Article

Improvements in the Wavelet Transform and Its Variations: Concepts and Applications in Diagnosing Gearbox in Non-Stationary Conditions

Trong-Du Nguyen * and Phong-Dien Nguyen

Fault Diagnostics Lab, School of Mechanical Engineering, Hanoi University of Science and Technology, Hanoi 10000, Vietnam; dien.nguyenthong@hust.edu.vn
* Correspondence: du.nguyentrong@hust.edu.vn

Abstract: Wavelet transform is a powerful time-frequency-based analysis method often used in gear fault diagnostics. The development of wavelet transform is closely linked to the improvement of resolution. When the high-frequency resolution allows for easy observation of different frequency components, it is a symptom of damage to an individual part of the machine. This study effectively applied the Wavelet analysis technique to diagnose faulty gearboxes operated in non-stationary conditions. This is a complex problem that usual diagnostic approaches need help to solve due to its non-linear character. This work conducted a simulation and real-world testing to show that the newest wavelet analysis techniques work well, showing that they can accurately find gear faults in gearboxes in non-stationary conditions. A thorough overview of the cutting-edge applications of wavelet transform in diagnosing faults in industrial gearbox systems was also given. This work also explained in detail the mathematical ideas behind the continuous wavelet transform, discrete wavelet transforms, and wavelet packet transform.

Keywords: wavelet; gear; vibration analysis; fault diagnosis; wavelet-based synchrosqueezing

1. Introduction

With their vital role in transferring and changing force and motion from the engine to the working parts, gearboxes are indispensable components of mechanical drive systems [1–4]. A faulty gearbox component might potentially halt the functioning of the entire production line, resulting in significant financial losses [5–10]. The mechanism’s vibrations reflect the technical condition of the gearbox, and any faults caused by the operation can be detected early and accurately using signal analysis tools [11]. Due to the above reasons, the gear clusters and shaft bearings in the gearbox are the most extensively studied objects in technical diagnostics using vibration analysis, with many publications in international scientific journals. The methods for analyzing oscillatory signals for diagnostic purposes can be classified into three groups.

The first group of analysis methods belongs to conventional methods in the frequency domain, which are verified and widely applied in the industry. Endo H [12] applies the least entropy deconvolution method to improve the capability of the current autoregressive model-based filtering methodology in identifying localized faults in gears. Antoni [13] described the program as a robust analysis tool for detecting transients buried in intense background noise in non-stationary analysis. Combet and Gelman [14] utilized the methodology of time synchronous averaging to identify and analyze specific faults occurring within gears. Barszcz and Randall [15] describe using the spectral kurtosis method to locate a tooth break in a wind turbine’s planetary gear. Loutas et al. [16] compared the performance of mean frequency and standard deviation utilizing different non-destructive inspection methodologies and processing the recorded waveforms with
modern signal processing techniques to monitor gear condition. Liu et al. [17] presented a novel approach for adaptive spectral kurtosis filtering utilizing the Morlet wavelet.

The second method, recently published in scientific journals, has higher diagnostic accuracy and is now only applied in a limited scope. Al-Bedoor et al. [18] evaluate the possibility of quantitatively determining the vibration state of the rotating blades by analyzing the measurement of shaft torsional vibration. Zhou C. [19] employs the parameter-optimized variational mode decomposition technique to recover the instantaneous frequency characteristics of the torsional vibration signals. Zhang et al. [20] enhanced full ensemble empirical mode decomposition using adaptive noise, wavelet transform, and robust independent component analysis approaches using the denoised torsional vibration data.

The third group consists of newly proposed analysis methods, primarily based on time-frequency analysis, with wavelet analysis being a typical example [21–25]. The time-frequency analysis provides an overview of both the time and frequency domains, revealing hidden information due to overlap in each domain. The wavelet transform has been extensively researched and utilized in various disciplines, focusing on time-frequency analysis. Stéphane Mallat [21] presents a fundamental and practical explanation of the wavelet transform, highlighting its potential to examine non-stationary signals using multiresolution analysis. This approach offers substantial benefits compared to typical Fourier transforms. Building on this theoretical foundation, Gao [22] explores the applications of wavelet analysis in the manufacturing industry. This research highlights how wavelet analysis can be used for fault detection and condition monitoring, demonstrating its practical advantages in an industrial environment. Yan [23] presents the specific use of wavelet analysis for diagnosing faults in rotary machinery. The authors review various case studies and applications, demonstrating the effectiveness of wavelet-based methods in identifying and diagnosing mechanical faults. Ngui et al. [24] focus on the critical aspect of choosing the appropriate mother wavelet for particular applications. An in-depth tutorial on choosing wavelets based on signal properties was given, resulting in optimal analysis and accurate results. Seshadrinath [25] investigates using complex wavelets to diagnose multiple faults in variable frequency drives. This paper emphasizes the advanced capabilities of complex wavelets in detecting detailed vibration indicators, leading to more precise fault diagnosis.

Recent advancements in machine learning and artificial intelligence have significantly impacted the field of gearbox fault detection. Habbouche et al. [26] proposed a multisensor data-fusion approach, combining linear predictive coefficients and mel-frequency cepstral coefficients with convolutional neural networks and long short-term memory networks for effective fault diagnosis, emphasizing the importance of addressing transient conditions in gearboxes. Additionally, Saucedo-Dorantes et al. [27] developed a methodology involving statistical time feature reduction for diagnosing gearbox wear, highlighting the challenges of detecting faults during transient operations. Furthermore, Azamfar et al. [28] investigated multisensor data fusion for gearbox fault diagnosis using a 2-D convolutional neural network and motor current signature analysis. This study demonstrated significant improvements in detecting transient fault conditions and the effectiveness of automatic feature extraction from motor current signals.

Wavelet analysis is a potentially powerful diagnostic technique due to its ability to flexibly represent signals in the time-frequency domain with high resolution. However, its verification and application are limited to experimental models or simulated vibration signals.

This work conducts a numerical analysis and experiments to validate the effectiveness of the recently updated wavelet analysis as a gear diagnostic tool. Overall, the article has specific contributions and novelty:

- A thorough overview of the analysis of Wavelet and its variations: Update the publication on Wavelet analysis, focusing on the development of improving time-
frequency resolution. Particularly, the application of Wavelet analysis in fault diagnostics in industrial gearboxes is noteworthy.

- Construct a multi-component signal’s mathematical model to identify gear faults’ symptoms in a gearbox operating in non-stationary conditions: Propose a solution to identify fault symptoms via the time-frequency representation by integrating multiple wavelet analysis methods.
- Construct a proposal method combining the latest wavelet transforms and other methods.
- The proposed method provides the generation of input data sets for the neural network to autonomously classify faults, matching with the developments of the 4.0 industrial era.

The remainder of the paper is structured as follows. Section 2 provides an overview of the development of wavelet analysis from discrete wavelet transforms to wavelet-based synchrosqueezing transforms and generalized wavelet-based synchrosqueezing transforms. Section 3 provides a review of works on the use of wavelet analysis to diagnose gearbox faults. The methodology is presented in Section 4, which provides the mathematical foundation of the proposed method. Section 5 describes the experimental test conducted to validate the effectiveness of the proposed method. Finally, the conclusion is presented in Section 6.

2. An Overview of Wavelet Analysis

When interacting with a multi-component signal with a frequency that changes over time, it is necessary to represent that signal in the time-frequency domain [29]. The main advantage of time-frequency analysis compared to other methods is its ability to represent a vibration signal simultaneously in the time and frequency domains (called the time-frequency domain), providing crucial information about the frequency structure of the signal and allowing the observation of the temporal variations of the frequency [30]. The signal is transformed by an integral transform similar to the Fourier transform, known as a time-frequency represented (TFR). Time-frequency transformations are classified into linear and nonlinear. However, time-frequency non-linear transformations have drawbacks regarding algorithmic complexity, resulting in slow computation due to many operations and difficulties in interpreting the results. As a result, they have rarely been used to analyze mechanical vibration.

The wavelet transform is categorized as a linear transformation. It is divided into two groups, continuous wavelet transform (CWT) and discrete wavelet transform (DWT), based on the computation method. Recent advancements in CWT and DWT have improved signal resolution to obtain wavelet packet transform (WPT), second-generation wavelet transform, and wavelet-based synchrosqueezing transform (WSST). The methods of signal analysis based on CWT, DWT, or WPT are known as wavelet analysis. The CWT is performed using a base function known as the basic wavelet function. CWT of the signal \( x(t) \) is defined as [31]:

\[
\text{CWT}_x(\tau, s) = \sqrt{s} \int_{-\infty}^{+\infty} X(f) \Psi^*(sf) e^{i2\pi ft} df
\]

where \( \Psi(f) \) is the Fourier transform of \( \psi_{\tau,s}(t) \), \( \Psi^*(f) \) is the complex conjugate function of \( \psi(f) \). Wavelet transform alters signals function \( x(t) \) to become a bivariate function \( \text{CWT}_x(\tau, s) \). Parameter \( s \) performs to stretch \( \psi(t) \) function, and the parameter \( \tau \) performs to shift \( \psi(t) \) function on the time axis. The wavelet transform has mapped the \( x(t) \) signal function from a one-variate function to a bivariate function \( s \) and \( \tau \). By changing the scale coefficient \( s \) and shifting the base wavelet function along the time axis of a quantity \( \tau \), the wavelet coefficients \( |\text{CWT}_x(\tau, s)| \) may generate a three-dimensional graph representing the change of amplitude simultaneously along the time (matching \( \tau \)) and frequency axis (ratio with factor \( s \)). Several base wavelet functions \( \psi(t) \) are widely used in mechanical
vibration analysis, such as Morlet-wavelet, Haar-wavelet, and Gabor-wavelet. In 2010, Daubechies [32] introduced an improved algorithm called the wavelet-based synchrosqueezing transform.

WSST generates a higher-resolution TFR of the signal compared to CWT. The theoretical basis of the WSST transformation is presented in detail in [33]. Subsequently, the generalized wavelet-based synchrosqueezing transform (GST) is a method that combines the generalized Fourier transform and the WSST transform. The main benefit of GST is its ability to identify the frequency of a signal changes over time within a narrow band, making it a promising tool for analyzing gear meshing vibrations in gearboxes with very close frequency bands [34,35].

This section reviewed the development of wavelet analysis from traditional methods like DWT and CWT to advanced methods such as WPT and WSST. These advancements have significantly improved signal resolution, making wavelet analysis a powerful tool for diagnosing faults in complex, non-stationary conditions. The continuous evolution of these techniques highlights their increasing importance in fault diagnostics.

3. Wavelet Analysis for Gearbox Diagnostics

As previously stated, the wavelet coefficients contain information about vibration amplitude in the time-frequency domain. Many studies use wavelet coefficients derived from CWT, DWT, or WPT computations, which are then integrated with other mathematical and statistical analysis approaches to provide signal analysis tools for gearbox diagnostics in non-stationary conditions. Zuo [36] and colleagues used the independent component analysis (ICA) approach using wavelet coefficients to monitor and diagnose tooth faults in gears using a single vibration sensor. To obtain characteristic values for gearbox faults, Rafiee and Tse [37] approximated the wavelet coefficients as simple sine functions to estimate the autocorrelation function.

The method of frequency averaging is applied in the time-frequency domain to detect the presence and progression of tooth surface pitting, as indicated in [38]. In [39], wavelet coefficients are utilized to quantitatively assess the severity of tooth fault under varying load conditions. Researchers used the CWT to extract various statistical parameters, such as standard deviation and kurtosis coefficient, from the wavelet coefficients of vibration data. This analysis employed multiple base wavelet functions to detect faults in gears and rolling bearings [40–42]. Li and colleagues [43] used the Hermitian basis wavelet function to perform CWT in order to detect common local faults in gearboxes, such as teeth cracks and chipped gear peaks. Furthermore, the amplitude and phase values of CWT using the Hermitian wavelet function are also employed simultaneously to identify faults in the rolling bearings in the gearbox [44–46]. By using the low-pass filtering characteristics of the Haar wavelet function, Li et al. [43] used the CWT to identify the low-frequency regions, which are symptoms of gears and rolling bearings faults. Combet et al. [47–50] presented a technique for assessing the correlation among wavelet coefficients of measured signals to detect several types of localized faults in gearboxes. Ohue [51] and colleagues analyzed gearbox vibration using DWT and CWT to investigate the correlation between the wavelet coefficient and the distribution of faults on the gear. Ayad et al., applied CWT and DWT with optimized parameters combined with autocorrelation functions to diagnose faults in industrial gearboxes [52].

Researchers also employed wavelet analysis as an efficient preprocessing method to enhance the quality of diagnostic information in signals and reduce noise. Al-Raheem and Ayad M. et al., diagnosed the rolling bearing fault in the gearbox using the noise-reduced signal’s autocorrelation function, which was processed with CWT [53,54]. Jafarizadeh and colleagues have combined the noise removal method through time-domain averaging with the Morlet wavelet “filter” to diagnose tooth faults in gears from measured raw signals [55]. Li and colleagues [56] have developed a method for simultaneously detecting multiple faults on gears using DWT combined with Principal Component Analysis (PCA). In this method, DWT is employed as a signal-preprocessing tool. Chen [57], Wang [58],
and Qiang Z [59] proposed highly effective denoising solutions based on CWT and DWT to detect weak periodic pulse signals caused by tooth meshing impact.

The wavelet analysis outputs from the recorded vibration signal during gearbox operation can also be used as input for intelligent monitoring and diagnostic systems, including genetic algorithms, neural networks, or support vector machine classifiers (SVM). Notably, a signal analysis system based on CWT with Morlet wavelet and genetic algorithm is proposed to automatically detect rolling bearing and gear faults from vibration signals [60–62]. Kankar and colleagues [63] derived statistical indicators using the wavelet coefficient to automatically detect and classify faults in rolling bearings using neural networks. Using genetic algorithms, Rafiee et al. [37,40,64,65] improved WPT’s decomposition, selected the optimal Daubechies wavelet and calculated the number of hidden neurons in the ANN network to accurately detect slight wear, medium wear, and broken gear teeth. Gao [66] and their research group have developed an automatic system for detecting tooth cracks and broken teeth in gearboxes, utilizing DWT in conjunction with SVM. Rajeswari [67] proposed a method that combines DWT, genetic algorithms, and neural networks to detect tooth decay and cracks in the teeth automatically. Du and Dien used artificial neural networks and wavelet packet transforms to identify gear faults [68]. Jedlinski and colleagues [69] combined feedforward neural networks and SVM using CWT to detect the local faults of gears in gearboxes early.

Another research direction is effectively applying wavelet analysis in gearbox diagnosis, enhancing the quality of time-frequency distributions through resolution and representation. Halim et al. [70] and Bravo-Imaz et al. [71] employed time synchronous averaging (TSA) and CWT to detect faults in gear by analyzing characteristic components in the time-frequency distribution. Zhu [72] has represented the wavelet coefficients in polar coordinates to visualize better the periodic vibration signals for detecting a local fault of gears caused by meshing impact and fault elements in rolling bearings during operation. Meltzer and [73] also utilized this type of graph to enhance the ability to detect tooth damage in cases when the gearbox operates at variable rotational speeds. Li and Liang [34,74] were the first authors to exploit the benefits of GST to create high-resolution time-frequency distributions within a narrow frequency range for identifying gear meshing frequency (GMF) and characteristic modulation sidebands related to tooth wear. Several studies have recently proposed complex analysis methods for the third group of methods. These include Yu J [75], who diagnosed the fault in gear shafts, and Yang [76], who diagnosed gearbox faults in wind turbines. Some other researchers [77–80] proposed new analysis methods for the early detection of gear faults in multi-stage gearboxes based on wavelet analysis.

Alongside research on applying wavelet analysis in technical diagnostics, researchers continue to propose new forms of wavelet basis functions that are more suitable. Jiang and Liu [81] used the t-test to validate a relationship between the feature values derived from the DWT coefficients and the vibration signals. Subsequently, the estimated probability obtained from the t-test was employed to make choices on wavelet functions. This encouraged research focused on developing novel wavelet functions for engineering applications. Li and colleagues [82] have introduced an innovative approach for calculating the DWT, where a wavelet function was immediately generated based on the statistical characteristics of the analyzed signal. A new family of base wavelets was proposed in [83] for detecting small changes in the vibration signal of a gear. A set of basic wavelet functions combined with multiple wavelet functions (multi-wavelet) has been applied to analyze oscillating signals to detect gearbox faults [84–89].

This section examined the application of wavelet analysis for diagnosing gearbox faults, highlighting how wavelet coefficients from CWT, DWT, or WPT are used alongside mathematical and statistical methods. These approaches have proven effective in detecting and diagnosing faults under non-stationary conditions, showcasing the potential of wavelet analysis in enhancing diagnostic accuracy and reliability in industrial applications. Despite their effectiveness, these methods often struggle with noise and closely
spaced frequency components. This underscores the need for new methods, providing better resolution and fault detection capabilities.

4. Methodology

Based on the above analysis and evaluation, the main diagnostic symptom of gearbox fault is the detection of sidebands that appear around a gear meshing frequency. Only the meshing frequency will be present when a gearbox is operated in good condition. On the contrary, a gearbox with any fault will result in sidebands associated with the gear meshing frequency. As the intensity of sidebands increases, the level of fault also increases.

Assume that the faulty gearbox operated stationary, causing sidebands to be observable in the frequency spectrum. Conversely, with non-stationary conditions, the spectrum’s sidebands become blurred. Identification of the time-varying frequency changes requires representation of the signal in the time-frequency domain. This section will outline the mathematical model of a faulty gearbox operated in non-stationary situations via simulation and experimentation. Based on the proposed approach, a suitable method is suggested to identify fault symptoms quickly.

A measured signal from faulty gearboxes is a multi-component signal formed by modulation of amplitude and frequency. Assume that the signal $x(t)$ is a multi-component of a faulty gearbox representation in the following [90]

$$x(t) = A(t) \cos(\varphi(t))$$  \hspace{1cm} (2)

where the instantaneous amplitude of the signal:

$$A(t) = A(1 + B \cos(2\pi f_s(t)t))$$  \hspace{1cm} (3)

and instantaneous phase of a signal:

$$\varphi(t) = 2\pi f_s(t)t + \phi(t)$$  \hspace{1cm} (4)

Regarding this situation, $Z$ is the number of the gear teeth, $\phi(t)$ presents a frequency modulation function; $f_s(t)$ is shaft rotating frequency.

Based on the characteristics of the signal, the instantaneous frequency can be determined as follows:

$$f(t) = \frac{d\varphi(t)}{2\pi dt}$$  \hspace{1cm} (5)

By putting Equations (3) and (4) into Equation (2), it is obtained:

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$  \hspace{1cm} (6)

in which the component signals are computed in the following manner:

$$\begin{cases} 
x_1(t) = A \cos(2\pi Z f_s + \phi(t)) \\
x_2(t) = \frac{1}{2} AB \cos(2\pi (Z + 1) f_s + \phi(t)) \\
x_3(t) = \frac{1}{2} AB \cos(2\pi (Z - 1) f_s + \phi(t)) 
\end{cases}$$  \hspace{1cm} (7)

By utilizing Equation (5), the instantaneous frequencies of the component signals can be calculated, resulting in the following outcomes:

$$\begin{cases} 
f_1(t) = Z f_s + \phi'(t) \\
f_2(t) = (Z + 1) f_s + \phi(t) \\
f_3(t) = (Z - 1) f_s + \phi(t)
\end{cases}$$  \hspace{1cm} (8)

As was said above, it is easy to extract the sidebands when the frequencies do not overlap. However, in cases of time-varying frequency, there will be a phenomenon of frequency overlap that is difficult to extract. In order to overcome the limitation, this study
utilizes the Generalized Fourier Transform (GFT) developed by S. Olhede [91]. The GFT is utilized to transform the varying frequency into a constant frequency, hence preventing any overlap. The GFT transformation is performed by multiplying a mapping function $e^{-i2\pi x_0(t)}$ with an analytical signal $\tilde{x}(t)$ of the original signal $x(t)$. At that time, the time-varying frequency is mapped to a constant frequency. Assume a multi-component signal $x(t)$, the GFT transform is defined as:

$$X_g(f) = \int_{-\infty}^{+\infty} \tilde{x}(t)e^{-i2\pi ft + x_0(t)} dt$$

By employing transformations as described in the document [92], it is possible to extract the constant frequency component $f_0$ from the instantaneous frequency according to the following formula:

$$f(t) = \frac{dx_0(t)}{dt} + f_0$$

The above equation proves that the signal $x(t)$ with time-varying frequency can be straightened out using the mapping function with phase $x_0(t)$ determined by (10). After straightening the frequency, the phenomenon of frequency overlap will no longer exist. Thus, this work proceeds to bandpass filter around the straightened frequency. Afterward, the original frequency is restored by applying the inverse GFT transformation according to Equation (9). Based on this idea, the proposed method is employed as presented in the summary chart in Figure 1. Hence, the sidebands next to the meshing frequency can be efficiently explored by employing the Generalized Fourier Transform in conjunction with the Wavelet Synchrosqueezed Transform. This is a fault symptom of a faulty gearbox.

In summary, the proposed method, which can be divided into three parts, is implemented step by step as follows:

(a) Execute CWT:
- Step 1 (CWT): Apply FFT to the mother wavelet function to obtain the base wavelet in the frequency domain. Then, the time-frequency representations from the raw signal and the base wavelet function in the frequency domain are calculated.
- Step 2 (Estimate frequency): Apply derivative to the TFR to obtain the instantaneous rotation frequency.

(b) Execute WSST:
- Step 2 (Estimate frequency): Apply derivative to the TFR to obtain the instantaneous rotation frequency.
- Step 3 (Execute WSST): with the instantaneous rotation frequency, it is possible to squeeze the CWT’s TFR. As a result, the resolution is much better.

(c) Execute the proposed method:
- Step 4 (analytic signal): select const frequency $f_0$ to calculate phase $x_0(t)$. Then, the Hilbert transform is applied to obtain an analytic signal.
- Step 5 (execute GFT): apply GFT by multiplying the analytic signal with a reference phase $e^{-i2\pi x_0(t)}$. Apply the Hilbert transform again to obtain a new analytic signal.
- Step 6 (execute CWT): convert the new signal to TFR.
- Step 7 (execute iGFT): apply inverse GFT to reconstruct the signal with separated frequency component $f_0$.
- Step 8 (execute WSST): apply WSST to obtain the high-resolution GMF and its sidebands. Then, conclude the gear condition based on the obtained.
Figure 1. Flowchart of the proposed method.

In order to clarify the method’s concept, a numerical simulation example is demonstrated. This example involves creating a multi-frequency signal and exploring techniques to filter and extract various frequency components, even when they overlap. Figure 2 displays the outcomes of examining a signal with three distinct frequency components that fluctuate over time within a limited frequency region.

\[
\begin{align*}
  f_1(t) &= 10(-25.8t^2 + 40t + 24) - 0.24 \sin(0.6\pi t) \\
  f_2(t) &= 11(-25.8t^2 + 40t + 24) - 0.24 \sin(0.6\pi t) \\
  f_3(t) &= 9(-25.8t^2 + 40t + 24) - 0.24 \sin(0.6\pi t)
\end{align*}
\]

This multi-component signal is represented by the CWT transformation, as shown in Figure 2 on the left and by the WSST transformation, as shown in Figure 2 on the right.
Figure 2. Comparing the TFR of the signal using CWT, WSST.

It is evident that the CWT transformation yields a high time resolution but needs to clearly distinguish the three frequency components $f_1$, $f_2$, and $f_3$. With the WSST transformation, having frequency concentration around the central frequency is better, as shown in Figure 3. However, determining the boundary frequencies around the central frequency is challenging.

Figure 3. TFR of the signal using combined GFT and WSST.

The results of the simulation verification demonstrate that applying the updated Wavelet analysis in the case of multi-component signals yields better results than traditional Wavelet analysis, especially for signals with time-varying frequencies.

5. Experimental Test

In order to validate the practical use of the efficient Wavelet analysis method, a gearbox practical model with fault is presented, as shown in Figure 4. This experimental model consists of a Dewetron 3000 multi-channel measuring device (Dewetron Services, Grambach, Austria), a one-stage helical gearbox (Hanoi, Vietnam) with two gear condition types: healthy (Figure 4a) and fault (Figure 4b), and an electromagnetic brake (Warner Electric, South Beloit, IL, USA) used to generate load. Two vertical axis acceleration sensors (Murata Manufacturing Co., Ltd., Kyoto, Japan) are mounted outside on the input shaft’s rolling bearing sleeve. The gear is installed on the gearbox’s input shaft for testing purposes. The signal is sampled at a frequency of 10,000 Hz. The rotational speed of shaft 1 is adjusted using a frequency Converter drive, with the frequency of rotation fluctuating between 8–25 Hz. The Meshing frequency is determined to fluctuate from 115 Hz to 355 Hz, as shown in Figure 5.
Figure 4. An experimental model of a single-stage gearbox with healthy gear (a), fault gear (b), and fault gear position (red box).

Figure 5. Meshing frequency of input shaft.

Spectrum analysis is a popular technique in modern industries, as per conventional signal analysis approaches [93]. Upon observing Figure 6, one can clearly see the signal waveform in the time domain (Figure 6a) and amplitudes in the frequency domain (Figure 6b). Due to non-stationary conditions, the frequency spectrum may become blurred, and the mechanical characteristics, such as gear meshing frequency and sidebands, may not be well defined. This instability could be caused by load variations, leading to rotational speed changes. Hence, the spectrum analysis method [94] is less effective when the gearbox is in a non-stationary condition.

The signal is then given a continuous Wavelet transformation to obtain the healthy and fault gear results, as shown in Figure 7. Figure 7a shows that in the case of a healthy gear, there are slight increases in energy density corresponding to meshing frequency and its harmonics, without sidebands present. On the contrary, in Figure 7b, corresponding to the case of a fault gear, it can be observed that the energy density region around the meshing frequency increases. However, evaluating the energy level on TFR around the gear meshing frequency is not enough to make decisions about the gear fault condition. It is necessary to determine the sidebands around the meshing frequency. If there are any
sidelbands present, it is a symptom of faults, whereas the absence of sidelbands indicates a healthy gear.

Figure 6. Waveform and Spectrum of measured vibration signal.

Figure 7. Depicts the TFR of the vibration of the gearbox using traditional Wavelet Analysis case in both situations: healthy (a) and gear fault (b).

Figure 8 depicts the TFR of gearbox case vibrations in a faulty gear tooth using these methods of WSST [95–97]. Compared to Figure 7, although the TFR of WSST has a higher signal concentration than that of CWT, the GMF is hard to detect, and the frequency curves are not smooth. The cause of the above problem is the high degree of complexity of the signal. In this experiment, the rotational shaft speed increases rapidly, which makes many closely spaced and overlapping vibration frequency components. So, the ability of WSST to distinguish and isolate the individual components is reduced.
To solve the mentioned challenges, the proposal method is applied to extract the sidebands around the initial gear meshing frequency harmonic order. If the gear is healthy, there is no frequency sideband around the mesh frequency, as indicated in Figure 9a. In contrast, clear sidebands can be observed as symptoms in the TFR of the signal when having a gear fault, as shown in Figure 9b. From this, it can be concluded that updating knowledge about Wavelet transformation, specifically, and Wavelet analysis, in general, allows experts to easily identify fault symptoms in a gearbox operating in non-stationary conditions. This is the foundation and prerequisite for automatic fault classification using artificial intelligence networks shortly.

Based on Figures 6–9, it can be observed that the proposed method is compared with traditional methods (spectrum analysis) and WSST methods. For cases of stable rotational speed, traditional methods such as spectrum analysis can easily detect sidebands of gear fault. On the contrary, it has proved ineffective in this case due to the difficulty of observing the fault frequency indicator. The WSST approach, which was developed recently, has demonstrated the ability to represent time-varying frequencies. However, there are still challenges when it comes to differentiating separate frequencies. The proposed method, which integrated various approaches of many methods, including the latest wavelet analysis, can easily extract frequency components around the gear meshing frequency. Therefore, it can be qualitatively affirmed that the proposed method is highly effective in processing vibration signals in non-stationary conditions. It is a better tool than traditional spectrum or WSST methods.
6. Conclusions

The article provides an in-depth review of the wavelet analysis method, including its variations and state-of-the-art publications. Based on analysis and evaluation of research on the application of Wavelet transform in practice, it can be observed that this is an effective method for diagnosing faults in gearboxes operated in non-stationary conditions. The authors combined many methods and the latest wavelet transform variant to create the proposed method. In order to validate the proposed method, we constructed a mathematical model of a fault gearbox in non-stationary conditions, which consists of multi-component signals. The results demonstrated that the proposed method effectively detects fault symptoms based on the time-frequency representation. Moreover, an experimental configuration is being built to evaluate the efficacy of the suggested approach in reality. However, the proposed method has a limitation in cases where the shaft rotational speed changes too rapidly. Under these conditions, the phenomenon of frequency overlap occurs continuously, making it challenging to separate the frequency components of interest accurately. The analysis of the proposed approach in this study shows that it can provide input data sets for neural networks, enabling automatic fault detection in rotating machines. Therefore, it aligns with the advancements in Industry 4.0. The next research direction that has to be focused on is the adjustment of parameters and techniques to enhance the time-frequency resolution of signals in cases of machines operated in speed-up or speed-down conditions.

Author Contributions: T.-D.N.: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Writing—original draft, Writing—review and editing, Supervision, Validation. P.-D.N.: Data curation, Funding acquisition, Methodology, Project administration, Resources, Supervision, Validation, Visualization, Writing—review and editing. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Acknowledgments: The authors would like to thank the editors and the anonymous reviewers for their constructive comments, which greatly improved this paper.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>CWT</td>
<td>Continuous Wavelet Transform</td>
</tr>
<tr>
<td>DWT</td>
<td>Discrete Wavelet Transform</td>
</tr>
<tr>
<td>GFT</td>
<td>Generalized Fourier Transform</td>
</tr>
<tr>
<td>iGFT</td>
<td>Inverse generalized Fourier Transform</td>
</tr>
<tr>
<td>GMF</td>
<td>Gear meshing frequency</td>
</tr>
<tr>
<td>GST</td>
<td>Generalized Wavelet-Based Synchrosqueezing Transform</td>
</tr>
<tr>
<td>ICA</td>
<td>Independent Component Analysis</td>
</tr>
<tr>
<td>multi-wavelet</td>
<td>Multiple Wavelet Functions</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal Component Analysis</td>
</tr>
<tr>
<td>SVM</td>
<td>Support Vector Machine</td>
</tr>
<tr>
<td>TSA</td>
<td>Time Synchronous Averaging</td>
</tr>
<tr>
<td>TFR</td>
<td>Time-Frequency Represented</td>
</tr>
<tr>
<td>WPT</td>
<td>Wavelet Packet Transform</td>
</tr>
<tr>
<td>WSST</td>
<td>Wavelet-Based Synchrosqueezing Transform</td>
</tr>
</tbody>
</table>


Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.