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Raw Material Purchasing Optimization Using Column Generation

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Abstract: The raw material purchasing (RMP) problem is to determine the purchasing quantities of raw materials in given periods or cycles. Raw material purchasing optimization is crucial for large-scale steel plants because it is closely related to the supply of raw materials and cost savings. The raw material purchasing of large-scale steel enterprises is characterized by many varieties, large quantities, and high costs. The RMP objective is to minimize the total purchasing cost, consisting of the price of raw materials, purchasing set-up costs, and inventory costs, and meet product demand. We present a mixed integer linear programming (MILP) model and a column generation (CG) solution for the resulting optimization problem. The column generation algorithm is the generalization of the branch and bound algorithm while solving the linear programming (LP) relaxation of MILP using column generation (CG), and its advantage is to handle large-sized MILPs. Experimental results show the effectiveness and efficiency of the solution.

Keywords: purchasing; optimization; MILP; column generation

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1. Introduction

Raw material purchasing is crucial for steel enterprises because it is related to the supply of raw materials and cost savings [1]. The raw material purchasing of large-scale steel enterprises is characterized by many varieties, large quantities, and high costs. The raw materials purchased by steel enterprises include three types: iron, coal, and auxiliary materials. Iron includes iron ore, pellets, sinter, pig iron, scrap steel, and so on. Coal includes all pulverized coal, lump coal, and coke. Auxiliary materials, also known as solvents, include limestone, quicklime, serpentine, etc. In a large-scale iron and steel enterprise, the purchase volume of raw materials reaches tens of millions of tons and the purchase expenditure reaches tens of billions of yuan. Therefore, it is very important to make the optimal raw material purchasing decisions.

Raw material procurement optimization aims to determine the purchasing quantities of raw materials in some given periods or cycles, make the purchasing total cost minimal, and meet the production demand. The purchasing cost includes material price, procurement set-up cost (including negotiation and communication, transportation, etc.), and inventory cost (including capital occupation, management, maintenance, etc.). Raw material purchasing is often complex because it involves many factors, such as supplier selection, raw material price fluctuations, inventory control, product demand, etc., and its solution also involves many areas [2–4], and it has been a hot area of study in management and operations research [5–7].

Determining the purchase amount of raw materials is closely related to the number of varieties of raw materials, which affects the complexity of the problem. According to the number of varieties purchased, raw material purchasing problems can be naturally divided into two categories: single-item procurement and multi-item procurement. Single-variety purchasing decisions determine the purchase of a single raw material for many periods. The famous economic order quantity (EOQ) model is a single-item purchasing example. The EOQ assumes the demand is steady and constant, the price of raw materials is unchanged, and the supply of raw materials is enough. The EOQ model has an analytic solution. However, when the assumption of steady-state demand is dropped, the single-item procurement problem becomes difficult to solve. The W-W [8] problem is a dynamic version of the EOQ. It allows demand to vary from period to period and can be solved with dynamic programming; however, as the number of periods increases (e.g., 300), the solution is time-consuming. The W-W problem still assumes the satisfaction of requirements is unlimited. When there is a limitation on the availability of raw materials, the problem becomes NP-hard [9] and is called the single-item capacitated lot-size problem (SCLP). Most early research interest is focused on this kind of problem [10].

For the multi-item purchasing problem, since complex material structure and process relationships often need to be considered, mathematical programming models are the most common choice [1,11,12]. Mathematical programming models have the advantage of being able to handle complex constraints. There are also some other methods to study the multi-item procurement problem, such as fuzzy mathematics-based methods and data-driven methods [13,14]. Fuzzy mathematics based on fuzzy logic has been widely used in many fields, like industrial process control, production planning and scheduling, image processing, etc. [15]. Data-driven methods are a hot research topic nowadays and are used in almost all fields [7].

The main contributions of this paper are: We present a mixed integer programming for this large-scale raw material procurement problem. We present a column generation solution for the RMP problem.

The rest of this paper is organized as follows. Section 2 gives a literature review. Section 3 states the model and the solving method—the column generation. Section 4 gives the experimental results. Section 5 concludes this paper.

2. Literature Review

Research on raw material purchasing can be generally divided into two categories: single-item procurement and multi-item procurement. The single-item raw material purchasing problem is the first studied problem [8]. However, general single-item procurement problems are NP-hard, so most solutions are heuristic [9]. Blackburn and Millen [16] examine several heuristics for the single-item uncapacitated lot-size version for single-item purchasing and show that the Silver–Meal heuristic can provide cost-effective performance superior to that of the W-W algorithm. DeMatteis [17] tests a heuristic called the part-period algorithm for the above problem and shows that the part-period algorithm is well-suited for industries whose demand forecast extends for a limited number of periods. Ekici et al. [18] studied cyclic ordering policies for single-item purchasing to minimize total periodic ordering costs consisting of fixed ordering costs, variable purchasing costs, and inventory holding costs. They show that their heuristic provides better results compared to other methods in the literature. Tempelmeier [19] proposes a heuristic for supplier selection and purchase order sizing. The heuristic has been tested as a component of the advanced planner and optimizer (APO) of software of SAP. Hamid Mirmohammadi et al. [20] propose a branch and bound-based optimal approach for single-item purchasing with quantity discounts. Experimental results show the performance of the optimal approach in computational time measurement. Kania et al. [21] propose an integration model for lot-sizing and safety strategy placement for handling demand uncertainty. The goal is to store a proper quantity of items to satisfy demand but concurrently avoid shortages and excess inventory. The multi-objective model is selected and a computation-effective method is used for the solution. There is much literature on single-item procurement and interested readers can refer to relevant publications.

For the multi-item raw material purchasing problem, there are more literature studies on related issues. Gao and Tang [1] constructed a multi-objective linear programming (MOLP) model for RMP purchasing and supplier selection, where three optimization objectives, the minimization of the total purchasing cost, tardy-delivery ratio, and scrap fraction are used. The constraints considered include the purchasing budget, production demand, inventory capacity, and technology specifications. The model is solved using the point estimate weight-sums (PEWS) method. Cunha et al. [11] proposed a mixed integer linear programming (MILP) model for production lot size and raw materials purchasing based on the process industry background. The optimization objective is to minimize the raw materials costs, ordering costs, inventory holding costs, setup costs, and production costs. The constraints include purchasing, inventory control, packing tasks, and multipurpose storage tank control. The model is solved with the CPLEX solver. Arnold et al. [12] considered raw material purchasing with price fluctuations. Their objective is to maximize the net present value (NPV). The constraints include inventory movement and backorders. The model is solved by Pontryagin's maximum principle. Ahmed et al. [22] considered a multi-product and multi-period production system for minimizing the purchasing cost and inventory costs. They propose a genetic algorithm (GA) for the solution of the problem. Kazemi and Davari-Ardakani [23] proposed a multi-objective model that integrates project scheduling and raw material purchasing simultaneously and solved it with NSGA-II and Taguchi. Kannan et al. [13] proposed a multi-criteria decision-making (MCDM) approach called fuzzy axiomatic design (FAD) to select the best green supplier for a Singapore-based plastic manufacturing company for purchase and supply cycle to ensure the supplier of goods and meeting of green criteria. Muteki and MacGregor [14] developed a data-driven approach for raw materials purchasing. A partial least squares (PLS) regression is presented to extract the latent purchasing raw materials as input for a sequential quadratic programming (SQP) model that optimizes the raw materials purchasing.

To sum up, although there are some studies on the problem of multi-item procurement, there are very few studies on iron–steel raw materials, and the existing solution methods are mainly heuristic, or the optimal solution is only for small-scale problems. The model and solution proposed in this paper can effectively solve the large-scale iron–steel raw material purchasing problem that fills the gap.

3. Method

3.1. Model

The real-world RMP problem is from a certain large-scale iron and steel enterprise (below called M-Steel) in China. M-Steel purchases about 15 million tons of raw materials per year with about 4.5 billion of CNY of purchase expenditure and the purchased number of raw material items is over 3000. The total raw materials purchasing cost accounts for more than 70% of the total operating cost of M-Steel. The purchasing cost is composed of the raw materials price, purchasing set-up cost, and inventory cost. The optimization problem is to minimize the sum of the above three costs.

Assumptions:

A1: a single period is set as a month; therefore, a one-year length includes 12 periods.

A2: the inventory of each item of raw materials in each period is known.

A3: the demand quantity of each item in each period is known.

The RMP model is as follows:

$$\min \sum_{i=1}^{N} \sum_{t=1}^{T} (p_{it}x_{it} + s_{it}Y_{it} + h_{it}I_{it}) \tag{1}$$

subject to

$$I_{i,t-1} + x_{it} + I_{it} = d_{it}, \quad i = 1, \ldots, N, \ t = 1, \ldots, T, \tag{2}$$

$$x_{it} \le MY_{it}, \qquad \forall\, i, t, \tag{3}$$

$$\sum_{i=1}^{N} r_{ki}x_{it} \le C_{kt}, \qquad t = 1, \ldots, T;\ k = 1, \ldots, K, \tag{4}$$

$$I_{i0} = 0,\ I_{iT} = 0, \qquad i = 1, \ldots, N, \tag{5}$$

$$x_{it} \geq 0, I_{it} \geq 0, \qquad i = 1, \ldots, N, t = 1, \ldots, T \tag{6}$$

The model is a MILP. Its symbolic meanings are shown in Table 1. The objective of (1) is to minimize the sum of raw materials price, purchasing set-up costs, and inventory holding costs. Constraint (2) represents supply-demand balance. Constraint (3) represents the incidence relation of Y_{it} and x_{it}, where M is a sufficiently big number (e.g., $M \geq max\sum_{k=t}^{T} d_{ik}$) and $Y_{it} = 1$ if and only if $x_{it} > 0$. Constraint (4) represents the capacity absorption of each resource not exceeding the corresponding resource restriction available in each period. Constraint (5), without loss of generality, assumes that the starting inventory of period 1 and the ending inventory of period T are 0. Constraint (6) indicates that all purchase quantities and inventory are non-negative.

Table 1. The meanings of notations in the model.

Notion	Type	Meaning
i	index	index of the items
j	index	index of the periods
k	index	index of the resources
N	set	number of items
T	set	number of periods
K	set	number of resources
d_{it}	parameter	the demand for item i in period t
M	parameter	a enough large positive number
r_{ki}	parameter	unit absorption of item i for resource k
C_{kt}	parameter	available amount of resource k in period t
s_{it}	parameter	purchasing set-up cost of item i in period t
p_{it}	parameter	unit price of item i in period t
h_{it}	parameter	unit inventory holding cost of item i in period t
x_{it}	variable	the purchase amount of item i in period t
I_{it}	variable	inventory level of item i at the end of period t
Y_{it}	variable	binary variables, =1, if the item i is purchased in period t, otherwise =0

The above MILP model is a single-level, multi-item capacitated lot-sizing problem (CLSP) [24,25]. However, it is a large-scale mixed integer programming problem and is thus difficult to solve.

3.2. Column Generation Solution

Solving a CLSP is NP-hard [9]; therefore, most existing solutions are heuristic [26–28], and those exact solutions are only for small-sized problems [29,30]. However, in the actual industrial production environment, optimization problems are mostly large-sized, such as the RMP problem in this paper. We developed a column generation (CG) solution [31–33] for the RMP because it can handle large-sized problems. In the CG algorithm, we first convert the original problem to an equivalent set partitioning (SP) one and then use column generation to solve the resulting SP problem.

3.2.1. Set-Partitioning Reformulation

We note that for the original MILP problem, if for each item $i \in \Omega$ we introduce a set of schedules, denoted by Ω_i, where a schedule is a T-dimension vector with the purchasing quantities in T periods for the item i, then, deciding the purchasing quantity of item i in each period t is equivalent to identifying a schedule $j \in \Omega_i$ for that item. We consider a specific set of schedules that satisfy the extreme flow conditions for each item i, i.e., $I_{it-1} \times x_{it} = 0$ [8]. These schedules are sometimes also called W-W schedules with the property that the demand at a period is completely supplied by either inventory or purchasing and/or a purchasing activity happens if and only if the purchased quantity satisfies the demands of an integral number of periods for any item. Based on the idea, the RMP model can be reformulated as a set partitioning problem:

$$\min \sum_{i\in\Omega} \sum_{j\in\Omega_i} b_{ij}\theta_{ij} \tag{7}$$

subject to

$$\sum_{j\in\Omega_i} \theta_{ij} = 1, \forall i \in \Omega \tag{8}$$

$$\sum_{i\in\Omega} \sum_{j\in\Omega_i} R_{ijt}^k \theta_{ij} \le C_{kt}, \forall t, k = 1, \ldots, K \tag{9}$$

$$\theta_{ij} \in \{0, 1\}, \forall i \in \Omega, j \in \Omega_i \tag{10}$$

where, decision variable θ_{ij} is equal to 1 when schedule j is selected, and 0 otherwise. K is the number of resources, parameter b_{ij} is the cost associated with the jth schedule for item i and parameter R_{ijt}^k is the k^{th} resource requirement in period t for schedule j, and parameter C_t^k is the kth resource capacity available in period t. Let x_{ijt}, Y_{ijt}, and I_{ijt} denote the purchasing quantity, purchasing set-up, and inventory variables associated with the schedule j for item i in period t, respectively. Consequently, hold

$$b_{ij} = \sum_{t=1}^{\mathrm{T}} (p_{it}x_{ijt} + s_{it}Y_{ijt} + h_{it}I_{ijt}) \tag{11}$$

$$R_{ijt}^k = r_{ki}x_{ijt},\ t = 1, \ldots, T,\ k = 1, 2 \tag{12}$$

In the SP model, the objective function (7) minimizes the total sum of purchasing costs, set-up costs, and inventory holding costs. The partitioning constraint (8) requires that for each item i, only a schedule can be selected. Constraint (9) requires the resource absorption of all schedules not to exceed the capacity of each resource in the period t available. Constraint (10) indicates the decision variable θ_{ij} is a 0–1 variable.

For the W-W schedules Manne [30] called dominant schedules because there are 2^{T-1} dominant schedules per item i, generating them may be rather time-consuming or impractical and the resulting SP problem is far too large to solve. Therefore, a column generation algorithm is introduced to solve the SP problem, in which not all $N{\cdot}2^{T-1}$ of schedules are generated explicitly. Alternatively, many schedules are handled implicitly by column generation techniques [34], and, thus, the large-sized SP problem is effectively resolved.

3.2.2. Column Generation Principle

Column generation algorithms are used to solve a MILP problem with numerous columns. For handling the huge number of variables (columns) of the MILP, the column generation algorithm instead of explicitly enumerating all columns in the constraint matrix, called the master problem (MP), only a very small subset of columns for the initial solution is used and the remaining columns can be added only when needed. We refer to an MP with only a subset of columns as the restricted master problem (RMP). The column generation algorithm solves the large-sized MILP by solving some LP relaxations of RMPs. For finding the added columns to an RMP, we check if there is a potential adding column with a negative reduced cost by solving an optimization subproblem, called the pricing problem. If none can be found, the current LP relaxation solution to the RMP is optimal for the original MILP. If one or more such columns are found, they will be added to RMP, and the solution process is repeated.

3.2.3. LP Relaxation Solution

Suppose that for each item $i \in \Omega$, a subset of feasible schedules $\Omega_{i'}$ is already found and the RMP has a feasible solution $\boldsymbol{\theta}$, and let $\mathbf{U}$, $\mathbf{V}$ be the associated dual solutions and that the unrestricted dual variables u_i ($i \in \Omega$) are associated with the partitioning constraints and the negative dual variables v_{kt} ($k = 1, \ldots, K$; $t = 1, \ldots, T$) are associated with the k^{th} resource constraint. From linear programming duality we know that $\boldsymbol{\theta}$ is the optimal solution for LP if and only if for any $i \in \Omega$ and any $j \in \Omega_i$, the reduced cost β_{ij} is non-negative, i.e.,

$$\beta_{ij} = b_{ij} - \sum_{t=1}^{T} \sum_{k=1}^{K} v_{kt} r_{ki} x_{ijt} - u_i \geq 0, \text{ for all } i \in \Omega, \ j \in \Omega_i \tag{13}$$

Testing the optimality of $\boldsymbol{\theta}$ for LP can thus be done by solving the following pricing problem:

$$\beta_i = \underset{j}{\text{minarg}} \sum_{t=1}^{T} \left(\left(p_{it} - \sum_{k=1}^{K} v_{kt} r_{ki} \right) x_{ijt} + s_{it} Y_{ijt} + h_{it} I_{ijt} \right) - u_i, \text{ for all } i \in \Omega \tag{14}$$

If $\beta \geq 0$ for all i, then $\boldsymbol{\theta}$ is optimal for the LP relaxation, otherwise the i, j define a new entering basis column (clearly $\beta_{ij} < 0$), Let $\Omega_{i'} = \Omega_{i'} \cup \{j\}$, resolving the RMP.

3.2.4. Pricing Problem

The pricing problem consists of N minimization subproblems.

$$\beta_i = \underset{j}{\text{minarg}} \sum_{t=1}^{T} \left(\left(p_{it} - \sum_{k=1}^{K} v_{kt} r_{ki} \right) x_{ijt} + s_{it} Y_{ijt} + h_{it} I_{ijt} \right) - u_i, \text{ for } i = 1, \ldots, N.$$

Note that for fixed i, the subscription may be eliminated. Instead of enumerating all schedules j for the negative reduced costs for item i, the pricing problem instead solves an uncapacitated single-item lot-sizing problem (ULS_i) to find the column with the most negative reduced cost:

$$\min \beta \tag{15}$$

s.t.

$$I_{t-1} + x_t - I_t = d_t, \ \forall\, t \tag{16}$$

$$x_t \leq MY_t, \ \forall\, t \tag{17}$$

$$I_0, I_T = 0 \tag{18}$$

$$x_t, \ I_t \geq 0, \forall t \tag{19}$$

where the meanings of the occurred notations are the same as before. This ULS_i problem can be easily solved by a dynamic programming algorithm [8]. If the cost for all $i \in \Omega$ holds $\beta \geq 0$, then the LP relaxation is solved optimally. Otherwise, the solution (column) (with $\beta < 0$) is added to the RMP.

3.2.5. Integer Solution

The LP relaxation solution from (Section 3.2.4) is not the final integer solution we needed. To obtain integer solutions, we make use of the branch and bound technique to obtain the integer solutions with a branching scheme designed as follows [35]:

Provided that the LP relaxation solution of the SP problem, $\sum_{j \in \Omega_i} x_{ijt} \theta_{ij} = \alpha$, is fractional, then we can implement a branching scheme for the MILP: on one branch we let $\sum_{j \in \Omega_i} x_{ijt} \theta_{ij} \leq \lfloor \alpha \rfloor$ and on the other branch we require $\sum_{j \in \Omega_i} x_{ijt} \theta_{ij} \geq \lceil \alpha \rceil$. The branching scheme can be performed in the restricted main problem (RMP) by deleting columns for item i that violates the upper bound on component t on the first branch or the lower bound on the other branch. When a new column is generated for item i an upper bound of $\lfloor \alpha \rfloor$ on component t is added to the pricing problem in the first branch and a lower bound of $\lceil \alpha \rceil$ on the second branch. Then, the traditional branch and bound procedure is executed until the optimal integer solution is obtained. Now, we can give the complete column generation algorithm as Algorithm 1.

Algorithm 1. Column generation algorithm

Input: $p_{it}, s_{it}, h_{it}, r_{ki}, d_{it}, C_{kt}, M, N, T, K.$

1 Solve ULS_i, $i = 1, \ldots, N$, to get the initial solution, x_{it}, $i = 1, \ldots, N$, $t = 1, \ldots, T$.
2 Solve the relaxed RMP of the SP problem with x_{it}, getting the dual solution u_i, $i = 1, \ldots, N$, v_{kt}, $k = 1, \ldots, K$, $t = 1, \ldots, T$.
3 Solve the updated pricing problem:

$$\beta_i = \min_j \arg \sum_{t=1}^{T} ((p_{it} - \sum_{k=1}^{K} v_{kt} r_{ki}) x_{ijt} + s_{it} Y_{ijt} + h_{it} I_{ijt}) - u_i, \text{ for } i = 1, \ldots, N,$$

4 for finding the negative reduced cost $\beta_i < 0$.
5   if $\beta_i > 0$ for all i then
6   if all θ_{ij}, $i \in \Omega$, $j \in \Omega_i$, is integral
7   stop
8   else
9   go to 14
10   else
11   execute step 12.
12 Generate the adding column:

$$\begin{bmatrix} x_i^T \\ e_i^T \end{bmatrix},$$

where, $x_i = [x_{i1}, \ldots, x_{iT}]$, e_i, N-dimensional unit row vector with its ith component equal to 1
13 Execute the branch and bound procedure
14 Add constraints $\sum_{j \in \Omega_i} x_{ijt} \theta_{ij} \leq \lfloor \alpha \rfloor$ and $\sum_{j \in \Omega_i} x_{ijt} \theta_{ij} \geq \lceil \alpha \rceil$ to the pricing problem for the two branches, generate two new RMPs, and solve, until the optimal integer solution is obtained.

Output: x_{it}, $i = 1, \ldots, N$, $t = 1, \ldots, T$.

4. Experimental Results

4.1. Algorithm Test

We have implemented the column generation algorithm with Visual C++. For testing the effectiveness and rightness, we have used the CG algorithm to solve two examples: No. 1 from the literature [25] is a 12-item × 10-period CLSP problem, and No. 2 is a group of examples randomly generated for a 10-task × 4-machine generalized assignment problem (GAP) [33]. The original data are left out and here we only list the computational results for verifying the effectiveness; see Tables 2 and 3.

Table 2. Computational results for example No. 1 [25].

Algorithm	Cost 10 Periods	Feasible to Extra Constraint
LV	13,924,130	No
DS	13,113,060	No
ABC1	14,523,330	No
ABC10	13,855,930	No
ABC72	13,080,230	No
CG	13,290,690	No

Remark: LV means LV heuristic [10], DS means DS heuristic [36], ABC_1, ABC_{10}, and ABC_{72} heuristics come from the literature [37]. The CG algorithm with a dual gap of 2.52%; 27 nodes fathomed.

Table 3. Computational results for example No. 2 (GAP) [33].

Problem	LP Value	MILP Value	Columns	Nodes	Dual Gap (%)	CPU Time (s)
1	57.4	56	1139	22	0.0024	0.0236
2	57.0	57	2046	38	0.0000	0.4630
3	56.5	56	183	2	0.0089	0.0039
4	57.0	57	379	6	0.0000	0.0091

In Table 2, the LV, DS, ABC_1, ABC_{10}, and ABC_{72} are five heuristics for comparison; the interested reader is referred to the related literature. In Table 3, LP value indicates the optimal objective function value of the LP relaxation of the root node of the MILP; MILP value is the optimal objective function value of the MILP; nodes is the number of nodes fathomed; dual gap is defined as the value [(MILP value − LP value)/LP value] × 100%; and CPU time is the running time of the algorithm. It can be seen that the column generation solution is very effective for most computational examples and at least we can evaluate the quality of the solution, for example, in example No. 1, although the solution we obtained is not the best, the maximum deviation is 2.52%.

4.2. Case Study

Now, we make use of the CG algorithm for solving the RMP in Section 2. M-Steel's plant purchases its production raw materials from month to month. The number of items of raw materials is over 3000. However, in the actual decision-making process, it is not necessary to optimize the procurement of all raw materials. In fact, just choosing those raw materials with large quantities and high prices can greatly reduce the total procurement cost. These materials include iron ore and some coal. We selected 50, 100, 150, and 200 raw materials for purchasing optimization, respectively. The data are from the M-Steel plant and the computational results are shown in Table 4.

Table 4. Computational results for the RMP.

Problem	LP Value	MILP Value	Columns	Nodes	CPU (s)	Gap (%)
P/50/12/1	439,090,828.8491	439,148,890	861	4	0.4100	0.0132
P/50/12/2	436,811,265.4539	436,983,237	18,543	22	2.8230	0.0394
P/50/12/3	436,495,314.6475	436,992,295	4768	6	0.7600	0.1139
P/100/12/1	862,867,896.6292	862,868,090	3068	4	3.7140	2.24×10^{-5}
P/100/12/2	862,858,277.1114	862,868,090	5907	4	2.7462	0.0011
P/100/12/3	862,831,596	862,831,596	660	1	0.8774	0.0000
P/150/12/1	1,311,460,255.2423	1,313,850,978	1680	2	4.1843	0.1823
P/150/12/2	1,309,469,137.4146	1,319,050,475	233	2	5.4000	0.7317
P/150/12/3	1,296,957,435	1,296,957,435	334	1	13.7027	0.0000
P/200/12/1	2,106,130,786.3546	2,109,031,624	207	2	0.8354	0.1377
P/200/12/2	2,094,315,188.635	2,096,631,105	211	2	1.1225	0.1106
P/200/12/3	2,053,716,895.0005	2,055,909,857	226	2	2.8562	0.1068

The computational results show the proposed CG solution is effective. According to different procurement varieties and quantities, we give four raw material procurement patterns, corresponding to 50, 100, 150, and 200 raw materials respectively. The computational times are not more than 90 min, and the dual gaps are less than 0.2% for all patterns but one. This shows the efficiency and effectiveness of the algorithm. It is estimated that the application of the algorithm will save at least 80 million yuan in procurement costs for the enterprise every year.

5. Conclusions

Raw material purchasing optimization in steel plants is important because the RMP cost accounts for a significant fraction. In this paper, we proposed a MILP model and a column generation solution for the RMP for a large-scale steel plant. The computational results show that this solution is quite effective. For different varieties and quantities, we recommend several patterns for the selection of raw materials. The results show that it can save a lot in procurement costs, and it is estimated that it can save more than 80 million yuan in procurement costs every year

During the development of the solution, we note that two issues deserve more attention in the future. First, developing effective heuristics for finding feasible solutions for the lot-size problem is still a good topic, especially with tight capacity constraints. The second is the combination of column generation and neighborhood search techniques.

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