

Article

Parameter Sensitivity Analysis for Machining Operation of Autofrettaged Cylinder Using Taguchi Method

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Abstract: This study investigates the impact of material parameters such as yield strength (S_y), Young's modulus (E), and tangent modulus (T) on the safety factor (SF) of autofrettaged cylinders under 400 MPa working pressure, considering the three scenarios: no machining, internal machining, and external machining. Finite element (FE) simulations were conducted based on the Taguchi experimental design and converted into signal-to-noise (S/N) ratios to determine the optimal settings. ANOVA was utilized to evaluate the significance and percentage contributions of each factor. The analysis indicated that S_y is the most influential parameter on SF , contributing approximately 98.20% across all scenarios, including no machining, internal machining, and external machining. The contributions of E and T were minimal, but T had a slightly greater effect than E . The analytical validation of the FE model showed good agreement, with a maximum deviation of 4.37% for no machining, 4.75% for internal machining, and 5.20% for external machining. Regression analysis further confirmed the high prediction capability of the model, validated using AISI 4340 steel. The study concludes that internal machining results in higher residual stress loss compared to external machining. Overall, the analytical method tends to provide lower SF values than the numerical method, highlighting its conservative nature.

Keywords: autofrettage; residual stress; Taguchi analysis; analysis of variance (ANOVA); finite element method (FEM)



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1. Introduction

In modern engineering applications, various techniques are employed to enhance the durability of components under pressure and increase safety levels. One of these techniques is autofrettage, a crucial manufacturing process that involves subjecting a cylinder to high internal pressure, inducing plastic deformation. Upon pressure release, the cylinder's material attempts to revert to its original shape, resulting in beneficial residual compressive stresses within and residual tensile stresses outside the cylinder. These residual stresses enhance the cylinder's ability to withstand higher pressures by reducing material stress. Hydraulic autofrettage, utilizing fluid pressure, and mechanical autofrettage, employing mandrels, are the two primary methods used to enhance the pressure-bearing capacity and fatigue life of cylinders. Widely adopted across industries like aerospace, automotive, and pressure vessel manufacturing, autofrettage significantly improves the durability and safety of high-pressure components [1–7].

The most significant factors that can affect the residual stress field obtained from hydraulic autofrettage are autofrettage pressure, wall thickness, and material properties [8].

The variability of these factors can create a high level of variability in the residual stress response. Therefore, knowing the sensitivity of the residual stress response obtained from the autofrettage process to these factors is crucial in the design of pressure vessels. Fielder et al. utilized the complex variable finite element method to compute the sensitivity of the residual stress field obtained from autofrettage applied to a thick-walled spherical pressure vessel to material parameters and autofrettage pressure. According to the results obtained, it was observed that the most influential parameter on residual stresses is the autofrettage pressure, while the yield stress stands out as the most influential among material parameters [8].

In the production process of a component resistant to high pressure, machining is often required after the autofrettage process. Machining is employed to achieve specific manufacturing tolerances in components such as the piston–cylinder pair of a high-pressure pump. Additionally, it can be applied to the outer surface of a firearm barrel to lighten the barrel in regions where the internal pressure of the barrel is low or to create grooves on the inner bore to impart a rotational motion to the bullet. As a result of these processes, some of the beneficial residual stresses created by the autofrettage process are lost, and a new residual stress distribution is formed in the cylinder wall. This affects the load-bearing capacity of the component during service. Therefore, determining the residual stress loss caused by the machining process is crucial for accurately estimating the pressure-carrying capacity of the designed cylinder [9,10].

Parker et al. examined the residual stress fields resulting from internal and external material removal operations applied to a thick-walled autofrettaged cylinder for materials exhibiting both the Bauschinger effect and those that do not show it [11]. Perl et al. developed an analytical model for calculating the new residual stress field resulting from the internal, external, and combined processing of a firearm barrel after autofrettage. The model treats partially autofrettaged and fully autofrettaged barrels separately. Additionally, in this study, they validated the analytical model developed with finite element analysis [12]. Jahed et al. investigated the effect of material removal on residual stress using an analytical method that accounts for the material's true unloading behavior [13]. Hameed et al. examined the effect of internal and external machining on residual stress using the finite element method. The results indicated that the maximum circumferential stress was more sensitive to internal machining than external machining and that material removal did not alter the elastic–plastic interface [14]. Bhatnagar et al. analyzed the effect of internal and external machining applied to a compound tube after autofrettage on residual stress loss analytically and numerically. The analysis revealed that the sequence of operations led to different results in residual stress loss when both internal and external machining were applied together [15].

Despite the extensive research on autofrettage and its effects on residual stress, there is a notable gap in the literature regarding the quantitative impact of material parameters on the safety factor of autofrettaged cylinders, especially when different machining processes are applied post-autofrettage. This study advances the field by providing a detailed analysis of how yield strength, Young's modulus, and tangent modulus influence the safety factor under various scenarios. By integrating the Taguchi experimental design, finite element simulations, ANOVA, and regression analysis, this research offers a comprehensive approach to understanding and predicting the behavior of autofrettaged cylinders. The novel contribution lies in quantifying the effects of these material parameters and validating the findings with both numerical and analytical methods, filling a critical gap in the existing research.

In this study, the effect of material parameters such as yield strength, Young's modulus, and tangent modulus on the safety factor for the working pressure of autofrettaged cylinders has been investigated for the following three scenarios: with no machining, only internal machining, and only external machining. The working pressure was chosen as 400 MPa, which is a pressure value commonly experienced in pressure vessels and artillery guns [9,16,17]. Initially, a series of finite element (FE) simulations were conducted based on

the Taguchi experimental design, where three levels were set for each of the three selected material parameters, creating a standard orthogonal array L_9 . The machining simulation after autofrettage simulation was performed using the element birth and death method available in the ANSYS 2023 R1. The results of nine FE simulations, conducted according to the Taguchi experimental design, were transformed into signal-to-noise (S/N) ratios to determine the best settings. Analysis of variance (ANOVA) was used to understand how much each factor contributed and how significant they were. To ensure the accuracy of the FE model, its outcomes were compared with an analytical method based on the deformation theory of plasticity, showing close agreement. Finally, a regression analysis predicted responses for different parameter values, with confirmation simulations confirming the accuracy of the regression model. The analysis indicated that among autofrettaged cylinders across three scenarios—without machining, with only internal machining, and with only external machining—the parameter most significantly affecting the safety factor is S_y . Hameed et al. also highlighted the crucial role of yield strength in residual strain outcomes, noting that materials with higher yield strength exhibit better performance in terms of residual strain upon pressure removal, which aligns with our findings [14]. In all three scenarios, the influence of E and T on the safety factor is relatively minimal. However, T exhibits a greater impact than E . As E decreases and T increases, there is a slight tendency for the safety factor to increase. A strong agreement is noted between the numerical and analytical findings. Generally, the safety factor tends to show lower values in the analytical approach compared to the numerical approach across all scenarios. The numerical analysis results suggest that internal machining results in a higher loss of residual stress compared to external machining.

2. Material and Method

2.1. Analytical Method

In this study, the analytical method is based on the deformation theory of plasticity. The stress–strain relationship of the material is defined using the bilinear kinematic hardening model shown in Figure 1. This model simplifies the stress–strain behavior in the plastic region linearly and incorporates the Bauschinger effect, making it an appropriate model [18].

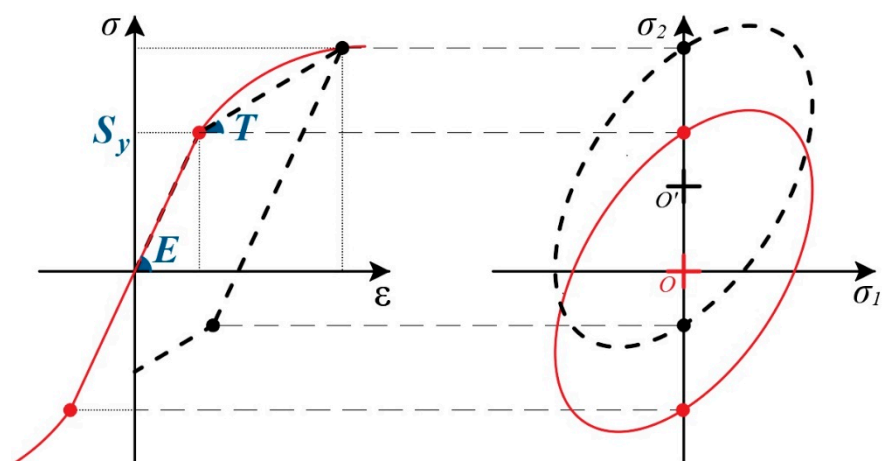


Figure 1. Schematic representation of bilinear kinematic hardening model.

In the analytical method, the plain strain approach was utilized, which is compatible with the actual physical conditions as it accounts for the placement of the cylinder between holders that limits longitudinal extension from both ends in the hydraulic autofrettage process. Furthermore, the incompressible volume assumption was applied in the model ($\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$), and the Von Mises yield criterion was used for the plastic region [18].

2.1.1. Residual Stresses after Autofrettage

The residual stresses developed as a result of the autofrettage process are examined for two regions of the cylinder: the region experiencing plastic deformation in the inner part of the cylinder and the elastic region outside the plastic zone. These regions are schematically represented in Figure 2. In this schematic depiction, a represents the inner radius of the cylinder, b represents the radius where the elastic region ends and the plastic region begins, and c represents the outer radius of the cylinder [18].

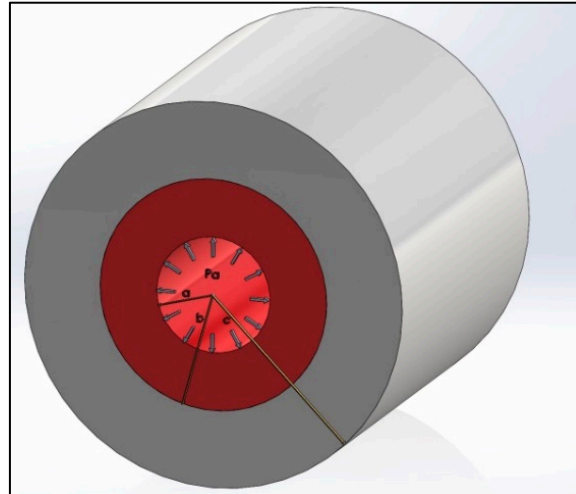


Figure 2. Schematic representation of elastic and plastic regions after autofrettage pressure.

Residual stresses developed in the elastic part of the cylinder ($b \leq r \leq c$)

$$\sigma_{re}^R = \frac{1}{\sqrt{3}} S_y b^2 \left(\frac{1}{c^2} - \frac{1}{r^2} \right) - P_a \frac{a^2}{c^2 - a^2} \left(1 - \frac{c^2}{r^2} \right) \quad (1)$$

$$\sigma_{\theta e}^R = \frac{1}{\sqrt{3}} S_y b^2 \left(\frac{1}{c^2} + \frac{1}{r^2} \right) - P_a \frac{a^2}{c^2 - a^2} \left(1 + \frac{c^2}{r^2} \right) \quad (2)$$

$$\sigma_{ze}^R = \frac{1}{\sqrt{3}} S_y \frac{b^2}{c^2} - P_a \frac{a^2}{c^2 - a^2} \quad (3)$$

where S_y and P_a represent the yield stress and autofrettage pressure, respectively. The autofrettage pressure P_a is expressed by the following equation.

$$P_a = \frac{1}{\sqrt{3}} \left[2A \ln \frac{b}{a} + (S_y - A) b^2 \left(\frac{1}{a^2} - \frac{1}{b^2} \right) - S_y b^2 \left(\frac{1}{c^2} - \frac{1}{b^2} \right) \right] \quad (4)$$

Residual stresses developed in the plastic part of the cylinder ($a \leq r \leq b$)

$$\sigma_{rp}^R = \frac{2}{\sqrt{3}} A \ln \frac{r}{a} + \frac{b^2}{\sqrt{3}} (S_y - A) \left(\frac{1}{a^2} - \frac{1}{r^2} \right) - P_a - P_a \frac{a^2}{c^2 - a^2} \left(1 - \frac{c^2}{r^2} \right) \quad (5)$$

$$\sigma_{\theta p}^R = \frac{2}{\sqrt{3}} A \left(1 + \ln \frac{r}{a} \right) + \frac{b^2}{\sqrt{3}} (S_y - A) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) - P_a - P_a \frac{a^2}{c^2 - a^2} \left(1 + \frac{c^2}{r^2} \right) \quad (6)$$

$$\sigma_{zp}^R = \frac{1}{\sqrt{3}} A + \frac{2}{\sqrt{3}} A \ln \frac{r}{a} + \frac{1}{\sqrt{3}} (S_y - A) \frac{b^2}{a^2} - P_a - P_a \frac{a^2}{c^2 - a^2} \quad (7)$$

The term A in the above equations is a constant expressed by the following equation.

$$A = S_y - T \times \epsilon_y \quad (8)$$

where T and ε_y are tangent modulus and yield strain, respectively.

2.1.2. Final Stresses after Working Pressure in Autofrettaged Cylinder

When working pressure is applied to an autofrettaged cylinder, the total stresses σ_r^T , σ_θ^T , σ_z^T are obtained by adding the elastic stress components σ_r^w , σ_θ^w , σ_z^w , known as Lamé equations, to the residual stress components σ_r^R , σ_θ^R , σ_z^R created by autofrettage [19].

$$\sigma_r^T = \sigma_r^R + \sigma_r^w \quad (9)$$

$$\sigma_\theta^T = \sigma_\theta^R + \sigma_\theta^w \quad (10)$$

$$\sigma_z^T = \sigma_z^R + \sigma_z^w \quad (11)$$

The elastic stress components, also known as Lamé equations, are expressed as follows:

$$\sigma_r^w = P_w \frac{a^2}{c^2 - a^2} \left(1 - \frac{c^2}{r^2} \right) \quad (12)$$

$$\sigma_\theta^w = P_w \frac{a^2}{c^2 - a^2} \left(1 + \frac{c^2}{r^2} \right) \quad (13)$$

$$\sigma_z^w = P_w \frac{a^2}{c^2 - a^2} \quad (14)$$

where P_w is the working pressure.

2.1.3. Optimum Autofrettage Pressure

The optimum autofrettage pressure is the pressure that minimizes the maximum total stress occurring at the elastic–plastic radius of the autofrettaged cylinder under the working pressure. In other words, it is the pressure value at the working pressure that maximizes the safety factor of the autofrettaged cylinder, and it can be calculated using the following equation derived from the Von Mises yield criterion [19]:

$$P_{a,opt} = \frac{S_y}{2} \left(1 - \frac{e^{n\sqrt{3}}}{k^2} + n\sqrt{3} \right) \quad (15)$$

Here, n and k are constants expressed by the following equations:

$$n = \frac{P_w}{S_y} \quad (16)$$

$$k = \frac{c}{a} \quad (17)$$

2.1.4. Residual Stresses after Machining

After the machining process, the residual stresses in an autofrettaged cylinder redistribute. This new residual stress distribution can be obtained for a cylinder with inner and outer radii of a' and c' , respectively, by following the procedure below [13]:

1. Residual stresses obtained from autofrettage are calculated in a cylinder with an inner radius and an outer radius (i.e., $\sigma_r^R(a')$ and $\sigma_r^R(c')$).
2. The radial residual stresses at a' and c' are determined.
3. The stress field generated when the internal pressure $\sigma_r^R(a')$ and the external pressure $\sigma_r^R(c')$ are applied to the cylinder with inner radius a' and outer radius c' is calculated.
4. In the third stage, the stresses obtained are superimposed with the stresses obtained in the first step.

2.2. Finite Element Analysis

The numerical analyses were conducted using the ANSYS Mechanical 2023 R1 Implicit Solver (Ansys, Inc., Canonsburg, PA, USA). In each of the nine FE simulations applied according to the Taguchi experimental design, the optimum autofrettage pressure calculated with Equation (15) was considered. For the analyses, the Poisson's ratio of the materials was taken as 0.3. The analyses were performed under the assumption that the materials exhibit bilinear kinematic hardening behavior. The finite element analysis was carried out as 2D Plain Strain.

Mesh convergence was ensured by using mesh sizes of 10, 5, 4, 2, and 1 mm. When the analysis results were compared using mesh sizes of 2 mm and 1 mm, it was observed that the results varied by less than 2%. Therefore, to expedite the solution in the analyses, the general mesh size of the model given in Figure 3a was selected as 2 mm. MultiZone Quad4 elements were used in the finite element model to better conform to the geometry, ensure uniform distribution of the lateral loads, require fewer elements, and obtain more accurate results. Additionally, "Share Topology" was used between meshes to ensure that the neighboring elements share nodes, aiming for more accurate results. The generated finite element model consists of 4161 elements and 4380 nodes.

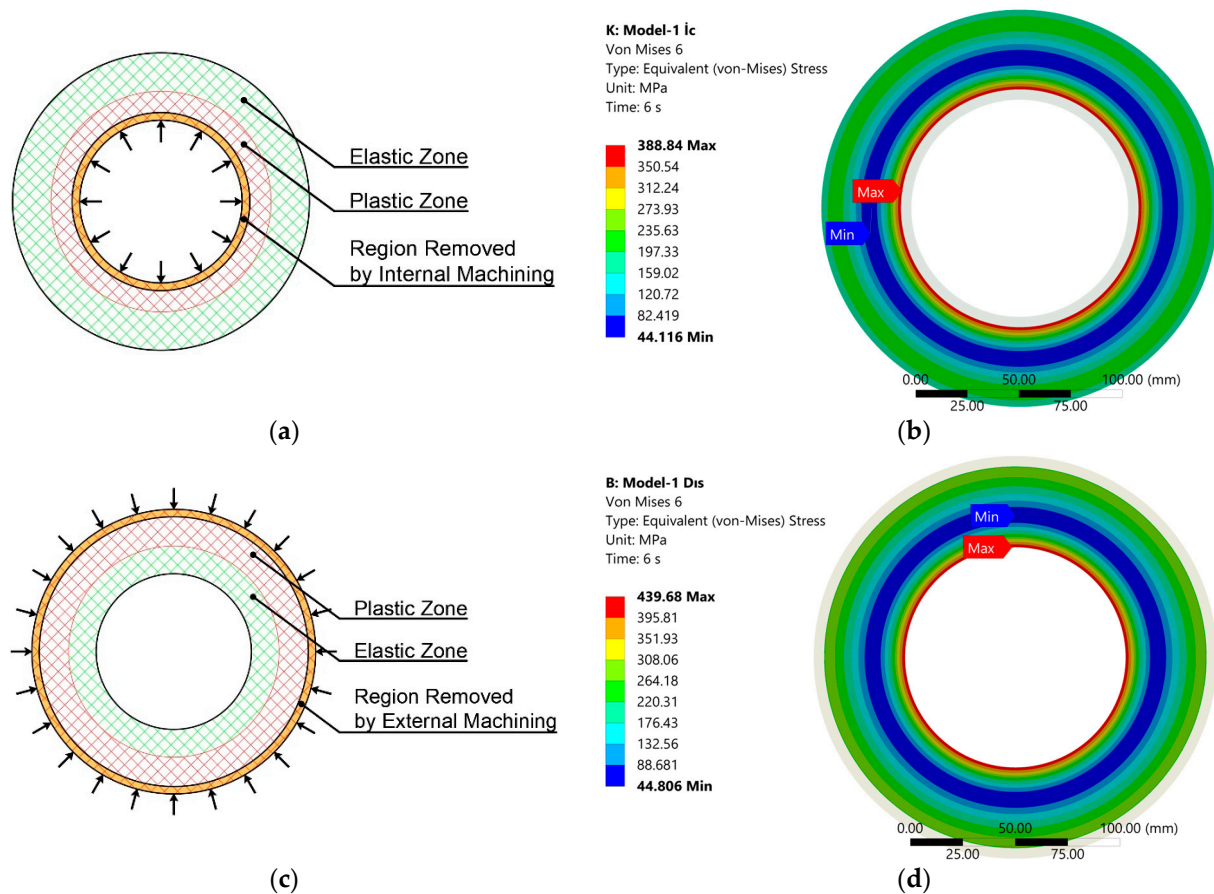


Figure 3. (a) Schematic representation of the material removed region for internal machining operation (b) Contour plots of the equivalent stress for the FEM results of the internal machining (c) Schematic representation of the material removed region for the external machining operation (d) Contour plots of the equivalent stress for the FEM results of the external machining.

As a result of the mesh generation process, a minimum element quality of 0.87987 and an average element quality of 0.95181 were obtained. When the aspect ratio was examined, the maximum value was found to be 1.6301, and the average value was 1.3082. The data related to the element quality and aspect ratio values obtained from the analysis support the accuracy and stability of the solution.

No supports were used during the analysis, and the “weak springs” feature was activated to conduct the simulations. The numerical analyses related to the machining were performed using the “Element Kill” and “Element Birth” commands of ANSYS APDL. In this context, a material removal of 5 mm from the inner and outer surfaces of the cylindrical model, as shown in Figure 3b, and the application of a pressure of 400 MPa to the inner surface of the part, as shown in Figure 4, were conducted for numerical analysis.

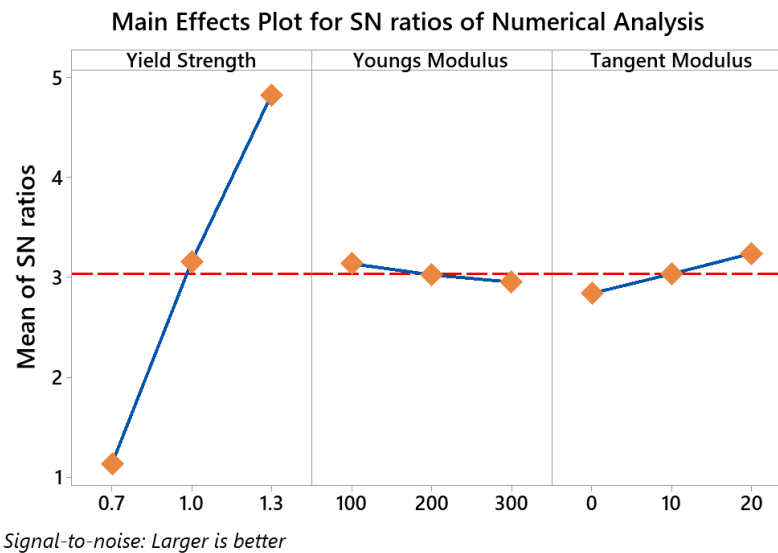


Figure 4. Main effects plots for S/N ratios of the safety factor of the autofrettaged cylinder without machining under working pressure of 400 MPa.

2.3. Taguchi Analysis

In many engineering problems, full factorial and fractional factorial design techniques are frequently preferred [20]. However, as the number of factors increases, the statistical process becomes complex and time-consuming, foremost. To simplify this laborious process of examining the entire space, describing the process, and finding the best combination of parameters, the Taguchi experimental design method proposes the use of an orthogonal array. In this way, the Taguchi approach is able to overcome the problem by using a minimum number of experiments to obtain information such as the main and interaction effects of these parameters. For this reason, the Taguchi approach is considered one of the most effective methods for evaluating the effects of process variables needed to address challenges in engineering fields [20,21]. In the Taguchi technique, a loss function is utilized to measure the target value change and interpret the performance characteristics, and then this function value is expressed as the signal-to-noise (S/N) ratio [22]. These S/N ratios have three categories: “higher is better” (HiB), “nominal is best” (NiB), and “lower is better” (LiB), depending on the nature of the problem during the analysis or experiment performed [23].

It is obvious that the Taguchi method can be preferred as a design of experiment (DoE) due to the many advantages mentioned before, especially since determining the most appropriate parameters is of critical importance in applications such as the autofrettage process where mechanical behavior in high-pressure cylinders needs to be improved. In addition, with this method, it is possible to systematically evaluate the effects of various process parameters and their different levels, and with this approach, the computational cost will also be reduced with fewer simulations. Additionally, this method enables a systematic analysis of the effects of the various process parameters and their different levels, thereby reducing computational costs through fewer simulations. As a consequence, this research employed the Taguchi method to enhance product quality and process efficiency by identifying the optimal parameter combinations for the autofrettage process, both with and without machining. Since the safety factor was taken as the control parameter in

order to evaluate the efficiency in the analyses, HiB was preferred for the S/N ratio. The signal-to-noise (S/N) ratio for the HiB case is described as follows:

$$\frac{S}{N} = 10 \log \frac{1}{N} \sum_{i=1}^n \frac{1}{y_i^2} \quad (18)$$

In this work, Taguchi's L_9 orthogonal array method was employed due to its recognized utility and user-friendly nature. This method facilitates a reduction in computational time by conducting a limited number of analyses while allowing for the evaluation of parameters and their cross-effects. The factors and their levels for the Taguchi L_9 orthogonal array are summarized in Table 1.

Table 1. Material parameters and levels.

Factors	Levels		
	1	2	3
Yield Strength (S_y , MPa)	700	1000	1300
Young's Modulus (E , GPa)	100	200	300
Tangent Modulus (T , GPa)	0	10	20

3. Results and Discussion

3.1. Taguchi Analysis

In this study, the safety factor for three different scenarios of an autofrettaged cylinder was investigated: without machining, with only internal machining at a thickness of 5 mm, and with only external machining at a thickness of 5 mm. The effect of the factors listed in Table 1 was examined using the Taguchi L_9 (3^3) orthogonal array design. The experimental matrix created using the Minitab 19 statistical software is presented in Table 2. The safety factor for the operating pressure of autofrettaged cylinders without machining, with only internal machining, and with only external machining was taken as the response parameter. Since the goal is to maximize each response parameter, a type of S/N ratio where larger is better was considered.

Table 2. Taguchi's L_9 orthogonal array.

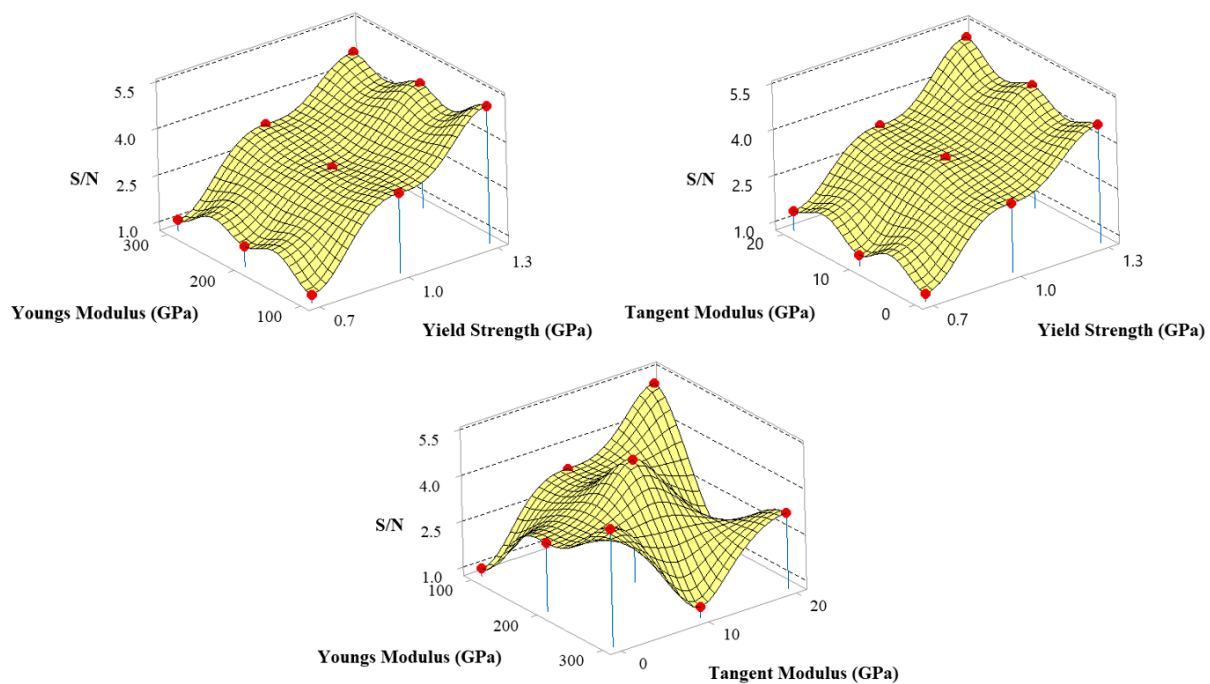
Order	Young's Modulus (GPa)	Tangent Modulus (GPa)	Yield Strength (GPa)
1	100	0	0.7
2	200	20	0.7
3	300	10	0.7
4	100	10	1
5	200	0	1
6	300	20	1
7	100	20	1.3
8	200	10	1.3
9	300	0	1.3

FE Analyses were conducted for each set of parameters specified in Table 2 of the experimental matrix. Subsequently, the calculated simulation responses were transformed into signal-to-noise (S/N) ratios using the Minitab 19 software. Regardless of the objective function, a larger S/N ratio corresponds to better performance characteristics. The calculated signal-to-noise (S/N) ratio values (mean) for the safety factor of the autofrettaged cylinder without machining are provided in Table 3 and graphically presented in Figure 4.

Table 3. S/N response table for the safety factor of the autofrettaged cylinder without machining under a working pressure of 400 MPa.

Level	S_y	E	T
1	1.140	1.462	1.407
2	1.438	1.434	1.439
3	1.742	1.424	1.475
Delta	0.602	0.038	0.068
Rank	1	3	2

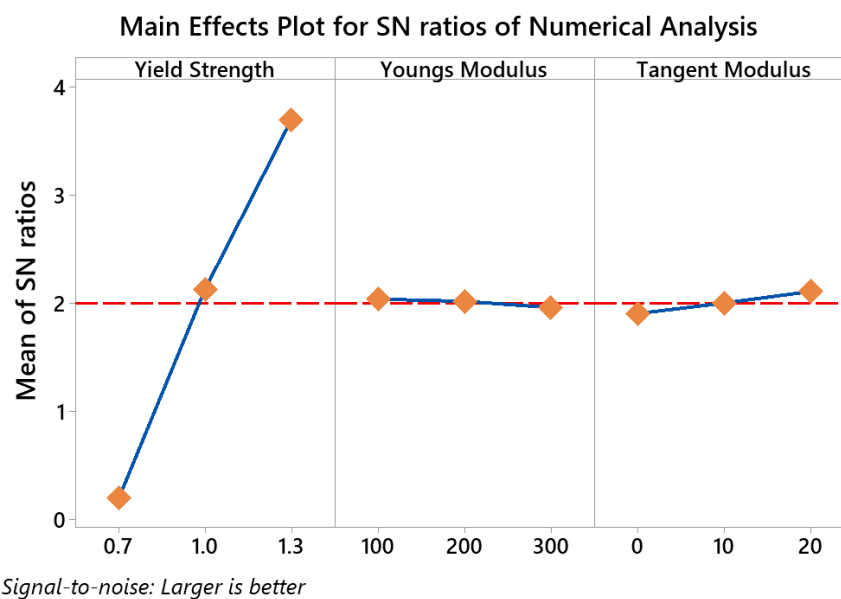
The analysis of Figure 5 reveals that an increase in the yield strength of the material undergoing the autofrettage process without machining can enhance the S/N ratio by 4 to 5 times, irrespective of the Young's modulus value. Conversely, at a tangent modulus value of 0, the S/N ratio rises proportionally with the increase in Young's modulus. Notably, the optimal S/N ratios are observed with the maximum levels of yield strength and tangent modulus within the examined range, along with the lowest value of Young's modulus.

**Figure 5.** Surface plot of S/N versus Young's modulus, yield strength, and tangent modulus for the case without machining.

The maximum effectiveness ranks were calculated based on delta values representing the variability between the highest and lowest average response values for each factor. According to the delta values in Table 3, the most influential parameter on the response for the autofrettaged cylinder with only internal machining is S_y . It is clearly seen from the graphical data (Figure 4) that the average S/N ratio increases as S_y and T increase. This indicates that the safety factor of the autofrettaged cylinder without machining increases as S_y and T increase. However, the safety factor decreases as E values increase. The dashed line in Figure 4 represents the overall average of all S/N ratios. The calculated S/N ratio values (average) for the safety factor of the autofrettaged cylinder with only internal machining applied are provided in Table 4 and presented graphically in Figure 6.

Table 4. S/N response table for the safety factor of the autofrettaged cylinder with only internal machining under working pressure of 400 MPa.

Level	S_y	E	T
1	1.022	1.284	1.262
2	1.277	1.276	1.276
3	1.530	1.269	1.291
Delta	0.507	0.015	0.030
Rank	1	3	2

**Figure 6.** Main effects plots for S/N ratios of the safety factor of the autofrettaged cylinder with only internal machining under working pressure of 400 MPa.

A similar outcome to the autofrettage process without machining is evident in the case of the process involving internal or external machining, as depicted in Figures 7 and 8. The distinction between them becomes apparent at the maximum value that the S/N ratio can attain.

According to the delta values in Table 4, the most influential parameter on the response for the autofrettaged cylinder with only internal machining is S_y . It is clearly seen from the graphical data (Figure 6) that the average S/N ratio increases as S_y and T increase. This means that the safety factor of the autofrettaged cylinder with only internal machining increases as S_y and T increase. However, the safety factor decreases as E values increase. The calculated S/N ratio values (average) for the safety factor of the autofrettaged cylinder with only external machining are provided in Table 5 and presented graphically in Figure 9.

According to the Delta values in Table 5, the most influential parameter on the response for the autofrettaged cylinder with only external machining is S_y . As observed from the graphical data (Figure 9), the average S/N ratio increases with the increase in S_y and T . This implies that the safety factor of the autofrettaged cylinder with only external machining increases with increasing S_y and T . However, the safety factor decreases with increasing E values.

According to the analysis results obtained using Taguchi's robust process design, in order to achieve the maximum safety factor in all three scenarios, the values of S_y , E , and T should be 0.7 GPa, 100 GPa, and 20 GPa, respectively.

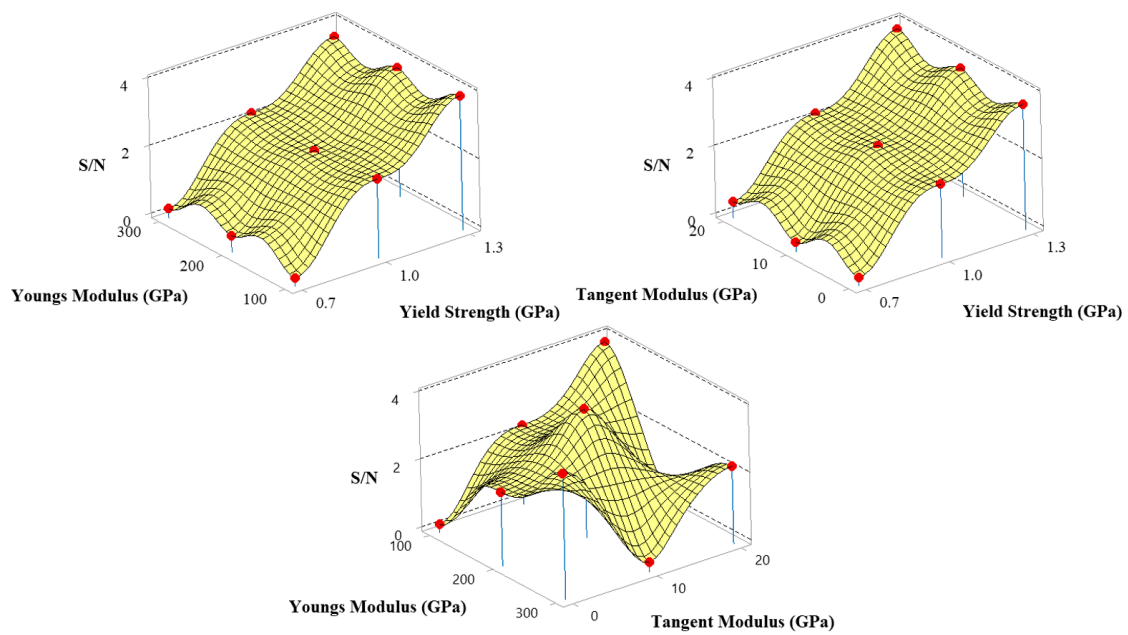


Figure 7. Surface plot of S/N versus Young's modulus, yield strength, and tangent modulus for the case with only internal machining.

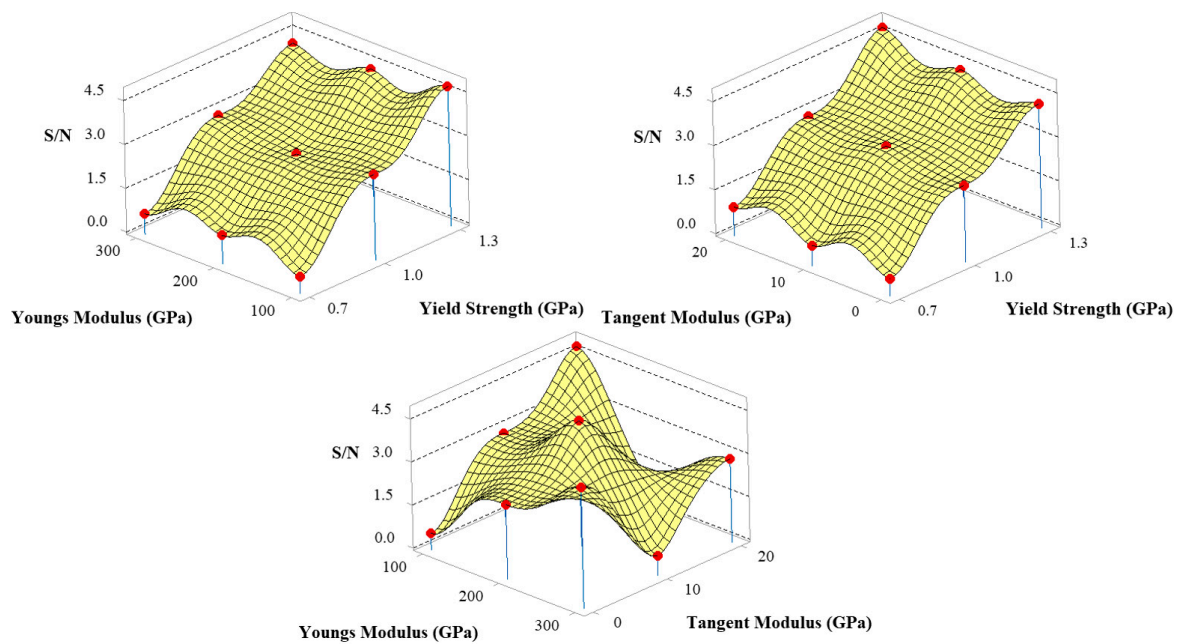


Figure 8. Surface plot of S/N versus Young's modulus, yield strength and tangent modulus for the case with only external machining.

Table 5. S/N response table for the safety factor of the autofrettaged cylinder with only external machining under working pressure of 400 MPa.

Level	S_y	E	T
1	1.079	1.388	1.335
2	1.369	1.361	1.366
3	1.656	1.356	1.404
Delta	0.576	0.032	0.069
Rank	1	3	2

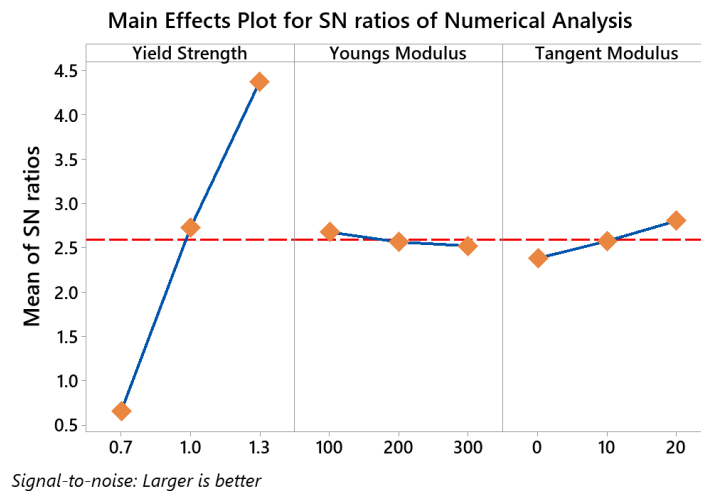


Figure 9. Main effects plots for S/N ratios of the safety factor of the autofrettaged cylinder with only external machining under working pressure of 400 MPa.

3.2. Analysis of Variance

The Taguchi methodology does not directly provide the impact of process parameters on specific outputs. Therefore, statistical tools are especially required to calculate the relative importance and effects of each process parameter on specific outputs. The effectiveness of each parameter was evaluated using analysis of variance (ANOVA). This method can clearly identify how much each process parameter influences the response and determine the contribution of each parameter. The ANOVA analysis was conducted at a 95% confidence level. The importance of process parameters was determined by comparing the p -values. The lower the p -value, the greater the effect of that parameter on the response. Generally, if the p -value is less than 0.05, it indicates that the parameter has a significant effect on the process response [24]. Furthermore, it is necessary to verify whether assumptions such as independence and homogeneity of variance are met for ANOVA [25–27]. In this context, the Shapiro–Wilk test was used to check the normality of the samples for the analyses conducted in this study.

Table 6 shows the ANOVA results for the safety factor of the autofrettaged cylinder without machining. When examining the ANOVA results, it is observed that the p -values for the S_y and T parameters are less than 0.05. Therefore, since the p -value is less than 0.05 at a 95% confidence level, it is concluded that the factors are statistically significant in their effect on the response. Additionally, the contribution of each material parameter was calculated by dividing each individual sum of squares by the total sum of squares. The contributions of the S_y , E , and T parameters to the response are 98.20%, 0.41%, and 1.29%, respectively. Yield strength is the most influential factor on the safety factor of the machined autofrettaged cylinder, while the effects of E and T are very low.

Table 6. Results of ANOVA for the safety factor of the autofrettaged cylinder without machining under working pressure of 400 MPa.

Source	DF	Adj SS	Adj MS	F-Value	p -Value
Regression	3	0.553602	0.184534	1188.37	0.000
S_y	1	0.544380	0.544380	3505.73	0.000
E	1	0.002220	0.002220	14.30	0.013
T	1	0.007001	0.007001	45.09	0.001
Error	5	0.000776	0.000155		
Total	8	0.554378			

Table 7 displays the ANOVA results for the safety factor of the autofrettaged cylinder with only internal machining. Upon examining the ANOVA results, it is observed that the p -values for all material parameters are less than 0.05. Therefore, since the p -value is less than 0.05 at a 95% confidence level, the factors are statistically significant in their effect on the response. The contributions of the S_y , E , and T parameters to the response are 98.20%, 0.4%, and 1.26%, respectively. Yield strength is the most influential factor on the safety factor of the autofrettaged cylinder with only internal machining, while the effects of the other two factors are very low.

Table 7. Results of ANOVA for the safety factor of the autofrettaged cylinder with only internal machining under working pressure of 400 MPa.

Source	DF	Adj SS	Adj MS	F-Value	p -Value
Regression	3	0.553602	0.184534	1188.37	0.000
S_y	1	0.544380	0.544380	3505.73	0.000
E	1	0.002220	0.002220	14.30	0.013
T	1	0.007001	0.007001	45.09	0.001
Error	5	0.000776	0.000155		
Total	8	0.554378			

Table 8 shows the ANOVA results for the safety factor of the autofrettaged cylinder with only external machining. Upon examining the ANOVA results, it is observed that the p -values for the S_y and T parameters are less than 0.05. Therefore, since the p -value is less than 0.05 at a 95% confidence level, it is concluded that the factors have a statistically significant effect on the response. The contributions of the S_y and T parameters to the response are 98.19% and 1.39%, respectively. While S_y is the most influential factor on the safety factor of the autofrettaged cylinder with only external machining, the effect of T is very low. E , on the other hand, does not have any effect on the response.

Table 8. Results of ANOVA for the safety factor of the autofrettaged cylinder with only external machining under working pressure of 400 MPa.

Source	DF	Adj SS	Adj MS	F-Value	p -Value
Regression	2	0.505400	0.252700	710.18	0.000
S_y	1	0.498357	0.498357	1400.57	0.000
T	1	0.007043	0.007043	19.79	0.004
Error	6	0.002135	0.000356		
Total	8	0.507535			

3.3. Comparison of the Numerical and Analytical Methods

For the validation of the FE model, analytical calculations were performed according to the plan in Table 2, and they were compared with the FE simulation results. Table 9 shows the safety factors numerically and analytically calculated for the autofrettaged cylinder without machining at the operating pressure according to the plan in the experiment matrix. Upon examining the results, it is observed that there is a maximum deviation of 4.37% between the numerical and analytical predictions.

Table 10 presents the safety factors calculated numerically and analytically for the autofrettaged cylinder subjected to only internal machining, following the experimental matrix plan. Upon review, we find a maximum deviation of 4.75% between the numerical and analytical estimations.

Table 9. Comparison of the numerical and analytical results for the safety factor of the autofrettaged cylinder without machining under working pressure of 400 MPa.

Order	Young's Modulus (GPa)	Tangent Modulus (GPa)	Yield Strength (GPa)	Factor of Safety (Numerical)	Factor of Safety (Analytical)	Deviation %
1	100	0	0.7	1.12	1.08	3.70
2	200	20	0.7	1.17	1.12	4.37
3	300	10	0.7	1.13	1.09	3.64
4	100	10	1	1.46	1.40	3.88
5	200	0	1	1.41	1.36	3.11
6	300	20	1	1.45	1.39	3.81
7	100	20	1.3	1.81	1.74	3.97
8	200	10	1.3	1.72	1.68	2.86
9	300	0	1.3	1.69	1.66	2.26

Table 10. Comparison of the numerical and analytical results for the safety factor of the autofrettaged cylinder with only internal machining under working pressure of 400 MPa.

Order	Young's Modulus (GPa)	Tangent Modulus (GPa)	Yield Strength (GPa)	Factor of Safety (Numerical)	Factor of Safety (Analytical)	Deviation %
1	100	0	0.7	1.01	0.96	4.53
2	200	20	0.7	1.04	0.99	4.75
3	300	10	0.7	1.02	0.97	4.28
4	100	10	1	1.29	1.24	3.91
5	200	0	1	1.26	1.22	3.81
6	300	20	1	1.28	1.23	3.93
7	100	20	1.3	1.56	1.50	3.52
8	200	10	1.3	1.52	1.48	3.22
9	300	0	1.3	1.51	1.47	2.85

Table 11 displays the safety factors calculated numerically and analytically for the autofrettaged cylinder subjected to only external machining, following the experimental matrix plan. Upon examination of the results, we observe a maximum deviation of 5.20% between the numerical and analytical predictions.

Table 11. Comparison of the numerical and analytical results for the safety factor of the autofrettaged cylinder with only external machining under working pressure of 400 MPa.

Order	Young's Modulus (GPa)	Tangent Modulus (GPa)	Yield Strength (GPa)	Factor of Safety (Numerical)	Factor of Safety (Analytical)	Deviation %
1	100	0	0.7	1.06	1.01	4.32
2	200	20	0.7	1.11	1.05	4.97
3	300	10	0.7	1.07	1.03	4.32
4	100	10	1	1.39	1.33	4.39
5	200	0	1	1.34	1.29	3.66
6	300	20	1	1.38	1.31	5.20
7	100	20	1.3	1.72	1.64	4.50

Table 11. Cont.

Order	Young's Modulus (GPa)	Tangent Modulus (GPa)	Yield Strength (GPa)	Factor of Safety (Numerical)	Factor of Safety (Analytical)	Deviation %
8	200	10	1.3	1.64	1.58	3.36
9	300	0	1.3	1.61	1.57	2.75

3.4. Effect of Machining Process on Factor of Safety

Table 12 has been created to compare the changes in the safety factor determined for the autofrettaged cylinder after the internal and external machining operations. The values comprise the results obtained from numerical analysis.

Table 12. The effect of internal and external machining processes on the safety factor.

Order	E (GPa)	T (GPa)	S _y (GPa)	Without Machining	Internal Machining	External Machining	Variation for Internal Machining %	Variation for Internal Machining %
1	100	0	0.7	1.12	1.01	1.06	−9.82	−5.36
2	200	20	0.7	1.17	1.04	1.11	−11.11	−5.13
3	300	10	0.7	1.13	1.02	1.07	−9.73	−5.31
4	100	10	1	1.46	1.29	1.39	−11.64	−4.79
5	200	0	1	1.41	1.26	1.34	−10.64	−4.96
6	300	20	1	1.45	1.28	1.38	−11.72	−4.83
7	100	20	1.3	1.81	1.56	1.72	−13.81	−4.97
8	200	10	1.3	1.72	1.52	1.64	−11.63	−4.65
9	300	0	1.3	1.69	1.51	1.61	−10.65	−4.73
Mean Variation							−11.94	−4.97

Upon examination of Table 12, it is observed that the safety factor determined for the autofrettaged cylinder with only internal machining decreases by an average of 11.94%, whereas the safety factor for the autofrettaged cylinder with external machining decreases by an average of 4.97%. This indicates that the internal machining operation leads to a greater residual stress loss compared to the external machining operation. This effect can be attributed to the distribution of equivalent residual stresses obtained after the autofrettage process, which decrease from the inner surface towards the outer surface of the cylinder. Consequently, internal machining, which directly affects the region with the highest residual stresses, causes a more significant reduction in residual stresses than external machining.

3.5. Regression Analysis

In this study, regression analyses were also employed to investigate the relationship between dependent and independent variables. A regression analysis based on numerical data was conducted for the safety factors of the autofrettaged cylinder without machining. The following equation defines the regression model.

$$SF_1 = 0.4405 + 1.0040 S_y - 0.000192 E + 0.003416 T \quad (19)$$

A regression analysis based on numerical data was applied to the safety factors of the autofrettaged cylinder with only internal machining. The equation below defines the regression model.

$$SF_2 = 0.43030 + 0.84555 S_y - 0.000073 E + 0.001499 T \quad (20)$$

A regression analysis based on the numerical data was conducted for the safety factors of the autofrettaged cylinder with only external machining. The equation below defines the regression model.

$$SF_3 = 0.4053 + 0.9607 S_y - 0.000161 E + 0.003426 T \quad (21)$$

Upon examining the equations derived from the regression analysis, it is observed that E is negatively signed in all equations. This indicates that E has a decreasing effect on the response. By evaluating the magnitudes of the coefficients preceding each parameter in the equations, we can determine the relative influence of each parameter on the response. Specifically, S_y has the most significant impact on the safety factor, followed by T , and finally, E . In practical terms, this means that increasing the yield strength of the material will result in the most substantial improvement in the cylinder's ability to withstand pressure, while changes in the tangent modulus and Young's modulus will have progressively lesser effects.

In our analysis, we also considered the potential for multicollinearity among the independent variables. Multicollinearity can lead to unreliable and unstable estimates of regression coefficients, inflating the variance and making it difficult to determine the individual effect of each predictor.

To assess and address multicollinearity, we calculated the variance inflation factor (VIF) for each independent variable. A VIF value below 5 generally indicates that multicollinearity is not a serious concern. In all three scenarios, the models without interaction terms showed high R^2 values and statistically significant p -values (below 0.05) for each parameter. Additionally, the VIF values for all parameters in these non-interaction models were 1.0, indicating no multicollinearity issues. Therefore, we chose to use the simpler models without interaction terms for clarity and consistency, ensuring robust and valid findings.

3.6. Validation of the Regression Model

The next step involves conducting validation tests to confirm the predictive capability of the equations obtained from the regression analysis [24]. For this purpose, the safety factor values obtained from simulations conducted with inputs other than the material parameters used in the Taguchi analyses are compared with the values predicted by the regression equations. Since AISI 4340 steel is commonly used in autofrettage applications, it was utilized in this validation test [28]. The material properties of AISI 4340 steel are provided in Table 13 [18].

Table 13. Mechanical Properties of the AISI 4340 Steel [18].

Designation	Yield Strength	Tangent Modulus	Young's Modulus	Poisson's Ratio
AISI 4340	1200 (MPa)	1489 (MPa)	200 (GPa)	0.3

The safety factors calculated using numerical methods and regression equations are provided in Table 14. When comparing numerical and regression results, it is observed that there is a maximum deviation of 0.0279%. Therefore, it can be concluded that the regression model has a high predictive power.

Table 14. Comparison of the numeric model and regression model.

	Factor of Safety (Numerical)	Factor of Safety (Regression)	Deviation %
Without machining	1.6057	1.6120	0.3908
Internal machining	1.4322	1.4326	0.0279
External machining	1.5251	1.5310	0.3854

4. Conclusions

In this study, the effect of material parameters such as yield strength, Young's modulus, and tangent modulus on the safety factor determined for autofrettaged cylinders under a working pressure of 400 MPa was investigated. A series of FE simulations were conducted based on the Taguchi experimental design, and then, the data were analyzed through ANOVA and regression analysis. For the regression analysis conducted based on the numerical results for all three scenarios, a validation simulation was performed using data for AISI 4340 steel. Additionally, analytical calculations were performed to validate the finite element model and compared with the FE simulation results. The following conclusions are drawn:

- According to the Taguchi analysis, the most influential parameter on the safety factor determined for autofrettaged cylinders under a working pressure of 400 MPa is the yield strength. For all three scenarios, the effect of Young's modulus and tangent modulus on the safety factor is very low. Although the effect of Young's modulus and tangent modulus on the response is minimal, tangent modulus is more influential on the response than Young's modulus for all three scenarios. While Young's modulus decreases and tangent modulus increases, there is a slight tendency for the safety factor to increase.
- The numerical and analytical results show a very good agreement. The safety factors calculated using the analytical method are lower than those obtained using the numerical method in all three scenarios. It can be said that the analytical method is more conservative in calculating safety factors compared to the numerical method.
- According to the analysis results, it was observed that the internal machining process leads to more residual stress loss compared to the external machining process.
- It was found that selecting materials with higher yield strength and tangent modulus and lower stiffness would have a positive impact on the safety factor of the structure in all three scenarios investigated in the study.
- According to the results of ANOVA, using materials with higher yield strength maximizes the benefit obtained from the autofrettage process.
- When comparing the results obtained from the validation simulation with those from the regression equation, it is evident that the regression equations exhibit a high prediction capability.

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