Article

2D Numerical Analysis of Prefabricated Vertical Drains Using Different Matching Methods

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Abstract: A full-scale embankment on soft clays improved with prefabricated vertical drains (PVDs) have to be analyzed in 3D conditions due to a great number of vertical drains under an embankment. However, 3D analysis is very complex, time-consuming, and needs a powerful computer. Therefore, axisymmetric vertical drains have to be converted into equivalent plane-strain conditions for 2D analysis. Different matching approaches based on unit cell concept have been developed in the literature and the matching can be achieved by modifying the drain distance and/or soil permeability according with relatively simple instructions. This paper investigates verification of three different matching approaches to be used in the numerical analysis of full-scale embankment built on multiple vertical drains. The elasto-plastic soft soil model was used in the numerical analysis, and the results are compared with the laboratory and field measurements. The results of numerical analysis demonstrate that the matching methods are in extremely good agreement with the measurements if the effect of both the smear zone and discharge capacity are taken into consideration. It is seen that these methods provide practical solutions and important advantages to geotechnical engineers.

Keywords: prefabricated vertical drain; numerical analysis; consolidation; matching; embankment

1. Introduction

In recent years, construction projects around the world tend to focus more on low-lying areas, such as peat and organic soil, as a result of rapid population growth. However, these soil layers have a significant impact on vital infrastructures consisting of roads, buildings, and railways due to their relatively low load-bearing capacity [1,2]. Thus, it is extremely important to consolidate the soil clay layers before starting any construction projects to prevent large settlement. The ground can be consolidated by applying loads to stabilize superstructure work. However, the low vertical permeability requires a considerable amount of time to achieve the target settlement compared to the available construction period [3,4]. In this case, various vertical drainage systems, such as sand compacting piles and prefabricated vertical drains, have become extensively employed to enhance the soft ground. This idea was initially presented in the 1920s and published in 1926 by an American engineer, Daniel Moran. According to Johnson [5], Moran proposed using sand drains for the first time to stabilize the mud soil below the highway approach to the San Francisco Oakland Bay Bridge. However, Aboshi [6] explained that vertical sand drain behavior was not well-known, since the foundation-bearing capacity was assumed to be adequate for a full load right away following installation, resulting in common foundation collapse. Consequently, Kjellman [7] introduced ground improvement techniques using PVDs, utilizing cardboard wick drains. The technique of installing prefabricated vertical drains (PVDs) under embankment is among one of the most widely employed ways for shortening the consolidation process time which, to dissipate excess pore pressure rapidly, also provide horizontal drainage in addition to vertical, [3]. By using PVDs, the drainage line is reduced from the soil layer’s thickness to the drain’s effect zone’s radius, which
speeds up consolidation [8]. The durability and performance of PVD are dependent on the specific polymers employed to create the drain and soil properties at the place of installation, which is determined by its discharge capacity [9,10]. The discharge capacity of PVD is affected by a variety of parameters, such as the type and form of the core, the cross-sectional area of the drain, and the effect of lateral pressure [9,11,12]. Some researchers evaluated various types of band-shaped drains in the laboratory to assess the performance of a bent PVD [9,13–17]. Chai et al. [15] discovered that the discharge capacity of PVD decreased by up to 10% approximately 200 days of installation. Faisal [9] stated that, to ensure that PVD perform properly, the flow capacity must meet a minimum discharge capacity requirement, considering the effects of lateral pressure, drain deformation, and creep. Jamiolkowski et al. [13] found that discharge capacity should be at least 10–15 m/year under 300–500 kPa lateral stress and for drains up to 20 m in length. However, the average laboratory test values of discharge capacity evaluated by Jamiolkowski et al. [13] were inadequate, and the amount of acceptable discharge capacity was subsequently increased to 100–150 m$^3$/year in soft clay [14]. Several researchers have highlighted the benefits of PVDs, which include an improvement in soil strength [18], reduce the amount of surcharge material needed to attain the specified precompression, and show a reduction in total settlement time as well [19]. Moreover, this method is also capable of improving the rate of road construction while simultaneously having a favorable influence on the environment [20].

However, despite improvements in comprehending the behavior of embankment on soft soil in last decade [21–24], it remains a problem, and it is difficult in geotechnical engineering to estimate the settlement and excess pore pressure (EPP) of improved subsoils with PVDs. Furthermore, field testing is frequently both a time-consuming and expensive process [25]. Therefore, sophisticated numerical approaches, like Finite Element Analysis (FEA), are highly beneficial. In the design phase, engineers tend to prefer numerical methods, as it can reduce project costs and duration when compared to analytical approaches [26]. Moreover, this approach can help engineers solve extremely complex soil, as well as for its overall ability to model deformations and to determine collapse [26,27]. Apart from the numerical simulations, geotechnical engineering researchers have created and used artificial intelligence (AI) approaches over the last three decades. These approaches were selected for their capacity to anticipate complicated nonlinear interactions [28].

2. Objectives and Scope

During the consolidation of subsoil-improved-PVDs, the water flows both radially and vertically, requiring three-dimension (3D) numerical analysis to model the effects of PVDs [29]. However, 3D modelling necessitates a significant amount of time and an efficient computer. In reality, hundreds of drains are installed under an embankment in a large project. FEA of PVDs frequently encounters problems with their size in relation to the overall dimensions of the issue [30]. PVDs typically have a square section of 100 mm × 4 mm, which is significantly less than the width of a normal road embankment, so they are commonly constructed at a spacing of 1–2 m. Furthermore, through a 3D analysis, the exact shape of the drain section has to be depicted in multiple soil layers [30]. Therefore, finite element meshes must be extremely fine in order to correctly represent the geometry of the PVDs, which results in significant model preparation and computational costs and mesh error as well.

Therefore, in order to reduce the computation time, most available finite element analyses on embankment stabilized with PVDs are conducted for 2D plane-strain conditions [24,31,32]. Barron [33] and Hansbo [8] were introduced the initial consolidation concept of fine-grained soils using single drain wells. Nevertheless, certain negative aspects are found regarding this research, such as how the influence of depth on the well was not taken into consideration, as well as the distinction in elasticity between soil surrounding and elements in the sand well [34]. Following that, various approaches were developed to match the equivalent value of the axisymmetric analysis with the plane-strain conditions [27,35]. Shinsha et al. [36], Hird et al. [37], Bergado et al. [38], Lee et al. [39], Indraratna
et al. [40], Lin et al. [41], Chai et al. [15], Tran et al. [42], and Ba-Phu [43] proposed different approaches for analyzing the soft soil stabilized with PVD in plane-strain conditions. These approaches were basically applied to unit cell concept using a single PVD, and FEA converts from axial symmetry to plane-strain conditions by altering the distance between vertical drains and/or the permeability of the ground around the vertical drain.

The main objective of this research is to analyze the accuracy and feasibility of these matching methods in FEA by considering the smear zone, which is the soil disturbance due to the placement of PVDs.

All matching methods were developed based on the unit cell concept with single vertical drain. For this purpose, large-scale consolidometer experimental research was carried out by Rujikiatkamjorn [44] and Saowapakpiboon et al. [45] in the laboratory. For this reason, in this paper, the experimental of laboratory consolidometer was examined with FEA to analyze the accuracy of these matching methods. In this section, the laboratory consolidometer experiment in the literature (Saowapakpiboon et al. [45]) is first modeled as axisymmetric, and the parameters used in the analyzes are validated. Then, the experimental setup is converted to plane-strain conditions using matching methods, and the numerical analyses are compared with the experimental measurements.

In the second part of this study, a real embankment with PVD-improved multilayered ground and under numerous vertical drains was converted into plane-strain conditions to examine the effectiveness of these matching methods. Following that, FEA results were obtained by comparing the settlement and EPP distributions to field measurements.

3. Materials and Methodology

3.1. Large Scale Consolidometer Experiment

Saowapakpiboon et al. [45] designed a large-scale consolidation test setup in a laboratory environment to study the influence of a vertical drain on clay consolidation behavior. The investigation used soft samples from the second Bangkok International Airport (SBIA) area, which is around 30 km southeast of Bangkok. Samples were recovered at a depth of 3.0 to 4.0 m beneath an exhausted crust layer. The thickness of the clay samples prepared in this way was measured to be approximately 0.7 m, as shown in Figure 1. During the consolidation test, 100 kPa pressure was applied vertically to the ground and time-dependent settlements were measured. However, according to Bergado et al. [46], a shaft was installed in the center and top side of the piston, and the real average pressure exerted to the model surface was around 95 kPa. During the consolidation, the drainage was only allowed on the upper surface, while the bottom side was closed to drainage [45].

![Figure 1. Large-Scale consolidometer test (Saowapakpiboon et al. [45]).](image-url)
3.1.1. Soil Properties and Testing Procedures

The index properties of the soft soil used in the large-scale consolidometer experiment carried out in the laboratory is shown in Table 1. In the consolidometer experiment, CT-D911 model PVD material produced by CeTeau company was used. The properties of PVD material used during the experiment test are summarized in Table 2.

Table 1. Physical characteristics of soft Bangkok clay (Saowapakpiboon et al. [45]).

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid limit (%)</td>
<td>102.24</td>
</tr>
<tr>
<td>Plastic limit (%)</td>
<td>39.55</td>
</tr>
<tr>
<td>Water content (%)</td>
<td>112.69</td>
</tr>
<tr>
<td>Plasticity index</td>
<td>62.69</td>
</tr>
<tr>
<td>Total unit weight (kN/m³)</td>
<td>14.70</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.66</td>
</tr>
</tbody>
</table>

Table 2. Characteristics of CeTeau CT-D911 drain (Saowapakpiboon et al. [45]).

<table>
<thead>
<tr>
<th>Drainage</th>
<th>Polypropylene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels</td>
<td>44</td>
</tr>
<tr>
<td>Filter</td>
<td>Polypropylene</td>
</tr>
<tr>
<td>Weight (g/m)</td>
<td>78</td>
</tr>
<tr>
<td>Width, W (mm)</td>
<td>100</td>
</tr>
<tr>
<td>Thickness, td (mm)</td>
<td>3.5</td>
</tr>
</tbody>
</table>

3.1.2. Soil Model Parameters and Drain Properties

The soil sample was modeled using an elasto-plastic model, the Soft Soil Model (SSM). This model is appropriate for material with excessive levels of compressibility, such as typically consolidated clays, clayey silts, and peat [47]. As described by Janbu [48] in his Rankine lecture, this model has been well proven using oedometer tests. The fundamental model’s theory is based on the Mohr–Coulomb and Modified Cam-Clay Models [49]. Logarithm relationships comparable with the ones for the Modified Cam-Clay Model are used, and failure behavior is evaluated according to the Mohr–Coulomb Models criteria [49]. Model parameters derived from the findings of laboratory test are presented in Table 3, whereas the characteristics of the PVD are contained in Table 4.

Table 3. Model parameters used in FEA for soft soil (Saowapakpiboon et al. [45]).

<table>
<thead>
<tr>
<th>H (m)</th>
<th>γ (kN/m³)</th>
<th>K*</th>
<th>λ*</th>
<th>ν</th>
<th>ε₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td>14.70</td>
<td>0.0167</td>
<td>0.173</td>
<td>0.3</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Where ε₀ is the initial void ratio; λ* is the modified compression index; K* is the modified swelling index; and ν is the Poisson’s ratio.

Table 4. Characteristics of PVD used during consolidation (Saowapakpiboon et al. [45]).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain diameter</td>
<td>d_w</td>
<td>m</td>
<td>0.0268</td>
</tr>
<tr>
<td>Diameter of smear zone</td>
<td>d_s</td>
<td>m</td>
<td>0.087</td>
</tr>
<tr>
<td>Discharge capacity</td>
<td>q_w</td>
<td>m³/year</td>
<td>100</td>
</tr>
<tr>
<td>Ratio of k_h to k_s</td>
<td>k_h/k_s</td>
<td>m³/year</td>
<td>3</td>
</tr>
</tbody>
</table>

Where k_h is the coefficient of horizontal permeability, and k_s is the horizontal permeability of the smear zone.

3.2. Full Test Embankment in Eastern China

The Hangzhou-Ningbo (HN) highway runs along the southern the ocean of Hangzhou Bay. It begins from Hangzhou, Zhejiang province’s capital, and ends in Ningbo, the province’s largest harbor city. The HN highway is about 145 km long, with around 92 km
passing over a soft soil clay. According to Wang et al. [50], 12 full-scale field test embankments with an overall dimension of 3.15 km were constructed and studied to collect credible field information and personal experience that would influence the planning and building of the complete highway. As a result, one of these PVD-improved embankments is evaluated in this paper.

3.2.1. Soil Profile and Properties for Test Embankment

After Ba-Phu [43], the soil profile of the eastern China embankment can be summarized as follows: the top layers of soil consisted of a thin weathered crust (TC) with a thickness of 1 to 1.5 m above a silty clay layer (SC1) that was 4 m deep. The third layer is exceedingly soft mucky clay (MC), with an overall thickness about 10 m. The fourth layer is likewise a soft clay, known as mucky-silty clay (MSC), with an average thickness of approximately 4 m. The fifth layer is a medium-to-stiff silty clay layer (SC2) that is around 3 to 5 m thick. Then, there is a clayey sand layer below the fifth layer. Prior to embankment construction, the subsoils were in mildly to moderately consolidated states. The upper crust had an overconsolidated ratio (OCR) of around 5. The ground water table was approximately 1.5 m beneath the ground surface.

The cross-sectional and soil characteristics of the eastern China test embankment with PVD-improved subsoil are illustrated in Figure 2. The top of the soft soil was covered with 0.5 m thick sand, and then the granite was used as the fill material which was consolidated in layers to a unit weight of around 20 kN/m². The constructed embankment has a height of 5.88 m. PVDs were constructed to have a length of 19 m in a triangular arrangement with 1.5 m intervals.

![Cross-section of the test embankment and the placement of PVD (Ba-Phu [43]).](image)

3.2.2. Model Parameters of Soft Clay and Characteristics of PVDs

The PLAXIS 2D Version 8.6 [47] software program was used, and the soft soils were modelled with SSM, as indicated in Table 5. The fill material was modeled using Mohr–Coulomb Models (MCM), in which the value of Young’s modulus \( E = 30,000 \text{kPa} \) and Poisson’s ratio \( \nu = 0.2 \) were considered. Throughout the consolidation process, the conductivity of water was modified with void ratio using the equation of Taylor [51]:

\[
k = k_0 \times 10^{-(e_0 - e)/C_k}
\]

(1)
where \( k_0 \) is the initial hydraulic conductivity, \( e_0 \) is the initial void ratio, \( k \) is the current hydraulic conductivity, \( e \) is the current void ratio, and \( C_k \) is a constant (the value of \( C_k = 0.45 \, e_0 \) to \( 0.5 \, e_0 \), according to Tavenas et al. [52]).

Table 5. Model parameters for soil clay layers in FEA (Ba-Phu [43]).

<table>
<thead>
<tr>
<th>Layer</th>
<th>H (m)</th>
<th>E (kPa)</th>
<th>( \gamma_{sat} ) (kN/m(^3))</th>
<th>( K^* )</th>
<th>( \lambda^* )</th>
<th>( \nu )</th>
<th>( e_0 )</th>
<th>( C_k )</th>
<th>OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>1</td>
<td>–</td>
<td>19.3</td>
<td>0.004</td>
<td>0.044</td>
<td>0.3</td>
<td>0.81</td>
<td>0.365</td>
<td>5</td>
</tr>
<tr>
<td>SC1</td>
<td>4</td>
<td>–</td>
<td>18.5</td>
<td>0.008</td>
<td>0.077</td>
<td>0.35</td>
<td>1.07</td>
<td>0.482</td>
<td>1</td>
</tr>
<tr>
<td>MC</td>
<td>10</td>
<td>–</td>
<td>17.3</td>
<td>0.012</td>
<td>0.119</td>
<td>0.35</td>
<td>1.36</td>
<td>0.612</td>
<td>1</td>
</tr>
<tr>
<td>SMC</td>
<td>4</td>
<td>–</td>
<td>17.9</td>
<td>0.009</td>
<td>0.086</td>
<td>0.35</td>
<td>1.1</td>
<td>0.495</td>
<td>1</td>
</tr>
<tr>
<td>SC2</td>
<td>5</td>
<td>–</td>
<td>19.3</td>
<td>0.006</td>
<td>0.055</td>
<td>0.3</td>
<td>0.81</td>
<td>0.365</td>
<td>1</td>
</tr>
<tr>
<td>CS</td>
<td>5</td>
<td>25,000</td>
<td>19.5</td>
<td>–</td>
<td>–</td>
<td>0.25</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Fill</td>
<td>5.88</td>
<td>30,000</td>
<td>20</td>
<td>–</td>
<td>–</td>
<td>0.2</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

The properties of PVDs and the equivalent horizontal permeability in the plane strain model for different matching methods are shown in Tables 6 and 7.

Table 6. Properties of PVD used for test embankment (Ba-Phu [43]).

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain diameter</td>
<td>( d_{iw} )</td>
<td>m</td>
<td>0.053</td>
</tr>
<tr>
<td>Unit cell diameter</td>
<td>( d_e )</td>
<td>m</td>
<td>1.58</td>
</tr>
<tr>
<td>Space of PVD</td>
<td>S</td>
<td>m</td>
<td>1.5</td>
</tr>
<tr>
<td>Smear zone diameter</td>
<td>( d_s )</td>
<td>m</td>
<td>0.355</td>
</tr>
<tr>
<td>Ratio of ( k_h ) over ( k_s ) in field</td>
<td>( (k_h / k_s)_f )</td>
<td>–</td>
<td>13.8</td>
</tr>
<tr>
<td>Length of PVD</td>
<td>L</td>
<td>m</td>
<td>19</td>
</tr>
<tr>
<td>Discharge capacity</td>
<td>( q_{iw} )</td>
<td>m(^3)/year</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 7. Comparison of calculated equivalent and laboratory permeability properties of soft soil.

<table>
<thead>
<tr>
<th>Equivalent Permeability</th>
<th>Laboratory Test</th>
<th>Method A</th>
<th>Method B</th>
<th>Method C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_h ) (m/day)</td>
<td>6.30 \times 10^{-5}</td>
<td>2.04 \times 10^{-5}</td>
<td>4.18 \times 10^{-6}</td>
<td>1.49 \times 10^{-4}</td>
</tr>
<tr>
<td>( k_v ) (m/day)</td>
<td>6.30 \times 10^{-5}</td>
<td>2.04 \times 10^{-5}</td>
<td>4.18 \times 10^{-6}</td>
<td>7.26 \times 10^{-6}</td>
</tr>
</tbody>
</table>

3.3. Methodology: Matching Methods

As mentioned earlier, various researchers have developed matching methods. In this study, three different approaches were adopted to investigate PVD-improved subsoil in FEA, which are Method A proposed by Indraratna et al. [40], Method B proposed by Chai and Miura [15], and Method C proposed by Ba-Phu [43]. These methods provide a simple theory and an adequate accuracy for multi-drain analysis as compared to the other approaches, such as the quadrilateral macro-element method proposed by Sekiguchi et al. [53], which are exceptionally time-consuming and exceedingly inconvenient in engineering practice. Moreover, these methods are more preferred due to their concepts which are completely different from each other. According to Method A, both the smear zone around each vertical drain must be defined in the numerical analysis, and the values of horizontal permeabilities in the smear zone and undisturbed soil must be input separately for each layer. However, by way of Method B, PVD-improved subsoil can easily be evaluated in the same way, with the similar solid constituents, as unimproved soil. Nevertheless, based on Method C, the smear zone surrounding each vertical drain is not modelled in the simulation. Firstly, these methods are briefly described below.
3.3.1. Method A

Indraratna et al. [40] adjusted the relationship of soil permeability to transform the vertical drain system seen in Figure 3 into a corresponding parallel drain well. They considered that the half-widths of unit cell $B$, of drains $b_w$, and of smear zone $b_s$ are identical as their axisymmetric radius $R$, $r_w$, and $r_s$, correspondingly. Indraratna et al. [40] proposed the following relationship between $k_h$ and $k_p$:

$$\frac{k_p}{k_h} = \frac{k_p}{k_h} \left[ \frac{\beta}{\frac{h}{k}} \right] \left[ \frac{\left( \frac{h}{k} \right)}{\left( \frac{h}{k} \right)} \right] - \alpha$$

(2)

where $n = D_e/d_w$ and $s = r_s/r_w$ as $D_e$, $d_w$, and $r_s$ are the equivalent diameter of unit cell, the equivalent diameter of the drain, and the radius smear zone, respectively, $k_h$ is the horizontal permeability of the undisturbed soil, and $k_s$ is the horizontal permeability of disturbed soil, and the letter $p$ denotes the plane strain condition.

Where $\alpha$ and $\beta$ parameters can be related by:

$$\alpha = \frac{2}{3} - \frac{2b_s}{B} \left( 1 - \frac{b_s}{B} + \frac{b_s^2}{3B^2} \right)$$

(3)

$$\beta = \frac{1}{B^2} \left( b_s - b_w \right)^2 - \frac{b_s}{3B^3} \left( 3b_w^2 - b_s^2 \right)$$

(4)

![Figure 3. Axisymmetric and plane-strain unit cell models (Indraratna et al. [40]). (a) Axisymmetric; (b) plane strain.](image)

3.3.2. Method B

Chai and Miura [15] proposed a simple approximation technique for evaluating the effect of PVD. This approach provides an estimate of vertical permeability that roughly represents both the influence of vertical flow of natural subsoil and radial permeability due to PVD [15]. This equivalent vertical permeability ($k_{ve}$) was calculated using an identical average degree of consolidation within the 1D condition. The equivalent vertical permeability $k_{ve}$ can be stated as follows:

$$k_{ve} = \left( 1 + \frac{2.5 \mu^2 k_h}{\mu D_e^2 k_v} \right) k_v$$

(5)
where \( k_v \) is the hydraulic conductivity in the vertical direction, \( l \) is the drain length, and \( D_e \) is the equivalent diameter of unit cell. The value of \( \mu \) can be expressed as:

\[
\mu = l_n \left( \frac{n}{s} \right) + \frac{k_h l_n(s)}{k_h} - \frac{3}{4} + \frac{\pi 2l^2 k_h}{3q_w} \]

(6)

where \( n = D_e/d_w \) (drain equivalent diameter), \( s = d_s/d_w \) (smear zone equivalent diameter), and \( L \) is the drainage length.

3.3.3. Method C

Ba-Phu [43] proposed an equivalent plane-strain (2D) theory of the PVD-improved ground, which included the impacts of the smear zone. Ba-Phu [43] calculates the corresponding horizontal permeability of the soft soil in the plane-strain concept as follows:

\[
k_{hp} = -\frac{\pi B U_h k_h}{8T_h} \]

(7)

where \( U_h \) is the degree of radial consolidation of the soft soil, \( B \) is the half width of the plane-strain unit cell, \( k_h \) is the natural soil’s horizontal permeability, and \( T_h \) is the time factor of the horizontal consolidation, which was determined using the approach of Hansbo [54]. The value of \( \mu \) can be described in Equation (6). Then, the time factor \( T_h \) was formulated as:

\[
T_h = -\frac{\mu}{8l_n} (1 - U_h) \]

(8)

where, by substituting Equation (8) into Equation (7), then the equivalent horizontal permeability of soil in the (2D) model can be obtained:

\[
k_{hp} = -\frac{\pi B U_h k_h}{\mu l_n (1 - U_h)} \]

(9)

4. Results and Discussion

4.1. FEA of Large-Scale Consolidometer Experiment

In the model, boundary conditions and lower boundary displacements are not allowed in both directions \( u_x = u_y = 0 \). The right and left edges do not allow for horizontal displacements \( (u_x = 0) \) (Figure 4a). The right and bottom edges are closed to drainage, and the groundwater level is defined at the lower boundary (Figure 4c).

Figure 5a shows the predicted vertical settlement of axisymmetric unit cell in the FEA analysis and laboratory measurements. It can be seen that, in FEA, the time-dependent settlement value was obtained around 119 mm after 47 days, while the experimental measurement result was determined to be approximately 120 mm. It is clear that FEA results and experimental data match extremely well.

Moreover, EPP versus time of the large-scale consolidometer in FEA result and the laboratory measurement are also presented in Figure 5b. Although the simulations slightly overestimated the EPP values, numerical analysis generally matches up fairly well with the measurements. As can be seen, the numerical analysis and the experimental measurement are in very good agreement. Afterwards, they differed slightly beyond 20 days. According to Rujikiatkamjorn [44], it is not surprising to see a discrepancy of EPP between the numerical analysis and the laboratory measurement. This is because the EPP experimental is installed near the PVD, which means that the maximum EPP closer to the PVD under cyclic load was considerably lower compared to that near the cell border, and EPP at the vicinity of the drain reduced dramatically as a result of the permeable barrier established using PVD [44]. From these results, it was seen that the consolidation behavior of PVD on soft clay was successfully modeled in the FEA program with the SSM model.
Hansbo [54]. The value of $\mu$ can be described in Equation (6). Then, the time factor $T_4$ was formulated as:

$$ T_4 = -\mu \frac{8}{l_2(1 - U_\text{th}_4)} $$

(8)

where, by substituting Equation (8) into Equation (7), then the equivalent horizontal permeability of soil in the (2D) model can be obtained:

$$ k_4 \theta = -\frac{\pi B U_\text{th}_4 k_4 \mu l_2(1 - U_\text{th}_4)}{\pi} $$

(9)

4. Results and Discussion

4.1. FEA of Large-Scale Consolidometer Experiment

In the model, boundary conditions and lower boundary displacements are not allowed in both directions ($u_x = u_z = 0$). The right and left edges do not allow for horizontal displacements ($u_x = 0$) (Figure 4a). The right and bottom edges are closed to drainage, and the groundwater level is defined at the lower boundary (Figure 4c).

![Figure 4](image-url)

Figure 4. Finite Element Modeling of the large-scale consolidometer experiment. (a) Geometric model; (b) axisymmetric mesh; (c) initial conditions.

As mentioned previously, in the next part of the numerical analysis of consolidometer, the experiment which was carried out with axisymmetric is analyzed into two-dimensional plane-strain conditions using three different matching methods, and the results are compared to axisymmetric and laboratory measurements. Table 7 shows the experimental permeability of soft soil and equivalent permeability in plane-strain conditions.

In view of these results presented in Figure 6a,b, although the real behavior of the consolidation of soft soil deposit stabilized with PVD is in 3D, 2D plane-strain conditions using matching methods are able to predict the settlement and the EPP in FEA. Generally, it can be seen that, even though a relatively differences, all predicted vertical settlements of matching methods are extremely good agreement with the axisymmetric result and the experimental measurement as well. The discrepancy between Method A and the axisymmetric method was 6% at the beginning of the consolidation. However, it was decreased to 1.5% at the end of 47th day, as shown in Figure 7. According to Method B, the gap was roughly 4.15% at the end of 47th day of consolidation, whereas Method C showed a 4% difference at the end of 47th day. Based on Method B, Chai et al. [35] demonstrated that the difference is due to the fundamental approximation approach, which incorporates the effect of both vertical and radial drainages into vertical drainage.
Moreover, these results show that the matching EPP analysis agree well with both the axisymmetric and laboratory measurements. However, a quite discrepancy can be seen between EPP in axisymmetric and plane-strain analysis results. As indicated by Hird et al. [55], the error in EPP calculation was probably due to a decrease in global permeability. Furthermore, Russell [56] demonstrated that, although perfect matching is achieved, EPP at a similar location in the axisymmetric and plane-strain unit cells might differ, which is caused by an underestimating of geometry and global permeability.

Figure 5. Comparison of FEA axisymmetric unit cell and experimental data. (a) Settlement, (b) excess pore pressure.
Figure 6. FEA for axisymmetric, three matching methods, and experimental measurement. (a) Settlement, (b) EPP.

Figure 7. Errors of the computational results of three matching methods.
4.2. FEA of Full Test Embankment in Eastern China

In this section, as previously described, 2D FEA of a real embankment built on numerous vertical drains is analyzed because multiple drain behavior is neglected in unit cell analysis. To evaluate the behavior of PVDs, a full-trial embankment in eastern China was analyzed using three different matching methods, as previously discussed. The equivalent permeability in plane-strain conditions for all matching approaches, and in situ taken from Ba-Phu [43], are summarized in Table 8. During consolidation, bottom boundary displacements are not permitted in both directions \(u_x = u_y = 0\). Horizontal displacements are not allowed on the right and left boundaries \(u_x = 0\), and they are closed to the consolidation boundary. Finite element mesh for three matching methods in plane-strain conditions are shown in Figure 8.

Table 8. Comparison of calculated equivalent and in situ permeability properties of soft soil.

<table>
<thead>
<tr>
<th>Layers</th>
<th>(k_h) (m/day)</th>
<th>(k_v) (m/day)</th>
<th>(k_{hp}) (m/day)</th>
<th>(k_{vp}) (m/day)</th>
<th>(k_{ve}) (m/day)</th>
<th>(k_{hp}) (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>(3.03 \times 10^{-3})</td>
<td>(3.03 \times 10^{-3})</td>
<td>(7.68 \times 10^{-4})</td>
<td>(3.56 \times 10^{-5})</td>
<td>(3.41 \times 10^{-2})</td>
<td>(1.46 \times 10^{-4})</td>
</tr>
<tr>
<td>SC1</td>
<td>(5.60 \times 10^{-4})</td>
<td>(2.24 \times 10^{-4})</td>
<td>(1.42 \times 10^{-4})</td>
<td>(6.57 \times 10^{-6})</td>
<td>(7.36 \times 10^{-2})</td>
<td>(3.34 \times 10^{-5})</td>
</tr>
<tr>
<td>MC</td>
<td>(2.54 \times 10^{-3})</td>
<td>(1.69 \times 10^{-3})</td>
<td>(6.44 \times 10^{-4})</td>
<td>(2.98 \times 10^{-5})</td>
<td>(2.88 \times 10^{-2})</td>
<td>(1.27 \times 10^{-4})</td>
</tr>
<tr>
<td>SMC</td>
<td>(2.06 \times 10^{-3})</td>
<td>(1.03 \times 10^{-3})</td>
<td>(5.22 \times 10^{-4})</td>
<td>(2.42 \times 10^{-5})</td>
<td>(2.39 \times 10^{-2})</td>
<td>(1.07 \times 10^{-4})</td>
</tr>
<tr>
<td>SC2</td>
<td>(3.90 \times 10^{-4})</td>
<td>(1.82 \times 10^{-4})</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>SC</td>
<td>(2.60 \times 10^{-2})</td>
<td>(2.60 \times 10^{-2})</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Fill</td>
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<td>(2.0 \times 10^{-1})</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8. Cont.
As seen from these results in Figure 9, although the matching methods are based on unit cell with single vertical drain, they are successfully capable to predict the settlement and the EPP in full-scale embankment. Moreover, despite the real behavior of consolidation of soft soil deposit stabilized with PVDs is in 3D, 2D plan strain conditions using matching methods are capable to predict the vertical settlement and EPP in FEA. Furthermore, even though these three approaches are completely different, their FEA results were close together and are very comparable with the field measurements. As seen in Figure 9a, the time-dependent settlement for all matching methods were attained about 200 cm after 900 days. However, there is a minor variation among the predicted outcomes and field measurement within the early stage (from 0 to around 120 days). According to Leroueil et al. [57], the settlements are small due to the subsoil clay being overconsolidated throughout the early phase of execution, which behaves in a quasielastic manner and has a high stiffness.
Based on Method A, the predicted vertical settlement most accurate agreement compared with other methods with the field measurement. However, the application of this approach requires complex works because the smear zone around each vertical drain must be defined in the numerical analysis, and the values of horizontal permeabilities in the smear zone and undisturbed soil must be input separately for each layer.

According to Method B, FEA produced a somewhat slower consolidation rate. However, this method is a much simpler concept, and does not require modeling of vertical drains. This approach is more convenient and simpler to employ in real-world engineering practice due to its simplicity.

Method C yielded a somewhat faster vertical settlement rate. However, differences between the two other methods are quite small. This approach has a slight advantage over Method A, since the smear zone surrounding each vertical drain is not modelled in the simulation.

Moreover, as shown in Figure 9b, the simulated EPP are also compared with the field measurements. Generally, the accurate prediction of EPP is more challenging than the prediction of settlement [2]. It can be seen that the predicted EPP were in good agreement from the initial stage up to 200 days or so, and afterwards they underestimated the field measurement. It is possible that the large discrepancies between the numerical analysis and measurements are due to potential errors in piezometric measurement [2]. Piezometers are commonly used to measure pore water pressure in deep soil. As a result, these piezometers can get clogged when dirt particles pass into the filter’s pores. Thus, a certain quantity of excess pore pressure is trapped in the piezometers [15,58]. Furthermore, the geometrical differences between the numerical analysis, performed using plane strain and the field condition, that is three-dimensional might have been an alternative explanation for the discrepancy [59]. According to Sundström [60], the error in the EPP prediction could be attributed to creep effects in the layer, as the volume of the voids filled with the water decreases when soil particles rearrange.

From these results, the matching methods were accurately able to predict the settlement and EPP in FEA. These approaches are useful in engineering practice, since they allow for the simulation of the behavior of PVD-enhanced subsoil. In any case, we need to conduct a 3D full-scale simulation to evaluate the behavior of PVD-improved subsoil. However,
applying 3D analyses to an actual embankment project with a significant number of vertical drains requires a great deal of computational effort due to their sophisticated nature.

5. Conclusions

In this article, the application of different matching methods created for converting vertical drain systems built on soft clay soils into plane strain conditions in 2D was analyzed. Three different matching methods were used to transform vertical drains from axisymmetric into plane strain conditions. The numerical analysis was performed using FEA PLAXIS 2D Version 8.6 [47] software, with SSM representing the soft soil. The smear zone was considered in the research, and caution should be exercised while analyzing soft soil stabilized with PVDs. To evaluate the accuracy of these methods, large-scale consolidometer experiment results conducted under laboratory settings were analyzed, as well as the long-term behavior of eastern China on PVD-improved soft soil. For comparison, the numerical analysis is compared to experimental and field measurements. Although the matching methods are based on unit cell with single vertical drain, they are successfully capable of predicting the settlement and the EPP in the full-scale embankment. The comparisons of numerical models and observations show that the settlement and excess pore pressure predicted by SSM are typically in agreement with laboratory and field measurements.

It is possible to convert the real 3D behavior of vertical drains into equivalent plane strain conditions by using three different matching methods. It was observed that the settlement predictions created using these three different methods were satisfactory. The use of these techniques is beneficial in engineering practice, and can be used to simplify the analysis of the behavior of PVD improved subsoil. Method A achieves the most accurate agreement compared to other approaches with the measurements. However, this method is most computationally demanding, since each drain and associated smear zone are represented discretely in the simulations. Method B produced a somewhat slower fit than the other two matching approaches. Nevertheless, this method is a much simpler concept, and does not require modeling of vertical drains. This approach is more convenient and simpler to employ in real-world engineering practice due to its simplicity.

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