

Article

Damping and Stiffness Responses of Silica Rock under Constant Amplitude and Variable Rate Cyclic Loading

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Abstract: In this paper, the shear modulus and damping ratio of silica rock under cyclic loading were experimentally analyzed using two loading modes, constant amplitude and increasing amplitude, combined with three increasing loading rates. Observations have indicated a decrease in the shear modulus of specimens as the number of cycles increased during the loading and unloading phases and an overall increase with larger amplitude intervals. The change in loading rate significantly affects the damping ratio of the specimens, leading to a stepwise decrease within the same cyclic group, while the damping ratio of a single specimen exhibits a ‘concave’ distribution throughout the cyclic interval. Based on the axial strain and dissipation energy, this paper develops two damage models that can effectively predict the damage accumulation process in rocks under cyclic loading. These findings have significant implications for a deeper understanding of the mechanical behavior of rocks under dynamic loading and offer theoretical guidance and technical support for rock engineering.

Keywords: cyclic loading; loading rate; shear modulus; damping ratio; damage model



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1. Introduction

Earthquakes are among the most destructive geological hazards, with the resulting ground motion characteristics being closely related to the dynamic parameters of the rock mass [1], including shear modulus [2] (modulus of elasticity) and damping ratio [3], which are essential for exploring the intrinsic connections between earthquakes and rock behavior. Shear modulus measures a rock mass’s resistance to deformation under seismic forces, and this relationship is proportional; that is, a larger shear modulus indicates higher material stiffness [4–6]. Theoretical analyses indicate that seismic waves propagate with reduced energy attenuation in materials of higher stiffness, consequently increasing the amplitude of vertical ground fluctuations [7–9]. The damping ratio quantifies a material’s energy dissipation during vibration; a higher damping ratio indicates effective energy dissipation, which suppresses the amplitude of vibrations caused by earthquakes and reduces their destructive impact on rock structures [10–12]. Studying the dynamic properties of rocks under various cyclic modes holds both theoretical value and practical significance for enhancing the safety of underground engineering constructions.

In mechanical tests, the shear modulus and damping ratio of rocks are influenced by factors such as material composition, stress amplitude, and loading frequency [13–15]. Scholars have conducted extensive theoretical and experimental research on the fatigue deformation [16], strength, and damage of intact rocks. In studies on the fatigue of soft rocks, such as sandstone, shale, and salt rock, water content and stress amplitude significantly influence the dynamic shear modulus and damping characteristics. Under conditions of

low strain amplitude, the effect of water content on the dynamic shear modulus is subtle, but it significantly affects the damping ratio and damping coefficient [17]. However, as stress amplitude increases, the impact of water content on the dynamic shear modulus becomes more pronounced. This indicates a complex interaction between water content and stress amplitude that collectively determines the dynamic response characteristics of soft rocks [18,19]. Under graded cyclic loading, the damping parameter of soft rocks increases with rising stress amplitude, and this trend varies under different stress paths. Specifically, the damping parameter values measured under stress paths with incrementally increasing stress levels are higher than those under paths with incrementally increasing stress amplitudes. This may be attributed to the nonlinear response of the internal structure of the rock and the associated damage accumulation mechanism [20,21]. Under constant amplitude cyclic loading, the damping ratio and damping coefficient of soft rock change as the number of cycles increases. These changes adhere to a specific evolutionary law, particularly when the stress level surpasses the fatigue damage threshold of the rock. The evolution of the damping parameter exhibits a three-stage semi-U curve, encapsulating the entire process from the rock's initial elastic response through the accumulation of fatigue damage to its ultimate destruction. This phenomenon encapsulates the entire process from the rock's initial elastic response to the accumulation of fatigue damage, culminating in its destruction [22–24].

For hard rocks, such as marble, granite, and schist, their mechanical fatigue properties are influenced by cyclic loading conditions. Hard rocks typically exhibit higher shear moduli and lower damping ratios, attributed to their internal structural compactness and mineral composition. Under cyclic loading, the fatigue damage and damping properties of hard rocks demonstrate an evolutionary pattern distinct from that of soft rocks. The fatigue damage process in hard rocks is often more abrupt, with stress–strain curves displaying a more pronounced linear elastic phase before the onset of damage. Moreover, the damping behavior of hard rocks can be influenced by internal defects, like cracks and voids, potentially enhancing their damping capacity [25–27]. In summary, research on soft and hard rocks primarily focuses on the effects of rock type, stress level, stress path, and number of cycles, frequently overlooking the link between loading rate and the shear modulus and damping ratio of the test material.

This study focuses on the dynamic properties of hard rock (silica) under variable-rate cyclic loading, analyzing the effects of loading rate and stress amplitude on the evolution patterns of the dynamic shear modulus and damping ratio of silica.

2. Materials and Methods

2.1. Material Characterization

The test material originated from a lead–zinc mine in Yunnan, China. Drill a 110 mm diameter borehole into the roof of the rock drilling chamber at the top of the mining site to obtain core samples. Then, cut, grind, and polish the core samples to prepare standard specimens. Given that the mineral composition and structure are critical in influencing the rock's mechanical properties, X-ray powder diffraction (XRD) and polariscope tests were conducted on the material prior to testing to identify the microscopic composition of the specimen. As shown in Figure 1a, the primary structural components of the material include actinolite, garnet, diopside, and quartz particles that are cemented together. Figure 1b displays the XRD analysis results of the crushed specimen; the principal components are stilbite, actinolite, and calcite, comprising 26.1%, 42.6%, and 16.9% respectively.

The mechanical behavior of siliceous mudstone is significantly influenced by the stiffness of its primary mineral grains, particularly tourmaline and epidote, which are stiffer and comprise 60% of the rock. The presence of these hard minerals effectively impedes microcrack propagation. Under monotonic loading, the intergranular tension within the rock can lead to the formation of vertical cracks. Under cyclic loading, misalignment displacements among mineral grains of differing stiffnesses lead to stress concentrations, particularly at the edges of the mineral grains. As the load cycles, damage accumulates in

these stress concentration areas, leading to the coalescence of cracks and the formation of macroscopic shear fractures along the rock's diagonals, ultimately leading to rock failure.

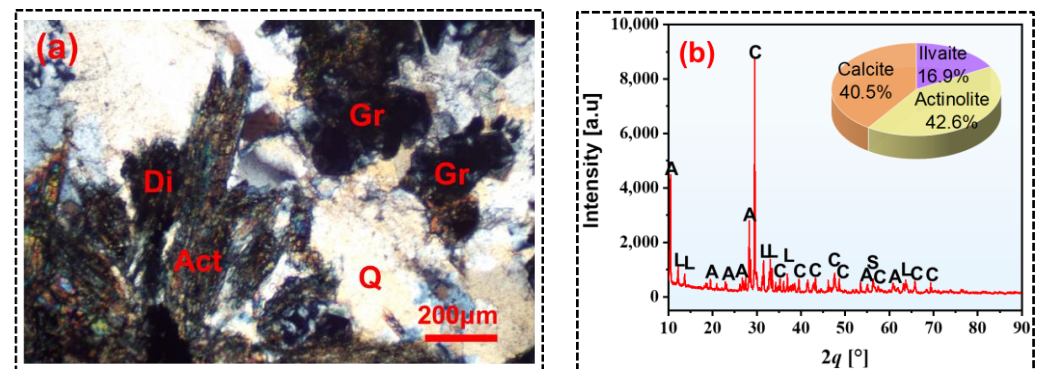


Figure 1. Analysis of the microstructure of skarn rock. (a) Polarization microscope micrograph of the sample crushed block processed into a cast thin film, Gr (Garnet), Di (Diopside), Act (Actinolite), Q (Quartz); (b) analysis results of the powder XRD test after sample failure.

2.2. Sample Preparation

The tester has strict requirements for the morphology of the test material. In order to ensure the scientific rigor and validity of the rock mechanics test, it is recommended to carry out the pre-preparation of the test specimens according to ISRM requirements so as to improve the quality of the test data and provide a reliable basis for the study of the mechanical properties of the rock and its engineering applications.

Based on the purpose and requirements of the test, samples free of cracks, voids, or other visible defects were selected from the parent core, and the shape and size of the specimens were required to meet the specifications of the experimental equipment to avoid boundary effects (the top and bottom of the specimens were maintained to be less than 0.02 mm non-parallel). Figure 2a illustrates the selection of test specimens, with three specimens chosen as representatives due to their density and wave speed being near the batch mean; another four specimens were randomly selected for the fatigue test. Figure 2b–d present the measurements of wave speed, mass, and size of the specimens, respectively, which are foundational for selecting the test specimens. The measuring equipment is an NM-4A non-metallic ultrasonic testing analyzer. The physical parameters of the specimens are detailed in Table 1.

Table 1. Physical parameters of diorite specimens (experimental part).

No.	Mass [g]	High [mm]	Diameter [mm]	Density [g/m ³]	Axial Longitudinal Wave Velocity [m/s]	Radial Longitudinal Wave Velocity [m/s]
X10	675.1	100.29	49.82	3.45	4811	3906
X15	675.6	99.63	50.09	3.44	4630	3676
X49	676.0	100.15	50.19	3.41	4690	3788
X43	718.9	100.30	50.23	3.62	4314	3472
X48	635.9	100.15	50.19	3.41	4690	3788
X53	645.6	99.78	50.03	3.29	4606	3472
X54	658.8	100.57	49.58	3.39	4438	3205

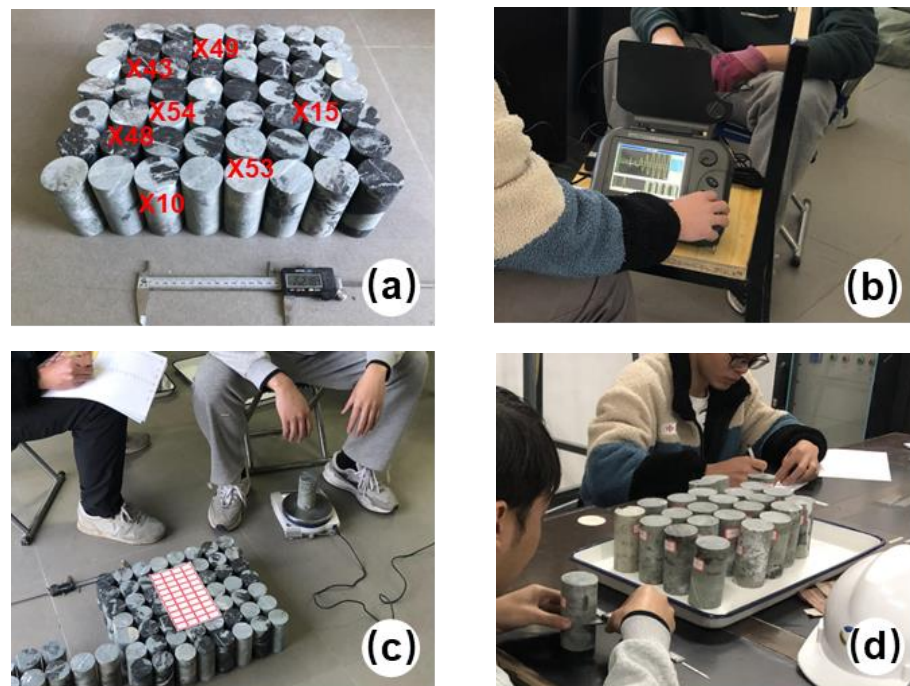


Figure 2. Specimen pre-treatment: (a) selection of specimens; (b) wave velocity measurement; (c) mass measurement; (d) dimensional measurement.

2.3. Experimental Setup

The test equipment is the YAW-2000 servo rigid mechanical testing machine (Chaoyang Testing Machine Co., Ltd, Changchun, China), which has a stiffness of more than 10GN/m and a maximum load capacity of 2000 kN, making it suitable for uniaxial and fatigue testing of hard rock materials, such as granite and marble. The test path and control are commanded by the German DOLI electronic system, which allows for free switching between multimodal controls, such as strain, displacement, and load. The axial and radial deformations are measured by the model YYSJ50 extensometer (Beijing Zhongxi Huada Science and Technology Co, Beijing, China). The test equipment is depicted in Figure 3.



Figure 3. Experimental setup: (a) overview of the test loading system; (b) control interface of the operating system; (c) schematic of the installation of the system for measuring the deformation of rock samples.

2.4. Testing Scheme

2.4.1. Uniaxial Compressive Strength Test

According to the pre-test preparation, three specimens, X10, X45, and X49, were selected for uniaxial compressive strength testing, with the results serving as a benchmark for the design of the cyclic loading test path. The test was controlled by stress mode, with the contact force between the testing machine and the specimen surface maintained at 2 kN. The specimen was loaded at a rate of 0.2 kN/s until its failure. The peak strengths of the three specimens were 180.86 MPa, 155.95 MPa, and 167.06 MPa, respectively, yielding an average peak strength of 168 MPa. The stress–strain curves are depicted in Figure 4.

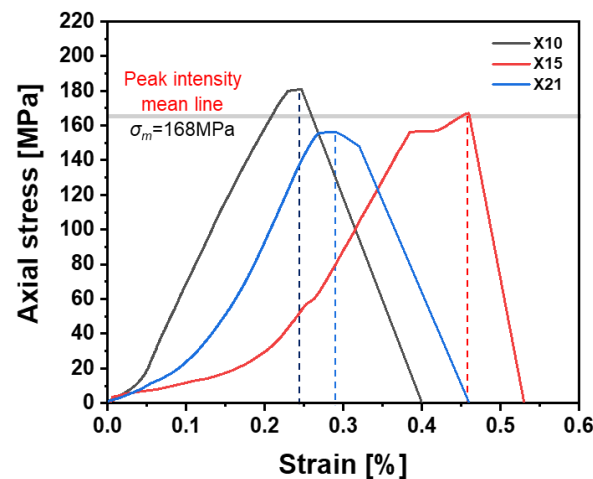


Figure 4. Uniaxial stress–strain curve.

2.4.2. Cyclic Loading Test

This part of the test was divided into a constant amplitude cyclic loading test and an increased amplitude cyclic loading test, with 15 cycles in each cyclic group. It featured rates of 2 kN/s ($f = 0.006$ Hz) for the first five cycles, 4 kN/s ($f = 0.012$ Hz) for the middle five cycles, and 6 kN/s ($f = 0.018$ Hz) for the last five cycles. The amplitude intervals for the three specimens of the constant amplitude loading test were as follows: $50\%\sigma_m$ – $100\%\sigma_m$ for mode 1, $60\%\sigma_m$ – $110\%\sigma_m$ for mode 2, and $70\%\sigma_m$ – $120\%\sigma_m$ for mode 3; each was loaded monotonically at a rate of 0.2 kN/s until specimen damage occurred after surviving more than six cycle sets within the designated cyclic interval. The increasing value cyclic loading test was mode 4, with the lower amplitude starting at a constant $10\%\sigma_m$ and increasing by $15\%\sigma_m$, continuing until specimen destruction. The peak strengths of the four specimens were 206.8 MPa, 258.8 MPa, 260.3 MPa, and 163.5 MPa, respectively. The loading paths for the four modes are depicted in Figure 5.

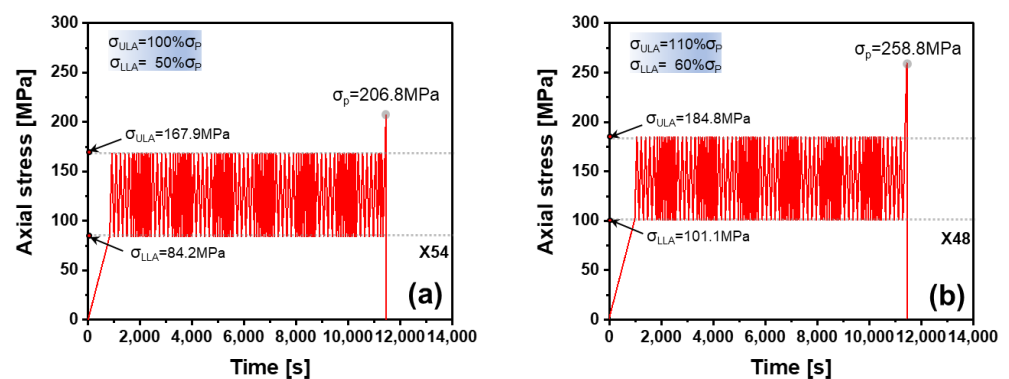


Figure 5. Cont.

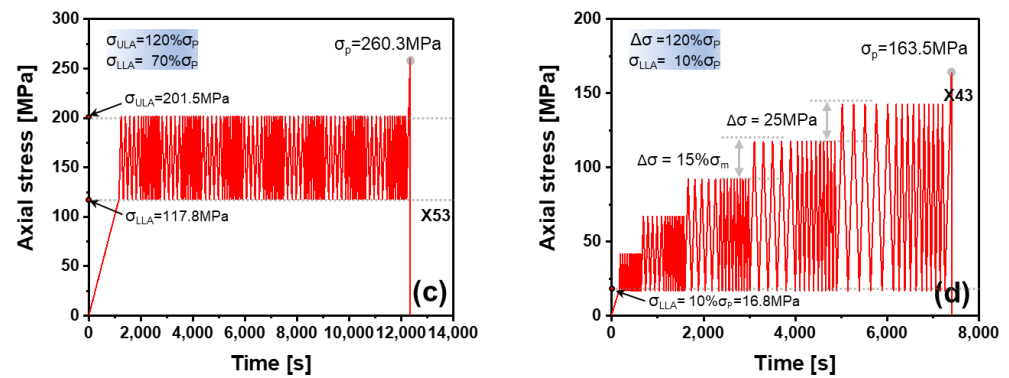


Figure 5. Cyclic loading path diagram: (a) mode 1; (b) mode 2; (c) mode 3; (d) mode 4.

3. Testing Results

3.1. Determination of the Shear Modulus and Damping Ratio

As a multiphase material, the interaction between mineral particles, such as stilbite, actinolite, and quartz, as well as the dynamic reorganization of the microstructure, significantly influences the shear modulus. Under cyclic loading conditions, the relative positions and contact states between particles alter, leading to the transfer and redistribution of stress concentration areas. Such changes may lead to variations in the trend of the shear modulus. Specifically, the shifting of stress concentration, changes in the contact state, microstructural reorganization, damage accumulation, and changes in the inter-particle forces significantly affect the shear modulus of rocks.

In cyclic loading and unloading tests, the loading and unloading curves do not coincide and usually form a hysteresis loop, indicating irreversible mechanical actions that lead to energy dissipation [28,29] during a single cyclic loading cycle. The shear modulus and damping ratio are usually determined from the stress–strain curves of rock specimens, reflecting the nonlinearity and hysteresis of the material [30]. In this test, the modulus of elasticity E is defined as the ratio of the stress difference (the peak stress minus the minimum stress during the loading stage) to the strain difference (the strain corresponding to the peak stress minus the strain corresponding to the minimum stress during the loading stage), calculated over a single cycle. The damping ratio relates to the hysteresis loop formed by the stress–strain curve, and the magnitude of this ratio intuitively reflects the material's energy dissipation under loading effects due to damping. The dynamic shear modulus G and damping ratio λ are calculated as follows:

$$E = \frac{(\sigma_{\max} - \sigma_{\min})}{(\varepsilon_{\max} - \varepsilon_{\min})} \quad (1)$$

where $\sigma_{\max}$ is the maximum stress in a single cycle, $\varepsilon_{\max}$ is the strain corresponding to the maximum stress, $\sigma_{\min}$ is the minimum stress during unloading in this cycle, and $\varepsilon_{\min}$ is the strain corresponding to the minimum stress.

$$G = \frac{E}{2(1 + \nu)} \quad (2)$$

where ν is Poisson's ratio.

$$\lambda = \frac{A_R}{4\pi A_S} \quad (3)$$

where A_R is the area of ABCDA, and A_S is the area of AOE. The horizontal coordinate in Figure 6 is ε_d , and the vertical coordinate is the stress σ_d .

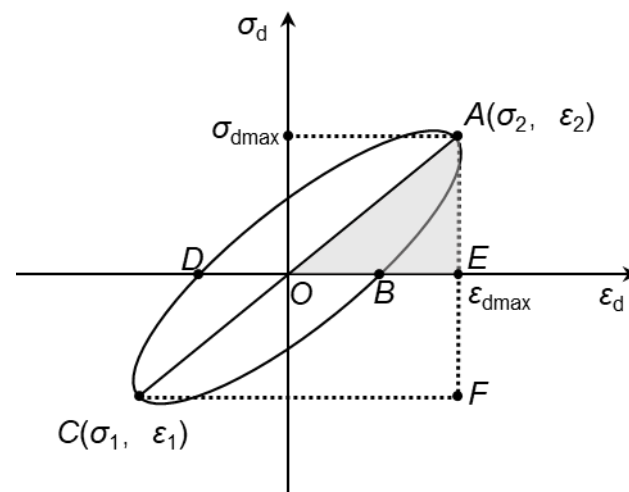


Figure 6. Hysteresis loop of axial stress σ_d – ε_d .

3.2. Shear Modulus Analysis

The shear modulus of the rock shows a corresponding trend with the accumulation of cyclic loading and unloading times, attributed primarily to the gradual expansion and coalescence of microcracks and fissures within the rock during the loading process. This results in internal damage that progressively diminishes the rock's strength and stiffness, ultimately leading to its destruction [31–33]. The changing trend of the shear modulus of sillimanite in cyclic loading and unloading tests primarily exhibits characteristics of initial decrease, subsequent stabilization, and eventual attenuation, influenced by numerous factors such as the rock's microstructure, loading frequency, and stress level. Understanding this change pattern deeply is crucial for predicting the engineering behavior of the rock.

Figure 7 displays the shear modulus of the four specimens versus the number of cycles. Analysis of the four graphs reveals that both the loading and unloading shear moduli increased equally as the amplitude interval was incremented. During the loading phase, the average shear modulus was 28.22 GPa for X43, 34.78 GPa for X54, 40.91 GPa for X48, and 45.04 GPa for X53. The growth rates relative to the X43 specimen were 23.3%, 45%, and 59.5%, respectively. In the unloading stage, the average shear modulus was 27.58 GPa for X43, 34.01 GPa for X54, 40.07 GPa for X48, and 44.04 GPa for X53. The growth rates relative to the X43 specimen were 23.3%, 45.3%, and 59.7%, respectively. In the growth rate test, the loading and unloading shear moduli at the same cycle level decrease as the number of cycles increases, and the transverse value test indicates a general decline in both shear moduli throughout the stage.

Gr is defined as the ratio of the loaded cutline modulus to the unloaded cutline modulus, as illustrated in Figure 7a. Initially, Gr rapidly increases with the number of cycles, and within the same cycle group, the ratio increases as the loading rate increases. Between two adjacent cycle groups, Gr suddenly drops when the loading rate changes from 6 kN/s to 2 kN/s, highlighting that the loading rate significantly impacts the cutline modulus. This phenomenon is consistent across the three specimens undergoing transverse value loading. In the increased load test, Gr's trend at the same cycle level mirrors the transverse amplitude load's performance, initially growing and then stabilizing after a certain value is reached. At different cycle levels, Gr initially shows a marked increase, which gradually diminishes as the cycle level increases.

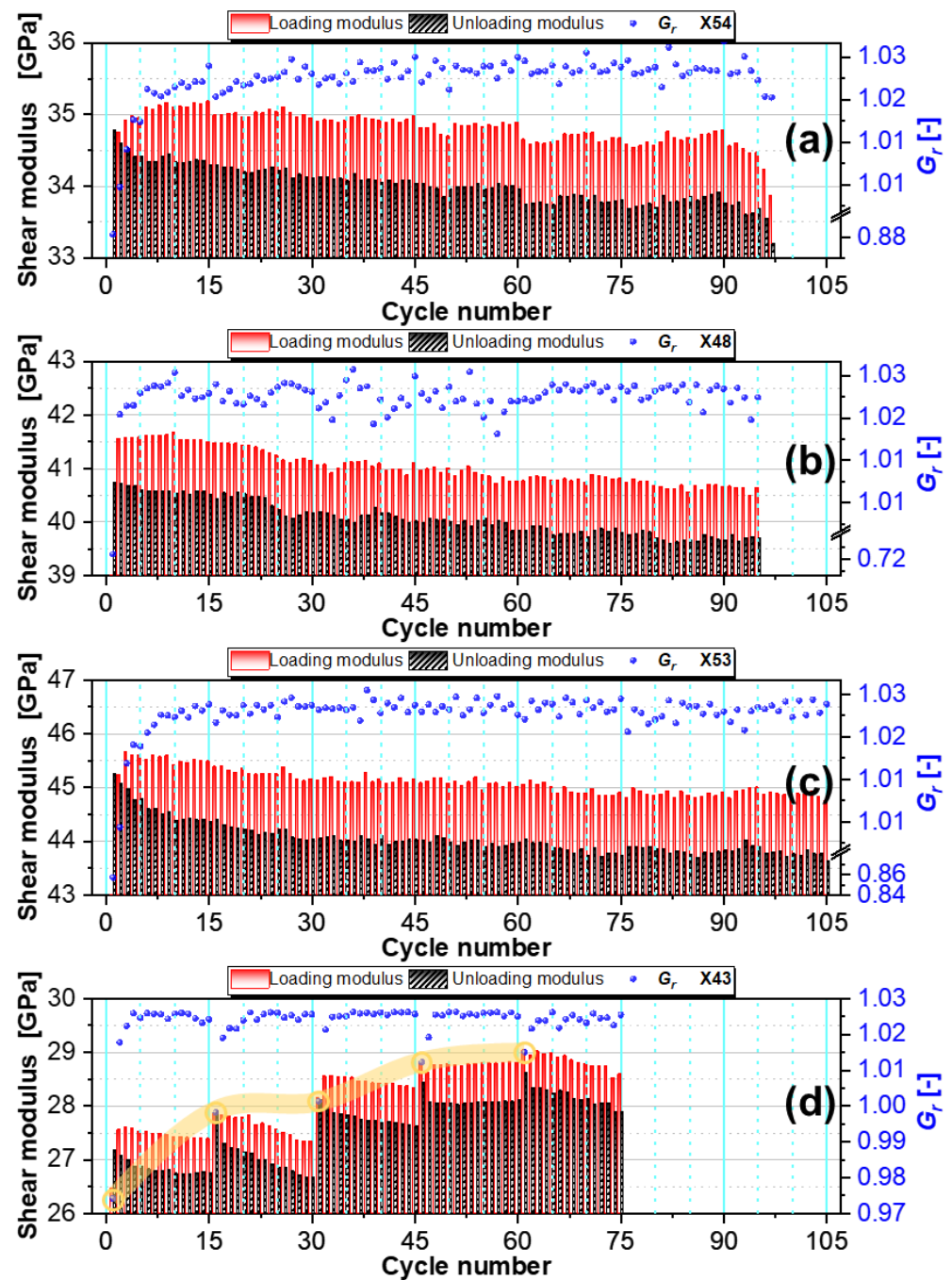


Figure 7. Shear modulus versus number of cycles: (a) evolutionary pattern of shear modulus for specimen X54; (b) evolutionary pattern of shear modulus for specimen X48; (c) evolutionary pattern of shear modulus for specimen X53; (d) evolutionary pattern of shear modulus for specimen X43.

3.3. Damping Ratio Analysis

In the process of cyclic loading and unloading, the sillimanite specimen has already shown obvious local damage, resulting in irrecoverable macroscopic cracks. This may promote the expansion of these cracks during further cyclic loading and unloading, leading to a decreased damping ratio as crack expansion diminishes the rock's overall energy absorption capacity [34,35]. As the number of cyclic loadings increases, the damping ratio of silica rocks gradually decreases, attributed to the progressive expansion and coalescence of microscopic cracks and fissures within the rocks. This process results in a gradual reduction in the rocks' stiffness and strength, subsequently leading to a decrease in the

damping ratio. After reaching a certain number of cyclic loadings, the damping ratio of skarn tends to stabilize. At this stage, the cracks and microstructure within the rock become relatively stable, and the development and coalescence rate of the cracks slow, resulting in slower changes to the damping ratio. Prolonged cyclic loading may lead to continual expansion [36] and coalescence of cracks within the rock, drastically reducing its strength and stiffness, which typically leads to a sharp decline in the damping ratio as the rock nears a state of damage, and this decline may coincide with the rock's destruction. Under cyclic loading, the damping ratio of the rock will follow a 'concave' pattern, decreasing and then gradually increasing, which reflects the rock's progression from compaction and elastic deformation to crack expansion and destruction.

Figure 8 illustrates the relationship between the damping ratio and the number of cycles for the four specimens, and the comparison indicates that the damping ratio of silica exhibits a trend to first decrease and then increase with the increase in the number of cycles throughout the cycling stage. As illustrated in Figure 8a, the damping ratio initially decreases in the first five-cycle stage, and with the increase in the loading rate to 4 kN/s, the damping ratio experiences a sharp decrease and then stabilizes, and the damping ratio experiences a discontinuous decrease when the rate reaches 6 kN/s. Observations of the damping ratio change in the subsequent unused cycle group indicate that the rate-dependent behavior is consistent with that of the first cycle group, and this behavior is also observed in the other three specimens. The red bar graph in Figure 8 represents the mean damping ratio at each rate, whether at constant or increasing amplitude. The mean damping ratio at the three rates in each cycle group shows a stepwise decrease, suggesting that the loading rate significantly influences the damping ratio; specifically, higher rates result in lower damping ratios and correspondingly lower energy dissipation. In summary, the trends in the damping ratios of silica rocks under cyclic loading correlate with internal damage and crack development, as well as the loading rate. These trends are crucial for understanding and predicting the long-term stability of rocks under dynamic loading.

Table 2 displays the fitting equation for the mean damping ratio of a single sample at a consistent rate. The fitting curve represents a logarithmic function, and the correlation coefficients listed in the table suggest a high degree of fit between them.

Table 2. Fitting of the mean damping ratio of a single sample at the same rate.

No.	Loading Rate	Fitting Equation	R ²
X43	2 kN/s	$y = -0.063\ln(x) + 0.1167$	0.772
	4 kN/s	$y = -0.05\ln(x) + 0.0895$	0.718
	6 kN/s	$y = -0.044\ln(x) + 0.0789$	0.677
X48	2 kN/s	$y = 0.0071\ln(x) + 0.0181$	0.833
	4 kN/s	$y = 0.0096\ln(x) + 0.0084$	0.968
	6 kN/s	$y = 0.0104\ln(x) + 0.006$	0.961
X53	2 kN/s	$y = -0.018\ln(x) + 0.0549$	0.520
	4 kN/s	$y = 0.0021\ln(x) + 0.0151$	0.752
	6 kN/s	$y = 0.003\ln(x) + 0.0095$	0.983
X54	2 kN/s	$y = -0.02\ln(x) + 0.0553$	0.768
	4 kN/s	$y = 0.0071\ln(x) + 0.0181$	0.992
	6 kN/s	$y = -0.004\ln(x) + 0.017$	0.987

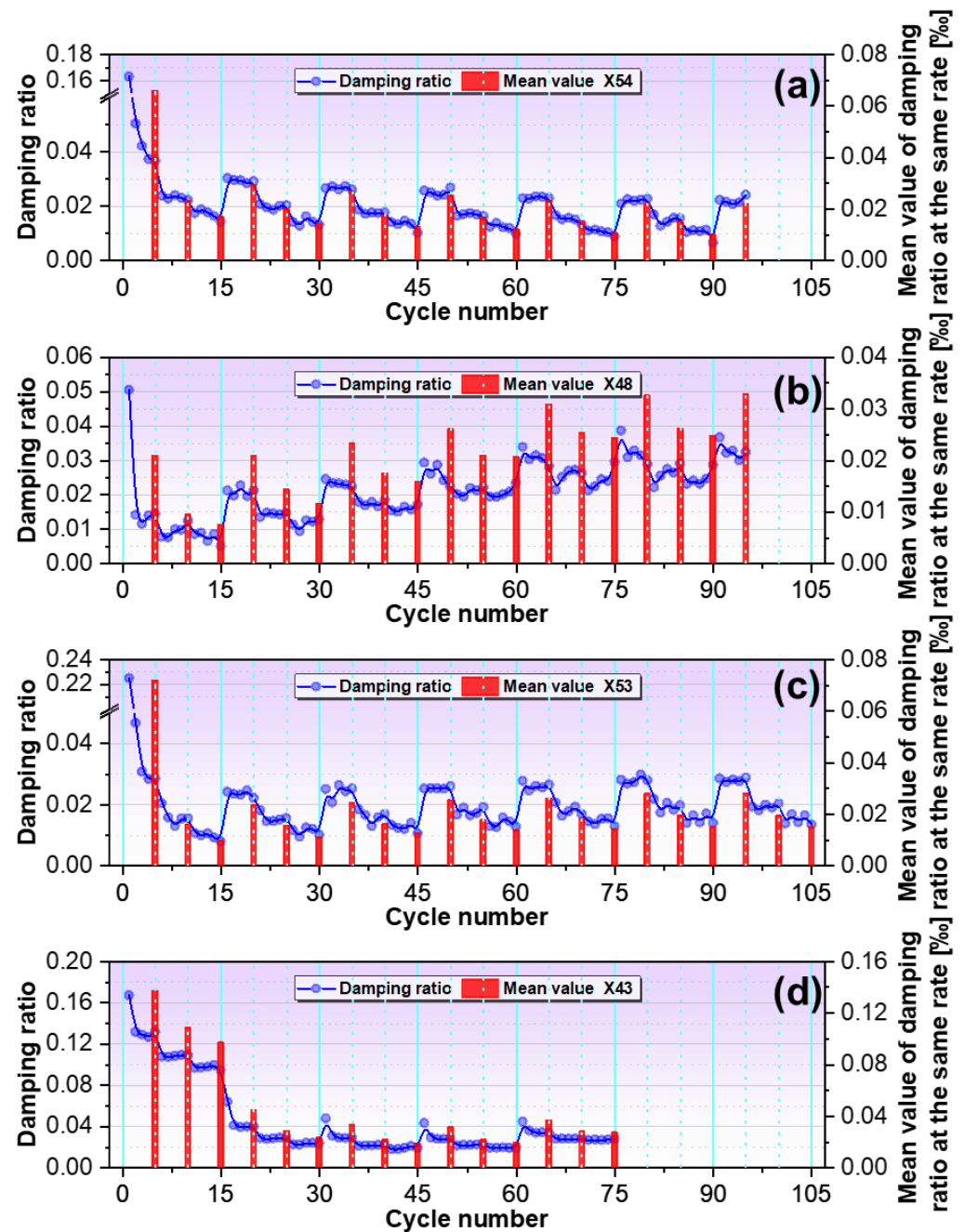


Figure 8. Damping ratio versus number of cycles: (a) evolutionary pattern of the damping ratio for specimen X54; (b) evolutionary pattern of the damping ratio for specimen X48; (c) evolutionary pattern of the damping ratio for specimen X53; (d) evolutionary pattern of the damping ratio for specimen X43.

4. Discussion

Modeling the damage evolution of rocks [37] under cyclic loading and unloading conditions is complex, covering the entire course of deformation, damage accumulation, and eventual destruction. The damage variable, a core concept in rock mechanics, quantifies the degradation of the rock from integrity to damage. Under cyclic loading, microstructural changes, including microcrack initiation, extension, and penetration, are manifested as macroscopic damage through cumulative damage effects. Damage evolution models aim to capture these behaviors, defining the damage variable D from 0 (no damage) to 1 (complete damage) [38]. The model is constructed based on experimental observations and validated

through theoretical analyses and numerical simulations to ensure accuracy and reliability in rock mechanics applications.

4.1. Damage Characterization Analysis Based on Axial Strain

At the beginning of cyclic loading, the strain hardening phenomenon in rock is evident as a decrease in the axial strain increment, even at a constant stress level, suggesting that the rock possesses damage resistance. However, as the loading cycles increase, the cumulative effect of damage begins to dominate the rock's behavior, leading to the strain softening phase. In this stage, the axial strain increment increases at the same stress level, indicating an accelerated damage accumulation within the rock and a degradation of its macroscopic mechanical properties. This transition from hardening to softening [39] is a critical feature of the damage evolution process in rocks under cyclic loading, enhancing the understanding and prediction of rock fatigue life. In this study, rock damage is quantified in terms of axial strain using the following equation:

$$D = \frac{\varepsilon_p}{\varepsilon} \frac{\varepsilon - \varepsilon_0}{\varepsilon_p - \varepsilon_0} \quad (4)$$

where D is the damage variable, ε represents the strain corresponding to the minimum unloaded stress in a single cycle, ε_0 is the axial initial strain in the first cycle, and ε_p is the strain at the axial minimum unloaded stress in the last cycle.

Based on the calculation results in Equation (4), Figure 9 illustrates the trend of damage evolution in skarn specimens under fatigue loading. The S-shaped curve in the figure demonstrates the correlation between the damage value and the relative number of cycles, defined as the ratio n/N of the number of cycles n to the total number of cycles N . In the early stages of cyclic loading, compaction of the rock's internal pores leads to a rapid increase in the damage value. As the number of cycles accumulates, the expansion of cracks and further damage accumulation become the predominant processes within the rock. When damage accumulates to a certain extent, the growth of the damage value significantly accelerates in the late stages of cycling, typically indicating the rock's critical state prior to undergoing significant damage. As shown in Figure 9a, within a single cyclic group, the damage effects on the rock vary with different loading rates. The data points marked by rectangular boxes show a trend of increasing damage with the number of cycles, with higher damage values observed under low-rate loading conditions, suggesting the impact of the loading rate on the damage accumulation process. Comparing adjacent cycle groups, a loading rate of 6 kN/s versus 2 kN/s shows that the latter induces a more significant damage increment, highlighting the role of the loading rate in rock damage behavior. Additionally, the effect of an increasing loading rate on rock damage in the cyclic group shown in Figure 9d aligns with the previously described trends. As the stress amplitude increases, the damage increment exhibits a decreasing trend, possibly due to the rock's greater deformation capacity and damage tolerance at higher stress levels, indicating that stress amplitude is another crucial factor influencing rock damage evolution.

In this study, we develop a mathematical model based on the stress–strain relationship of rocks to describe the damage evolution of rocks under cyclic loading. The model is derived through parameter fitting and can quantify and digitally characterize the damage state of rocks. The expression for the proposed mathematical model of rock damage evolution is presented as follows:

$$D = 1 - \left(1 - \left(\frac{n}{N}\right)^a\right)^b \quad (5)$$

where n is the number of cycles, N is fatigue life, and a and b are material-related parameters.

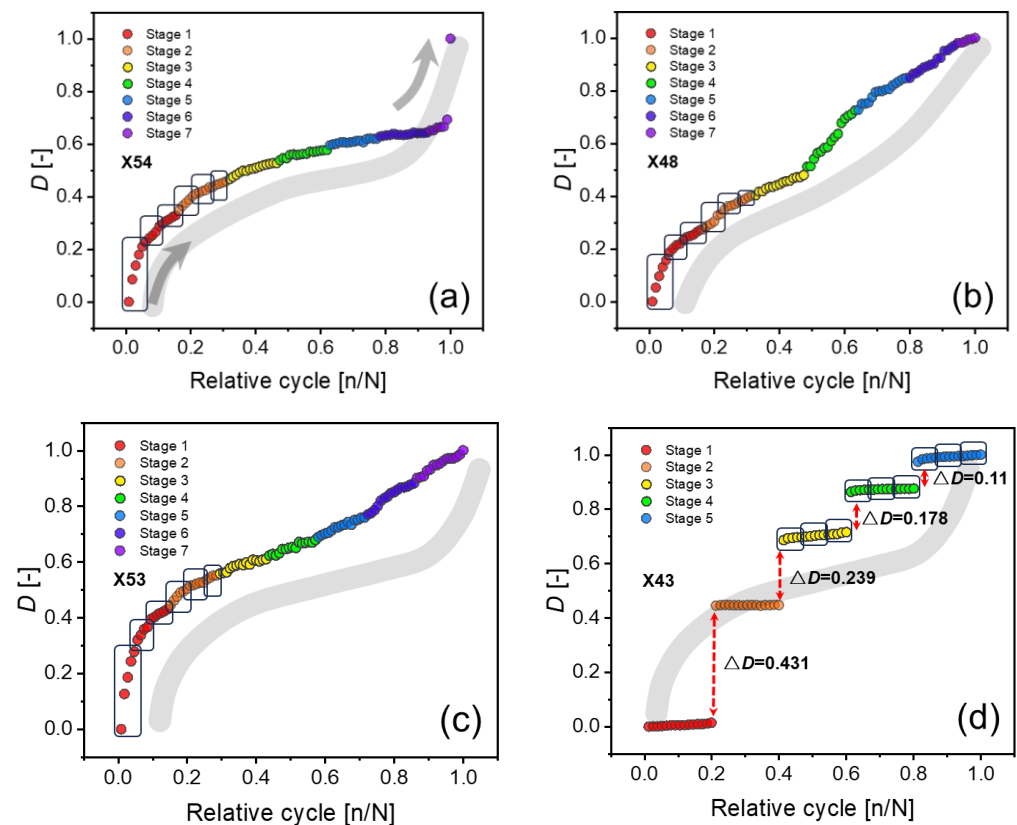


Figure 9. Trend of rock damage evolution under cyclic loading: (a) specimen X54; (b) specimen X48; (c) specimen X53; (d) specimen X43.

Figure 10 details the relationship between maximum damage and the relative number of cycles for four specimens across different cycle sets, as well as the evolutionary trend of damage accumulation versus the relative number of cycles. By fitting an S-function to these data points, the dynamics of the damage variable as the number of cycles increased were captured, with the parameters of the fitted results listed in the boxed squares in the upper left corner of the Figure 10a–d. The fitted correlation coefficients for all cyclic groups exceed 0.97, indicating that the strain-based proposed fatigue damage model is highly reliable and accurate for describing the damage evolution of rocks under cyclic loading.

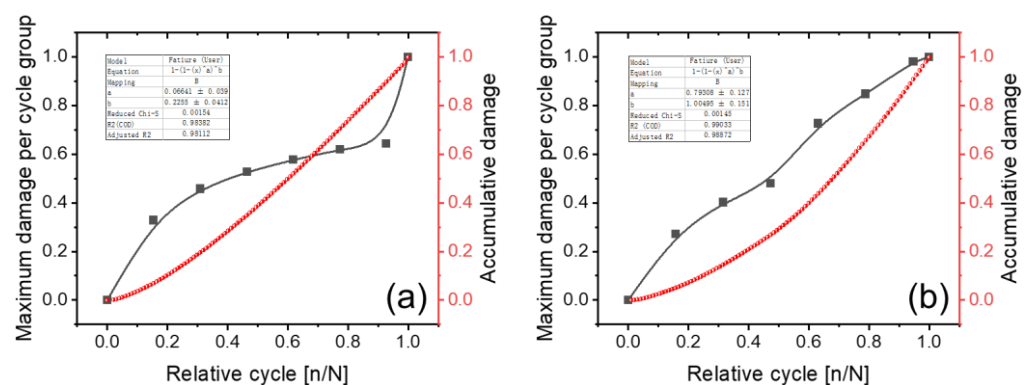


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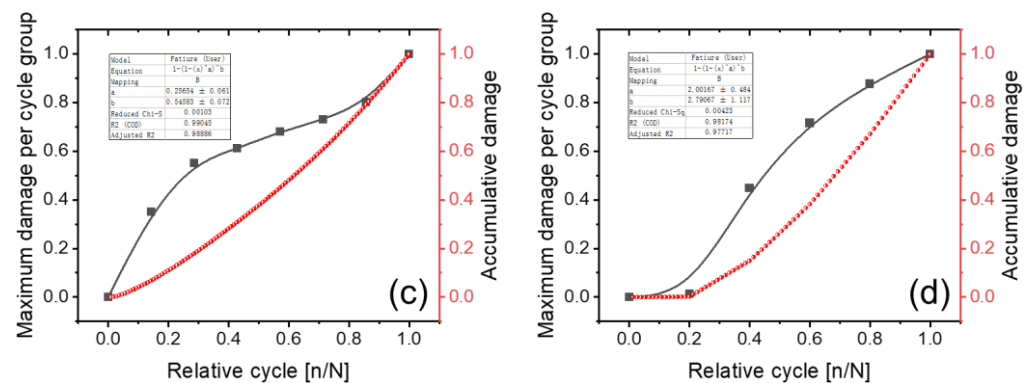


Figure 10. Cumulative damage evolution model for rocks: (a) specimen X54; (b) specimen X48; (c) specimen X53; (d) specimen X43.

4.2. Damage Characterization Analysis Based on Dissipative Energy

The damage process in rocks is usually accompanied by energy dissipation, released in the form of heat, acoustic energy, etc. [40], indicating the degradation of the internal structure of the rock. Under cyclic loading conditions, the damage variable of a rock accumulates with increasing loading cycles, and its growth is closely related to the initiation, extension, and penetration of microcracks within the rock. The accumulation of dissipated energy, a key indicator [41,42] for evaluating the evolution of rock damage, reflects the intensification of damage within the rock. By monitoring changes in dissipated energy, the damage state of rocks can be quantitatively analyzed, providing a scientific basis for predicting fatigue life and assessing the engineering stability of rock materials.

To quantify the degree of damage in rock samples, many early researchers adopted the concept of dissipated energy density. Based on the cumulative change characteristics of dissipated energy density, the damage variable D is defined as the ratio of dissipated energy density to the cumulative dissipated energy density, expressed as follows:

$$D_e = \frac{\sum_{i=1}^n E_D}{\sum_{i=1}^N E_D} \quad (6)$$

where D_e denotes the dissipated energy damage variable, n denotes the n th cycle in cyclic loading and unloading, and E_D is the dissipated energy in a single cycle.

Using the method defined in Equation (6), the dissipated energy damage variables of the four rock specimens were calculated in this study, and the results are displayed in Figure 11. The trends of the dissipated energy damage variables under different amplitude intervals were consistent, showing an increasing trend with the increase in the relative number of cycles. When comparing the same cycle groups, lower loading rates resulted in higher levels of dissipated energy, which indicated an increase in the degree of rock damage. Additionally, for a given relative number of cycles, the increase in dissipated energy damage variables is more pronounced at higher amplitude intervals, as shown in Figure 11d, indicating the contribution of higher stress amplitude intervals to the accumulation of rock damage.

By integrating the relationship between dissipated energy and rock damage, a model can be developed for predicting rock damage, which is used to calculate the cumulative damage of rocks under multi-stage cyclic loading and unloading. The model can quantify the cumulative process of rock damage and predict the evolution of rock damage over the number of cyclic loading cycles. The damage accumulation model is expressed mathematically as follows:

$$D = a\left(\frac{n}{N}\right)^2 + b\left(\frac{n}{N}\right) + D_0 \quad (7)$$

where n is the number of cycles, N is fatigue life, a and b are the fitting coefficients, and D_0 is the initial damage.

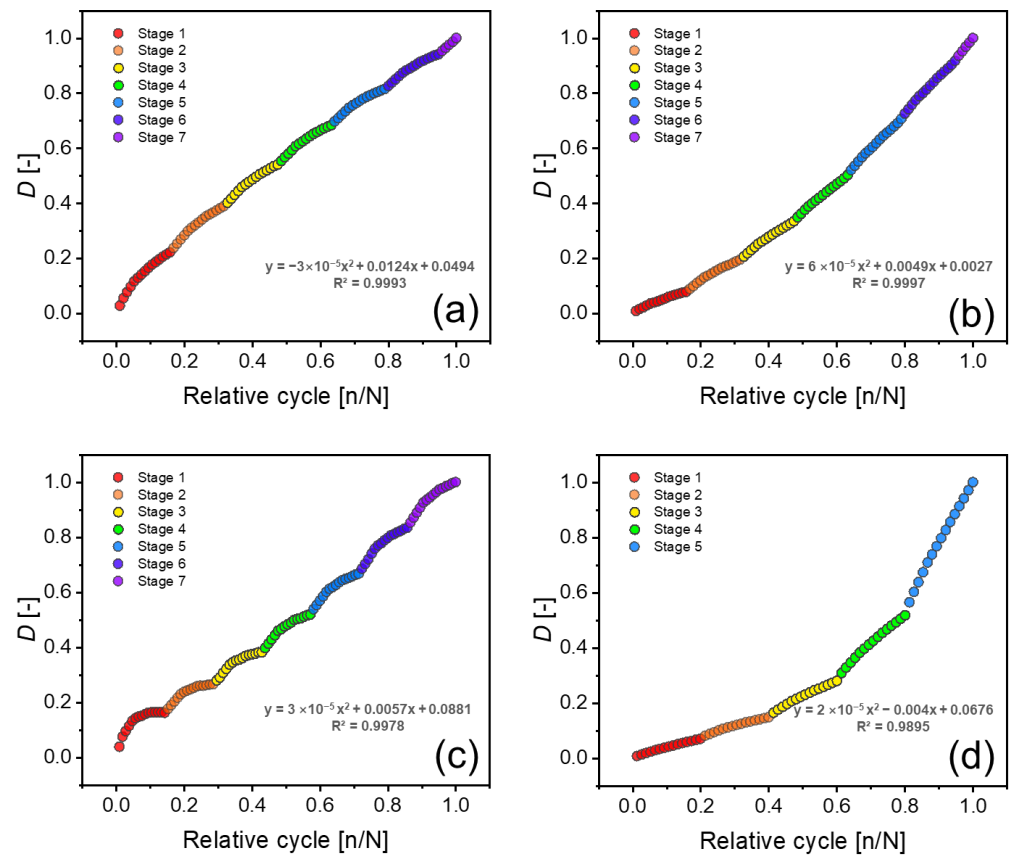


Figure 11. Evolution of dissipative energy damage variables with relative number of cycles: (a) specimen X54; (b) specimen X48; (c) specimen X53; (d) specimen X43.

In this study, Equation (7) was utilized to fit and quantify the dissipated energy damage variable, enabling the quantification of the cumulative damage of rocks under multi-stage cyclic loading and unloading. As shown in Figure 11, the fitting results of this equation exhibit high consistency with the experimental data for specimens of different amplitude intervals, and the coefficients of determination of all the fitted curves exceed 0.98, indicating a very high goodness of fit. This result validates the applicability of Equation (7) in describing the variation of dissipated energy damage variables and provides a reliable mathematical tool for the numerical analysis of rock damage evolution. Using this model, the damage accumulation process of rocks under cyclic loading can be more accurately predicted.

Figure 12 demonstrates the damage pattern of the silica specimens after testing, where the cracks are mainly distributed along the diagonal in a shear manner, representing a typical phenomenon of shear damage characteristics in rock compression tests. As the cyclic loading amplitude increased, a tendency for the width of primary cracks to become narrower in the four specimens was noted. This narrowing may be attributed to the closure or rearrangement of microcracks at higher stress levels. Furthermore, the number of secondary cracks decreased with increasing cyclic loading amplitude, likely due to the merging of microcracks or the increase in rock fragmentation at higher stress levels, resulting in less observable secondary cracks.

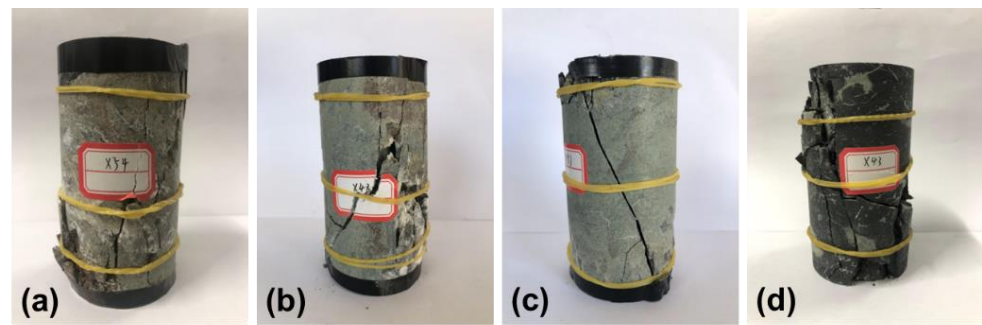


Figure 12. Specimen damage form diagram: (a) specimen X54; (b) specimen X48; (c) specimen X53; (d) specimen X43.

5. Conclusions

In this study, three different loading rates were designed, and cyclic loading tests were conducted under two paths of increasing and constant amplitude. The evolution of the dynamic shear modulus and damping ratio of silica rock under different strain amplitudes and loading rates was thoroughly investigated, and two damage models were subsequently established. The findings of this study demonstrate a complex relationship between the loading rate, damping ratio, and the mechanical behavior of rocks, which is crucial for understanding the response of rocks under various loading conditions. In particular, gaining a deeper understanding of the relationship between the loading rate and damping ratio is crucial for assessing the response of rocks under dynamic loads, such as earthquakes, and enhancing the safety of earthquake engineering. Future studies should continue to explore how multiple factors, including the loading rate, amplitude, and frequency, comprehensively influence the mechanical behavior of rocks to develop a more accurate rock mechanics model. This model can serve as a scientific basis for engineering design in earthquake engineering and related fields. The conclusions of this study are as follows:

1. Under constant amplitude loading conditions, the shear modulus tended to increase during both loading and unloading as the amplitude interval increased. Conversely, the shear modulus in the transverse amplitude test exhibited a decreasing trend throughout the test phase. Additionally, the shear modulus ratios (G_r) of two neighboring cyclic groups exhibited a sudden decrease, suggesting that higher loading rates could result in larger G_r values.
2. The damping ratio of the diorite initially decreases and then increases with the number of cycles during the cycling process, a trend observed under both constant amplitude and increasing value loading paths. The mean values of the damping ratios for each cyclic group at the three different rates exhibited a stepwise decrease in response to the loading rate.
3. The strain-based damage model revealed that the rock damage was larger under low-rate loading conditions, suggesting that the loading rate had a significant effect on the damage accumulation process. The model fitting captures that the damage variables dynamically change with an increasing number of cycles following an S-shaped function, offering a quantitative description of the damage evolution process.
4. Drawing on the correlation between the calculated dissipated energy and the relative cyclic number, a new damage model was proposed in this study. The model accurately predicts the dissipated energy density damage accumulation process of silica rock under cyclic loading, offering an effective tool for assessing the damage of rocks under cyclic loading.

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