Determination of the Ground Reaction Curve for an Elasto-Plasto-Fractured Rock Mass

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Identification of various models for underground excavations support design outline the deformational pressure model for assessing loads acting on the support systems of deep underground excavations. They distinguish two different rock mass models, highlighting the pivotal role of the critical longitudinal strain of the rock mass in appropriate model selection. A comparison between the design method given by Polish Standards and the widely recognized convergence–confinement method, consisting of a ground reaction curve (GRC), longitudinal displacement profile (LDP), and support characteristics curve (SCC), reveals the advantages of the latter in capturing the three-dimensional nature of underground excavations. The following study presents a method for establishing a GRC curve for the elasto-plasto-fractured rock mass model, featured in Polish Standards, demonstrating its applicability through analyses of a typical circular roadway under varying rock mass conditions. Practical implications are discussed, including the design of yielding steel arches as the primary support system and the calculation of safety factors for both the support system and the surrounding rock mass, considered as a natural support component. Overall, the study contributes to a deeper understanding of the actions of rock masses in the vicinity of excavations located at great depths. Furthermore, it provides practical insights for engineering applications.

Keywords: rock mechanics; convergence–confinement method; ground reaction curve; elasto-plasto-fractured model; excavation support

1. Introduction

The development of the pressure of a rock mass that acts on an excavation support depends on various factors, including the physical and mechanical properties of the rock mass, the mining technology used, and the characteristics of the support [1]. As the excavation face advances, the rock mass gradually moves toward the excavation axis. The increment of the displacement of the rock mass is influenced by both the distance to the excavation face and the time. However, in practical applications, the influence of time is often overlooked [2]. An analysis of the impact of the time factor on the growth of the displacement of rock masses requires the utilization of various rheological models—as discussed, for example, in [3–7].

The development of a deformational pressure acting on an excavation support is the result of the suppression of the increase in the displacement of the rock mass increment caused by the installation of the support [1]. The magnitude of the deformational pressure

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acting on a support system decreases as the ground displacement increases. Consequently, its value is a function of the distance between the excavation face and the support location during installation [2,8,9].

Up to a certain level of ground displacement, the rock mass in the vicinity of an excavation acts as an elastic medium. Once the displacement of the ground exceeds a certain threshold, inelastic zones begin to form around the excavation, which could lead to a reduction in the rock mass within these zones [4,10]. Therefore, four different types of inelastic zones can be identified:

- The compressive strength of the rock mass within the inelastic zone is equal to the compressive strength in the elastic zone, satisfying the condition:

  \[ R_{cg} = R'_{cg} \]  
  \( (1) \)

  This scenario characterizes the elasto-perfectly plastic medium.

- The compressive strength of the rock mass within the inelastic zone is lower than the compressive strength in the elastic zone, meeting the following condition:

  \[ R_{cg} > R'_{cg} \]  
  \( (2) \)

  This represents the softening elasto-plastic medium.

- The rock mass within the inelastic zone exhibits no compressive strength:

  \[ R'_{cg} = 0 \]  
  \( (3) \)

  This represents the elasto-fractured medium.

- Two coaxial inelastic zones form around the excavation. In the immediate vicinity of the excavation, there exists a fractured rock mass zone without compressive strength. At a certain distance from the excavation boundary, a zone of plastic rock mass develops, retaining some residual compressive strength. This scenario denotes an elasto-plasto-fractured medium [11].

The development of inelastic zones around the excavation corresponds to the occurrence of a static rock mass pressure, which arises from the gravitational load of plasticized and fractured roof rocks. As the value of the ground displacement increases, the extent of the inelastic zones, and thus the level of static pressure, also increases. The elasto-plastic rock mass exhibits a boundary displacement value at which the static pressure reaches its maximum, while the deformational pressure diminishes. Therefore, the elasto-plastic model produces a paradox, since it suggests that the value of the support pressure always decreases along with an increase in ground displacement, whereas according to practical engineering, the support pressure is expected to increase significantly after reaching a certain value of ground displacement as a result of the loss of confinement and the detrimental loosening of the rock mass.

Meanwhile, in elasto-fractured and elasto-plasto-fractured media, as displacements increase, the static pressure theoretically tends toward infinity, while the deformational pressure approaches zero. Therefore, the final value of the support pressure increases after reaching a certain value of ground displacement, which seems to be consistent with engineering practice [12,13].

2. Deformational Pressure Model According to Polish Standards

In the late 1970s, the Industrial Standards BN-78/0434-07 [14] and BN-79/0434-04 [15] were introduced into Polish engineering practice. Their aim was to regulate and organize the various methods for calculating loads acting on the support systems of underground roadways and chambers. Previously used rheological models were replaced with the so-called deformational pressure model, which was later implemented in the National Standards PN-G-05600 [16] and PN-G-05020 [17], which remain in use up to today. The equations used to calculate the radii of the inelastic zones and the values of deformational
and static pressure acting on excavation supports were based on the solution of a disk of uniform thickness initially subjected to a hydrostatic far-field (primary) stress. Calculations were performed under the following assumptions [18]:

- the plain strain condition,
- the hydrostatic far-field (primary) stress condition,
- the circular shape of the excavation section,
- the homogeneity, isotropy and weightlessness of the rock mass,
- that rock mass destruction occurs in cases of exceeding its compressive strength (based on the Mohr–Coulomb yield criterion).
- The Polish Standards PN-G-05600 and PN-G-05020 distinguish between the following rock mass models [16,17], depending on the characteristics of the rock mass within the inelastic zone:
- the elasto-plastic model with softening (referred to simply as elasto-plastic model),
- the elasto-plasto-fractured model.

A schematic representation of the rock mass models as provided by the Polish Standards is depicted in Figure 1.

![Diagram of rock mass models](image)

**Figure 1.** Deformational pressure model according to Polish Standards.

The choice of the appropriate rock mass model is dependent on the value of the critical longitudinal strain of the rock mass. This strain value marks the point at which the rock mass transitions from a plastic to a fractured medium in terms of its strength properties [18]. The critical longitudinal strain of the rock mass can be calculated using the equation provided by the PN-G-05020 standard [17]:

$$\varepsilon_{ng} = 1.5 \cdot \varepsilon_{n5}$$

(4)

where:

$\varepsilon_{n5}$ — the value of critical longitudinal strain of the intact rock, evaluated by loading the rock sample to a value of around 95% of its compressive strength.
The PN-G-05020 Standard recommends employing the elasto-plasto-fractured model when the following condition is satisfied [17]:

\[ u_w > r_w \cdot \varepsilon_{ng} \]  

(5)

where:

- \( u_w \) — value of ground displacement, m,
- \( r_w \) — excavation radius, m.

Similarly, the PN-G-05600 Standard advises using the elasto-plasto-fractured model under the condition outlined as follows [16]:

\[ 0 < p_o \leq p_g \]  

(6)

where:

- \( p_g \) — radial stress on the boundary of plastic and elastic zone, MPa,
- \( p_o \) — radial stress on the boundary of fractured and plastic zone, MPa.

In accordance with the PN-G-05600 Standard, the dimensioning of the so-called shell lining (comprising a thin layer of shotcrete with supplementary steel arches and rockbolts) is based solely on the value of the static roof pressure. Deformational pressure can be disregarded since the entire displacement of the rock mass must be transferred by the supporting structure. Therefore, the following condition must be satisfied [16]:

\[ u^{ob} \geq k \cdot u_w \]  

(7)

where:

- \( u^{ob} \) — radial deformability of the support, m.

To incorporate a portion of the displacement of the rock mass that occurs before support installation, a coefficient \( k \) is introduced into Formula (7). This introduction can be viewed as an attempt to crudely include the three-dimensional effect of road excavation in an otherwise two-dimensional analysis. The PN-G-05600 standard assumes a fixed value of the \( k \) factor, set at 0.9. However, the authors of this article regard such an assumption as oversimplified, as it may lead to either overestimation or underestimation of the required deformability of the roadway support. The actual value of the \( k \) coefficient depends on numerous factors, including the rock mass and support properties, as well as the distance between the location of the support installation and the excavation face [8,9,12,19]. This topic has been extensively discussed in [20].

The dimensioning of the so-called vaulted lining (constructed of brick or a thick layer of unreinforced or reinforced concrete) according to the PN-G-05020 Standard should be carried out while considering the following load combinations [17]:

- simultaneous loading of the lining’s self-weight and the static pressure of the rock mass,
- simultaneous loading of the lining’s self-weight and the deformational pressure of the rock mass,
- simultaneous load of the lining’s self-weight and the injection pressure.

Since the PN-G-05020 standard does not provide suitable formulas, the influence of deformational pressure on the vaulted lining is sometimes overlooked by engineers [21]. This practice may be justified due to the utilization of a yieldable primary lining, which is capable of accommodating the entirety of the initial displacement of the rock mass (associated with the advancement of the excavation face). According to the recommendation outlined in the Polish Standard, secondary support can only be constructed once an equilibrium state is reached between the primary layer and the rock mass [17]. Therefore, if a primary lining with appropriate deformability is installed beforehand, the deformational pressure acting on the secondary lining should be eliminated or significantly reduced.

In summary, according to Polish National Standards, the primary lining of an underground roadway or chamber must always be designed as a yieldable structure in
order to eliminate the occurrence of deformational pressure. This assumption accounts for a significant weakness of the discussed solution. Determining the value of the deformational pressure acting on the support of limited deformability (for which condition (7) is not met) is not possible using the equations provided by Polish Standards only. Additionally, it must be noted that in engineering practice, the geotechnical properties of rock masses are never constant, as they may be reduced due to mining activities, rock delamination, water inflow, and contact with the mine’s atmosphere [10]. Therefore, in many cases, the use of shell support with limited deformability, which allows for the rapid suppression of rock mass displacement, may be highly desirable. This solution is widely employed in the worldwide tunneling industry, constituting one of the basic principles of the new Austrian tunneling method [22].

3. Theoretical Background for the Convergence Confinement Method

The convergence confinement method (CCM) serves as a straightforward analytical tool capable of estimating the deformational load acting on an excavation support. Initially, the CCM method was based on the stress state solution of anelastic disc of a uniform thickness proposed by Kirsch [23]. Over subsequent years, the method underwent further refinement and enhancement, including considerations such as the development of the plastic zone around the excavation [24–28]. Regardless of the specific yield criterion used, these adaptations were based on the fundamental assumption of rock mass continuity [8]. Pacher [29] conducted research on potential disruptions in this continuity, leading to the formulation of the hypothesis that appropriate radial deformability and the timing of support installation are critical factors in ensuring both safety and cost-effectiveness in excavation design.

CCM is a graphical method consisting of three basic components [22,30]:

- Ground reaction curve (GRC)—illustrates the correlation between fictitious support pressure and the radial displacement of the excavation boundary. In essence, fictitious pressure refers to the internal support pressure required to prevent further ground displacement.
- Support characteristics curve (SCC)—depicts the relationship between increases in the radial displacement of the support and the external radial stress applied to the support (deformational pressure).
- Longitudinal displacement profile (LDP)—depicts the correlation between the radial displacement of the excavation boundary and the longitudinal distance from the excavation face.

In comparison to the design method advocated by Polish Standards, the convergence–confinement method is capable of:

- More accurately considering the three-dimensional character of an underground roadway or chamber excavation.
- Considering rigid support systems, which enable the quick suppression of rock mass displacement.
- Considering the correlation between support system stiffness and the value of the excavation boundary displacement.
- Considering the deformational load acting on the designed support system.

4. Algorithm for the Establishment of a Ground Reaction Curve for an Elasto-Plasto-Fractured Rock Mass

An important issue connected with the use of the CCM method is the difficulty in evaluating the real actions of rock masses within inelastic zones [31]. The simplest procedure for determining a GRC curve is based on the model of an elasto-perfectly-plastic medium [2]. Hoek and Brown claim that such a model can only be used for rock masses of low quality (GSI < 30) [32]. Therefore, for a rock mass of medium (30 < GSI < 70)
or high (GSI > 70) quality, a number of alternative equations have been introduced over the years, considering such issues as post-failure acting, including rock mass softening, stiffness degradation, and dilatancy [31,33].

The development of a three-phase rock mass model, based on the model of elasto-plasto-fractured medium, was considered an important achievement of Polish mining engineering [18]. This three-phase model was introduced into Polish Standards [14–17] and successfully applied in the design of underground roadways and chambers located at significant depths in the vicinity of low and medium-quality rock masses. The properties of an elasto-plasto-fractured medium may be depicted by polyline (Figure 2) [18]:

- Within the elastic, the zone action of the rock mass is depicted by a straight line inclined by the angle \( \arctg(E_g) \), where \( E_g \) is a rock mass deformation modulus.
- Within the plastic zone a following condition, depicting rock mass action, is met:
  \[
  \frac{R_{cg}}{E_g} < \varepsilon_t < \varepsilon_{ng},
  \]
  where:
  \( R_{cg} \) — rock mass compressive strength in the elastic zone, MPa,
  \( E_g \) — rock mass Young modulus, MPa,
  \( \varepsilon_t \) — longitudinal strain of the rock mass.
- Within the fracture zone a following condition, depicting rock mass action, is met:
  \[
  \varepsilon_t > \varepsilon_{ng}.
  \]

\[\text{(9)}\]

Figure 2. Stress–strain characteristics for a rock mass described by the elasto-plasto-fractured model.

This paper aims to present a guideline for the determination of a ground reaction curve for an elasto-plasto-fractured rock mass model, employed by Polish Standards, which, however, neither incorporate the convergence confinement method of support design nor the concept of the ground reaction curve.

The presented solution of a ground reaction curve for a rock mass described by an elasto-plasto-fractured model is based on the Mohr–Coulomb yield criterion. The compressive strength of the rock mass is determined by the following equation:
\[ R_{cg} = \frac{2 \cdot c \cdot \cos \phi}{1 - \sin \phi} \]  
(10)

where:
- \( c \) — cohesive strength of the rock mass, MPa,
- \( \phi \) — angle of internal friction of the rock mass, ‘.

According to the PN-G-05020 Standard, the residual compressive strength of the rock mass is given by the following equation [17]:

\[ R'_{cg} = (0.4 \div 0.6) \cdot R_{cg} \]  
(11)

Alternatively, if the mechanical properties of a rock mass are determined based on the Geological Strength Index (GSI), a residual GSI factor may be applied to estimate the residual compressive strength of the rock mass [34]. Such an approach has been introduced in various tunneling projects worldwide [35–38].

In order to simplify the calculation further, factor \( \beta \), given by the following equation, shall be introduced:

\[ \beta = \frac{2 \sin \phi}{1 - \sin \phi} \]  
(12)

The critical radial stress (or radial stress on the boundary between the elastic and plastic zones) is determined by the following equation:

\[ p_g = \frac{2p_z - R_{cg}}{2 + \beta} \]  
(13)

where:
- \( p_z \) — in situ hydrostatic stress, MPa.

The radial stress on the boundary between the fractured and plastic zones is determined by the following formula:

\[ p_o = \frac{p_g \cdot \beta + R_{cg}'}{\beta} \cdot \left[ \frac{(1 + \nu)(p_z - p_a)}{E_g \epsilon_{cg}} \right]^\frac{\beta}{\beta} - \frac{R_{cg}'}{\beta} \]  
(14)

where:
- \( \nu \) — rock mass Poisson’s ratio.

The value of the ground displacement is a function of the radial stress that needs to be applied to the excavation boundary to prevent further movement (fictitious pressure \( p_a \)):

- if \( p_a > p_g \), the rock mass acts as an elastic medium;
- if \( p_g > p_a > p_o \), the rock mass acts as a plastic medium;
- if \( p_a < p_o \), the rock mass acts as a fractured medium.

The value of the ground displacement for an elastic medium is given by the following equation:

\[ u(p_a) = \frac{r_w \cdot (1 + \nu)}{E_g} \cdot (p_z - p_a) \]  
(15)

where:
- \( r_w \) — excavation radius, m.

For a plastic medium, the plastic zone radius \( r_1 \) (which is a function of the fictitious pressure \( p_a \)) must be calculated beforehand:

\[ r_1(p_a) = r_w \cdot \left( \frac{p_a \cdot \beta + R_{cg}'}{p_a \cdot \beta + R_{cg}'} \right)^{1/\beta} \]  
(16)

The fracture zone radius \( r_0 \) is a function of the fictitious pressure \( p_a \) and may be calculated using the following equation:
\[ r_a(p_a) = r_w \cdot \left( \frac{p_a}{p_w} \right)^\frac{1}{\beta} \]  

(17)

As shown in Formula (17), with reductions in the fictitious pressure, the radius of the fracture zone tends to infinity. At the equilibrium state, the value of the deformational pressure is equal to the value of the static pressure of the fractured rock mass within the fracture zone. The radius of the fracture zone in the equilibrium state \( r_{aeq} \) may be determined based on the equation given by the PN-G-05600 Standard [16]:

\[
\left( \frac{r_a}{r_w} \right)^{\beta+1} - \left( \frac{r_a}{r_w} \right)^\beta = \frac{p_a}{\gamma_s \cdot r_w} \]

(18)

where:

\( \gamma_s \) — average bulk density of the roof rocks, MN/m³.

The static pressure of the rock mass within the fracture zone for the radius determined from Formula (18) is equal to the value of the deformational pressure at the equilibrium state:

\[ p_{amin} = q_{za} = (r_a - r_w) \cdot \gamma_s \]

(19)

The radius of the plastic zone developed outside the range of the fracture zone is a function of the fracture zone radius:

\[ r_p(r_a) = r_a \cdot \left( \frac{\rho_g \beta+\rho_f\beta}{\rho_o \beta+\rho_f \beta} \right)^\frac{1}{\beta} \]

(20)

The static pressure of the plastic and fractured rock masses within the inelastic zones (both plastic and fracture zone) is a function of the plastic zone radius:

\[ q_{za}(r_p) = (r_p - r_w) \cdot \gamma_s \]

(21)

Ultimately, the value of the ground displacement for both plastic and fractured media is a function of the plastic zone radius:

\[ u(r_p) = \frac{r_w \cdot (1 + \nu)}{E_g} \cdot \left[ 2 \cdot (1 - \nu) \cdot (p_z - p_g) \cdot \left( \frac{r_p}{r_w} \right)^2 - (1 - 2 \cdot \nu) \cdot (p_z - p_o) \right] \]

(22)

Formulas (10) to (22) constitute all the essential relationships for the establishment of a GRC curve for an elasto-plasto-fractured medium. The featured algorithm may be introduced to the calculation sheet for rock masses fulfilling the conditions given by Formulas (5) or (6).

5. Application of the Developed Approach

In order to verify the algorithm featured in Section 4, a series of analyses were performed involving a typical circular roadway of radius \( r_w = 3 \) m excavated at a depth of 1000 m, where the in situ hydrostatic stress equaled \( p_z = 25 \) MPa. Three cases considering rock mass classes with different mechanical properties were investigated for comparative purposes:

- class I—competent rock mass,
- class II—fair rock mass,
- class III—weak rock mass.

Input parameters for different rock mass classes were selected based on Polish Industrial Standard BN-78/0434-07 [14], which provides guidelines for the crude approximation of basic rock mass parameters such as an internal friction angle, cohesive strength, Young modulus, Poisson’s ratio and critical longitudinal strain. These approximations are derived from the uniaxial compressive strength (UCS) of an intact rock and the Rock Quality Designation (RQD) of the rock mass. Input parameters selected for different rock mass class parameters are presented in Table 1.
Table 1. Input parameters for exemplary calculation of GRC curves for different rock mass classes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Rock Mass Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive strength of the rock mass</td>
<td>c</td>
<td>MPa</td>
<td>I   6.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  5.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 2.65</td>
</tr>
<tr>
<td>Internal friction angle of the rock mass</td>
<td>φ</td>
<td>°</td>
<td>I   38.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  33.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 29.30</td>
</tr>
<tr>
<td>Rock mass Young modulus</td>
<td>E</td>
<td>MPa</td>
<td>I   7667</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  5409</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 3338</td>
</tr>
<tr>
<td>Rock mass Poisson’s ratio</td>
<td>ν</td>
<td>-</td>
<td>I   0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 0.20</td>
</tr>
<tr>
<td>Critical longitudinal strain of the rock mass</td>
<td>ε&lt;sub&gt;εg&lt;/sub&gt;</td>
<td>-</td>
<td>I 0.0046</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II 0.0051</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 0.0054</td>
</tr>
</tbody>
</table>

The GRC curves for different rock mass classes were calculated based on the elasto-plastic-fractured model of the rock mass and the procedure featured in Section 4. The calculation results of basic GRC curves parameters are presented in Table 2.

Table 2. Calculation results of basic GRC parameters for elasto-plasto-fractured model for different rock mass classes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Rock Mass Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength of the rock mass within the elastic zone</td>
<td>R&lt;sub&gt;e&lt;/sub&gt;</td>
<td>MPa</td>
<td>I   26.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  19.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III  9.05</td>
</tr>
<tr>
<td>Compressive strength of the rock mass within the plastic zone</td>
<td>R'&lt;sub&gt;e&lt;/sub&gt;</td>
<td>MPa</td>
<td>I   13.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  9.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III  4.53</td>
</tr>
<tr>
<td>Computational factor</td>
<td>β</td>
<td>-</td>
<td>I   3.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  2.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III  1.92</td>
</tr>
<tr>
<td>Radial stress on the boundary between plastic and elastic zone</td>
<td>p&lt;sub&gt;g&lt;/sub&gt;</td>
<td>MPa</td>
<td>I   4.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  6.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 10.45</td>
</tr>
<tr>
<td>Radial stress on the boundary between fracture and plastic zone</td>
<td>p&lt;sub&gt;o&lt;/sub&gt;</td>
<td>MPa</td>
<td>I   0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  4.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 10.03</td>
</tr>
<tr>
<td>Deformational pressure at the equilibrium state</td>
<td>p&lt;sub&gt;eq&lt;/sub&gt;</td>
<td>MPa</td>
<td>I   0.071</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II  0.284</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 0.354</td>
</tr>
<tr>
<td>Fracture zone radius at the equilibrium state</td>
<td>r&lt;sub&gt;eq&lt;/sub&gt;</td>
<td>m</td>
<td>I   5.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II 10.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 17.17</td>
</tr>
<tr>
<td>Plastic zone radius at the equilibrium state</td>
<td>r&lt;sub&gt;eq&lt;/sub&gt;</td>
<td>m</td>
<td>I   6.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II 11.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 17.47</td>
</tr>
<tr>
<td>Rock mass displacement at the equilibrium state</td>
<td>u&lt;sub&gt;p&lt;sub&gt;eq&lt;/sub&gt;&lt;/sub&gt;</td>
<td>m</td>
<td>I   0.076</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II 0.217</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III 0.833</td>
</tr>
</tbody>
</table>

The results of the GRC calculations based on the proposed elasto-plasto-fractured model were compared with the typical elasto-plastic solution for the displacement of a circular roadway. GRC curves obtained for both the elasto-plasto-fractured and elasto-plastic model for different rock mass classes are depicted on Figures 3–5.
**Figure 3.** GRC curves for the competent rock mass (class I).

**Figure 4.** GRC curves for the fair rock mass (class II).
The analysis of Figures 3–5 highlights the importance of the proper selection of the rock mass model used in geotechnical calculations. The initial portions of the GRC curves for both the elasto-plasto-fractured model and the elasto-plastic model were identical. According to the elasto-plasto-fractured model, once the fictitious pressure reached the theoretical value of the radial stress at the boundary between the fracture and plastic zone $p_o$ (which is dependent on the value of the critical longitudinal strain of the rock mass $\varepsilon_{ng}$), a fracture zone began to develop in the vicinity of the roadway. Since the rock mass within the fracture zone lacked cohesion (and thus compressive strength), a portion of the GRC curve associated with the fractured rock mass exhibited a rapid displacement increase.

Along with the decrease in rock mass strength, a simultaneous increase was observed in both the displacement of the rock mass and the values of deformational pressure occurring at the equilibrium state between the static and deformational pressures. In practice, the roadway support has to be installed before reaching an equilibrium point—especially in the case of fair and weak rock mass. Otherwise, excess displacement may lead to the dissipation of rock mass confinement, thereby allowing detrimental loosening and ultimately ground failure.

Figure 6 illustrates the comparison of GRC (ground reaction curve) plots derived from the elasto-plasto-fractured model for different rock mass classes.
It is evident that with decreases in the mechanical parameters of the rock mass, there was an increase in rock mass displacement observed at consistent values of fictitious pressure. Regardless of the rock mass class under consideration, along with the reduction in fictitious pressure, the radius of the fracture and the plastic zones—and thereby the value of the displacement—tended to infinity, thereby indicating the need for support installation. This feature of the GRC curve based on the elasto-plasto-fractured model distinguishes it from a typical elasto-plastic GRC curve. In the latter, there is always a theoretical equilibrium point where the fictitious pressure equals zero, while the radius of the plastic zone and the value of ground displacement reach their maximum value (in most cases, in practical applications, before such an equilibrium point is reached, excessive displacements may lead to detrimental loosening and ground failure).

In the subsequent stage of the analysis, in order to determine the optimal moment for the installation of the support, longitudinal deformation profiles (LDP) were established for different rock mass classes. The determination of the LDP curve followed the algorithm outlined in [8]. Considering that for the rock mass model based on an elasto-plasto-fractured medium, the plastic zone radius and the value of the ground displacement consistently tended towards infinity, a maximum value of these parameters could not be unambiguously determined. Therefore, a plastic zone radius and the value of the ground displacement at the equilibrium point between the static and deformatonal pressures were adopted for the calculations. Although this assumption was simplified, further exploration into the solution of LDP for elasto-plasto-fractured media seems to be necessary. An exemplary LDP curve based on the solution given by [8] is illustrated in Figure 7.
Figure 7. Exemplary LDP curve for a fair rock mass (class II).

The extent of the plastic and fracture zones, as well as the magnitude of rock mass displacement, can be mitigated by the application of an appropriate support system. In Polish underground coal mines, the most prevalent support system for long-lasting roadways and chambers consists of yielding steel arches, sometimes combined with a thin layer of shotcrete.

Since a large amount of support deformation is to be expected, it was assumed that the roadway support system should consist of yielding steel sets fitted with sliding joints. These sets are installed immediately behind the roadway face, ensuring safety for personnel working at the face.

Support characteristics curves (SCC) for yielding steel sets, determined using the algorithm outlined in [39], consist of four parts:

1. First part involves elastic deformation, where steel arches act as rigid elements.
2. Second part involves yielding, where overlapped segments of the steel sets start to slide and the arch section diminishes.
3. Third part once again involves elastic deformation, wherein the maximum yielding capacity of the steel arch is reached, and therefore, it acts again as a rigid element.
4. Fourth part involves plastic deformation, wherein the bearing capacity of the rigid steel support is reached, and therefore, a certain amount of plastic deformation occurs.

The input parameters and calculation results for SCC curves are shown in Table 3. The resulting SCC curves, in conjunction with GRC curves, obtained for different mass classes are depicted in Figures 7–9.

Table 3. Input parameters and calculation results of the basic parameters of SCC curves for different rock mass classes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross sectional area of the section</td>
<td>$A_x$</td>
<td>m²</td>
<td>0.00452</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Young’s modulus of the steel $E_y$ MPa 210,000
Yield strength of the steel $\sigma_{ys}$ MPa 440
Yielding force of the sliding joint $N_{lim}$ MN 0.35
Maximum sliding length $s_{lim}$ m 0.30
Number of sliding joints in the steel set $n$ 3
Initial overlapped length $s_0$ m 0.6
Steel set spacing $d$ m 1.0
Support capacity at 1st part of elastic deformation $p_1$ MPa 0.117 0.199 0.307
Maximum support capacity at yielding part $p_2$ MPa 0.176 0.299 0.461
Support capacity at 2nd part of elastic deformation $p_3$ MPa 0.701 1.192 1.842

Figure 8. SCC curve coupled with GRC curve for a competent rock mass (class I).
Figure 9. SCC curve coupled with GRC curve for a fair rock mass (class II).

The graphical interpretation of the SCC and GRC curves along with the ground-support equilibrium point included in Figures 8–10 allows for the estimation of the deforming pressure applied to the roadway support ($p_{asup}$). The factors of safety for both the designed support system ($F_{sup}$) and the surrounding rock mass, considered as the natural support component ($F_{srm}$), can be derived from the following equations:

$$F_{sup} = \frac{p_3}{p_{asup}}$$

$$F_{srm} = \frac{p_{asup}}{p_{eq}}$$

In cases of competent and fair rock masses (classes I and II), the ground-support equilibrium point was located on the yielding part of the SCC curve. This is in contrast to weak rock masses, where the ground-support equilibrium point was situated on the 2nd elastic deformation part of the SCC curve. This indicates that the maximum yielding capacity of the support was reached, due to the large value of the ground displacement. Consequently, the steel sets are no longer able to slide, thus acting like a rigid element. As a result, the factor of the safety of the roadway support ($F_{sup} = 2.00$) for a weak rock mass was significantly lower than in case of a fair ($F_{sup} = 4.17$) or competent rock mass ($F_{sup} = 5.31$). Meanwhile the factor of the safety of the surrounding rock mass component was greater in the case of the weak rock mass ($F_{srm} = 2.54$) than in cases of a fair ($F_{srm} = 1.32$) or competent rock mass ($F_{srm} = 1.33$).

In all analyzed cases, the support capacity at the yielding part of the SCC curve exceeded the static load exerted on the steel arches at a certain value of ground displacement. This observation, connected with the calculated values of the factors of safety, suggests that the designed support system effectively ensures roadway stability—thereby mitigating ground failure resulting from the loss of confinement and the eventual collapse of the fractured rock mass.
Figure 10. SCC curve coupled with GRC curve for the weak rock mass (class III).

After reaching the equilibrium point between the rock mass and yielding steel support, a more rigid and durable support may be employed, such as a thin layer of shotcrete or a thick layer of in situ-made concrete. Embedding steel sets in shotcrete or concrete may yield some additional advantages:

- creation of a favorable, triaxial state of stress on the roadway boundary,
- protection of steel arches from corrosion,
- shielding the rock mass from the influence of water inflow and the mine’s atmosphere.

6. Engineering Application—Ventilating a Roadway in the “Knurów” Coal Mine

The “Knurów” coal mine is located in the western part of the Upper Silesian Coal Basin in Poland. An elasto-plasto-fractured model of the rock mass was employed for the initial support design of a ventilating roadway situated near the “Aniołki” ventilation shaft. This roadway is to be excavated at a depth of 474 m using the conventional “drill and blast” method. The roadway has a horseshoe shape with a maximum height of approximately 6.05 m and a width of approximately 6.09 m, corresponding to a circular geometry with a radius of 3.13 m (Figure 11).

According to the geological survey, the roadway will be excavated in a thick layer of claystone characterized by an uniaxial compressive strength (UCS) of at least 21.98 MPa. Other geotechnical parameters of the rock mass, such as its cohesive strength, internal friction angle, and Young’s modulus, were determined using the guidelines provided by the Polish Industrial standard BN-78/0434-07 [14]. The input parameters and basic calculation results of the Ground Reaction Curve (GRC), utilized in the initial design of the roadway support structure, are listed in Table 4.
Figure 11. Cross-section of the ventilating roadway in the “Knurów” coal mine.

Table 4. Input parameters for the calculation of the GRC curve for the ventilating roadway in the “Knurów” coal mine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excavation radius</td>
<td>$r_e$</td>
<td>m</td>
<td>3.13</td>
</tr>
<tr>
<td>In situ hydrostatic stress</td>
<td>$p_z$</td>
<td>MPa</td>
<td>11.39</td>
</tr>
<tr>
<td>Cohesive strength of the rock mass</td>
<td>$c$</td>
<td>MPa</td>
<td>2.65</td>
</tr>
<tr>
<td>Internal friction angle of the rock mass</td>
<td>$\phi$</td>
<td>$^\circ$</td>
<td>29.30</td>
</tr>
<tr>
<td>Rock mass Young’s modulus</td>
<td>$E$</td>
<td>MPa</td>
<td>3347.83</td>
</tr>
<tr>
<td>Rock mass Poisson’s ratio</td>
<td>$\nu$</td>
<td>-</td>
<td>0.24</td>
</tr>
<tr>
<td>Critical longitudinal strain of the rock mass</td>
<td>$\varepsilon_{ng}$</td>
<td>-</td>
<td>0.0054</td>
</tr>
<tr>
<td><strong>Results</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive strength of the rock mass within the elastic zone</td>
<td>$R_{cg}$</td>
<td>MPa</td>
<td>9.05</td>
</tr>
<tr>
<td>Compressive strength of the rock mass within the plastic zone</td>
<td>$R_{cg}'$</td>
<td>MPa</td>
<td>3.62</td>
</tr>
<tr>
<td>Computational factor</td>
<td>$\beta$</td>
<td>-</td>
<td>1.92</td>
</tr>
<tr>
<td>Radial stress on the boundary between plastic and plastic zone</td>
<td>$p_o$</td>
<td>MPa</td>
<td>1.103</td>
</tr>
<tr>
<td>Deformational pressure at the equilibrium state</td>
<td>$p_{aeq}$</td>
<td>MPa</td>
<td>0.143</td>
</tr>
<tr>
<td>Fracture zone radius at the equilibrium state</td>
<td>$r_{aeq}$</td>
<td>m</td>
<td>9.09</td>
</tr>
<tr>
<td>Plastic zone radius at the equilibrium state</td>
<td>$r_{eq}$</td>
<td>m</td>
<td>12.35</td>
</tr>
<tr>
<td>Rock mass displacement at the equilibrium state</td>
<td>$u\left(p_{aeq}\right)$</td>
<td>m</td>
<td>0.210</td>
</tr>
</tbody>
</table>
The support structure of the roadway under consideration consists of yielding steel arches made from V36 profiles, spaced 0.50 m apart. After reaching an equilibrium point between the rock mass and steel yielding support, additional steel arches shall be embedded in fiber-reinforced shotcrete (Figure 12). The input parameters and basic calculation results of SCC curve for the designed support structure are listed in Table 5.

Table 5. Input parameters and calculation results of the basic parameters of the SCC curve for the ventilating roadway in the “Knurów” coal mine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross sectional area of the section</td>
<td>(A_x)</td>
<td>m(^2)</td>
<td>0.00452</td>
</tr>
<tr>
<td>Young’s modulus of the steel</td>
<td>(E_x)</td>
<td>MPa</td>
<td>210,000</td>
</tr>
<tr>
<td>Yield strength of the steel</td>
<td>(\sigma_{ys})</td>
<td>MPa</td>
<td>440</td>
</tr>
<tr>
<td>Yielding force of the sliding joint</td>
<td>(N_{lim})</td>
<td>MN</td>
<td>0.35</td>
</tr>
<tr>
<td>Maximum sliding length</td>
<td>(s_{lim})</td>
<td>m</td>
<td>0.15</td>
</tr>
<tr>
<td>Number of sliding joints in the steel set</td>
<td>(n)</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>Initial overlapped length</td>
<td>(s_0)</td>
<td>m</td>
<td>0.60</td>
</tr>
<tr>
<td>Steel set spacing</td>
<td>(d)</td>
<td>m</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Support capacity at 1st part of elastic deformation</td>
<td>(p_1)</td>
<td>MPa</td>
<td>0.227</td>
</tr>
<tr>
<td>Maximum support capacity at yielding part</td>
<td>(p_2)</td>
<td>MPa</td>
<td>0.299</td>
</tr>
<tr>
<td>Support capacity at 2nd part of elastic deformation</td>
<td>(p_3)</td>
<td>MPa</td>
<td>1.364</td>
</tr>
</tbody>
</table>

The SCC curve coupled with the GRC curve calculated for the geological conditions of the ventilation roadway in the “Knurów” coal mine is depicted in Figure 13.
As a result of the application of the convergence–confinement method and the employment of the elasto-plasto-fractured rock mass model, the initial choice of the support scheme—consisting of yielding steel arches made from V36 profiles with a spacing of 0.50 m—was confirmed to be correct. The factors of safety for the roadway support \( F_{\text{so}} = 5.39 \) and the rock mass \( F_{\text{srn}} = 1.78 \) indicate that the designed support structure can be successfully employed in a ventilation roadway under the given geological conditions.

7. Conclusions

Based on the conducted study, the following conclusions can be inferred:

(1) Polish National and Industrial Standards introduced the deformational pressure model for calculating loads acting on the support systems of underground excavations located at significant depths. These Standards distinguish between the softening elasto-plastic and elasto-plasto-fractured models. The choice of the appropriate rock mass model is contingent upon the value of the critical longitudinal strain of the rock mass, which marks the point at which the rock mass transitions from a plastic to a fractured medium. Since Equation (4), provided by the National Standards to calculate the value of the critical longitudinal strain of the rock mass, is not supported by any empirical studies, further investigation on this topic may be necessary.

(2) The three-phase rock mass model outlined in the Polish National and Industrial Standards is based on the assumption that under certain conditions, a rock mass may act as an elasto-plasto-fractured medium. In this scenario, two coaxial inelastic zones form around the excavation: a fracture zone, where the rock mass lacks cohesion and therefore is devoid of its compressive strength, and a plastic zone, where the rock mass can retain its residual compressive strength. For an elasto-plasto-fractured model of the rock mass,
the support pressure value increases after reaching a certain ground displacement threshold, which appears to align with engineering practice.

(3) The convergence–confinement method (CCM) is a widely recognized, straightforward analytical tool for predicting the ground action and support design of underground roadways, chambers, and tunnels. Compared to the method outlined in Polish National Standards, it can more accurately capture the three-dimensional nature of underground excavations. Furthermore, it facilitates the design of a rigid support system, unlike the method prescribed by Polish Standards, which requires the support system to be able to transfer the entirety of the predicted rock mass displacement.

(4) In Section 4 of this paper, a method for the establishment of the ground reaction curve for an elasto-plasto-fractured rock mass model was presented; this detailed equations for determining the value of ground displacement and the radii of the plastic and fracture zones, which are dependent on the value of fictitious pressure. These formulations provide a basis for establishing the ground reaction curve and can be applied in practical engineering calculations.

(5) In Section 5 of this paper, a series of analyses were conducted on a typical circular roadway to verify the algorithm proposed in Section 4. Three rock mass classes with different mechanical properties were considered. Ground reaction curves for each rock mass class were calculated based on the elasto-plasto-fractured model. These curves were compared with the GRC curves obtained for the typical elasto-plastic model. The elasto-plasto-fractured model showed distinct action compared to the elasto-plastic model—most importantly in terms of displacement increments.

(6) In the second part of the analyses conducted for the various rock mass classes, exemplar calculations of a support system were performed. Yielding arches were considered as the primary support system due to the expected large deformations. Support characteristics curves were determined for different rock mass classes, taking into account the action of the yielding steel sets. Based on ground-support equilibrium points, the factors of safety for the designed support system and the surrounding rock mass were calculated, indicating the correct choice of support parameters. Additionally, a real engineering case of a ventilation roadway in the “Knurów” coal mine was introduced. The engineering calculations, based on the convergence–confinement method and elasto-plasto-fractured model of the rock mass, indicated that the designed support structure can be successfully employed in the roadway under the given geological conditions.

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References

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Appl.  

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