

Article

Study on Water Entry of a 3D Torpedo Based on the Improved Smoothed Particle Hydrodynamics Method

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Abstract: The water entry of a torpedo is a complex nonlinear problem, involving transient impact, free surface deformation, droplet splashing, and fluid–structure coupling, which poses severe challenges to traditional mesh methods. The meshless smoothed particle hydrodynamics (SPH) method shows unique advantages in capturing the complex features of the water entry of the torpedo at different entry angles. However, it still suffers from some inherent shortcomings, such as low surface discretization accuracy, poor discretization flexibility, and low calculation efficiency. In this study, an improved adaptive SPH algorithm is proposed to investigate the water entry of the torpedo accurately and efficiently. This method integrates meshless point generation and adaptive techniques simultaneously. The numerical results demonstrate that when the torpedo vertically enters the water at different velocities, the induced impact loads acting on the head of the torpedo fluctuate significantly with two peak values in the initial stage and thereafter stabilize in a later stage. The impact load acting on the torpedo, the entry depth of the torpedo, the splash height of the droplets, and the size of the cavity generated around the torpedo increase with the increment in the entry velocity. When the torpedo enters the water at different entry angles under the same initial entry velocity, both the vertical and the horizontal movements of the torpedo are observed, which results in more complex variations in parameters along the *x*- and *y*-axes. The findings and the corresponding numerical method in this study can provide a certain basis for the future designs of the entry trajectory and the structural bearing capacity of torpedoes.

Keywords: water entry; entry velocity; entry angle; spatial discretization; dynamic refinement and coarsening



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1. Introduction

The water entry of a torpedo is a typical example of water entries of structures, relating to large deformation of the free liquid surface. The impact load and the corresponding moving status induced by the water entry of the torpedo may result in strength damage to the torpedo structure or internal device failure. Thus, they should be carefully considered in the design and analysis of a torpedo. Three types of research methods are widely employed in investigating water entry processes of torpedoes, including the experimental method [1], theoretical method [2], and numerical method [3]. With the rapid development of computer technology, the numerical method has become a mainstream research method owing to its essential advantages of low cost and fast calculating speed compared to the other two methods. The traditional mesh-based numerical methods, such as the volume of fluid (VOF) [4,5], the finite volume method (FVM) [6], the boundary element method (BEM) [7], and the constrained interpolation profile (CIP) [8], are widely applied in simulating phenomena of fluid–structure interactions. However, meshes generated by these methods are prone to distortions, leading to inaccurate calculation results when applied

to problems involving large deformations of free interfaces, such as the water entry of a torpedo. In recent years, the meshless numerical method has been extensively developed owing to its unique natural advantages in overcoming the problems above [9]. Rather than using meshes as the computational basis, it discretizes the computational domain into particles and then solves various integral equations or partial differential equations (PDEs) on the particles.

The smoothed particle hydrodynamics (SPH) method is a typical meshless numerical method that uses a collection of particles with specific physical properties, such as density, viscosity, mass, velocity, and temperature, to represent fluids [10]. The movement and the changes in the properties of particles are controlled via mass, momentum, and energy conservation equations. The numerical solution to these equations is obtained by employing the kernel approximation and the weighted integration of particle interpolation [11]. The SPH method has an inherent advantage in solving strongly nonlinear problems of the water entry of the torpedo, induced by complex free surface splashing and large deformation. Lots of work has been conducted to improve the numerical accuracy and stability of the SPH method in capturing the moving boundary, the large deformation of the fluid, and the free surface breakage in water entry processes [12–25]. However, there remain certain shortcomings in the SPH method. First, the SPH method performs calculation processes based on a series of discrete particles. Most studies have, first, commonly applied meshes over the computational domain and then extracted particles from the meshes to discretize the computational domain [26]. This existing particle generation method has low surface discretization accuracy and poor discretization flexibility. Second, the SPH method widely adopts a uniform particle distribution over the entire computational domain. To accurately capture the dramatic variations in the flow field around a moving three-dimensional (3D) torpedo, the total number of particles allocated in the computational domain is often very large. It consumes large amounts of computer memory and processing power and thus makes the SPH method less efficient.

This work thoroughly investigates the complex water entry of a 3D torpedo by applying an improved adaptive SPH algorithm. First, a meshless and efficient point distribution method is established for the 3D computational domain of the water entry of the 3D torpedo. Second, based on the adaptive SPH technique reported by Chiron et al. [27], a dynamic refinement and coarsening scheme of particles within the target region is proposed. Third, the improved adaptive SPH algorithm is validated by conducting a simulation of the water entry of a 3D horizontally placed cylinder. Finally, a parametric study on the effects of different entry velocities and angles of the torpedo on the impact loads and the flow fields around the torpedo is conducted using the validated adaptive SPH algorithm.

2. Numerical Method and Model Development

2.1. Mathematical Model Based on the SPH Method

The SPH method is a Lagrangian particle method, wherein the computational domain is composed of a set of discrete particles. Each particle is assigned physical properties such as mass, density, pressure, and velocity. Based on these particles, the time derivatives of various physical quantities can be obtained using the SPH kernel approximation algorithm, which allows the time integration of the physical process of the flow. The kernel approximation of a field function $f(\mathbf{r})$ at a spatial position $\mathbf{r}$, $\langle f(\mathbf{r}) \rangle$, can be expressed as

$$\langle f(\mathbf{r}) \rangle = \int_{\Omega} f(\mathbf{r}^*) W(\mathbf{r} - \mathbf{r}^*, h) dV_{r^*} \quad (1)$$

where $W(\mathbf{r} - \mathbf{r}^*, h)$ is the smoothing kernel function [28], h is the smoothing length [29], and V is the volume. Ω represents the support domain of the smoothing kernel function, which is a function of h . A good tradeoff between the computational accuracy and the kernel approximation cost can be realized using the Wendland kernel function [30], which is expressed as

$$W(\mathbf{r} - \mathbf{r}^*, h) = \alpha_D \left(1 - \frac{q}{2}\right)^4 (2q + 1) \quad (2)$$

where D is the dimensions of the computational domain and q is the dimensionless variable, expressed as $q = (\mathbf{r} - \mathbf{r}^*)/h$. In addition, α_D is a normalization factor which is given as $7/(4\pi h^2)$ for a two-dimensional (2D) domain and $21/(16\pi h^3)$ for a 3D domain. As the computational domain is composed of a set of discrete particles with particular physical properties, the continuous format of the kernel approximation, as expressed in Equation (1), should be transformed into a discrete one:

$$\langle f(\mathbf{r}_i) \rangle = \sum_{j=1}^N f(\mathbf{r}_j) W(\mathbf{r}_i - \mathbf{r}_j, h) V_j \quad (3)$$

where N is the total number of particles located in the support domain of particle i . V_j is the virtual volume of the neighboring particle j , which is the ratio of the mass m_j to the density ρ_j . Consequently, the kernel approximation of the derivative of the field function in the discrete format, $\langle \nabla \cdot f(\mathbf{r}) \rangle$, can be derived as

$$\langle \nabla \cdot f(\mathbf{r}_i) \rangle = \sum_{j=1}^N f(\mathbf{r}_j) \cdot \nabla_i W(\mathbf{r}_i - \mathbf{r}_j, h) \frac{m_j}{\rho_j} \quad (4)$$

Equation (4) indicates that the derivative of the field function can be transferred to the derivative of the smooth kernel function, thereby reducing the smoothness requirements for the field function and producing stable results in numerical approximations of PDEs [31]. Thus, the governing equations in the form of weakly compressible SPH are derived as

$$\begin{cases} \frac{d\rho_i}{dt} = -\rho_i \sum_{j=1}^N (\mathbf{u}_i - \mathbf{u}_j) \cdot \nabla_i W(\mathbf{r}_i - \mathbf{r}_j, h) \frac{m_j}{\rho_j} \\ \frac{d\mathbf{u}_i}{dt} = \frac{1}{\rho_i} \sum_{j=1}^N (P_i + P_j) \nabla_i W(\mathbf{r}_i - \mathbf{r}_j, h) \frac{m_j}{\rho_j} + \mathbf{g} \\ \frac{d\mathbf{r}_i}{dt} = \mathbf{u}_i \\ P_i = \frac{\rho_0 c_0^2}{\gamma} \left(\left(\frac{\rho_i}{\rho_0} \right)^\gamma - 1 \right) \end{cases} \quad (5)$$

where $\mathbf{u}$ is the velocity. The pressure of particle i , P_i , is expressed in the form of the Tait equation, where ρ_0 is the density of the fluid when $P = 0$ Pa, c_0 is the speed of sound [32,33], and γ is a constant [34,35] that usually takes the value of 7 in hydrodynamic problems. The continuity equation employs an asymmetric SPH approximation scheme to minimize errors arising from discontinuities [36], and the momentum equation adopts symmetric SPH approximation schemes to ensure better conservation [37].

2.2. Six-Degree-of-Freedom (6DOF) Motion Algorithm

Assuming that the structure of the torpedo is a rigid body, the structure is discretized into a series of particles which participate in the computation of the flow field [38]. The force per unit mass of each rigid particle is given as

$$\mathbf{f}_i = \sum_{j \in FP}^N \mathbf{f}_{ij} \quad (6)$$

where $\mathbf{f}_i$ denotes the force acting on the rigid body particle i per unit mass, FP represents the fluid particle, and N is the total number of particles within the support domain. $\mathbf{f}_{ij}$ denotes the force per unit mass exerted by the fluid particle j on the particle i .

For a moving rigid body, the motion equations of the six-degree-of-freedom (6DOF) rigid body can be written as follows:

$$\begin{cases} M \frac{d\mathbf{V}}{dt} = \sum_{i \in BP}^N m_i \mathbf{f}_i \\ \mathbf{I} \frac{d\boldsymbol{\omega}_0}{dt} = \sum_{i \in BP}^N m_i (\mathbf{r}_i - \mathbf{R}_0) \times \mathbf{f}_i \end{cases} \quad (7)$$

where M denotes the mass of the rigid body, $\mathbf{V}$ is the velocity of the center of mass, and BP represents the rigid body particle. $\mathbf{r}_i$ denotes the position vector of particle i . $\mathbf{I}$ denotes the moment of inertia of the rigid body, $\boldsymbol{\omega}_0$ is the angular velocity of the rigid body, and $\mathbf{R}_0$ is the position vector of the center of mass of the rigid body. The velocity of each mass point on the rigid body can be obtained by

$$\mathbf{V}_i = \mathbf{V} + \boldsymbol{\omega}_0 \times (\mathbf{r}_i - \mathbf{R}_0) \quad (8)$$

2.3. Spatial Discretization and Adaptive Algorithm

2.3.1. Spatial Discretization

In contrast to the point generation for a curved line boundary in a 2D space, that in a 3D space is considerably more complex. In addition, owing to the meshless property of the algorithm, the generated points over the curved surface lack connecting relationships and thus lack the structure required for the traditional ray method [39]. To address these challenges, as shown in Figure 1, this work firstly applies a mapping method [40] to discretize the curved surfaces into points. Consequently, an improved method of the vector dot product is employed to evaluate the attributes of background Cartesian points. Finally, attribute-based background grids are used to pre-mark the computational domain and thus accelerate the spatial discretization.

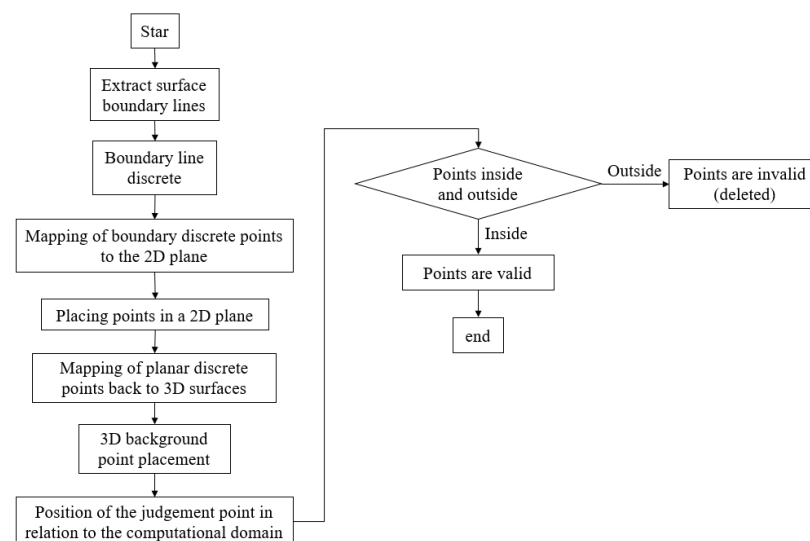


Figure 1. Flowchart of the spatial discretization of a 3D space.

The process of generating the discrete points using the mapping method is shown in Figure 2. It comprises the following four steps. First, the boundary curves of the surface are extracted and then discretized into a series of points. Second, the discrete points of the boundary curves are mapped onto an x_1 – x_2 2D plane to form a boundary of a 2D domain. Third, the meshless point generation algorithm for the 2D domain is applied to generate a series of discrete points within the domain. Fourth, all the discrete points on the x_1 – x_2 plane are mapped back onto the 3D surface to obtain the meshless discrete point set.

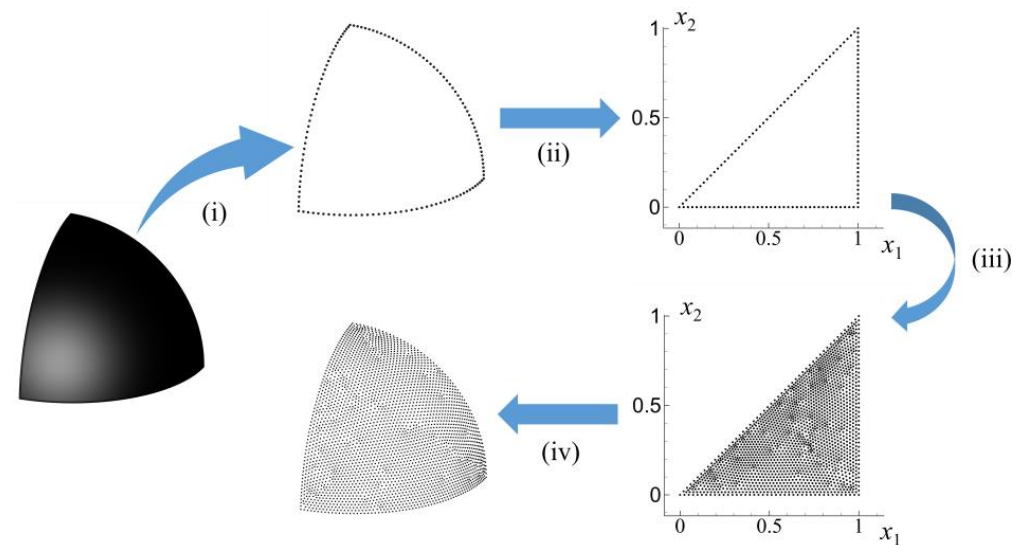


Figure 2. Generation of discrete points using the mapping method.

The coordinates of the background points in the 3D domain are calculated as

$$\begin{aligned} x_{i,j,k} &= x_{start} + (i-1)\Delta x, (1 \leq i \leq N_x) \\ y_{i,j,k} &= y_{start} + (j-1)\Delta y, (1 \leq j \leq N_y) \\ z_{i,j,k} &= z_{start} + (k-1)\Delta z, (1 \leq k \leq N_z) \end{aligned} \quad (9)$$

where the subscript *start* represents the background points at the start; Δx , Δy , and Δz represent the spacings between two neighboring background points along the *x*, the *y*, and the *z* directions, respectively; N_x , N_y , and N_z represent the total number of rows of all background points generated along the *x*, the *y*, and the *z* directions, respectively; and the subscripts *i*, *j*, and *k* denote the orders of the rows with certain background points along the *x*, the *y*, and the *z* directions, respectively.

After generating the background points, the spatial relationship between the background points and the boundary points must be determined. A vector dot product method is introduced in this work to determine the attribute of the background points and thereby accelerate the generation of the discrete points. Considering a computational domain partially composed of a hemispherical surface, as shown in Figure 3, as an example, it is defined that the right side and the left side of the hemispherical surface are the outside and the inside of the computational domain, respectively. Point A is a point on the surface, whose normal vector is $\vec{n}$, and point B beside the surface is a background Cartesian point to be evaluated. Point A is strictly selected by following two principles. First, point A is inside the 3D background points where point B is located. Second, point A is the closest surface boundary point to point B.

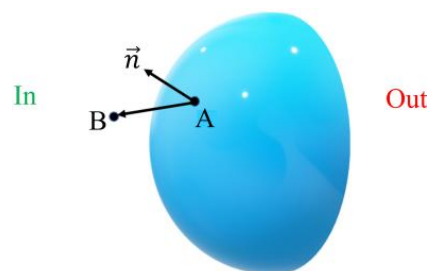


Figure 3. Schematic of the vector dot product method used for determining the spatial relationship between boundary point A and background Cartesian point B. $\vec{n}$ is the normal vector of the surface.

Considering the vector pointing from point A to point B as $\vec{AB}$, the attribute of the background Cartesian point B can be determined as

$$\begin{aligned} \vec{AB} \cdot \vec{n} &> 0, & \text{In} \\ \vec{AB} \cdot \vec{n} &< 0, & \text{Out} \end{aligned} \quad (10)$$

where the indicators In and Out represent situations of point B being inside and outside the computational domain, respectively. As each background Cartesian point needs to be evaluated at least once, the corresponding efficiency is very low in the case of a large number of background Cartesian and boundary points. Therefore, the accelerated ray method developed in one of our previous studies [41] is utilized to improve the efficiency of the evaluation of the background Cartesian points and consequently to accelerate the point generation.

2.3.2. SPH Adaptive Algorithm

- Dynamic refinement and coarsening algorithm

Following the initial discretization of the overall computational domain, the refinement of particles within the region of interest is performed at the beginning of the numerical calculations to achieve high calculation accuracy. Inspired by previous studies on the refinement algorithm [42,43], a parent particle, within the initially and uniformly discretized computational domain, requiring refinement is split into eight child particles by using a pre-defined cubic refinement pattern as shown in Figure 4. The parent particle is located in the center of the cubic comprising the eight child particles. Thus, the distance between two neighboring child particles, Δd_c , is determined by

$$\Delta d_c = \varepsilon \Delta d_p \quad (11)$$

where the subscripts c and p, respectively, represent the child particle and the parent particle, Δd_p is the initial distance between neighboring parent particles, and ε is the separation ratio ranging from 0 to 1. In addition, the smooth length of the child particle h_c is expressed as

$$h_c = \zeta h_p \quad (12)$$

where h_p is the smooth length of the parent particle and ζ denotes the smooth length ratio ranging from 0 to 1. The two refinement parameters, $\varepsilon = 0.5$ and $\zeta = 0.5$, are utilized in this work to warrant the same number of neighboring particles before and after refinement.

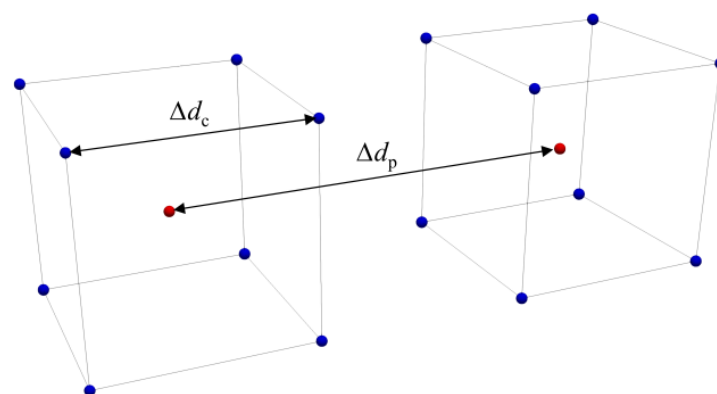


Figure 4. Schematic of the cubic refinement pattern for a 3D computational domain. The black and the red dots denote the parent and the child particles, respectively. Δd_c and Δd_p are the distance between two neighboring child particles and the distance between two neighboring parent particles, respectively.

The physical properties of the child particle, the mass m_c and the velocity u_c , are derived from the parent particle as

$$m_c = \lambda_c m_p \quad (13)$$

$$u_c = u_p \quad (14)$$

where λ_c is the refinement coefficient. This is determined based on the refinement pattern and set as 0.125 to achieve a cubic refinement pattern in this work. Consequently, the mass, the energy, the linear momentum, and the angular momentum conservation equations are expressed as

$$m_p = \sum_{c=1}^8 \lambda_c m_c \quad (15)$$

$$\frac{1}{2} m_p u_p^2 = \sum_{c=1}^8 m_c u_c^2 \quad (16)$$

$$m_p u_p = \sum_{c=1}^8 m_c u_c \quad (17)$$

$$r_p \times m_p u_p = \sum_{c=1}^8 r_c \times m_c u_c \quad (18)$$

To further increase the efficiency of the SPH adaptive algorithm, a coarsening algorithm can be reasonably developed to dynamically remove the numerous child particles that move out of the interested region. Inspired by our previous study on the dynamic coarsening algorithm for a 2D computational domain [41], this work further improves the dynamic refining and coarsening algorithm to render it feasible for a 3D computational domain. Once a parent particle enters the region of interest (refinement region), it is split into eight child particles. To conserve all parameters, the parent particle is inactivated and the eight affiliated child particles are simultaneously activated. The inactivated parent particle passively moves with the flow field and all the parameters of the parent particle are not involved in any calculations. Once the parent particle exits the refinement region, it is immediately activated to be involved in the calculations and the eight affiliated child particles are simultaneously removed from the calculation domain. In general, the improved dynamic refinement and coarsening algorithm can simultaneously realize high accuracy, high efficiency, and better conservations during calculations.

- Adaptive particle refinement (APR) technology

In SPH calculations, two neighboring regions of different resolutions interact with each other through particles in the guard area [41]. The field function f of the particles in the guard area is interpolated from the fluid as

$$\langle f(r) \rangle_{\text{guard}} = \frac{\sum_{j=1}^N f(r_j) W(r_i - r_j, h)}{\sum_{j=1}^N W(r_i - r_j, h)} \quad (19)$$

where $\langle f(r) \rangle_{\text{guard}}$ denotes the approximation of the field function f for particles in the guard area. Interpolations of the field functions are conducted before each time step and the corresponding values are assigned equally to each particle in the guard area using Equation (16). In addition, an enhanced shifting algorithm [41] is also applied to correct the erroneous displacement of particles in the non-guard area and thus further increase the computational accuracy.

3. Results and Discussion

3.1. Numerical Model Validation

A classic study on water entry of a 3D horizontally placed cylinder [44] is conducted to validate the mathematical model and the SPH numerical method developed in this work. As shown in Figure 5, the 3D computational domain contains a large pool, filled with water, and a cylinder, horizontally located upon the pool. The pool is sufficiently large with dimensions of 1 m in length, 0.4 m in width, and 0.6 m in height to ensure that the calculation of the fluid flow is not affected by boundaries. The diameter and the length of the cylinder are 0.05 and 0.2 m, respectively. The densities of the water and the cylinder are 1000 and 1370 kg/m³, respectively. The kinematic viscosity of water is 10^{−6} m²/s and the initial velocity of the cylinder entering the water is set as 6.22 m/s.

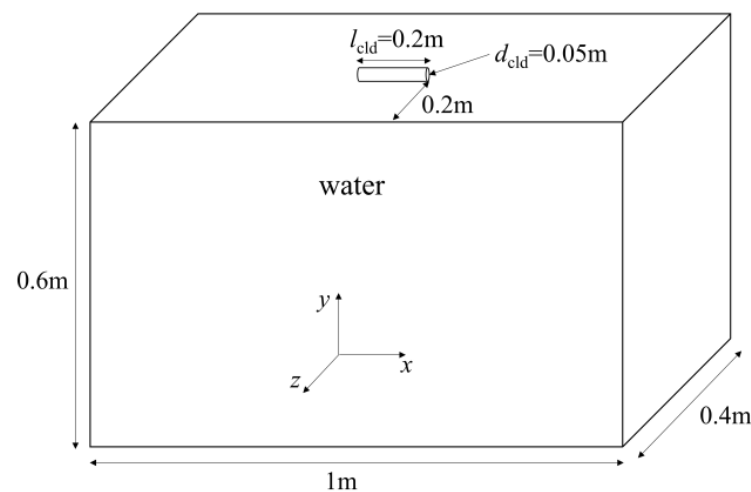


Figure 5. Three-dimensional computational domain of water entry of a horizontally placed cylinder.

First, grid independence verification is performed by applying four different particle resolutions, including $d_{\text{cld}}/\Delta d_p = 5, 10, 15$, and 20. The corresponding time evolutions of the penetration depth $y - y_0$ and the vertical velocity u_y of the cylinder entering water achieved from this numerical work and the reported experimental studies [44,45] are summarized in Figure 6. As evident, the numerical variation trends of both parameters gradually converge to the corresponding experimental ones with increasing particle resolution. Considering the numerical results achieved at a particle resolution of $d_{\text{cld}}/\Delta d_p = 20$ as an example (Table 1), the errors between the numerical and the reported experimental results at 0.12 s are only 2.11% and 4.15% for the penetration depth $y - y_0$ and the vertical velocity u_y , respectively. This comparison result validates that the mathematical model and the SPH numerical method developed in this work can effectively give rise to accurate convergence results.

Table 1. Errors between the numerical results achieved at a particle resolution of $d_{\text{cld}}/\Delta d_p = 20$ and the reported experimental results.

	$y - y_0/\text{m}$	$u_y/\text{m}\cdot\text{s}^{-1}$
Exp. Wei and Hu [44]	−0.285	\
Sun [45]	\	−1.421
SPH	−0.291	−1.362
Error (%)	2.11	4.15

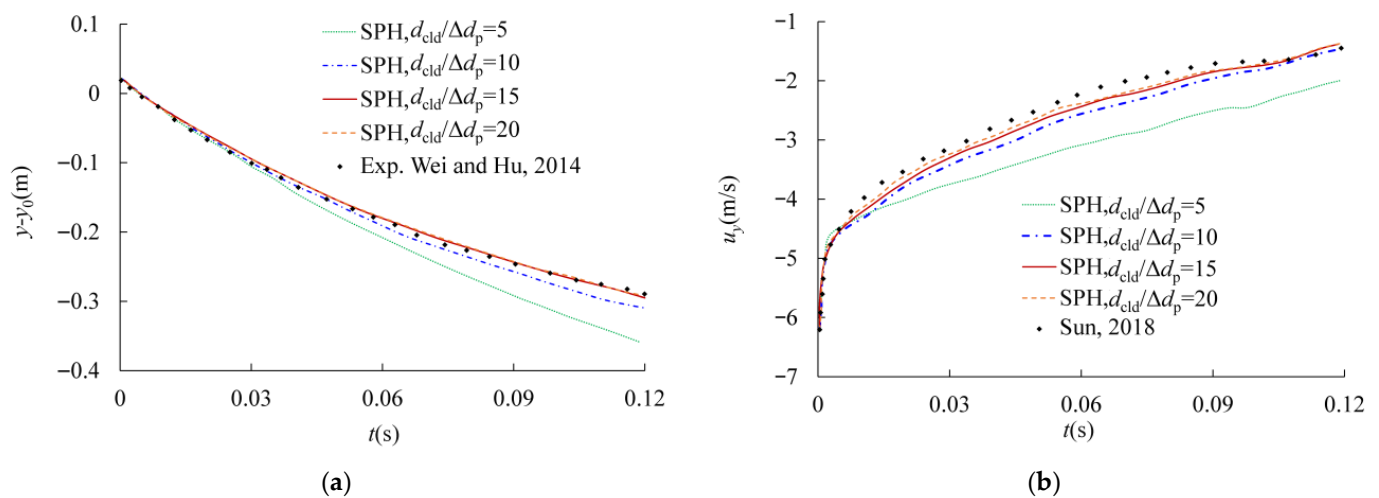


Figure 6. Time evolutions of the penetration depth $y - y_0$ (a) and vertical velocity u_y (b) of the cylinder achieved from this numerical work and the reported experimental studies [44,45].

As calculated, the total number of particles is as high as 1.7×10^7 if the particle resolution of $d_{cld}/\Delta d_p = 20$ is applied for the whole 3D computational domain, which greatly reduces the computational efficiency. Thus, three adaptive refinement areas with three different particle resolutions of $d_{cld}/\Delta d_p = 20, 10$, and 5 are assigned outward from the cylinder to skillfully reduce the total number of particles and thus improve the computational efficiency. In particular, the distance from the boundary of the area of the highest particle resolution to the boundary of the cylinder is controlled at $30\Delta d_{p,max}$ to capture the force of the fluid on the cylinder more accurately. Here, $\Delta d_{p,max}$ is the initial distance between the two neighboring particles within the area of the highest particle resolution. Based on this adaptive technique, the total number of particles is reduced to 1.4×10^6 , which is approximately 92% less than that generated under the condition of the uniformly distributed particle resolution of $d_{cld}/\Delta d_p = 20$. Therefore, the adaptive technique significantly reduces the usage of the computer memory and thus improves computational efficiency.

Figure 7 shows the evolutions of the velocities within the middle plane along the z direction, as achieved from this work and the reported experimental work [44]. The results showed significant consistency. In the initial stage of the cylinder entering water, the fluid splashes around the cylinder, forming a thin water curtain, as shown in Figure 7b. As the cylinder continues to sink, an open cavity forms and gradually grows above the cylinder. Simultaneously, the thin water curtain also gradually expands to stand vertically. Owing to the large initial splash velocity of the fluid, the tail of the thin water curtain eventually separates from the thin water curtain, forming a number of small splashed droplets, as shown in Figure 7c. As the cavity continues to expand, the free surface of the fluid forms four obvious cross lines along four corners of the cylinder when observed from the top of the computational domain. Two pairs of cross lines at the left and the right ends of the cylinder shrink inward faster than the other parts of the free surface. Two cross lines at each end approach each other and eventually merge to form a new cross line at the corresponding end, as shown in Figure 7c–f. The two newly formed cross lines continue to approach each other along the axial direction at a faster speed, as shown in Figure 7f–h. The phenomenon observed above is typically referred to as cavity inversion [46]. The remarkable consistency of the results achieved from this work and the reported experimental work proves that the mathematical model and the SPH numerical method developed in this work can accurately and efficiently capture all the physical phenomena of the water entry problems of 3D entities.

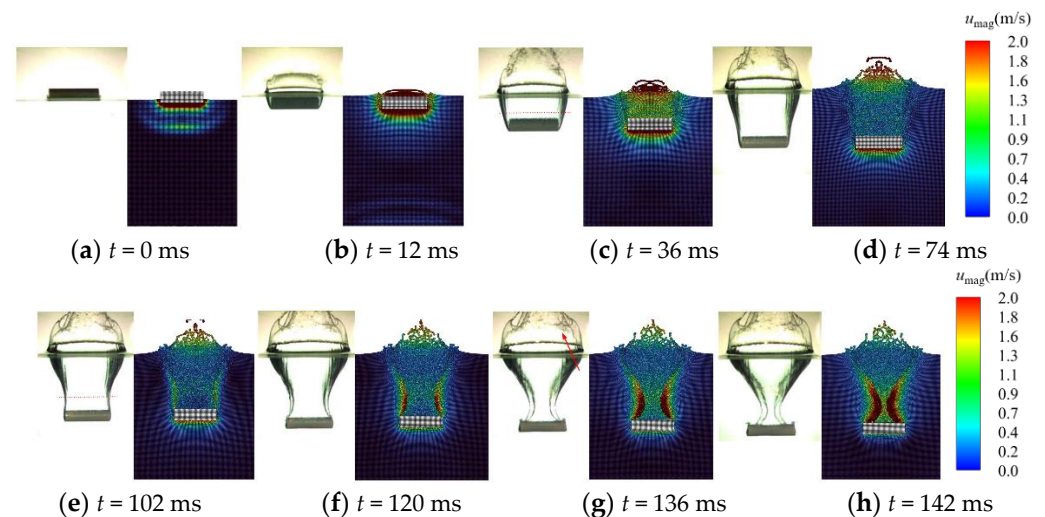


Figure 7. Evolutions of the velocities, within the middle plane along the z direction, as achieved from this work and the reported experimental work [44].

3.2. Water Entry of 3D Torpedo

This section systematically investigates the water entry process of a 3D torpedo at different entry velocities and entry angles. The corresponding schematic and detailed parameters are shown in Figure 8. The length, the width, and the height of the pool filled with water are 8, 8, and 4 m, respectively. The length and the diameter of the torpedo are 2.79 and 0.32 m, respectively. The center of mass of the torpedo is located at a distance of 1.18 m from its head. The density of the torpedo is assumed as 1000 kg/m^3 and the rotational inertias of the torpedo are set as $I_x = I_z = 88.01 \text{ kg}\cdot\text{m}^2$ and $I_y = 2.31 \text{ kg}\cdot\text{m}^2$. The torpedo is initially placed above the water with its head placed tangentially to the surface of the water. In addition, its center of mass is fixed at (x_0, y_0, z_0) . At the beginning of the water entry, the entry velocity of the torpedo is set along the same direction as the axis of the torpedo. Therefore, the entry angle β formed between the entry velocity and the water surface is the same as the attitude angle θ of the torpedo formed between the axis of the torpedo and the water surface.

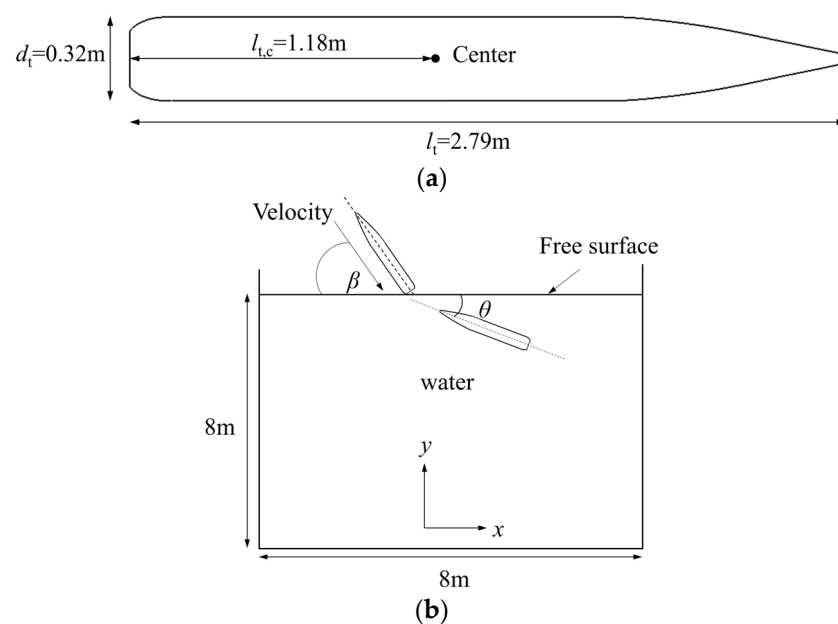


Figure 8. Schematic of (a) the torpedo and (b) the calculation domain of the water entry of the torpedo.

3.2.1. Effect of Entry Velocity

Three different entry velocities of 5, 10, and 15 m/s, at the same entry angle of $\beta = 90^\circ$, are applied to investigate the effects of the entry velocity on the water entry of a torpedo. A total of 1.52×10^7 initialization particles are employed as the optimum strategy of particle discretization along with the adaptive refinement performed for the area around the torpedo. Figures 9 and 10 summarize the evolution of the velocity fields within the middle plane of the calculation domain along the z direction and the development processes of the cavities during the water entry of the torpedo at three different entry velocities, respectively. As evident, three stages of evolution are observed.

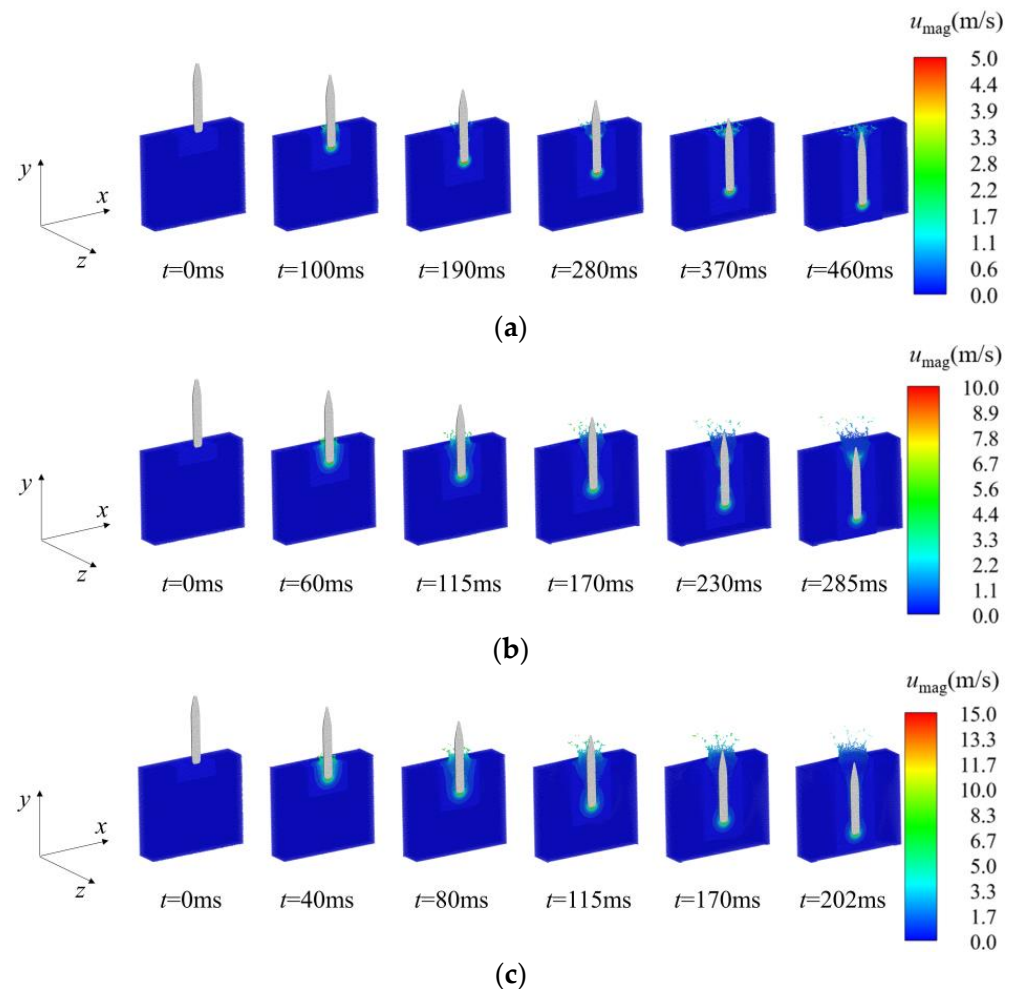


Figure 9. Evolution of velocity fields of the water domain induced by the torpedo vertically entering water at different speeds of (a) 5 m/s, (b) 10 m/s, and (c) 15 m/s.

The first stage is the impact stage. When a torpedo starts to collide with the water surface in a very short time, intensive compression waves are induced and dissipate rapidly to the surroundings. Meanwhile, the head of the torpedo is significantly impacted by the instantaneous impact load, which may result in considerable damage to the torpedo head. Typically, the impact process is accompanied by the exchange of momentum between the torpedo and the surrounding fluid near the impact surface. Consequently, the surrounding fluid, obtaining momentum from the torpedo, is squeezed away from its original location and splashes around at a high velocity, as shown in Figure 9a–c for $t = 100$, 60, and 40 ms, respectively. In addition, with a gradual increase in the entry velocity from 5 to 15 m/s, the surrounding fluid obtains growing momentum and thus splashes away at an increasing speed.

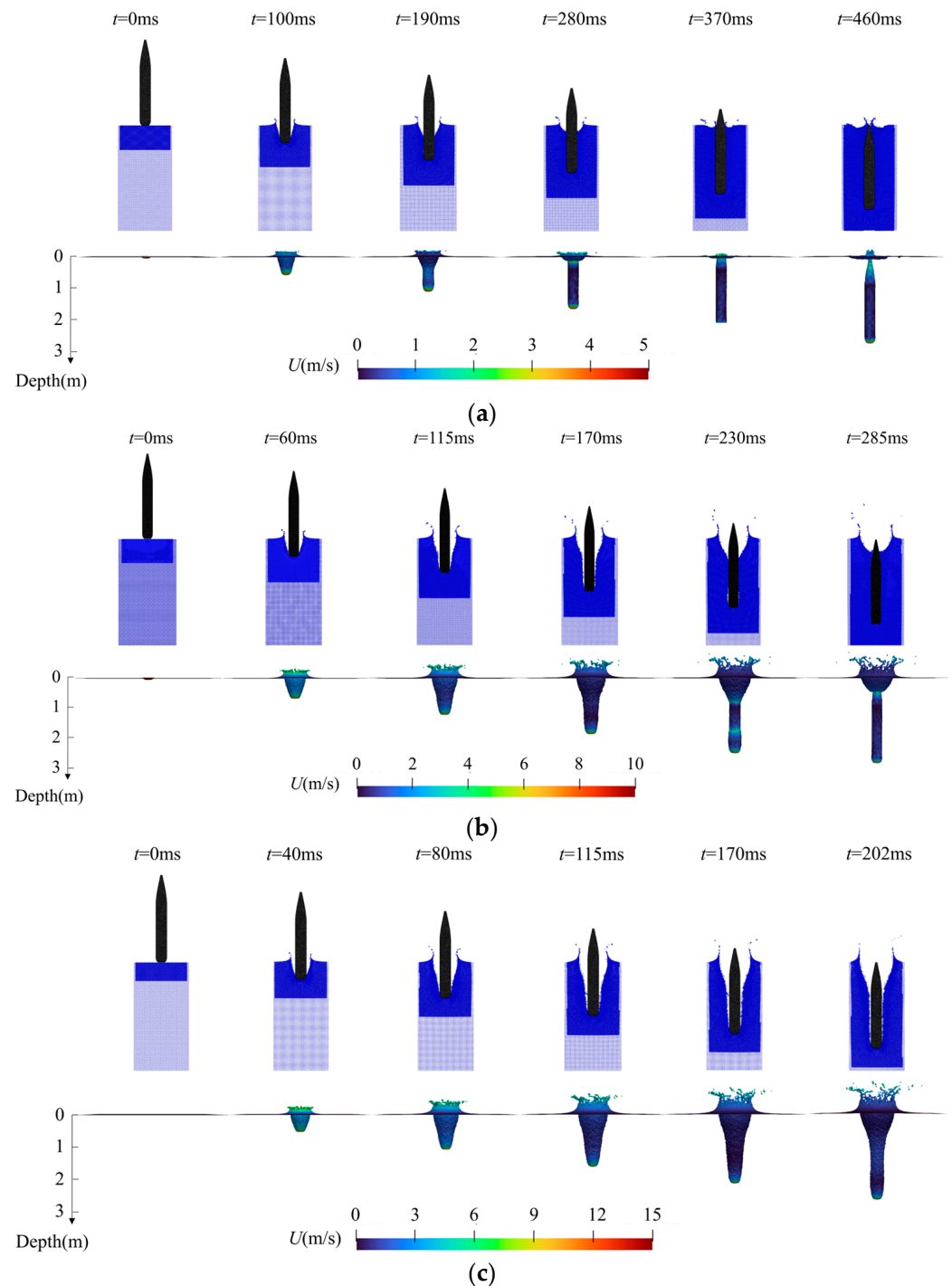


Figure 10. Development of the water cavity induced by the torpedo vertically entering the water at different speeds of (a) 5 m/s, (b) 10 m/s, and (c) 15 m/s. As for the cavity development at a certain entry velocity, the figures within the upper row show the cross-sectional view of the water phase within the middle plane of the computational domain along the z direction and those within the bottom row show the corresponding 3D views of the free surface of the water generated around the torpedo.

The second stage is the cavity formation stage. After the impact stage, the torpedo continues to dive downward. Thus, more fluid around the head of the torpedo is squeezed out and gradually forms a growing V-shaped cavity, as shown in Figure 10. In addition, it is also found that a larger entry velocity results in a larger cavity formed at the same depth of the water domain.

The third stage is the cavity closure stage. As the torpedo further enters into the water, the fluid, forming the free surface of the cavity around the torpedo, moves radially toward the torpedo under the hydrostatic force. Consequently, it gradually closes the space formed by the cavity and the torpedo surfaces as shown in Figure 10. For a low entry velocity, the cavity is small and thus is closed earlier. As for the case of a low entry velocity of 5 m/s (Figure 10a), the cavity starts to close from the head to the rear of the torpedo at $t = 100$ ms, and it is fully closed at $t = 370$ ms. Subsequently, the free surface of the fluid around the previous cavity continues to be squeezed upward, forming new splashing droplets (Figures 9a and 10a). As shown in Figure 10b, when the entry velocity increases to 10 m/s, the cavity requires approximately 170 ms to close, and the area where the cavity closes is from the middle part instead of the head of the torpedo. In addition, the induced droplets with a series of higher splashing heights are observed (Figures 9b and 10b). When the entry velocity continues to increase to 15 m/s (Figure 10c), the cavity does not close until 202 ms and the induced droplets with a series of highest splashing heights are captured (Figures 9c and 10c). In general, when the entry velocity is low, the free surface of the cavity closes with the torpedo earlier at the front part of the torpedo. In addition, a small quantity of splashing droplets is also generated and they occupy a small space above the water domain. With increasing entry velocity, the cavity size increases significantly, and numerous droplets are simultaneously generated and splash above the water domain.

Figure 11 summarizes the variations in the impact load projecting onto the y -axis F_y , the velocity u_y , and the vertical displacement of the center of mass $y - y_0$ of the torpedo with time when the torpedo vertically enters the water at different speeds of 5, 10, and 15 m/s. As shown in Figure 11a, once the torpedo enters the water at a certain entry velocity, it is subjected to an instantaneous impact load along the positive y -direction, F_y , which increases to two peak values in a short period and stabilizes afterward. In addition, it is also observed that a higher entry velocity results in a higher peak and stabilized impact loads. In addition, the gravity force G is calculated as 1.848 kN. Subsequently, the vertical resultant forces and the corresponding accelerations, that occur around the moment of the first peak impact loads, point to the positive direction of the y -axis. Therefore, the absolute values of the velocities decrease at the beginning, as shown in Figure 11b. Although the impact loads along the positive y -direction slightly fluctuate, the regions of $F_y < G$ between two peak loads occur in a very short time. Therefore, the fluctuations in the velocities can be reasonably neglected during the corresponding period. Figure 11b also shows that different stabilized impact loads, induced by different entry velocities, induce different stabilized accelerations and variation trends of velocities. For a low entry velocity of 5 m/s, as the vertical stabilized impact load acting on the torpedo is less than the gravity force, the resultant force along the vertical direction is in the same direction as that of the velocity. Therefore, the torpedo accelerates and the absolute value of velocity of the torpedo linearly increases during the stabilization period. For an entry velocity of 10 m/s, as the vertical stabilized impact load is almost equal to the gravity force, the resultant force and the acceleration along the vertical direction are zeros and thus the corresponding velocity is maintained at a relatively stable value during the stabilization period. Contrary to the findings observed from the case of a low entry velocity of 5 m/s, the large entry velocity of 15 m/s induces a high stabilized impact load, which is greater than the gravity force. Thus, the corresponding resultant force is in the opposite direction from the directions of the gravity force and the velocity. Therefore, the absolute value of the velocity linearly decreases during the stabilization period. Although the variation trends of the velocities are opposite for different entry velocities of 5 and 15 m/s, the displacements of the center of mass are mainly dominated by the initial entry velocity. Therefore, the displacements of the center of mass at the initial entry velocities of 15 and 5 m/s increase the fastest and the slowest, respectively, as shown in Figure 11b.

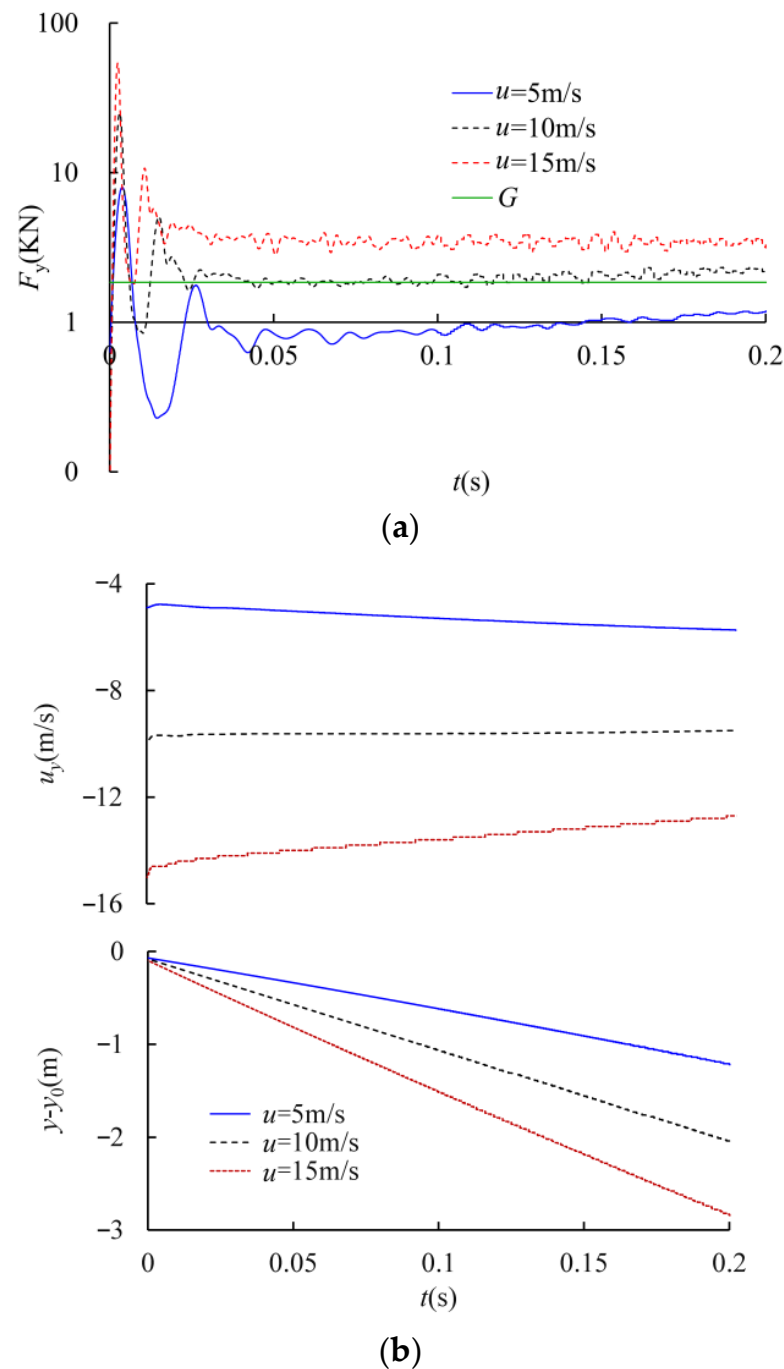


Figure 11. Variations in (a) the impact load projecting onto the y -axis F_y and (b) the velocity u_y and the vertical displacement of the center of mass $y - y_0$ of the torpedo with time when the torpedo vertically enters the water at different speeds of 5, 10, and 15 m/s.

3.2.2. Effect of Entry Angle

In addition to the case of the entry angle of $\beta = 90^\circ$ under 10 m/s studied above, two more entry angles of $\beta = 75^\circ$ and 60° under the same entry velocity are also investigated to determine the effects of the entry angle on the water entry of the torpedo. Figures 12 and 13 summarize the evolution of the velocity fields within the middle plane of the calculation domain along the z direction and the development processes of the cavities during the water entry of the torpedo at different entry angles, respectively.

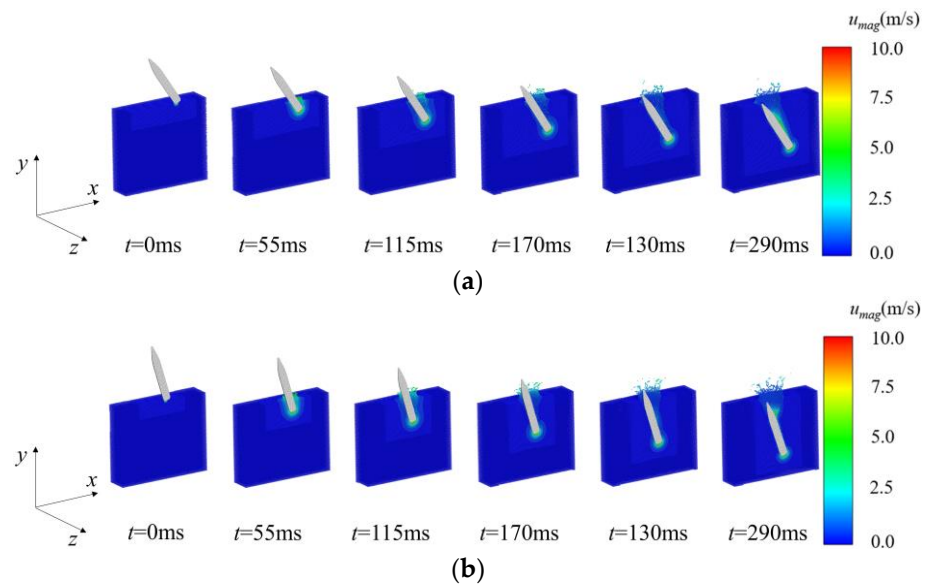


Figure 12. Evolution of velocity fields of the water domain induced by the torpedo entering the water at different entry angles of (a) $\beta = 75^\circ$ and (b) $\beta = 60^\circ$ under the same entry velocity of 10 m/s.

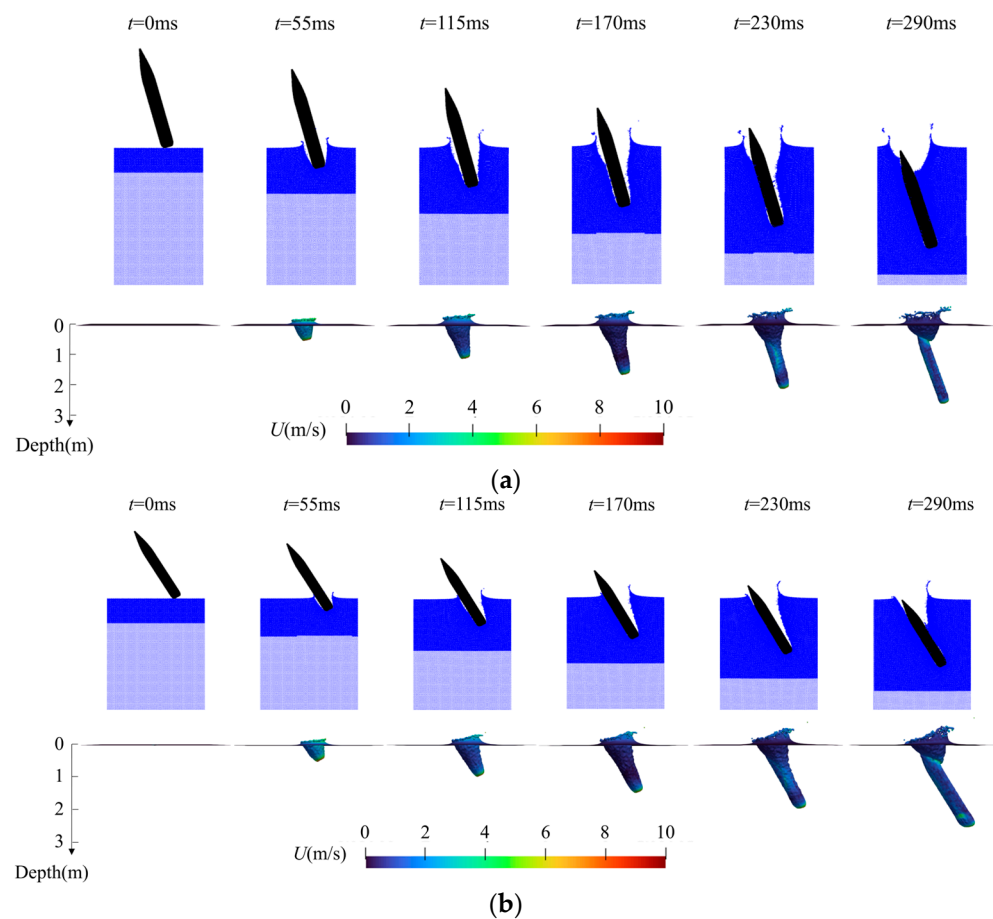


Figure 13. Development of the cavity within the water domain induced by the torpedo entering the water at different entry angles of (a) $\beta = 75^\circ$ and (b) $\beta = 60^\circ$ under the same entry velocity of 10 m/s. As for the cavity development at a certain entry angle, the figures within the upper row show the cross-sectional view of the water phase within the middle plane of the computational domain along the z direction and those within the bottom row show the corresponding 3D views of the free surface of the water generated around the torpedo.

Once the torpedo collides with water in a very short time, it causes the nearby fluid to splash radially outward with high velocities, as seen from the figures of $t = 55$ ms in Figure 12a and $t = 55$ ms in Figure 12b. However, in contrast to the symmetric splash curtain generated by the vertical water entry of the torpedo, the velocity along the x -positive component of the torpedo u_x , occurring when the torpedo enters water at angles of $\beta = 75^\circ$ or 60° , results in a larger impact force on the fluid in front of the torpedo. This results in an asymmetric splash curtain, as shown in Figures 12 and 13. Correspondingly, an asymmetric cavity is also simultaneously generated, which is larger at the front of the torpedo owing to the higher momentum transferred to the fluid there. As the torpedo moves downward, the fluid gradually flows back to contact with the torpedo from the bottom to the upper part of the torpedo. In addition, the cavity closes faster at the back of the torpedo than at the front of the torpedo. Furthermore, the cavity at the back of the torpedo closes faster for a smaller entry velocity.

The variations in the impact loads, the velocities, and the displacements of the center of mass of the torpedo with time when the torpedo enters water at different entry angles of $\beta = 60^\circ$, 75° , and 90° under the same entry velocity of 10 m/s are drawn in Figure 14. Compared to the impact load of the x -component of zero at the entry angle of $\beta = 90^\circ$ depicted in Figure 14a, entry angles of $\beta = 60^\circ$ and 75° lead to non-zero impact loads of the x -component. These two non-zero horizontal impact loads initially point to the negative direction of the x -axis, impeding the horizontal movement of the torpedo. In addition, a larger impact load of the x -component occurs at the smaller entry angle of $\beta = 60^\circ$ in the earlier stage of the torpedo entering the water. Correspondingly, the velocity of the x -component at the smallest entry angle of $\beta = 60^\circ$ is largest and it decreases the most sharply in the earlier stage, as shown in Figure 14c. When the torpedo continues to enter water more deeply in the later stage, the impact loads of the x -component at entry angles of $\beta = 60^\circ$ and 75° gradually approach zero, as shown in Figure 14a; then, they increase toward the direction of the positive x -axis owing to the momentum introduced by the backflow of the fluid, as observed in Figure 13. Thus, the velocities of the x -component at those two entry angles slightly increase in the later stage of the water entry of the torpedo, as shown in Figure 14c.

The torpedo is initially subjected to fluctuating vertical impact loads along the positive direction of the y -axis at all three entry angles and the larger entry angle yields a higher vertical impact load, as shown in Figure 14b. Correspondingly, the torpedo achieves higher downward velocity for a larger entry angle, as shown in Figure 14d. All those vertical impact loads increase slowly after the fluctuations. Considering the gravity force, the vertical resultant forces at those three entry angles are mostly along the opposite direction with the direction of the vertical velocity during the fluctuation period. Therefore, the absolute values of the velocities decrease first. In the later stage after $t = 0.02$ s, the variation trends of the vertical velocities at those three entry angles are slightly different, as shown in Figure 14b. The absolute value of the vertical velocity of the torpedo entering the water at $\beta = 90^\circ$ slowly decreases after $t = 0.02$ s as the induced vertical impact load is always slightly larger than the gravity force of the torpedo during this period. However, for entry angles of $\beta = 60^\circ$ and 75° , the corresponding vertical impact loads increase from the loads being smaller than the gravity force to the loads being larger than the gravity force. Thus, the resultant forces at those two entry angles in the later stages first exhibit the same direction with the vertical velocities and then vary to the opposite direction. Correspondingly, the absolute values of the velocities at entry angles of $\beta = 60^\circ$ and 75° first increase and thereafter decrease in the later stages, as shown in Figure 14d.

Variations in the horizontal displacement $x - x_0$ and the vertical displacement $y - y_0$ of the center of mass of the torpedo with time are shown in Figure 14e,f, respectively. In general, the displacements along the different dimensions are mainly determined by the corresponding initial velocities as the velocities at different dimensions and different entry angles fluctuate within relatively small ranges during such a short period from 0 s to 0.2 s. At the same entry velocity, the smaller entry angle results in a larger absolute value of the

horizontal velocity and a smaller absolute value of the vertical velocity according to the principle of velocity orthogonal projection. Therefore, the smaller entry angle ultimately induces a larger absolute value of the horizontal displacement and, simultaneously, a smaller absolute value of the vertical displacement.

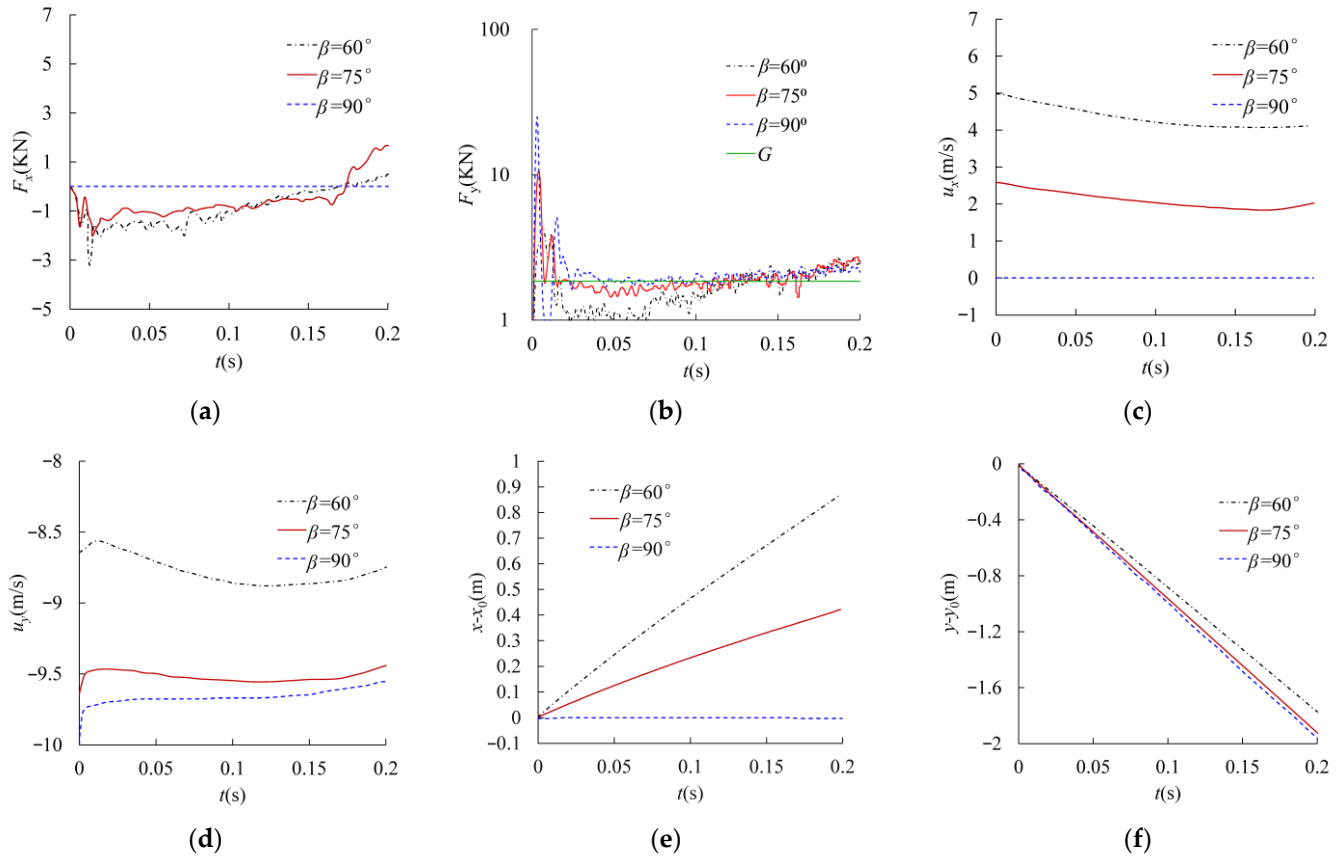


Figure 14. Variations in (a) the impact load projecting onto the x -axis F_x , (b) the impact load projecting onto the y -axis F_y , (c) the velocity projecting onto the x -axis u_x , (d) the velocity projecting onto the y -axis u_y , (e) the horizontal displacement $x - x_0$, and (f) the vertical displacement $y - y_0$ of the center of mass of the torpedo with time when the torpedo enters water at different entry angles of $\beta = 60^\circ$, 75° , and 90° under the same entry velocity of 10 m/s.

Figure 15 shows the variations in the moment about the z -axis m_z , the angular velocity ω_z , and the variation in the attitude angle of the torpedo $\theta - \theta_0$ with time when the torpedo enters the water at different entry angles of 60° , 75° , and 90° under the same entry velocity of 10 m/s. When the torpedo vertically enters water at $\beta = 90^\circ$, there is no rotating movement of the torpedo observed, owing to the relatively uniform force acting around the torpedo body. Therefore, the angular velocity and the variation in the attitude angle at $\beta = 90^\circ$ are always maintained at zero. However, for the torpedo entering the water at the other two entry angles, similar and remarkable fluctuations in different parameters of the torpedo are observed. Owing to the induced complex impact loads acting on the torpedo body at different periods of the water entry, as shown in Figure 14a,b, the moments of the torpedo first decrease to negative values and then increase to positive values and finally decrease again. Affected by the fluctuating moments, the corresponding angular velocities and the variations in the attitude angles exhibit similar variation trends with those of the moments. In general, as the entry angle decreases, the fluctuation in the moment acting on the torpedo becomes more significant, thereby resulting in greater changes in the angular velocity and the attitude angle of the torpedo.

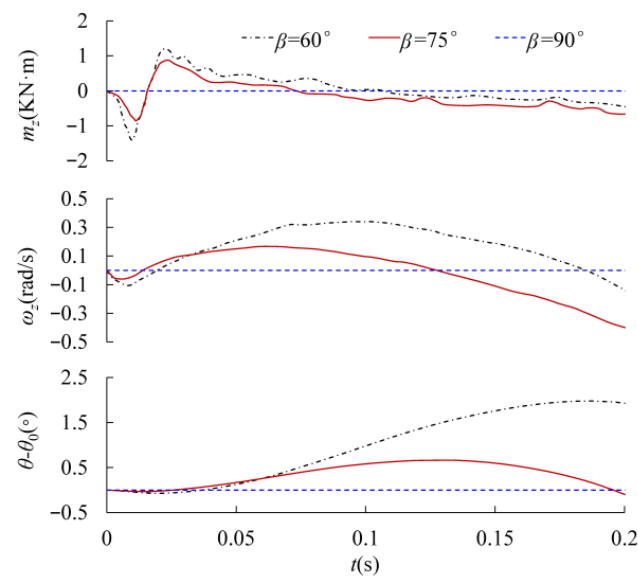


Figure 15. Evolutions of the moment about the z-axis m_z , the angular velocity ω_z , and the variation in the attitude angle $\theta - \theta_0$ of the torpedo when the torpedo enters the water at different entry angles of $\beta = 60^\circ$, 75° , and 90° under the same entry velocity of 10 m/s.

4. Conclusions

This work systematically investigates the complex water entry process of the 3D torpedo by applying an improved adaptive SPH algorithm. The initial flow field is generated by an algorithm of 3D meshless point generation. Based on adaptive particle refinement (APR) technology, the dynamic refinement/coarsening algorithm of particles is improved, avoiding the flow field noise in the dynamic refinement process and maintaining the mass conservation of the flow field. The guard area adopted in APR technology facilitates the realization of the coupling of two regions of different resolutions, avoiding the numerical error caused by the interaction between particles of different smooth lengths. In addition, the enhanced shifting algorithm is used to ensure the uniform distribution of particles during calculations, effectively preventing non-physical oscillations of the flow field and improving the accuracy of local SPH numerical simulations.

Based on the improved SPH method, a parametric study on the water entry of the torpedo is conducted from two aspects: entry velocity and entry angle. When the torpedo vertically enters water at different velocities, all the induced impact loads acting on the head of the torpedo fluctuate significantly with two peak values in the initial stage and thereafter stabilize in the later stage. With the growth in the entry velocity, the impact load acting on the torpedo, the entry depth of the torpedo, the splash height of the droplets, and the size of the cavity generated around the torpedo increase. When the torpedo enters the water at different entry angles under the same initial entry velocity, both the vertical and the horizontal movement of the torpedo are observed, leading to increased and more complex variations in parameters along both the x - and the y -axes. First, the torpedo entering water at angles except $\beta = 90^\circ$ results in one side of the cavity being larger than another side, forming an asymmetric cavity. Second, with a decrement in the entry angle, the fluctuation in the horizontal impact load becomes more significant, the horizontal velocity increases, and the corresponding horizontal displacement of the center of mass of the torpedo increases, throughout the entire water entry process of the torpedo. On the contrary, opposite conclusions about variations in the fluctuation in the vertical impact load, the absolute value of the vertical velocity, and the absolute value of the vertical displacement of the center of mass of the torpedo with decreasing entry angle are observed. Third, the moment, the angular velocity, and the variation in the attitude angle of the torpedo obliquely entering water initially decrease to negative values, then increases to positive values, and finally decreases again. In addition, the decreasing entry

angle increases fluctuations in the moment, the angular velocity, and the attitude angle of the torpedo.

This work systematically investigates the complex nonlinear fluid–structure coupling performance of the water entry of the torpedo, which provides valuable guidance for the future design of the entry trajectory and the structural bearing capacity of the torpedo. However, there are three challenges for future works. Firstly, when a torpedo enters the water, the head will be subjected to a transient peak load during the impact stage, which may damage the structure. This aspect is not fully investigated along with the water entry process in this work. Therefore, it is necessary to develop an SPH-FEM coupling algorithm to solve the elastic deformation problem of the structure in the future, which will give more complicated guidance on the design of the torpedoes. Secondly, when a torpedo enters the water at a high speed, a low-pressure area will be formed at the junction of the torpedo head and the torpedo body, which may cause local cavitation. The existing SPH method lacks an effective cavitation model. Therefore, it is an important research direction to establish an SPH algorithm of a two-phase flow coupled with a cavitation model. Finally, it is also worth studying how to improve the search efficiency of neighboring particles after adaptive refinement.

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