Article

Safety Evaluation of Existing R.C. Buildings: Uncertainties Due to the Location of In Situ Tests

Vincenzo Sepe 1,2, Mariella Diaferio 3,* and Roberta Caraccio 4

1 Department of Engineering and Geology, University “G. d’Annunzio” of Chieti-Pescara, 65127 Pescara, Italy; vincenzo.sepe@unich.it
2 UdA-TechLab, Research Center, University “G. d’Annunzio” of Chieti-Pescara, 65127 Pescara, Italy
3 Department of Civil, Environmental, Land, Building Engineering and Chemistry, Polytechnic University of Bari, 70125 Bari, Italy
4 Independent Researcher, 00100 Rome, Italy; ing.caraccioroberta@gmail.com
* Correspondence: mariella.diaferio@poliba.it

Abstract: The paper aimed to investigate the influence, on the assessment of the structural safety level of an existing r.c building, of the different choices that the technician in charge of a structural evaluation (the "analyst") can make regarding the structural elements to be tested to obtain a prescribed level of knowledge. To this end, the case study of a reinforced concrete framed structure built in the 1960s in Italy was investigated by means of numerical analyses. The probability distribution of the estimated safety levels was evaluated in the paper by means of a Monte Carlo approach, considering the alternative selections of elements done by a large number of analysts, and the probability of unsuccessful safety estimations is discussed for the knowledge levels considered in the Italian technical codes and the Eurocodes.

Keywords: confidence factor; assessment of seismic safety index; Monte Carlo simulation; existing buildings

1. Introduction

The quantitative assessment of the safety of an existing building requires, as is known, the implementation of a numerical model that is able to describe the geometric and mechanical characteristics of the building. For a reinforced concrete (r.c.) building, according to the Italian technical codes [1,2], ASCE 41-17 [3], and the Eurocodes [4], this requires, in particular, knowledge of the construction details (e.g., the quantity and arrangement of reinforcements) and the strength of the materials (concrete and steel).

The formulation of a numerical model, in all its phases, is inevitably affected by epistemic uncertainties, e.g., those due to the possible alternatives in the choice of the representative model of the building [5–7] or of the actions to be considered, and by random uncertainties, e.g., those due to the intrinsic variability of the mechanical parameters throughout the structure, which can therefore be described only from a probabilistic point of view.

This is the consequence of the variability of these properties within the building, which may be due to the different characteristics of the materials used, corresponding to different days during which the concrete was poured, different casting conditions, etc.

It is worth noting that, in practical investigations, modern survey technologies can ensure the exact description of the geometrical features of the investigated building, while the design documentation, if available, may provide data regarding the dimensions and positions of the steel bars inside each structural element. Of course, the acquisition of a given level of knowledge of the investigated building depends on the number of tests that are conducted to assess the mechanical properties of the materials.

For this reason, the codes usually suggest a minimum number of tests to be performed, but even if the number of such tests and the experimental technique are the same, a huge
number of combinations of investigated elements and of points within the same element may be chosen, depending on the technician’s criteria and/or for logistic reasons.

This is an intrinsic source of uncertainty, even neglecting other aspects that can have effects on the results of experimental investigations, such as the environmental conditions in which the tests are performed, the instruments utilised, and several other elements depending on the chosen technique [8–10].

Therefore, for the same knowledge level, a huge number of different experimental results may be obtained, and, consequently, for each different combination of experimental data, this may correspond to a different value of the assessed safety level of the building with respect to seismic events.

For random uncertainties, usually, codes suggest discrete knowledge levels with the aim of describing the availability of information on the existing building examined. Based on the acquired knowledge level, two possible approaches are suggested by standards: the amplification of the force demand or a reduction in the materials’ resistance. For example, FEMA-356 [11] prescribes the introduction of a knowledge factor, dependent on the acquired knowledge level, which amplifies the force demand.

However, for the random uncertainty of mechanical parameters—which is the focus of this paper—the Italian technical codes [1,2], ASCE 41-17 [3], and the Eurocodes [4] introduce a confidence factor (CF) which reduces the mean resistance of materials as provided by on-site tests, with the CF depending on the knowledge level (KL). A larger value of CF is suggested by such codes for a lower number—and therefore a lower representativeness—of the tests carried out. In detail, the codes define three knowledge levels based on the number of acquired data: KL1 is limited knowledge, KL2 is extended knowledge, and KL3 is full knowledge. It is worth noting that different values for the confidence factor correspond to the different knowledge levels. According to the Italian technical code [2] and the Eurocodes [4], the confidence factor related to limited knowledge is \( CF_{KL1} = 1.35 \), while the confidence factor corresponding to extended knowledge is \( CF_{KL2} = 1.20 \) and the confidence factor corresponding to full knowledge is \( CF_{KL3} = 1.00 \).

The possibility of taking into account the many uncertainties that arise in the procedure for the seismic safety assessment of existing buildings with a single confidence factor applied to material strengths has been, and is still, widely debated in the scientific literature [12–16]. In detail, starting from the approach proposed in [12], the present research explores the effects of uncertainties related to the choice of on-site test locations on the assessment of seismic safety indexes.

Specifically, this paper investigates, through the case study of a typical reinforced concrete structure of the 1960s built in Italy, the probability that a safety assessment based on a pre-defined knowledge level and the value consequently assigned to the confidence factor can actually lead to the technician in charge obtaining a precautionary estimate of the building’s capacity to resist seismic actions.

To this end, the actual level of seismic safety of the building—evaluated by assuming perfect knowledge (never available in practice) of the mechanical properties of all the structural elements [12]—is compared with the safety level evaluated under the assumption that the tests were carried out, and thus that the mechanical properties were estimated, only for a small number of structural elements. In this case, as is usual, the safety index was assessed by introducing the elements’ average strengths divided by a CF whose value was fixed according to the level of knowledge corresponding to the number of tests.

For a given knowledge level, the probability distribution of the estimated safety level was evaluated in the paper by means of a Monte Carlo approach, which allows phenomena with significant uncertainties to be modelled by considering randomly generated scenarios and by evaluating the outcomes of the investigated model for each of them. In this way, the mean value of an output, its distribution, and its minimum or maximum values can be estimated. This method is used widely to evaluate the impact of risk and uncertainties in the prediction of outcomes. In the case examined here, the Monte Carlo method was applied to simulate a large set of alternative selections of the structural elements to be
tested, with the same number of such elements randomly picked out, and by estimating the safety index of the building for each scenario.

Moreover, to take into account that in real cases perfect knowledge of structural characteristics can never be attained, nine alternative realisations of the reference model—with equivalent statistical properties—were considered, and therefore a total number of \( 9 \times 10^3 \) analyses were carried out for each one of the three knowledge levels prescribed by the codes.

This procedure was intended to evaluate the effect on safety estimation uncertainty of different possible choices of structural element locations that, ceteris paribus, can be made by the technician in charge (denoted as the analyst in the following, according to [12]), even when the number of such tests is assumed to be constant.

In this way, the paper deals with an intrinsic uncertainty in safety evaluation, starting from the observation that no strategy can assure that the different positions of the tests chosen by different analysts—even when they perform the same number of tests—will lead to unique “best” values of the average strengths of concrete and reinforcing steel. Thus, the paper explores how the unavoidable uncertainty of these values affects the evaluation of the safety index obtained by a given analyst (i.e., what the probability is for a given analyst of underestimating or overestimating the “true”—but never really knowable—safety index).

The rest of the paper is organised in four parts: Section 2 describes the case study and the main assumptions of the performed analyses, Section 3 describes the numerical model implemented to assess building safety levels, Section 4 reports the statistical analyses and discusses the effects of the confidence factor values on the accuracy of the assessed vulnerability index, and Section 5 presents the conclusions.

2. Case Study

The sample case described in the manuscript is a five-floor r.c. structure representative of a widespread typology in the existing building stock built in Italy in the 1960s, with a structure designed for gravitational loads only. Although its geometrical and mechanical characteristics have been suggested by typical real situations, the mechanical model analysed here does not reproduce—and therefore cannot be referred to—any specific existing building. The building plane covers about 800 m², with global dimensions of 58 m \( \times \) 13 m and a total height of 16 m.

The building is realised by infilled r.c. frames and hollow bricks, with floors which can be considered as rigid in their plane. In the 1960s, the main part of the Italian territory was not classified as a seismic-prone zone; thus, the technical codes prescribed the design of buildings that would be capable of resisting gravitational loads and static loads. Typical of many buildings at that time, the loads of the case study building are carried by frames in the longitudinal direction, while only a few of the columns are connected by beams in the transversal direction and belong, therefore, to frames that are able to resist horizontal actions. In detail, as shown in Figure 1a, four longitudinal frames and four transversal frames are present. The span lengths of the beams range between 2.10 m and 5.30 m, the inter-story height is 3.30 m, and the mass distribution is uniform in plan and in elevation.

![Figure 1. A schematic axonometric view of the FE model (a) and a plan view (b) of the building. A longitudinal and a transversal frame have been highlighted in the FE model (with beams shown in red and green colour, respectively). The letters in the plan view represent the different parts considered in the analyses, each one characterised at a given floor by constant mechanical properties of concrete and steel, as reported in Tables 1 and 2.](image-url)
Table 1. Reference model M₀ (actual configuration): concrete compressive strength for each part of the building in Figure 1b.

<table>
<thead>
<tr>
<th>Concrete Strength [MPa]</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>First storey</td>
<td>16</td>
<td>28</td>
<td>16</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>Second storey</td>
<td>16</td>
<td>23</td>
<td>29</td>
<td>22</td>
<td>10</td>
</tr>
<tr>
<td>Third storey</td>
<td>28</td>
<td>22</td>
<td>22</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>Fourth storey</td>
<td>17</td>
<td>22</td>
<td>24</td>
<td>28</td>
<td>9</td>
</tr>
<tr>
<td>Fifth storey</td>
<td>22</td>
<td>18</td>
<td>21</td>
<td>28</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2. Reference model M₀ (actual configuration): steel strength for each part of the building in Figure 1b.

<table>
<thead>
<tr>
<th>Steel Strength [MPa]</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>First storey</td>
<td>360</td>
<td>440</td>
<td>360</td>
<td>440</td>
<td>460</td>
</tr>
<tr>
<td>Second storey</td>
<td>460</td>
<td>340</td>
<td>380</td>
<td>500</td>
<td>400</td>
</tr>
<tr>
<td>Third storey</td>
<td>360</td>
<td>380</td>
<td>480</td>
<td>500</td>
<td>400</td>
</tr>
<tr>
<td>Fourth storey</td>
<td>320</td>
<td>460</td>
<td>500</td>
<td>460</td>
<td>360</td>
</tr>
<tr>
<td>Fifth storey</td>
<td>340</td>
<td>460</td>
<td>340</td>
<td>380</td>
<td>320</td>
</tr>
</tbody>
</table>

The elements’ cross-sections range between 40 cm × 30 cm and 25 cm × 25 cm for the columns and between 40 cm × 60 cm and 20 cm × 50 cm for the beams.

The geometry, load conditions, and design criteria of the case study are representative of typical residential r.c. buildings of the 1960s in Italy. In particular, the quantity and arrangement of steel reinforcements have been designed for gravitational loads according to the allowable stress approach.

Assuming a reference period (\(V_R\)) equal to 50 years, according to the Italian Building Code [1], a seismic hazard corresponding to medium–high seismic level was considered, with a peak ground acceleration (PGA) equal to 0.18 g at the Life-Safety Limit State (SLV).

3. Finite Element Model of the Building

A 3D finite element model of the building was implemented with beam elements and floors acting as rigid diaphragms, assuming the columns to be fixed at the base (Figure 1a).

In order to represent the possible variation in the mechanical properties, corresponding, for example, to different conditions of the preparation, casting, or curing of the concrete or to different batches of steel, the value of the concrete compressive strength (\(f_c\)) was considered to be in the range of 9 MPa to 29 MPa, while for the steel strength (\(f_s\)), the range of variability was considered to be between 300 MPa and 500 MPa.

On this basis, each floor of the building was divided into five parts (see Figure 1b). Each part was considered representative of a different casting day or batch of steel, and, for each part and for each floor, the values of the concrete and steel strengths were randomly chosen in the mentioned ranges, as summarised in Tables 1 and 2, respectively.

The aforementioned procedure aimed to fix the “real” distribution of strengths in the examined structure and, on that basis, to evaluate the “true” seismic safety index of the “real” structure, which represents the final scope of seismic vulnerability assessment.

For the “real” structure, the randomly extracted concrete strength values were characterised by a mean value of 19.6 MPa, a standard deviation equal to 6.1 MPa, and a coefficient of variation equal to 0.3, while the chosen steel strengths had a mean value of 396 MPa, a standard deviation of 63 MPa, and a coefficient of variation equal to 0.16. As it is reasonable to expect, the steel strength was characterised, therefore, by a variability lower than that of the concrete.
These values were assigned to the beams and columns of each of the different regions (Figure 1b) of the reference model of the structure, from now on denoted as M_0 (i.e., model 0) to distinguish it from alternative realisations (Section 4) of the same “true” model, and represent the actual mechanical strengths of these elements. The perfect knowledge of these—never attainable in practice, but here introduced as a hypothesis—allowed evaluation of the “true” level of seismic safety of the building once all the other geometrical and loading details were also known, as assumed in the following, in order to focus attention on the random variation of mechanical properties.

The dynamic analyses of the finite element model, obtained by introducing the values in Tables 1 and 2, were performed considering the spectral acceleration corresponding to the Life-Safety Limit State and assuming a behaviour factor (q) equal to 2, and the seismic safety index (α) of the model was evaluated as the ratio between the peak ground acceleration capacity (PGA_C) and the peak ground acceleration demand (PGA_D).

It is worth noting that the assessed seismic safety index was evaluated by considering “exact” strength values for each of the structural elements and not a mean value constant for the whole structure.

The obtained seismic safety index for the reference model was 0.315, and the analysis highlighted that the safety of the structure was mostly conditioned by the combined axial compression and bending moment capacity. Of course, the structural element capacity that conditions the safety of buildings in other cases studies may be different from the one considered here. However, the results discussed in Section 4 highlight some issues that are also expected to be observed in other possible structural typologies and/or different mechanisms of limit state attainments.

The subsequent step of the numerical investigation described here aimed at evaluating the impact that, during the acquisition of a fixed knowledge level, the alternative locations of the in situ tests selected by different analysts can have on the estimated safety level, all other conditions being equal. Different locations may in fact correspond to different experimentally acquired strengths and, consequently, to different average values of the strengths adopted by the technician in charge of the structural analyses on the basis of the available data. The obtained mean values and the corresponding confidence factors were then utilised to assess the seismic safety indexes.

To compare the evaluated indexes, it was assumed that the attainment of the Life-Safety Limit State is always conditioned by the combined axial compression and bending moment capacity of the columns. Without affecting the more general validity of the proposed approach, which will be further explored in the following part of this research, such an assumption allows, by means of the interpolating surfaces introduced in Section 4, a reduction in the computational effort required.

4. Effects of Uncertainties Related to Test Locations

According to the Italian technical codes [1,2], the seismic vulnerability assessment of an existing structure always requires the execution of in situ tests on the structural elements. Their number depends on the choice of the knowledge level, which in turn is related to the confidence factor (CF) (see Section 1) introduced to pass from the average values of test results to the design values of materials’ strengths. All other factors being equal (the geometry of the building, the quantity and arrangement of the reinforcements, etc.), as assumed in this paper, the design strength of concrete and steel, on the other hand, affect the safety level of the building provided by the structural analysis.

However, even when the number of tests can be assumed as fixed (e.g., according to the indications of technical codes), the technician in charge may make alternative choices regarding the position of the tests. Therefore, for the same knowledge level, different analysts may estimate different mechanical properties for the same building.

As a consequence, even if the numerical model developed to assess the response of the investigated structure and the confidence factor are the same with respect to any other characteristic of the building, different analysts may estimate different values of the seismic
safety index because of the variation in the mean values of the concrete and steel strengths that each analyst assumed to be representative of the existing building on the basis of the available in situ tests. Mean values of concrete and steel strength can therefore be viewed as samples for a statistical analysis of the seismic safety index, as discussed below.

The procedure here proposed is based on a Monte Carlo simulation to select the structural elements whose strengths are assumed to be measured in situ, with the aim of quantifying the uncertainty in the evaluated safety index as the location of the tests varies while keeping their number fixed.

In detail, the probability distribution of the safety indexes evaluated by 100 different analysts was obtained, i.e., considering the mean values of concrete and steel strengths corresponding to 100 different combinations of test locations.

To reduce the computational effort, however, for each given knowledge level, only 15 numerical FEM models were analysed, each one corresponding to a different combination of the mean strengths \( (f_c, f_s) \) virtually obtained by different analysts. As is usual in engineering practice, the obtained mean values were considered to be constant throughout the whole structure during the assessment of the safety index, which is different from the procedure (Section 3) for the evaluation of the actual index, where the knowledge of the local strength of each element was assumed and introduced in the model. The evaluated safety indexes for each \( f_c, f_s \) pair are listed in Table 3 for knowledge level KL2 and a confidence factor \( (\text{CF}_{KL2}) \) equal to 1.2 [2]. The safety indexes \( (\alpha_{KL2}) \) corresponding to other combinations of \( f_c \) and \( f_s \) may be obtained by fitting the obtained value by means of a polynomial law (see Figure 2). Similarly, Table 4 shows the safety indexes \( (\alpha_{KL1}) \) corresponding to knowledge level KL1 and \( \text{CF}_{KL1} = 1.35 \).

| Table 3. Seismic safety index \( (\alpha_{KL2}) \) for 15 different combinations of mean concrete and steel strength values corresponding to knowledge level KL2. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( f_c \) [MPa] | 300 MPa | 340 MPa | 380 MPa | 420 MPa | 460 MPa |
| 15 MPa | 0.242 | 0.268 | 0.283 | 0.293 | 0.315 |
| 20 MPa | 0.253 | 0.278 | 0.288 | 0.302 | 0.323 |
| 25 MPa | 0.258 | 0.283 | 0.297 | 0.310 | 0.327 |

![Figure 2. Polynomial surface (of order 3) fitting the values of the seismic safety indexes \( (\alpha_{KL2}) \) in Table 3 (black dots). \( f_c \) and \( f_s \) are the mean concrete compressive strength and the mean steel strength, respectively.](image)

At first the analyses with knowledge level KL2 are described below, assuming that no other information (e.g., results of tests during the building process) was available on the strength of materials. Under this hypothesis and with reference to the columns, i.e., the elements by which the safety level is conditioned in this case study (see Section 2), the Italian technical codes [1,2] suggest extended in situ testing that involves the testing of two samples of steel bars per storey and two concrete cores for each 300 m² of floor, and then
six cores per storey for the investigated building (about 800 m² plan). For the five-floor building considered here, therefore, the strengths ($f_c$ and $f_s$) to be considered in the safety evaluation were obtained for each analyst as the average of the strengths of 30 cores and 10 steel bars extracted by columns.

Table 4. Seismic safety index ($\alpha_{KL1}$) for 15 different combinations of mean concrete and steel strength values corresponding to knowledge level KL1.

<table>
<thead>
<tr>
<th></th>
<th>300 MPa</th>
<th>340 MPa</th>
<th>380 MPa</th>
<th>420 MPa</th>
<th>460 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 MPa</td>
<td>0.230</td>
<td>0.247</td>
<td>0.263</td>
<td>0.283</td>
<td>0.293</td>
</tr>
<tr>
<td>20 MPa</td>
<td>0.236</td>
<td>0.253</td>
<td>0.268</td>
<td>0.288</td>
<td>0.297</td>
</tr>
<tr>
<td>25 MPa</td>
<td>0.242</td>
<td>0.258</td>
<td>0.273</td>
<td>0.293</td>
<td>0.302</td>
</tr>
</tbody>
</table>

A Monte Carlo approach was adopted to define 100 different cases. For each case, the procedure simulated the extraction of six values of concrete strength and two values of steel strength for each floor, corresponding to a random subset of the columns, and, based on such values, the mean values of the concrete compressive strength ($f_c$) and steel strength ($f_s$) were evaluated. The obtained values are plotted in Figure 3.

By means of the interpolating surface in Figure 2, the seismic safety index was then evaluated for each combination of strength values in Figure 3, and their cumulative distribution is shown in Figure 4. Moreover, in Figure 4, the cumulative distribution of the Gaussian curve which fits the evaluated safety indexes is plotted.

The red broken line in Figure 4 corresponds to the safety index evaluated by means of the reference model ($M_0$) (Section 2), which assumed perfect knowledge (never attainable in practice) of the effective strength of each structural element. The safety index of the reference model can therefore be assumed as an ideal target, whose comparison with the indexes evaluated by different analysts may allow estimation of the reliability of the safety level attainable when a finite (and usually small) number of in situ tests are available, as in real cases.
Cumulative distribution functions of seismic safety indexes obtained for knowledge level KL2 by performing a Monte Carlo simulation: the red broken line represents the actual safety index evaluated by means of the reference model (M₀); the blue stars represent the indexes corresponding to each of the 100 cases; the black line represents the Gaussian curve approximating the numerical data.

Figure 4 highlights that for the randomly selected locations of in situ tests corresponding to the average values of $f_c$ and $f_y$ in Figure 3, the probability of overestimating the seismic safety is negligible once the confidence factor $\text{CF}_{KL2} = 1.2$ is adopted in the analysis, as suggested by the Eurocodes [4] and the Italian codes [2] for the level of knowledge KL2.

To check with a wider set of analysts the probability of obtaining an unsafe evaluation of the safety index, the aforementioned procedure was repeated another nine times for different sets of 100 randomly selected values of concrete and steel strength, leading to another nine cumulative distribution functions of the safety index, which are shown in Figure 5 together with the curve in Figure 4. A total number of $10^3$ combinations were therefore analysed to obtain the data plotted in Figure 5.

This confirms that for the reference model (M₀) corresponding to the concrete and steel resistance in Tables 1 and 2, this $\text{CF}_{KL2}$ value is well calibrated because it effectively guarantees, in almost all cases, a slightly precautionary estimation of the structural capacity.
Due to the inhomogeneity of the mechanical properties in Tables 1 and 2, however, the capacity of the reference model (M₀) considered up to this point—and therefore its safety risk (α₀ = 0.315)—was conditioned by the effective distribution of such characteristics in the different regions of each storey, which is never completely knowable in practice.

To evaluate the impact of such a distribution on the capacity of the model assumed as a reference to describe the real structure, therefore, eight alternative models (M₁–M₈) were considered to be representative of eight different realisations of the building described so far. These were obtained by randomly re-distributing the steel and the concrete properties of the elements between the different 25 regions of the building and thus obtaining different combinations of \( f_c, f_s \) without changing the parameters (i.e., the mean values and standard deviations) of their values. The procedure performed for the M₀ model was also carried out for each of the eight new models, leading to 10 cumulative distribution functions for each model, i.e., a total number of \( 9 \times 10³ \) analyses.

For each model, Figure 6 compares the cumulative distribution functions of the safety indexes evaluated by 10 different sets of analysts with the actual safety index of the model.

![Figure 6](image-url) Cumulative distribution functions of seismic safety indexes obtained for knowledge level KL2 by performing the Monte Carlo simulation 10 times for nine different models (M₀–M₈) with the same geometrical features and mean values of material strengths. The red broken lines represent the actual safety indexes of each model.

Figure 7a shows, for each model and for each set of analysts, the probability of underestimation (\( p_* \)) of the actual safety index (blue circles) and the average \( \mu_{p*} \) (red crosses)—also reported in Table 5—as a function of the actual safety index of each model. Although for one of the models (model M₇) the mean probability of overestimating the actual safety index was about 50%, the overall mean value of the probability of underestimation (\( p_* \)) for the nine models was 92% (Table 5), thus confirming that the confidence factor CFKL2 allows a cautionary evaluation of the seismic safety index.

Based on the statistical distribution of the safety indexes of the nine models (M₀–M₈) and denoting their average values and standard deviations \( \mu_\alpha \) and \( \sigma_\alpha \), respectively, Figure 7a also reports, as a vertical green line, the average value (\( \mu_\alpha = 0.313 \)), and it can be verified that, in this case, the probability of underestimation of the safety index by the analysts is almost equal to 100%. Figure 7a also shows with a green broken line the value of the safety index \( \mu_\alpha - \sigma_\alpha = 0.299 \) with an 84% probability of exceedance, and in this case the probability of underestimation of the safety index by the analysts is equal to 87%.
The same results can be shown by plotting the probability ($p_{\text{exc}}$) of exceeding the actual safety index, as reported in Figure 7b.

It is worth noting that for a major part of the analysed models (i.e., seven models out of the total number of nine), almost all the analysts underestimated the building’s actual structural capacity with an average probability of at least 95% and that, considering all of the nine models, the mean value of the probability of underestimation was equal to 92%. Thus, adopting a confidence factor $CF_{KL2} = 1.2$, the probability of overestimating the real structural capacity of the sample case discussed here is limited to about 8%. This value can be considered acceptable when compared with other sources of uncertainties which are usually involved in structural capacity assessment, such as those related to the choice
of software, the method adopted for the analysis (linear or non-linear), and those strictly connected to the experimental tests.

Table 5. Actual safety index (α) and the corresponding average probability of underestimation (μF) for each of the nine models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Actual Safety Index</th>
<th>Average Probability of Underestimation (μF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M0</td>
<td>0.315</td>
<td>99%</td>
</tr>
<tr>
<td>M1</td>
<td>0.324</td>
<td>100%</td>
</tr>
<tr>
<td>M2</td>
<td>0.328</td>
<td>100%</td>
</tr>
<tr>
<td>M3</td>
<td>0.333</td>
<td>100%</td>
</tr>
<tr>
<td>M4</td>
<td>0.298</td>
<td>83%</td>
</tr>
<tr>
<td>M5</td>
<td>0.302</td>
<td>95%</td>
</tr>
<tr>
<td>M6</td>
<td>0.310</td>
<td>99%</td>
</tr>
<tr>
<td>M7</td>
<td>0.293</td>
<td>51%</td>
</tr>
<tr>
<td>M8</td>
<td>0.314</td>
<td>99%</td>
</tr>
<tr>
<td>Mean value</td>
<td>0.313</td>
<td>92%</td>
</tr>
</tbody>
</table>

To investigate the influence of the number of in situ tests, the procedure was repeated by assuming that three concrete cores and one steel bar were extracted at each of the five storeys, with a total number of 15 values of concrete strength and 5 values of steel strength randomly selected in a subset of the columns for each of the 100 simulated samples.

In this case, corresponding to the knowledge level KL1 of the Italian technical code, a confidence factor CFKL1 = 1.35 was suggested [2]. As shown in Figure 8, which reports the cumulative distribution functions of the seismic safety indexes obtained by means of an interpolating surface based on Table 4 (confidence factor CFKL1 = 1.35), the probability of an unsafe evaluation becomes negligible, as also shown in Figure 9, which was obtained by means of the same procedure adopted to evaluate Figure 7.

Figure 8. Cumulative distribution functions of seismic safety indexes obtained for knowledge level KL1 by performing the Monte Carlo simulation 10 times for nine different models (M0–M8) with the same geometrical features and mean values of material strengths. The red broken lines represent the actual safety indexes of each model.

This confirms that for the lowest level of knowledge allowed by the codes, the value of CFKL1 = 1.35 is well calibrated in the case study described here.
Figure 9. (a) Probability of underestimation ($p^*$) vs. actual safety index and (b) probability of exceedance ($p_{\text{exc}} = 1 - p^*$) for knowledge level KL1. The green line represents the mean value ($\mu_\alpha$) of the actual safety indexes related to the nine models (M0–M8); the green broken line represents a safety index equal to $\mu_\alpha - \sigma_\alpha$.

The above-described procedure was also carried out to investigate the case of the full-knowledge level, i.e., KL3, when the confidence factor is equal to 1.0 [2], and, in this case, the seismic safety index of the sample building discussed here was always overestimated. Although this aspect deserves to be further explored in the future by examining different structures, what seems to emerge clearly from these circumstances is that the choice of the confidence factor cannot always and only be related to the number of available tests and that it should instead explicitly take into account their higher or lower homogeneity. Accordingly, it should eventually be assumed to be equal to 1 only when the dispersion of experimental results is particularly low, unlike the case reported here.

For the building considered here, it was in fact observed that, even if the nine alternative (and statistically equivalent) models M0–M8 are characterised by the same mean values of concrete and steel strength, i.e., $f_{c*} = 19.64$ MPa and $f_{s*} = 396$ MPa, their safety indexes
indexes vary with respect to their mean values ($\mu_\alpha$) in an interval $\pm1.46$ times their standard deviations $\sigma_\alpha$. Moreover, the safety indexes of models $M_0$–$M_8$ (probably because they are conditioned by the structural elements with the lowest resistance) were generally lower than the “ideal” safety index ($\alpha^* = 0.337$), corresponding to the hypothesis of absolute homogeneity of the mechanical characteristics of concrete and steel, i.e., equal to $f_{c^*}$ and $f_{s^*}$ for each structural element, similarly to the results of [12]. As a consequence, assuming the average values of experimental tests as design strengths, as when $CF = 1$ is assigned, can lead to the overestimation of safety indexes in cases of significant material inhomogeneity, as in those of models $M_0$–$M_8$.

This suggests that, in such cases, the confidence factor should be precautionarily assumed to be greater than 1, independently of the number of in situ tests, in order to take into account that the structural safety of a building can be highly conditioned by the structural elements with the lowest capacities. The suggestion of adopting in such cases a confidence factor greater that 1 is in accordance with results in [12].

Future research will therefore be devoted to applying the above-described procedure to different probability distributions of mechanical properties, with the aim of exploring how and to what extent the calibration of the confidence factor should include the dispersion of experimental results, an issue that is only qualitatively considered in the Italian codes [2].

Extending the results discussed here, which were obtained by adopting the linear analysis with behaviour factor, the future steps of the research will also consider the effect of the introduction of non-linear analyses (for example, push-over analyses, etc.) for the assessment of the seismic safety indexes of buildings with mechanical parameter distributions similar to those examined here, with the aim of verifying the influence of their mean values and their variations on safety indexes.

5. Conclusions

In this paper, the case study of a framed reinforced concrete building designed to resist only gravitational and static loads, representative of a widespread typology of structures built in the 1960s in Italy, is examined. Indeed, buildings of this kind usually stand in seismic-prone zones, according to the current codes, and this requires improvements to their structural and building performances to guarantee acceptable levels of safety and habitability conditions. The actuality of this topic is also confirmed by the attention devoted by building codes to the analysis of existing buildings.

These codes prescribe, among other procedures, the execution of experimental tests, the numbers of which depend on the desired knowledge level.

Although this topic is still debated in the scientific literature [12–16], the Italian technical codes [1,2] and the Eurocodes [4] prescribe the introduction of a confidence factor, based on the acquired knowledge level, in the assessment of seismic safety indexes, which serves to reduce the mean values of experimentally evaluated mechanical strengths of structural materials.

It is worth noting that the positions in which the tests are performed depend on several factors, and, clearly, different analysts may choose different locations; consequently, they may assess different values of the mechanical properties and obtain different safety indexes.

In the present paper, starting from the approach in [12], the case study of a residential r.c. building built in the 1960s in Italy is discussed with the aim of investigating the implications of the different choices that the technician in charge (the “analyst”) can make regarding the structural elements to be tested for obtaining a prescribed level of knowledge.

The case study of an r.c. building was examined, assuming, initially, perfect knowledge of the material mechanical properties of each structural element and, on this basis, evaluating the “actual” safety index.

Then, numerical analyses were performed by considering the three knowledge levels prescribed by the building technical codes and, for each level, numerically simulating the acquisition of experimental strengths.
In detail, a Monte Carlo approach was adopted to consider the effects of the different locations of the tests. A numerical simulation of $10^3$ possible combinations of concrete and steel mechanical properties, as virtually assessed for each knowledge level suggested by the codes, was carried out, and the corresponding seismic safety indexes were evaluated by introducing the confidence factor suggested by technical codes for each knowledge level.

To evaluate the capacity of the model assumed as a reference to describe the real structure, the same procedure was applied to nine different realisations of the same building, and thus a total number of $9 \times 10^3$ combinations were analysed for each knowledge level. The analyses performed for the sample case show that the confidence factors corresponding to KL1 and KL2 [2] could effectively provide a precautionary assessment of the structural capacity of the building. With the level of knowledge KL2, in fact, for the analysed models (representative of nine alternative realisations of the reference model with equivalent statistical properties), almost all the analysts underestimated the actual building’s structural capacity with an average probability of 92%. Thus, adopting a confidence factor $\text{CF}_{KL2} = 1.2$, the probability of overestimating the real structural capacity of the sample case discussed here was limited to about 8%, while for the level of knowledge KL1 ($\text{CF}_{KL1} = 1.35$), the probability of an unsafe evaluation became negligible.

For the full-knowledge level, the safety index was instead overestimated when a value of $\text{CF}_{KL3} = 1$ was assumed, thus suggesting that the calibration of this parameter should not only depend on the number of available in situ tests but must consider their dispersion.

As is widespread in engineering practice, in the present paper a linear numerical analysis was performed. However, further studies must be performed to extend the findings to the assessment of safety indexes by means of non-linear approaches.

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