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Proposal of a New Control System Making Use of AI Tools to Predict a Ship's Behaviour When Approaching the Synchronism Phenomenon

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Abstract: In spite of the IMO's efforts to improve the safety of intact stability on ships, synchronous rolling still causes large rolling angles, resulting in a very dangerous situation on board. The implementation of automatic tools to predict this undesirable situation has not been widely implemented. Furthermore, the safety of ship stability is not the only responsibility of people involved in the design process, so ship operators must have enough knowledge to predict and avoid these dangerous situations well in advance. Therefore, from a theoretical point of view, in the first part, this paper aims to present a valuable guiding tool for ship operators in order to predict synchronous rolling and avoid undesirable situations on board by making use of only empirical observations of the wave profile and moments. With this purpose, mathematical models are first proposed, with the ship sailing in the worst condition, i.e., with and without considering the damping factor, at zero speed and considering the influence of any pure beam and trochoidal waves. Relevant results shown provide the exact time and wave profile at which the maximum rolling angles are reached. In the second part of the paper, a new control system making use of AI tools is proposed in order to be used by ship operators on board, avoiding dangerous situations. Finally, the results are validated using a set of ship rolling simulations for the most common and representative ship loading conditions and wave periods.

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Keywords: artificial intelligence (AI); control system on board; synchronism phenomenon; ship's rolling motion; ship's stability; beam and trochoidal waves

1. Introduction

The behaviour of ships sailing in waves is of utmost importance for safety from an intact stability point of view [1]. For this reason, the International Maritime Organization (IMO) developed the Second Generation Intact Stability Criteria (SGISC) to cover five failure modes that may occur in all vessels [2,3]. However, even on ships that meet existing stability criteria, accidents still occur [4]. However, in most cases, the seakeeping operability criteria related to roll motion and lateral acceleration can help in preventing the ship from experiencing dangerous situations due to roll synchronism, since they set limits low enough to allow people on board to work. Of the six degrees of freedom, the rolling motion is the most important from the point of view of periodicity, amplitude of angles and accelerations [5]. In fact, one of the five failure modes in SGISC is the excessive acceleration criterion, which deals with the lateral acceleration experienced on board. This criterion was introduced in the SGISC framework later than the other four failure modes and as a consequence of a submission from Germany, which noted that lateral acceleration due to synchronous rolling was not addressed by the other four SGISC [6]. This rolling motion is directly related to the above-mentioned intact ship stability because the ship's natural rolling period (T_d) is inversely proportional to the square root of the metacentric

height (GM), i.e., the ship's loading condition, and directly proportional to the beam of the ship. Rosano et al. [7] proposed an interesting system to monitor lateral acceleration and estimate the extreme values that could occur, especially sailing in beam waves and at low speed. To test the system, a mid-sized stern trawler was selected, showing that the operability criterion for lateral acceleration is exceeded in many sea-state conditions.

If the ship encounters a series of waves where its natural rolling period matches the wave period, synchronous rolling can occur, and the vessel will not have time to right itself before the next wave strikes [8]. If this situation is not previously detected by ship operators, it can result in the ship's capsizing, with disastrous consequences for the safety of the ship, its crewmembers, its cargo and the environment [9,10]. Therefore, when ships are close to reaching the transversal synchronism phenomenon, they are in serious danger because they may roll over beyond a point from which they cannot return to the upright position, resulting in capsizing [11]. Other dangers associated with this problem are water washing over the deck, which is especially dangerous for small fishing vessels with low weather decks [3], or cargo shifting on merchant ships, which can result in loss of stability and, finally, capsizing. Furthermore, excessive rolling can cause cargo lashings to give way, resulting in cargo damage and, in some cases, structural damage to the ship [12]. Therefore, in spite of not being studied as much as other phenomena such as parametric resonance, synchronous rolling is a typical experience for almost all sailors. For this reason, researchers such as Terada et al. [13] studied the procedure for the measurement of the ship's natural rolling period (T_d) during navigation. Furthermore, in order to connect the relationship of T_d with the sea state, other scholars such as Nielsen et al. [14] addressed the estimation of sea state during the sea route in order to predict wave-induced ship motions.

In view of the development of autonomous merchant ships, Acanfora et al. [15] proposed a new method for detecting and avoiding dangerous phenomena on board these vessels due to excessive motion as a consequence of the synchronous roll produced in rough seas. With this approach, remote operators have sufficient time to react to avoid large rolling angles and capsizing. However, although it is very interesting research, it is important to notice that, at present, the consequences of synchronism still occur in non-autonomous ships. Other researchers such as Davidson et al. [16] designed a control system to mitigate the onset of parametric rolling in a real-time detection system. Using this tool and on-board measurements of the angular displacement and the velocity of the ship's roll, the system provides an early warning with sufficient time for the control system to respond and take the best action, avoiding an increase in the rolling amplitude. Previously, Galeazzi et al. [17] proposed signal-based methods to detect the onset of parametric roll in the time domain. Luthy et al. [18] also presented an innovative real-time detection method to predict the parametric roll in regular longitudinal waves. Other authors [10] proposed a decision support system to be used during the operation of fishing vessels in waves to improve vessel safety. After processing several parameters and measurements, this tool provides the ship master with the consequences of various ship-handling decisions. However, all these very interesting proposals, and other similar ones, have not been widely implemented in the world fleet, due to the difficulties associated with control system design as a consequence of the characteristics of wave-induced roll motion [9].

As some relevant research indicates [19], the safety of ship stability cannot only be ensured by measures taken during the design process of naval architects. What is more, it is essential that ship operators (ship master and officers on watch) assume a certain level of knowledge, skills, experience and prudence during ship operation to avoid dangerous situations for the ship's safety. Then, in some potentially dangerous operational situations, the officer on watch must have the knowledge to predict and make the most appropriate decision to reduce the risk level. This philosophy is included in the so-called operational measures of the SGISC framework, where operational limitations and operational guidance are provided to ship masters to support decision making on board and ensure

the safety of navigation [20]. As a consequence of this IMO resolution, important works about operational guidance were published. Researchers such as Begovic et al. [21] were focused on rolling motion and proposed a new procedure to be easily implemented in a user's code to assess simplified operational guidance without the use of commercial software at any ship speed and heading. Petacco [22] proposed a new guideline which relies on a simplified numerical tool capable of assessing the rolling motion due to non-linear irregular wave effects in the time domain, keeping computational effort as low as possible. Regarding the operational limitation, according to MSC.1/Circ.1627 [20], it is possible to design a ship to be operated in a specific operational area. For example, Bulian et al. [23] carried out an interesting study on the design of a vessel taking into account the specific conditions that the ship will operate in, particularly with regards to parametric roll. Therefore, the importance of predicting excessive rolling motion is shown in other works such as [6], where excessive lateral acceleration was investigated in naval vessels in order to determine the agreement of their stability criteria with the new SGISC, although the SGISC do not refer to naval vessels.

Although the synchronism phenomenon can be experienced with any wave influence direction with respect to the ship's heading, it is most likely to happen when the ship is sailing in pure beam seas. Moreover, a ship is most vulnerable to this undesirable phenomenon when it is in static condition, such as when a fishing vessel is engaged in fishing tasks, or when it has suffered engine failure. For this reason, the Level 2 criterion of SGISC considers a ship at zero speed in beam waves [21]. Therefore, in the present paper, as in other relevant research on parametric resonance [24], the influence of the most unfavourable condition for synchronous rolling was studied, i.e., receiving purely regular beam waves.

The effective prediction of this situation results in being profitable, specifically in the fishing industry, which is most prone to labour accidents [10]. A fishing vessel is a type of ship that usually has a low weather deck, so when it is sailing or engaged in fishing tasks in adverse sea conditions, especially with a roll amplitude larger than a certain angle, water can submerge part of the deck and then flow back into the sea from the deck. Studies about how the officer on watch can empirically predict these dangerous situations without complex calculations or specific tools installed on board are insufficient and rare. Most of the previous research has focused on high freeboard ships such as container ships, Ro-Ro ships and passenger ships affected by parametric roll [3]. Therefore, the results presented in the present paper can be a valuable guiding tool for officers on watch in order to predict synchronous rolling and avoid undesirable situations.

As the rolling angles when the ship is approaching a synchronism condition build up very fast, the present work tries to explain the basic mechanism of ship rolling with simplified theoretical and numerical models, using only one degree of freedom, similar to procedures followed by other scholars [8,25]. The prediction of the synchronism phenomenon is strictly related to the mathematical modelling of ship motion. Therefore, in this paper, novel and simplified mathematical models were obtained. Furthermore, papers about the estimation of roll damping [26] point out that the estimation of roll damping is a difficult task due to the complexity of quantifying the viscous effect influencing roll motion. However, in this research, an average value of damping factor is used in order to be as simple as possible.

Afterwards, in this paper, a new control system making use of feedforward neural network tools is proposed in order to correct the effect of new waves (T_w) encountered in the sea passage and alter the ship's natural roll period (T_d) determined by specific loading conditions. With this control system of neural networks, ship operators can be alerted about dangerous situations on board well in advance, so that they can alter T_d accordingly. For this mission, it is necessary that weather forecasts are received on board on a regular and updated basis so the ship master and officers can act on the loading condition in advance.

The remainder of the present paper is structured as follows: in Section 2, the mathematical models previously used by authors are presented and new ones obtained, together with the presentation of the artificial intelligence used. Section 3, making use of a set of ship rolling simulations, shows the results and application of the present work, obtaining interesting conclusions that are as simple as possible. Section 4 discusses the models and the results, and Section 5 concludes the paper.

2. Materials and Methods

The theoretical-practical analysis of a ship's rolling motion in waves is subject to some initial conditions and assumptions. Therefore, it has to be understood as an orientation of its behaviour in beam waves.

This paper studies the special case when T_d is equal to T_w , i.e., when the synchronism phenomenon is reached. With this goal, in the first sub-section, the mathematical models used to define the full spectrum of sea state conditions are shown; secondly, a novel mathematical model for the rolling motion of a ship sailing in trochoidal and beam waves, without considering roll damping, is obtained. In the third sub-section, the mathematical equations previously published by authors are shown in order to analyse and compare, in the Results section, a ship's behaviour and trends before it reaches the synchronism phenomenon studied in the present paper. In the fourth sub-section, the effect of the damping factor of water resistance in the synchronism phenomenon is studied, obtaining a new mathematical model of a ship's rolling angles. Finally, the neural network and the process of simulation used and followed in the present paper are shown.

2.1. Trochoidal Waves

The waves generated on the sea free surface have a definite importance for a ship's behaviour, as they affect its stability and the structural stresses. The knowledge and control of these parameters to ensure the safety of sea navigation, the ship, and its crew and cargo is the responsibility of the ship master and deck officers.

The sea free surface presents a very complex irregular appearance, showing a certain regularity in the crests and the troughs as soon as regular waves move in the same direction. The time between crests, their heights and the distances between them vary widely and erratically. To evaluate the effect of this irregular surface on a ship's behaviour, it is necessary to perform a systematic analysis, decomposing a complex physical phenomenon into simpler parts to study its effects. This method assumes that irregular waves are formed by the superposition of a number of regular waves of different characteristics and that the ship's motion will be the result of the superposition of the effects of each of these regular waves. For this reason, one of the simplest and most common ways of describing the wave profile of the sea free surface is to consider the trochoidal wave following the first theory developed by Franz Gerstner in 1802 [27,28]. However, although the intention of the present paper is not to analyse the influence of the full spectrum of waves on a ship's behaviour, in this sub-section it is necessary to show the mathematical models followed referring to sea state conditions.

According to the authors' previous work, the following mathematical model was followed, where a precision above 99% for the determination factor was observed [29].

$$L_w(m) = 1.5838 \cdot T_w^2 - 0.5558 \cdot T_w + 1.4737, \quad (1)$$

2.2. Synchronism Phenomenon When Sailing between Beam Regular Waves in a Non-Resistant Environment

If a ship is in an upright position in a wave's trough, it receives the effect of the wave's slope and then the rolling motion starts, according to the induced movement of Equation (2), which represents the rolling motion of a ship sailing in regular waves in a non-resistant environment [27]:

$$\theta = \frac{d\varphi}{dt} \cdot \frac{1}{\omega} \cdot \sin \omega \cdot t + \varphi \cdot \cos \omega \cdot t + \frac{\theta_{MW}}{1 - \frac{Td^2}{Tw^2}} \cdot \sin \omega_w \cdot t, \quad (2)$$

The first two terms of Equation (2) corresponding to the rolling motion in calm waters (free motion) are equal to zero.

$$0 = \frac{d\varphi}{dt} \cdot \frac{1}{\omega} \cdot \sin \omega \cdot t + \varphi \cdot \cos \omega \cdot t, \quad (3)$$

Therefore, the induced rolling motion is

$$\theta = \frac{\theta_{MW}}{1 - \frac{Td^2}{Tw^2}} \cdot \sin \omega_w \cdot t, \quad (4)$$

After carrying out mathematical operations, Equation (2), of the rolling motion of a ship sailing without resistance in regular waves, when $Td = Tw$, can be concluded as follows:

$$\theta = \frac{d\varphi}{dt} \cdot \frac{1}{\omega} \cdot \sin \omega \cdot t + \varphi \cdot \cos \omega \cdot t - \frac{\theta_{MW}}{2} \cdot \omega \cdot t \cdot \cos \omega_w \cdot t, \quad (5)$$

Then, the ship's rolling angle due to induced motion during the synchronism phenomenon, i.e., when $Td = Tw$ ($\omega = \omega_w$) and without resistance, can be expressed as follows:

$$\theta = -\frac{\theta_{MW}}{2} \cdot \frac{2\pi}{Td} \cdot t \cdot \cos \left(\frac{2\pi}{Td} \cdot t \right) = -\theta_{MW} \cdot \frac{\pi}{Td} \cdot t \cdot \cos \left(\frac{2\pi}{Td} \cdot t \right), \quad (6)$$

2.3. Rolling Motion of a Ship at Zero Speed and Sailing in Regular Beam Waves in a Resistant Environment

As previously published, the rolling angle of a ship in static conditions under the influence of regular beam waves, considering the resistance of the environment, is defined by the following equation [27,30]:

$$\theta = \theta_M \cdot e^{\frac{-\lambda_1 \cdot t}{Td}} \cdot \left[\cos \frac{2\pi}{Td} \cdot t + \frac{\lambda_1}{2\pi} \cdot \sin \frac{2\pi}{Td} \cdot t \right] + \frac{\theta_{MW} \cdot \cos \beta \cdot \sin \left[\frac{2\pi}{Tw} \cdot t - \beta \right]}{1 - \frac{Td^2}{Tw^2}}, \quad (7)$$

where λ is the rolling damping factor, λ_1 is the result of $(\lambda \cdot t)$ and β is the angle obtained as follows:

$$\tan \beta = \frac{2\lambda_1}{Td} \cdot \frac{\omega_w}{\omega^2 - \omega_w^2} = \frac{\lambda_1}{\pi} \cdot \frac{\frac{Td}{Tw}}{1 - \frac{Td^2}{Tw^2}}, \quad (8)$$

2.4. Influence of the Damping Factor on the Synchronism Phenomenon

In this sub-section, the effect of the damping factor on the synchronism phenomenon is studied, i.e., when $Td = Tw$. From Equation (7), the component of the induced motion between waves and with resistance is obtained:

$$\theta_f = \frac{\theta_{MW} \cdot \cos \beta \cdot \sin \left[\frac{2\pi}{T_w} \cdot t - \beta \right]}{1 - \frac{T_d^2}{T_w^2}}, \quad (9)$$

After carrying out mathematical operations, the equation of the induced rolling motion as a function of the ship's natural roll period (T_d) and wave period (T_w) is as follows [27]:

$$\theta_f = \frac{\theta_{MW} \cdot \sin \left[\frac{2\pi}{T_w} \cdot t - \beta \right]}{\sqrt{\left(1 - \frac{T_d^2}{T_w^2}\right)^2 + \left(1 - \frac{T_d^2}{T_w^2}\right) \cdot \frac{\lambda_1^2}{\pi^2} \cdot \frac{T_d^2}{T_w^2}}} = \frac{\theta_{MW} \cdot \sin \left[\frac{2\pi}{T_w} \cdot t - \beta \right]}{\sqrt{\left(1 - \frac{T_d^2}{T_w^2}\right)^2 + \frac{\lambda_1^2}{\pi^2} \cdot \frac{T_d^2}{T_w^2}}}, \quad (10)$$

Now, Equation (10) is introduced in Equation (7) in order to obtain the equation of the rolling angle of a ship sailing in beam waves in a resistant environment, which allows analysis of the influence of the damping factor on the synchronism phenomenon.

$$\theta = \theta_M \cdot e^{\frac{-\lambda_1 \cdot t}{T_d}} \cdot \left[\cos \frac{2\pi}{T_d} \cdot t + \frac{\lambda_1}{2\pi} \cdot \sin \frac{2\pi}{T_d} \cdot t \right] + \frac{\theta_{MW} \cdot \sin \left[\frac{2\pi}{T_w} \cdot t - \beta \right]}{\sqrt{\left(1 - \frac{T_d^2}{T_w^2}\right)^2 + \frac{\lambda_1^2}{\pi^2} \cdot \frac{T_d^2}{T_w^2}}}, \quad (11)$$

2.5. Artificial Intelligence (AI) for the Ship's Stability Control System

The aim of this work is to define if it is possible to compensate for the effect of new sea conditions (T_w) by modifying weight distributions on board (T_d), which, initially, is not considered in daily voyage considerations. Despite this, due to the complex resolution of the ship's rolling equation in a control system, a neural network curve fitting was proposed to be employed in a future control system that may alert ship operators about risky situations and possible T_d modifications of interest.

The first step was to define the steady equation of ship balance, as shown in Figure 1.

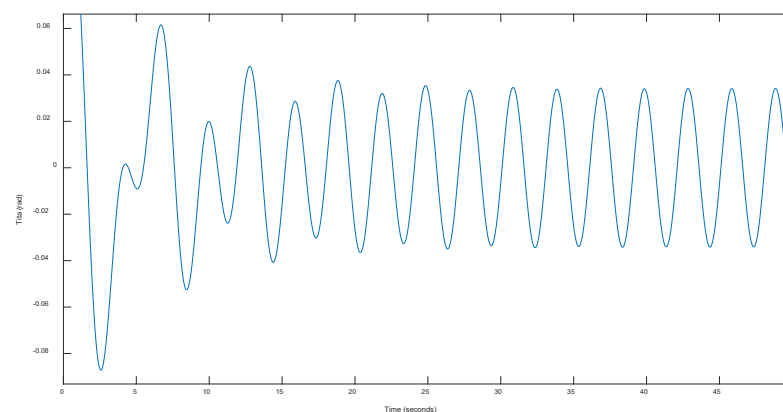


Figure 1. A ship's rolling motion.

The previous novel mathematical models obtained were introduced in the Matlab Simulink toolbox to analyse the transient process when a ship under a loading condition, represented by the ship's natural roll period of $T_d = 6$ s, for instance, is sailing in a sea that changes its T_w from 1 to 21. This initial process was modelled as shown in Figure 2.

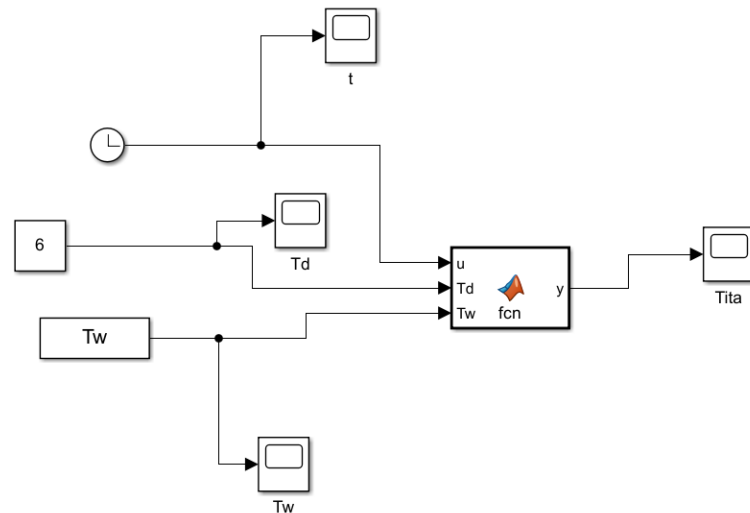


Figure 2. Process simulation in Matlab Simulink.

In Figure 2, it can be observed that time (t), T_w and t_d are input variables in the function that defines the final value ($Tita$). The more important advantage of this tool is its ability to solve equations graphically in transient processes, as was previously done in control systems such as the one proposed in this work.

3. Results

Using a set of ship rolling simulations, this section presents, point by point, the results obtained according to the above novel mathematical models. For that, the results are expressed based on parameters most easily measurable on board by the ship operators, i.e., T_d , T_w and the ship's position along the wave profile (crest and trough).

In order to establish the maximum and minimum values of the ship's double natural period (T_d) used in the present work (defined by the ship's loading condition), the following empirical equations available in the literature are followed [31].

$$T_d = 0.77 \cdot \frac{B}{\sqrt{GM}}, \quad (12)$$

Furthermore, according to the IS code [32], T_d (s) can be calculated as follows:

$$T_d = \frac{2 \cdot C \cdot B}{\sqrt{GM}}, \quad (13)$$

where

$$C = 0.373 + 0.023 \left(\frac{B}{d} \right) - 0.043 \left(\frac{Lwl}{100} \right), \quad (14)$$

considering that B is the beam (m), d the draught (m) and Lwl the length at the waterline (m). Moreover, some classification societies, such as DNV (2016) [33], state that the relationship between GM and B is

$$GM = 0.07 \cdot B \quad (15)$$

In addition, according to DNV (2016) [33], knowing the beam (B) and the metacentric height (GM), Td can also be calculated as follows:

$$T_d = \frac{2 \cdot 0.39 \cdot B}{\sqrt{GM}} \quad (16)$$

Therefore, for the most common particulars of a ship, regardless of the type of vessel involved, the range $6 \text{ s} < T_d < 16 \text{ s}$ can be considered sufficiently representative, under normal conditions, for any ship and loading condition.

3.1. Synchronism Phenomenon When Sailing in Regular Beam Waves in a Non-Resistant Environment

Analysing Equation (6) corresponding to the induced motion during the synchronism phenomenon, it is found that the maximum value $\pm \theta_{MW}$ is reached when $\cos\left[\frac{2\pi}{T_d} \cdot t\right] = \pm 1$, which is achieved with a periodicity of $t = \left(\frac{n+1}{2}\right) \cdot T_d$. Furthermore, if different values are given for t (time) in Equation (6), the ship's rolling angles as a function of the maximum angle of the wave slope are deduced, as shown in Table 1.

Table 1. Moments of reaching the ship's rolling angles as a function of the wave slope.

t	θ
$\frac{1}{2} \cdot T_d$	$\theta_{MW} \cdot \frac{\pi}{2}$
T_d	$-\theta_{MW} \cdot \pi$
$\frac{3}{2} \cdot T_d$	$\theta_{MW} \cdot \frac{3\pi}{2}$
$2 \cdot T_d$	$-\theta_{MW} \cdot 2\pi$
$\frac{1}{2} \cdot T_d + n \cdot T_d$	$\theta_{MW} \cdot \pi \left(\frac{1}{2} + n\right)$
$n \cdot T_d$	$-\theta_{MW} \cdot n \cdot \pi$

According to the results of Table 1, it can be concluded that for each half- T_d ($\frac{1}{2} \cdot T_d$), the ship's rolling angle is increased by $\frac{1}{2} \cdot \theta_{MW} \cdot \pi$. This interesting conclusion can be verified in the representation of the free rolling motion in Figure 3, where a usual ship's loading condition, represented by the natural roll period, is simulated ($T_d = 10 \text{ s}$). Therefore, as T_d is theoretically a constant value, the angular velocity will be greater each time the ship reaches an upright position.

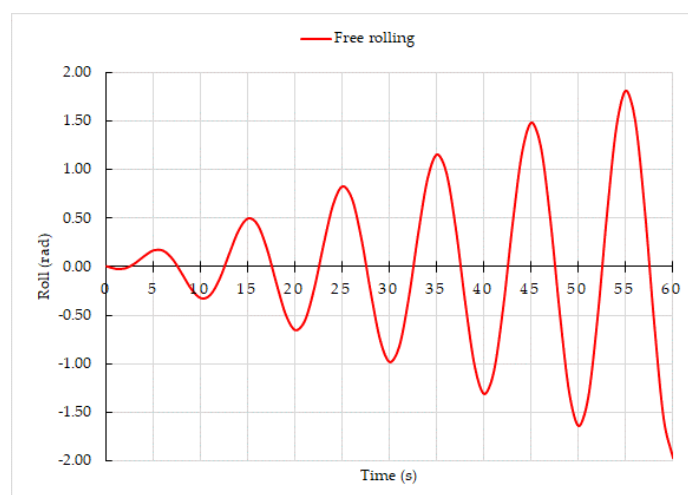


Figure 3. Free rolling motion for a ship's natural roll period of $T_d = 10$ s.

3.2. Rolling Motion of a Ship at Zero Speed and Sailing in Regular Beam Waves in a Resistant Environment

In this sub-section, two cases of rolling motion in beam waves are studied, depending on whether T_d is very small or large.

3.2.1. First Case: Very Small T_d

When the T_d of a ship is very small, $T_d/T_w \rightarrow 0$, and according to Equation (8), $\tan\beta \rightarrow 0$; $\beta \rightarrow 0$; $\cos\beta \rightarrow 1$. Furthermore, the factor $e^{\frac{-\lambda_1 \cdot t}{T_d}}$ of the ship's free rolling motion causes this rolling decrease quickly due to the damping factor, so that the rolling motion is sufficiently small for it to be neglected compared to the rolling motion induced by the waves. Therefore, the term of the induced rolling motion of Equation (7) can be reduced to

$$\theta_f \rightarrow \theta_{MW} \cdot \sin\left[\frac{2\pi}{T_w} \cdot t\right], \quad (12)$$

In Table 2, for some values of t (time), the position of the ship in the wave profile and the corresponding value of the induced motion are included. On applying Equation (17), the range of this induced motion is between zero and the maximum angle of the wave slope, θ_{MW} .

Table 2. A ship's rolling motion for very small T_d .

Time (t)	Ship's Position in the Wave	$\sin\left[\frac{2\pi}{T_w} \cdot t\right]$	Induced Motion, θ_f
0	Trough	0	0
$\frac{1}{4} \cdot T_w$	Middle of the slope	+1	$+\theta_{MW}$
$\frac{1}{2} \cdot T_w$	Crest	0	0
$\frac{3}{4} \cdot T_w$	Middle of the slope	-1	$-\theta_{MW}$
T_w	Trough	0	0

Figure 4 represents the simulation for a sea state condition of $T_w = 10$ s and a very small ship's loading condition ($T_d = 6$ s), which can be found in some small fishing vessels. Figure 4 can be corroborated by the relevant results expressed in Table 2. Therefore, it is concluded that the masts of a ship closely follow the normal to wave profile, which is

consistent with the response of a large ship's righting moment (a very small T_d corresponds to a large GM) [5].

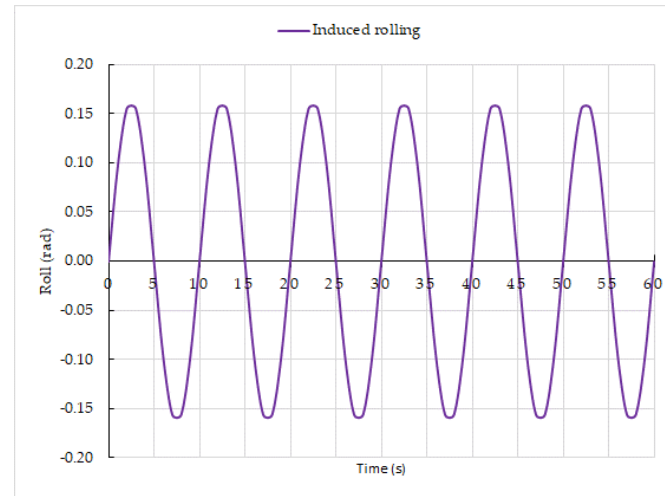


Figure 4. Induced rolling angles for $T_w = 10$ s and $T_d = 6$ s.

The absolute rolling angle is zero at the trough and the crest of the wave, having its maximum value at the middle of the wave slope. Therefore, a ship rolls according to the wave slope, with the relative rolling practically zero and keeping its deck parallel to the wave profile.

3.2.2. Second Case: Large T_d

In this case, when the T_d of a ship is large, $T_w/T_d \rightarrow 0$ and $\beta \rightarrow 0$. Considering that $\omega^2 = (2\pi/T_d)^2 \rightarrow 0$, Equation (8) can be expressed as follows:

$$\tan \beta = \frac{2\lambda_1}{T_d} \cdot \frac{1}{(-\omega_w^2)} = -\frac{2\lambda_1}{T_d} \cdot \frac{T_w}{2\pi} = -\frac{\lambda_1}{\pi} \cdot \frac{T_w}{T_d}, \quad (13)$$

Then, $\tan \beta \rightarrow 0$; $\beta \rightarrow 0$; $\cos \beta \rightarrow 1$.

As in the first case, the effect of the induced rolling motion is analysed. Therefore, according to Equation (7), for the induced motion, the following equation is obtained:

$$\theta_i = \frac{\theta_{MW} \cdot \sin \left[\frac{2\pi}{T_w} \cdot t \right]}{1 - \frac{T_d^2}{T_w^2}}, \quad (14)$$

The denominator takes a high value, and consequently, the induced rolling motion is small. Moreover, according to the results shown in Table 2, corresponding to the first case (very small T_d), it is observed that in the trough and the crest of the wave, the induced rolling motion is also zero in the present case. Furthermore, the maximum value of the induced rolling angle is also reached at the middle of the wave slope, although this value is smaller than in the first case. These conclusions can be verified in Figure 5, where the same sea state condition ($T_w = 10$ s) is simulated, but with a ship natural roll period of $T_d = 16$ s, which can be found in passenger vessels. Therefore, considering the same sea state conditions, the maximum rolling angle for a 'stiff' ship with $T_d = 6$ s is 0.155 rad (Figure 4), while for a 'tender' ship with $T_d = 16$ s, it is 0.063 rad (Figure 5).

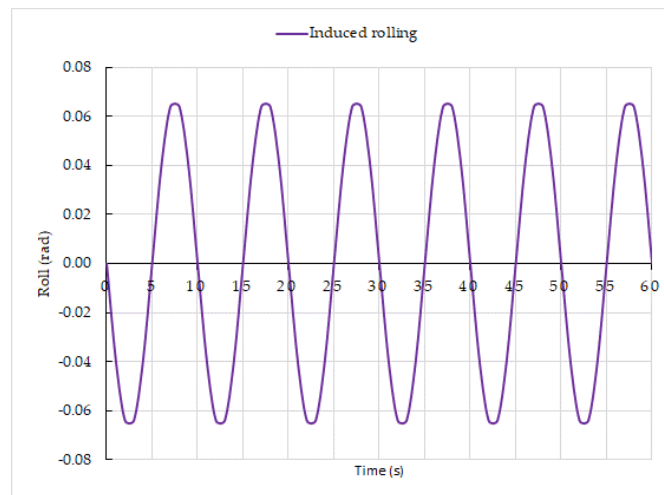


Figure 5. Induced rolling angles for $T_w = 10$ s and $T_d = 16$ s.

As in this case study T_d is large, i.e., the metacentric height (GM) is small, the ship has a reduced righting moment. As a consequence, during the rolling motion, the masts of the ship tend to slowly follow the normal of the wave. The maximum angle of absolute rolling is small compared to the relative rolling motion with respect to the normal. This circumstance can lead to seawater washing over the deck easily.

3.3. Influence of the Damping Factor on the Synchronism Phenomenon

Using Equation (10) for the synchronism phenomenon, it can be deduced that $\tan\beta = \infty$, so $\beta = \frac{\pi}{2}$. The second term of Equation (11) reaches the maximum value $\pm\theta_{MW}$ when the numerator is $\sin\left[\frac{2\pi}{T_w} \cdot t - \beta\right] = \pm 1$, which is achieved with a periodicity of $t = \left(\frac{n+1}{2}\right) \cdot T_w$.

This remarkable conclusion can be observed in the results shown in Figure 6, where the induced rolling motion generated by two different sea state conditions is simulated, considering 0.015 as an average value of the damping factor, according to the literature [34,35]. In these simulations, this representative average value was used, although it is important to note that this parameter changes with the amplitude of different rolling angles. In the case of using another damping factor, the amplitude of the results shown in the figures will be different, but the appearance, and then the ship's behaviour, will be the same.

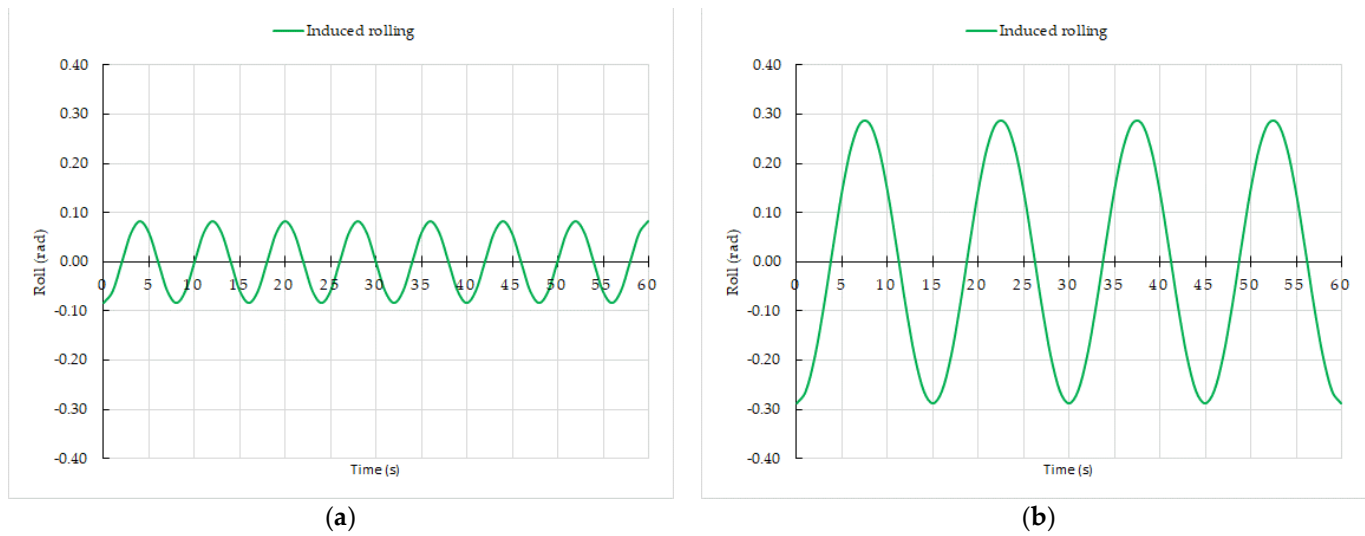


Figure 6. Representation of induced rolling motion for $T_d = 12$ s. (a) $T_w = 8$ s; (b) $T_w = 15$ s.

This periodicity indicates that the maximum rolling angles are reached in the crests and the troughs of the waves. What is more, for the T_d considered in the previous subsection (very small and large T_d), the maximum rolling angles were reached in the middle of the wave slope, while during the process of approaching the synchronism phenomenon, these maximum rolling angles are shifted towards the crest and the trough.

Taking into account the previously obtained equations, the relationship between the induced rolling motion and the wave slope maximum angle, θ_{MW} , is as follows:

$$\frac{\theta}{\theta_{MW}} = \pm \frac{1}{\sqrt{\left(1 - \frac{T_d^2}{T_w^2}\right)^2 + \frac{\lambda_1^2}{\pi^2} \cdot \frac{T_d^2}{T_w^2}}}, \quad (20)$$

θ_{MW} does not usually exceed 10° , and the wave slope maximum angle increases up to a finite value, which is achieved when there is equilibrium between the damping energy during a single rolling motion (port-starboard) and the energy induced by the wave in the passage of the trough or crest, i.e., half of wavelength.

Therefore, once the new mathematical models of the rolling motion were independently analysed and represented, Figure 7 depicts the combination of both effects for a sea state condition of $T_w = 15$ s and a usual ship's loading condition of $T_d = 12$ s.

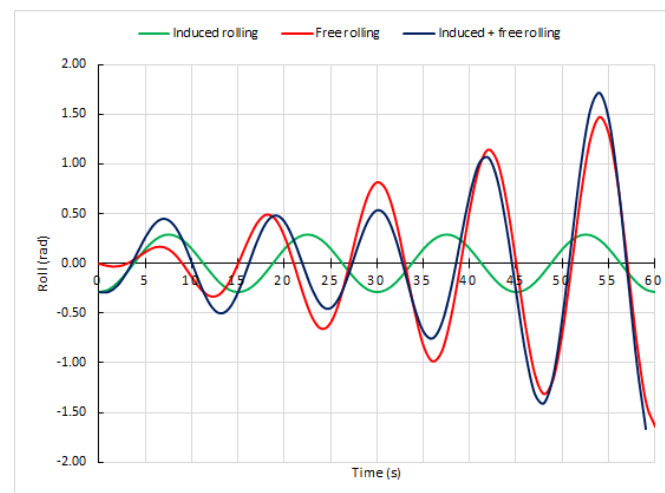


Figure 7. Rolling motion of a ship approaching synchronism.

From Figure 7, it is deduced that, as time goes on, during the synchronism phenomenon, the ship's tendency is to roll without considering the periodicity of the waves. Furthermore, the amplitude of the rolling angles increases according to the interesting conclusions mentioned in the previous sub-sections. What is more, according to this theoretical approach, any ship in less than a minute reaches a rolling angle where flooding occurs as a consequence of a non-watertight element being submerged in water. For this reason, it is essential that officers on watch detect this dangerous situation in order to take action and avoid potential problems. According to the present results, this warning can be easily detected by carefully checking the time on the wave profile where the maximum rolling angles occur.

3.4. Artificial Intelligence (AI) for Ship Stability Control System

Despite the fact that a symbolic mathematic resolution would let us define the derivative of this function, it is not possible to define an equation that defines T_d as a function of the sea conditions (T_w), the time (t), and the maximum rolling angle (θ). This is the reason why artificial intelligence may be a solution for defining this new relationship and, consequently, designing a control system or an alarm system to alert ship operators if needed.

To create a neural network, it is necessary to obtain a detailed combination of all the variables involved in the process in order to generate a map of it with adequate precision. Consequently, a training simulation was developed combining periods of 201 s for each T_d value (which varies from 6 to 16 s) with each possible sea condition (T_w from 1 to 21 s); see Figures 8–10. It is interesting to highlight that, from previous studies, synchronism appears before 200 s, so all the simulations were designed for 201 s.

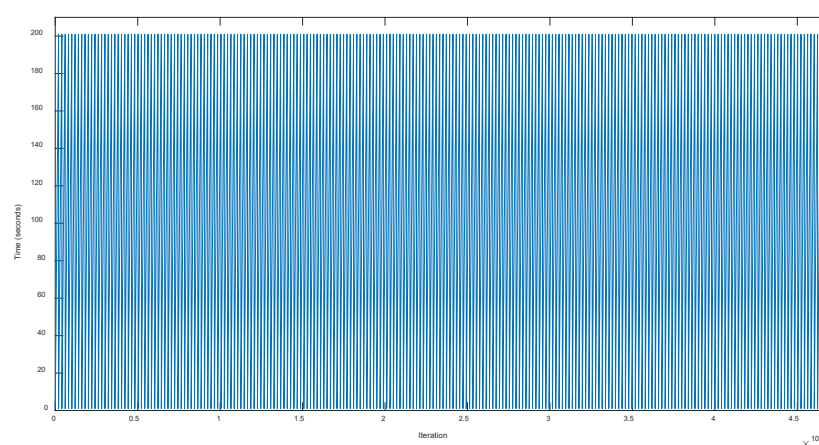


Figure 8. Time cycles of 201 s.

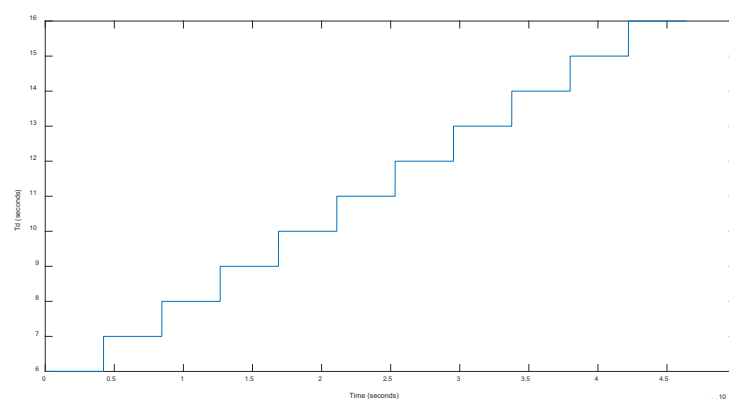


Figure 9. T_d change during 201 s for each T_w value (4222 s for each T_d value from 6 to 16 s).

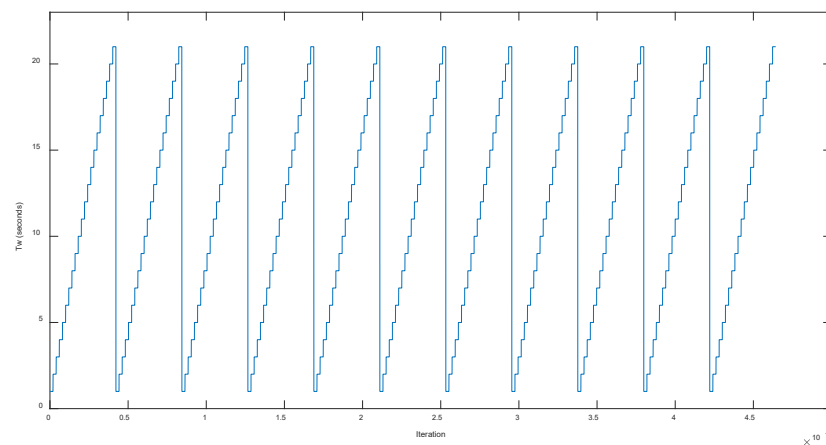


Figure 10. Tw increase for each Td value during 201 s (a total of 4222 s for each Td).

Consequently, Figure 11 shows the map of the process for each of the 11 Td values (from 6 to 16 s) changing with each sea condition from Tw = 1 s to T = 21 s.

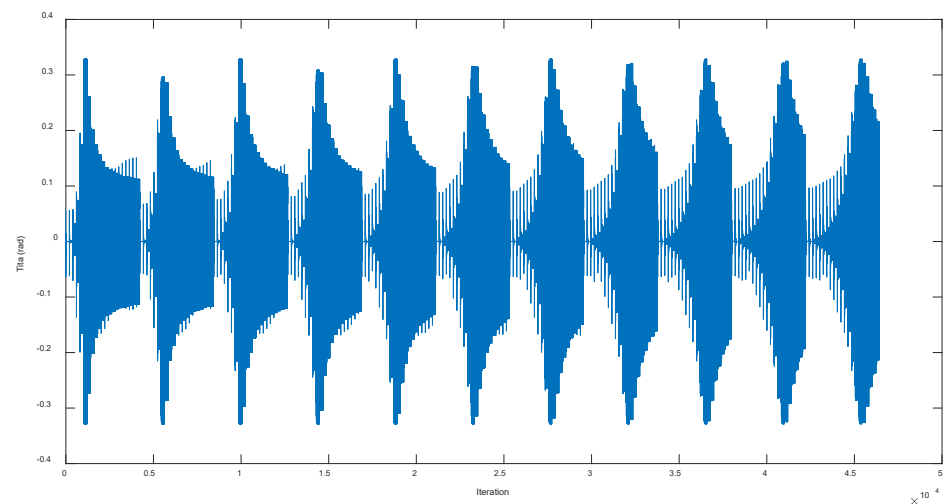


Figure 11. Rolling angle for each Td value (from 6 to 16 s) changing with each sea condition from Tw = 1 s to Tw = 21 s.

This information was employed to train a two-layer feedforward neural network with more than 46,000 values. The neural network was designed with 10 neurons in the hidden layer. The error assumed after the training process showed adequate precision, as shown in the regression curves in Figures 12 and 13.

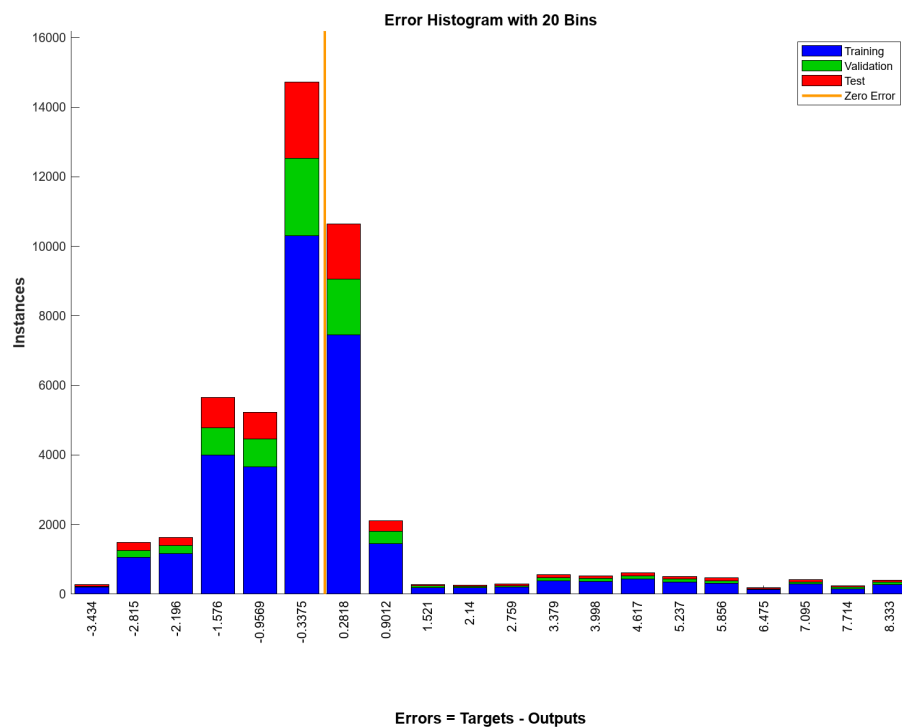


Figure 12. Error histogram of a feedforward neural network.

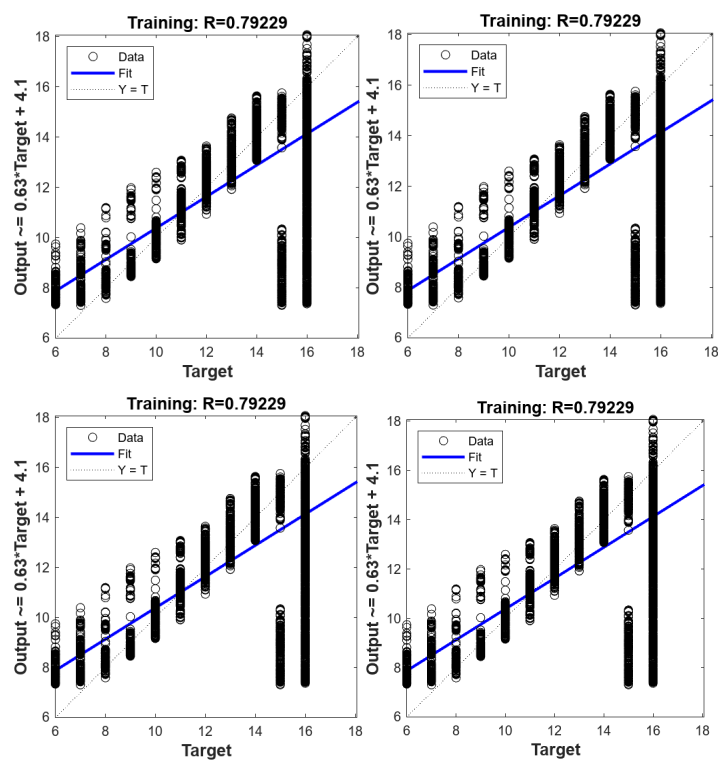


Figure 13. Curve fitting precision of the neural network.

Due to the high determination factor obtained in the training and validation of the neural network, it was possible to implement this in the control system, as shown in Figure 14.

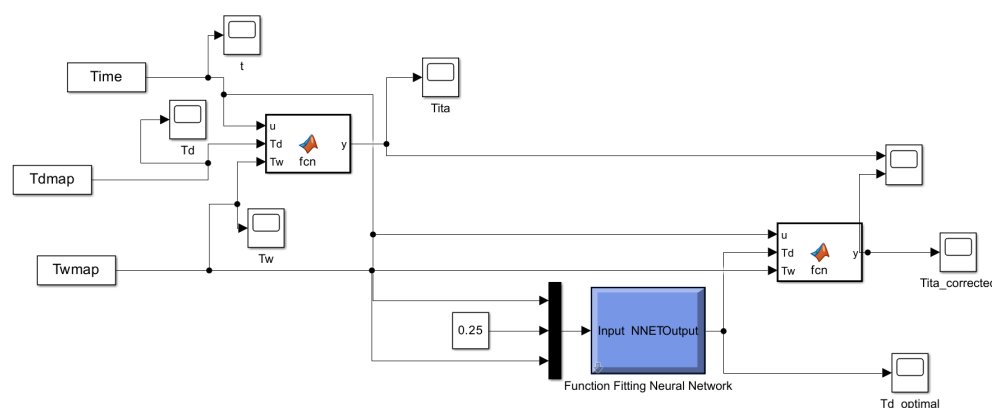


Figure 14. Stability control system employing NN as a control algorithm.

In this sense, the results in Figure 15 show the change in T_d that must be done in each moment to compensate for the change in T_w and the moment (t) and the maximum rolling angle allowed (0.35 rad).

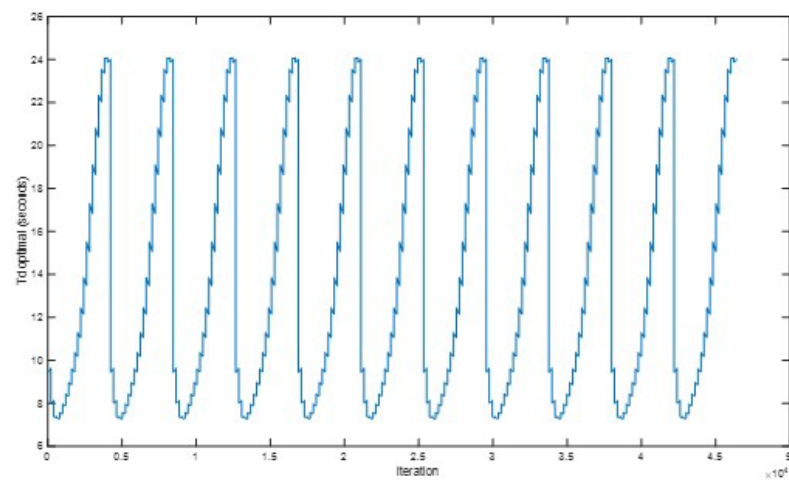


Figure 15. Proposed T_d to ensure 0.35 rad in each T_w and moment (t).

As shown in Figure 15, it is possible to compensate for the change in sea conditions by modifying T_d , but the proposed values are over 16, which is outside of normal working conditions. Consequently, several simulations of this control system were done to define the optimal rolling angle and obtain adequate T_d responses, as shown in Figure 16.

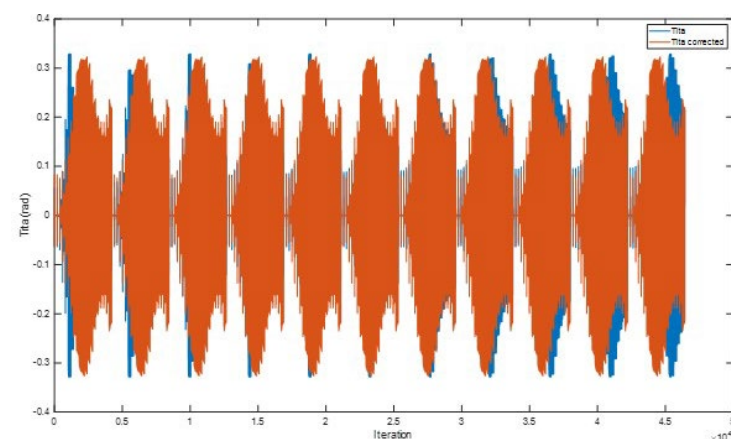


Figure 16. Comparison between the initial rolling angle (blue line) and the rolling obtained under the limitation of 0.35 radians (red line).

As we can see from these figures, lower peaks of rolling angles are obtained and a lateral displacement of the curves is obtained, too. As explained before, due to the need to reduce the proposed Td values of the control systems, more simulations were done for 0.30, 0.25 and 0.20 rad, as shown in Figures 17–19.

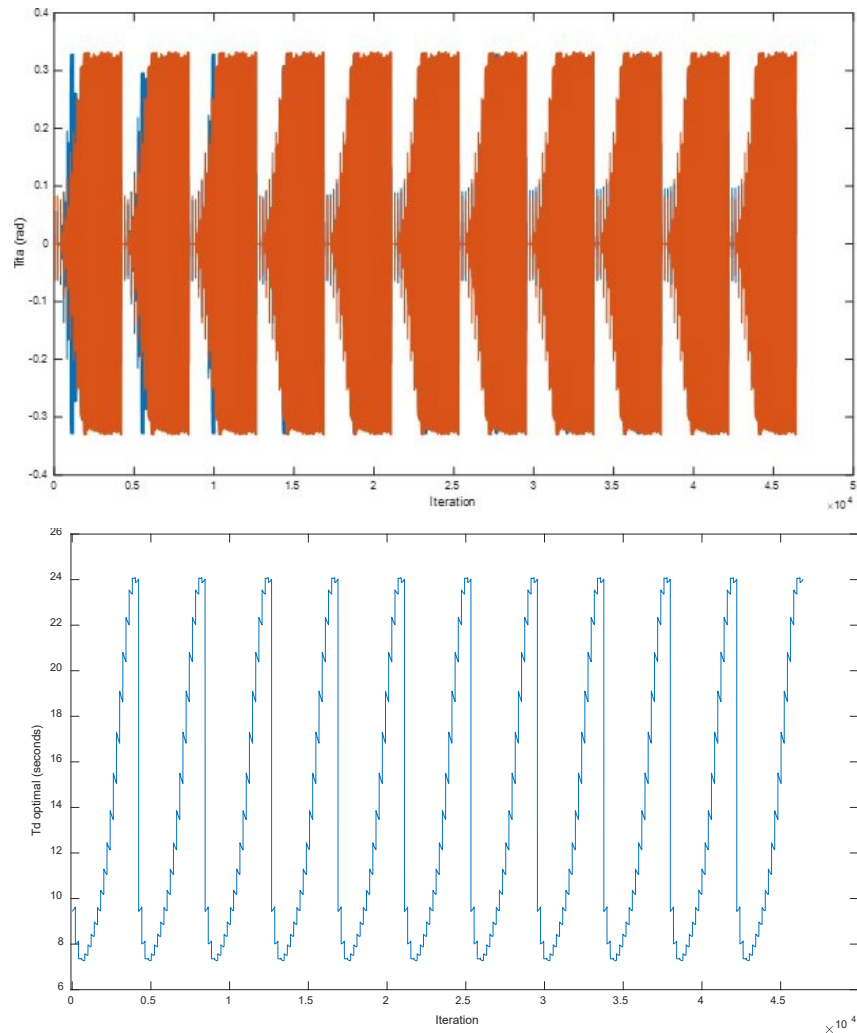
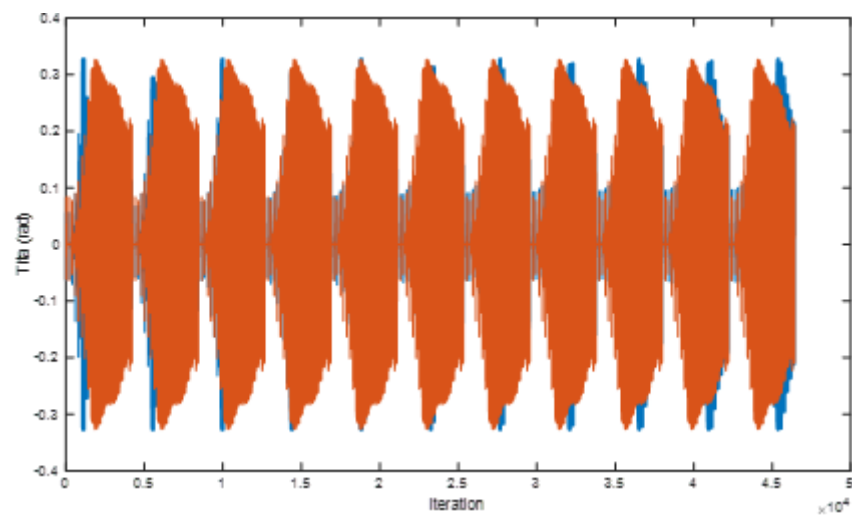


Figure 17. Comparison of initial and limited to 0.30 rad rolling angles.



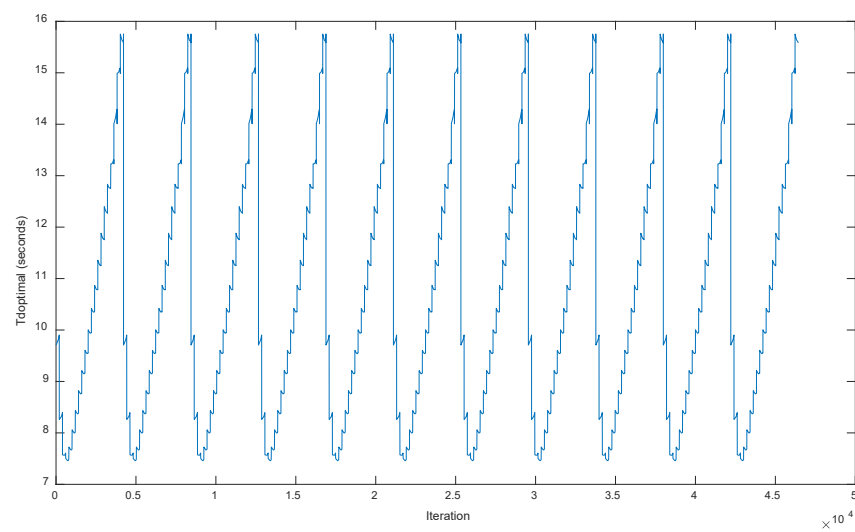


Figure 18. Comparative of initial and limited to 0.25 rad rolling angles.

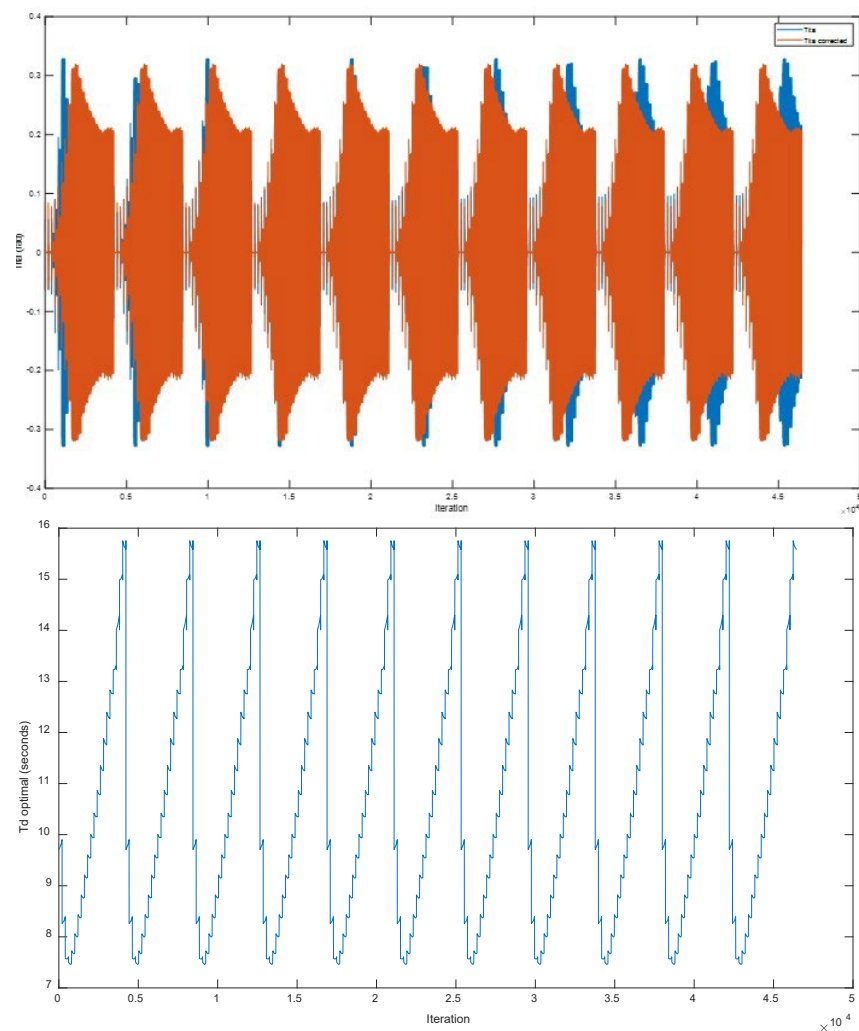


Figure 19. Comparison of initial and limited to 0.20 rad rolling angles.

From these results, it can be concluded that 0.25 rad is the best value to be employed as a control limit in this kind of algorithms, ensuring adequate response for all situations. What is more, this same study shows that it is possible to develop a control system with adequate precision by employing neural networks.

4. Discussion

After obtaining the new mathematical models and analysing the obtained results about the moments within the wave profile, when the maximum rolling angles are reached in different conditions, and determining the increase in rolling amplitude as a function of T_d , the results consider other scenarios in order to ascertain the sensibility of this study.

4.1. Synchronism Phenomenon for a Ship Sailing in Regular Beam Waves in a Non-Resistant Environment

In the previous sections, the behaviour of a ship rolling freely was analysed considering a wave slope most common, of about 6° (0.1046 rad). However, as previously mentioned, according to the literature, the maximum wave slope can reach up to 10° (0.1745 rad). For this reason, Figure 20 now simulates and compares the influence of different wave slopes on a ship's behaviour from a theoretical point of view.

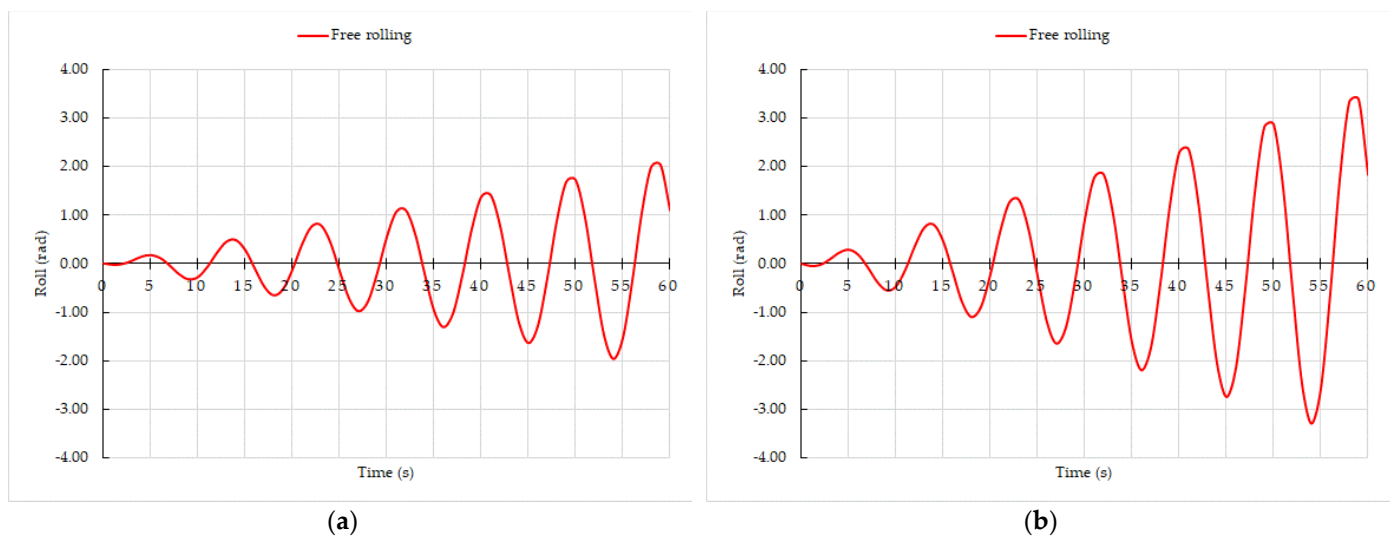


Figure 20. Free rolling motion for $T_d = 9$ s. (a) $\theta_{MW} = 0.1046$ rad; (b) $\theta_{MW} = 0.1745$ rad.

For a different ship natural roll period ($T_d = 9$ s) from previous simulations, it is observed that the moments of reaching the maximum ship rolling angles are the same, as was concluded in the previous section. However, relevant differences are revealed when different maximum wave slopes (θ_{MW}) are simulated. For instance, in Figure 20a, where $\theta_{MW} = 0.1046$ rad (6°) when $t = 9$ s, the relative maximum rolling angle is 0.3286 rad, while for $\theta_{MW} = 0.1745$ rad (10°), it is 0.5482 rad, i.e., about 66% higher. Therefore, it is evident that, from a theoretical point of view, the differences between the wave slopes can be decisive in the time available before the capsizing. However, in any case, the time available for ship operators to alter, for example, the ship's loading conditions (T_d) is minimal. Once again, the importance of predicting this undesirable situation well in advance is concluded, regardless of a certain wave slope.

4.2. Rolling Motion of a Ship at Zero Speed and Sailing in Regular Beam Waves in a Resistant Environment

Having concluded that, regardless of T_d , i.e., a 'stiff' or a 'tender' ship, the maximum rolling angle is reached at the middle of the wave slope (although the relative rolling angle is different in each case), the influence of the same ship's loading condition is now being discussed. With this purpose, on the one hand, Figures 21a and 22a depict the rolling angles for a very smooth sea state condition ($T_w = 4$ s) corresponding to $T_d = 6$ s and $T_d = 16$ s, respectively, i.e., the studied cases representing a very 'stiff' and 'tender' ship. In both cases, the ship rolls with the same periodicity, although the maximum values of rolling

are very different. What is more, a very ‘stiff’ ship, with a large metacentric height (small T_d), gives a good sense of safety from the intact stability point of view, although the accelerations sustained on board can be too excessive, especially in passenger vessels or some cargo vessels, due to the effects of sloshing in tanks or potential problems with the lashing arrangements.

On the other hand, Figures 21b and 22b represent, for the same ship’s loading conditions, a sea state condition corresponding to rough seas ($T_w = 14$ s). In this case, a similar ship behaviour is observed from a periodicity point of view (and according to previous results). However, the differences between the maximum rolling values reached are not proportional to the differences between the previous assumption for both T_d values. Therefore, for the same ship’s loading conditions analysed, the differences in the expected behaviour of the ship sailing in smooth and rough seas are not the same from the point of view of amplitude of angles and accelerations.

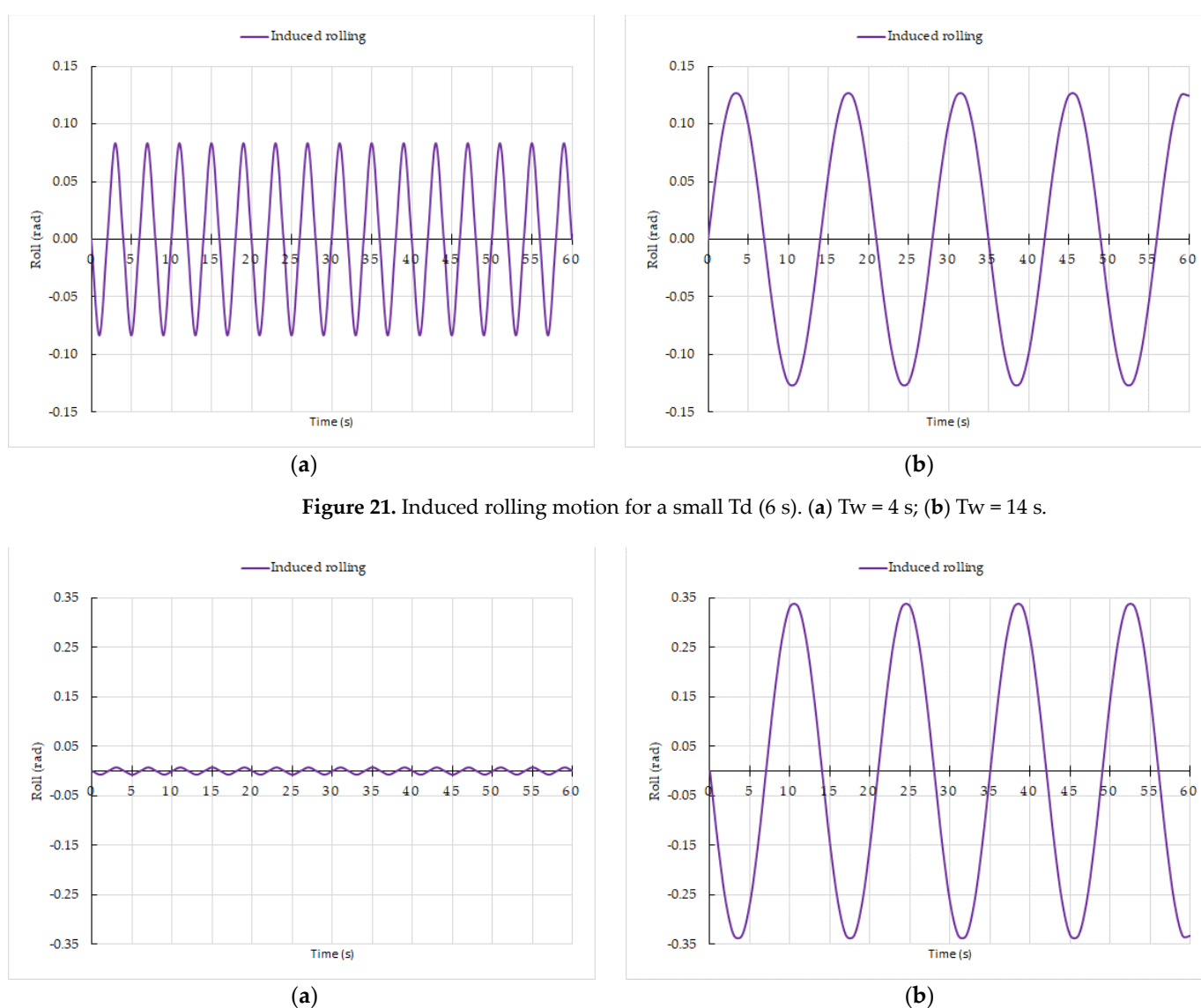


Figure 21. Induced rolling motion for a small T_d (6 s). (a) $T_w = 4$ s; (b) $T_w = 14$ s.

Figure 22. Induced rolling motion for a very large T_d (16 s). (a) $T_w = 4$ s; (b) $T_w = 14$ s.

Obviously, if the exact moment when $T_d = T_w$ is simulated, the ship is just in the synchronism condition, so the graphical representation is null. In any case, the ship capsizes before reaching the exact moment of synchronism.

4.3. Influence of the Damping Factor on the Synchronism Phenomenon

Although it is known that no ship will roll just in the moment of synchronism ($T_d = T_w$) because it capsizes before reaching this moment, from a theoretical point of view, it can be interesting to represent the sum of all forces involved (free and induced motion) at this moment. For this reason, Figure 23 shows the ship's behaviour assuming that the ship starts to roll just when $T_d = T_w$, in this case, 12 s. A perfect synchronism with the induced and free motion is noted, which is reached from the first moments.

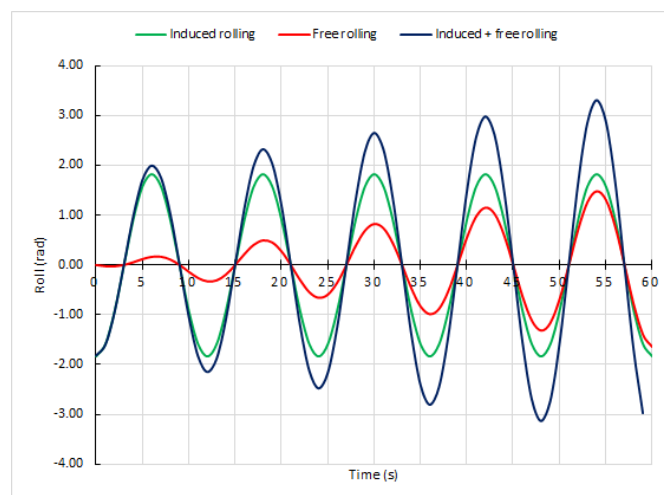


Figure 23. A ship's rolling motion just as the synchronism phenomenon occurs.

Although the accurate calculation of roll damping is essential for roll motion prediction [36], relevant papers can be found in the literature that deal with the difficulty of predicting this roll damping as a consequence of highly non-linear components experienced on board ships and barges, regardless of type and size [37]. For this reason, in the present research, an average and constant damping factor is considered (0.015), although it is not an exact and constant value throughout the whole rolling process. This value is a subject of naval architecture, and the present paper, keeping the focus on the ship's operation, does not pretend to address it. However, when many fishing vessels are engaged in fishing activities, they are at zero speed and receive beam waves, i.e., the assumptions considered in the present study. In these circumstances, the weight of nets and fishing catches causes an induced delay in the rolling motion. In other words, they increase the damping factor. In this case, the ship's behaviours are expected to be similar to those presented in the previous figures, and although the amplitude of the rolling angles is smaller, it can ensure adequate safety for any ship.

5. Conclusions

In this study, mathematical models were obtained based on several theoretical assumptions of a ship's synchronous rolling. These models refer to the worst conditions considered, i.e., the ship in static condition and under the influence of regular pure beam and trochoidal waves. After analysing and carrying out a set of simulations for ships with different intact stability, expressed as the ship's natural roll period (T_d), determined by the specific loading condition, and wave periods (T_w), relevant results were obtained.

It was determined that for each half- T_d ($1/2 \cdot T_d$), the amplitude of the rolling angle increases by a constant value that depends on the maximum slope of the wave. Therefore, as a function of T_d , the margin available to reach a given maximum allowed angle varies according to the ship's loading condition.

Furthermore, it is important to notice that as the ship approaches synchronous rolling, the relative maximum rolling angle reached changes from occurring in the middle of the wave slope to the crest or trough of the wave. Then, if the ship operators have good

knowledge and previous training about this circumstance, with good observation of the clinometer on board and the wave state, undesirable situations can be avoided. For that, the officer on watch should recognize as soon as possible the conditions conducive to synchronism phenomena in order to alter the ship's heading to change the encounter period and eliminate this danger. Therefore, the effort by the officer on watch to bring the ship's heading into the waves should be made well in advance. Another theoretical solution that requires more time would be to alter the position of the vertical centre of gravity of the ship (KG) in order to change the metacentric height (GM), as well as ballasting, de-ballasting or transferring ballast. By doing so, the ship's natural rolling period (T_d) also changes, making it a non-synchronous value. However, the proposed solution to be used as a countermeasure is only feasible from a theoretical point of view, because according to the present results, synchronous roll occurs so rapidly that it is not technically possible to alter the vertical position of the centre of gravity sufficiently in advance. Finally, and although it may not be as useful and simple to be used by ship operators, the generic rule of the periodicity with which the maximum rolling angles are reached was obtained, as a function of two known parameters, the ship's loading condition, represented by the ship's natural roll period (T_d), and the sea state (T_w).

The proposed control system does not ensure an exact rolling angle because the effect of time does not allow for obtaining an exact rolling angle. However, setting 0.25 rad allows for not exceeding the $T_d = 16$ s maximum in this study. Similarly, the results shown in the red curves have shifted and overlapped with the initial map at different T_d values according to the value of rolling angle. Therefore, it is possible to solve the ship's rolling motion (ship's stability) inversely and accurately by means of 10-layer neural networks.

In order to rectify the limitations of this study, which was carried out under theoretical conditions, future studies could be guided to perform simulations in towing tanks and validate the results. Regarding putting the results and the proposed control system into practice by ship operators, virtual simulators for training could be used in such a manner that with the visual observation of the clinometer and the ship's position along the wave profile, they can predict when the ship is approaching dangerous situations.

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