

Article

Derivation of the Ultimate Bearing Capacity Formula for Layered Foundations Based on Meyerhof's Theory

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Abstract: In this paper, based on Meyerhof's theory of homogeneous foundation, the limit equilibrium analysis method and unified logarithmic spiral sliding surface assumption are used to derive the theoretical formula for the ultimate bearing capacity of a layered foundation when the foundation is completely rough. It should be noted that this formula is only applicable to strip foundations of upper soft clay and lower sandy soil. In addition, a comparative analysis is conducted between theoretical formulas and semiempirical formulas for layered foundations. On the basis of verifying the reliability of the theoretical formula results, numerical simulation is carried out to further explore and analyze the influence of the width to depth ratio of the foundation, the strength parameters of the double-layer soil, and the thickness of the upper soft soil on the bearing capacity of the foundation. Research has shown that the formula for the bearing capacity of a layered foundation derived in this paper has a certain degree of error compared to Meyerhof's semiempirical formula, but it is in good agreement with numerical simulation results and Hansen's weighted average method results. The ratio of the width to depth of the foundation, the ratio of the cohesive force of the double-layer soil, and the tangent ratio of the internal friction angle have a significant positive correlation with the ultimate bearing capacity of the foundation. The increase in thickness of the overlying cohesive soil has a negative impact on the ultimate bearing capacity of the foundation, and the thicker the soil, the smaller the foundation's bearing capacity.



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Keywords: layered foundation; limit equilibrium analysis method; theoretical derivation; completely rough; numerical simulation

1. Introduction

The determination of the ultimate bearing capacity of foundations is an important topic in classical soil mechanics and foundation engineering that has received widespread attention and research from the academic and engineering communities [1,2]. The ultimate bearing capacity of homogeneous strip foundations calculated by various classic theoretical analysis methods varies greatly, and even theoretical formulas derived from the same theory will have significant differences in their calculation results. Moreover, layered soil of upper soft clay and lower sandy soil in coastal areas are relatively common [3]. For the ultimate bearing capacity of layered foundations, scholars mainly apply the formula for calculating the bearing capacity of homogeneous foundations to layered foundations in the form of empirical coefficients, correction coefficients, etc., without fully considering the actual situation of layered soils, and the calculation results lack reliability [4].

Regarding the ultimate bearing capacity of homogeneous foundations, many international scholars have conducted extensive theoretical formula research. Due to different analysis methods and basic assumptions, the bearing capacity coefficients they derived have significant differences. Prandtl derived the formula for the ultimate bearing capacity of a rigid foundation in weightless soil based on plastic theory [5]. Due to Prandtl's failure to consider the weight of the soil in the foundation bearing layer, the formula has limitations. Scholars represented by Terzaghi have proposed a widely used formula for

calculating the ultimate bearing capacity of foundations by considering the weight of the soil in the foundation bearing layer based on Prandtl's basic principle [6]. However, Terzaghi's theory assumes that the sliding surface extends to the bottom of the foundation, without considering the influence of burial depth. Continuous uniformly distributed loads on both sides of the foundation bottom are used to replace the shear strength of soil within the buried depth range. When the foundation is buried deep, this simplified method will underestimate its bearing capacity, making the foundation design biased conservatively. Meyerhof [7,8] made modifications to this by assuming that the sliding surface extends to the surface of the foundation and equivalently converting the interaction between the foundation side and the soil, as well as the weight of the soil on both sides, into stresses on equivalent free surfaces. This is done to reasonably consider the shear strength of the soil on both sides, which is more in line with the actual state of foundation failure [9]. Meyerhof and Hanna [10] assume that the ultimate bearing capacity of a double-layer foundation is obtained by placing the foundation separately on soft and hard foundations, and then using the difference analysis method to derive a semiempirical formula for the ultimate bearing capacity of the double-layer foundation. Hansen [11] used the weighted average method to represent the strength parameters of a layered foundation using the average index, thus simplifying the layered foundation into a homogeneous foundation for the ultimate bearing capacity calculation.

In recent years, international scholars have conducted in-depth research on the bearing characteristics of foundations from theoretical derivation, model testing, numerical simulation, and other aspects. The related research has guiding significance for the optimization design of foundations and engineering construction. Zhao [12] introduced reduction coefficients and depth coefficients to derive the theoretical formula for the ultimate bearing capacity of skirt foundations on soil slopes under undrained conditions and established a numerical model to study the influencing factors of bearing capacity. Chen [13] studied the effect of saturation on the strength and deformation index of soft clay through indoor experiments, analyzed the mechanism of groundwater's influence on the bearing capacity of shallow foundations, and proposed a method to determine the critical depth of groundwater. Keba [14] studied the influence of the position and shape of internal cavities in soil on the bearing characteristics and failure modes of a cohesive soil foundation based on the rigid plastic finite element method. In addition, he proposed a critical equation based on numerical simulation results to determine the influence area of internal cavities in the soil. Eshkevari [15] studied the failure mode and ultimate bearing capacity of layered sand strip foundations based on the finite element limit analysis method and explored the influence of the width to depth ratio of the foundation on the geometric shape of the failure mode. Ou [16] combined the theoretical research of self-balancing test piles with indoor model tests, proposed an analytical method to convert the results of self-balancing test piles into static load test curves, and verified the effectiveness of the analytical method through model tests. Ahmad [17] used Gaussian Process Regression (GPR) to predict the ultimate bearing capacity of non-cohesive soil foundations and compared the results with classical theoretical calculation methods. In addition, he explored the influence of the width, burial depth, length to width ratio, soil cohesion, and internal friction angle of the foundation on the ultimate bearing capacity and found that the burial depth of the foundation had the highest degree of influence. Wei [18] proposed a lower-bound solution for the bearing capacity of circular foundations under uniformly distributed loads by establishing a half space static application field and studied the influence of soil shear strength on the lower limit solution. Ter-Mattirosyan [19] pointed out the nonlinear behavior of foundation soil and proposed a calculation method for the bearing capacity of rigid rectangular foundations considering the weight of soil and different pre-overburden pressure values. Zhao [20] established a numerical model for a strip foundation on sandy soil slopes, studied the influence of slope angle, relative distance, internal friction angle and other factors on the bearing capacity of the foundation, and provided a range of values for the distance of a strip foundation to the slope on sandy soil slopes. Moayedli [21] optimized the neural

network (ANN) for predicting the bearing capacity of circular foundations based on two algorithms, the Imperialist Competition Algorithm (ICA) and Particle Swarm Optimization (PSO). He found that compared with the ICA-ANN hybrid model and ANN model, the PSO-ANN hybrid model had higher advantages in predicting the bearing capacity of circular foundations. Yu [22] summarized the results of triaxial tests on friction angles for 106 types of soil; summarized the quantitative relationship between failure criteria, plane strain, and soil material parameters; and proposed a new method for determining soil failure criteria.

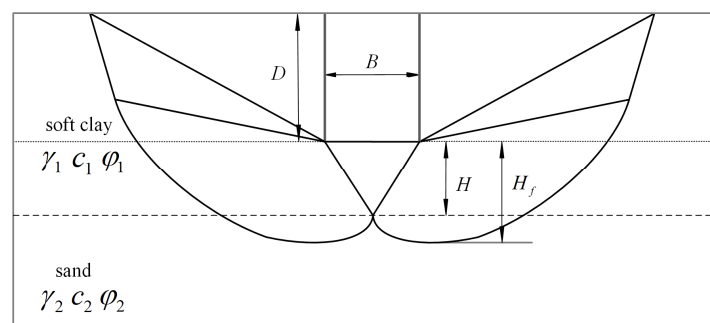
In this paper, the unified assumption of a logarithmic spiral sliding surface is adopted to derive the bearing capacity coefficient N_γ , and the difference analysis method is used to extend Meyerhof's homogeneous foundation theory formula to a double-layer foundation and derive the formula for calculating the ultimate bearing capacity of a layered foundation when the foundation is completely rough. On this basis, after verifying the reliability of the theoretical formula results, a numerical model is established to further explore and analyze the influence of soil and foundation parameters on the bearing capacity of the foundation. Finally, grey correlation analysis is conducted based on MATLAB R2021a to study the degree of influence of various parameters on the bearing capacity of the foundation.

2. Theoretical Solution to the Ultimate Bearing Capacity of a Completely Rough Layered Foundation

There is both soft clay and sandy soil below the foundation, and the distribution of soil layers and the location of failure sliding surfaces below the foundation are shown in Figure 1. Meyerhof suggests calculating the ultimate bearing capacity of double-layer foundations with upper soft clay and lower sandy soil using the semiempirical Formula (1). The formula is obtained by placing the foundation separately on soft clay and sandy soil, and then taking the difference in the bearing capacity based on the proportional relationship between the thickness of the soft clay layer and the depth of the failure sliding surface.

$$q_u = q_t + (q_b - q_t) \left(1 - \frac{H}{H_f} \right)^2 \geq q_t \quad (1)$$

where H_f is the depth of influence caused by the failure of the sliding surface. For loose sand and clay, $H_f = B$, and for tight sand, $H_f = 2B$ [10]. H is the thickness of the soft clay layer under the foundation; q_u is the ultimate bearing capacity of the layered foundation under the central load; and q_t and q_b , respectively, represent the ultimate bearing capacity when the foundation is assumed to be located on the surface of the upper and lower soil layers.



additional comments: c , ϕ and γ are cohesion, internal friction angle and density of soil.

H is the thickness of the lower sandy soil.

H_f is the depth of influence caused by the failure of the sliding surface.

D and B are the burial depth and width of the foundation.

Figure 1. Schematic diagram of the failure sliding surface of a layered foundation.

The bearing capacity of a strip foundation is provided by factors such as soil overload within the depth range of the foundation, soil cohesion, soil gravity, and passive soil pressure. In this paper, the limit equilibrium method is used to superimpose the bearing capacity caused by cohesion, overloaded soil, and the gravity of the soil under the foundation bottom and obtain the formula for the ultimate bearing capacity of the foundation.

2.1. The Bearing Capacity of the Foundation Caused by Cohesive Force and Overload

As shown in Figure 2, according to Meyerhof's bearing capacity theory, it is assumed that the sliding surface extends to the surface and slides out from E. When considering the bearing capacity caused by soil overload within the depth range of the foundation and soil cohesion, it is assumed that the weight of the bearing layer of the foundation is zero, and the center point of the logarithmic spiral curve is point B' in Figure 2. Consider the plane B'E as an equidistant free surface and use the equidistant method to make equidistant the influence of the weight of the soil inside the B'EF and the frictional force on the foundation side B'F as the stress on the B'E surface (σ_0 and τ_0). Assuming that the normal pressure σ_a on the foundation side is calculated based on the static soil pressure, and the friction angle between the foundation side and the soil is δ , the average normal stress and tangential stress on the foundation side are:

$$\sigma_a = \frac{1}{2}k_0\gamma D \quad (2)$$

$$\tau_a = \frac{1}{2}k_0\gamma D \tan \delta \quad (3)$$

where k_0 is the coefficient of earth pressure at rest; and δ is the friction angle between the foundation side and the soil mass.

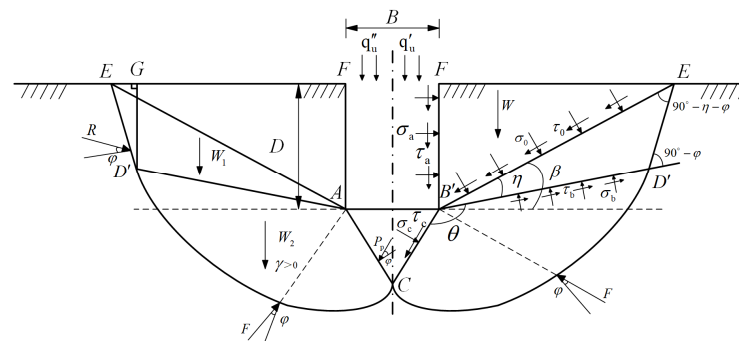


Figure 2. The strip foundation model of Meyerhof's theory.

Assuming that the angle between the equivalent free surface B'E and the base horizontal plane is β , the sliding point E on the surface can be determined. Based on the normal and tangential equilibrium conditions of the soil on the B'EF, the normal and tangential stresses on the equivalent free surface B'E can be obtained.

$$\sigma_0 = \frac{1}{2}\gamma_1 D \left(k_0 \sin^2 \beta + \frac{1}{2}k_0 \tan \delta \sin 2\beta + \cos^2 \beta \right) \quad (4)$$

$$\tau_0 = \frac{1}{2}\gamma_1 D \left[\frac{1}{2}(1 - k_0) \sin 2\beta + k_0 \tan \delta \sin^2 \beta \right] \quad (5)$$

According to the properties of the logarithmic spiral curve, and the geometric relationship of triangle B'D'E, the length of line B'D' and B'E can be obtained as:

$$B'D' = B'C \cdot e^{\theta \tan \varphi_1} = \frac{Be^{\theta \tan \varphi_1}}{2 \cos \varphi_1} \quad (6)$$

$$B'E = \frac{B'D' \sin(90^\circ + \varphi_1)}{\sin(90^\circ - \eta - \varphi_1)} = \frac{Be^{\theta \tan \varphi_1} \sin(90^\circ + \varphi_1)}{2 \cos \varphi_1 \sin(90^\circ - \eta - \varphi_1)} \quad (7)$$

simultaneously affected by the cohesive forces of both soft clay and sandy soil. The torque caused by this force on point B' is:

$$M'(c) = M(c_1) + M(c_2) = c_1 r_0^2 \int_{\theta_1}^{\theta} e^{2\theta \tan \varphi_1} d\theta + c_2 r_0^2 \int_0^{\theta_1} e^{2\theta \tan \varphi_2} d\theta \quad (12)$$

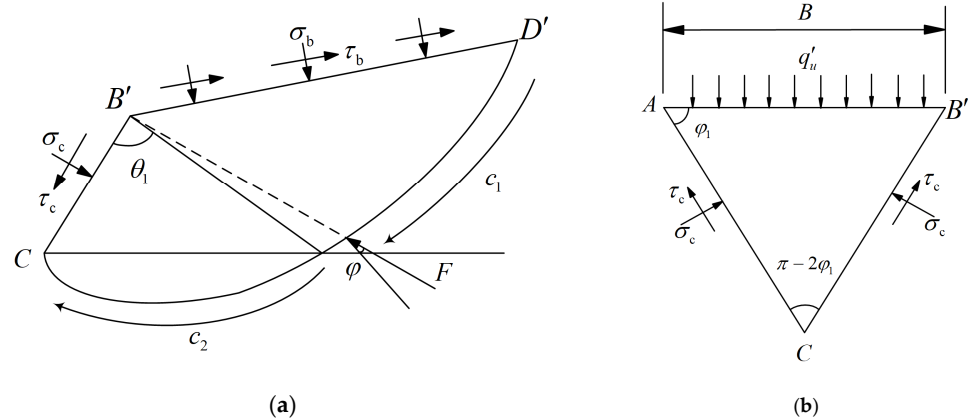


Figure 4. Force analysis of isolation body considering the cohesion of foundation soil and lateral load of foundation. (a) Force analysis of isolation body $B'CD'$; (b) Force analysis of triangular wedge $AB'C$.

Referring to the difference analysis method in Meyerhof's semiempirical formula, the coefficient n is introduced to simplify the above equation as follows:

$$\begin{aligned} M'(c) &= M(c_1) + n[M(c_2) - M(c_1)] \\ &= \frac{(1-n)c_1 r_0^2}{2} \cot \varphi_1 (e^{2\theta \tan \varphi_1} - 1) + \frac{nc_2 r_0^2}{2} \cot \varphi_2 (e^{2\theta \tan \varphi_2} - 1) \end{aligned} \quad (13)$$

where $n = \left(\frac{H_f - H}{H_f} \right)^2$.

Similarly, the moment at point B' caused by stress on the plane $B'D'$ is:

$$M(\sigma_b) = \frac{1}{2} \sigma_b r_0^2 \left[(1-n)e^{2\theta \tan \varphi_1} + ne^{2\theta \tan \varphi_2} \right] \quad (14)$$

So, based on the moment balance at point B' on the isolation body $B'CD'$ and the Mohr Coulomb failure criterion, the stress on the plane $B'C$ is obtained:

$$\sigma_c = \sigma_b \left[(1-n)e^{2\theta \tan \varphi_1} + ne^{2\theta \tan \varphi_2} \right] + (1-n)c_1 \cot \varphi_1 (e^{2\theta \tan \varphi_1} - 1) + nc_2 \cot \varphi_1 (e^{2\theta \tan \varphi_2} - 1) \sigma_c \quad (15)$$

$$\tau_c = c_1 + \sigma_b \left[(1-n)e^{2\theta \tan \varphi_1} + ne^{2\theta \tan \varphi_2} \right] \tan \varphi_1 + (1-n)c_1 (e^{2\theta \tan \varphi_1} - 1) + nc_2 \cot \varphi_2 \cdot \tan \varphi_1 (e^{2\theta \tan \varphi_2} - 1) \quad (16)$$

The angle between the straight line $B'C$ and the horizontal plane is φ_1 . Based on the balance relationship of vertical forces on the triangular wedge $AB'C$ in Figure 4b, the ultimate bearing capacity of the foundation q'_u is calculated, and σ_b and Equations (15) and (16) are substituted and organized to obtain:

$$q'_u = c_1 N_c + c_2 N'_c + \sigma_0 N_q \quad (17)$$

$$N_c = \tan \varphi_1 \left\{ \frac{(1-n) \sin(2\eta + \varphi_1) \cos^2 \varphi_1 e^{2\theta \tan \varphi_1} + [\sin(2\eta + \varphi_1) - \sin \varphi_1] ne^{2\theta \tan \varphi_2}}{\sin \varphi_1 [1 - \sin \varphi_1 \sin(2\eta + \varphi_1)]} + n \right\} + (1-n) \cot \varphi_1 (e^{2\theta \tan \varphi_1} - 1) \quad (18)$$

$$N'_c = \frac{n \cot \varphi_2 (e^{2\theta \tan \varphi_2} - 1)}{\cos^2 \varphi_1} \quad (19)$$

$$N_q = \frac{(1-n)e^{2\theta \tan \varphi_1} + ne^{2\theta \tan \varphi_2}}{1 - \sin \varphi_1 \sin(2\eta + \varphi_1)} \quad (20)$$

where N_c and N'_c are, respectively, the cohesive bearing capacity coefficients of soft clay and sandy soil, and N_q is the overload bearing capacity coefficient.

2.2. The Bearing Capacity of the Foundation Caused by the Weight of the Sliding Zone Soil

At this point, soil cohesion and overloading on both sides of the foundation are not considered, i.e., $c = 0$, $\sigma_0 = \tau_0 = 0$. To obtain the most unfavorable passive soil pressure, the starting center point of the logarithmic spiral curve needs to be determined through trial calculation. Due to the difficulty of completing the calculation, a unified assumption of a logarithmic spiral curve is adopted to simplify the calculation. It is assumed that the initial center point of the logarithmic spiral curve is at point A in Figure 2. Taking $ACD'E$ on the left side of Figure 2 as the research object, the moment of each force on point A is taken, and the passive earth pressure on the plane AC is obtained based on the moment balance.

(1) Moment of soil gravity on point A in the passive zone $AD'E$ is:

$$M(G_1) = \frac{\gamma_1 r_0^3 \sin \eta \cos \varphi_1 \cos(\beta - \eta)}{3 \cos(\varphi_1 + \eta)} \left[(1-n)e^{\theta \tan \varphi_1} + ne^{\theta \tan \varphi_2} \right]^3 \quad (21)$$

(2) Moment of soil gravity on point A in the logarithmic spiral curve area ACD' is:

$$M(G_2) = (1-n) \left\{ \frac{\gamma_1 r_0^3}{3(1+9 \tan^2 \varphi_1)} \{ 4 \sin \varphi_1 - e^{3\theta \tan \varphi_1} [3 \tan \varphi_1 \cos(\varphi_1 + \theta) + \sin(\varphi_1 + \theta)] \} \right\} \\ + \frac{\gamma_2 r_0^3 n}{3(1+9 \tan^2 \varphi_2)} \{ 4 \sin \varphi_2 - e^{3\theta \tan \varphi_2} [3 \tan \varphi_2 \cos(\varphi_2 + \theta) + \sin(\varphi_2 + \theta)] \} \quad (22)$$

(3) The angle between the radial reaction force F on the failure surface of the logarithmic spiral curve and the normal line of the action point is φ_1 . According to the properties of the logarithmic spiral curve, the action line of this force passes through the initial center point A of the curve, so its moment at point A is zero.

(4) Moment of the resultant force R on the plane $D'E$ on point A is:

$$M(R) = -\frac{\gamma_1 r_0^3 \sin^3 \eta \cos \varphi_1}{6 \cos^3(\varphi_1 + \eta)} \left[(1-n)e^{\theta \tan \varphi_1} + ne^{\theta \tan \varphi_2} \right]^3 \sin \theta \tan \theta \quad (23)$$

According to the moment balance of each force on point A in the sliding zone ACD' of the logarithmic spiral curve, the passive earth pressure on the AC surface can be obtained as:

$$P_p = \frac{\gamma_1 r_0^2 \sin \eta \cos(\beta - \eta)}{2 \cos(\varphi_1 + \eta)} \left[(1-n)e^{\theta \tan \varphi_1} + ne^{\theta \tan \varphi_2} \right]^3 \\ + (1-n) \left\{ \frac{\gamma_1 r_0^2}{2(1+9 \tan^2 \varphi_1) \cos \varphi_1} \{ 4 \sin \varphi_1 - e^{3\theta \tan \varphi_1} [3 \tan \varphi_1 \cos(\varphi_1 + \theta) + \sin(\varphi_1 + \theta)] \} \right\} \\ + n \frac{\gamma_2 r_0^2}{2(1+9 \tan^2 \varphi_2) \cos \varphi_1} \{ 4 \sin \varphi_2 - e^{3\theta \tan \varphi_2} [3 \tan \varphi_2 \cos(\varphi_2 + \theta) + \sin(\varphi_2 + \theta)] \} \\ - \frac{\gamma_1 r_0^2 \sin^3 \eta}{4 \cos^3(\varphi_1 + \eta)} \left[(1-n)e^{\theta \tan \varphi_1} + ne^{\theta \tan \varphi_2} \right]^3 \sin \theta \tan \theta \quad (24)$$

The angle between the line AC and the horizontal plane is φ_1 . Based on the balance relationship of vertical forces on the triangular wedge $AB'C$ in Figure 5, the ultimate bearing capacity of the foundation q''_u is calculated, and Equation (24) is substituted and organized as follows:

$$q''_u = \frac{\gamma_1 B}{2} N_\gamma + \frac{\gamma_2 B}{2} N'_\gamma \quad (25)$$

$$N_\gamma = \frac{\tan \varphi_1}{2 \cos \varphi_1} \left\{ \frac{\sin \eta [(1-n)e^{\theta \tan \varphi_1} + ne^{\theta \tan \varphi_2}]^3}{\cos(\varphi_1 + \eta)} \left[\cos(\beta - \eta) - \frac{\sin^2 \eta \sin \theta \tan \theta}{2 \cos^2(\varphi_1 + \eta)} \right] + \frac{(1-n) \{ 4 \sin \varphi_1 - e^{3\theta \tan \varphi_1} [3 \tan \varphi_1 \cos(\varphi_1 + \theta) + \sin(\varphi_1 + \theta)] \}}{1 + 9 \tan^2 \varphi_1} - 1 \right\} \quad (26)$$

$$N'_\gamma = \frac{n \cdot \tan \varphi_1 \{ 4 \sin \varphi_2 - e^{3\theta \tan \varphi_2} [3 \tan \varphi_2 \cos(\varphi_2 + \theta) + \sin(\varphi_2 + \theta)] \}}{2(1 + 9 \tan^2 \varphi_2) \cos^2 \varphi_1} \quad (27)$$

where N_γ and N'_γ are, respectively, the weight bearing capacity coefficients corresponding to the weight of soft clay and sandy soil.

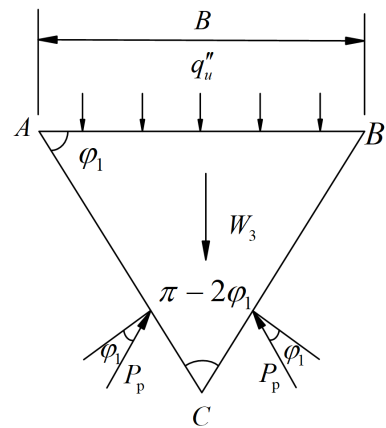


Figure 5. Force analysis of isolation body considering the weight of soil and passive soil pressure.

By superimposing Equations (17) and (25), the formula for the ultimate bearing capacity of a layered foundation with a completely rough foundation is obtained:

$$q_u = c_1 N_c + c_2 N'_c + \sigma_0 N_q + \frac{\gamma_1 B}{2} N_\gamma + \frac{\gamma_2 B}{2} N'_\gamma \quad (28)$$

3. Numerical Simulation Verification and Comparative Analysis

This paper uses the finite difference software FLAC3D 6.0 to model and analyze the bearing capacity of layered foundations with upper soft clay and lower sandy soil layers, verifying the rationality of the theoretical derivation in the previous text. Furthermore, the theoretical solution of this paper is compared with the existing formula results of previous scholars.

For layered foundations considering burial depth, a numerical model in Figure 6 is established to verify the rationality of the previously derived formulas. The foundation consists of a double-layer soil composed of soft clay in the upper part and sandy soil in the lower part, both of which adopt the Mohr Coulomb constitutive model. The specific soil parameters are shown in Table 1. The basic material is concrete, set as a linear elastic model, and the Young's modulus and Poisson's ratio are taken as 20 GPa and 0.2, respectively. The friction angle between the side of the foundation and the soil is 12° . The horizontal displacement at the bottom of the foundation is limited to simulate the situation where the bottom of the foundation is completely rough. The width of the foundation is 3 m.

Table 1. Soil strength parameters.

Soil Mass	Weight ($\text{kN} \cdot \text{m}^{-3}$)	Cohesive Force (kPa)	Internal Friction Angle ($^\circ$)	Young's Modulus (kPa)	Poisson's Ratio
Upper soft clay	14	30	10	5×10^3	0.3
Lower sandy soil	19	8	35	1×10^4	0.2

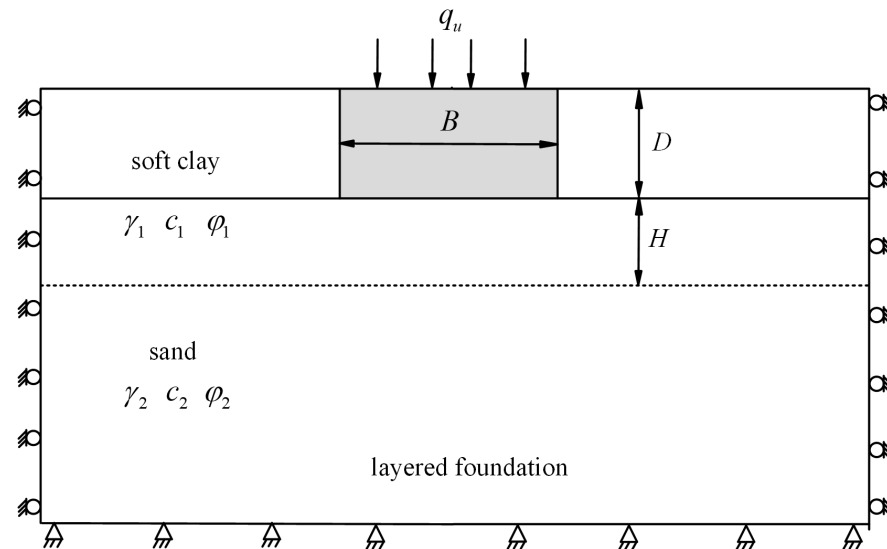


Figure 6. The model of foundation for numerical simulation.

Based on the above parameters, numerical simulation and theoretical formula calculations were conducted, and the results were compared with Meyerhof's semiempirical Formula (1) and Hansen's weighted average method. The comparison results are shown in Figure 7. From Figure 7, it can be seen that the numerical simulation results and the calculation results of Formula (28) in this paper are relatively similar to the results of Hansen's weighted average method, with a maximum error of no more than 5%. The numerical simulation results are very close to the formula results in this paper, although both of them are lower than Meyerhof's formula results, with a maximum error of about 20%, but the overall trend is basically consistent. Moreover, the larger the foundation burial depth and the thickness of the overlying soft clay layer, the more significant the error. The reason is as follows: as mentioned in the second section of the previous text, Formula (1) takes the difference in the bearing capacity of the foundation between soft clay and sandy soil based on the ratio of the thickness of the soft clay layer to the depth of the failure sliding surface. However, the normal stress σ_0 on the equivalent free surface is derived from the equivalent conversion of frictional force and the weight of soil on both sides of the foundation, which is independent of the depth of the sandy soil in the bearing layer of the foundation. For the terms $\sigma_0 N_q$, the difference analysis method overestimates the ultimate bearing capacity of layered foundations. However, the Formula (28) derived in this paper is a correction to it, so the calculated foundation bearing capacity will be less than the result of Formula (1), which is reasonable. Therefore, the unified logarithmic spiral assumption adopted in this paper not only simplifies the calculation but also conforms to the actual situation of a unique sliding surface. The derived theoretical formula has a certain degree of credibility, and the established numerical model can accurately reflect the ultimate bearing capacity of layered foundations when the foundation is completely rough.

In addition, it can be observed from the figure that the ultimate bearing capacity of the foundation decreases approximately linearly with the increase of the thickness of the overlying cohesive soil and increases linearly with the increase of the foundation burial depth. When the burial depth of the foundation increases from 1 m to 5 m, the ultimate bearing capacity of the foundation increases from 1558 kN to 3156 kN, with an increase of nearly 102%. When the burial depth of the foundation is 2 m, the thickness of the overlying soft soil layer increases from 0.2 m to 1.5 m, and the ultimate bearing capacity of the foundation decreases from 2581 kN to 1018 kN, with a decrease of nearly 60%. It can be seen that the burial depth of the foundation and the thickness of the overlying soft soil layer have a significant impact on the ultimate bearing capacity.

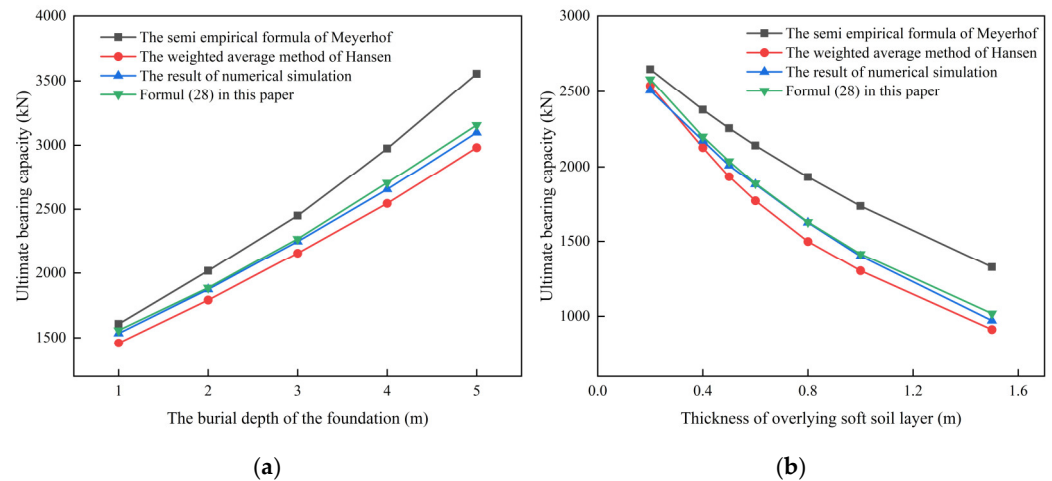


Figure 7. Comparison between numerical and theoretical results of ultimate bearing capacity of completely rough foundation. (a) Different burial depths of foundations; (b) Different thicknesses of overlying soft clay.

4. Sensitivity Analysis of Bearing Capacity Parameters of a Layered Foundation

In order to systematically explore the influence of the size of the foundation and the inherent properties of the soil on the bearing characteristics of the foundation, a numerical simulation scheme containing multiple working conditions was designed to study the variation law of the ultimate bearing capacity of the foundation under each working condition. This numerical simulation scheme designed an orthogonal experiment with 16 working conditions including four factors and four levels [23]. The specific parameter settings are shown in Table 2, where the upper soil layer is a soft clay layer, and the lower soil layer is a sandy soil layer.

Table 2. Numerical Simulation Scheme.

Condition Number	B/D	c_1/c_2	$\tan\phi_1/\tan\phi_2$	H (m)
1	1	3	0.2	1
2	1.5			
3	2			
4	2.5			
5	1.5	1	0.2	1
6		1.5		
7		2		
8		3		
9	1.5	3	0.2	1
10			0.4	
11			0.6	
12			0.8	
13	1.5	3	0.2	0.5
14				1
15				1.5
16				2

4.1. The Width to Depth Ratio of the Foundation and the Thickness of the Overlying Soft Clay Layer

Figure 8a shows the variation of the ultimate bearing capacity of the foundation under different width to depth ratios. It can be seen from the figure that when the foundation burial depth is kept constant, the ultimate bearing capacity of the foundation increases approximately linearly with the increase of the foundation width to depth ratio, and the larger the foundation burial depth, the higher the growth rate of the foundation's ultimate

bearing capacity. The reason is as follows: the increase in foundation width and burial depth expands the contact area between the side and bottom of the foundation and the soil, thereby increasing the lateral constraint force of the soil on the foundation and the friction force on the bottom. In addition, increasing the width of the foundation can effectively reduce the bearing pressure of the soil per unit area, avoid stress concentration in the soil, and thus improve the ultimate bearing capacity of the foundation.

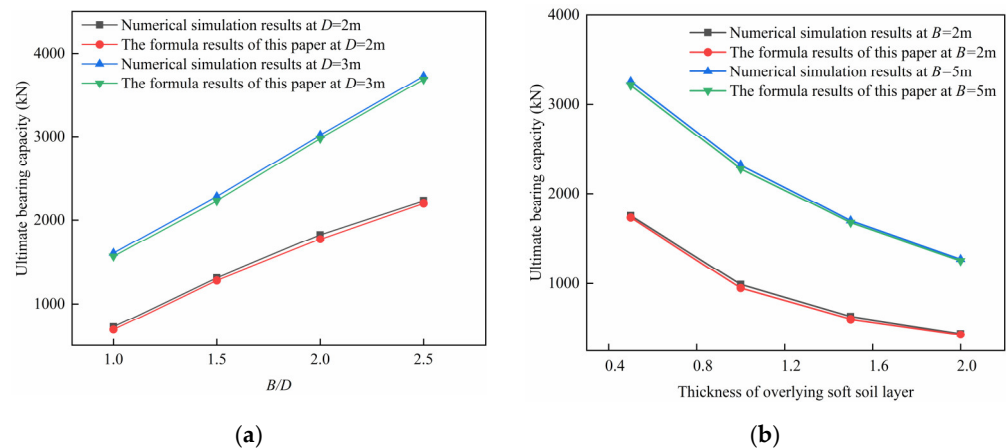


Figure 8. The influence of the width to depth ratio of the foundation and the thickness of the overlying soft clay layer. (a) Different width to depth ratios; (b) Different thicknesses of overlying soft clay layer.

Figure 8b shows the variation of the ultimate bearing capacity of the foundation under different thicknesses of overlying soft clay layers. It can be seen from the figure that the ultimate bearing capacity of the foundation decreases approximately linearly with the increase of the thickness of the overlying cohesive soil under the foundation. When the burial depth of the foundation is 2 m, the thickness of the overlying soft clay layer increases from 0.5 m to 2 m, and the ultimate bearing capacity of the foundation decreases from 1751 kN to 440 kN, with a decrease of nearly 74.7%. It can be seen that the thickness of the soft clay layer has a significant adverse effect on the ultimate bearing capacity of the foundation. The reasoning is as follows: the deformation modulus and internal friction angle of the overlying cohesive soil layer are smaller compared to sandy soil. As its thickness increases, the ability of the overlying soil to resist loads weakens, leading to a downward trend in the ultimate bearing capacity of the foundation. Furthermore, it can be observed from Formula (13) that the thicker the soft clay under the foundation, the smaller the coefficient n , which in turn leads to a decrease in the bearing capacity coefficient, ultimately resulting in a decrease in the bearing capacity of the layered foundation. In practical engineering, reinforcement should be considered for the soil with weak bearing layers to reduce engineering costs.

4.2. The Shear Strength of Soil

Figure 9a shows the variation of the ultimate bearing capacity of the foundation under different ratios of cohesive forces c_1/c_2 of the double-layer soil. It can be seen from the figure that there is a weak linear increase between the ultimate bearing capacity of the foundation and the ratio of cohesive forces. The larger the ratio of c_1/c_2 , the higher the growth rate of the bearing capacity. The reasoning is as follows: the cohesion of soil characterizes the attraction between soil particles. As the cohesion of soil increases, the interaction force between soil particles also increases. As a result, the internal structural stability and shear strength of the soil are enhanced, thereby improving the ultimate bearing capacity of the foundation.

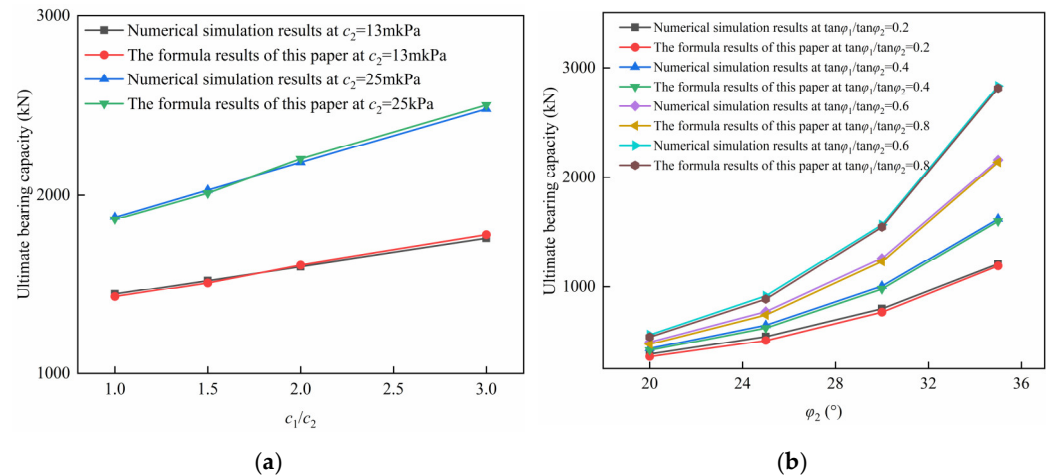


Figure 9. The influence of strength parameters. (a) Different c_1/c_2 ; (b) Different $\tan\phi_1/\tan\phi_2$.

When the cohesive force of the underlying soil is 8 kPa and a is 1, 1.5, 2, and 3 respectively, the ultimate bearing capacity of the foundation is 1263.9 kPa, 1312 kPa, 1360.1 kPa, and 1456.4 kPa, with the highest increase of 7%. As the cohesion of the lower soil increases, although the growth rate shows an upward trend, the maximum growth rate is only 13.8%. It can be seen that although the ratio of cohesive forces of double-layer soil has an improvement effect on the ultimate bearing capacity of the foundation, the degree of influence is relatively small compared to the impact of changes in the size of the foundation.

Figure 9b shows the variation of the ultimate bearing capacity of the foundation under different tangent ratios $\tan\phi_1/\tan\phi_2$ of the internal friction angle of the double-layer soil. It can be seen from the figure that the ultimate bearing capacity of the foundation shows a strong nonlinear upward trend with the increase of the internal friction angle of the double-layer soil. When the internal friction angle of the underlying soil is less than 25° , the increase in the ultimate bearing capacity is relatively small. However, when the internal friction angle of the underlying soil is greater than 25° , the increase in the ultimate bearing capacity begins to increase. Moreover, the larger the tangent ratio of the internal friction angle of the double-layer soil, the more significant the increase in ultimate bearing capacity. The reasoning is as follows: when the friction angle inside the soil increases, the mutual friction force between the soil increases, thereby improving the shear strength of the soil. When subjected to compression from the upper load, the soil on the side of the foundation will prevent the transfer of shear force, achieving the effect of improving the bearing capacity of the foundation.

4.3. Grey Correlation Analysis of Influencing Parameters

The previous section discussed and analyzed the effects of the width to depth ratio of the foundation, the ratio of the cohesive force, the ratio of the tangent value of the internal friction angle, and the thickness of the overlying soft clay on the bearing capacity of the foundation. To gain a more intuitive understanding of the impact of various factors on the ultimate bearing capacity of the foundation, in this section, grey correlation analysis is conducted based on MATLAB to understand the correlation level of each influencing factor. The specific formulas used are as follows:

$$\Delta = |X_i - Y_i(j)| \quad (29)$$

$$A_{ij} = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ij} + \rho\Delta_{\max}} \quad (30)$$

$$R_j = \left(\sum_{i=1}^m A_{ij} \right) / m \quad (31)$$

where Δ is the difference coefficient matrix; X_i is the column matrix, and is the i -th row element of the X matrix; $Y_i(j)$ represents the i -th element of the influencing factor in column j ; A_{ij} is the correlation coefficient matrix; $\Delta_{\min}$ is the minimum value of the coefficient of difference matrix, and $\Delta_{\max}$ is the maximum value of the coefficient of difference matrix; Δ_{ij} is the i -th row and j -th column element of the coefficient of difference matrix; and ρ is the resolution coefficient, with a value of 0–1. In this paper, a value of 0.5 is taken [24].

In this paper, there are a total of 16 sets of operating conditions in the orthogonal experiment, including 3 sets of repeated operating conditions and 13 sets of non-repeated operating conditions. Therefore, $Y_i(j)$ is a 13×4 matrix composed of normalized input variables of various influencing factors. X_i is the corresponding sequence of the ultimate bearing capacity of the foundation, and the ultimate bearing capacity of each row in X_i is the normalized output value corresponding to each variable in $Y_i(j)$. In this paper, each column of $Y_i(j)$ from left to right represents the ratio of foundation width to depth, the ratio of cohesive force, the ratio of tangent value of internal friction angle, and the thickness of overlying soft clay. A_{ij} is the correlation coefficient matrix obtained from grey correlation analysis between various influencing factors and the ultimate bearing capacity of the foundation. By taking the mean of all values in column j of A_{ij} , the grey correlation degree of the influencing factor can be obtained.

The final grey correlation matrix is obtained as $R = [0.7977 \ 0.5613 \ 0.6820 \ 0.7243]$. Figure 10 shows the correlation between the parameters selected for numerical simulation in this paper and the ultimate bearing capacity of the foundation. From Figure 10, it can be seen that the influencing factors considered in the numerical simulation in this paper are highly correlated with the ultimate bearing capacity of the foundation, so the selection of influencing factors is relatively reasonable. Among them, the width to depth ratio of the foundation has the greatest impact on the ultimate bearing capacity of the foundation, followed by the thickness of the overlying soft clay, the tangent ratio of the internal friction angle, and the ratio of the cohesive force. Therefore, in practical engineering, when designing foundations, it is necessary not only to consider the size design of the foundation but also to fully analyze the inherent properties of the foundation soil and the thickness of the soft clay interlayer.

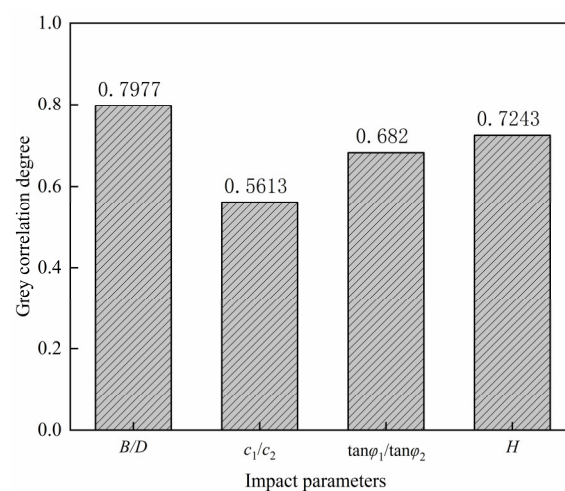


Figure 10. Correlation between different influencing parameters and ultimate bearing capacity of the foundation.

5. Conclusions

This paper is based on Meyerhof's theory of homogeneous foundation and extends it to a layered foundation using the unified sliding surface assumption and difference analysis method. In this paper, a theoretical formula for the ultimate bearing capacity of layered foundations with completely rough foundations is derived and compared and analyzed in relation to the classical semiempirical formula for layered foundations. On

the basis of verifying the reliability of the theoretical formula results, a numerical model is established to further explore and analyze the influence of relevant parameters of soil and foundation on the bearing capacity of the foundation. The main conclusions are as follows:

- (1) In this paper, the unified assumption of a logarithmic spiral sliding surface is adopted to derive the bearing capacity coefficient, which not only simplifies the calculation but also conforms to the actual situation of a unique sliding surface. Meyerhof's theory of homogeneous foundations has been extended, and the derived theoretical formulas have a certain degree of credibility, providing a new approach for predicting the bearing capacity of layered foundations.
- (2) The numerical simulation results are in good agreement with the derived formulas, and the comparison with different theoretical formula solutions shows that the theoretical formula for the ultimate bearing capacity of layered foundations derived in this paper has certain rationality and applicability. Meyerhof's semiempirical formula for a double-layer foundation overestimates the ultimate bearing capacity of the foundation. However, the results of the bearing capacity formula for the upper soft clay and lower sandy soil derived based on this correction in this paper are relatively close to the results of Hansen's weighted average method.
- (3) The numerical simulation results obtained from the single factor analysis method show that the width to depth ratio of the foundation, the ratio of cohesive force of the double-layer soil, and the tangent ratio of the internal friction angle have a positive correlation with the ultimate bearing capacity of the foundation, and the influence is significant. The increase in the thickness of the overlying cohesive soil has an adverse effect on the ultimate bearing capacity of the foundation, manifested as the larger the thickness, the smaller the foundation bearing capacity. Among the four influencing factors mentioned above, grey correlation analysis shows that the width to depth ratio of the foundation has the greatest impact on the ultimate bearing capacity of the foundation, followed by the thickness of the overlying clay layer.

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