. On this basis, based on the gray direct modeling method, the first-order linear differential equation of the model is discretized, and the model is transformed into the first differential equation of the model with $x(t) - x(t-1)$ as the whitening value of $\frac{dx_i}{dt}$ and $[x_i(t) + x_i(t-1)]/2$ as the background value of $\frac{dx_i}{dt}$ as follows:
\[
x_i(t) - x_i(t-1) = a_i \left( \frac{x_i(t) + x_i(t-1)}{2} - b_i \left( \frac{x_i(t) + x_i(t-1)}{2} \right)^2 - c_i \left( \frac{x_i(t) + x_i(t-1)}{2} \right)^3 - d_i \left( \frac{x_i(t) + x_i(t-1)}{2} \right)^4 \right), \quad i = 2, 3, \ldots, n
\]

Bringing the data at time \( t=2, 3, \ldots, n \) into the above equation yields an n-dimensional system of linear equations:

\[
\begin{align*}
x_2(t) - x_2(t-1) &= a_2 \left( \frac{x_2(t) + x_2(t-1)}{2} - b_2 \left( \frac{x_2(t) + x_2(t-1)}{2} \right)^2 - c_2 \left( \frac{x_2(t) + x_2(t-1)}{2} \right)^3 - d_2 \left( \frac{x_2(t) + x_2(t-1)}{2} \right)^4 \right), \\
&\vdots \\
x_n(t) - x_n(t-1) &= a_n \left( \frac{x_n(t) + x_n(t-1)}{2} - b_n \left( \frac{x_n(t) + x_n(t-1)}{2} \right)^2 - c_n \left( \frac{x_n(t) + x_n(t-1)}{2} \right)^3 - d_n \left( \frac{x_n(t) + x_n(t-1)}{2} \right)^4 \right),
\end{align*}
\]

which is expressed in matrix form as

\[
\begin{align*}
\begin{bmatrix}
X_N \\
X_N
\end{bmatrix}
&= B \hat{A}
\end{align*}
\]

\[
X_N = \begin{bmatrix}
(x_1(1) + x_1(2)) / 2 \\
(x_2(1) + x_2(2)) / 2 \\
\vdots \\
(x_n(1) + x_1(n-1)) / 2
\end{bmatrix}^T
\]

\[
B = \begin{bmatrix}
\begin{bmatrix} x_1(1) + x_1(2) \end{bmatrix} / 2 & \begin{bmatrix} x_2(2) + x_2(1) \end{bmatrix} / 2 & \begin{bmatrix} x_3(2) + x_3(1) \end{bmatrix} / 2 & \cdots & \begin{bmatrix} x_n(2) + x_n(1) \end{bmatrix} / 2 \\
\begin{bmatrix} x_1(1) + x_1(2) \end{bmatrix} / 2 & \begin{bmatrix} x_2(2) + x_2(1) \end{bmatrix} / 2 & \begin{bmatrix} x_3(2) + x_3(1) \end{bmatrix} / 2 & \cdots & \begin{bmatrix} x_n(2) + x_n(1) \end{bmatrix} / 2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\begin{bmatrix} x_1(n-1) + x_1(n) \end{bmatrix} / 2 & \begin{bmatrix} x_2(n) + x_2(n-1) \end{bmatrix} / 2 & \begin{bmatrix} x_3(n) + x_3(n-1) \end{bmatrix} / 2 & \cdots & \begin{bmatrix} x_n(n) + x_n(n-1) \end{bmatrix} / 2
\end{bmatrix}
\end{bmatrix}^{10}
\]

According to the least squares criterion, \( \hat{A} \) is estimated as

\[
\hat{A} = [a_i, b_i, c_i, d_i]^T = (B^T B)^{-1} B^T X_i
\]

Similarly, \( \hat{A}_2, \hat{A}_3 \) can be solved in the same way, which will not be repeated here.

On this basis, according to the Formula (11), the competing coefficients between conventional buses, rail transit, and cabs in the urban bus system in cold areas and the scale thresholds of each mode’s capacity can be obtained, and the competing coefficients matrix between the public transportation sub-systems in cold areas can be obtained, on the basis of which the competing relationships among the three sub-systems can be analyzed.

(1) When \( \alpha_i = 0 \) and \( \alpha_j = 0 \), it represents that the two systems are independent of each other and do not affect each other;

(2) When \( \alpha_i > 0 \) and \( \alpha_j = 0 \), it represents that system \( i \) will gradually replace the status of system \( j \), but system \( j \) will not affect the status of system \( i \). It is a biased relationship, and vice versa;
(3) When \( \alpha_i < 0 \) and \( \alpha_j = 0 \), it represents that system \( i \) will promote system \( j \), but system \( j \) will not affect the status of system \( i \), which is a partial-benefit relationship, and vice versa;

(4) When \( \alpha_i > 0 \) and \( \alpha_j > 0 \), it represents that the two systems are in competition with each other and vice versa in cooperative symbiosis;

(5) When \( \alpha_i < 0 \) and \( \alpha_j > 0 \), it represents that system \( i \) has a facilitating effect on system \( j \), but system \( j \) has an inhibiting effect on system \( i \), which belongs to a biased symbiotic relationship, and vice versa.

2.3. Analysis of the Evolutionary Mechanism of Competition between Conventional Bus and Rail Transit

In the urban public transportation system in the cold region, it is mainly dominated by conventional buses and rail transit, so this paper further analyzes the evolution mechanism of the competition between conventional buses and rail transit.

(1) Competitive Synergy Modeling

Conventional public transport and rail transit competition is the essence of the competition and occupation of passenger flow. Those unable to adapt to the fierce competition in the market will be eliminated or they will need to reform and upgrade to enhance their competitiveness, to achieve the separation of market positioning, and ultimately to achieve each other’s competitiveness in the state of symbiosis. In order to study the evolution mechanism of conventional bus and rail transportation under the competitive environment, the synergistic set of evolution equations is based on the basic competitive relationship model with the simultaneous introduction of competitive influence effect coefficient \( \alpha_{ij} \) and cooperative influence effect coefficient \( \beta_{ij} \), and its synergistic evolution equations are

\[
\begin{align*}
\frac{dx_1}{dt} &= f(x_1, x_2) = r_1 x_1 \left( 1 - \frac{k_1 x_1}{K_1} - \frac{\alpha_{12} x_2}{K_2} + \frac{\beta_{12} x_2}{K_2} \right) \\
\frac{dx_2}{dt} &= g(x_1, x_2) = r_2 x_2 \left( 1 - \frac{k_2 x_2}{K_2} - \frac{\alpha_{12} x_1}{K_1} + \frac{\beta_{12} x_1}{K_1} \right)
\end{align*}
\]

where \( x_1, x_2 \) are the existing demand for conventional public transportation and rail transit, as a function of time; \( \frac{dx_1}{dt}, \frac{dx_2}{dt} \) are the growth rate of the two modes at the moment \( t \); \( r_1, r_2 \) are the natural growth rate of the volume of traffic, respectively; \( K_1, K_2 \) are the scale thresholds of the transportation volume of the two modes under the level of economic development of the cold region in a certain period of time and are constants; \( k_1, k_2 \) are the coefficients of influence of the two modes themselves; \( \alpha_{12}, \alpha_{21} \) are the coefficients of influence of the regional average competition between the two modes under the level of economic development of the cold region in a certain period of time; \( \beta_{12}, \beta_{21} \) are the coefficients of influence of the regional average cooperation...
between the two modes under the level of economic development of the cold region in a certain period of time; and \( \alpha_2, \alpha_{21}, \beta_2, \beta_{21} \geq 0 \).

The four equilibrium points in the Equation are

\[
M_0(0,0), \quad M_1(0,K_2), \quad M_2(K_2,0), \quad M_3\left(\frac{K_1(1+\beta_2-\alpha_{21})}{1-(\beta_2-\alpha_{21})}, \frac{K_2(1+\beta_{21}-\alpha_{21})}{1-(\beta_{21}-\alpha_{21})}\right),
\]

and the evolutionary outcome depends on the interrelationships of \( \alpha_2, \alpha_{21}, \beta_2, \) and \( \beta_{21} \).

(2) Stability analysis of competing synergies

The stability of the four equilibrium points can be judged by the Jacobi matrix if \( \det(J) \neq 0 \), the four equilibrium points satisfy \( \det(J) > 0 \) and \( -\text{tr}(J) = \left( \frac{\partial f}{\partial \alpha_1} + \frac{\partial g}{\partial \alpha_2} \right) > 0 \), which are the stabilization points. The Jacobi matrix is

\[
J = \begin{bmatrix}
\gamma_1\left(1 - \frac{x_1}{K_1} - \frac{x_2}{K_1} + \beta_2 \frac{x_2}{K_1}\right) & \gamma_1\frac{x_1}{K_2}(-\alpha_{21} + \beta_{21}) \\
\gamma_2\frac{x_1}{K_2}(-\alpha_{21} + \beta_{21}) & \gamma_2\left(1 - \frac{x_1}{K_1} - \frac{x_2}{K_1} + \beta_2 \frac{x_2}{K_1}\right)
\end{bmatrix}
\]

(13)

Substituting the four equilibrium points \( M_0, M_1, M_2 \) and \( M_3 \) into Equation (13) and calculating \( \det(J) \) and \(-\text{tr}(J)\), the stability or stability conditions can be found, as shown in Table 2.

<table>
<thead>
<tr>
<th>( M_i )</th>
<th>( \det(J) )</th>
<th>(-\text{tr}(J))</th>
<th>Stability Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_0 )</td>
<td>( \gamma_1\gamma_2 )</td>
<td>(-\gamma_1 - \gamma_2)</td>
<td>instability</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>(-\gamma_1\gamma_2(1+\beta_2-\alpha_{21}))</td>
<td>(\gamma_1 - \gamma_1(1+\beta_2-\alpha_{21}))</td>
<td>( \alpha_{21} - \beta_2 &gt; 1, \quad \alpha_{21} - \beta_2 &lt; 1 )</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>(-\gamma_1\gamma_2(1+\beta_{21}-\alpha_{21}))</td>
<td>(\gamma_1 - \gamma_2(1+\beta_{21}-\alpha_{21}))</td>
<td>( \alpha_{21} - \beta_{21} &gt; 1, \quad \alpha_{21} - \beta_{21} &lt; 1 )</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>(\gamma_1\gamma_2(1+\beta_{21}-\alpha_{21})(1+\beta_2-\alpha_{21}))</td>
<td>(\gamma_1(1+\beta_{21}-\alpha_{21}) + \gamma_2(1+\beta_2-\alpha_{21}))</td>
<td>( \alpha_{21} - \beta_{21} &gt; 1, \quad \alpha_{21} - \beta_{21} &lt; 1 )</td>
</tr>
</tbody>
</table>

According to Table 2, \( M_0(0,0) \) is not a stabilization point, when \( \alpha_{21} - \beta_{21} > 1 \) and \( \alpha_{21} - \beta_{21} < 1 \), \( \det(J) > 0 \), \(-\text{tr}(J) > 0 \), \( M_1(0,K_2) \) is the stabilization point; when \( \alpha_{21} - \beta_{21} > 1 \) and \( \alpha_{21} - \beta_{21} < 1 \), \( \det(J) > 0 \), \(-\text{tr}(J) > 0 \), \( M_2(K_2,0) \) is the stabilization point; and when \( \alpha_{21} - \beta_{21} < 1 \) and \( \alpha_{21} - \beta_{21} < 1 \), \( \det(J) > 0 \), \(-\text{tr}(J) > 0 \), \( M_3 \left(\frac{K_1(1+\beta_{21}-\alpha_{21})}{1-(\beta_{21}-\alpha_{21})}, \frac{K_2(1+\beta_{21}-\alpha_{21})}{1-(\beta_{21}-\alpha_{21})}\right) \) is the stabilization point.

The conventional bus and rail transit competition and cooperation evolution trend is shown in Figure 1; different competition, cooperation impact effect coefficients, and conventional bus and rail transit competition and cooperation evolution trends tend to have different equilibrium points. As shown in Figure 1a, when \( \alpha_{21} - \beta_{21} > 1 \) and \( \alpha_{21} - \beta_{21} < 1 \)
, region I is $\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} > 0$; region II is $\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} > 0$; and region III is $\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} < 0$. Over time, evolutionary trajectory lines eventually converge to an equilibrium point $M_1$. With the initial state in I, conventional bus and rail transit have a growth rate greater than zero, the evolutionary trajectory will move to II; with the initial state in II, the growth rate of rail transit is positive, the growth rate of the conventional bus is negative, the size of the conventional bus is gradually reduced, and the size of rail transit is gradually expanding, and they ultimately move to $M_1$.

![Figure 1](image1.png)

**Figure 1.** Evolution of the competitive equilibrium. (a) $\alpha_2 - \beta_2 > 1, \alpha_2 - \beta_2 < 1, M_1$ stable. (b) $\alpha_2 - \beta_2 > 1, \alpha_2 - \beta_2 < 1, M_2$ stable. (c) $\alpha_2 - \beta_2 < 1, \alpha_2 - \beta_2 < 1, M_3$ stable. (d) $\alpha_2 - \beta_2 > 1, \alpha_2 - \beta_2 > 1$, no stabilization point.

If the initial state is in III, the growth rates of both conventional and rail transit are negative, moving first to II and eventually reaching the equilibrium point $M_1$.

As shown in Figure 1b, when $\alpha_2 - \beta_2 > 1$ and $\alpha_2 - \beta_2 < 1$, the regional characteristics are similar to Figure 1a, and the trend of competitive evolution eventually converges to the equilibrium point $M_2$. At this point, the competitive impact of mutual cooperation and rail transit on the market is smaller than that of the conventional buses on the market, and the conventional buses will gradually occupy the whole market.
As shown in Figure 1c, when $\alpha_2 - \beta_2 < 0$ and $\alpha_1 - \beta_1 < 0$, similarly, the trend of competitive evolution eventually converges to the equilibrium point $M_3$. At this point, the competition between conventional buses and rail transit is more moderate, and together with the cooperative relationship between them, they can promote each other to reach an equilibrium state in the market.

As shown in Figure 1d, when $\alpha_2 - \beta_2 > 0$ and $\alpha_1 - \beta_1 > 0$, $M_3$ is the saddle point, and there is no stabilizing point for competing with each other.

3. The Case Study of Harbin

Harbin is a typical city in the cold zone and also a national model city for public transportation city construction and a national model city for green travel, so this paper chooses Harbin as the research object, which is typical for studying the evolution mechanism of the multi-modal public transportation competitive relationship in cities in the cold zone.

The model simulation data are provided by the Harbin Statistical Yearbook, China Urban Statistical Yearbook, National Major Cities Inspection Report, China Urban Construction Statistical Yearbook, and Harbin Municipal Bureau of Transportation from 2011 to 2021.

3.1. Simulation Analysis of Multi-Modal Public Transportation Competitive Relationships

The passenger volume data of urban mass transit, conventional buses, and cabs of the public transportation subsystem from 2012 to 2021 are brought into the previously constructed multi-modal travel competitiveness model to empirically analyze the competitiveness of multi-modal travel in Harbin. According to the model solving algorithm of Equations (4)–(11), the parameters of the model are solved, and the results of $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$ are obtained as

$\hat{A}_1 = [0.747782, 0.000015, 0.000007, 0.000096]$

$\hat{A}_2 = [0.289466, -0.000002, 0.000020, -0.000031]$

$\hat{A}_3 = [0.656746, -0.000001, 0.000055, 0.000043]$

In turn, the L-V model of multi-modal public transportation in Harbin City is obtained as follows:

$$\begin{align*}
\frac{dx_1}{dt} & = 0.747722x_1 - 0.000015x_1^2 - 0.000070x_1x_2 - 0.000096x_1x_3 \\
\frac{dx_2}{dt} & = 0.289466x_2 + 0.000002x_1x_2 - 0.000020x_2^2 + 0.000031x_2x_3 \\
\frac{dx_3}{dt} & = 0.656746x_3 + 0.000001x_1x_3 - 0.000055x_2x_3 - 0.000043x_3^2
\end{align*}$$

Based on the above model, the coefficients of the three modes of transportation are solved according to Equation (5), and the matrix of the coefficients of passenger transportation among the three modes of transportation is obtained:

$$\begin{bmatrix}
0.000019 & -0.000007 & -0.000003 \\
0.000093 & 0.000067 & 0.000083 \\
0.000128 & -0.000108 & 0.000065
\end{bmatrix}$$
where $C_{ij}$ is $\frac{a_{ij}}{K_i}$, the competing coefficient of the $i$th travel mode to the $j$th travel mode, and when $i = j$, it denotes the self-influence coefficient of that travel mode.

Based on the results of the above model calculations, the three modes of travel in Harbin City are analyzed in terms of competing relationships.

The coefficient of competitiveness of bus travel to rail travel is $-0.000007$, which shows a facilitating effect; the coefficient of competitiveness of rail to bus is $0.000093$, which shows an inhibiting effect. The two competing relationships show a biased symbiosis.

The coefficient of competitiveness of bus travel to cab travel is $-0.000003$, which shows a facilitating effect; the coefficient of competitiveness of cab travel to bus is $0.000128$, which shows an inhibiting effect. The two competing relationships show a biased symbiosis.

The coefficient of competitiveness of rail travel to cab travel is $0.000083$, which shows an inhibiting effect; the coefficient of competitiveness of cab travel to bus is $-0.000108$, which shows a facilitating effect. The two competing relationships show a biased symbiosis.

From the above coefficient matrix analysis, it is known that rail transit and cab travel modes have an inhibiting effect on the development of conventional transportation. In recent years, with the development of the economy and the rapid development of rail transit in major cities in China, rail transit in the city of medium- and long-distance transportation has the advantages of rapidity, punctuality, and comfort, so the medium- and long-distance passenger transportation occupies a dominant position and impacts on the conventional public transport market. Relative to public transportation, cabs do not have fixed stations, provide point-to-point services, do not have fixed routes, in accordance with the needs of passengers, and have destinations for direct delivery, are more flexible and convenient, have better accessibility, suitable for individual rides or specific needs, and therefore, the development of urban cabs in short- and medium-distance passenger transportation will be an inhibitor of the development of conventional public transportation. The rail transit travel mode has an inhibiting effect on the development of cab travel. Rail transit has a large capacity, no right-of-way problem, and can ensure speed. However, the current ground traffic congestion is serious, and the original speed advantage of cabs is obviously lost. Therefore, the development of rail transit will have an impact on the cab market.

3.2. Simulation Analysis of the Evolution Mechanism of Competition between Conventional Bus and Rail Transit

In accordance with the above statistics for 2019, Harbin’s rail transit carries 104 million passengers, while ground transportation carries 1.06 billion passengers. In accordance with the Harbin city rail transit planning and bus system passenger flow historical data, as well as rail transit as the backbone of the future of the city’s passenger transportation, the rail transit growth rate can be set to 0.5, while the ground bus growth rate can be set to 0.05. This defines the stable scale of urban rail transit development as $N = 2.5$ billion passenger trips, and the scale of surface bus development is $N = 1.5$ billion passenger trips, and it analyzes the evolution mechanism and the characteristics of passenger flow growth in each competing period with the passenger volume in 2019 as the initial value. In Figures 2–5, the time unit is year, and billion passenger trips are the unit of passenger traffic.

As shown in Figure 2, the passenger volume of rail transit is growing rapidly and approaching its own stabilized passenger volume, while the passenger volume of surface buses is rapidly declining due to its influence and impact.

As shown in Figure 2, $\alpha_2 > 1$ indicates that rail transit has a stronger ability to compete for and occupy the market, which makes the passenger volume of rail transit grow...
rapidly and approach its own stable passenger volume, while the passenger volume of conventional buses declines rapidly due to its influence and impact. This is due to the rail transit in the urban passenger transport system giving full play to the characteristics of large capacity, high speed, and not being subject to the influence of conventional ground traffic jams to bring convenience to public travel, so the performance of the strong competitiveness of the initial period to attract a large number of passengers from various sources, so as to survive in the environment of the city’s public transportation and as a new mode of travel in the passenger market to occupy a certain advantage and continue to grow and develop. The ground public transportation is affected by it, and due to the homogeneity of the service and the gap of operation ability, the passenger traffic drops rapidly in a short period of time. At this time, the surface transit system should make appropriate strategic adjustments, such as improving fares, increasing departure frequency, and adjusting routes, to improve its competitiveness in order to attract patronage.

![Figure 2](image)

**Figure 2.** \( \alpha_1 = 1.2 \quad \alpha_2 = 0.2 \quad \beta_1 = 0.1 \quad \beta_2 = 0.1 \).

As shown in Figure 3, keeping the competition impact coefficient \( \alpha \) in Figure 2, but increasing the cooperation impact coefficient \( \beta \) between the two modes of travel, makes moderate changes to the passenger volume of the two modes of travel, which reduces the loss due to excessive competition but also makes full use of each other’s advantages.

As can be seen from Figures 2 and 3, when one party is more competitive, it is enough to dominate the whole market and eliminate the other party; although, this law is also satisfied in a competitive environment, but because of the existence of some cooperation with each other, the demise of the inferior party will be greatly delayed. However, this phenomenon does not occur in the transportation environment, and although this evolutionary pattern is consistent with biological populations, there is no extinction of one party among the modes of transportation, and there is some cooperative behavior between conventional bus and rail transit.
As shown in Figure 4, $\alpha_2 < 1, \alpha_3 < 1$ indicates that the competition between conventional traffic and rail transit is more moderate, and coupled with each other’s cooperative relationship, the two have a greater space for development; although, the rapid growth period of rail transit has a certain impact on public transportation, with the passage of time, the two can ultimately reach a state of stable development, but it will not exceed the size of the city’s conventional traffic and rail transit thresholds. On this basis, as shown by Figure 5a–d, when increasing the competition coefficient of one of the transportation modes to the other, the development of the other transportation mode will be constrained, showing the trend of this trend. When the coefficient of competitiveness of one mode of transportation over the other is lowered, the two modes of transportation eventually evolve into a state of equilibrium over time.
Figure 5. Simulation analysis of competitive synergy evolution. (a) Increase the competition factor of urban mass transit over surface public transit. (b) Reducing the competition factor of urban mass transit to surface public transit. (c) Increasing the competition factor of surface public transit for urban mass transit. (d) Reducing the competition factor of surface public transit for urban mass transit.

From the above analysis, in an urban public transportation system, the development of the traffic mode is closely related to the intensity of the competition coefficient, and the stable result of competition between public transportation and rail transit is related to $\alpha$, $\beta$. In the actual traffic environment, a series of traffic management measures should be taken to enhance its own competitiveness. For example, improving the service level of public transport and optimizing the departure interval can enhance the competitiveness of public transport to rail transit. When the competition coefficient of two traffic modes in the urban public transportation system is small at the same time, the competition between traffic modes is too gentle, the two traffic modes do not reach saturation state, and there is a large development space. At the same time, it will aggravate urban road traffic congestion, occupy urban space, and cause waste of existing available resources. When the competition coefficient between the two modes is large, the competition between them is too fierce, and the urban traffic mode cannot have benign development. Therefore, maintaining moderate competition among transportation modes is conducive to promoting their own reform and optimization, and at the same time, cooperation between transportation modes should be formed to promote urban transportation to provide more convenient services for the majority of travelers and form sustainable development of urban public transportation.
4. Discussion and Conclusions

Few studies have analyzed the competing relationships among conventional bus, rail, and cab subsystems from the perspective of urban public transportation systems, while fewer or insufficiently in-depth studies have been conducted on the advantageous spacing of various public travel modes in cities in cold regions. Existing studies are as follows: Wei J et al. [13] (2020) used the competition and cooperation index to define the competitiveness of bus and metro lines from a geospatial perspective based on the geographic location and service scope of metro and bus stations, but they lacked the consideration of passenger demand and service capacity of bus lines. Chen Qixiang et al. [20] (2022) analyzed the competitive relationship between cabs and the metro mainly considering the spatial heterogeneity of the built environment, characterizing it as competing with, extending, and complementing the metro, but they lacked a study of the intrinsic dynamics of competition. Feng Tianjun et al. [23] (2019) applied a two-layer planning model to describe the competition problem among private cars, cabs, buses, and rail transit in terms of mode choice and constructed a computational model for the rail transit sharing rate in cities in cold regions, but they did not further investigate the competition characteristics among multi-modal transit in cities in cold regions. Based on this, this paper considers the level of economic development of the cities in the cold area, the climate characteristics of the cities in the cold area during the ice and snow period, and the travel characteristics of the residents. We construct the model of multi-modal public transportation Lotka–Volterra competing relationship in the cities in the cold area, calibrate the model coefficients with the average competition coefficients and the role coefficients of the cities in the cold area, and combine them with the actual passenger volume and growth rate of the various modes of transportation in the cold area city. We simulate and analyze the multi-modal public transportation competing relationship in the cities in the cold area, as well as the mechanism of the evolution of the competing between the conventional buses and the rail transportation. The major contributions of this study are summarized as follows.

1. The Lotka–Volterra model is transformed with transportation significance, so that the parameters in the model have more significance in the field of transportation, which is suitable for analyzing the complex competitive relationship between multiple modes of transportation;

2. Considering that conventional buses and rail transit are mainly dominant in urban transit systems in cold regions, the competition influence effect coefficients and cooperation influence effect coefficients are introduced to analyze the mechanism of the evolution of the competition between conventional buses and rail transit in cities in cold regions;

3. Rail transit has the advantages of rapidity, punctuality, and comfort in urban medium- and long-distance transportation, and cabs are more flexible and convenient in urban medium- and short-distance transportation, with better accessibility, which is suitable for individual rides or specific needs, and therefore, rail transit and cab travel modes have an inhibiting effect on the development of conventional transportation;

4. The system dynamics of the direction of the evolution of the competitive relationship between rail transit and surface buses lies in the strength of competition or cooperation between the two sides;

5. In the competition environment, due to the existence of a certain degree of cooperation with each other, even if the competition is too intense, the disadvantaged party extinction time will be greatly delayed. In the cold region of the transportation environment, due to the regional specificity of the conventional public transport and rail transit, there will not be an extinction of the situation between the conventional bus and rail transit, and there is also a certain degree of cooperation between the conventional public transport and rail transit behavior.
(6) When raising the competition coefficient of one of the modes of transportation to the other, the development of the other mode of transportation will be constrained, showing the tendency of both sides; when lowering the competition coefficient of one of the modes of transportation to the other mode of transportation, the two modes of transportation will eventually evolve into an equilibrium state with the passage of time.

The findings of this paper can, to a certain extent, provide a theoretical basis for the overall planning and operation policy orientation of the urban public transportation system in cold regions. However, this paper does not consider the impact of temperature changes in cold regions during snow and ice periods on the competition of their transportation modes, the relationship between the level of economic development of cities in cold regions and the development of public transportation, as well as the relationship between multi-modal public transportation competition in cities in cold regions in the non-snow and ice periods, and further research is still needed to improve the accuracy of the model in the future.

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