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A Novel OFDM Approach Using Error Correcting Codes and Continuous Phase Modulation for Underwater Acoustic Communication

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Abstract: In this work, the performance of coded continuous phase modulation (CPM) transceivers for orthogonal frequency-division multiplexing (OFDM) in underwater acoustic channels is evaluated. The proposed technique employs the Bose–Chaudhuri–Hocquenghem (BCH) class of cyclic codes, and a CPM-based mapper is being used in place of a traditional OFDM mapper. Bit-error-rates (BERs) for various distances are evaluated. We use the Bellhop acoustic channel simulator that is considered to be very close to a real underwater channel. The performance of the proposed system is evaluated for 23 values of the CPM modulation index h as a function of the distance between the transmitter (Tx) and receiver (Rx). Based upon the error performance, we identified several best-performing CPM indices. We also observed that for Tx–Rx distances of 100 and 250 m, the proposed system gives acceptable performance without the need for equalization. We also compared the out-of-band radiation of the proposed system with PSK-OFDM and observed that the uncoded CPM-OFDM has a better out-of-band (OOB) performance than the traditional OFDM. Moreover, a comparison of BERs with the traditional PSK-OFDM system shows that the proposed system outperforms the traditional OFDM system by a wide margin.

Keywords: CPM; OFDM; UWA; underwater acoustic channel; BCH



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1. Introduction

Underwater acoustic (UWA) communication plays a pivotal role in addressing the challenges of transmitting information through the complex underwater environment [1]. It leverages sound waves as a means of communication, making it a viable solution for various underwater applications, ranging from oceanographic research and environmental monitoring to defense and offshore industries [2]. UWA communication relies on the propagation of sound waves through the water medium. Sound travels faster in water than in air, with a speed of approximately 1500 m per second, allowing it to serve as an efficient carrier of information over long distances [3]. Acoustic waves are also preferred because light waves scatter underwater [4], whereas radio waves undergo absorption. Key factors affecting underwater acoustic communication include the acoustic channel's characteristics, which are affected by factors such as temperature, salinity, and depth, as well as the presence of acoustic noise [5].

Underwater communication is one of the most challenging tasks due to the very harsh nature of the underwater medium [6]. The UWA channel has a very large delay spread and suffers from frequency selectivity and multipath as well as time selectivity and Doppler spread. Usually, acoustic communication takes place at low frequencies and bandwidth is

limited to the order of a few kilohertz, realizing a channel that is wideband in nature [7]. Single-carrier communication approaches, such as SC-FDE, are susceptible to inter-symbol interference (ISI), which requires complex equalization techniques. The increased complexity associated with such equalizers poses a significant obstacle to achieving high data rates in these systems [8,9].

OFDM is the scheme of choice in traditional wireless communication and is reliably used for high-data-rate communication [10]. It has been employed in UWA channels for high-data-rate communication over longer distances [11]. A key benefit of OFDM in underwater acoustics is its ability to subdivide the available bandwidth into orthogonal subcarriers. The orthogonality in subcarriers eliminates ISI, allowing for the use of longer symbol durations, which enhances robustness against the multipath fading commonly encountered in underwater channels [12]. The equalizer complexity is significantly reduced; however, with the increase in high-data-rate communication requirements and limited bandwidth, such as in an UWA channel, the symbol duration must be reduced, which will, in turn, increase the equalizer complexity [13]. Another benefit of employing OFDM in an UWA channel is that the equalization can be performed in the frequency domain [14]. Due to Doppler shifts, the orthogonality of sub-carriers can be severely affected in a time-variant channel, and it must be countered at the receiver [15]. A time-frequency synchronization method for UWA OFDM is proposed in [16]. Authors compared their technique with the traditional methods and showed that their proposed method could obtain the signal timing position more accurately and estimate the carrier frequency offset in complex multipath channels. In [17], the authors propose an effective deep-learning-based channel estimation technique. Their BER results demonstrate the superiority of their algorithm over other methods.

OFDM systems have traditionally relied on memoryless modulators such as QAM and PSK [18]. Memory-based modulation schemes such as continuous phase frequency shift keying (CPFSK) and continuous phase modulation (CPM) have been the subject of extensive research for several decades and possess a well-established foundation [19]. CPM is a digital modulation technique where the phase of the carrier signal is continuously varied according to the data to be transmitted, offering spectral efficiency and robustness. Scientists have employed it for its spectral efficiency and fewer out-of-band (OOB) leakages and for it having constant envelope together with phase continuity [20–22]. Combining CPM with OFDM creates a hybrid modulation technique called OFDM-CPM [23,24]. This approach leverages the strengths of both methods to enhance performance in wireless communication systems [25].

Bose–Chaudhuri–Hocquenghem (BCH) codes are a popular type of error-correcting code that are often used in wireless communication systems to enhance the reliability of data transmission [26]. BCH codes have been used in a variety of wireless communication systems, including cellular networks, Wi-Fi, and satellite communications. They are a valuable tool for ensuring that data is transmitted reliably and accurately. They are called block codes because they work on data in blocks, or groups, of binary bits. They can find and correct multiple errors within those blocks using a mathematical formula (polynomial) and the code's size, which is determined by the Galois field [27]. A direct linear relationship exists between the decoding complexity and the energy consumption of error-correcting codes. This dependency scales positively with both the code length and the number of errors to be corrected. Simply put, longer codes and a higher number of correctable errors necessitate significantly more complex decoding algorithms and, consequently, increase energy consumption [28].

In this paper, we propose a coded CPM modulator for OFDM in doubly-selective [29] UWA channels. Due to the complex nature of underwater channels and the high error rates in transmission, we observed that the presence of a block coder module significantly improves the error rate performance with regard to the spectral efficiency and smooth phase transitions of a CPM modulator. Since in a CPM, the data symbols are correlated, the receiver has more information when detecting the transmitted data [30,31]. Hence, it is nat-

ural for CPM-OFDM to have a better error performance than conventional OFDM. Channel codes are an integral part of a typical communication system because the error performance isn't good enough for practical purposes. Hence, it is a normal practice to evaluate the performance of an established scheme with channel coding schemes. Some examples of papers that investigated the performance of OFDM with channel codes are [32–35]. BCH codes are in use in underwater acoustic systems not only theoretically [36] but in industry as well [37]. The performance of CPM-OFDM with channel codes, especially BCH, has not been studied for underwater acoustic channels. Hence, investigation of CPM-OFDM with BCH codes is a natural extension. Moreover, in this work, we used a well-known statistical channel model [3] that closely resembles a real, practical underwater channel.

The following are some of the key contributions of this work:

- The performance evaluation of BCH-encoded CPM-OFDM over a statistical channel [38] that closely resembles a practical underwater channel.
- A detailed complexity analysis of BCH-encoded CPM-OFDM that takes into account the complexity of BCH encoding, BCH decoding, and CPM detection, and compares it with a conventional OFDM system.
- Simulation of a BCH-encoded CPM-OFDM system using 23 different values of the modulation index h , comparison with a conventional OFDM system, and evidence of the clear superiority and effectiveness of the proposed scheme.
- Simulation of a BCH-encoded CPM-OFDM system and performance evaluation by varying the transmitter–receiver distance from 100 m to 1000 m in steps of 150 m, and identification of the range for which the proposed system performs error-free transmission.
- Identification of the best performing modulation indices (h) as being $(\frac{7}{16}, \frac{9}{16}, \frac{5}{8}, \frac{3}{5}, \frac{3}{8}, \frac{2}{5}, \frac{5}{16})$.
- Evaluation of the spectrum efficiency of the proposed scheme, where it is concluded that the proposed scheme does not degrade the spectrum efficiency of the system.

The remaining paper is organized as follows. Section 2 details the state-of-the-art literature review. Section 3 explains the proposed coded CPM-OFDM scheme. A computational complexity analysis is presented in Section 4, while in Section 5, the simulation setup and the channel simulator used are described, along with presentation of our results. Finally, the paper is concluded in Section 6.

2. Literature Review

Underwater acoustic communication is a challenging area due to the unique characteristics of underwater channels, including multipath propagation, high propagation delays, and the presence of ambient noise [7]. Due to its ability to effectively address the challenges mentioned, OFDM has emerged as a leading contender in the domain of multicarrier modulation [29,39]. OFDM divides the transmission bandwidth into multiple orthogonal subcarriers, providing robustness against multipath fading and enabling efficient data transmission [40]. OFDM's effectiveness in combating the adverse effects of an underwater channel has been explored extensively. It offers advantages such as resistance to multipath fading and support for high data rates [40]. Research has focused on adapting traditional OFDM to an underwater channel's unique characteristics, including the use of frequency-domain equalization and adaptive modulation schemes [41,42]. OFDM systems in underwater environments require robust channel estimation and equalization techniques due to time-varying and frequency-selective channels [43].

In [44], the authors proposed a low-complexity scheme for PAPR reduction in underwater acoustic OFDM systems using XOR-ciphered BCH codes. The system enhances the reliability of communication through the use of error-correction codes and improves the PAPR of the system. In CPM-based modulators, correlation among the symbols is exploited through the continuity of phase from one symbol to another adding memory to the system [31]. Some of the recent works in the wireless communication domain [45–47] have focused on CPM modulators. The CPM modulator is a kind of finite-state machine that sends a continuous phase, constant envelope waveform to the channel based on the

input stream and the internal state. Moreover, the authors in [24] evaluated a CPM-OFDM transceiver for the first time in the context of UWA channels. The traditional memoryless mapper was replaced with a CPM-based mapper and a Viterbi decoder was employed at the receiver side. Monte Carlo simulations over a realistic acoustic channel model suggested that the proposed setup can reliably communicate at distances of up to 1.5 KM without an equalizer and channel coding. Most of the previous implementations have been in the domain of radio frequency transmissions, such as [48], where the authors demonstrated the viability of an OFDM transceiver for aeronautical telematics using convolutionally encoded CPM. By exploiting space-time coding in a multiple input multiple output (MIMO) context, the CPM decoding complexity can be significantly reduced [49]. The authors in [50] had evaluated a CPM-based OFDM system for added a white Gaussian noise (AWGN) channel and proposed that the modulation index of 0.75 had superior performance as compared to the other modulation indexes used in their proposed system. The results were validated using a software defined radio system. According to a systematic review of the literature, no study has been conducted to date that examines the performance of coded CPM-OFDM for BER enhancement across UW channels. As a result, the proposed effort is the first of its type and adds considerably to the body of knowledge.

3. System Architecture

This section explains the fundamentals of continuous phase modulation and different components of the CPM-OFDM transceiver.

3.1. Continuous Phase Modulation (CPM)

Two advantages of CPM have led to its widespread application. The transmitted signal's constant envelope and lack of amplitude variations are the first of these. Constant envelope modulations are required in applications where completely saturated, non-linear RF power amplifiers are required due to power supply constraints. Constant envelope modulations are also helpful in situations requiring low-cost, basic transmitters. Bluetooth technologies and digital FM land mobile radios are two instances of this. Overall, CPM is particularly transmitter-friendly because of the modulation's constant envelope feature [51]. The constant envelope of CPM happens in single carrier communication. For the proposed OFDM system, since CPM is employed in the mapper, the transmitted signal does not have a constant envelope.

Power efficiency and bandwidth are CPM's other advantages. The size of the data alphabet (binary, M-ary, etc.), the modulation index or indexes, and the frequency pulse's duration and shape define a specific CPM format. One can regulate the signal's spectrum and power efficiency by carefully choosing these parameters. Because of this, CPM is incredibly spectrum-friendly and adaptable [51].

The transmitter and the transmission medium, which make up the first two components of the communications link, benefit from these advantages (constant envelope, power efficiency, and spectrum efficiency). CPM, however, does not handle the receiver—the last component of the communications link—well. The signal is nonlinear because of its constant envelope, which indicates that it is not a linear function of the transmitted data. This complicates the process of demodulating and synchronizing the signal. Moreover, employing longer, smoother frequency pulses and expanding the size of the data alphabet are common ways to increase spectral efficiency. These also make the receiver more complex. Despite having a complex receiver, CPM still possesses a lot of potential in improving wireless and underwater acoustic communication.

In this study, we will utilize complex-baseband notation to represent various signals, operating on the understanding that the outcomes are applicable in a carrier-modulated environment. The complete set of CPM signals can be denoted as follows [20]:

$$s(t; \alpha) = \sqrt{\frac{E}{T}} \exp(j\psi(t; \alpha)) \quad (1)$$

Here, E stands for symbol energy, T represents symbol duration, and $\psi(\cdot)$ denotes the phase of the signal. As evident from this equation, the CPM signals exhibit a constant envelope, a characteristic that adds to the appeal of this modulation type. All the information is encoded in the signal phase, which is given by [52]:

$$\psi(t; \alpha) \equiv 2\pi \sum_{i=-\infty}^n \alpha_i h_i q(t - iT), \quad nT \leq t < (n+1)T \quad (2)$$

In this equation, $\{h_i\}$ is a set of modulation indexes with N_h elements, $\alpha = \{\alpha_i\}$ represents data symbols drawn from an M -ary alphabet, and $q(t)$ signifies the phase pulse. The subscript notation with an underline in Equations (1) and (2) is defined as modulo- N_h , i.e.,

$$\underline{i} \triangleq i \bmod N_h \quad (3)$$

We assume that the modulation indexes are rational numbers of the form [27]:

$$h_{\underline{i}} \triangleq \frac{K_i}{p} \quad (4)$$

Here, p is determined by expressing all modulation indexes as fractions and selecting p as the smallest common denominator. The case where $N_h = 1$ is recognized as single- h CPM, and the less common scenario where $N_h > 1$ is termed multi- h CPM. When the numerator of $h_{\underline{i}}$ is even, the phase states $\psi(t; \alpha)$ are given as

$$\left\{ 0, \pi \frac{K_i}{p}, 2\pi \frac{K_i}{p}, \dots, (p-1)\pi \frac{K_i}{p} \right\} \quad (5)$$

If the numerator of h_i is odd, the phase states $\psi(t; \alpha)$ are given as

$$\left\{ 0, \pi \frac{K_i}{p}, 2\pi \frac{K_i}{p}, \dots, (2p-1)\pi \frac{K_i}{p} \right\} \quad (6)$$

For example, if $h = 2/5$, there will be $p = 5$ phase states given by $\{0, 2\pi/5, 4\pi/5, 6\pi/5, 8\pi/5\}$. Similarly, if $h = 1/4$, there will be $2p = 8$ phase states given by $\{0, \pi/4, 2\pi/4, 3\pi/4, 4\pi/4, 5\pi/4, 6\pi/4, 7\pi/4\}$. In this work, we use a single- h CPM employing a fixed h .

The phase pulse $q(t)$ can be understood as the time integral of a frequency pulse $f(t)$ with an area of $1/2$ as shown below:

$$\int_0^{LT} f(\tau) d\tau = q(LT) = \frac{1}{2} \quad (7)$$

When the time interval is outside of $(0, LT)$, the frequency pulse is zero. The signal is referred to as a full response when $L = 1$ and a partial response when $L > 1$.

3.2. BCH Codes

To reduce bit error rates in communication systems, BCH has been found to be an effective technique for error correction [26]. Instead of working on individual data bits, BCH codes are a category of cyclic codes that operate on groups of data bits or blocks [27]. They find utility in a variety of applications because of their versatility in fixing many errors and the ease in coding and decoding implementations. The decoding energy consumption of BCH codes is a linear function of both the code word length and the capability of a code to correct errors [28].

Although BCH codes are derived from Hamming codes, these codes can remedy multi-bit errors while Hamming codes can only correct one-bit faults. Their code generators are well documented [53], making it easy to employ many kinds of BCH codes. This makes them one of the more potent groups of cyclic codes. BCH code generators that are

frequently used to create BCH codes can be found for a range of values of n , k , and t , up to a block length of 255 [54], using the table provided in [53].

3.3. CPM-OFDM Transmitter

The CPM-OFDM transmitter is shown in Figure 1. Let a stream of binary bits denoted by a matrix B be the input into a BCH encoder. The matrix B can be represented as

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_k \end{bmatrix} \quad (8)$$

where

$$B_i = [b_{i,1}, b_{i,2}, \dots, b_{i,m}] \quad (9)$$

$b_{i,u}$ is the u th data bit out of the m total bits in i th uncoded binary word while k is the total number of uncoded binary words that will be sent to the transmitter. These uncoded binary words pass through an (n, m) BCH encoder as m -bit words at a time. The encoder outputs n -bit encoded words represented by E_i while the matrix of all the encoded words is represented by E , as shown below.

$$E = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_k \end{bmatrix} \quad (10)$$

where

$$E_i = [e_{i,1}, e_{i,2}, \dots, e_{i,n}] \quad (11)$$

$e_{i,u}$ is the u th data bit out of the n total encoded bits in i th encoded binary word while k is the total number of encoded binary words that will be output by the BCH encoder. Next, the encoded bits pass through the serial to parallel converter—shown as S/P block in Figure 1. This block has N outputs, where N is the total number of subcarriers in the system. Each output of the S/P block is a data symbol α_i drawn from an M -ary alphabet, where M is the CPM modulation order. The data symbols α_i then pass through the CPM mapper. The outputs of the N CPM mappers can be represented as a vector of complex numbers C_u given by

$$C_u = \begin{bmatrix} c_{u,0} \\ c_{u,1} \\ \vdots \\ c_{u,N-1} \end{bmatrix} \quad (12)$$

where, for the u th OFDM block and for the l th subcarrier, the complex number $c_{u,l}$ is defined as [55].

$$c_{u,l} = \cos \psi_{u,l}(t; \alpha) + j \sin \psi_{u,l}(t; \alpha) \quad (13)$$

The phase $\psi_{u,l}(t; \alpha)$ for u th OFDM block and l th subcarrier has been defined in Equation (2). The complex numbers from the output of the CPM mapper are input into the IFFT block. The output of the IFFT block is a discrete time signal Φ whose p th sample for the u th OFDM symbol is given by

$$\Phi_{u,p} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \phi_{u,l} e^{j \frac{2\pi l p}{N}}, \quad p = 0, 1, \dots, N-1 \quad (14)$$

where $\phi_{u,l}$ is the output of the IFFT block for the u th OFDM symbol and the l th subcarrier and $\sqrt{N}$ is a scaling factor to normalize the symbol energy. To facilitate symbol synchro-

nization and to minimize the effects of multipath, the last G samples ($G \leq N/4$) of the OFDM symbol are copied and put as a preamble in the OFDM symbol called cyclic prefix. The resulting u th OFDM symbol can then be represented as

$$\Phi_u = [\Phi_{u,N-G+1}, \Phi_{u,N-G+2}, \dots, \Phi_{u,N-1}, \Phi_{u,0}, \Phi_{u,1}, \dots, \Phi_{u,N-1}] \quad (15)$$

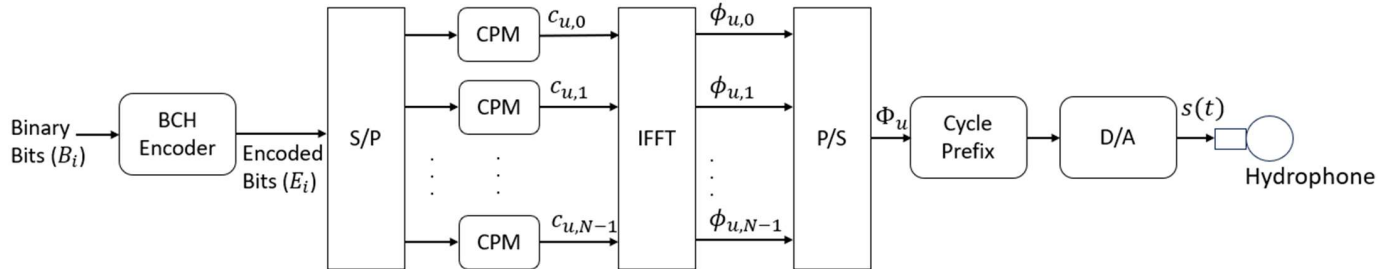


Figure 1. The proposed coded CPM-mapper-based OFDM transmitter.

To convert the digital signal into an analog signal, a D/A converter is used that outputs the analog signal represented as

$$s(t) = \frac{1}{\sqrt{T}} \sum_{l=0}^{N-1} \phi_{u,l} e^{j2\pi f_l t} \quad (16)$$

In this equation, $f_l = l/NT_s$, NT_s is the OFDM symbol duration, T_s is the sampling time, and T is the total symbol duration with guard interval. The signal $s(t)$ is then transmitted and passes through an acoustic underwater channel.

3.4. CPM-OFDM Receiver

With reference to Figure 2, the received signal $r(t)$ passes through the A/D converter, the cyclic prefix remover, the S/P converter, and the FFT blocks. The output of the FFT block is the estimated complex numbers that can be represented as a vector $\hat{C}_u$, as shown below.

$$\hat{C}_u = \begin{bmatrix} \hat{c}_{u,0} \\ \hat{c}_{u,1} \\ \vdots \\ \hat{c}_{u,N-1} \end{bmatrix} \quad (17)$$

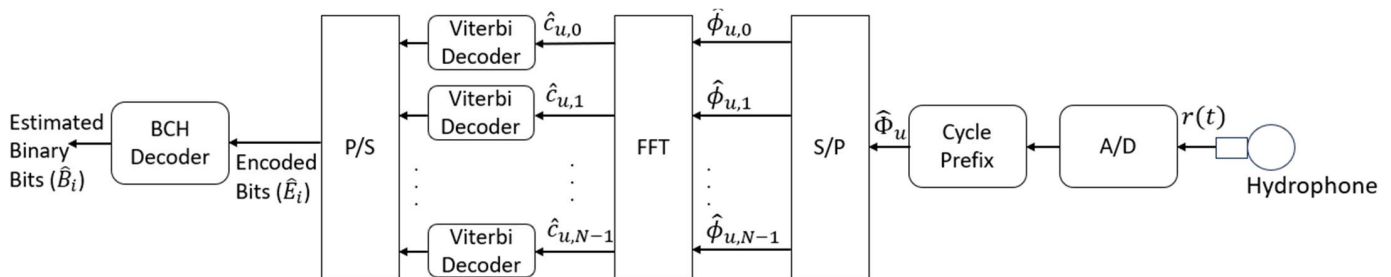


Figure 2. The proposed coded CPM-OFDM receiver.

Each of these estimated complex numbers are input into a Viterbi decoder which is a maximum likelihood decoder [56–58]. For all practical applications, the Viterbi decoder is used to estimate the symbols that have been transmitted using CPM [59]. The output of the Viterbi decoder passes through the P/S converter before being input into the BCH decoder. The (n, m) BCH decoder takes n encoded bits as its input and outputs m decoded bits, which is the estimated binary transmitted data represented as $\hat{B}_i$. When simulating this

system, the difference between B_i and $\hat{B}_i$ is used to compute the bit error rate (BER). It is noted that the proposed system does not employ any channel estimation and equalization techniques. This is to demonstrate the effectiveness of using a CPM mapper with OFDM. As will be shown in the section on simulation results, the performance of the proposed system is drastically better than the traditional systems even without incorporating channel estimation and equalization.

4. Complexity Analysis

In this section, we analyze the complexity of the proposed system and compare it with that of the conventional OFDM system. In the proposed system, there are three main blocks where the computational burden is significant. On the transmitter side, these are the BCH encoder, the CPM mapper, and the FFT block. On the receiver side, the main computational burden is due to the BCH decoder, the FFT block, and the Viterbi decoder used to decode the CPM-mapped symbols. The complexity of FFT will be the same for the proposed system and the traditional OFDM system, which is known to be $O(N \log N)$ when N is in powers of two [60].

Traditional OFDM systems incorporate channel estimation and equalization. As will be shown in the section on simulation results, certain configurations of our proposed system give error-free performance without any equalization. Hence, the computational burden due to equalization will only be for the traditional OFDM systems, which is $O(N^2)$ whether a zero-forcing (ZF) equalizer is used or a minimum mean square error (MMSE) equalizer is used [61]. It is evident that this is too high a computation burden for even moderate values of N . When compared with the additional computation burden due to BCH and CPM, it can be concluded that the conventional OFDM that uses equalizers is comparable to the proposed scheme, if not worse, in terms of computational complexity. Table 1 summarizes the complexity involved with these blocks, while a detailed analysis of the complexity of BCH and CPM is given below.

Table 1. Computational Complexity.

	Proposed OFDM	Conventional OFDM
BCH Encoder	$O(kL) + O(nL \deg(g))$	-
BCH Decoder	$O(Lnt)$	-
FFT and IFFT	$O(N \log N)$	$O(N \log N)$
Viterbi Decoder	$O(pN)$	-
Equalizer	-	$O(N^2)$

4.1. BCH Encoder and Decoder

With generator polynomial $g(x)$, let C be an $[n, m]$ BCH codeword over $GF(q)$. The number of message bits is denoted by m , and the number of encoded bits is represented by n . The Galois field (GF) has q elements, where $q = 2^k$ for BCH codes [31]. The integer k is such that $n = 2^k - 1$ and $k \geq 3$. The following is a description of the process used to generate a codeword C [62]:

1. The message vector $(u_0, u_1, \dots, u_{m-1})$ and its corresponding polynomial, $u(x)$, are defined.
2. The remainder $r(x)$ is determined by computing $x^{n-m}u(x) \bmod g(x)$
3. This means that $c(x) = x^{n-m}u(x) - r(x)$ is the transmitted codeword.

One can obtain the residual $r(x)$ by doing a long division of $x^{n-m}u(x)$ by $g(x)$ [63]. In this procedure, $O(n \deg(g))$ arithmetic operations over $GF(q)$ are needed to compute the remainder, while $O(k)$ operations are needed to compute the first term of the codeword. In an OFDM system, each subcarrier carries $\log_2 M$ bits, where M is the alphabet size. Therefore, the total number of bits in one OFDM symbol will be $N \log_2 M$, where N is the

number of subcarriers. It is then easy to show that when using BCH encoding in OFDM, the number of codewords required for one OFDM symbol is given by:

$$L = \text{floor} \left[\frac{N \log_2 M}{n} \right] + 1 \quad (18)$$

In the above expression, $\text{floor}[x]$ will output the greatest integer less than or equal to x . Hence, the arithmetic operations required to generate codewords for one OFDM symbol will be:

$$(O(kL) + O(nL \deg(g))) \quad (19)$$

Moreover, since the code rate is m/n , the data rate has to be increased by a factor of n/k to compensate for the additional $(n - m)$ redundant bits being sent with the message.

The complexity of the BCH decoder is $O(nt)$, where t is the maximum number of errors corrected by the code [64,65]. For OFDM, it is clear that the complexity to decode one OFDM symbol will be $O(Lnt)$.

4.2. CPM Mapper and Decoder

The complexity due to CPM comes mainly from the Viterbi decoder. For rational values of the modulation index h , the number of trellis states in a Viterbi decoder are equal to p (p has been defined in (5) and (6)), which is also the number of operations required for decoding one symbol, i.e., $O(p)$ [52]. For one OFDM symbol, the total number of operations will be $O(pN)$.

5. Simulation Results

The performance of the proposed system is evaluated by simulating the coded CPM-OFDM transceiver over a realistic underwater acoustic channel. In this work, we evaluated the performance of a binary CPM-OFDM system. Other simulation parameters are shown in Table 2.

To generate a realistic UWA channel, we used a Bellhop simulator [3,7]. We generated seven typical UWA channel profiles for seven transmitter–receiver distance scenarios. The channel scenarios for four of these distances are shown in Figure 3, while a typical power delay profile is shown in Figure 4. One of the reasons for these varying profiles is the multipath effect of the channel, which increases with increasing distance [66]. When a signal travels over unguided media such as a radio channel or underwater acoustic channel, the antenna (in the case of radio transmission) and hydrophone (in the case of underwater channels) transmits the signal in more than one direction, thus, creating several copies of the signal. The farther away the receiver, the more the transmitted signals get reflected from obstacles in the environment and the more multipaths are created.

Table 2. Simulation parameters.

Parameters	Values
Number of subcarriers	256
CP Length	64
Modulation index	23 rational values
Bandwidth	10 kHz
OFDM symbol duration	25.6 ms
Channel Encoder	BCH(63,10), BCH(63,16), BCH(31,6), BCH(31,11)
Tx–Rx distance	[100, 250, 400, 550, 700, 850, 1000] m
Transmitter depth	100 m
Receiver depth	100 m
Speed of sound in water	1500 m/s
Speed of sound in bottom	1200 m/s
Coherence time	60 s

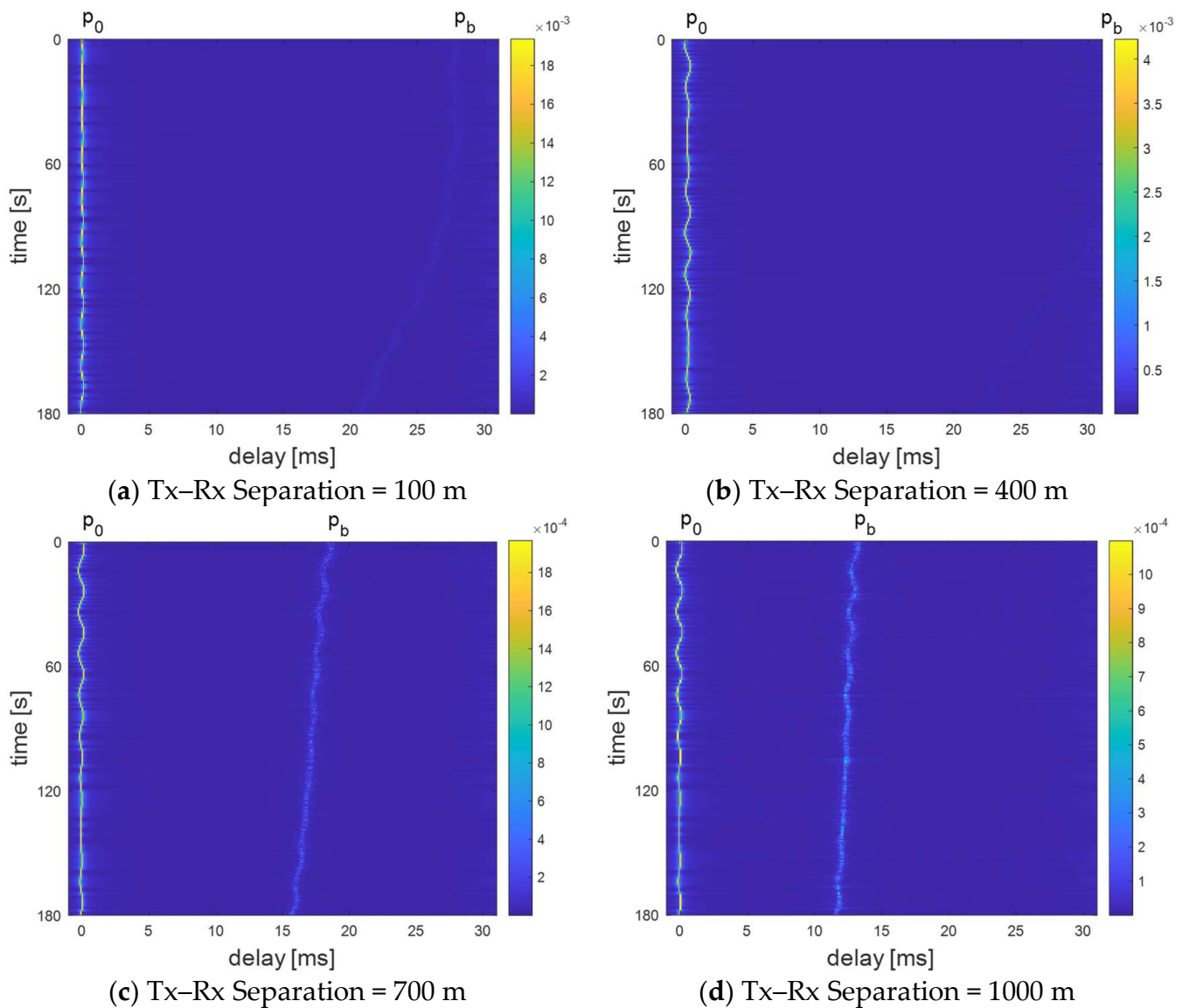


Figure 3. Channel profiles for four typical channel distances.

To simulate the proposed system, 5000 OFDM blocks were transmitted. A BCH(63,10) encoder is used for encoding the data. With reference to Equation (4), the numerator K_i is varied between 1 and 15 to choose the values of the modulation index h . To maintain the rationality of the modulation index, we varied the denominator p between 2 and 16. As a result, we had 23 values of h in total that are shown in Table 3. We determined that this set is adequate to evaluate the intended system's performance, and that any other numbers outside this range do not result in any better performance. Furthermore, choosing non-rational values of h , such as $1/3$ results in a constellation with too many points that increases the complexity of the Viterbi decoder manifolds, is without any benefit.

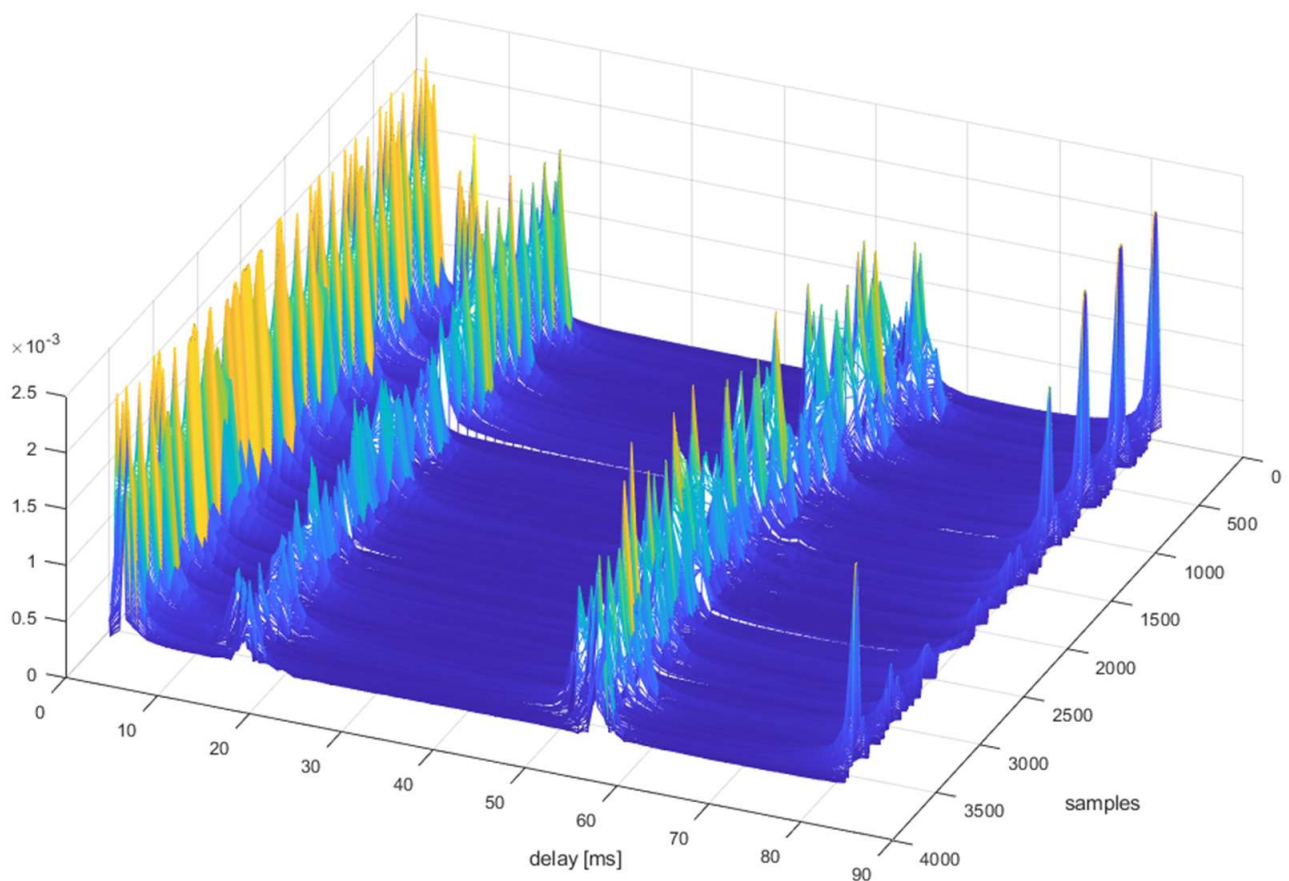


Figure 4. Power delay profile of the UWA channel generated using Bellhop for the channel of Figure 3c.

Table 3. Values of modulation indices.

Numerator (K_l)	Denominator (p)	Modulation Index (h)
1	2	0.5
1	4	0.25
1	5	0.2
1	8	0.125
1	10	0.1
1	16	0.0625
2	5	0.4
3	4	0.75
3	5	0.6
3	8	0.375
3	10	0.3
3	16	0.1875
4	5	0.8
5	8	0.625
5	16	0.3125
7	8	0.875
7	10	0.7
7	16	0.4375
9	10	0.9
9	16	0.5625
11	16	0.6875
13	16	0.8125
15	16	0.9375

In Figure 5, the BER of the proposed system is plotted against the channel distance. Each of the 23 curves of this figure is plotted for a fixed value of h . The legend on the right-hand side of the figure enlists the h values used for each curve. The curves have been plotted starting from the best BER to the worst BER and the h values have been listed in the legend in the same order. For example, the first curve is for $h = 7/16$, which gives the best BER performance and is the first value in the legend. It is observed that there is no error when transmitting the signals over a channel distance of 100 and 250 m for the first five values of h . Hence, these should be considered as error-free transmissions. It is also noted that the values of h that give superior performance are

$$\left\{ \frac{7}{16}, \frac{9}{16}, \frac{5}{8}, \frac{3}{5}, \frac{3}{8}, \frac{2}{5}, \frac{5}{16}, \frac{11}{16}, \frac{3}{10}, \frac{7}{10}, \frac{3}{16}, \frac{1}{4}, \frac{13}{16}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}, \frac{1}{5}, \frac{1}{8}, \frac{7}{8}, \frac{1}{10}, \frac{9}{10}, \frac{1}{16}, \frac{15}{16} \right\}$$

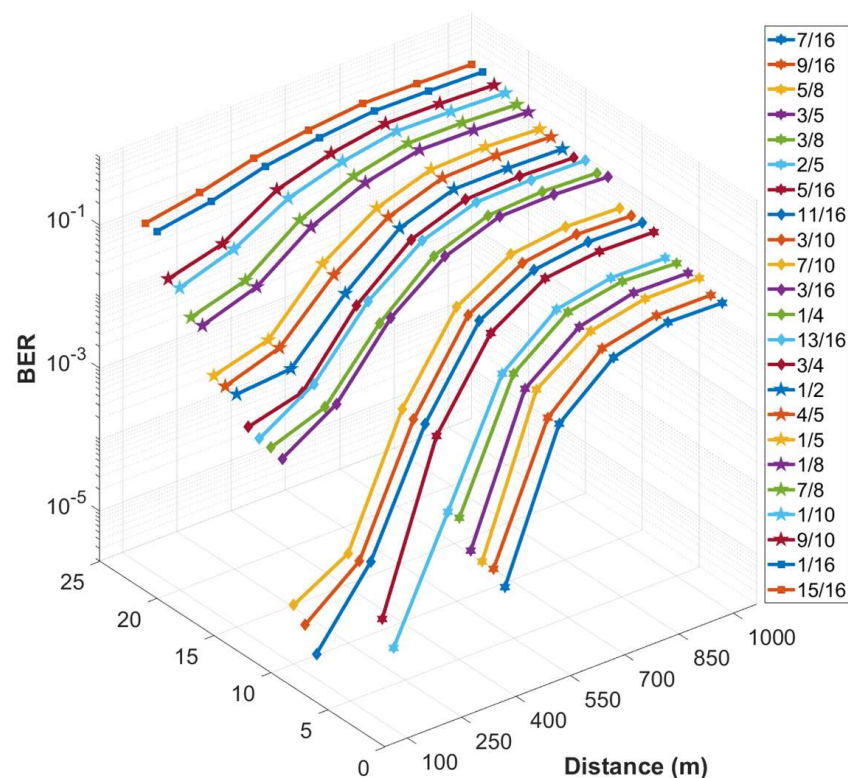


Figure 5. BER performance of coded CPM-OFDM for 23 values of h over seven transmitter–receiver distances.

In the above set, only the sixth value of h , i.e., $2/5$, will have five phase states, as the value in the numerator of h is even. On the other hand, the numerators for the rest of the h values are odd and will have $2p$ phase states, which is, relatively, a much higher number, making the Viterbi decoder complex.

To compare the performance of the proposed system with that of a traditional OFDM system, we plot coded CPM-OFDM alongside PSK-OFDM for four typical h values in Figure 6. In all these plots, for each value of h , BER is plotted against the channel distance. Each plot shows the performance of four scenarios as follows:

1. Uncoded PSK-OFDM
2. BCH-encoded PSK-OFDM
3. Uncoded CPM-OFDM
4. BCH-encoded CPM-OFDM

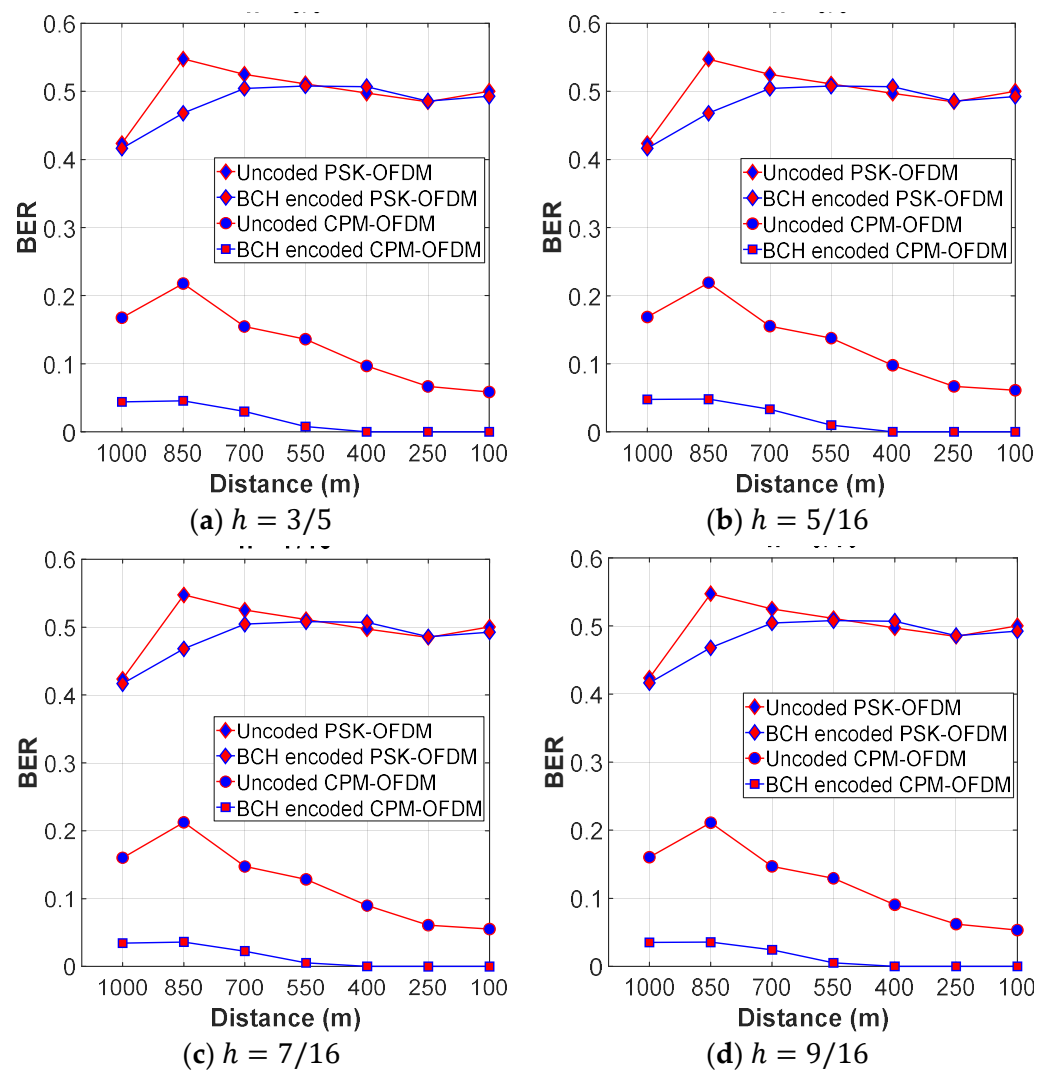


Figure 6. Comparison of BER performance of the proposed system with traditional one.

The superiority of the proposed system is clearly visible. Moreover, it is observed that the difference in performance for the four h values is barely noticeable. This gives an added advantage to the designer selecting such a value of h that would optimize the implementation of the proposed system, especially on the receiver side. Furthermore, for these h values, there is a significant difference in performance between the uncoded and BCH-encoded systems. It is also observed that for channel-free distances of 100 and 250 m there is no need for equalization as the transmission is error-free. At other distances, equalization will be required for acceptable error performance.

Figure 7 compares the BER performance of the proposed system with four different types of BCH codes, viz. a viz., BCH(63,10), BCH(63,16), BCH(31,11), and BCH(31,6). It is observed that except for BCH(31,11), all the other BCH codes achieve zero-error performance for distances of 250 and 100 m.

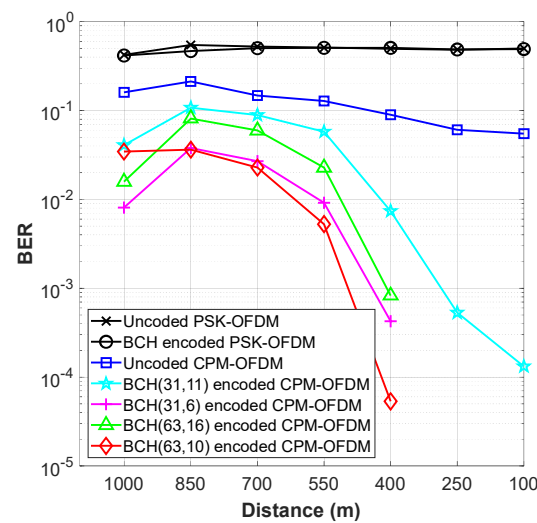


Figure 7. Comparison of BER performance of the proposed system using four different BCH codes when $h = 7/16$.

A comparison of OOB radiation appears in Figure 8. It is observed that, by and large, CPM-OFDM has a better OOB performance than the traditional PSK-OFDM, especially for the uncoded systems. For the BCH-encoded system, the performance of both the systems is pretty much the same.

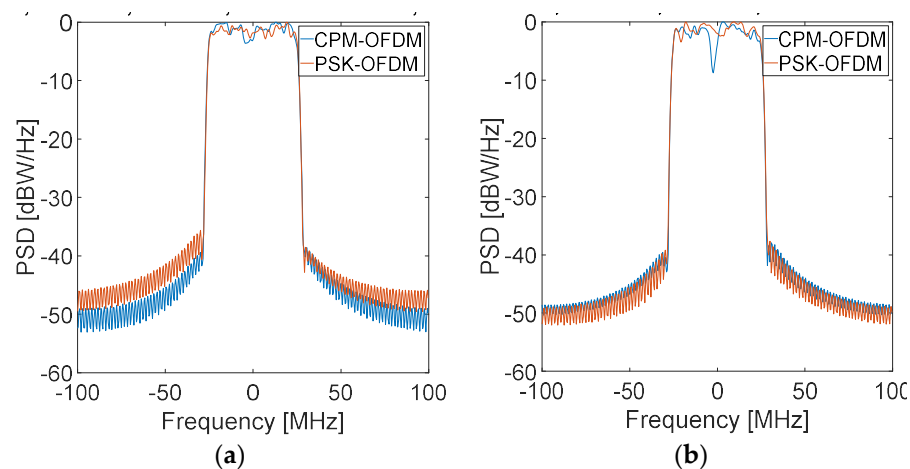


Figure 8. (a) OOB comparison of uncoded CPM-OFDM with that of PSK-OFDM and (b) OOB comparison of BCH-encoded CPM-OFDM with that of PSK-OFDM.

6. Conclusions

It has been demonstrated via computer simulations that BCH-encoded CPM-OFDM is a promising option for underwater acoustic communication. Without using a complex equalization method, this system provides good error performance. There are several values of modulation index h that yield good error performance. Furthermore, for a channel distance of 100 meters between the transmitter and the receiver, the error performance is so good that no equalization is required. This is at the expense of a complex receiver due to CPM and the reduction in effective information rate due to the use of BCH encoding, which can be overcome by increasing the overall bit rate. An analysis of the computational complexity of the proposed system has also been presented and compared with that of conventional OFDM. In this study, as proof of concept, we assessed the performance of a binary CPM-OFDM system using a BCH encoder as an example. The performance of an M-ary BCH-encoded CPM-OFDM system remains to be explored. Moreover, several other

variants of BCH encoding schemes can be used to evaluate the system performance and identify more effective encoding schemes for reliable underwater acoustic communication.

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