applied sciences

MDPI

Article

Prediction of Brake Pad Wear of Trucks Transporting Oversize Loads Based on the Number of Drivers' Braking and the Load Level of the Trucks—Multiple Regression Models

Grzegorz Basista *, Michał Hajos *, Sławomir Francik and Norbert Pedryc

Department of Mechanical Engineering and Agrophysics, Faculty of Production and Power Engineering, University of Agriculture in Cracow, Balicka 120, 30-149 Cracow, Poland; slawomir.francik@urk.edu.pl (S.F.); norbert.pedryc@urk.edu.pl (N.P.)

* Correspondence: grzegorz.basista@urk.edu.pl (G.B.); michal.hajos@urk.edu.pl (M.H.)

Abstract: Brake pad wear forecasting, due to its complex nature, is very difficult to describe using engineering formulas. Therefore, the aim of this publication is to create high-quality brake pad wear forecasts based on three stochastic quantitative models based on multiple regression models (linear model, inverted linear model, and power model). The matrix of explanatory variables was extracted from the controllers of 29 vehicles: A—the driver's style of using the brake pedal specified on a 4-point scale and B—the number of vehicle load ranges specified on a 5-point scale. Methodology: A matrix of explanatory variables was obtained over a 2-year period from trucks carrying oversize loads via OBD2 socket. The trucks operated under similar operating conditions. The created models were verified in terms of their fit to the source data and by analyzing the residuals of the models. It should be emphasized that only the linear model met all the required criteria. The inverted linear and power-law models were rejected. Results: The verified linear model is characterized by very small MAPE errors. The model was validated on 4 trucks and the brake pad wear prediction errors ranged from −0.39% to 7.03%.

Keywords: wear estimation; brake pads prediction; truck operation; multiple regression

check for updates

Citation: Basista, G.; Hajos, M.; Francik, S.; Pedryc, N. Prediction of Brake Pad Wear of Trucks Transporting Oversize Loads Based on the Number of Drivers' Braking and the Load Level of the Trucks—Multiple Regression Models. *Appl. Sci.* **2024**, *14*, 5408. https://doi.org/10.3390/app14135408

Received: 10 May 2024
Revised: 17 June 2024
Accepted: 18 June 2024
Published: 21 June 2024

1. Introduction

The basic task of the braking system is to stop the vehicle, reduce its speed, and prevent the vehicle from rolling away when stationary [1]. Friction disc brakes are a common solution today due to the effectiveness of the braking process, low cost, and high reliability [2]. Despite the enormous progress in the development of the automotive industry, brake pads are commonly used actuators of braking systems [3]. Currently, vehicles are becoming heavier and faster, so their operational efficiency must increase [4]. The wear of pads used in real conditions is characterized by a number of non-linear, multi-dimensional factors [5]. When braking, the vehicle generates friction force in the actuator systems, which results in a reduction of its kinetic energy. The change in kinetic energy leads to an increase in thermal energy occurring mainly on brake discs and pads [6].

The mechanisms of wear and degradation of brake discs and pads have received considerable attention in the literature, highlighting the importance of the materials used in production. D.K. Kolluri et al. [7] examined the effect of graphite particle size on the heating of brake discs. They observed that in composites, the use of small graphite particles compared to large particles improves the thermal properties of discs. The use of a copper-metal matrix for brake pads shows better tribological properties [8,9]. New materials are also used. The authors of Ref. [10] used the newly developed DB-1 material and compared it to the materials currently used. In laboratory tests, they simulated the loads of high-speed trains. The new material has a very good coefficient of friction and is characterized by a lower wear rate due to the fine particles. There are publications in the literature regarding the use of natural material admixtures in effective braking systems. The authors [11] replaced synthetic fibers with hemp fibers. The use of natural hemp fibers reduced the specific wear rate while obtaining a consistent coefficient of friction for brake pads.

The aspect of accumulating significant heat energy in the braking system can prove very dangerous and shorten the life of brake pads and discs. The overheating phenomenon can occur due to the design of brake discs or pads made of materials with low thermal conductivity.

The paper [12] compared brake pads made from three asbestos-free composites. These composites contained different proportions of steel fibers 30, 35, and 41% and synthetic materials 11, 6, and 0%. The composite with 41% and 0% showed the best thermal stability and thermal conductivity. The effect of prolonged thermal loads on hardened steel causes a loss of mechanical properties [13]. As the authors point out, this is due to the material's susceptibility to tempering after heating. Improving the resistance to tempering is to increase the amount of such chemical elements as molybdenum and vanadium.

Articles [14] describe the problem of overuse of the braking system, which is caused by an incorrect driving style. Prolonged use of the brake while driving downhill can cause this and contribute to complete pad wear [15]. This problem is particularly dangerous for trucks with trailers or semi-trailers [16]. As the authors point out [17] (p. 1) "... when using the lowest-priced brake discs and brake pads, a substantial reduction in their efficiency can occur if braking intensively or over a long period". Overheating of the brake system significantly reduces the friction coefficient of the brake pad against the brake disc. This forces an increase in braking force to achieve the same braking torque and results in accelerated wear [18,19]. Corrosion of the braking system also adversely affects the vehicle's braking behavior. The primary corrosion factor is the composition of the brake pad and disc materials [20]. The formation of corrosion intensifies during frequent changes in humidity and temperature difference between the brake disc and pad, which promotes a reduction in the friction coefficient [21,22].

Precision in the installation of new brake pads is also of great importance. When replacing new brake pads, a caliper that has not been cleaned of corrosion can result in faulty seating of the friction element and faster wear and vibration [23]. The research topics presented above show the complexity of the braking phenomenon and the presence of many factors that affect brake pad wear. It should be noted that most of the research conducted was carried out in the laboratory and not on vehicles in real operating conditions. In the case of managing a fleet of multiple vehicles, the ability to estimate brake pad wear would make it possible to optimize the replacement schedule and identify the causes of rapid brake system wear. Currently, the most commonly used predictor is the vehicle distance traveled [24].

In a very interesting study [25], the authors measured the brake pad and disc wear on real objects. The study lasted 2 years, and 20 cars were analyzed. The influence of the type of traffic (urban–urban) and calendar month on the wear of the above-mentioned elements was determined. The study concluded that the wear and tear of the studied brake system components are influenced by the type of vehicle traffic and the season and are significantly statistical. However, the above work did not take into account many operational factors such as the driver's driving style, kilometers traveled, and vehicle load.

Some researchers have based their brake pad wear estimation results on machine learning methods [26,27]. Good results were obtained for XGBoost + Logistic Regression and XGBoost + Deep Recurrent Neural Network—accuracy of 70% and 85%. The disadvantages of these methods are the need to collect a very large number of data and the selection of optimal configurations of processing methods. In the study [28], frictional thermal energy and car braking analysis were used to determine wear. The results showed that the decisive factor in pad wear is the vehicle's initial speed.

The Archard equation is a popular method for estimating brake pad wear. Kenneth Ma et al. tried to estimate brake pad wear based on the Archard equation. The estimation error on a real-world car proved to be very large, and as the authors stated "...without access to the associated usage data, accurate validation of the prediction cannot be carried out" [24] (p. 12). Trucks carrying oversize loads experience frequent changes in the load carried. The weight of the load can vary up to 300% of the vehicle's weight. Therefore, in this case, estimating brake pad wear is important and should be based on reliable data. A key factor for fleet managers is the use of data stored in the vehicle's controllers. This allows verification of the driver's driving style and monitoring of the truck's operating data. OBD2 On-board vehicle diagnostics installed by manufacturers in each vehicle were used to download the data. Chunyu Yu et al. stated that "It is difficult to describe wear by using general formulas fully in engineering, because the wear characterization is related to several factors that are complex, nonlinear and multidimensional" [5] (p. 2).

Therefore, the purpose of this publication is to create highly qualitative predictions of brake pad wear based on three stochastic quantitative models based on multiple regression. The matrix of explanatory variables based on real data takes into account the number of vehicle load ranges determined on a 5-point scale and the driver's style of using the brake pedal determined on a 4-point scale. In this work, "the driver's style of using the brake pedal" is understood as the number and intensity of pressing the brake pedal.

2. Materials and Methods

Modeling of the brake pads' wear system is understood as a list of procedures, leading to the above-stated goal (Figure 1).

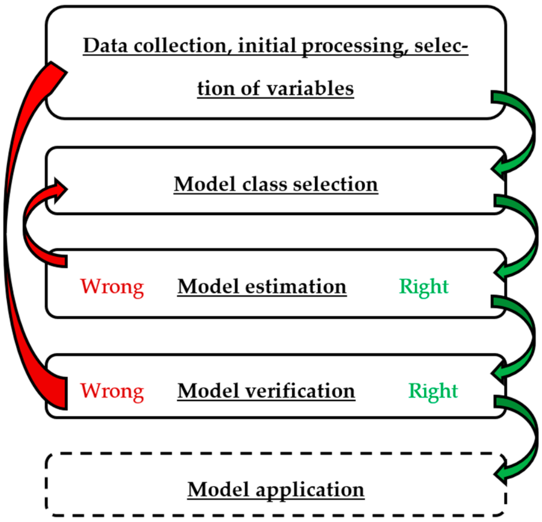

Figure 1. Procedure for determining the statistical model.

2.1. Method of Collecting Real-World Data to Determine Model Parameters

Operational data were collected from a transportation company that owns 34 trucks adapted for the transportation of oversized cargo. The selected vehicles were a homogeneous group moving in a closed area (trucks without trailers or semi-trailers). The gross vehicle weight (GVW) of the trucks was 50,000 kg, 10×6 (11 units) and 10×4 (24 units) configuration. All trucks had 4 torsion axles and disc brake systems on non-driving axles. A drum brake system was installed on the drive axles. All trucks had the same tire size installed.

The tests were conducted from February 2020 to November 2021 and began after the brake pads were replaced on a particular truck. The brake pads used were from the same manufacturer. Completion of the study was defined as the thickness of the brake pads outside the serviceable range designated by the manufacturer (new 30 mm thick, worn < 8 mm thick). During the study, 5 trucks were eliminated from the analysis (3 trucks had mechanical or thermal damage to the brake pads, and 2 had driver changes).

The source data were read via diagnostic equipment from Knorr-Bremse (Munchen, Germany), Wabco (Friedrichshafen, Germany) and specialized CL2000 CAN 2.0A software from CSS Electronics (Aabyhoej, Denmark). Air brake system pressure signals and vehicle load were recorded via the CAN-BUS (J1939 protocol, OBD2). The recording of variable data was grouped in the recorder at 1000 km intervals. During brake pad replacement, the number of kilometers was read and rounded up to 100 km. The collected source data were contained in two matrices A = [129 × 88] and B = [145 × 88], where A—brake system pressure data matrix and B—vehicle load range data matrix. The collected source data were clustered according to the established algorithm (Table 1).

Table 1. Algorithm for clustering source data.

Period of Exploitation of the Brake Pads [km]	A = Brake System Pressure Range [MPa]	B = Vehicle Load Range GVW [%]
Y	X_1: <0.15 X_2: 0.15 to 0.24 X_3: 0.25 to 0.40 X_4: >0.40	X_5: <20% X_6: 20% to 39% X_7: 40% to 59% X_8: 60% to 80% X_9: >80%

Due to the large number of data, the source material was converted to statistically significant information [29,30]. In this study, the value of the coefficient of variation related to a range of 1000 km was used for each explanatory variable. The critical value of the coefficient of variation (Var-co.) was set at <10% [31]. The explanatory variable Y determines the brake pad wear period in kilometers for individual trucks $T_i = \{T_1, T_2, \ldots, T_{29}\}$. The explanatory variables $X_i = \{X_1, \ldots, X_4\}$ represent the number of brake pedals used within a certain range of brake system operating pressures per 1000 km. The explanatory variables $X_i = \{X_5, \ldots, X_9\}$ represent the number of individual truck load ranges per 1000 km with respect to GVW.

2.2. Real Data Collection Method for Model Testing

Based on the created models, brake pad wear was predicted for four example trucks K1, K2, K3, and K4.

Trucks K1 and K2 are trucks that were not subject to testing, and their parameters are the same as T1 to T29. Trucks K3 and K4 belonged to another company specializing in bulk material transportation (2 trucks, 8 × 4 system, brake pads on 2 front non-drive axles, GVW—32,000 kg). The same equipment and methodology were used to acquire data as in the main study described in Section 2.1.

3. Regression Models

3.1. Model Class Selection

Finding the key relationships between the phenomena under consideration is the goal of the presented statistical model. Understanding of cause–effect relationships was realized using the linear model, inverted linear model, and power model [29,30,32]. An important aspect is to carry out calculations for all forms of models. The following models were used in this study:

Linear model (1):

$$Y = \propto_0 + \propto_1 X_1 + \propto_2 X_2 + \ldots + \propto_n X_n + \varepsilon \quad (1)$$

Inverted linear model (2):

$$Y^{-1} = \propto_0 + \propto_1 X_1^{-1} + \propto_2 X_2^{-1} + \ldots + \propto_n X_n^{-1} + \varepsilon \quad (2)$$

Power model (3):

$$Y = \propto_0 + X_1^{\propto_1} + X_2^{\propto_2} + \ldots + X_n^{\propto_n} + e^{\varepsilon} \quad (3)$$

The power model described by Equation (3) was linearized using the natural logarithm. The structure of the quasi-linear model is shown in Equation (4):

$$\ln Y = \ln\alpha_0 + \alpha_1 \ln X_1 + \alpha_2 \ln X_2 + \ldots + \alpha_8 \ln X_8 + \epsilon \tag{4}$$

where Y—dependent variable [km], $X_{1,2,\ldots,n}$—explanatory variables, α_0, α_1, . . ., α_n—unknown parameters of the model, and ε—randomness component of the model.

The models were recognized as high-quality models after meeting the following criteria:

- Valve of adjusted determination coefficient $\tilde{R}^2$ > 90%,
- full analysis of the random components of the residuals of the models—correctly verified statistically.

3.2. Parameter Estimation Methods and Models Verification

Methods for Estimating Structural Parameters of Models

The general structural form of the model was chosen to provide the best possible fit [29,30,32]. For this purpose, the parameters of the linear model α_i (i = 0, 1, 2, . . ., n) were estimated using the classical method of least squares (5).

$$\sum_{i=1}^{n} (y_i - \overline{y}_i)^2 \to \min \tag{5}$$

where y_i—the actual value of the explanatory variable and $\overline{y}_i$—the value of the explanatory variable determined from the model.

The coefficient of convergence φ^2 (6) describes what part of the data is not explained by the statistical model.

$$\varphi^2 = \frac{\sum_{i=1}^{n} (y_i - \overline{y}_i)^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2} \tag{6}$$

Coefficient of determination (7):

$$R^2 = 1 - \varphi^2 \tag{7}$$

Adjusted determination coefficient (8):

$$\tilde{R}^2 = 1 - \left(1 - R^2\right)\frac{n-1}{n-k} \tag{8}$$

where n—number of observations and k—degrees of freedom.

In the process of verifying an econometric model, it is first necessary to check whether there is a linear relationship between the explanatory variable Y and any of the explanatory variables X_i of the model. We test the significance of the determined regression coefficients and formulate hypotheses (9):

$$H_0 : \sum_{i=0}^{n} \alpha_i^2 = 0; H_1 : \sum_{i=0}^{n} \alpha_i^2 \neq 0 \tag{9}$$

We verify the set of hypotheses with statistics, Formula (10):

$$F = \frac{R^2(n-k-1)}{k\left(1 - R^2\right)} \tag{10}$$

In a valid econometric model, the explanatory variable Y must significantly depend on each of the explanatory variables Xi of the model. For each coefficient of the regression model, we pose hypotheses (11):

$$H_0 : \alpha_i = 0;\ H_1 : \alpha_i \neq 0 \tag{11}$$

We verify the set of hypotheses with statistics (12):

$$t_i = \frac{a_i}{S(\alpha_i)} \quad (12)$$

where a_i—the estimator of the coefficient α_i and $S(\alpha_i)$—the estimator of the dispersion of the coefficient α_i.

3.3. Methods for Analyzing the Random Components of the Model

In the method of least squares, in order for the obtained estimators of the coefficients α_i (i = 0, 1, 2,. . .,n) to be effective, the Gauss–Markov assumptions [30,32,33] must be met:

- The values of the explanatory variables are fixed (they are not random).
- The randomness of the values of the explanatory variable y follows from the randomness of the component ε.
- The random components ε for the individual values of the explanatory variables have a normal (or very close to normal) distribution with an expected value of zero and a constant variance: N(0, δε).
- The random components are not correlated with each other.

Fulfillment of the Gauss–Markov assumptions was verified using the appropriate statistical tests and relationships presented below.

3.3.1. The Hypothesis of Normality of the Random Components of the Residual

The normality of residuals was assumed a priori when deriving all test statistics. If random errors in small samples are not normally distributed, the distributions of the test statistics differ from the values resulting from the normality of the distribution of residuals.

Hypothesis H_0 was set: The random components have a normal distribution. Hypothesis verification was performed using the Shapiro–Wilk test. The value of the test statistic was determined by Formula (13):

$$W = \frac{\left[\sum_{i=1}^{\frac{n}{2}} a_{n,i}\left(e_{(n-i-1)} - e_i\right)\right]^2}{\sum_{i=1}^{n}(e_i - \overline{e})^2} \quad (13)$$

where $a_{n,i}$—Shapiro–Wilk coefficients, $e_1 \ldots e_n$—values of the model residuals, and $\overline{e}$—the mean value of the model residuals.

If $W > W_\alpha$ value, there is no basis for rejecting hypothesis H_0.

3.3.2. The Hypothesis of Autocorrelation of the Random Components of the Residuals

Autocorrelation is the interdependence of random components and is clearly undesirable. The hypothesis about the lack of autocorrelation of random components was verified using the Durbin–Watson test. Hypotheses (14) were formulated as follows:

$$H_0 : \rho(\varepsilon_i, \varepsilon_{i-1}) = 0; H_1 : \rho(\varepsilon_i, \varepsilon_{i-1}) > 0 \cup H_1 : \rho(\varepsilon_i, \varepsilon_{i-1}) < 0 \cup H_1 : \rho(\varepsilon_i, \varepsilon_{i-1}) \neq 0 \quad (14)$$

where ρ—autocorrelation coefficient of random components of order one.

The empirical value of the Durbin–Watson statistic was determined by Formula (15).

$$d = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2} \quad (15)$$

3.3.3. The Hypothesis of Randomness of the Components of the Residuals

Verification of the hypothesis of the randomness of the distribution of deviations of the model's residuals is aimed at assessing the appropriateness of the choice of the analytical form of the model. To check the randomness of the residuals, the number of series tests (16) was used.

$$\theta = \{n\varepsilon : P(n\varepsilon \leq n_{0.975} = \alpha/2)\} \cup \{n\varepsilon : P(n\varepsilon \geq n_{0.025} = \alpha/2)\} \quad (16)$$

where nε—the number of residuals of the same signs (even or odd).

Hypothesis H_0 was set: The error of the model residuals is random.

3.3.4. The Hypothesis of the Symmetry of the Random Components of the Residuals

The random components should have a normal distribution, which is a symmetric distribution. The test checks the number of residuals in plus ρ_+ and in minus ρ_-. We pose hypotheses (17) as follows:

$$H_0 : \ \rho_+ = 0.5; H_1 : \ \rho_+ \neq 0.5 \quad (17)$$

where ρ_+—the number of residuals in plus.

To test the hypotheses, the symmetry statistics of the random components were used in the form (18):

$$\mathrm{t} = \frac{\frac{\mathrm{m}}{\mathrm{n}} - 0.5}{\sqrt{\frac{\frac{\mathrm{m}}{\mathrm{n}}\left(1-\frac{\mathrm{m}}{\mathrm{n}}\right)}{\mathrm{n}-1}}} \quad (18)$$

where m—the number of residuals in plus.

The statistic, with the null hypothesis being true, has a t-Student's distribution with (n − 1) degrees of freedom. The critical area of the test is two-sided.

3.3.5. The Hypothesis of Homoskedasticity of the Random Components of the Residuals

The Breusch–Pagan test (19) was used to determine the presence of equality of variance of random components (homoskedasticity).

$$\chi^2 = n \cdot R_\varepsilon^2 \quad (19)$$

where R_ε^2—fit of the regression model residuals.

The hypotheses posed were as follows: H_0: Homoskedasticity is present $\chi_2 < \chi_{2\alpha}$ (constancy of variance); H_1: Heteroskedasticity is present $\chi^2 > \chi^{2\alpha}$ (no constancy of variance).

3.4. A Method for Evaluating the Use of Models

MAPE (Mean Absolute Percentage Error) was used to compare model results and actual values [29,30]. MAPE reports the average magnitude of forecast errors for the test period expressed as a percentage. The MAPE value allows comparing the accuracy of forecasts of different models and was calculated by Formula (20):

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - y_i^p}{y_i}\right| \cdot 100\% \quad (20)$$

where y_i^p—predicted value.

The primary predictive criterion for evaluating models is the minimization of MAPE error. The acceptable error must not exceed <15%. Scaling of the correctness of the models was performed according to the following criteria: MAPE < 5%—excellent, MAPE < 10%—very good, and MAPE < 15%—good.

4. Results and Discussion

4.1. Results of the Initial Grouping of Source Data

The first part of sorting the data was to group them according to the assumptions shown in Table 1. The results of grouping the data according to the adopted algorithm are shown in Table 2.

Table 2. Results of source data grouping.

Truck	Y [km]	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
T_1	68,100	199	311	322	9	143	76	36	186	537
T_2	56,400	98	176	389	44	129	67	74	101	570
T_3	72,400	201	413	179	33	146	50	123	105	505
T_4	68,300	193	265	325	29	137	87	73	174	562
T_5	68,800	201	311	246	6	151	116	39	186	499
T_6	63,400	101	222	366	47	146	49	171	215	493
T_7	64,900	194	257	351	77	138	69	75	266	538
T_8	68,500	188	319	300	33	147	77	170	195	489
T_9	73,500	131	400	245	9	170	50	20	247	559
T_{10}	59,700	171	183	337	32	138	88	50	196	550
T_{11}	66,100	151	260	304	10	146	86	50	195	552
T_{12}	78,000	193	462	121	29	179	90	75	275	457
T_{13}	53,100	78	148	387	67	125	50	31	235	543
T_{14}	77,300	124	386	149	3	166	51	20	244	554
T_{15}	84,200	186	472	94	29	182	44	120	109	510
T_{16}	83,800	279	449	105	11	190	70	78	155	485
T_{17}	81,100	280	479	125	4	177	116	51	407	478
T_{18}	83,100	277	460	116	24	178	51	21	193	444
T_{19}	61,700	106	201	384	35	134	88	75	257	511
T_{20}	79,100	185	450	100	5	177	89	73	253	494
T_{21}	72,400	200	401	183	28	173	63	64	203	559
T_{22}	77,000	198	423	152	7	169	113	42	363	469
T_{23}	68,600	194	320	259	41	150	83	66	198	503
T_{24}	78,900	260	408	118	16	179	70	83	152	492
T_{25}	75,200	234	389	168	33	162	56	155	206	570
T_{26}	88,500	178	482	76	26	189	89	77	169	483
T_{27}	88,300	167	461	92	23	187	68	78	154	522
T_{28}	69,100	83	403	194	11	157	50	24	250	549
T_{29}	56,800	147	186	402	54	135	51	20	179	552

A preliminary analysis of the grouped data is shown in Table 3, which involves calculating the coefficient of variation value as a measure of dispersion [29,31].

Table 3. Analysis of grouped data for the linear model.

	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Min	53,100	78	148	76	3	125	44	20	101	444
Max	88,500	280	482	402	77	190	116	171	407	570
Mean	71,927.6	179.2	348.1	227.2	26.7	158.6	72.5	70.3	209.3	518.2
Median	72,000	188.2	389.0	194.0	28.0	156.6	69.8	72.7	196.2	511.0
Std. dev.	9379.3	54.9	105.4	108.6	18.6	19.6	20.8	41.9	66.2	35.7
Var—co.	13.0%	30.6%	30.3%	47.8%	69.4%	12.4%	28.6%	59.6%	31.6%	6.9%

The results obtained in the range of less than Var-co. < 10% were eliminated [32]. Analysis of the results presented in Table 3 postulates the elimination of the X_9 variable. Therefore, after the first stage of selection, the variables $X_i = \{X_1, X_2, \ldots, X_8\}$ were used for further model construction.

4.2. Linear Model Estimation and Verification Results

The constructed linear model, Formula (1) for $X_i = \{X_1, X_2, \ldots, X_8\}$, was verified at the significance level $\alpha = 0.05$. The model's coefficient of determination is $\tilde{R}^2 = 0.926$ (coefficient of convergence $\varphi^2 = 5.3\%$). The model explains 92.6% of the variability of the studied trait, this indicates a good fit of the model to the empirical data (Table 4).

Table 4. Multivariate regression results for the linear model.

R^2	$\tilde{R}^2$	Std Error	Observation		
0.947	**0.926**	**2603.251**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	24,459.688	16,525.257	1.480	0.154	
X_1	4.778	12.076	0.396	0.697	
X_2	40.385	19.317	2.091	0.049	
X_3	−7.390	19.666	−0.376	0.711	
X_4	7.867	38.877	0.202	0.842	
X_5	211.851	74.906	2.828	0.010	
X_6	37.612	32.499	1.157	0.261	
X_7	8.061	13.108	0.615	0.546	
X_8	−13.710	9.380	−1.462	0.159	
ANOVA	df	SS	MS	F	Significance F
Regression	8	2,415,852,024	301,981,503	44.56	4.413×10^{-11}
Residual	20	135,538,320.4	6,776,916.021		
Total	28	2,551,390,345			

The F statistic, given the truth of the null hypothesis, has an F Snedecor distribution with 8 degrees of freedom of the numerator and 20 degrees of freedom of the denominator. The empirical value of the statistic is F = 44.56, and the corresponding critical level of significance F = 4.413×10^{-11}, which is less than the accepted significance level $\alpha = 0.05$. We therefore reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables Xi = $\{X_1, X_2, \ldots, X_8\}$. We test the significance of the individual regression coefficients.

The statistic at the truth of the null hypotheses has a Student's t-distribution with 20 degrees of freedom. The empirical values of the t-Student's statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 4. There are no grounds for rejecting the hypothesis that the model constants $a_i = \{\alpha_0, \ldots, \alpha_8\} \backslash \{\alpha_2, \alpha_5\}$ are insignificant, i.e., equal to zero (the values of the critical level of significance for these coefficients are greater than the accepted level of significance $\alpha = 0.05$). There is no basis for rejecting the hypothesis that the variables $X_i = \{X_1, \ldots, X_8\} \backslash \{X_2, X_5\}$ are insignificant. The current model structure is flawed.

Re-Selection of the Linear Model Class

For the new linear model, Formula (1) for $X_i = \{X_2, X_5\}$, model fitting was carried out at a significance level of $a = 0.05$. The coefficient of the model is $\tilde{R}^2 = 0.927$ (coefficient of convergence $\varphi^2 = 6.8\%$). The model explains 92.7% of the variation in the studied trait. This shows a good fit of the model to the empirical data (Table 5).

The F statistic, with the null hypothesis being true, has a Snedecor F distribution with 2 degrees of freedom of the numerator and 26 degrees of freedom of the denominator. The empirical value of the statistic is F = 178.177. The critical level of significance F = 6.65×10^{-16} is less than the accepted level of significance $\alpha = 0.05$. We therefore reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables $X_i = \{X_2, X_5\}$. For each coefficient of the regression model α_i, we test the hypothesis of its significance. We verify it with a statistic of Student's t-distribution with 26 degrees of freedom. The empirical values of the t-Student's statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 5. All the coefficients of the model are significantly different from zero (the values of the critical level of significance are less than the accepted level of significance $\alpha = 0.05$). There are no grounds for rejecting the hypothesis that all coefficients of the tested model are significantly different from zero. The current structure of the model is correct and is represented by the relation (21) and geometric interpretation in Figure 2:

$$\hat{y}_1 = 44.004x_2 + 234.194x_5 + 19,472.544 \tag{21}$$

Table 5. Multivariate regression results for the new linear model.

R^2	$\tilde{R}^2$	Std Error	Observation		
0.932	**0.927**	**2583.179**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	19,472.544	6607.045	2.947	0.007	
X_2	44.004	12.043	3.654	1.14×10^{-3}	
X_5	234.194	64.799	3.614	0.001	
ANOVA	df	SS	MS	F	Significance F
Regression	2	2.38×10^9	1.19×10^9	178.177	6.65×10^{-16}
Residual	26	173,493,185.4	6,672,814.821		
Total	28	2.55×10^9			

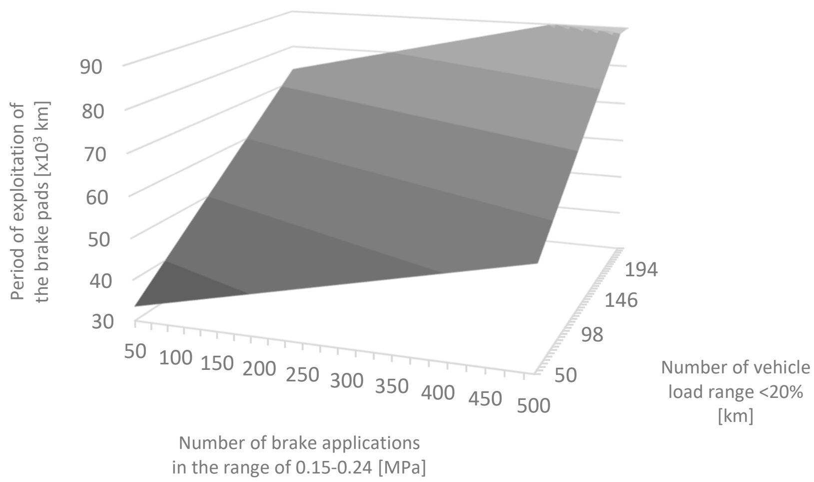

Figure 2. Geometric interpretation of the linear model (21). Y_1—Period of exploitation of the brake pads [km]; X_2—Number of brake applications in the range of 0.15–0.24 [MPa]; X_5—Number of vehicle load range < 20% [km].

4.3. Estimation and Verification Results of Inverted Linear Model

For the inverted linear model, Formula (2) for $X_i = \{X_1, \ldots, X_8\}$, the results of parameter estimation are presented in Table 6.

The constructed linear inverse model was verified at the significance level α = 0.05. The model's coefficient of determination is $\tilde{R}^2 = 0.961$ (coefficient of convergence φ^2 = 2.8%). The model explains 96.1% of the variation in the studied trait, this indicates a good fit of the model to the empirical data. The significance of the regression coefficients was tested, and we formulated hypotheses for a linear model. The F statistic, with the null hypothesis being true, has a Snedecor F distribution with 8 degrees of freedom of the numerator and 20 degrees of freedom of the denominator. The empirical value of the statistic is F = 86.598, and the corresponding critical level of significance F = 8.01×10^{-14}. The level is less than the accepted significance level α = 0.05. We reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables $X_i = \{X_1, X_2, \ldots, X_8\}$.

For each coefficient of the regression model, we hypothesize as for a linear model. The empirical values of the t-Student's statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 6. No basis for rejecting the hypothesis that the model constants $\alpha = \{\alpha_0, \ldots, \alpha_8\} \backslash \{\alpha_2, \alpha_3\}$ are insignificant, i.e., equal to zero; incorrect model structure.

Table 6. Multiple regression results for the inverted linear model.

R^2	$\tilde{R}^2$	Std Error	Observation		
0.972	**0.961**	**3.897×10^{-7}**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	1.02×10^{-5}	2.14×10^{-6}	4.780	1.14×10^{-4}	
X_1	7.12×10^{-5}	4.61×10^{-5}	1.546	0.137	
X_2	8.18×10^{-4}	1.43×10^{-4}	5.740	1.29×10^{-5}	
X_3	-1.96×10^{-4}	6.20×10^{-5}	−3.162	0.005	
X_4	-8.79×10^{-7}	1.29×10^{-6}	−0.683	0.502	
X_5	3.00×10^{-4}	3.36×10^{-4}	0.894	0.381	
X_6	-7.08×10^{-6}	2.63×10^{-5}	−0.269	0.791	
X_7	5.37×10^{-6}	7.24×10^{-6}	0.742	0.466	
X_8	1.72×10^{-5}	5.73×10^{-5}	0.300	0.767	
ANOVA	df	SS	MS	F	Significance F
Regression	8	1.05×10^{-10}	1.32×10^{-11}	86.598	8.01×10^{-14}
Residual	20	3.04×10^{-12}	1.52×10^{-13}		
Total	28	1.08×10^{-10}			

New Inverse Linear Model

For the new inverted linear model, Formula (2) for $X_i = \{X_2, X_3\}$, model fitting was carried out at a significance level of $\alpha = 0.05$. Based on the verified data, we determine the parameter values of the new inverted linear model (Table 7).

Table 7. Multiple regression results for the new inverted linear model.

R^2	$\tilde{R}^2$	Std Error	Observation		
0.965	**0.962**	**3.830×10^{-7}**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	1.22×10^{-5}	4.41×10^{-7}	27.598	8.69×10^{-21}	
X_2	1.02×10^{-3}	8.22×10^{-5}	12.463	1.80×10^{-12}	
X_3	-2.34×10^{-4}	3.47×10^{-5}	−6.734	3.81×10^{-7}	
ANOVA	df	SS	MS	F	Significance F
Regression	2	1.04×10^{-10}	5.22×10^{-11}	355.962	1.29×10^{-19}
Residual	26	3.82×10^{-12}	1.47×10^{-13}		
Total	28	1.08×10^{-10}			

The model fit was verified at a significance level of $\alpha = 0.05$. The coefficient of the model is $\tilde{R}^2 = 0.962$ (coefficient of convergence $\varphi^2 = 3.5\%$). Conclusion: The model explains 96.2% of the variability of the studied trait. This shows a very good fit of the model to the empirical data. We hypothesize that the coefficients of the regression model are not significant. The F statistic, with the null hypothesis being true, has a Snedecor F distribution with 2 degrees of freedom of the numerator and 26 degrees of freedom of the denominator. The empirical value of the statistic is F = 355.962, and the corresponding critical significance level $F = 1.92 \times 10^{-19}$. The level is less than the accepted significance level $\alpha = 0.05$. We therefore reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables $X_i = \{X_2, X_3\}$.

For each coefficient of the regression model α_i, we test the hypothesis of its significance. We verify it with a statistic with a Student's t-distribution with 26 degrees of freedom. The empirical values of the t-Student's statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 7. All the coefficients of the model are significantly different from zero (the values of the critical level of significance are less than the accepted level of significance $\alpha = 0.05$). There is no basis for rejecting the hypothesis that all model coefficients are significantly different from zero. The current structure of the inverse linear model is correct and is represented by the relation (22) and geometric interpretation in Figure 3:

$$\hat{y}_2 = \frac{x_2 x_3}{1.22 \times 10^{-5}(x_2 x_3) - 2.34 \times 10^{-4}(x_2) + 1.02 \times 10^{-3}(x_3)} \tag{22}$$

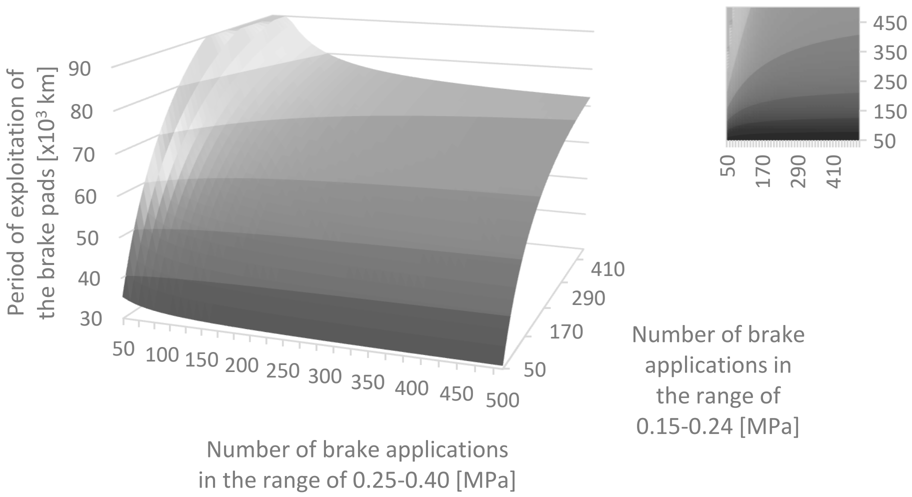

Figure 3. Geometric interpretation of the inverse linear model (22). Y_2—Period of exploitation of the brake pads [km]; X_2—Number of brake applications in the range of 0.15–0.24 [MPa]; X_3—Number of brake applications in the range of 0.25–0.40 [MPa].

4.4. Results of Estimation and Verification of the Power Model

For the power (quasi-linear) model specified by Formula (14), the parameter values are shown in Table 8.

Table 8. Multivariate regression results for the power model.

R^2	**$\tilde{R}^2$**	**Std Error**	**Observation**		
0.958	**0.941**	**0.033**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	8.929	0.987	9.050	1.65×10^{-8}	
X_1	0.025	0.027	0.911	0.373	
X_2	0.162	0.051	3.149	0.005	
X_3	−0.062	0.038	−1.621	0.121	
X_4	−0.004	0.011	−0.349	0.730	
X_5	0.299	0.174	1.721	0.101	
X_6	0.013	0.032	0.427	0.674	
X_7	0.009	0.013	0.738	0.469	
X_8	−0.016	0.026	−0.643	0.527	
ANOVA	df	SS	MS	F	Significance F
Regression	8	0.494	0.062	56.656	4.62×10^{-12}
Residual	20	0.022	0.001		
Total	28	0.516			

We will verify the constructed model at the significance level $\alpha = 0.05$. The model's coefficient of determination is $\tilde{R}^2 = 0.941$ (coefficient of convergence $\varphi^2 = 4.1\%$ $\varphi^2 = 4.2\%$). The model explains 94.1% of the variation of the studied trait, this is a good fit of the model to the empirical data. The significance of the regression coefficients was checked. We put hypotheses as for previous models. The F statistic, with the null hypothesis being true, has an F Snedecor distribution with 8 degrees of freedom of the numerator and 20 degrees of freedom of the denominator. The empirical value of the statistic is F = 56.656, and the corresponding critical level of significance F = 4.62×10^{-12}. The level is less than the accepted significance level $\alpha = 0.05$. We reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables $X_i = \{X_1, X_2, \ldots, X_8\}$.

For each coefficient of the regression model, we pose hypotheses as for previous models. The empirical values of the Student's *t*-statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 8. There are no grounds for rejecting the hypothesis that the model constants $a = \{\alpha_0, \ldots, \alpha_8\} \backslash \{\alpha_2\}$ are insignificant, i.e., equal to zero (the values of the critical level of significance for these coefficients are greater than the accepted level of significance $\alpha = 0.05$). There are no grounds for rejecting the hypothesis that the variables $X_i = \{X_1, \ldots, X_8\} \backslash \{X_2\}$ are insignificant. The current model structure is incorrect.

Determination of the New Power Model

The new quasi-linear power model is described by Equation (23):

$$\ln Y = \ln \alpha_0 + \alpha_2 \ln X_2 + \epsilon \quad (23)$$

Estimation of structural parameters for the presented model is provided in Table 9.

Table 9. Multiple regression results for the new power model.

R^2	$\tilde{R}^2$	Std Error	Observation		
0.904	**0.901**	**0.043**	**29**		
	Coefficients	**Std Error**	***t* Stat**	***p* Value**	
Intercept	9.075	0.132	68.877	7.01×10^{-32}	
X_2	0.362	0.023	15.964	2.82×10^{-15}	
ANOVA	df	SS	MS	F	Significance F
Regression	1	0.466	0.466	254.865	2.821×10^{-15}
Residual	27	0.049	0.002		
Total	28	0.516			

We verify the fit of the model at a significance level of $\alpha = 0.05$. The coefficient of the model is $\tilde{R}^2 = 0.901$ (convergence rate $\varphi^2 = 9.6\%$). The model explains 90.1% of the variability of the studied trait. This indicates a good fit of the model to the empirical data. We hypothesize that the coefficients of the regression model are not significant. The F statistic, given the truth of the null hypothesis, has an F Snedecor distribution with 1 degree of freedom of the numerator and 27 degrees of freedom of the denominator. The empirical value of the statistic is F = 254.865, and the corresponding critical level of significance F = 2.821×10^{-15} is less than the accepted significance level $\alpha = 0.05$. We reject hypothesis H_0 in favor of H_1. There is no basis for rejecting the hypothesis that brake pad wear depends on at least one of the variables X_2.

For the regression model coefficient α_2, we test the hypothesis of its significance. We verify it with a statistic of *t*-Student's distribution with 27 degrees of freedom. The empirical values of the *t*-Student's statistic and the corresponding values of the critical level of significance (*p*-value) are shown in Table 9. The coefficients of the model are significantly different from zero (the values of the critical level of significance are less than the accepted level of significance $\alpha = 0.05$). There are no grounds for rejecting the hypothesis that the coefficients of the tested model are significantly different from zero. The current structure of the quasi-linear power model is correct. After simple transformations, we obtain a power model in the form (24) and geometric interpretation in Figure 4:

$$\hat{y}_3 = 8725.971 \cdot x_2^{0.362} \tag{24}$$

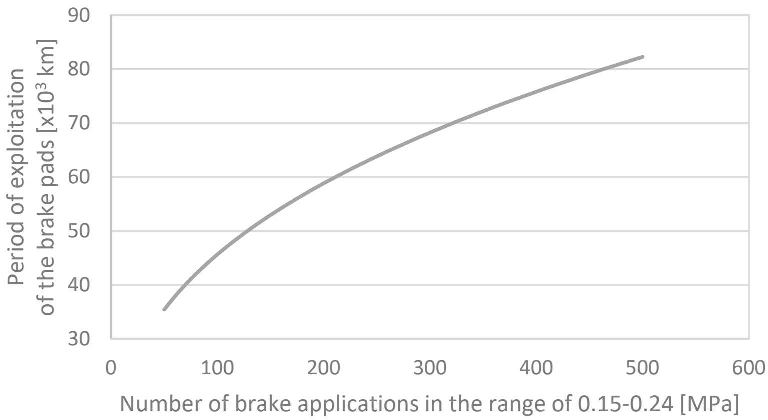

Figure 4. Geometric interpretation of the power model (24). Y_3—Period of exploitation of the brake pads [km]; X_2—Number of brake applications in the range of 0.15 to 0.24 [MPa].

4.5. Results of Random Component Analysis of the Models

In the least squares method, as mentioned earlier, the Gauss–Markov assumptions must be met.

The random components of the residuals of the obtained models (21), (22), and (24) were analyzed, and the results are shown in Table 10.

Table 10. Random components of the residuals of models (21), (22), and (24).

	Linear Model			Inverted Linear Model			Power Model		
Observation	Y_1	**Residual**	**Std. Residual**	Y_2	**Residual**	**Std. Residual**	Y_3	**Residual**	**Std. Residual**
1	66,647.389	1452.611	0.584	1.473×10^{-5}	-5.244×10^{-8}	−0.142	11.154	−0.025	−0.603
2	57,428.175	−1028.175	−0.413	1.738×10^{-5}	3.409×10^{-7}	0.924	10.948	−0.008	−0.182
3	71,838.350	561.650	0.226	1.334×10^{-5}	4.688×10^{-7}	1.270	11.257	−0.067	−1.591
4	63,218.056	5081.944	2.042	1.531×10^{-5}	-6.742×10^{-7}	−1.826	11.096	0.036	0.847
5	68,520,938	279.062	0.112	1.451×10^{-5}	2.225×10^{-8}	0.060	11.154	−0.015	−0.359
6	63,433.638	−33.638	−0.014	1.614×10^{-5}	-3.722×10^{-7}	−1.008	11.032	0.025	0.602
7	63,100.220	1799.780	0.723	1.54×10^{-5}	-8.079×10^{-8}	−0.219	11.085	−0.004	−0.104
8	67,936.193	563.807	0.226	1.460×10^{-5}	-2.352×10^{-9}	−0.006	11.163	−0.029	−0.682
9	76,886.948	−3386.948	−1.361	1.377×10^{-5}	-1.700×10^{-7}	−0.461	11.245	−0.040	−0.956
10	59,843.944	−143.944	−0.058	1.707×10^{-5}	-3.236×10^{-7}	−0.877	10.962	0.035	0.836
11	65,105.780	994.220	0.399	1.534×10^{-5}	-2.116×10^{-7}	−0.573	11.089	0.010	0.232
12	81,722.921	−3722.921	−1.496	1.245×10^{-5}	3.658×10^{-7}	0.991	11.297	−0.033	−0.784
13	55,259.297	−2159.297	−0.867	1.848×10^{-5}	3.442×10^{-7}	0.933	10.885	−0.005	−0.123
14	75,334.121	1965.879	0.790	1.325×10^{-5}	-3.175×10^{-7}	−0.860	11.232	0.023	0.551
15	82,865.540	1334.460	0.536	1.185×10^{-5}	2.334×10^{-8}	0.063	11.305	0.036	0.852
16	83,727.003	72.997	0.029	1.222×10^{-5}	-2.915×10^{-7}	−0.790	11.287	0.049	1.170
17	82,002.598	−902.598	−0.363	1.243×10^{-5}	-1.072×10^{-7}	−0.290	11.310	−0.007	−0.168
18	81,400.720	1699.280	0.683	1.238×10^{-5}	-3.473×10^{-7}	−0.941	11.296	0.032	0.761
19	59.699.237	2000.763	0.804	1.665×10^{-5}	-4.499×10^{-7}	−1.219	10.996	0.034	0.811
20	80,726.490	−1626.490	−0.653	1.210×10^{-5}	5.338×10^{-7}	1.446	11.288	−0.009	−0.224
21	77,633.533	−5233.533	−2.102	1.344×10^{-5}	3.665×10^{-7}	0.992	11.246	−0.056	−1.337
22	77,664.840	−664.840	−0.267	1.305×10^{-5}	-6.584×10^{-8}	−0.178	11.265	−0.014	−0.331
23	68,682.778	−82.778	−0.033	1.446×10^{-5}	1.096×10^{-7}	0.297	11.164	−0.028	−0.675
24	79,346.720	−446.720	−0.179	1.269×10^{-5}	-2.488×10^{-8}	−0.067	11.252	0.024	0.561
25	74,529.358	670.642	0.269	1.341×10^{-5}	-1.131×10^{-7}	−0.306	11.235	−0.007	−0.171
26	84,944.932	3555.068	1.428	1.121×10^{-5}	7.986×10^{-8}	0.216	11.313	0.078	1.858
27	83,552.467	4747.533	1.907	1.185×10^{-5}	-5.259×10^{-7}	−1.425	11.297	0.092	2.188
28	73,974.442	−4874.442	−1.958	1.350×10^{-5}	9.659×10^{-7}	2.617	11.248	−0.105	−2.490
29	59,273.374	−2473.374	−0.994	1.709×10^{-5}	5.097×10^{-7}	1.381	10.968	−0.021	−0.490

4.5.1. Results of the Hypothesis of Normality of the Random Components of the Residuals

Hypothesis H_0 was set: The random components have a normal distribution—linear model N(0; 2583.179); inverse linear model N(0; 3.83×10^{-7}); power model N(0; 0.048).

An analysis of Table 11 shows that there is no basis for rejecting the hypothesis that the random components of the models have a normal distribution, for the N(0, 2583.179) linear model and the N(0, 0.048) power model. Note that for the inverse linear model N(0; 0.048) value $W < W_\alpha$, and this means that there are grounds for rejecting hypothesis H_0. The large difference between the distribution of the residuals and the normal distribution may disturb the assessment of the significance of the coefficients of the individual variables of the model. Therefore, the inverted linear model was rejected.

Table 11. Values of the Shapiro–Wilk statistic.

	Linear Model	Inverted Linear Model	Power Model
W	0.973	0.864	0.946
W_α	0.926	0.926	0.926

4.5.2. Results of the Hypothesis of Autocorrelation of the Random Components of the Residuals

Autocorrelation is the interdependence of random components and is clearly undesirable. The results are presented in Table 12.

Table 12. Values of Durbin–Watson statistic.

	Linear Model	Power Model
d	1.688	1.977
d_L	1.27	1.341
d_U	1.563	1.483

There is no basis for rejecting hypothesis H_0 about the lack of autocorrelation of random components of order one.

4.5.3. Results of the Hypothesis of Randomness of the Components of the Residuals

Hypothesis H_0 was set: The error of the model residuals is random. The data are shown in Table 13.

Table 13. Results of verification of the randomness of the distribution of the model residuals.

	Linear Model	Power Model
$n_{\text{reszty nieparzyste}}$	14	17
$n_{\text{reszty parzyste}}$	15	12
$n_{0.025} < n\varepsilon < n_{0.975}$	<9–20>	<9–21>

The empirical value of the statistic does not fall into the critical area. There is no basis for rejecting the hypothesis H_0 that the distribution of the components of the model residuals is random.

4.5.4. Results of the Hypothesis on the Symmetry of the Random Components of the Residuals

The critical area of the test is two-sided, and the results are shown in Table 14.

Table 14. Results of verification of the symmetry of the distribution of the residuals of the models.

	Linear Model	Power Model
m	15	12
n	29	29
t	0.165	0.839
t_{kr}	2.048	2.048

The determined empirical value of the statistics is smaller in absolute value t than the critical value t_α. There are no grounds to reject hypothesis H_0 in favor of hypothesis H_1.

4.5.5. Results of the Hypothesis of Homoskedasticity of the Random Components of the Residuals

The results of the Breusch–Pagan test are shown in Table 15.

Table 15. Breusch–Pagan test results.

	Linear Model	Power Model
$R^2{}_\varepsilon$	0.037	0.142
χ^2	1.089	4.130
$\chi^2{}_\alpha$	5.99	3.84

Analyzing Table 15, it should be noted that only the linear model meets the criterion set. The computational value of the χ^2 statistic is less than the critical value of $\chi^2{}_\alpha$. Therefore, there is no basis for rejecting hypothesis H_0 about the constancy of the variance of the model's residuals. For the power model, the value of χ^2 is greater than the critical value of $\chi^2{}_\alpha$. Therefore, there are grounds to reject the hypothesis of constancy of variance in favor of hypothesis H_1. Therefore, the power model was rejected.

The verification shows that only the linear model described by Equation (21) is characterized by high cognitive quality.

The linear model shows that the explanatory variable X5 has the greatest impact on brake pad wear. The coefficient for X_5 is 5 times greater than the coefficient for X_2. The correct interpretation of the regression model requires compliance with the ceteris paribus condition, i.e., changing only one explanatory variable. The presented model shows that each braking of the X_2 translates into 44 km of brake pad durability. Each increase in kilometers traveled <20% GVW increases the durability of the brake pads by a further 234 km.

4.6. Results of the Evaluation of the Models Used

Based on the created models, brake pad wear was predicted for four example trucks, which are presented in Table 16.

Table 16. Brake pad wear prediction results for sample trucks.

	X_2 [-]	X_5 [-]	MAPE [%]	Prediction [km]	Reality [km]
			Linear model		
K1	250	158	−0.39%	67.476	67.214
K2	487	165	2.38%	79.545	81.481
K3	307	168	5.21%	72.326	76.303
K4	386	152	7.03%	72.056	77.507

The forecast results are presented in Table 16, and they show high compliance between the linear model and the actual wear of brake pads for K1, K2, K3, and K4 trucks. The model is characterized by very low MAPE prediction errors, less than <15%. For K1 and K2 vehicles operating in the same conditions as $T_1 \ldots T_{29}$ trucks, the error is less than <5%, which means that MAPE is excellent. For K3 and K4 trucks, which are operated in different conditions and with different loads, the MAPE error was 5.21% and 7.03%, respectively. This result indicates a very good prediction of brake pad wear.

5. Conclusions

Based on the conducted research and statistical modeling, it was found that:

1. The paper presents three models based on the multiple regression method. Only the linear model met all criteria. The inverse linear and power-law models did not meet the criteria.
2. To predict the wear of brake pads of vehicles carrying oversized loads, a linear model can be used based on the variables X_2—number of brake applications in the range of 0.15–0.24 [MPa] and X_5—vehicle load range <20% GVW.
3. Model validation showed that MAPE forecast errors ranged from −0.39% to 7.03%.
4. The work shows how important it is to perform full verification of regression models. Verification should be based on meeting all Gauss–Markov assumptions.

Additional achievement of the work—The methodology for building regression models presented in the work, based on statistical foundations, ensures correct verification and determination of the quality of the models. It should be used in all kinds of regression model creation tasks in engineering problems.

In further work, it is planned to expand the selection of data for modeling to include long-distance transport trucks. Time will tell whether it will be possible to create a generalized model of brake pad wear in oversized and long-haul trucks.

Author Contributions: Conceptualization, G.B.; methodology, G.B.; software, S.F.; validation, S.F., G.B. and M.H.; investigation, G.B.; data curation, M.H. and N.P.; writing—original draft preparation, G.B.; writing—review and editing, S.F., G.B., M.H. and N.P.; visualization, N.P. All authors have read and agreed to the published version of the manuscript.

Funding: Article processing charges were financed from the subsidy of the Ministry of Education and Science for the Agricultural University of Hugo Kołłataja in Krakow for the year 2024.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available within the article.

Acknowledgments: The authors would like to thank Zbigniew and Iwona Jatczak for providing the trucks for research.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Regulation No 13 of the Economic Commission for Europe of the United Nations (UN/ECE)—Uniform Provisions Concerning the Approval of Vehicles of Categories M, N and O with Regard to Braking [2016/194]. Available online: https://op.europa.eu/en/publication-detail/-/publication/0a43f880-d612-11e5-a4b5-01aa75ed71a1/language-en (accessed on 6 March 2024).
2. Bellini, C.; Di Cocco, V.; Iacoviello, D.; Iacoviello, F. Temperature Influence on Brake Pad Friction Coefficient Modelisation. *Materials* **2024**, *17*, 189. [CrossRef] [PubMed]
3. Mege-Revil, A.; Rapontchombo-Omanda, J.; Serrano-Munoz, I.; Cristol, A.-L.; Magnier, V.; Dufrenoy, P. Sintered Brake Pads Failure in High-Energy Dissipation Braking Tests: A Post-Mortem Mechanical and Microstructural Analysis. *Materials* **2023**, *16*, 7006. [CrossRef]
4. Ilie, F.; Cristescu, A.-C. Experimental Study of the Correlation between the Wear and the Braking System Efficiency of a Vehicle. *Appl. Sci.* **2023**, *13*, 8139. [CrossRef]
5. Yu, C.; Li, W.; Guo, Y.; Sun, X.; Hong, F.; Sun, N.; Zhang, Q. Research on wear rate of train brake pads driven by small sample data. *Wear* **2024**, *536*, 20516. [CrossRef]
6. Jeong, P.; Lee, J.; Oswald, M.; Kellner, S. *Model-Based Brake Disc Temperature Prediction on High Speed Testing Mode and Circuit*; SAE Technical Paper; SAE: Warrendale, PA, USA, 2020.
7. Kolluri, D.K.; Boidin, X.; Desplanques, Y.; Degallaix, G.; Ghosh, A.K.; Kumar, M.; Bijwe, J. Effect of Natural Graphite Particle Size in Friction Materials on Thermal Localisation Phenomenon during Stop-Braking. *Wear* **2010**, *268*, 1472–1482. [CrossRef]
8. Xiao, Y.; Zhang, Z.; Yao, P.; Fan, K.; Zhou, H.; Gong, T.; Zhao, L.; Deng, M. Mechanical and Tribological Behaviors of Copper Metal Matrix Composites for Brake Pads Used in High-Speed Trains. *Tribol. Int.* **2018**, *119*, 585–592. [CrossRef]
9. Borawski, A. Study of the Influence of the Copper Component's Shape on the Properties of the Friction Material Used in Brakes—Part One, Tribological Properties. *Materials* **2023**, *16*, 749. [CrossRef]
10. Wang, N.; Yin, Z. The Influence of Mullite Shape and Amount on the Tribological Properties of Non-Asbestos Brake Friction Composites. *Lubricants* **2022**, *10*, 220. [CrossRef]

11. Naidu, M.; Bhosale, A.; Munde, Y.; Salunkhe, S.; Hussein, H.M.A. Wear and Friction Analysis of Brake Pad Material Using Natural Hemp Fibers. *Polymers* **2022**, *15*, 188. [CrossRef]
12. Thiyagarajan, V.; Kalaichelvan, K.; Vijay, R.; Singaravelu, D.L. Influence of thermal conductivity and thermal stability on the fade and recovery characteristics of non-asbestos semi-metallic disc brake pad. *J. Braz. Soc. Mech. Sci. Eng.* **2016**, *38*, 1207–1219. [CrossRef]
13. Boniardi, M.; D'Errico, F.; Tagliabue, C.; Gotti, G.; Perricone, G. Failure Analysis of a Motorcycle Brake Disc. *Eng. Fail. Anal.* **2006**, *13*, 933–945. [CrossRef]
14. Peng, D.; Fang, K.; Tian, Z.; Zhang, Y.; Tan, G. *Speed Planning System for Commercial Vehicles in Mountainous Areas*; SAE Technical Paper; SAE: Warrendale, PA, USA, 2021.
15. Feng, J.; Tian, Z.; Cui, J.; Zhou, F.; Tan, G. *Downhill Safety Assistant Driving System for Battery Electric Vehicles on Mountain Roads*; SAE Technical Paper; SAE: Warrendale, PA, USA, 2019.
16. Quan, J.; Zhao, Y.; Tan, G.; Xu, Y.; Huang, B.; He, T. *A Study on Safety Intelligent Driving System for Heavy Truck Downhill in Mountainous Area*; SAE Technical Paper; SAE: Warrendale, PA, USA, 2018.
17. Synák, F.; Lenka Jakubovičová, L.; Klacko, M. Impact of the Choice of Available Brake Discs and Brake Pads at Different Prices on Selected Vehicle Features. *Appl. Sci.* **2022**, *12*, 7325. [CrossRef]
18. Putra, M.R.A.; Pratama, P.S.; Prabowo, A.R. Failure of friction brake components against rapid braking process: A review on potential challenges and developments. *Transp. Res. Procedia* **2021**, *55*, 653–660. [CrossRef]
19. Adamowicz, A. Effect of convective cooling on temperature and thermal stresses in disk during repeated intermittent braking. *J. Frict. Wear* **2016**, *37*, 107–112. [CrossRef]
20. Merlo, F.; Passarelli, U.; Pellerej, D.; Buonfico, P. *Effect of Gray Cast-Iron Microstructure and Brake Pad Formula on Wear Behavior and Corrosion Sticking Influenced by Thermal Preconditioning: The Copper Role*; SAE Technical Paper; SAE: Warrendale, PA, USA, 2012.
21. Hamid, M.A.; Kaulan, A.; Syahrullail, S.; Abu Bakar, A. Frictional characteristics under corroded brake discs. *Procedia Eng.* **2013**, *68*, 668–673. [CrossRef]
22. Motta, M.; Fedrizzi, L.; Andreatta, F. Corrosion Stiction in Automotive Braking Systems. *Materials* **2023**, *16*, 3710. [CrossRef] [PubMed]
23. Jacobsson, H. Aspects of disc brake judder. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* **2003**, *217*, 419–430. [CrossRef]
24. Jensen, K.M.; Santos, I.F.; Corstens, H.J. Estimation of brake pad wear and remaining useful life from fused sensor system, statistical data processing, and passenger car longitudinal dynamics. *Wear* **2024**, *538–539*, 205220. [CrossRef]
25. Świderski, A.; Borucka, A.; Jacyna-Gołda, I.; Szczepański, E. Wear of brake system components in various operating conditions of vehicle in the transport company. *Eksploat. I Niezawodn.—Maint. Reliab.* **2019**, *21*, 1–9. [CrossRef]
26. Jegadeeshwaran, R.; Sugumaran, V. Brake fault diagnosis using Clonal Selection Classification Algorithm (CSCA)—A statistical learning approach. *Eng. Sci. Technol. Int. J.* **2015**, *18*, 14–23. [CrossRef]
27. Burnaev, E. Time-series classification for industrial applications: A brake pad wear prediction use case. In *IOP Conference Series: Materials Science and Engineering*; IOP Publishing: Bristol, UK, 2020; Volume 904, p. 012012. [CrossRef]
28. Zhang, S.; Chen, W.; Li, Y. Wear of Friction Material during Vehicle Braking. In Proceedings of the SAE World Congress & Exhibition, Detroit, MI, USA, 22 April 2009; SAE Technical Paper 2009-01-1032. [CrossRef]
29. Sobczyk, M. *Statystyka: Aspekty Praktyczne i Teoretyczne*; Wydawnictwo Uniwersytetu Marii Curie-Skłodowskiej: Lublin, Poland, 2006.
30. Starzyńska, W. *Statystyka Praktyczna*; Wydawnictwo Naukowe PWN: Warszawa, Poland, 2012.
31. Santos, C.; Dias, C. Note on the coefficient of variation properties. *Braz. Electron. J. Math.* **2021**, *2*, 101–111. [CrossRef]
32. Bartosiewicz, S. *Ekonometria*; Wydawnictwo PWE: Warszawa, Poland, 1978.
33. Hansen, B.E. Challenges for econometric model selection. *Econom. Theory* **2005**, *21*, 60–68. [CrossRef]