Abstract: This paper delivers an examination of relative permeability hysteresis in porous media in the field of petroleum engineering, encompassing mathematical modeling, experimental studies, and their practical implications. It explores two-phase and three-phase models, elucidating the generation of scanning curves and their applications in various porous materials. Building on the research of traditional relative permeability hysteresis models, we have incorporated literature on forward calculations of relative permeability based on digital rock core models. This offers a new perspective for studying the hysteresis effect in relative permeability. Additionally, it compiles insights from direct relative permeability and flow-through experiments, accentuating the methodologies and key findings. With a focus on enhanced oil recovery (EOR), carbon capture, utilization and sequestration (CCUS), and hydrogen storage applications, the paper identifies existing research voids and proposes avenues for future inquiry, laying the groundwork for advancing recovery techniques in oil and gas sectors.

Keywords: relative permeability; hysteresis effect; carbon capture; utilization and sequestration; water alternating gas; hydrogen storage

1. Introduction

Relative permeability is one of the key properties of rocks, closely related to production. Relative permeability can be used to estimate production capacity, injection capability, hydrocarbon reserves, breakthrough times, and ultimate recovery rates [1,2]. The relative permeability curve depends on the direction of saturation changes and the maximum and minimum saturations reached [3,4]. It is generally recognized that the relative permeability increases with the increase in saturation and decreases with the decrease, which is a constant curve. In fact, the relative permeability changes with the displacement fluid and the cycle, and the relative permeability curve has a lag phenomenon between different saturation cycles. The irreversibility or path dependence of the physical properties is called “hysteresis”, so when observed in the relative permeability data, it is called “phase permeability hysteresis”, and the degree of this irreversibility depends on the fluid type [5–7], pore structure [8], and wettability [5,9].

The hysteresis effect of relative permeability in porous media has formed a relatively unified conclusion through years of experimental research. Ignorance of the relative permeability hysteresis effect of gas water alternation will misinterpret the production/experimental result [10]. Not considering hysteresis of relative permeability will
underestimate the recovery factor and storage capacity in WAG-related CCUS application [11].

There are two main causes for the occurrence of hysteresis in relative permeability [12–14]. First, wettability hysteresis: the advancing contact angle (the contact angle as the wetting phase displaces the non-wetting phase) is greater than the receding contact angle (the contact angle as the wetting phase retreats due to the intrusion of the non-wetting phase), closely related to surface chemical properties, roughness, and changes caused by adsorbed solution particles or solvents. Second, trapping of the non-wetting phase: during imbibition, part of the non-wetting phase breaks off in clumps or ganglia and becomes immobile (trapped). Since the non-wetting phase primarily occupies the centers of pores and throats in the porous medium, even a small amount of trapping can significantly affect the flow of fluids and the values of relative permeability.

The trapping of residual oil varies in different wettability systems. In water-wet systems, the mechanism of disconnection is the main reason for oil trapping. During the water injection process, due to the bridging of two water layers on either side of the throat, the water in the wetting phase will fill the smaller pores and disconnect the crude oil between two pores. Whereas in oil-wet systems, due to the difference in capillary forces caused by pore size differences, the smaller capillaries in interconnected capillaries will become the main sites for trapping residual oil.

There are two types of relative permeability [7,15] hysteresis, as shown in Figure 1: displacement-direction-dependent and saturation-history-dependent hysteresis [12,14,16]. Displacement direction-dependent hysteresis refers to the difference between drainage and imbibition permeability curve. Saturation-history-dependent means that the relative permeability curves of different cycles of continuous displacement in the same phase are different.

Figure 1. Different relative permeability hysteresis. (a) Displacement direction dependent. (b) Saturation history dependent.

In water-wet systems, it is generally believed that the relative permeability of the water phase increases with increasing water saturation, while the relative permeability of the non-wetting phase decreases with increasing water saturation. The value of the non-wetting phase relative permeability depends on the water saturation and its direction of movement; the process of increasing water saturation, known as waterflooding, is called the imbibition process, while the process of decreasing water saturation is called the drainage process.
In all models of relative permeability hysteresis, the scanning curves are very important. As shown in Figure 2, assuming the minimum water saturation is $S_{wc}$ and water injection begins, then $k_n$ should follow the imbibition curve until the water saturation reaches its maximum value $S_{w}^{\text{max}}$. If, after reaching the maximum value $S_{w}^{\text{max}}$, drainage begins, $k_n$ will follow the drainage curve until $S_{w} = S_{wc}$. If the drainage process is interrupted before reaching $S_{wc}$ and water injection starts again, $k_n$ now needs to stop following the drainage curve and instead follow the imbibition curve. However, the imbibition and drainage curves almost never intersect at any point, so to make the transition from one point to another smooth, we need a transitional curve that starts from that point to another relative permeability curve. These transitional curves are called scanning curves [17,18].

Similarly, during the imbibition process, a transition can be made to the drainage process from any point, generally the drainage curve and imbibition curve, also called boundary curves or relative permeability envelopes.

![Schematics of scanning curves.](image)

Figure 2. Schematics of scanning curves.

Research on the hysteresis effect of relative permeability has led to the development of various two-phase and three-phase models. Traditional studies on relative permeability hysteresis models generally rely on experimental results to establish the models. With the advancement of CT scanning technology, a novel method for studying relative permeability hysteresis models has emerged in recent years. This method establishes digital rock core models based on CT scanning, and uses geometric and calculus approaches to create models that enable forward calculations of relative permeability to describe the hysteresis phenomenon.

The discussion on relative permeability hysteresis will be divided into three parts: Mathematical Models, Experimental Studies, and Applications. These three Sections will together provide a comprehensive perspective on a deep understanding of the relative permeability hysteresis phenomenon, from the establishment and optimization of theoretical models, to the design and execution of experimental methods, to the practical applications and impact of these models and experimental results in the development of oil and gas fields.

2. Mathematical Models

This Section will cover the theoretical frameworks used to describe the phenomenon of relative permeability hysteresis. This may include models based on physical principles, such as extensions of the Land model and capture models, which attempt to estimate
the increasing capture saturation of intermediate-wet and non-wet phases by introducing
dynamic coefficients. Additionally, there are methods based on behaviors observed in
experiments to update and optimize the model, which uses innovative techniques to
develop new hysteresis models that predict the saturation of oil and water at the end of
each injection cycle.

The permeability model is the fundamental of the relative permeability hysteresis
model. After the calculation of the scanning curve of the hysteresis model, the perme-
ability model will be used for the construction of the whole relative permeability curve.

In 1956 [19], Corey introduced a method to calculate the oil-water permeability
of a three-phase system from the easily measurable gas relative permeability, related
to saturation:

$$ k_{ro} = \frac{(S_L - S_W)^3}{(1 - S_{Lr})^4} (S_W + S_L - 2S_{Lr}) $$

where $S_W$ is the water saturation, $S_L$ is the liquid saturation, and $S_{Lr}$ is the residual
liquid saturation.

In 1970, Stone [20] proposed a model based on channel flow theory, treating porous
media as a combination of flow channels and assuming that only one type of flowing fluid
exists in any given channel. Wettability determined the occupancy rate of channels of
different sizes. The intermediate-wetting phase separates the wet phase in the smallest
channels from the non-wet phase in the largest channels. Therefore, just like in two-phase
systems, three-phase systems have the same saturation history, and the wet phase (water)
saturation occupies the same flow channels. This also means that the water phase relative
permeability is a function of its own saturation, the same in both two-phase and three-
phase systems.

The relationship for oil phase relative permeability:

$$ k_{ro} = S_{onor} \cdot \beta_w \cdot \beta_g $$

$$ S_{onor} = S_o - S_{ow} - S_w $$

where $S_{ow}$ is the three-phase residual oil saturation, and $S_w$ is the connate water saturation.
The function $\beta_w$ depends only on water saturation, and $\beta_g$ depends only on gas saturation.

In 1973, Stone [21] introduced another model (Stone II model), which uses the relations-
ships of four two-phase relative permeabilities in oil/gas and oil/water systems to predict
the three-phase oil phase relative permeability. The main difference between Stone I and
Stone II is that Stone II does not require $S_o$ for calculations. To solve for $k_{ro}$, the following
equation is used:

$$ k_{ro} = (k_{row} + k_{rwo})(k_{rog} + k_{rgo}) - (k_{row} + k_{rgo}) $$

In 1976, Dietrich, J. and Bondor, P. [22] proposed that the oil-relative permeability
would be a function of gas-water saturation, and assumed that water and gas could flow
simultaneously with oil in the pore spaces. They considered the overall reduction in
oil-phase relative permeability caused by water and gas, deriving an expression for three-
phase oil-phase relative permeability. The reduction in oil-phase relative permeability is
as follows:

$$ \left[ k_{ro}^0 - k_{row} \right] + \left[ k_{ro}^0 - k_{rgo} \right] - (S_w + S_o) \left[ k_{ro}^0 - k_{row} \right] \left[ k_{ro}^0 - k_{rgo} \right] / k_{ro}^0 $$

where $k_{ro}^0$ is the oil phase relative permeability at irreducible water saturation and zero gas
saturation, $k_{row}$ is the oil phase relative permeability in oil-water two-phase flow, and $k_{rgo}$
is the oil phase relative permeability in oil-gas two-phase flow. The first and second terms
represent the reduction in oil phase relative permeability due to the presence of water and
gas, respectively, while the third term adjusts for the reduction caused by the interaction
of water blocking oil and gas blocking oil. The resulting expression for oil phase relative permeability becomes:

\[ k_{ro} = k_{row} + k_{rog} - k_{0ro} + (S_w + S_o) \left( \frac{k_{ro}^0}{k_{row}^0} - k_{rog}^0 \right) / k_{ro}^0 \]  

(6)

In 1979, Aziz and Settari [23] modified the Stone model to overcome the limitations of the original model, namely, that it could only be simplified to two-phase data when endpoint relative permeabilities were equal to 1. The modified Stone I model:

\[ k_{ro} = k_{onor}w_{row} \max \]  

(7)

\[ \beta_w = \frac{k_{row}}{k(1 - S_{wnor})_{row} \max} \]  

(8)

\[ \beta_g = \frac{k_{rog}}{k(1 - S_{gnor})_{row} \max} \]  

(9)

The modified Stone II model:

\[ k_{ro} = k \left[ \left( \frac{k_{row}}{k_{ro} \max w_{ro}} \left( \frac{k_{rog}}{k_{ro} \max g_{ro}} \left( k_{ro} + k_{ro} \right) \right) \right) \right]_{row} \max \]  

(10)

In 1989, Delshad [26] and Pope proposed a three-phase model independent of two-phase relative permeabilities, based on experimental data at two-phase limits. In this model, the residual oil saturation \( S_{or} \) in three-phase flow exhibits a linear relationship between the two limiting residual oil saturations \( S_{orw} \) and \( S_{org} \):

\[ k_{ro} = \frac{1}{2} \left[ (S_{onor})^{eog} (1 - S_{wnor})^{ro = eog} + (S_{onor})^{eaw} (1 - S_{gnor})^{eog = eaw} \right]_{row} \max \]  

(12)

In 1989, Kokal and Maini [27] discussed the Stone model modified by Aziz and Settari [23] (1979). If \( k_{row} \) and \( k_{rog} \) do not equal the oil phase relative permeability at zero connate water saturation (oil-water) and gas saturation (oil-gas), then it is not possible to match relative permeabilities at the two-phase boundaries. To address the following two limitations:

(1) The measurement of two-phase oil-gas relative permeability data does not necessarily occur at connate water saturation.

(2) Due to experimental errors, \( k_{row} \) often does not equal \( k_{rog} \).
Introducing another normalization factor to correct the Stone I model:

\[
k_{ro} = \frac{k_{row}}{k_{row\text{max}} (1 - S_{\text{onor}})} \times \frac{k_{rog}}{k_{rog\text{max}} (1 - S_{\text{onor}})} \times \frac{k_{ro\text{max}} S_{\text{onor}} + k_{row\text{max}} S_{\text{onor}}}{1 - S_{\text{onor}}} \tag{13}
\]

In 1997, Moulu [28] and others extended the research of Vizika (1993) by proposing a theoretical model. This model is based on a physically relevant description of phase distribution and flow mechanisms at the pore scale, considering only strong water-wet and extended conditions, but it can be expanded to oil-wet and even mixed-wet conditions. The porous medium is described as a set of fractal pores, with their linear fractal dimension and size distribution derived from the mercury intrusion capillary pressure curve. The following relative permeability for the three-phase oil phase was proposed:

\[
k_{ro} = k_{row} \left[ (S_w + S_o - S_{\text{org}}) \frac{4 - D_l}{2} - (S_w + S_{\text{orw}}) \frac{4 - D_l}{2} \right] \tag{14}
\]

The authors extended the model to oil-wet and intermediate-wet conditions (Moulu et al., 1999 [29]) using the wettability index WI (where a water-wet medium equals 1, and an oil-wet medium equals −1):

\[
k_{ro} = (1 - m) \times \left( \frac{S_o^b - S_w^b}{\alpha} \right) + m \times k_{row} \times \left( (S_o + S_w - S_{\text{org}})^\beta - (S_w + S_{\text{orw}})^\beta \right) \tag{15}
\]

\[
\beta = \frac{4 - D_l}{2 - D_l} \tag{16}
\]

In 1999 [30], Blunt introduced an empirical model for three-phase relative permeability that accounts for changes in hydrocarbon composition, different saturation paths, and the trapping of oil, water, and gas. The model is based on saturation-weighted interpolation, using flowing saturation as a parameter for two-phase permeability functions. The relative permeability for the three-phase oil phase is as follows:

\[
k_{ro}(S_o) = \frac{1}{\alpha} \left[ (S_w - S_{wc}) \frac{a_k S_{\text{org}}}{(S_w - S_{wc}) + (S_o - S_{gr})} + (S_w - S_{wc}) \frac{a_k S_{\text{org}}}{(S_w - S_{wc}) + (S_o - S_{gr})} \right] + a_k \frac{S_o}{S_w + S_o} \left[ (S_w + S_{\text{orw}}) (S_{hf}) + b_o k_{rg(w)} (S_{hf}) \right] \tag{17}
\]

\[a, \beta, a_o, \text{ and } b_o \text{ are functions of interfacial tension and fluid density when considering component consistency, that is, changes in fluid properties due to variations in pressure and components. If the changes are minimal, these function values are specified as: 1, 0, 1, and 0.} \]

In 2008 [31], Bevan and Alan introduced a new empirical model for three-phase oil relative permeability to provide flexibility in adjustments for improving simulator predictions. The new model is based on a saturation-weighted interpolation between two-phase values, similar to the Baker model, but with simple tuning coefficients, allowing for adjustments to match experimental measurements.

\[
k_{ro} = \frac{(S_w - S_{wc}) k_{row} + S_{ow} 2 \alpha (S_w - S_{gr}) k_{rog}}{(S_w - S_{wc}) + 2 \alpha (S_w - S_{gr})} \tag{18}
\]

\[
S_{ow} = \frac{(S_o - S_{om})}{(1 - S_{wc} - S_{gr} - S_{om})} \tag{19}
\]

\[
S_{om} = \frac{S_w S_{orw} + S_g S_{org}}{S_w + S_g} \tag{20}
\]

In 2010, Hustad and Hansen [32] proposed a model based on three sets of two-phase data, appropriately considering six types of saturation, namely, two-phase saturation,
residual saturation, or critical saturation. They used the following interpolation to plot permeability maps of the reservoir:

$$k_{ro}(S_{omnx}) = \frac{S_w}{S_w + S_g} \cdot k_{row}(S_{omnx}) + \frac{S_g}{S_w + S_g} \cdot k_{rog}(S_{omnx})$$  \hspace{1cm} (21)

In 2019, Shokoufeh and Mehran [33] modified the Stone I model, taking into account the flow in three-phase systems during immiscible and near-miscible gas-water alternating injection experiments to correct oil phase relative permeability. The results indicated that the Stone model overestimated oil phase relative permeability data in regions of low oil saturation. Additionally, it was found that Stone’s exponential model could not simulate the gradual decrease in oil phase relative permeability data. To improve the results, a constant parameter called the oil reduction factor ($\beta_o$) was introduced into the Stone I model at low oil saturations, which remained constant throughout the entire gas-water alternating experiment to consider the effect of separated oil clusters. Furthermore, this modified model no longer requires the end-of-cycle residual oil saturation ($S_{om}$) needed by the Stone model.

For $S_o > S_{orw}$ (Stone I):

$$k_{ro} = k_{rocw} \cdot SS_{o} F_{w} F_{g}$$  \hspace{1cm} (22)

For $S_o < S_{orw}$ (The modified Stone I model):

$$k_{ro} = \beta_o \cdot k_{rocw} \cdot SS_{o} F_{w} F_{g}$$  \hspace{1cm} (23)

$$SS_{o} = \frac{S_o - S_{om}}{1 - S_{om} - S_{wco}}$$  \hspace{1cm} (24)

$$SS_{w} = \frac{S_w - S_{wco}}{1 - S_{om} - S_{wco}}$$  \hspace{1cm} (25)

$$SS_{g} = \frac{S_g}{1 - S_{om} - S_{wco}}$$  \hspace{1cm} (26)

$$F_{w} = \frac{k_{row}}{k_{rocw}(1 - SS_{w})}$$  \hspace{1cm} (27)

$$F_{g} = \frac{k_{rog}}{k_{rocw}(1 - SS_{g})}$$  \hspace{1cm} (28)

Research on the current state and development history of relative permeability models reveals that relative permeability is a function of saturation. Three-phase permeability is derived from two-phase relative permeability through linear interpolation or saturation interpolation. Currently, the most widely applied three-phase relative permeability models are the Stone I model, Stone II model, and Baker model.

2.1. Two-Phase Relative Permeability Hysteresis Model

The concept of incorporating the effects of relative permeability hysteresis on the basis of three-phase relative permeability models is referred to as the relative permeability hysteresis model. Currently, the mathematical models for relative permeability hysteresis have been extensively developed, which can be divided into two-phase relative permeability hysteresis models and three-phase relative permeability hysteresis models. The individual model could either consider the direction of saturation or both the direction of saturation and the saturation history. Different models vary in their scanning curves, among which the most widely used models include the two-phase hysteresis models such as the Killough model [34] and the Carlson model [35], and the three-phase hysteresis model by Larsen and Skauge [36].
(1) Land Model [37]

Land discovered an empirical relationship between the residual non-wetting phase saturation after imbibition and the initial non-wetting phase saturation. Based on this empirical relationship and the theory related to the distribution of relative permeability pore sizes, he established expressions for the trapping coefficient and the movable non-wetting phase saturation. By describing the trapping behavior experienced by the non-wetting phase and explaining its impact on relative permeability, he formally established the concept of hysteresis:

\[
\frac{1}{S_{gr}} - \frac{1}{S_{gi}} = \frac{1}{(S_{gr})_{max}}
\]

(29)

where:

\[
S_{gr} = \frac{S_{gi}}{1 + CS_{gi}}
\]

(30)

\[
C = \frac{1}{(S_{gr})_{max}} - 1
\]

(31)

where \(S_{gi}\) is the initial gas saturation; \(S_{gr}\) is the residual gas saturation after imbibition; \((S_{gr})_{max}\) is the maximum residual gas saturation when the initial gas saturation is consistent; and \(C\) is a constant, known as the Land constant.

(2) Killough Model [34]

Building on the Land model, Killough assumed that no trapping occurs during the displacement process, and the relative permeability of the wetting phase does not exhibit hysteresis effects; the imbibition-relative permeability curve is reversible. Therefore, any imbibition curve, when reversed, will return precisely to its origin until we once again reach the main drainage curve.

Assuming the first drainage process reverses at saturation \(S_{ni}\). At this time:

\[
K_{n I}^{'}(S_{ni}) = K_{n D}(S_{ni})
\]

(32)

where \(K_{n I}^{'}(S_{ni})\) represents the permeability of the non-wetting phase during the imbibition stage at the reversal saturation \(S_{ni}\), and \(K_{n D}(S_{ni})\) represents the permeability of the non-wetting phase during the displacement stage at the reversal saturation \(S_{ni}\).

However, when the saturation reaches \(S_{nr}\), the new imbibition curve will reach \(K_{n} = 0\), which is related to \(S_{ni}\) in Equation (33).

\[
K_{n I}^{'}(S_{nr}) = 0
\]

(33)

To predict the intermediate curve \(K_{n I}^{'}\) located in (32) and (33), Killough considered two methods: (a) parameter interpolation, and (b) normalization of experimental data.

Specifically, the calculation of the non-wetting phase imbibition relative permeability is as follows:

\[
K_{ri} = \frac{K_{rnd}(S_{nh})}{K_{rnd}(S_{nmax}) r_{ni} (\bar{S}_{nh})}
\]

(34)

where:

\[
\bar{S}_{w} = \left( \frac{S_{n} - S_{nrh}}{S_{nh} - S_{nrh}} \right) (krm_{nmax})
\]

(35)

where \(K_{rnd}(S_{nh})\) represents the relative permeability during \(S_{nh}\) displacement, \(K_{rnd}(S_{nmax})\) represents the relative permeability during \(S_{nmax}\) displacement, and \(K_{rni}(S_{nh})\) represents the relative permeability during \(S_{nh}\) imbibition.

(3) Carlson Model [35]

Carlson focused on the study of the relative permeability of the non-wetting phase \(K_{n}\) in two-phase systems within the Killough model, and assumed that the relative permeability of the wetting phase \(K_{w}\) did not exhibit hysteresis behavior. It was assumed
that trapping only occurs during the imbibition period; thus, if the imbibition process is reversed, then the imbibition curve will be precisely retraced.

Let \( S_{nt} \) be the “trapped” fraction of saturation, and \( S_{nf} \) be the “free” fraction, so that:

\[
S_n = S_{nf} + S_{nt}
\]

(36)

The value of the imbibition curve that follows must be the value of the drainage curve calculated only for the free saturation, equivalently:

\[
K_n^I(S_n) = K_n^D(S_{nf})
\]

(37)

(4) Beattie Model [38]

Beattie [38] introduced a water-oil relative permeability hysteresis model to simulate the historically related hysteresis behavior exhibited by the Cold Lake reservoir. This model, which is based on laboratory measurements, takes into account the influence of saturation history on relative permeability. To use this model, boundary imbibition and drainage curves are input, and grid block relative permeabilities are determined such that the calculated values always lie on or below the drainage curve and always on or above the imbibition curve. This modeling approach allows for more realistic simulations and improves the accuracy of predicting reservoir behavior. Hysteresis is only applied to water/oil two-phase relative permeabilities. The three-phase permeability is calculated using gas/oil two-phase curves, hysteresis water/oil curves, and the Stone I model.

Display a pair of normalized water relative permeability boundary curves, one for increasing water saturation (imbibition), and one for decreasing water saturation (drainage). Assume that these curves depend only on water saturation. Normalized water relative permeability and water saturation are given:

\[
k^*_rw = k_{rw} / (k_{rw})_ro
\]

(38)

\[
S^*_w = \left[ S_w - (S_w)_{ir} \right] / \left[ (S_w)_ro - (S_w)_{ir} \right]
\]

(39)

As water saturation increases, the block’s water relative permeability will move towards the imbibition boundary curve. We define a ratio, \( K \), which compares the distance of the block’s permeability from the boundary curve to the total distance between boundary curves, all at the same water saturation:

\[
K = \frac{[k^*_rw - k^*_ww]}{[k^*_rw - k^*_ww]}
\]

(40)

Now, \( k^*_rw \) can be calculated using Equation (38). Assuming \( K \geq S^*_w \geq (S^*_w)_p \):

\[
K = A + B(1 - S^*_w)^n
\]

(41)

where A, B, and n are constants. There are three boundary conditions.

First, when \( S^*_w = 1 \), \( K = 0 \). This forces the scanning curve to approach the water imbibition boundary curve. So, \( A = 0 \), \( n > 0 \).

The second boundary condition forces the scanning curve to pass through the initial point \( P \):

\[
\text{When } S^*_w = (S^*_w)_p, K = K_p
\]

(42)

\[
K = K_p \left\{ (1 - S^*_w) / [1 - (S^*_w)_p] \right\}^n
\]

(43)

The third boundary condition requires that the derivative of \( K \) with respect to \( S^*_w \) is finite within the range \( 1 \geq S^*_w \geq (S^*_w)_p \) at \( S^*_W = 1 \). When \( n < 1 \), this derivative is infinite. Therefore, \( n \geq 1 \).
Therefore, by combining Equations (20) and (23) and using Equation (20) to define $K_P$, we obtain the expression for $k_{ru}^*$:

$$k_{ru}^* = k_{rwi}^* - \left( \frac{(k_{rwi}^*)_p - (k_{ru}^*)_p}{(k_{rwi}^*)_p - (k_{rwi}^*)_p} \right) \left( 1 - \frac{S_w^*}{(S_w^*)_p} \right)^n \left( k_{rwi}^* - k_{ru}^* \right) n \geq 1 \quad (44)$$

Because the other derivations are similar, only the final equation is provided. Normalized water relative permeability and normalized water saturation reduction are shown by:

$$k_{rw}^* = k_{rwi}^* + \left( \frac{(k_{rwi}^*)_p - (k_{rw}^*)_p}{(k_{rwi}^*)_p - (k_{rwi}^*)_p} \right) \left( 1 - \frac{S_w^*}{(S_w^*)_p} \right)^n \left( k_{rwi}^* - k_{rw}^* \right) n \geq 1 \quad (45)$$

Normalized oil relative permeability, normalized water saturation increase:

$$k_{ro}^* = k_{roi}^* + \left( \frac{(k_{roi}^*)_p - (k_{ro}^*)_p}{(k_{roi}^*)_p - (k_{roi}^*)_p} \right) \left( 1 - \frac{S_w^*}{(S_w^*)_p} \right)^n \left( k_{roi}^* - k_{ro}^* \right) n \geq 1 \quad (46)$$

where the normalized oil relative permeability is:

$$k_{ro}^* = k_{ro}/(k_{ro})_i$$

Normalized oil relative permeability and decreased normalized water saturation are shown by:

$$k_{ro}^* = k_{rod}^* - \left( \frac{(k_{rod}^*)_p - (k_{ro}^*)_p}{(k_{rod}^*)_p - (k_{ro}^*)_p} \right) \left( 1 - \frac{S_w^*}{(S_w^*)_p} \right)^n \left( k_{rod}^* - k_{ro}^* \right) n \geq 1 \quad (48)$$

(5) Delshad Model [26]

In 2003, Delhard obtained analytical expressions for water and oil relative permeabilities using Burdine’s relative permeability model and Brooks–Corey capillary pressure function for main drainage.

The saturation $M_{ow}$ characterizes the smallest pores where oil has displaced water within the required residence time, converting water-wet to oil-wet. $M_{ow}$ is the initial water saturation before reservoir oil recovery. In many reservoirs, this is equal to the bound water saturation. To establish the relationship for calculating residual oil saturation, it is assumed that $S_{or}$ has a maximum value when $M_{ow} = S_{iw}$, and it equals zero when $M_{ow} = 1$. The relationship between $S_{or}$ and $M_{ow}$ is:

$$S_{or} = S_{or}^{\text{max}} \left( 1 - M_{ow} \right)^2 \quad (49)$$

where $S_{or}^{\text{max}}$ is the residual oil saturation at $M_{ow} = S_{iw}$:

$$M_{ow} = \left( \frac{M_{ow} - S_{iw}}{1 - S_{iw} - S_{or}} \right) \quad (50)$$

$M_{ow}$ is used to distinguish between the saturation of pore throats that are water-wet from those that are oil-wet or intermediate wet. It is assumed that in mixed-wet reservoirs, the largest pores will be oil-wet or intermediate wet.

The relative permeability expressions for water and oil are as follows. For $S_w \leq M_{ow}$:

$$k_{rw} = S_w^{2+3\lambda}/\lambda \quad (51)$$

$$k_{ro} = (1 - S_w)^2 \left[ 1 - S_w^{2+3\lambda}/\lambda \right] \quad (52)$$
For \( S_w > M_{ow} \):

\[
k_{ow} = S_w^2 \left[ 1 + \frac{M_{ow}^{(2+\lambda)/\lambda} - \Omega^{(2+\lambda)/\lambda}}{\Omega^{(2+\lambda)/\lambda}} \right]
\]

(53)

\[
k_{oo} = (1 - S_o)^2 \left[ \Omega^{(2+\lambda)/\lambda} - \frac{M_{ow}^{(2+\lambda)/\lambda}}{\Omega^{(2+\lambda)/\lambda}} \right]
\]

(54)

where \( \Omega = M_{ow} + S_o \) and \( S_o = \frac{S_o - S_{ow}}{1 - S_{ow}} \).

\( \lambda \) is the pore size distribution index used in the Brooks–Corey model, and \( \Omega \) is the term in the integral of relative permeability.

(6) Kjosavik Model [39]

Kjosavik et al. established a two-phase model for mixed wettability relative permeability curves, which took into account the processes of primary drainage, spontaneous imbibition, and secondary drainage. They integrated two Corey-type relative permeability expressions and weighted them based on capillary pressure correlation, in order to comprehensively describe the characteristics of water-wet and oil-wet systems. Their model is based on the assumption that the relative permeability for each injection cycle should return to the starting point of the previous displacement and form a closed scanning loop.

The capillary pressure equation for the proposed mixed-wet system is an extension of the Brooks–Corey capillary relationship, as shown below:

\[
P_c = \frac{c_{ow}}{\left( \frac{\bar{s}_{ow} - \bar{s}_{ow}}{1 - \bar{s}_{ow}} \right) a_{ow}} + \frac{c_{od}}{\left( \frac{\bar{s}_{od} - \bar{s}_{od}}{1 - \bar{s}_{od}} \right) a_{od}}
\]

(55)

where \( 1/a_w \) is the exponent of the pore size distribution, \( c_{iw} \) is the inlet pressure, \( c_{wd} \) and \( c_{od} \) were the displacement parameters in the water-wet system, and \( a_{ow} \) and \( a_{od} \) were the displacement parameters in the oil-wet system. Replace the subscript “d” with “i” to make it applicable to the imbibition process.

In Equation (35):

\[
P_{cw} = \frac{c_{ow}}{\left( \frac{\bar{s}_{ow} - \bar{s}_{ow}}{1 - \bar{s}_{ow}} \right) a_{ow}}
\]

(56)

\[
P_{co} = \frac{c_{od}}{\left( \frac{\bar{s}_{od} - \bar{s}_{od}}{1 - \bar{s}_{od}} \right) a_{od}}
\]

(57)

\( P_{cw} \) and \( P_{co} \) represent the capillary pressures in the water and oil branches, respectively, \( a_{ow} \) and \( a_{od} \) are constants.

By integrating the water portion relationship in the capillary (Equation (35)), the following relative permeability equations for the water-wet system were obtained, applicable for drainage and imbibition:

\[
k_{rw,wd} = S_{nw}^{2a_{wd}+1+m_{wd}}
\]

(58)

\[
k_{rw,we} = S_{nw}^{2a_{we}+1+m_{we}}
\]

(59)

\[
k_{ro,wd} = (1 - S_{nw}^{2a_{wd}+1})(1 - S_{nw})^{m_{od}}
\]

(60)

\[
k_{ro,we} = (1 - S_{nw}^{2a_{we}+1})(1 - S_{nw})^{m_{oi}}
\]

(61)

\( k_{rw,wd} \) and \( k_{ro,wd} \) are the relative permeabilities of water and oil during the drainage process in the water-wet system. Similarly, the relative permeabilities of water and oil during the imbibition process in the oil-wet system:

\[
k_{rw,wd} = (1 - S_{no}^{2a_{wd}+1})(1 - S_{no})^{m_{od}}
\]

(62)

\[
k_{rw,we} = (1 - S_{no}^{2a_{we}+1})(1 - S_{no})^{m_{oi}}
\]

(63)

\[
k_{ro,wd} = S_{no}^{2a_{wd}+1+m_{od}}
\]

(64)
The parameter \( k \) has the same value in both the capillary pressure and relative permeability equations, while the curvature exponent \( (m) \) is only used in the relative permeability relationships. The definition of normalized saturation used in the relative permeability equations is as follows:

\[
S_{nw} = \frac{S_w - S_{wR}}{1 - S_{wR} - S_{oR}} \quad (66)
\]

\[
S_{no} = \frac{1 - S_w - S_{oR}}{1 - S_{wR} - S_{oR}} \quad (67)
\]

They argue that when \( S_{wR} \) is small, entering water-wet pores, fluid behavior can be predicted using the water-wet equation. When water saturation reaches \((1 - S_{oR})\) the oil-wet equation should be employed:

\[
k_{rwd} = \frac{p_{cwd}k_{ro,wwd} - p_{cod}k_{ro,owd}}{p_{cwd} - p_{cod}} \quad (68)
\]

\[
k_{rod} = \frac{p_{cwd}k_{ro,wwd} - p_{cod}k_{ro,owd}}{p_{cwd} - p_{cod}} \quad (69)
\]

\( k_{rwd} \) and \( k_{rod} \) are the relative permeabilities of water and oil, respectively, in the mixed-wet system during the drainage process. Substituting the subscript “d” with “i” in the equation yields the relationship for the imbibition relative permeability.

Once boundary drainage experiences its first reversal, the Land closure model is used to compute the residual oil saturation. The normalized saturation \( S_{oR} \) should be adjusted using the computed value, and the oil branch (from Equation (37)) should also be adjusted according to the new residual saturation \((p_{1}^{1}coi)\). The water branch remains unchanged.

Therefore, the relative permeability \( S_{oR} \) will be oil-wet at the initial saturation of the first imbibition, obtained through the following equation for the first imbibition:

\[
k_{rwi}^{1} = \frac{p_{cwi}k_{ro,wwi} - p_{1}^{1}k_{ro,owi}}{p_{cwi} - p_{1}^{1}} \quad (70)
\]

\[
k_{roi}^{1} = \frac{p_{cwi}k_{ro,wwi} - p_{1}^{1}k_{ro,owi}}{p_{cwi} - p_{1}^{1}} \quad (71)
\]

Their hysteresis model is based on the assumption that each scanning curve should move towards the initial point of the previous displacement and form a closed loop. Therefore, by having the relative permeability values of water at the initial and final saturations with water and putting them into the equation, \( k_{rwi}^{0}[j] \) and \( k_{rwi}^{1}[j] \) are obtained. The relative permeability of the first imbibition water can be calculated using Equation (52). The same procedure can be used to obtain the relative permeability of oil:

\[
k_{rwi}[j](S_w) = k_{rwi}^{0}[j]k_{rwi}^{1}(S_w) + k_{rwi}^{1}[j] \quad (72)
\]

\[
k_{roi}[j](S_w) = k_{roi}^{0}[j]k_{roi}^{1}(S_w) + k_{roi}^{1}[j] \quad (73)
\]

(7) Spiteri Model [10]

The obtained non-wetting saturation does not always change monotonically with the initial non-wetting saturation. Therefore, Spiteri et al. proposed the following equation to overcome the limitations of the Land trapping saturation model (only applicable to strong water-wet systems).
The Spiteri capture model is characterized by two parameters, $\alpha$ and $\beta$, which are obtained by fitting the results of pore network simulations. Parameters $\alpha$ and $\beta$ represent the initial slope and curvature of the curve, respectively.

$$S_{ot} = \alpha S_{oi} - \beta S_{oi}^2$$ (74)

where $S_{ot}$ is the captured oil saturation, and $S_{oi}$ is the initial oil saturation.

The constraints of the Spiteri model parameters are as follows:

$$0 \leq \alpha \leq 1, 0 \leq \beta$$ (75)

Spiteri’s hysteresis model is an improvement over the Land and Carlson models for predicting relative permeability. The Land model fails to predict the curvature of the relative permeability curve in oil-wet systems. Therefore, the Spiteri model proposes the following relationship:

$$\Delta S_o = S_{ot}(S_{oi}) - S_{ot}(S_{of}) - \gamma [S_o - S_{ot}(S_{oi})](S_o - S_{ot})$$ (76)

where $\Delta S_o$ refers to the intermediate capture saturation, $\gamma$ is an additional parameter depending on the rock type and wettability characteristics, and $S_{of}$ is the flowing oil saturation. When the flowing oil saturation $S_{of}$ is zero, the total oil saturation $S_o$ equals the maximum trapped oil saturation $S_{ot}(S_{oi})$. At the start of water injection ($S_o = S_{oi}$), and when the remaining oil saturation is zero, the oil saturation equals the flowing oil saturation ($S_o = S_{of}$).

By substituting (56) into the equation, $S_{of} = S_{ot} + S_{of}$:

$$S_{of} = S_o - S_{ot}(S_{oi}) + S_{ot}(S_{of}) + \gamma [S_o - S_{ot}(S_{oi})](S_o - S_{ot})$$ (77)

By substituting the residual oil saturation from Equation (54) into Equation (57) and solving it for flowing oil saturation, considering only positive values, we obtain the following equation:

$$S_{of} = \frac{1}{2\beta} \left[ (a-1) + \sqrt{(a-1)^2 + 4\beta[S_o - S_{ot} + \gamma(S_o - S_{ot})(S_o - S_{ot})]} \right]$$ (78)

2.2. Three-Phase Relative Permeability Hysteresis Model

(1) Parker and Lenhard Model [40,41]

Parker and Lenhard utilized two-phase wetting-saturation-capillary pressure and three-phase saturation-capillary pressure models to simulate a drainage and imbibition process, defining saturation paths with three parameters.

They proposed a theoretical model for predicting the relative permeability–saturation relationship in two-phase (gas-water) and three-phase (gas-oil-water) porous media systems influenced by arbitrary saturation paths. Integral expressions for the actual permeabilities of gas, water, and oil were given, extending Parker et al.’s non-hysteretic relative permeability model to accommodate the effects of gas trapped in water and oil phases, as well as pore blockage by trapped oil in the water phase. The closed-form expressions for relative permeabilities of gas, water, and oil were derived as functions of current fluid saturations and saturation history using saturation–pressure relationships and a parameter model for fluid entrapment based on the Stone I model in the integral equations. Three-phase relative permeability–saturation relationships assumed for the principal drainage and imbibition paths in hypothetical soils were computed to illustrate the model’s applicability and assess the degree of fluid trapping’s impact on relative permeabilities. The water permeability–saturation relationship is expected to exhibit mild hysteresis effects except at high saturations, while hysteresis in the air permeability–saturation relationship is more
pronounced. Predicted oil permeability hysteresis is lower at low water saturations, but becomes significant as water saturation increases.

Water Phase:

\[
k_{rw} = \frac{S_{of}^1}{2} \left[ 1 - \left( 1 - S_{or}^1 \right)^m \left( 1 - \frac{S_{or}}{1 - S_{ow}^\min} \right) - \frac{S_{or}}{1 - S_{ow}^\min} \left( 1 - S_{ow}^\min \right)^m \right]^2
\]

(79)

Oil phase:

\[
k_{ro} = \frac{S_{of}^1}{2} \left[ (1 - S_{or}^{1/m})^m - \left( 1 - S_{i}^{1/m} \right)^m \left( 1 - \frac{S_{gro}}{1 - S_{i}^{\min}} \right) - \left( 1 - S_{i}^{1/m} \right)^m \frac{S_{gro}}{1 - S_{i}^{\min}} \right]^2
\]

(80)

Gas phase:

\[
k_{rg} = \frac{S_{gf}^1}{2} \left( 1 - S_{i}^{1/m} \right)^{2m}
\]

(81)

\( m \) is the parameter in the Genchent model \([42]\), \( S_{\text{min}}^w \) is the minimum value of \( S_{ow} \), \( S_{of} \) is the flowing oil saturation, and \( S_{gro} \) is the residual gas saturation in the gas-oil system.

(2) Larsen and Skauge Model \([36]\)

Larsen and Skauge model addresses hysteresis in three-phase systems, allowing for obtaining irreversible relative permeability scanning curves in the second displacement and all subsequent reverse flow processes. These models depend on the initial saturations at the start of a given process. The new three-phase model uses experimental wetting and non-wetting relative permeabilities as inputs, and understands the relationship between maximum non-wetting saturation and captured non-wetting saturation.

Assumptions: Consistent gas-water phase permeability during both two-phase and three-phase flow processes. Oil-phase permeability in three-phase relative permeability calculated using the Stone I model. No hysteresis effect in the oil phase. Hysteresis phenomenon in gas-water phase permeability described by gas trapping and reduced permeability during three-phase flow. Enhanced oil recovery (decreased residual oil saturation) associated with gas trapping effects resulting from multiple gas-water cycles. Displacement of gas phase relative permeability:

\[
[k_{\text{drain}}^\text{g} (S_g)]_n = \left\{ \left[ K_{\text{rg}}^{2\text{Ph}} (S_g) - K_{\text{rg}}^{2\text{Ph}} \left( S_{\text{init}}^g \right) \right] \cdot \left( \frac{S_{i}^{2\text{Ph}}}{S_{iw}^{\init}} \right)^a \right\} + [K_{\text{imb}}^\text{g} \left( S_{\text{init}}^g \right)]_{n-1}
\]

(82)

where \( K_{\text{drain}}^\text{g} \) is the gas phase relative permeability during drainage; \( K_{\text{rg}}^{2\text{Ph}} \) is the two-phase gas phase relative permeability; \( S_g \) is the gas saturation; \( S_{\text{init}}^g \) is the gas saturation after water injection or before gas injection; \( S_{i} \) is the irreducible water saturation; \( S_{iw}^{\init} \) is the water saturation after water injection or before gas injection; \( a \) is the coefficient for reduction in gas phase relative permeability; and \( K_{\text{imb}}^\text{g} \) is the gas phase relative permeability during imbibition.

Gas-phase relative permeability during imbibition:

\[
[k_{\text{imb}}^\text{g} (S_g)]_n = \left[ k_{\text{imb}}^\text{g} \left( S_{\text{trans}}^g \right) \right]_{n-1}
\]

(83)

where

\[
\left\{ S_{gf} = \frac{1}{2} \left[ (S_g - S_{gr}) + \sqrt{(S_g - S_{gr})^2 + \frac{4}{C_{\text{trans}} (S_g - S_{gr})}} \right] \right\}_n
\]

(84)

\[
\left( S_{\text{trans}}^g \right)_n = \left( S_{gf} \right)_n + \left( S_{\text{end}}^g \right)_{n-1}
\]

(85)

In the Larsen and Skauge model, the primary parameters used are “\( a \)” and “\( \alpha \)”, which control and identify hysteresis effects. In the relative permeability between two adjacent cycles, “\( \alpha \)” is used to input the value of the reduction index when calculating the relative
permeability during drainage (increasing gas saturation), while “a” is used to modify the residual oil saturation required as input in the STONE1 oil-phase relative permeability model, referred to as the residual oil reduction index.

The Larsen and Skauge model provided a higher recovery factor compared to simulations without relative permeability hysteresis or with a two-phase gas hysteresis mode.

(3) Egermann Model [12]

Egermann proposed expressions for hysteresis to avoid any negative effects on numerical instability. $C_L$, $C_T$, and $C_U$ represent constants for Land, trapping, and untrapped phases, respectively. $C_T$ equals $C_L$ in all trapping stages (imbibition) because it corresponds entirely to conditions where Land constants can be directly used. As $S_{gr}$ decreases, $C_U$ approximates $C_L$ because there is almost no gas trapped in the largest pores, making it the easiest to trap. As $S_{gr}$ increases, it becomes harder to release trapped gas, causing $C_U$ to rise above $C_L$.

When a significant portion of gas has been trapped (high $S_{gt}$), reversibility can be assumed, attributed to the presence of trapped gas saturation that remains unapproachable in the smallest gas-occupied pores.

An empirical expression for $C_U$ has been established to meet the above requirements:

$$C_U = \left( \frac{K_{rg} - K_{rg\text{min}}}{K_{rg}^{\text{DL}}} \right) \frac{S_{gt}}{S_{gr2}}(C_{UM} - C_L) + C_L$$

where $\lambda$ is an empirical coefficient governing the evolution of $C_U$, $C_{UM}$ represents the maximum value of unmapped constants, $K_{rg\text{min}}$ is the low mobility curve, $K_{rg}^{\text{DL}}$ is the high mobility curve, and $S_{gr2}$ is the maximum trapped gas saturation.

The gas phase is the most easily modified expression because free gas flows in the center of the fractal pore. The apparent saturation of the liquid phase is $S_L$. It is a general expression.

$$k_{\text{hyst}}^{rg} = k \left( 1 - (S_L + S_{gt})^\alpha \right)^\beta$$

where $k_{\text{hyst}}^{rg}$ is the gas phase hysteresis relative permeability, $S_L$ is the liquid saturation, $S_{gt}$ is the trapped gas saturation, $\alpha = \frac{4}{2 - D_L}$, where $D_L$ is the linear fractal dimension of the porous medium and $S_L$ is the total liquid saturation.

When $K_{rg}^{\text{hyst}}(S_L) = K_{rg}(S_{gt})$, similar results to Carlson’s are obtained, but by calculating the trapped gas saturation, hysteresis effects can be fully accounted for.

The relative permeability of oil is also derived by considering the apparent saturation. As shown above, $S_L$ becomes $S_L + S_{gt}$, and $S_w$ becomes $S_w + (1 - a)S_{gt}$. Then:

$$k_{ro} = k_{ro}^{2P_W} \left( S_L + S_{gt} \right)^\beta - \left( S_w + (1 - a)S_{gt} + (S_w)_{S_{gt}=0} \right)^\beta$$

where $\beta = (4 - DL)/(2 - DL)$, $S_{gt}$ is the trapped gas saturation, $k_{ro}^{2P_W}$ is the oil relative permeability during water/oil imbibition tests, and $a$ is the residual oil saturation reduction factor. When reaching $S_{or}$, this expression ensures that $k_{ro}$ equals zero.

If a portion of the trapped gas contributes to a higher apparent saturation of water, its presence would influence the flow behavior of the wetting phase. In the following expression, it is assumed that the flowing $S_w$ is deprived of a quantity proportional to the saturation of gas trapped with water:

$$k_{rw} = (S_w - R(1 - a)S_{gt})^\beta - S_{wi}^\beta$$

where $R$ is the reduction factor for permeability.
(4) F. J. Fayers Model [43]

F. J. Fayers and colleagues proposed a consistent procedure for calculating three-phase relative permeability (3PRP) and capillary pressure (3PCP), known as the Rescaled Interpolation Method (RIM). The method starts with the use of Baker saturation-weighted interpolation for three phases, but modifies the oil phase by rescaling saturation to be compatible with the general form of the residual saturation function \( S_{om}(S_g, S_w) \). It is assumed that the hysteresis of each phase depends on whether the phase is increasing or decreasing, independent of the state of other phases.

The current captured gas saturation \( S_{gr} \) during secondary imbibition process is given by the following equation:

\[
S_{gr} = [(S_{gmax} - S_g)(S_{gt} - S_{gc})/(S_{gmax} - S_{gt})] + S_{gc}
\]  

where \( S_{gmax} \) is the historical maximum \( S_g \), and \( S_{gt} \) is the final captured gas saturation to be reached in the current hysteresis cycle:

\[
S_{gt} = [(S_{gtm} - S_{gc})(S_{gmax} - S_{gc})/(1 - S_{wc} - S_{gc})] + S_{gc}
\]  

The relative permeability curves for the gas phase, \( k_{rgo} \) and \( k_{rgw} \), are then readjusted to meet the current value of \( S_{gr} \):

\[
k_{rgi} = k_{mgi} S_{hi}
\]  

\[
S_{hi} = [(S_g - S_{gr})(1 - S_{wc} - S_{gc})/(1 - S_{wc} - S_{gc})] + S_{gc}
\]

The interpolation weights are calculated as follows:

\[
r_w^h = \frac{1 - S_{wc} - S_{orw}}{(1 - S_{wmax} - S_{orw})S_w + (S_{wmax} - S_{wc})} \]

\[
r_o^h = \frac{S_{omin} - S_{orw}}{(S_{omin} - S_{orw})(1 - S_w) + (S_o - S_{omin})(1 - S_{omin} - S_{wc})}
\]

(5) UTHYST Model [16]

Beygi provides a simple method to calculate the irreversible hysteresis behavior of any phase. In a multi-cycle process, considering the influence of saturation direction and saturation path, the relative permeability is determined by the following equation:

\[
k_{rf} = k_{rf}[S_j, (S_{\mu})_n]
\]

The method is applicable to any relative permeability model that includes captured saturation, even in multi-cycle processes with phase-wise monotonic increases where captured saturation remains constant, as shown below:

\[
(S_{\mu})_n = (S_{\mu})_{E,n-1} + \Delta S_{\mu} \left( \frac{S_{mc}}{S_{m}} \right)^{a_j}
\]
where \( (S_j^\mu)_{E,n-1} \) represents the captured saturation at the end of the previous cycle, \( \Delta S_j^\mu \) is the captured saturation for the current cycle, and parentheses introduce the influence of saturation pairs of conjugate phases at the beginning of the current cycle. Water, oil, and gas are defined as the conjugate phases of gas, water, and oil phases, respectively.

We use an improved Jerauld model (Jerauld 1997) to calculate the phase capture of non-wetting phases:

\[
S_j^\mu (S_j^{\text{max}}, L_j) = S_{jc} + \frac{\max(S_j, S_j^{\text{max}}) - S_{jc}}{1 + L_j \left[ \max(S_j, S_j^{\text{max}}) - S_{jc} \right]}^{1 + \frac{\rho_j}{\rho_i}} \tag{100}
\]

Different cycle’s trapping saturation is calculated as follows:

\[
(S_j^\mu)_{E,n-1} = S_j^\mu (S_j^{\text{max}})_{E,n-1} \tag{101}
\]

\[
\Delta S_j^\mu = S_j^\mu \left[ S_j^{\text{max}} (S_j^\mu)_{E,n-1} (L_j)_n \right] \tag{102}
\]

The proposed model addresses the cycle-related Land coefficients in a multi-cycle process:

\[
L_j = \frac{1}{S_j^{3P} - S_{jc}} - \frac{1}{1 - \sum_{i=1}^{3} S_{ic}} \tag{103}
\]

\( S_j^{3P} \) is corrected based on changes in composition and trapping number. Initially, the impact of two-phase residual saturations on trapping numbers is considered, then extended to include three-phase residual saturations in the equation, where \( S_{jc} \) and \( S_{ic} \) represent the co-existing water and critical oil or critical gas saturations, respectively.

(6) WAG-HW Model [16]

Shahverdi and Sohrabi proposed the WAG-HW model to simulate the cyclic hysteresis of three-phase relative permeability during WAG injection. This technique is a direct approach that utilizes measured three-phase relative permeability data obtained from the first cycle of WAG injection to predict the relative permeability of subsequent cycles. The basic equation is as follows:

\[
[k_{ri}(S_i)]_n = H \times [k_{ri}(S_i)]_{n-1} \tag{104}
\]

Here, \( k_{ri} \) represents the relative permeability of different fluid phases, \( n \) denotes different periods, and \( H \) is a constant coefficient.

The model derives different expressions for relative permeability in different stages, divided into the following sections:

The relative permeability of the gas phase during the gas injection stage:

\[
[k_{rg}^{GI} (S_g)]_n = \left( \frac{S_{g,start}^{n-1}}{S_{g,start}^n} \right) [k_{rg}^{GI} (S_g^*)]_{n-1} \tag{105}
\]

\[
S_g^* = S_g - \left( S_{g,start}^n - S_{g,start}^{n-1} \right) \tag{106}
\]

Here, \( k_{rg}^{GI} \) is the gas-phase relative permeability for gas injection, \( S_{g,start}^n \) is the initial gas saturation, and \( S_g^* \) is the adjusted gas saturation.

The relative permeability of the gas phase during the water injection stage:

\[
[k_{rg}^{WI} (S_g)]_n = \left( \frac{S_{g,start}^n}{S_{g,start}^{n-1}} \right)^{WI} [k_{rg}^{Wl} (S_g^*)]_{n-1} \tag{107}
\]
\[ S^*_g = S_g - (S^n_{gt} - S^{n-1}_{gt}) \]  
\[ \left( S^n_{g,start} - S^n_{gt} \right)_{WI} = \left( S^{n-1}_{g,start} - S^{n-1}_{gt} \right)_{WI} = \text{Constant} \]

Here, \( k_{rg}^{WI} \) is the gas-phase relative permeability during water injection.

The relative permeability of the water phase during the water injection stage:
\[ k_{rw}^{WI}(S_w)_{n} = \left( \frac{S^n_{r,start}}{S^n_{g,start}} \right)_{WI} \left( k_{rw}^{WI}(S_w) \right)_{n-1} \]

The relative permeability of the water phase during the gas injection stage:
\[ k_{rw}^{GI}(S_w)_{n} = \left( \frac{S^n_{r,start}}{S^n_{g,start}} \right)_{GI} \left( k_{rw}^{GI}(S_w) \right)_{n-1} \]

The relative permeability of the oil phase during the water injection stage:
\[ k_{ro}^{WI}(S_o)_{n} = \left( \frac{1}{S^n_{w,start} S^n_{o,start}} \right)_{WI} \left( k_{ro}^{WI}(S_o) \right)_{n-1} \]
\[ S^*_o = S_o + S^{n-1}_{or} - S^n_{or} \]

The relative permeability of the oil phase during the gas injection stage:
\[ k_{ro} \propto s_o^2 \]

\( k_{ro} \) is proportional to \( s_o^2 \).

If three-phase relative permeability data are not measured, one can utilize an existing three-phase relative permeability model such as the Stone or Baker model to generate input three-phase relative permeability data.

(7) Aghabozorgi Model \[44\]

Aghabozorgi et al. estimated the fluid saturations at the end of the WAG experimental cycles based on new coefficients implemented in the three-phase relative permeability model, and finally, they estimated changes in water and gas relative permeabilities. The model successfully simulated the hysteresis observed in WAG experiments in sandstone.

To calculate the final saturations for each injection cycle, a new parameter called the ‘relative difference’ was defined based on experimental values of oil saturation at the end of the injection cycle.
\[ \delta_o = \left( \frac{S^n_{of} - S^{n-1}_{of}}{S^n_{of}} \right) \]  
\[ S_{of} = (1 - \delta_{o,ave})S^n_{of}^{n-1} \]

Here, \( \delta_o \) is the rate of change of oil saturation, \( S_{of} \) is the residual oil saturation, \( S^n_{of} \) is the residual oil saturation in the \( n \)-th cycle, and \( \delta_{o,ave} \) is the average rate of change in oil saturation.

The main assumption in this model is that the water saturation fluctuates within a certain range of saturations, with known limits (water saturation at the beginning and end of the imbibition cycle). This assumption is applicable to systems where the decrease in water saturation is negligible (or assumed to be negligible). If the variation in water saturation is not negligible in the system, a relative difference term can also be defined for
the water phase. If the pore volume injected during different cycles is not constant, the average can be calculated and used in the simulation (if necessary):

\[ S_{wf} = (1 - \delta_{w,ave})S_{wf}^{n-1} \]  
(117)

\[ S_{gf} = 1 - S_{of} - S_{wf} \]  
(118)

Here, \( S_{wf} \) is the water saturation, and \( S_{gf} \) is the gas saturation.

The simulation results indicate that this new approach can capture the gradual variation of the captured gas saturation, even if it is not based on the calculation of the captured gas saturation.

As mentioned earlier, there are limitations to estimating the relative permeability of water and gas using existing WAG hysteresis models. Therefore, it is suggested to use two empirical equations to predict the changes in water and gas relative permeability. For the imbibition cycle, the change in water relative permeability is calculated as follows (assuming the obtained curves are reversible in the extraction cycle):

\[ k_{rw}^n = \beta_w k_{rw}^{n-1} \]  
(119)

\[ \beta_w = \frac{\Delta P}{S_{gf}^n - S_{gf}} \]  
(120)

Here, \( k_{rw}^n \) is the water-phase relative permeability in the \( n \)-th cycle, \( \beta_w \) is the scaling factor for adjusting the water-phase relative permeability, and \( \Delta P \) is the pressure difference.

During the WAG experiment, it is assumed that the gas relative permeability will decrease as follows. For the drainage cycle, it is recommended to use a reduction factor for gas \( k_r \) data to estimate the reduction in gas relative permeability:

\[ k_{rg}^n = \beta_g k_{rg}^{n-1} \]  
(121)

\[ \beta_g = \frac{\Delta P}{S_{gf}^n - S_{gf}} \]  
(122)

Use another equation to calculate the gas relative permeability during the imbibition cycle:

\[ k_{rg}^n = \left( \frac{S_g - S_{gf}^n}{S_{gf}^n - S_{gf}} \right) k_{rg}^{n-1} \]  
(123)

Before proceeding with the next simulation step, update the calculated final saturations at the end of each cycle. Assume that the relative permeability of gas and water depends only on their saturation levels.

(8) Sigmoid Model [45]

Ehsan Ranaee et al. developed a three-phase oil relative permeability prediction model based on the sigmoid function. This model incorporates the key effects of pore-scale phase distribution into the proposed effective Kro empirical model. It can reflect the reactivation of oil and gas due to gas injection in water-wet media, the smooth transition to a drainage system with low oil saturation formations, and the subsequent reduction of residual oil saturation in the three-phase system.

According to the Baker’s [25] model saturation weighted interpolation method, \( k_{ro} \) is:

\[ k_{ro} = \frac{(S_w - \overline{S}_{wc})\overline{k}_{row} + (S_g - \overline{S}_{gt})\overline{k}_{rog}}{(S_w - \overline{S}_{wc}) + (S_g - \overline{S}_{gt})} \]  
(124)

Here, \( \overline{k}_{row} \) and \( \overline{k}_{rog} \), respectively, are water and gas saturations for three-phase flow conditions; \( \overline{S}_{wc} \) and \( \overline{S}_{gt} \), respectively, are connate water saturation and oil relative permeability in an oil–gas environment; and \( \overline{S}_{gf} \) is the trapped gas saturation.
Modeling the primary gas injection:

\[ k_{ro}^G = \max \left( k_{ro}^{GS}, k_{rog} \right) \]

with

\[ k_{ro}^{GS} = \frac{M_{row}^M S_o}{S_{ow} + \exp \left[ \lambda - \beta \left( \frac{S_o}{S_{ow}} \right) \right]} \]

\[ \lambda = \ln \left[ \frac{M_{row}^M}{k_{ro}^M} - \frac{S_{ow}}{S_{ow}} \right] + \beta \left( \frac{S_o}{S_{ow}} \right) \]

\[ \beta = \frac{1}{S_{ow}} \left( m_{inf} \frac{S_o}{k_{ro}^M} - 1 \right) \left( 1 - \frac{S_{ow} k_{ro}^M}{M_{row}^M} \right) \left( S_o - S_{ow} \right) \]

Here, \( S_o \) is the oil saturation in the three-phase system, \( \lambda \) and \( \beta \) are model parameters, and \( S_{ow}^M \) and \( M_{row}^M \), respectively, are the largest oil saturation and relative permeability observed in a two-phase oil–water system.

Modeling secondary water injection:

\[ k_{ro}^W = \frac{(S_w - S_{uw}) k_{row}^S + (S_g - S_{gt}) k_{rog}}{(S_w - S_{uw}) + (S_g - S_{gt})} \]

with

\[ k_{row}^S = \frac{M_{row}^M S_{uw}}{S_{ow} + \exp \left[ \lambda_{ow} - \beta_{ow} \left( \frac{S_{ow}}{S_{ow}} \right) \right]} \]

Here, \( S_{ow} \) is oil saturation in an oil–water environment, all the remaining quantities being defined above.

A remarkably good agreement between model predictions and direct three-phase relative permeability measurements from the literature is observed when the model parameters are estimated solely on the basis of two-phase information.

### 2.3. Forward Calculation of Phase Permeability Based on Digital Core

The aforementioned two types of permeability models are mostly based on inverse modeling of relevant experimental data, which is the traditional approach to handling permeability data. In addition, digital rock core models can be established by scanning rock cores to obtain measurable data. This method primarily establishes models from geometric and calculus perspectives to describe the hysteresis phenomenon in relative permeability. Currently, this approach is mainly applied to the flow of two fluids.

This method has been studied for a considerable amount of time. Nowadays, in addition to conventional CT scans, some scholars have attempted to introduce neural networks for computation with good results. Liu et al. [46] visualized multiphase flow during core oil displacement experiments using X-ray microcomputer tomography. Quantify the obtained phase states using Minkowski Functionals and measure relative permeability using image-based methods, but only capture the effective permeability of the flow path and not the ganglion flow. McClure et al. [47] Validated, using synchrotron X-ray computed tomography (\( \mu \)CT), that the geometric state function can represent the microscopic fluid configuration generated by a wide range of simulated flow conditions on an average basis, while considering the geometric description of capillary pressure. From the results, it can be seen that the geometric state function can serve as a closed relationship in macroscopic models to effectively remove hysteresis effects attributed to fluid arrangement in porous media. P. Purswani and R. T. Johns [48] proposed a General Equation of State (EOS) method for relative permeability (kr) based on a set of geometric state parameters, normalized Euler properties (connectivity) and saturation, and verify that the new kr EoS matches experimental data better than the traditional Corey form. The fitting results are shown in Figure 3. The description of relative permeability hysteresis has been achieved in the two-phase flow of digital rock cores.
In the latest research progress, Al Zubaidi et al. [49] developed an Artificial Neural Network (ANN) that relies on fundamental geometric relationships to determine the dissipation of mechanical energy during creeping immiscible two-fluid flow. The developed ANN is based on a set of physically insightful specific state variables. This model predicts the effective permeability of numerous unseen pore geometric states with strong correlation. The prediction results are shown in Figure 4.

![Figure 3](image1.png)

**Figure 3.** P. Purswani and R. T. Johns using (a) kr-EoS with constant partial derivatives and (b) Corey form to best fit the literature data [48].

![Figure 4](image2.png)

**Figure 4.** (a) Predictability results of the ANN on the test set with all four Minkowski functionals as inputs. (b) Based on the predictive results of the ANN, simulations provide a variety of unique relative permeability curves for different wettability conditions [49].

### 2.4. Summary of the Model Study

Table 1 summarized the existing relative permeability models. The differences in the scanning curve are the core of these models.

<table>
<thead>
<tr>
<th>Hysteresis Model</th>
<th>Application Water</th>
<th>Application Oil</th>
<th>Application Gas</th>
<th>Wettability</th>
<th>Assumed Conditions/Applicable Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land et al. [37] (1968)</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Water-wet</td>
<td>Capture occurs only during imbibition.</td>
</tr>
<tr>
<td>Killough et al. [34]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Water-wet</td>
<td>Capture occurs only during imbibition; there is no hysteresis effect in the relative permeability curves of the wetting phase; imbibition relative permeability curves are reversible.</td>
</tr>
<tr>
<td>Carlson et al. [35]</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Water-wet</td>
<td>Capture occurs only during imbibition; there is no hysteresis effect in the relative permeability curves of the wetting phase; all imbibition curves are parallel.</td>
</tr>
<tr>
<td>Beattie et al. [36]</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Water-wet</td>
<td>Assume that the water-phase permeability depends only on water saturation.</td>
</tr>
</tbody>
</table>
### Table 1. Cont.

<table>
<thead>
<tr>
<th>Hysteresis Model</th>
<th>Application</th>
<th>Wettability</th>
<th>Assumed Conditions/Applicable Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kjosavik et al. [39] (2002)</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Assume that each injection cycle of kr should return to the starting point of the previous displacement and form a closed scanning loop.</td>
</tr>
<tr>
<td>Delshad et al. [26] (2003)</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Assume that Sor has a maximum value at $M_{ow} = S_{iw}$ and is zero at $M_{ow} = 1$.</td>
</tr>
<tr>
<td>Spiteri et al. [10] (2008)</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The captured non-wetting saturation does not always monotonically change with the initial non-wetting saturation.</td>
</tr>
</tbody>
</table>

Three-phase hysteresis (related to saturation history: saturation direction and path)

<table>
<thead>
<tr>
<th>Application</th>
<th>Wettability</th>
<th>Assumed Conditions/Applicable Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gas-water phase permeability is consistent during both two-phase and three-phase flow processes; oil phase permeability in three-phase flow is calculated using the Stone I model; there is no hysteresis effect in the oil phase.</td>
</tr>
<tr>
<td>Larse and Skauge et al. [36] (1998)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Egermann et al. [12] (2000)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>F.J.Fayers et al. [43] (2000)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UTHYST Beygi et al. [16] (2013)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sigmond et al. [45] (2014)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WAG-HW Shahverdi and Sohrabi et al. [50] (2015)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aghabozorgi et al. [44] (2019)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 2.4.1. Two-Phase Relative Permeability Model

The Land model studies gas-water two-phase relative permeability in strongly water-wet porous media, assuming a monotonic relationship between the captured non-wetting phase saturation (gas in the model) and the initial gas saturation. It also assumes that the imbibition relative permeability scanning curve is reversible. The Land model is widely used for calculating residual non-wetting phase saturation, but has only been studied for a single cycle without considering multi-cycle scenarios.

The Killough model introduces two methods for calculating imbibition-phase relative permeability, including parameter interpolation and normalization of experimental data. It considers the hysteresis effects in both the wetting and non-wetting phases. However, it does not account for multi-cycle experimental effects.

The Carlson model proposes a hysteresis model for non-wetting phase relative permeability, assuming theoretically parallel self-scan curves and no hysteresis effect in the wetting phase. It modifies the Land model.

The Beattie model considers hysteresis effects in both wetting and non-wetting phases and suggests that the imbibition scanning curve is irreversible. It is based on the normalization of boundary curves, assuming that each phase relative permeability scanning curve should approach its boundary curve during fluid injection processes. The Delshad model investigates oil-water two-phase hysteresis models developed based on capillary pressure and relative permeability in mixed-wet rocks. It assumes that the water saturation cannot be lower than the reverse water saturation from the main drainage to the main imbibition path.
The Kjosavik model is a hysteresis model for mixed-wet rocks. It extends the Corey–Burdine relative permeability correlation developed for water-wet and oil-wet systems to mixed-wet rocks using a weighted approach with capillary pressure data. The model assumes that each cycle’s relative permeability should return to the starting point of the previous displacement, forming a closed scanning loop.

The Spiteri model overcomes limitations of the Land trapping saturation model. It characterizes the Spiteri trapping model in strongly water-wet systems with two parameters obtained by fitting results from pore network simulations.

2.4.2. Three-Phase Relative Permeability Model

The Parker and Lenhard model predicts arbitrary phase scanning curves using empirical interpolation schemes and scaling procedures. It uses two-phase wetting-saturation-capillary force and phase-permeability-saturation-capillary force models for incremental and decremental processes, defining saturation paths with three parameters. The Larsen and Skaug model depends on the initial saturation at the start of the given process. It proposes an algorithm to implement a non-wetting phase hysteresis model in numerical simulators using experimental wetting and non-wetting relative permeability as input.

The Egermann model analyzes hysteresis in gas-water two-phase relative permeability developed through continuous gas and water injection experiments. It identifies cycle hysteresis on the relative permeability based on historical matches and applies different Land formula constants considering displacement and history.

The F.J. Fayers model is used for mixed-wet systems and accounts for oil-gas composition changes. It uses Baker saturation-weighted interpolation for three-phase relative permeability, modifying the oil phase by rescaling saturation. The partitioning of the mixed-wet pore space is derived from the cross-saturation of water/oil two phases.

The UTHYST model proposes new three-phase relative permeability and three-phase hysteresis models using multi-cycle three-phase water-alternating-gas experiments on non-water-wet rocks. It includes saturation history and composition effects, estimating three-phase parameters based on two-phase parameter saturation weighting interpolation. The hysteresis model is an extension of the Land trapping model, but introduces dynamic Land coefficients. The trapping model estimates continuously increasing capture saturations for intermediate wetting and non-wetting phases.

The WAG-HW model is applied in WAG experiments on water-wet cores to obtain three-phase relative permeability values for oil, water, and natural gas. It observes hysteresis during WAG injection cycles of water and gas, where oil, water, and gas relative permeability exhibit significant and irreversible hysteresis. The model combines hysteresis effects for simulating WAG injection-related relative permeability.

The Aghabozorgi model updates formulas for estimating hysteresis in water and gas phase relative permeability data to capture behavior observed in WAG experiments. It develops new hysteresis models based on innovative techniques to predict oil and water saturation at the end of each injection cycle. The main assumption of the model is that water saturation fluctuates within a certain saturation range with known limits (water saturation at the start and end of the water absorption cycles).

To calculate saturation during the WAG process using three-phase relative permeability models, it is necessary to determine relative permeability data under different saturation histories. Due to the complexity of flow paths in three-phase flow, it is challenging to measure three-phase permeability experimentally. Shokoufeh Aghabozorgi’s [44] research found that the Stone model overestimates the relative permeability in low oil saturation areas during the WAG process. Shahverdi’s [50] research identified several significant flaws in the WAG hysteresis model. Additionally, the Stone-I model exhibits certain errors when calculating the oil-phase relative permeability.
3. Experimental Studies

The experimental Section elaborates on the designs and methodologies employed to investigate the phenomena of relative permeability hysteresis. These experiments can be categorized into two types: direct relative permeability measurements, and flowthrough measurements. The flowthrough measurements involve standard core flow-through tests, which are predominantly utilized in studies of three-phase relative permeability hysteresis. In this approach, relative permeability is derived from the history matching of recorded pressure and flow rates for each phase during the core flow-through tests. On the other hand, direct relative permeability measurements are straightforward and direct, focusing on the assessment of relative permeability changes through specific experiments designed for this purpose, typically in two-phase flow conditions.

Experimental studies also include relative permeability experiments under different saturation histories, such as DDI (decreasing-decreasing-increasing) and IDI (increasing-decreasing-increasing), and the consideration that the relative permeability of the non-wetting phase in two-phase and multiphase flow is a function of hysteresis effects. The available experiments are summarized in Table 2.

3.1. Direct Relative Permeability Measurements

Previous studies on relative permeability experiments can be classified into three saturation history scenarios: DDI, DDI, and IDI. The naming convention proposed by Saraf et al. [51]. (i.e., D = Drainage and I = Imbibition) is used to name the saturation history. For example, DDI represents a scenario where water saturation decreases, oil saturation decreases, and gas saturation increases. IDI represents a scenario where water saturation increases, oil saturation decreases, and gas saturation increases, while IDIDID represents three cycles of imbibition followed by drainage. In relative permeability experiments, without considering capillary end effects, gas is trapped by water while gas permeability decreases after trapping. However, the capture of non-wetting phase during steady-state and transient drainage processes may differ. The trapped gas leads to a decrease in water relative permeability, and the slight variation in trapped gas saturation is due to changes in gas and water relative permeability. Hysteresis cycles are also irreversible, and significant errors in simulated relative permeability may occur when capillary effects are ignored (Batycky, Element, et al. [52,53]).

The relative permeability of the non-wetting phase in two-phase and multi-phase flows is a function of hysteresis effects. During imbibition after drainage, the hysteresis effect causes a significant decrease in CO₂ relative permeability. During CO₂ alternating injection, the increased trapped of CO₂ saturation increases the pressure drop during drainage and imbibition processes (Reza et al. [54]). Compared to the non-wetting phase (gas), the hysteresis of the wetting phase (oil) is much smaller, and with continuous alternating imbibition and drainage cycles, the hysteresis effect on oil relative permeability weakens. In high-permeability cores, the hysteresis of the wetting and non-wetting phases is smaller than in low-permeability ones. Compared to the water-wet system, the relative permeabilities of the wetting and non-wetting phases (oil and gas) in mixed-wet systems decrease, with reductions observed in both imbibition and drainage directions (Fatemi et al. [55]). As the number of WAG cycles increases, the relative permeability curves all show a downward shift of the equal permeability point and a narrowing of the co-flow region. The decrease in gas phase permeability with increasing cycles slows down and stabilizes, while the hysteresis effect on the water phase (Zhu Sinan et al. [56]) is not significant.
Homogeneity affects phase permeability experiments. Compared to sandstone, hysteresis effects are not dominant in carbonate rocks (Alnuaimi et al. [57]). In homogeneous systems, the hysteresis effect can be negligible. Due to the combined effects of different rock wettability and pore geometries, different types of rock samples exhibit different hysteresis patterns in imbibition relative permeability curves (Dernaika MR et al. [58]). Salinity also affects the hysteresis effects of phase permeability experiments, with lower salinity conditions leading to more significant capillary hysteresis (Wang et al. [59]). The reasons may be related to increased dissolution of minerals causing delayed contact angle or lower salinity brines making reservoir rocks more wettable.

3.2. Flowthrough Measurements

The flowthrough experiments mainly measure three-phase relative permeability. In these experiments, changes in recovery rate are determined through different displacement methods. After the displacement experiment, cumulative injected pore volumes (PV), experimental data of the core flooding are obtained. These data are then input into the simulator with proper relative permeability hysteresis model to perform the history matching and obtain the relative permeability curves, thereby analyzing the hysteresis phenomenon.

Unlike direct permeability experiments, the results of displacement-type experiments require numerical simulation. The accuracy of the numerical model directly affects the final experimental results, and the fitting results depend on the two-phase relative permeability curves. Therefore, different WAG hysteresis model and parameters are required for different relative permeabilities studies.

Gas trapping ultimately affects other characteristics of hysteresis modeling. In mixed-wet systems during gas injection, the cyclic hysteresis of water relative permeability can be neglected, while in water-wet systems, water relative permeability decreases as alternating imbibition and drainage processes occur. In oil-gas two-phase systems, gas relative permeability is much higher than in three-phase systems, mainly due to the significant impact of water presence on three-phase gas relative permeability. From the simulation results, it is observed that under the same gas saturation, gas-phase relative permeability exhibits cyclic decrease. The main reason is that during the drainage process after imbibition, some gas is trapped in the core, causing each new drainage cycle after imbibition to start with a higher content of trapped gas. Therefore, with the progress of WAG, the gas injection capacity significantly decreases and hysteresis occurs. Additionally, due to the cyclic water and gas injection, the trapped gas saturation continuously increases, and the residual oil saturation decreases accordingly.
<table>
<thead>
<tr>
<th>Author</th>
<th>Rock Property</th>
<th>Porosity (%)</th>
<th>Permeability (mD)</th>
<th>Wettability</th>
<th>Saturation Path</th>
<th>Temperature (°C)</th>
<th>Pressure (MPa)</th>
<th>IFT (mN/m)</th>
<th>Flow Rate (mL/min)</th>
<th>Hysteresis Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batycky et al. [52] (1981)</td>
<td>Sandstone</td>
<td>21.5</td>
<td>476</td>
<td>Water-wet</td>
<td>DIDI</td>
<td>/</td>
<td>/</td>
<td>50</td>
<td>0.92</td>
<td>Both</td>
</tr>
<tr>
<td>Van Spronsen et al. [60]</td>
<td>Sandstone</td>
<td>/</td>
<td>/</td>
<td>DDI</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Fulcher et al. [61] (1985)</td>
<td>Sandstone</td>
<td>19.02–26.5</td>
<td>123.2–531.6</td>
<td>Water-wet</td>
<td>DI</td>
<td>25</td>
<td>1.38</td>
<td>1</td>
<td>1.33–6.67</td>
<td>Both</td>
</tr>
<tr>
<td>Oak et al. [62] (1990)</td>
<td>Sandstone</td>
<td>/</td>
<td>210</td>
<td>Water-wet</td>
<td>DDI/IID</td>
<td>21 ± 0.6</td>
<td>5.5</td>
<td>/</td>
<td>0.001–30</td>
<td>Displacement</td>
</tr>
<tr>
<td>Hawkins et al. [57] (1992)</td>
<td>Synthetic core</td>
<td>/</td>
<td>/</td>
<td>Oil-wet</td>
<td>IDID</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>Both</td>
</tr>
<tr>
<td>Eleri et al. [63] (1995)</td>
<td>Sandstone</td>
<td>24.09</td>
<td>11.15</td>
<td>Water-wet</td>
<td>IDD</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0.5</td>
<td>Both</td>
</tr>
<tr>
<td>Braun &amp; Holland et al. [6]</td>
<td>Sandstone</td>
<td>22.3</td>
<td>512</td>
<td>Mix-wet/Water-wet</td>
<td>DID</td>
<td>22.8</td>
<td>6.9</td>
<td>/</td>
<td>2</td>
<td>Both</td>
</tr>
<tr>
<td>Bennion et al. [13] (1998)</td>
<td>Sandstone</td>
<td>6.2/9.8</td>
<td>0.19/0.26</td>
<td>Water-wet</td>
<td>/</td>
<td>/</td>
<td>3.6</td>
<td>/</td>
<td>/</td>
<td>Both</td>
</tr>
<tr>
<td>Element et al. [53] (2003)</td>
<td>Sandstone</td>
<td>19</td>
<td>393/328</td>
<td>Water-wet/Mix-wet</td>
<td>IDID/IDIDIDI/DIDIDIDID</td>
<td>/</td>
<td>2.5</td>
<td>/</td>
<td>0.07</td>
<td>Both</td>
</tr>
<tr>
<td>Fatemi et al. [64,65] (2012–2013)</td>
<td>Sandstone</td>
<td>18.3</td>
<td>65</td>
<td>Mix-wet</td>
<td>IDID/IDIDID</td>
<td>37.8</td>
<td>12.8</td>
<td>0.04</td>
<td>0.83–1.67</td>
<td>Both</td>
</tr>
<tr>
<td>Fatemi et al. [64,65] (2012–2013)</td>
<td>Dernaika MR et al. [66] (2013)</td>
<td>Sandstone</td>
<td>17</td>
<td>1000</td>
<td>Water-wet</td>
<td>37.8</td>
<td>12.8</td>
<td>0.04</td>
<td>0.62–1.25</td>
<td>Both</td>
</tr>
<tr>
<td>Fatemi et al. [64,67] (2012–2013)</td>
<td>Dernaika MR et al. [66] (2013)</td>
<td>Carbonate</td>
<td>17.4–27.9</td>
<td>0.98–46</td>
<td>Water-wet</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0.33, 0.5</td>
<td>/</td>
</tr>
<tr>
<td>Shahverdi et al. [50] (2014)</td>
<td>Sandstone</td>
<td>18</td>
<td>65</td>
<td>/</td>
<td>/</td>
<td>38</td>
<td>12.7</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Shahverdi et al. [50] (2014)</td>
<td>limestone</td>
<td>17</td>
<td>11</td>
<td>/</td>
<td>IDID</td>
<td>145</td>
<td>31</td>
<td>/</td>
<td>0.03/0.07/0.13</td>
<td>/</td>
</tr>
<tr>
<td>Shahverdi et al. [50] (2014)</td>
<td>Sandstone</td>
<td>18.2</td>
<td>65</td>
<td>Mix-wet</td>
<td>DIDIDIDIDDI</td>
<td>38</td>
<td>8.4/12.3/12.7</td>
<td>2.7/0.15/0.04</td>
<td>0.42</td>
<td>Both</td>
</tr>
<tr>
<td>Shahverdi et al. [5] (2016)</td>
<td>Sandstone</td>
<td>17.6/18.2</td>
<td>1000/65</td>
<td>Mix-wet/Water-wet</td>
<td>IDID/IDIDIDI</td>
<td>37.8</td>
<td>12.8</td>
<td>0.047/55/60</td>
<td>0.21–3.33</td>
<td>Both</td>
</tr>
<tr>
<td>Wang et al. [59] (2016)</td>
<td>Sandstone</td>
<td>15/12</td>
<td>50.42/56.68</td>
<td>Oil-wet/Water-wet</td>
<td>DI</td>
<td>70</td>
<td>6.9</td>
<td>/</td>
<td>0.35</td>
<td>/</td>
</tr>
<tr>
<td>Fatemi et al. [55] (2018)</td>
<td>Carbonate</td>
<td>20/16</td>
<td>260/40</td>
<td>Oil-wet</td>
<td>IIDIDIDIDID</td>
<td>58.7</td>
<td>12.7</td>
<td>/</td>
<td>0.33</td>
<td>/</td>
</tr>
<tr>
<td>Alkhazmi et al. [17] (2018)</td>
<td>Carbonate</td>
<td>15</td>
<td>45</td>
<td>Water-wet</td>
<td>IDIDIDIDIDIDID</td>
<td>38</td>
<td>12.7</td>
<td>0.04</td>
<td>0.18</td>
<td>/</td>
</tr>
<tr>
<td>Reza et al. [54] (2021)</td>
<td>Carbonate</td>
<td>17.7–27.1</td>
<td>2.3–64.5</td>
<td>Water-wet</td>
<td>DID</td>
<td>60</td>
<td>10.3</td>
<td>/</td>
<td>0.2</td>
<td>Displacement</td>
</tr>
<tr>
<td>Zhu Sinan et al. [69] (2021)</td>
<td>Sandstone</td>
<td>4.6/13.8/16.5</td>
<td>1.65/19.34/49</td>
<td>/</td>
<td>DIDIDIDIDIDIDID</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>Both</td>
</tr>
<tr>
<td>Alnuaimi et al. [57] (2021)</td>
<td>Carbonate</td>
<td>29</td>
<td>16.25</td>
<td>Mix-wet</td>
<td>IDIDIDIDIDIDIDID</td>
<td>133.3</td>
<td>12.7</td>
<td>/</td>
<td>/</td>
<td>Both</td>
</tr>
</tbody>
</table>
3.3. Summary of Experimental Research

To study the hysteresis phenomenon of relative permeability under different conditions, experts and scholars have conducted extensive experiments and numerical simulations. According to statistics in Table 2, it has been found that the hysteresis of relative permeability is influenced by various factors, such as rock wettability, permeability, and porosity.

(1) Permeability

Permeability, as one of the important properties of rocks, also has a significant impact on the hysteresis effect of relative permeability. Current research indicates that in reservoirs, low-permeability reservoirs compared to high-permeability reservoirs exhibit increased difficulty in gas-water alternating drive with increasing numbers of cyclic periods, leading to a stronger hysteresis effect of relative permeability \([13,65,70]\). A lower \(K_v/K_h\) ratio (vertical permeability to horizontal permeability ratio) results in earlier water breakthrough during the water injection phase, earlier gas breakthrough during the water injection period, lower ultimate recovery, greater reduction in recovery caused by the hysteresis effect of relative permeability, and stronger hysteresis effect \([12]\).

(2) Porosity

The larger the matrix porosity, the larger the geological reserves of natural gas, the more stable the reservoir pressure, the longer the stable production time, and the higher the cumulative gas production. However, the lower the ultimate recovery, the larger the matrix porosity, the greater the reduction in recovery caused by the hysteresis effect of relative permeability, and the stronger the hysteresis effect.

The impact of fracture porosity and matrix porosity on the ultimate recovery and hysteresis effect of relative permeability exhibits two opposite trends. The larger the fracture porosity, the higher the recovery, and the smaller the reduction in recovery caused by the hysteresis effect of relative permeability, resulting in a weaker hysteresis effect.

(3) Wettability

Compared to the water-wet system, the three-phase water relative permeability of the mixed-wet system exhibits less hysteresis during the gas injection cycle. In the water-wet system, the hysteresis effect of gas phase (non-wet phase) relative permeability is more pronounced than that of water phase (wet phase); compared to the drainage process, the wetting phase relative permeability increases during the imbibition process, while the non-wetting phase relative permeability during the imbibition process is lower than during the drainage process.

(4) Miscibility

Whether there is miscible flow does not affect the trend of relative permeability curves. Compared to miscible and near-miscible drives, a more significant decrease in gas relative permeability, greater decrease in recovery, and stronger hysteresis effect are observed in immiscible drives \([32]\).

(5) Cyclic times

With the increase in gas-water alternating cycle times, the water phase relative permeability continuously decreases as the saturation direction changes from imbibition to drainage, and continuously decreases with increasing cycle counts, resulting in a reduction in the hysteresis effect. Similarly, the gas phase relative permeability decreases continuously during drainage and imbibition cycles. In water and gas injection cycles, early-cycle gas phase relative permeability is higher, and late-cycle gas phase relative permeability is lower and decreases with increasing cycle count. The hysteresis phenomenon gradually decreases with cycle period and eventually stabilizes.
4. Applications

4.1. The Impact of Relative Permeability Hysteresis on Recovery

The application Section will discuss the practical applications of relative permeability hysteresis models in fields such as oilfield development and enhanced oil recovery. This may involve how these models are applied to simulate enhanced oil recovery processes, such as the significant and irreversible hysteresis phenomena of relative permeability observed in WAG (water-gas alternating injection) experiments, and how these observations can help improve the development strategies of oilfields and increase oil recovery rates.

Table 3 shows the CO₂ injection studies with consideration of relative permeability hysteresis. The hysteresis effect may be advantageous in reducing water coning and gas coning issues, increasing production in some cyclic operations (such as steam injection), and reducing gas mobility in certain processes (such as water alternating gas or simultaneous water and gas injection projects). During continuous displacement processes, hysteresis effects may significantly reduce the residual oil saturation in some heterogeneous carbonate reservoirs. Hysteresis effects can have either positive or negative impacts on production scenarios. In ultra-low permeability reservoirs, such as tight sandstones, tight carbonates, and shale formations, hysteresis effects in development strategies may be more crucial.

Positive impacts:

(1) Flow field control [71,72]

Hysteresis effects are often utilized for flow control to preferentially reduce the mobility of one phase compared to another, thereby achieving enhanced sweep efficiency and ultimate displacement efficiency, especially in cases with unfavorable viscosity ratios, reducing gas mobility in gas-water alternating processes. The interfacial effects between gas and liquid phases are employed to selectively retard the gas migration rate. Due to the lower viscosity of water, it tends to preferentially flow into higher permeability channels within the reservoir, selectively reducing the permeability of gas. Due to hysteresis and flow control effects, it becomes much more challenging for gas to displace water from this region, improving the overall sweep efficiency and conformance, particularly in thick oil zones, high-permeability zones.

(2) Reduction of water coning [71]

Hysteresis effects form the basis for techniques aimed at mitigating water coning in heavy oil reservoirs. In situations where the viscosity ratio between many heavy oils and underlying active water is highly unfavorable, rapid water coning occurs, resulting in high water cut and marginal or uneconomic production. In such cases, the presence of a flowing gas saturation seems to preferentially reduce the water-phase permeability rather than the oil-phase permeability.

(3) Enhancement of gas sequestration [73–75]

Injecting water after gas injection forces many connected gas segments to be disconnected. The hysteresis effect of water-wetting phases capturing non-wetting phases enhances the trapping and fixation of gas. In processes with strong flow reversals, significant hysteresis occurs, making gas-water alternation a potential effective CO₂ sequestration strategy.

(4) Gas blocking [13]

Gas-phase relative permeability may be preferentially affected by selective treatments of non-mixed fluids. Selective injection of water or surfactants into the upper part of high gas oil ratio wells may result in gas being hysteresis-trapped and lead to a temporary or permanent reduction in gas-oil ratio. Upon contact with gas saturation areas, the addition of surfactants can produce a high-viscosity, stable foam system, which will block most of the pores available for flowing gas, once again resulting in a preferential reduction in gas-phase permeability. Foam systems degrade over time due to adsorption (especially in clastic systems) and dispersion, and such treatments are often temporary to a certain extent.
Hysteresis effects can also be used to aid in water blocking. When using non-condensable gas for injection, water saturation can be preferentially reduced by selectively altering the relative permeability of water rather than oil, thus prioritizing the reduction in water saturation. A similar effect can be achieved by directly injecting immiscible, non-condensable gas into the water-saturated zone. This will establish a region of high-capillary gas saturation, significantly reducing the characteristics affecting water influx, depending on rock geometry and relative availability features. Establishing trapped gas saturation in oil-bearing formations is evidently undesirable and should be avoided, as this could severely damage oil-phase permeability.

Negative impacts:

1. Decrease in injection capacity [72, 76, 77]
   The hysteresis effect of phase permeability can lead to an increase in injection well pressure, especially during water injection after gas displacement. Once the pressure reaches a certain level, it may result in the inability to inject water and gas. In fact, instances of decreased injection capacity have been observed in real oilfield development scenarios, attributed to injection losses caused by the hysteresis effect. The potential loss of injection capacity can significantly impact the economic benefits of projects aimed at enhancing gas recovery.

   Several numerical simulation studies have found that models considering hysteresis in relative permeability provide predictions of recovery rates closer to actual conditions, with hysteresis in simulations leading to a decrease in recovery rates. Reasonably modeling hysteresis phenomena in the alternating gas-water process can significantly enhance the stability and accuracy of production outcomes. Failing to account for hysteresis effects may result in errors in predicting gas-water phase flow and distribution, leading to inaccurate recovery rate forecasts and irrational development scheme formulations.

   In many reservoir applications, phase trapping has been demonstrated in the literature as a significant factor for reducing productivity. The hysteresis effect of relative permeability has been found to cause trapping of the wetting phase by the non-wetting phase.

   Numerous experts have studied the impact of hysteresis on breakthrough times, with research indicating that considering hysteresis effects results in earlier breakthrough times for gas and water compared to conventional models that do not consider hysteresis effects. Failing to consider hysteresis effects may lead to erroneous estimates of the time for water breakthrough, preventing timely water shutoff measures and thereby hindering the maximization of economic benefits.

Table 3. The practical application of the Relative Permeability Hysteresis model.

<table>
<thead>
<tr>
<th>Author</th>
<th>Application</th>
<th>Two Phase or Three Phase</th>
<th>Injection Method</th>
<th>Gas</th>
<th>Phenomena and Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.R. Christensen et al. [78] (1998)</td>
<td>Reservoir</td>
<td>Two-phase</td>
<td>WAG</td>
<td>Natural gas</td>
<td>WAG injection has better accessibility and lower residual oil saturation than water injection.</td>
</tr>
<tr>
<td>D. J. Element et al. [53] (2003)</td>
<td>Core experiment</td>
<td>Three-phase</td>
<td>WAG</td>
<td>N2</td>
<td>The hysteresis cycle is irreversible. Reduce residual oil saturation by capturing gas with water.</td>
</tr>
<tr>
<td>Hamidreza Shahverdi et al. [79] (2011)</td>
<td>Core experiment</td>
<td>Three-phase</td>
<td>WAG</td>
<td>Mixture of methane (C1) and n-butane (n-C4)</td>
<td>The three-phase kr values of water and gas depend on two independent saturation levels. The kr values of three-phase water, oil, and gas are significantly lower than their corresponding two-phase values.</td>
</tr>
</tbody>
</table>
Table 3. Cont.

<table>
<thead>
<tr>
<th>Author</th>
<th>Application</th>
<th>Two Phase or Three Phase</th>
<th>Injection Method</th>
<th>Gas</th>
<th>Phenomena and Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Mobeen Fatemi et al. [80]</td>
<td>Core</td>
<td>Two-phase</td>
<td>WAG</td>
<td>Mixture of methane (C1) and n-butane (n-C₄)</td>
<td>As the WAG cycle increases, the hysteresis effect weakens. Compared to kᵣₑ, the cyclic hysteresis of kᵣᵣₑ is more pronounced.</td>
</tr>
<tr>
<td>Pedram Mahzari et al. [81]</td>
<td>Core</td>
<td>Three-phase</td>
<td>WAG</td>
<td>Mixture of methane (C1) and n-butane (n-C₄)</td>
<td>Adding WAG hysteresis parameters during fitting experiments instead of using the measured values of these parameters can significantly reduce simulation errors.</td>
</tr>
<tr>
<td>Ehsan Ranaee et al. [82]</td>
<td>Reservoir</td>
<td>Three-phase</td>
<td>WAG</td>
<td>Natural gas</td>
<td>The combined effect of relative permeability hysteresis and reservoir heterogeneity on WAG.</td>
</tr>
<tr>
<td>M. Kowsari et al. [83]</td>
<td>Reservoir</td>
<td>Three-phase</td>
<td>WAG</td>
<td>Methane (C1)</td>
<td>The optimal WAG parameters may be influenced by the choice of hysteresis models.</td>
</tr>
</tbody>
</table>

In the application results of these models, some scholars have conducted comparative validation work. The Killough model cannot accurately predict the hysteresis effect of relative permeability in gas/oil systems. The Land model used in the Killough hysteresis model is suitable for oil/water mixed-wet systems. A significant flaw of the Carlson model is that it does not assume the presence of hysteresis effects in the wetting phase, which can affect the model results. The Beattie model does not offer a trapping model. The advantage of the Spiteri model is that it is the proposed trapping model that is applicable to the diversity of wettability systems [84].

4.2. The Impact of Relative Permeability Hysteresis on CO₂ Sequestration

The effect of phase trapping is described by the gas trapping and reduction in relative permeability during three-phase flow processes, which is particularly pronounced during flow reversals and holds significant importance for CO₂ sequestration studies. Currently, there have been some studies on the impact of phase trapping on CO₂ sequestration, indicating that phase trapping is beneficial for CO₂ sequestration. In saline aquifers, which are the primary targets for CO₂ sequestration, the reservoirs are typically water-wet. During water flooding, the gas saturation decreases, and the non-wetting gas phase is trapped by the wetting phase. As the wetting phase invades the pores, the non-wetting gas phase also increases. During CO₂ injection into the reservoir, the gas saturation increases, and due to gravitational differentiation, the gas phase moves laterally from the injection well to the top of the aquifer, forming vertical pathways. Once injection ceases, CO₂ continues to migrate upwards. At the front edge of the CO₂ slug, gas continues to displace the wetting phase during drainage, while at the rear edge, water displaces gas during imbibition, leading to gas phase fragmentation and subsequent trapping. As the slug migrates upwards, some CO₂ is trapped as residual immobile phase in pore spaces, effectively isolating CO₂ in the rock until the immobile gas dissolves over geological time [11,73,74,76,77,85–87].

In recent years, numerical simulation techniques have been widely used, and many attempts have been made in simulating phase trapping. Table 4 shows the CO₂ sequestration with hysteresis consideration and various conclusions have been made:

(1) The choice of three-phase relative permeability models and phase trapping models significantly affects the prediction of CO₂ sequestration capacity. The impact of three-phase relative permeability models is less important than that of phase trapping models, and the choice of phase trapping models may affect the impact of three-phase relative permeability models.

(2) Injecting water after gas injection disrupts connected gas clusters, as the wetting phase traps the non-wetting phase due to phase trapping, enhancing gas trapping and immobilization. In processes with strong flow reversals, the effect of phase trapping is significant, making gas-water alternation a potentially effective CO₂ sequestration strategy.
Table 4. Relative permeability hysteresis applications with CO₂.

<table>
<thead>
<tr>
<th>Author</th>
<th>Time</th>
<th>Model</th>
<th>Research Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.R. Christensen et al. [89]</td>
<td>2000</td>
<td>Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>M. Delshad et al. [26]</td>
<td>2003</td>
<td>UTCHEM</td>
<td>√</td>
</tr>
<tr>
<td>Matthew Flett et al. [14]</td>
<td>2004</td>
<td>Land</td>
<td></td>
</tr>
<tr>
<td>Elizabeth J et al. [90]</td>
<td>2006</td>
<td>Carlson, Kilough, Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Juanc R et al. [91]</td>
<td>2008</td>
<td>Kilough</td>
<td>√</td>
</tr>
<tr>
<td>Liu Zixiong et al. [92]</td>
<td>2008</td>
<td>FJ Fayers</td>
<td>√</td>
</tr>
<tr>
<td>Hustad et al. [32]</td>
<td>2010</td>
<td>Hustad</td>
<td>√</td>
</tr>
<tr>
<td>Jiang Tao et al. [93]</td>
<td>2009</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Altundas et al. [75]</td>
<td>2011</td>
<td>Land</td>
<td>√</td>
</tr>
<tr>
<td>Hooman Karkooti et al. [94]</td>
<td>2011</td>
<td>Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Hoseini et al. [15]</td>
<td>2011</td>
<td>Carlson, Kilough, Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Shahrokhi et al. [95]</td>
<td>2014</td>
<td>Carlson, Killough and Jargon</td>
<td>√</td>
</tr>
<tr>
<td>Duchenne et al. [68]</td>
<td>2015</td>
<td>Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Wang et al. [59]</td>
<td>2016</td>
<td>Land</td>
<td>√</td>
</tr>
<tr>
<td>Fatemeh et al. [96]</td>
<td>2017</td>
<td>Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Wei Jia et al. [74]</td>
<td>2018</td>
<td>Larsen and Skauge, Carlson, Land</td>
<td>√</td>
</tr>
<tr>
<td>Huang Jianing et al. [97]</td>
<td>2018</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Zayer et al. [98]</td>
<td>2020</td>
<td>Larsen and Skauge</td>
<td>√</td>
</tr>
<tr>
<td>Rezaei, A. et al. [99]</td>
<td>2021</td>
<td>Kilough</td>
<td>√</td>
</tr>
<tr>
<td>Zhu Sinan et al. [56]</td>
<td>2021</td>
<td>Carlson, Kilough</td>
<td>√</td>
</tr>
<tr>
<td>Alhuaimi et al. [57]</td>
<td>2021</td>
<td>Aghabozorgi</td>
<td>√</td>
</tr>
<tr>
<td>Kowsari et al. [83]</td>
<td>2022</td>
<td>Carlson, Kilough</td>
<td>√</td>
</tr>
<tr>
<td>Ammar Al Ali et al. [100]</td>
<td>2022</td>
<td>Patterson</td>
<td>√</td>
</tr>
</tbody>
</table>

Overall, phase trapping research holds significance for CO₂ sequestration, but studies on CO₂ sequestration and phase trapping are currently limited, indicating a future research focus.

4.3. Effect of Relative Permeability Hysteresis on Hydrogen Storage

Unlike the injection process of carbon capture, utilization, and sequestration (CCUS), hydrogen storage is a cyclic process of injection and production. The cyclical nature of underground hydrogen storage processes makes them relevant to the concept of relative permeability hysteresis. Underground hydrogen storage refers to storing energy in the form of hydrogen in underground reservoirs, which can be salt caverns, aquifers, depleted oil and gas reservoirs, etc. Hydrogen is stored during periods when energy production exceeds demand, and when demand exceeds production, stored hydrogen will be recovered to the surface.

Within the range of hydrogen storage conditions, the relative permeability of the hydrogen-water system is not significantly affected by changes in temperature and pressure, as the viscosity of hydrogen changes little. Rezaei et al. [99] first used a non-steady-state method to obtain relative permeability curves of hydrogen-water systems under various temperatures, pressures, salinities, and rock types. The results showed that due to capillary...
forces overcoming viscous forces, higher pressures exhibited higher relative permeability of hydrogen.

Relative permeability measurements of hydrogen-water system in Berea sandstone, conducted by Lysyy et al. [101], showed a significant effect of hysteresis on the curves of water and hydrogen relative permeability. Hashemi et al.’s [102] PNM results indicate that due to the disconnect of hydrogen phase after residual water and secondary imbibition in porous media, the relative permeability values of secondary drainage are lower than primary drainage. However, hysteresis effects remain unchanged with more cycle occurrences.

From Table 5, the relative permeability and hysteresis effects in hydrogen-water systems differ from those in other gas and oil-water-gas systems. According to experimental measurements by Zhenkai Bo [103], a significant difference is observed in the hysteresis level during the wetting phase, typically showing smaller hysteresis effects in the wetting phase of other systems, but in hydrogen-brine systems, the hysteresis effect of liquid phase relative permeability is more pronounced.

**Table 5.** Relative permeability hysteresis on hydrogen storage application statistics.

<table>
<thead>
<tr>
<th>Author</th>
<th>Time</th>
<th>Software</th>
<th>Hydrogen Storage Site</th>
<th>Model</th>
<th>Whether Hysteresis Exists</th>
<th>Main Finding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pfeiffer et al. [104]</td>
<td>2015</td>
<td>Eclipse</td>
<td>Saline layer</td>
<td>Corey</td>
<td>No</td>
<td>Porous media hydrogen storage must address issues such as the resistance of caprock rocks to hydrogen permeation and gas diffusion.</td>
</tr>
<tr>
<td>Hagemann et al. [105]</td>
<td>2015</td>
<td>DuMux</td>
<td>aquifers</td>
<td>Brooks–Corey</td>
<td>No</td>
<td>For low injection rates, gravitational forces dominate hydrogen movement, while for high injection rates, viscous forces become dominant, and displacement becomes unstable.</td>
</tr>
<tr>
<td>Feldmann F. et al. [106]</td>
<td>2016</td>
<td>DuMux, COMSOL</td>
<td>Depleted gas reservoir</td>
<td>Brooks–Corey</td>
<td>No</td>
<td>The gravitational override and viscous fingering in the aquifer structure complicate the effective displacement of natural fluids.</td>
</tr>
<tr>
<td>Alvaro, A.S. et al. [107]</td>
<td>2017</td>
<td>COMSOL</td>
<td>saline layer</td>
<td>Brooks–Corey</td>
<td>No</td>
<td>If there are several shallow oil wells beneath the caprock, a large amount of H2 can be recovered. Rising appears to be the main limitation for brine aquifer storage in the absence of other buffering gases.</td>
</tr>
<tr>
<td>Katarzyna Luboset al [108]</td>
<td>2020</td>
<td>petraSim-TOUGH2</td>
<td>aquifers</td>
<td>Van Genuchten-Mualem</td>
<td>No</td>
<td>Maximum hydrogen saturation occurs around the injection well, and hydrogen diffuses along the top surface of the reservoir in contact with the caprock, likely due to the lack of viscous fingering and the low viscosity and density of hydrogen, which may impact the geometry of underground storage.</td>
</tr>
<tr>
<td>Lysyy et al. [101]</td>
<td>2021</td>
<td>Eclipse</td>
<td>Depleted field</td>
<td>Field data</td>
<td>No</td>
<td>When injecting pure hydrogen only, most injected hydrogen (&gt;84%) remains underground as a buffering gas. Unrecovered hydrogen accumulates in the uppermost layer. Injecting formation gases as buffering gases leads to higher hydrogen recovery rates.</td>
</tr>
<tr>
<td>D.S. Mahdi, et al. [109]</td>
<td>2021</td>
<td>TOUGH2</td>
<td>Water layer</td>
<td>van Genuchten-Mualem</td>
<td>Gas-liquid hysteresis</td>
<td>Caprock and injection rates have significant impacts on hydrogen leakage, as well as on the amount of hydrogen captured and recovered. Without caprock, higher injection rates increase H2 leakage.</td>
</tr>
</tbody>
</table>
Table 5. Cont.

<table>
<thead>
<tr>
<th>Author</th>
<th>Time</th>
<th>Software</th>
<th>Hydrogen Storage Site</th>
<th>Model</th>
<th>Whether Hysteresis Exists</th>
<th>Main Finding</th>
</tr>
</thead>
<tbody>
<tr>
<td>M. Kanaani et al. [110]</td>
<td>2022</td>
<td>CMG-GEM</td>
<td>Depletion of condensate reservoirs</td>
<td>Field data</td>
<td>No</td>
<td>Methane performs better as a buffering gas compared to nitrogen and carbon dioxide. Buffer gas injection reduces the negative effects of gravitational segregation.</td>
</tr>
<tr>
<td>Okoroaf et al. [111]</td>
<td>2022</td>
<td>Eclipse</td>
<td>Depleted gas reservoir</td>
<td>Experimental data</td>
<td>No</td>
<td>The main factors affecting hydrogen recovery are reservoir depth, dip angle, current pressure, and flow rate.</td>
</tr>
<tr>
<td>M. Delshad, et al. [112]</td>
<td>2022</td>
<td>CMG-GEM</td>
<td>Depleted reservoirs and aquifers</td>
<td>Experimental data</td>
<td>No</td>
<td>Lateral diffusion of hydrogen is greater in aquifers and natural gas scenarios, and in saline aquifers, the presence of reliable caprock/seal layers needs to be evaluated.</td>
</tr>
<tr>
<td>Wang et al. [113]</td>
<td>2022</td>
<td>CMG-GEM</td>
<td>Depleted field</td>
<td>Experimental data</td>
<td>Gas-hysteresis</td>
<td>Capillary pressure is not a key factor in H2 recovery performance, and the H2 recovery rate is not strongly dependent on gas sweep efficiency.</td>
</tr>
<tr>
<td>B. Pan et al. [114]</td>
<td>2023</td>
<td>CMG-IMEX</td>
<td>aquifers</td>
<td>Displacement and imbibition data</td>
<td>Yes</td>
<td>H2 recovery rate is reduced by hydrogen-brine relative permeability hysteresis, and higher H2 recovery rates result in greater hydrogen recovery, but also increased water production.</td>
</tr>
</tbody>
</table>

5. Conclusions

Based on the progress in mathematical modeling, experimentation, and application research of relative permeability hysteresis, researchers have gained some understandings of relative permeability hysteresis effects. Relative permeability hysteresis is a phenomenon that physically exists and is influenced by various factors such as rock properties and development parameters. It has a significant impact on production in reservoir development. A deeper understanding of relative permeability hysteresis could provide valuable guidance for on-site development.

Fluid flow under actual reservoir conditions of temperature and pressure differs greatly from room temperature conditions. Currently, there is limited research on relative permeability hysteresis under high temperature and pressure conditions. Future research on relative permeability hysteresis under high temperature and pressure should be a target, providing theoretical support for practical reservoir development. Furthermore, CCUS have become a recent hotspot. Considering relative permeability hysteresis in CCUS research is of great significance. Gas storage’s cyclical injection and production natural involves a lot of hysteresis phenomena. Experimental results and specific models applicable to large numbers repeated injection and production remain to be studied.

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