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Assessment of the Aerodynamic and Aeroelastic Performance of Long-Span Twin-Box Bridges Based upon Multidimensional Surrogate Models

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Abstract: Twin-box decks are being extensively used in the design of long-span bridges due to their superior flutter performance. Although the significant role played by the gap distance has been previously addressed in the frame of experimental studies, there is still a lack of understanding about the complex interplay between box geometry, gap distance and the aerodynamic force coefficients and flutter derivatives. In the present work, firstly, a surrogate model is developed, considering three design inputs for the geometry of the deck along with the angle of attack, providing the force coefficients as output. Afterwards, the work is then extended by developing another surrogate model, considering as inputs the reduced velocity and the same geometric variables of the deck, with the outputs being the flutter derivatives. The methodology, comprising the selection of the design domain, the definition of the samples, the CFD-based evaluation of the samples' response and the construction of the surrogate through the application of the neural network-based radial basis method, is reviewed. The surrogate models enable a quantitative description of the impact caused in the force coefficients and the flutter derivatives by modifications in the geometry of the twin-box deck. It has been found that flutter derivatives H_1^* , H_2^* and A_2^* are strongly dependent on the gap distance.

Keywords: twin-box; surrogate modelling; CFD; Latin Hypercube Sampling; radial basis; flutter; URANS; $k-\omega$ SST



Citation: Badhurshah, R.; Álvarez, A.J.; Nieto, F.; Jurado, J.Á.; Hernández, S. Assessment of the Aerodynamic and Aeroelastic Performance of Long-Span Twin-Box Bridges Based upon Multidimensional Surrogate Models. *Appl. Sci.* **2024**, *14*, 5531. <https://doi.org/10.3390/app14135531>

Academic Editors: Bin Sun, Rucheng Xiao, Chaolin Song and Wei Huang

Received: 7 May 2024

Revised: 14 June 2024

Accepted: 20 June 2024

Published: 26 June 2024



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1. Introduction

Ground transport networks have steadily developed worldwide, aiming to foster economic growth and development, link regions and communities separated by geography and create new opportunities for people. Technological advances in recent decades have turned into standard engineering practice former challenges, such as building bridges with spans well above 1 km. Nowadays, the engineering challenges in the design of long-span suspension bridges are in the range above 2000 m, while, for cable-stayed bridges, the equivalent mark could be set in span lengths above 1000 m.

Due to the exceptional flexibility of such long-span structures, safety with regard to aeroelastic effects becomes the dominant design factor, at least from a wind engineering perspective. Focusing on flutter performance, the safe design of ultra-long span bridges certainly benefits from twin-box deck design. Examples of long span suspension bridges constructed adopting a twin-box arrangement are the Xihoumen Bridge in China, with a main span of 1650 m [1], the Yi Sun-Sin Bridge in South Korea, featuring a 1545 m central span [2], or the 1915 Çanakkale Bridge in Turkey with a main span of 2023 m, which is the longest in the [3]. It can be noted that all of them, with main spans above 1500 m, must resort to the twin-box arrangement to meet the critical flutter speed requirements. At a fundamental level, the slot between boxes favours a higher flutter critical wind speed, as

the gap between boxes is intended to decrease the slopes of lift and moment coefficients, reducing aeroelastic coupling between the torsional and vertical modes [4] causing, in general, a decrease in the A_2^* flutter derivative [5].

Research addressing the aeroelastic performance of twin-box cable-supported bridges has mainly focused on parametric variation studies considering a small number of design variables. In [6], a parametric study based on the sensitivity of the flutter derivatives with respect to the gap distance was conducted for the box geometry of the Stonecutters bridge, finding a higher torsional damping ratio for larger gap widths. In a subsequent work [7], the authors studied several configurations with different gap widths and torsion–bending frequency ratios without introducing changes in the box geometry. Another parametric study based on wind tunnel testing is reported in [8], who studied two 20:1 side ratio rectangular cylinders in tandem arrangement, considering different gap distances along with gratings and vertical plates, demonstrating that the A_2^* flutter derivative is the one featuring a dependence on the gap length. More recently, additional design variables have been considered in parametric studies dealing with the flutter performance of cable-supported twin-box deck bridges. In [9], two different wind fairing shapes, symmetric and asymmetric, were considered along with several gap widths to investigate torsional divergence and flutter performance. The main conclusion of this study was that *“the favorable aerodynamic effects of the center slot on bridge decks depend on the aerodynamic shape of the box girders and on the slot widths rather than unconditionally improving the aeroelastic stability”*. Similarly, in [10], the authors adopted an experimental approach to analyse the effect of several vertical central stabilizers (VCSs) for a twin-box deck considering different gap distances. However, in [11], it is shown how minimal changes in the geometry of the deck, such as introducing a cut in the internal lower corner of the boxes facing the gap, produces a strong non-linear response affecting the aeroelastic torsional damping.

The review of the literature addressing the design of twin-box decks based on experimental testing shows the complex relationship between aeroelastic response and the deck shape, remarkably gap distance and box geometry. In addition, the large number of deck shape design variables precludes a thorough analysis based only on wind tunnel testing. As a matter of fact, in the last reference, it is stated *“Considering all the possible combinations of the main geometrical parameters, many configurations should be tested to find the best solution for the project. This would lead to a very large number of experimental tests . . . a common industrial procedure is to quickly sift and select 2 or 3 configurations with better performances, . . .”*. Such heuristic design procedure might provide a feasible design when relying on the previous experience of the design team; however, in the absence of a proper assessment of the full design domain under consideration, there is a true possibility for missing more efficient designs, given the complexity of the interplay between deck geometry and gap distance. This frames the target of this piece of research: improve wind tunnel-based design techniques by proposing a rigorous design framework founded upon the application of validated CFD (Computational Fluid Dynamics) simulations in the frame of surrogate-based design to explore an ample twin-box deck shape design domain, identifying the most efficient candidate designs for the ultimate detailed design of a cable-supported bridge.

A surrogate model might be considered as a “cheap-to-evaluate” function that emulates the “expensive-to-evaluate” continuous quality, cost or performance metric of a product or process. Since the evaluation of the output is so costly, only the output of a limited number of “samples” should be obtained, adopting this sparse set of samples to define an approximation that enables a cheap performance prediction [12]. The evaluation of aerodynamic and aeroelastic responses in wind engineering applications is a complex task, with heavy burdens, associated with experimental tests or CFD simulations.

Consequently, surrogate models have been applied in a relatively limited range of wind engineering applications. In [13], the authors defined a Kriging surrogate in the frame of the shape optimization of the cross-section geometry of high-rise buildings, considering both drag and lift coefficients. A multi-fidelity surrogate model was later presented in [14] for the drag coefficient and the standard deviation of the lift coefficient of

buildings, based on samples evaluated using RANS (Reynolds-averaged Navier–Stokes) and LES (Large Eddy Simulations) simulations. The surrogate model was applied in the context of an aerodynamic multi-objective optimization of tall buildings. In the study of vertical axis wind turbines, a Kriging model was defined, linking the power coefficient with two airfoil design variables, namely the maximum camber and its position [15]. In [16], the authors trained and compared six different neural network (NN) models to obtain surrogates for the aerodynamic loads on wind turbine blades. In [17,18], the multi-objective shape optimization of the cross-sections of tall buildings was addressed, considering a Kriging surrogate model for the aerodynamic response of the buildings, based on Eurocode stipulations.

Turning the focus now to surrogate modelling applications in the aeroelastic response of long span bridges, some of the authors of this work have been active in the surrogate-based optimal design of cable-supported bridges, co-authoring the articles [19–21]. These optimization works relied on the ability of the surrogate model to provide the data required for the evaluation of the aeroelastic responses of interest in the numerical optimization process for the modified designs. This framework was first reported in [22]. In this last reference, two design variables, namely the depth and the width of a box girder, were considered for the aerodynamic characterization of the deck. A set of 15 samples were defined deterministically over the design domain, and the force coefficients were obtained by means of 2D URANS (Unsteady Reynolds-averaged Navier–Stokes) simulations, validating the results of a subset of three designs using wind tunnel data. In a subsequent step, a Kriging surrogate model was defined for the force coefficients and their slopes at a 0° angle of attack, enabling the approximation of the flutter derivatives, adopting the quasi-steady theory. Finally, the surface response for the critical flutter speed over the considered design domain was obtained for two application cases. In [23], the previous framework was extended by including an additional design variable: the gap distance between girders. In that case, the definition of the samples over the three-dimensional design domain (box width, box depth and gap distance) was carried out by applying the random Latin Hypercube Sampling (LHS) method. Afterwards, a Kriging surrogate model was trained using the aerodynamic outputs obtained for 25 samples evaluated using a 2D URANS approach. In the application case, the gap to depth ratio range considered was $G/H = (0.51, 1.86)$ inside region 1, as defined in [24]. Given the relatively short gap-width considered, the flutter derivatives were successfully approximated by the quasi-steady formulation, requiring a correction in the values of the aerodynamic centres.

The present research effort extends the research on this subject by addressing the following limitations in previous studies: (i) In addition to modifications in the width and depth of the individual boxes of the twin-box deck, large gap-widths inside region 2 [24] were considered herein, in order to cover a more general design problem. Targeting large gaps in surrogate-based design is relevant, as experimental parametric studies have considered gap widths inside region 2, and, for instance, the Stonecutters Bridge features a gap to depth ratio of 3.7. (ii) The consideration of large slots between girders has required dropping the quasi-steady assumption for the approximation of the flutter derivatives adopted in previous research. Consequently, forced-oscillation simulations were implemented for the numerical evaluation of the flutter derivatives for the considered set of samples defining the surrogate model. In fact, this approach yields a multidimensional surrogate model for the approximation of the flutter derivatives at 0° angle of attack (AoA), which comprises the following four (4) input variables: box width, box depth and gap distance in the twin-box deck, along with reduced velocity. (iii) Furthermore, the computational burden associated with the forced-oscillation simulations at different reduced velocities for the heave and pitch degrees of freedom is substantially higher than that associated with the evaluation of the force coefficients at a given angle of attack. To the authors' knowledge, forced-oscillation simulations have previously never been adopted in surrogate modelling training. (iv) Finally, the nonlinearity of the aeroelastic response following changes in the deck geometry of twin-box decks has been highlighted by other

researchers without outlining the global patterns; therefore, a surrogate model based on the Radial Basis Function (RBF) has been developed herein in order to identify and shed some light on the general trends in the aeroelastic response.

It is important to note that twin-box decks are prone to vortex-induced vibration (VIV), which is a phenomenon highly dependent on the gap distance between girders [24–28].

The remainder of the article is organized as follows: A brief review of the fundamental formulation is provided, addressing force coefficients, flutter derivatives, Navier–Stokes equations and the Radial Basis Function (RBF) formulation for the definition of the surrogate model. In the next section, the application case is introduced, and the design of experiments methodology based upon the Latin Hypercube Sampling (LHS) method is reviewed. Afterwards, the experimental campaign conducted at the aerodynamic wind tunnel of the University of A Coruña is briefly described and the CFD modelling approach adopted for obtaining the outputs of the samples is explained, focusing on the validation of the force coefficients and the flutter derivatives with wind tunnel data for a subset of the considered samples. Finally, the multidimensional surrogate models obtained for the force coefficients and the flutter derivatives over the whole design domain considered for the twin-box deck are presented and discussed. The paper finishes summarizing the main findings in the conclusions section.

2. Formulation

2.1. Force Coefficients

The time-averaged aerodynamic action on bridge decks is assessed by means of the so-called force coefficients. These are defined as the time-averaged drag, lift and moment per unit of span-length, nondimensionalized by the dynamic pressure and a certain reference length, typically the width of the deck cross-section. The standard deviation is denoted with the symbol $\sim$. The mathematical formula for the time-averaged force coefficients is provided, along with the Strouhal number, as follows:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 B}; \quad C_L = \frac{F_L}{\frac{1}{2}\rho U^2 B}; \quad C_M = \frac{M}{\frac{1}{2}\rho U^2 B^2}; \quad St = \frac{fD}{U}. \quad (1)$$

In the above equation, F_D , F_L and M are the mean drag, lift and torsional moment per unit of span length. B is the total width of the deck, ρ is the air density, U is the reference flow speed and f is the dominant frequency in the lift force spectrum. In Figure 1a, the sign convention adopted in this work is schematically presented.

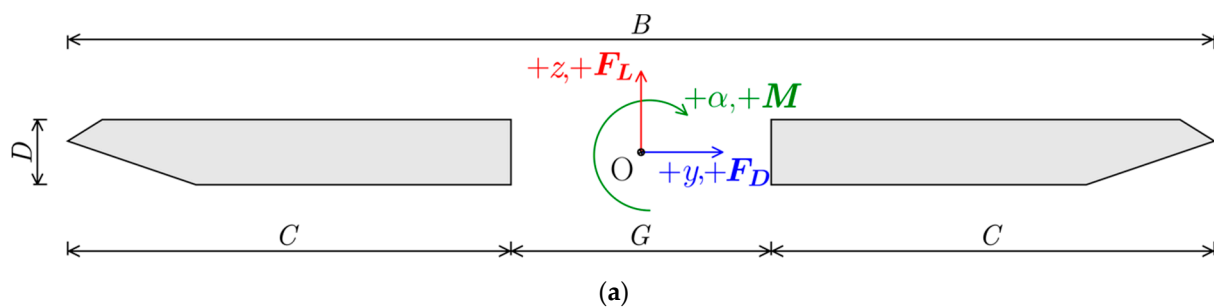


Figure 1. Cont.

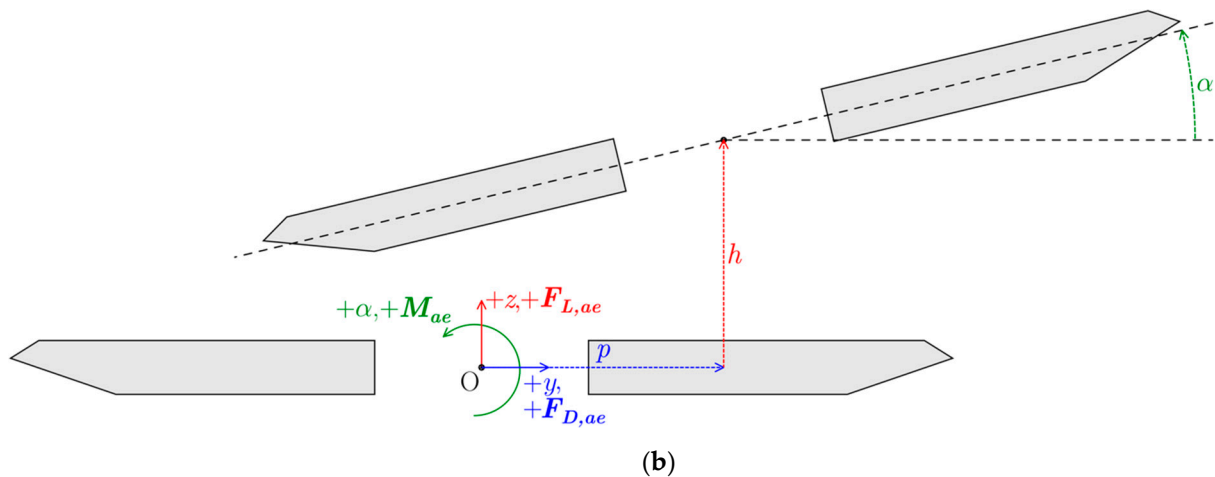


Figure 1. Sign criteria for (a) force coefficients and angle of attack; and (b) for aeroelastic forces and displacements.

2.2. Self-Excited Aeroelastic Forces

The aeroelastic forces acting on a bridge deck modelled as a two degree-of-freedom dynamical system can be expressed as a function of the dynamic pressure, the deck movements and velocities and the flutter derivatives. The formula proposed originally in [29] is as follows:

$$\begin{aligned} F_{L,ae} &= \frac{1}{2}\rho U^2 B \left(KH_1^* \frac{\dot{h}}{U} + KH_2^* \frac{B}{U} \dot{\alpha} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} \right), \\ M_{ae} &= \frac{1}{2}\rho U^2 B^2 \left(KA_1^* \frac{\dot{h}}{U} + KA_2^* \frac{B}{U} \dot{\alpha} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} \right), \end{aligned} \quad (2)$$

where $F_{L,ae}$ and M_{ae} are the self-excited lift force and torsional moment per unit of span length, $K = B\omega/U$ is the reduced frequency, ω is the circular frequency of oscillation, h and α represent the displacements in the heave and pitch degrees of freedom and $\dot{h}$ and $\dot{\alpha}$ are the corresponding time derivatives. In Equation (2), H_i^* and A_i^* ($i = 1, 2, \dots, 4$) are the flutter derivatives, which are empirical parameters dependent on the deck geometry and reduced velocity (U/fB ; f is the frequency of oscillation), which are identified by means of wind tunnel tests and, more recently, by means of CFD simulations [30]. The sign convention is depicted in Figure 1b.

2.3. Fluid Dynamics Numerical Formulation

CFD simulations are adopted in this piece of research as a high-fidelity model for assessing the aerodynamic and aeroelastic outputs of the set of samples used for training the surrogate model, which enables the exploration of the considered design domain. Taking into account the need for an approach capable of balancing computational burden and accuracy, the 2D URANS approach has been selected [31]. The turbulence model adopted is the current standard in bridge engineering applications two-equation $k - \omega$ SST (Shear Stress Transport) [32], based on the Boussinesq assumption, implemented in the open source CFD solver OpenFOAM.

For the simulations in which the deck is static (evaluation of the force coefficients), the URANS formulation, based upon the time-averaging of the conservation of momentum and mass, as described in [33], is adopted. For the forced-oscillation simulations, which are required to obtain the flutter derivatives, the Arbitrary Lagrangian–Eulerian (ALE) approach is applied, to take into account the movements of the mesh in contact with the moving walls of the bridge deck.

2.4. Radial Basis Surrogate Model

To populate the data obtained initially from high-fidelity CFD simulations, a low-fidelity surrogate model named Radial Basis Function (RBF) is adopted in the present

study. The principle behind RBF is to mimic human-like response functions, which are processed through acquired learning and experiences. The radial basis functions used for surrogate training are Gaussian, cubic, multi-quadratic and inverse multi-quadratic; for the present study, the multi-quadratic function is adopted. The mathematical expression for the multi-quadratic RBF is given by the following formula:

$$K(r) = \sqrt{r^2 + p^2}, \quad (3)$$

where the distance measured from the central point is represented as r , while p is a constant shape parameter. The directional independence of the r RBF adds to the mathematical complexity in the modelling between the input and output parameters.

The MATLAB inbuilt function ‘newrb’ is used to develop the surrogate. A set of input and output matrices obtained from the CFD simulations is fed into the model. The other parameters involved are as follows: spread constant, error goal, maximum number of neurons and the number of neurons in the display. The spread constant is chosen as unity so that the response by the neurons to the various input variables is similar and high sensitiveness over specific input variable or variables is avoided. The error goal aims to ensure that the surrogate is efficiently trained while not effecting the accuracy.

3. Design of Experiments

3.1. Choice of Design Domain—Design Variables

The motivation driving this work is to improve the understanding of the role played by the geometry of the twin-box deck in the aerodynamic and aeroelastic performance of cable-supported bridges. Several geometry-related design variables influence the wind-induced response of the twin-box deck; nevertheless, the consideration of a large number of design variables is usually cumbersome. The larger the number of considered design variables, the better the control is over the design of the twin-box deck; but it also introduces the difficulties associated with the management of a larger sample space, thereby increasing the process time and costs along with added complex interdependence between the design variables. Considering the expertise and the experience of the authors in this domain [23] and pondering the associated computational burden for the definition of the surrogate model, three geometric variables have been selected within the scope of the present study. The design variables chosen are the width of each deck, C , the depth of each deck, D , and the gap distance between the two decks, G . A simple schematic sketch highlighting the different design variables is shown in Figure 2. Although the deck appendages, such as the deck barriers, do play a certain role in the bridge design and aerodynamics, these have not been considered in this study, as the main goal is to ascertain the effects caused by the fundamental geometrical features of the twin-box deck.

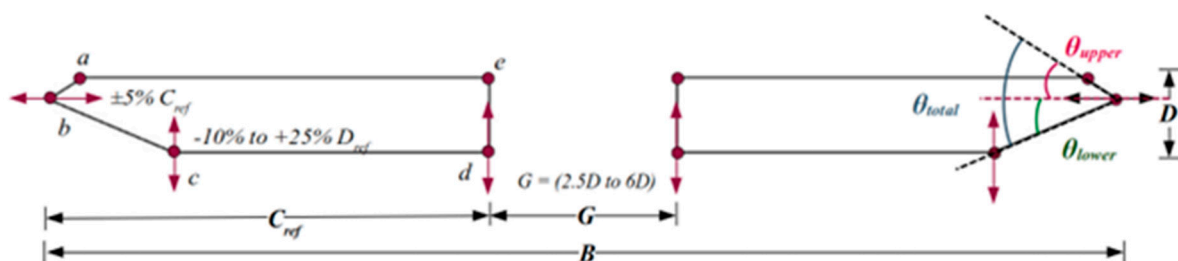


Figure 2. Cross-sectional view of the twin-box deck showing the 3 selected design variables and the chosen limit bounds.

In the present study, a twin-box deck design is always symmetric, thus, a change in one of the boxes is accompanied by a change in the symmetric box. Referring to Figure 2, the width of the deck can be varied by moving horizontally the corner point b . The displacement range of point b is restricted to $\pm 5\%$ of the reference single box width, C_{ref} .

This change in the left box is symmetrically replicated in the right box, thereby changing the total width of the twin-box deck. The depth of the deck can be modified by moving the points c and d simultaneously in the vertical direction. The lower limit is set as -10% , while the upper limit is set as $+25\%$ of the reference depth D_{ref} . The third variable considered herein is the gap distance between the two boxes. In [23], it was highlighted how the gap between the two boxes impacts the overall performance of the bridge. However, in those studies, the gap distance was restricted to relatively low values, setting the upper limit for G/D as 1.86. As this work focuses on larger gaps, the gap-to-depth ratio range considered in the present study is between 2.5 and 6, respectively. With the modifications in the positions of corner points b , c and d , there occurs a natural change in the side-corner angle, which remarkably influences the aerodynamic characteristics of the bridge. The total side-corner angle θ_{total} is the sum of the upper θ_{upper} and lower θ_{lower} corner angles as depicted in Figure 2. For carrying out the experimental as well as numerical studies, a geometric scale ratio of 1:70 has been adopted in the present study. The geometrical design variables for the model at scale are listed in Table 1.

Table 1. Design space with lower and upper bounds.

Variable	Symbol	Ref. Value	Lower Bound	Upper Bound
Width	C	$C_{\text{ref}} = 200 \text{ mm}$	$-5\% C_{\text{ref}}$	$+5\% C_{\text{ref}}$
Depth	D	$D_{\text{ref}} = 0.143 C_{\text{ref}}$	$-10\% D_{\text{ref}}$	$+25\% D_{\text{ref}}$
Gap/depth	G/D		2.5	6

3.2. Latin Hypercube Sampling

The design space created by the three design variables selected is certainly large; hence, an efficient and reliable method should be adopted to ensure that the sample points for the training of the surrogate model cover the whole design region. In the present study, the Latin Hypercube Sampling technique [34] has been applied to distribute the sample points evenly over the entire design space.

The fifteen unique sample points generated are shown in Figure 3, which presents a set of four sub-figures. In Figure 3a, a three-dimensional representation of the sample points is shown by black dots in a non-dimensionalized parametric design space. The bounds for each of the axis, associated with each geometric design variable, are as indicated in Table 1. Next, in Figure 3b, the sample points are projected over a two-dimensional space of C/C_{ref} versus D/C_{ref} , where both design variables have been non-dimensionalized by C_{ref} . Similarly, in Figures 3c and Figure 3d, the sample points have been plotted on the two-dimensional space with axes of C/C_{ref} versus G/C_{ref} and D/C_{ref} versus G/C_{ref} , respectively.

In order to obtain the data required for the training of the surrogate model, 2D URANS numerical simulations are performed for each of the fifteen sample points. With the purpose of validating the CFD results for the force coefficients and flutter derivatives with experimental data, prototypes of the twin-decks are built by means of additive manufacturing, including rapid 3D printing, and then experiments are performed in the wind tunnel. However, performing experiments on all of the fifteen points is expensive, considering the cost of the prototypes as well as the costs involved with the wind tunnel operation. Hence, from the fifteen selected samples, a subset of three samples is chosen to be tested in the wind tunnel. In Figure 3, these three chosen deck designs have been marked by squares. The selected three points are case g01, case g04 and case g07 and, as shown in the figures, these points are distributed evenly over the design space.

Table 2 shows the main geometrical features of the 15 samples, along with the corresponding corner angles, while, in Figure 4, the cross-section geometry of the set of samples adopted for the definition of the surrogate model are plotted (due to the symmetry of the cross-section geometry, only the left boxes are provided, along with the symmetry axis for reference). Based on the ample range of geometries considered, Figure 4 clearly depicts

the amplitude of the design domain that is being addressed in this application case and, therefore, the potential for surrogate-based aerodynamic and aeroelastic design.

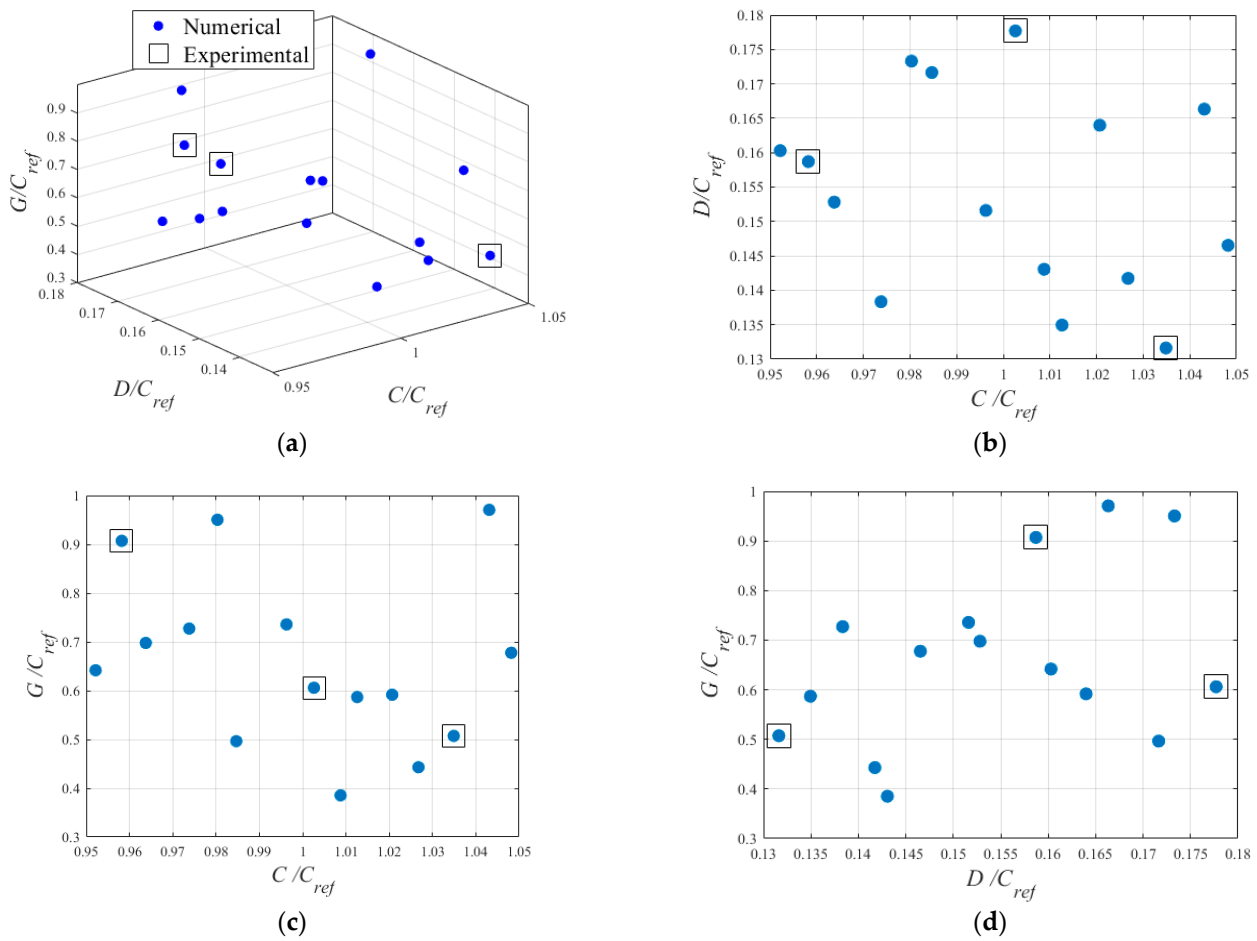


Figure 3. Samples definition in the geometric parametric space (a) 3D view, (b) projection on the D/C_{ref} vs. C/C_{ref} plane, (c) projection on the G/C_{ref} vs. C/C_{ref} plane and (d) projection on the G/C_{ref} vs. D/C_{ref} plane.

Table 2. Geometrical definition of the set of samples considered for the definition of the surrogate models.

Sample	C/C_{ref}	D/C_{ref}	G/C_{ref}	θ_{up}	θ_{down}	θ_{total}
g01	1.00	0.18	0.61	31.50	23.37	54.87
g02	0.96	0.15	0.70	49.46	21.83	71.29
g03	1.01	0.14	0.39	29.67	17.15	46.82
g04	1.03	0.13	0.51	23.70	13.98	37.68
g05	0.98	0.17	0.50	38.14	23.66	61.80
g06	1.03	0.14	0.44	25.31	16.01	41.32
g07	0.96	0.16	0.91	53.34	23.41	76.75
g08	1.02	0.16	0.59	26.64	19.98	46.61
g09	1.05	0.15	0.68	21.46	15.80	37.26
g10	1.04	0.17	0.97	22.27	19.10	41.37
g11	0.95	0.16	0.64	58.05	24.23	82.28
g12	0.97	0.14	0.73	43.44	18.33	61.77
g13	0.98	0.17	0.95	40.13	24.29	64.42
g14	1.01	0.14	0.59	28.63	15.54	44.17
g15	1.00	0.15	0.74	33.60	19.36	52.96

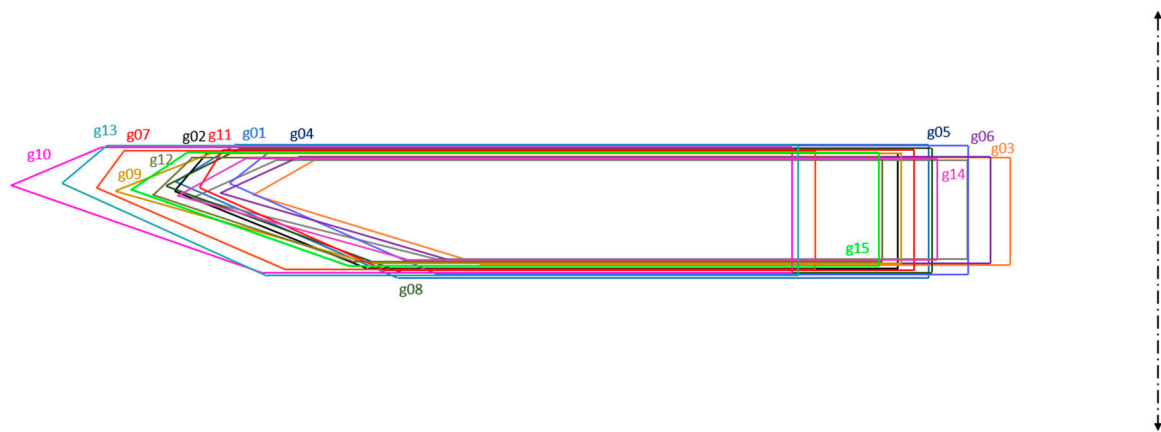


Figure 4. Geometrical definition for the left box of the 15 samples considered for the surrogate model definition (note the symmetry axis, providing the reference for the associated gap distance).

3.3. Surrogate Modelling Approach

The aerodynamic and aeroelastic responses of the fifteen sample points (g_i ; $i = 1, \dots, 15$), whose geometry was introduced above, are numerically simulated using CFD techniques. Surrogate models, often called by other names such as approximation models, metamodels or response surface models, are widely used numerical approaches which provide approximate results for design points inside the design space. Surrogate modelling involves training the model to generate analytical relationships between the inputs and outputs, thereby enabling the model to reproduce and replicate the results for any point inside the design space.

This approach, however, cannot completely replace experiments or a high-fidelity CFD methodology, as the surrogate model requires the initial population, or the initial data, to frame the analytical relationship between the inputs and the outputs. Therefore, in the present study, the initial dataset of aerodynamic and aeroelastic outputs is generated using the CFD-based methodology for the complete set of 15 samples, which is first verified and afterwards validated using experimental data for a subset of three samples. The realizations for the complete set of 15 samples are then fed into the surrogate model to populate the data over the entire design space. The combination of the high-fidelity time-consuming CFD techniques, along with the cheap and quick low-fidelity surrogate techniques, is sometimes also referred to as hybrid modelling.

In the present work, the surrogate model aims to evaluate the response function for the force coefficients and the flutter derivatives of the twin-box deck by assuming a smooth and continuous variation over the entire design space. The surrogate technique adopted in the present study is the radial basis function, whose formulation was introduced in Section 2.4, which is trained based on the outputs from the set of 15 samples introduced in Section 3.2. Consequently, for the definition of the surrogate model for the force coefficients, 45 CFD simulations will be completed for the static twin-box deck, corresponding to 15 different geometries, whose aerodynamic response is numerically evaluated at three different angles of attack. Similarly, for the evaluation of the flutter derivatives, the set of 15 different deck geometries are considered as samples, and the aeroelastic responses are obtained for two different degrees of freedom, heave and pitch at five different reduced velocities, which yields a total of 150 forced-oscillation simulations. The overall number of CFD simulations to complete is therefore 195, which is burdensome, although manageable when adopting a relatively inexpensive 2D URANS approach, but this would be intractable with a higher-fidelity CFD model such as 3D LES (Large Eddy Simulation), due to the remarkably larger computational demands.

4. Wind Tunnel Testing

In Section 3.2, it was mentioned that a subset of three deck designs among the considered set of samples for the surrogate model training were selected to be tested in the wind tunnel. In this manner the CFD simulations can be validated with experimental data.

The wind tunnel campaign was conducted at the aerodynamic wind tunnel of the University of A Coruña, which is a blown-type open circuit class facility, featuring a 1 by 1 m² test cross-section. The sectional models were manufactured at a geometric scale of 1/70, obtaining the required stiffness by placing one aluminium bar at the core of each box girder. In order to prevent end-effects, large circular end-plates were affixed to the ends of the sectional models.

The wind tunnel tests were conducted in smooth flow. For the evaluation of the force coefficients, the sectional model was connected to the force balances by means of stiff bars, measuring the aerodynamic forces. In Figure 5, general and more detailed views of the sectional models in the test chamber are provided. The force coefficients were obtained for the fixed models at a Reynolds number, Re , of 1.2×10^5 , after checking the insensitivity of the results for $Re > 8.0 \times 10^4$. The experimental values in the range $(-6^\circ, 6^\circ)$ are reported in Figure 7 for cases g01, g04 and g07.

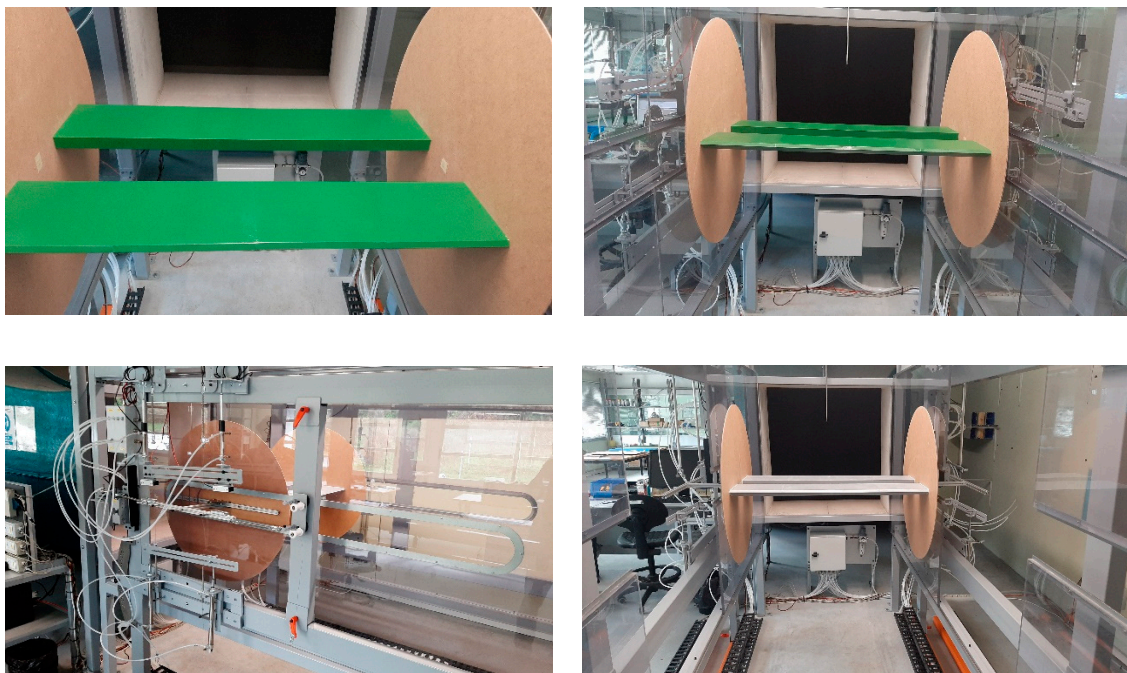


Figure 5. Images of models g01 and g04 placed in the aerodynamic wind tunnel of the University of A Coruña.

For the evaluation of the flutter derivatives, the sectional model was elastically supported by springs, behaving as a three degree-of-freedom dynamical system. For the identification of the flutter derivatives, the sectional model was released out of the balance position at different flow speeds. The Iterative Least Squares method (ILS), proposed by [35], was adopted for obtaining the experimental flutter derivatives from the model's displacement time-histories at different reduced velocities. The experimental flutter derivatives obtained for cases g01, g04 and g07 are reported in Figures 8–10.

5. Numerical Modelling

5.1. General Numerical Setup

2D URANS simulations, in combination with the $k - \omega$ SST turbulence model, are solved using the opensource CFD software OpenFOAM v8 (The OpenFOAM Foundation

distribution) for evaluating the aerodynamic and aeroelastic responses of the considered set of 15 samples. The Reynolds number for the CFD simulations is the same as the one considered for the experiments performed in the wind tunnel for validation: $Re = 1.2 \times 10^5$. For all 15 sample points, the CFD simulations are first simulated for the static deck at 0° AoA. To be computationally efficient, these simulations are later used as the initialization for the forced oscillation simulation that enables the evaluation of the flutter derivatives at different reduced velocities. It should be noted that a subset of three samples, namely cases g01, g04 and g07, were tested in the wind tunnel to provide validation for the numerical results (see Section 5).

The flow domain adopted for the CFD simulations has a length of $40C_{ref} + B$ in the streamwise direction and of $30C_{ref} + D$ along the directional orthogonal to the undisturbed flow. The windward corner of the twin-box deck is placed at $15C_{ref}$ from the inlet at mid height in the flow domain. Higher density mesh regions are created near the twin-deck box and in the wake region, while, away from the deck, coarser meshes have been used by creating different blocks. The meshing strategy adopted in the work is similar to the research group's previous work [23]. A uniform inlet speed with a turbulence intensity of 1% and a length scale of $0.1C_{ref}$ has been specified as the inlet boundary condition, whereas the outlet boundary condition is set as an atmospheric pressure outlet. It should be noted that corners b , d and e in Figure 1 are considered sharp in the CFD model, while corner a is modelled as slightly rounded, with a radius relative to the reference box width $R/C_{ref} = 0.008$, and corner c is modelled adopting a degree of sharpness ratio, R/C_{ref} , of 0.015. This modelling decision was taken in order to aim for better representation of the expected degree of sharpness in 3D-printed sectional models for streamlined twin-box decks.

For the time discretization, the backward scheme is adopted, based on the central differencing methodology. The gradient terms have been discretized using a second order Gauss linear scheme and the divergence terms (for U , k , as well as ω) have also been discretized using the Gauss scheme with a linear upwind scheme.

5.2. Sensitivity of Numerical Outputs with the Spatial and Temporal Discretizations

One of the key activities, which aims to provide confidence in CFD-based results, is to assess the sensitivity of numerical outputs depending on the levels of spatial and time discretization. To address this task, the g01 sample at a 0° angle of attack (AoA) was selected and three different levels of mesh refinement (coarse, medium and fine) were considered, comparing the values obtained for the force coefficients, the standard deviations of the force coefficients and the Strouhal number. For the three meshes, the characteristics of the boundary layer mesh region around the boxes are the same: a first layer mesh height, y_1/C , of 2.42×10^{-4} , an inflation ratio, r_{bl} , of 1.104, an element size along the wall, x_1/C , of 9.69×10^{-4} and an overall number of layers of 15. The results are reported in Table 3, which shows a minor impact of the mesh refinement, with only small changes in the absolute value of the lift coefficient and the Strouhal number, with consistent results for the medium and fine meshes. Based on this spatial discretization sensitivity study, the fine mesh characteristics are retained for the CFD simulations that are reported next.

Table 3. Force coefficients and standard deviation of the force coefficients for the g01 sample at 0° AoA considering different levels of mesh resolution (maximum Courant number: 1).

Mesh	# Cells	C_D	C_L	C_M	$\widetilde{C_D}$	$\widetilde{C_L}$	$\widetilde{C_M}$	St
Coarse	166,680	0.060	−0.071	0.015	0.011	0.089	0.022	0.079
Medium	200,460	0.062	−0.014	0.021	0.012	0.106	0.026	0.137
Fine	319,467	0.061	−0.010	0.021	0.012	0.106	0.025	0.134

Furthermore, the impact of the time-step resolution is further analysed by considering different levels of maximum Courant number in the time-marching simulations, also

for the g01 sample at 0° AoA and with fine mesh. The values obtained for the time-averaged, fluctuating force coefficients and Strouhal number are reported in Table 4 for maximum Courant numbers of 4, 2, 1 and 0.5, finding minor differences, remarkably, between maximum Courant numbers of 1 and 0.5; therefore, a maximum Courant number of 1 is retained for the simulations reported in the next sections.

Table 4. Force coefficients and standard deviation of the force coefficients for the g01 sample at 0° AoA, considering different levels of time discretization.

Max Co	C_D	C_L	C_M	$\widetilde{C_D}$	$\widetilde{C_L}$	$\widetilde{C_M}$	St
0.5	0.060	−0.014	0.021	0.012	0.110	0.025	0.139
1.0	0.061	−0.010	0.021	0.012	0.106	0.025	0.134
2.0	0.064	0.005	0.020	0.012	0.098	0.026	0.125
4.0	0.071	0.030	0.018	0.012	0.078	0.030	0.110

In Table 5, for case g01 with fine mesh and a maximum Courant number of 1, the mean and maximum values obtained for the y^+ distribution around the boxes are reported ($y^+ = (y_1 u_*)/v$; y_1 is the height of the first prismatic grid layer and u_* is the friction velocity). For the two deck boxes, the maximum values in Table 5 are below the generally accepted limit of eight, meeting the requirements for the low Reynolds wall modelling approach adopted herein.

Table 5. y^+ values for case g01, with fine mesh and a maximum Courant number of 1.

	$\overline{y^+}$	$y_{\max}^+$	% $y^+ > 2$	% $y^+ > 4$	% $y^+ > 6$
Windward box	2.02	6.99	36.81	1.57	0.13
Leeward box	0.79	5.09	0.65	0.08	0

In Figure 6, several images of the fine finite volume mesh for case g01 are depicted, where the refined wake region and the boundary layer mesh around the deck can be perused. In Table 6, the fundamental characteristics of the finite volume meshes for the 15 considered geometries are reported. It should be noted that the difference in the overall number of cells is related to the changes in the geometry of the samples, as the size of the flow domain depends on the deck total width B and depth D .

Table 6. Characteristics of the finite volume meshes for the set of 15 samples (y_1/C is the first layer mesh height, x_1/C is the first layer element size along the wall, n_{bl} is the number of layers in the boundary layer (bl) mesh, r_{bl} is the inflation ratio in the bl mesh and h_{bl}/C is the total height of the bl mesh).

Case	No. Elements	y_1/C	x_1/C	n_{bl}	r_{bl}	h_{bl}/C
g01	31,9467	2.42×10^{-4}	9.69×10^{-4}	15	1.104	7.95×10^{-3}
g02	294,175	2.52×10^{-4}	1.01×10^{-3}	15	1.104	8.27×10^{-3}
g03	356,262	2.41×10^{-4}	9.63×10^{-4}	15	1.104	7.90×10^{-3}
g04	368,172	2.35×10^{-4}	9.39×10^{-4}	15	1.104	7.70×10^{-3}
g05	362,604	2.47×10^{-4}	9.87×10^{-4}	15	1.104	8.10×10^{-3}
g06	368,871	2.37×10^{-4}	9.46×10^{-4}	15	1.104	7.76×10^{-3}
g07	295,849	2.53×10^{-4}	1.01×10^{-3}	15	1.104	8.32×10^{-3}
g08	331,608	2.38×10^{-4}	9.52×10^{-4}	15	1.104	7.81×10^{-3}
g09	306,320	2.32×10^{-4}	9.27×10^{-4}	15	1.104	7.61×10^{-3}
g10	308,364	2.33×10^{-4}	9.31×10^{-4}	15	1.104	7.64×10^{-3}
g11	290,430	2.55×10^{-4}	1.02×10^{-3}	15	1.104	8.37×10^{-3}
g12	295,644	2.49×10^{-4}	9.98×10^{-4}	15	1.104	8.19×10^{-3}
g13	298,393	2.48×10^{-4}	9.91×10^{-4}	15	1.104	8.13×10^{-3}
g14	334,589	2.40×10^{-4}	9.59×10^{-4}	15	1.104	7.87×10^{-3}
g15	296,766	2.44×10^{-4}	9.75×10^{-4}	15	1.104	8.00×10^{-3}

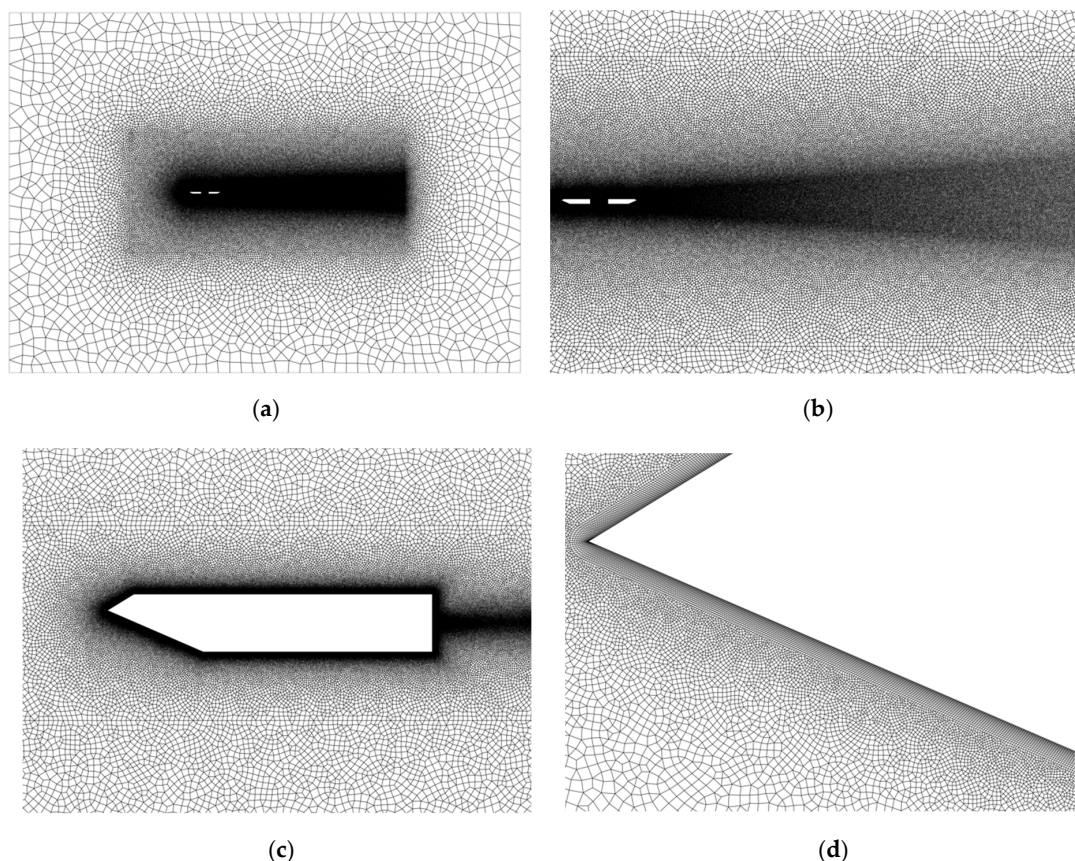


Figure 6. Fine mesh case g01: (a) complete flow domain, (b) refinement around the deck in the wake, (c) detail of mesh around the windward box and (d) detail of the boundary layer mesh around the deck.

6. Validation of CFD-Evaluated Force Coefficients and Flutter Derivatives

Standard practice in CFD modelling requires the validation of the computational results with available equivalent experimental data, aiming to assess the level of accuracy provided by the CFD simulations for the considered phenomena. To this end, three representative samples out of the set of the 15 samples adopted for the definition of the surrogate models were selected to be tested in the wind tunnel of the University of A Coruña, enabling a comparison between CFD-based and experimental force coefficients and flutter derivatives. The selected samples were g01, g04 and g07, whose geometrical definitions can be found in Table 2. These comparisons are reported next. Uncertainty in the evaluation of the force coefficients and flutter derivatives depending on the apparent AoA and incoming turbulent intensity has been previously studied in [36] for twin-box decks, finding higher uncertainty in the mean lift coefficient and in the high reduced velocity range of the flutter derivatives.

6.1. Force Coefficients

For the three geometries previously introduced, three different angles of attack have been considered in the CFD simulations: -2° , 0° and 2° . The evaluation of the force coefficients at these angles of attack enables the computation of the slopes of the force coefficients depending on the AoA, which are required, for instance, for the assessment of the buffeting response of the bridge [37]. The numerical and experimental results are reported in Figure 7, where the moment coefficient is given as the right secondary axis in the charts. The numerical simulations are capable of reproducing the trend in the force coefficients, as shown by the experimental values in the $(-6^\circ, 6^\circ)$ range, although some differences of small absolute value can be identified. The lower values obtained for drag

coefficients are due to the higher drag obtained in the wind tunnel tests due to the large end-plates in the sectional model (see Figure 5). Furthermore, the larger discrepancies that can be identified in the drag coefficient for the case g01 at -2° AoA and in the lift coefficient at $+2^\circ$ AoA for case g07 might be related to the subtle effect caused by the non-perfectly sharp corners of the sectional models tested in the wind tunnel, as these were manufactured using 3D printing; minor misalignments between girders slightly modify the exposure of the leeward box and consequently, the mean pressure distribution around the boxes between the experimental and the CFD models.

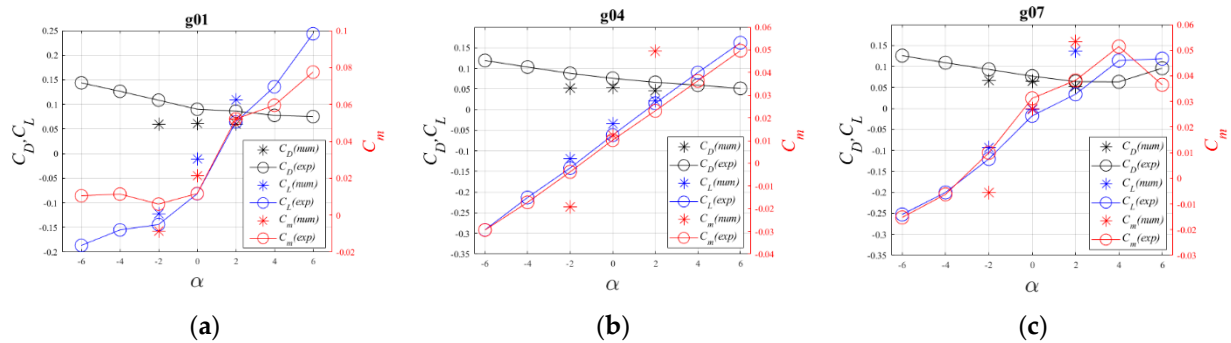


Figure 7. Experimental and CFD force coefficients vs. AoA for samples (a) g01, (b) g04 and (c) g07.

6.2. Flutter Derivatives

The same three samples were tested in the wind tunnel to obtain their flutter derivatives for the 0° AoA, and the experimental values are compared with the CFD-based forced-oscillation simulations. For the numerical evaluation of the flutter derivatives, the forced-oscillation simulations were conducted separately for the heave and pitch degrees of freedom, following the procedure in [31]. The amplitude of the forced oscillations in the pitch degree of freedom was 3° , while, in the heave degree of freedom, an amplitude of $h_0/D_{ref} = 0.7$ was adopted. The velocity of the incoming flow was adjusted within the range (3.7 m/s, 21.3 m/s) and the frequency of oscillation within the range (3.2 rad/s, 36 rad/s), aiming at covering a range of reduced velocities up to $U_r = U/(fB) = 20$.

In Figure 8, the experimental and CFD values for the flutter derivatives H_i^* and A_i^* , $i = 1, 2$ and 3, are reported for sample g01. It can be noted that the experimental values were obtained with the adoption of two different initial pitch angles at release.

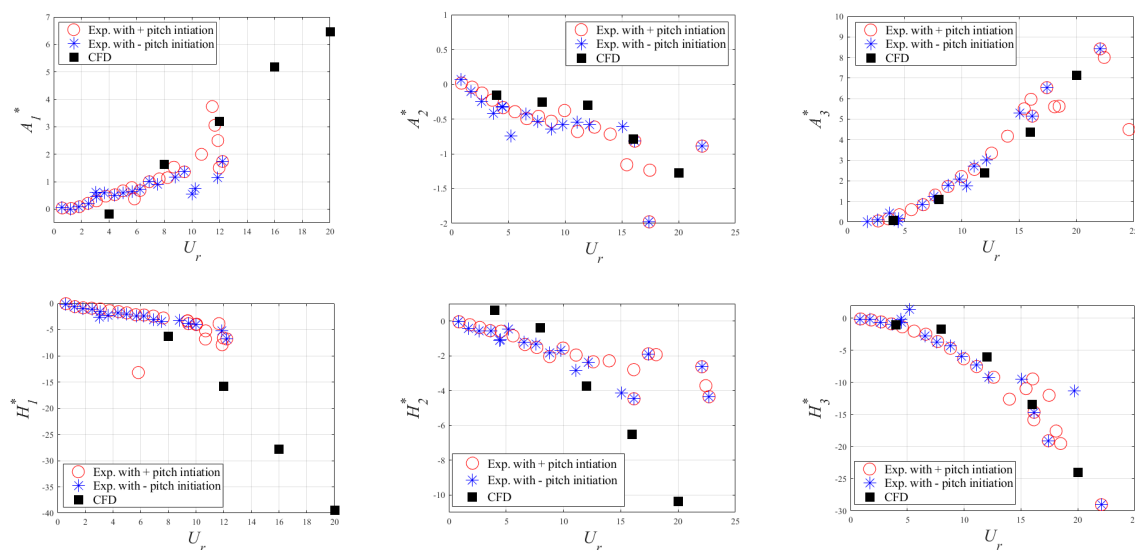


Figure 8. Experimental and CFD-evaluated flutter derivatives at 0° AoA for sample g01.

Similarly, in Figures 9 and 10, the experimental and numerical results for the g04 and g07 samples are reported. The general agreement between experimental and numerical values is good, presenting some differences at high reduced velocities, at which the experimental identification is challenging due to the very short time-histories of the sectional model displacement signal, remarkably for the A_2^* and H_2^* flutter derivatives.

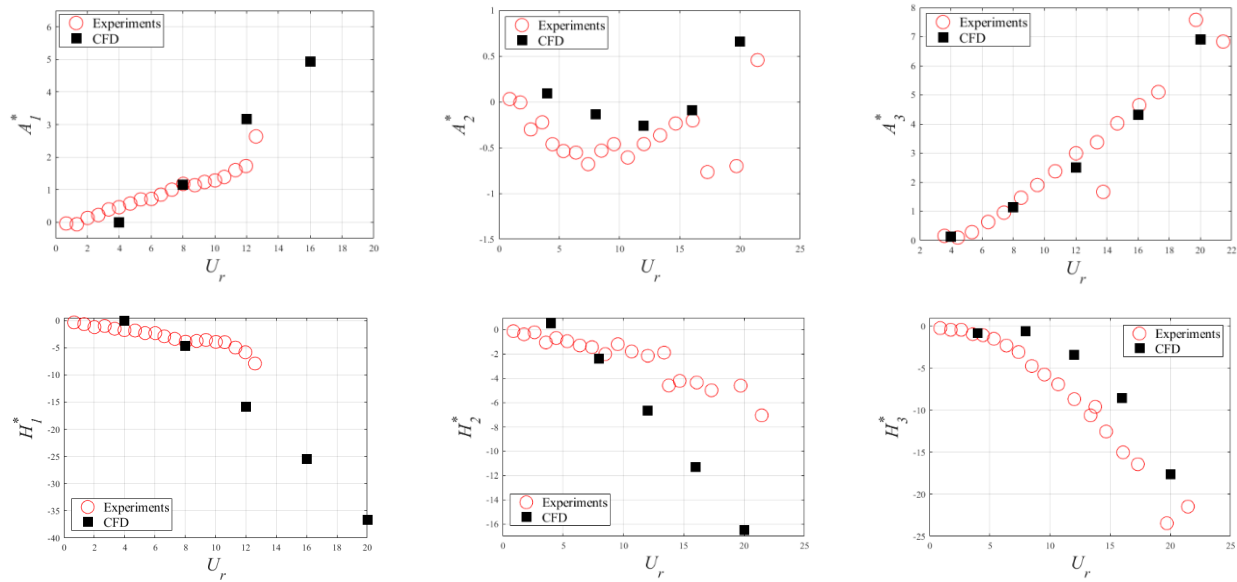


Figure 9. Experimental and CFD-evaluated flutter derivatives at 0° AoA for sample g04.

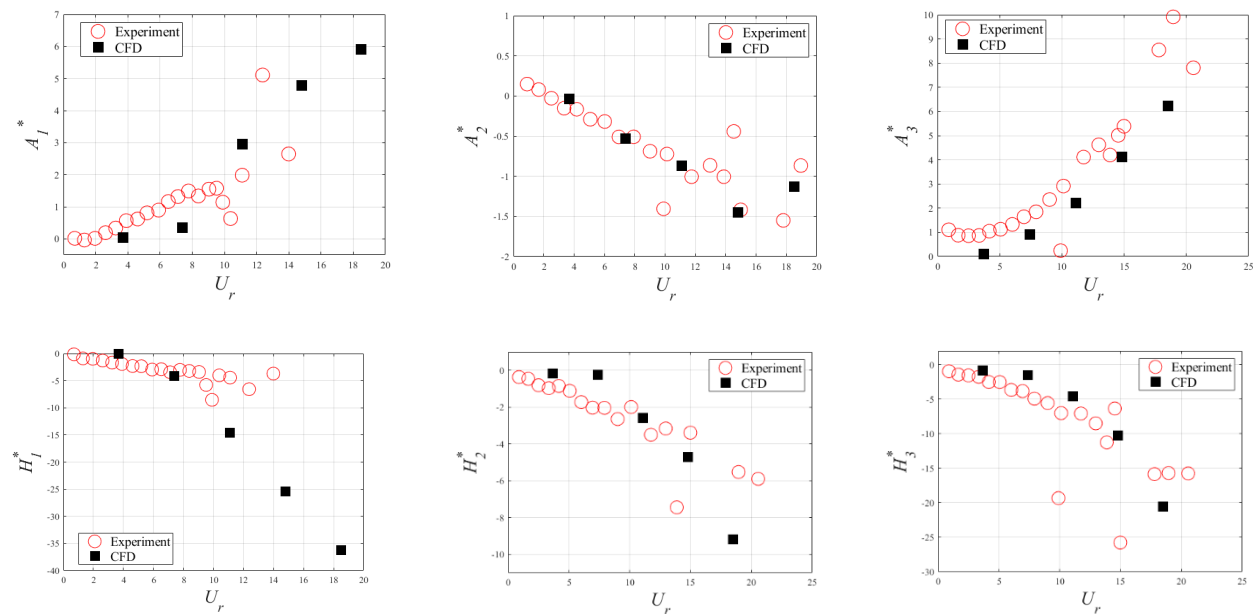


Figure 10. Experimental and CFD-evaluated flutter derivatives at 0° AoA for sample g07.

7. Aerodynamic and Aeroelastic Surrogate Models

7.1. Surrogate Model of the Force Coefficients

The surrogate model based on the radial basis method is trained by adopting the force coefficients obtained from the 2D URANS simulations for the set of 15 geometries covering the selected design domain. The input design variables are the width, depth and gap distance of the twin-box deck, as described in Table 2, along with the angle of attack in the range $(-2^\circ, +2^\circ)$. In Table 7, the time-averaged force coefficients obtained numerically for

the set of 15 samples considered in the definition of the surrogate model are reported. The application of the RBF surrogate enables the approximation of the drag, lift and moment coefficients for any angle of attack in the range $(-2^\circ, 2^\circ)$ and for any geometry within the considered deck shape design domain; therefore, a quantitative linkage is established between the twin-box deck geometry and angle of attack with the force coefficients. In the RBF model, the maximum number of neurons is set to 15, similar to the number of initial designs, and the number of display neurons is set to 25.

Table 7. CFD realizations of the time-averaged force coefficients for the 15 samples considered in the definition of the surrogate model.

Case	Angle of Attack								
	-2°			0°			$+2^\circ$		
	C_D	C_L	C_M	C_D	C_L	C_M	C_D	C_L	C_M
g01	0.060	−0.12	−0.01	0.061	−0.01	0.02	0.059	0.11	0.05
g02	0.061	−0.13	−0.01	0.061	−0.03	0.03	0.050	0.11	0.06
g03	0.052	−0.18	−0.02	0.053	−0.09	0.02	0.051	0.00	0.05
g04	0.051	−0.12	−0.02	0.054	−0.03	0.01	0.045	0.02	0.05
g05	0.060	−0.13	−0.01	0.066	−0.03	0.02	0.066	0.09	0.06
g06	0.047	−0.16	−0.02	0.051	−0.06	0.01	0.049	0.02	0.05
g07	0.066	−0.09	−0.01	0.064	0.00	0.03	0.052	0.14	0.05
g08	0.053	−0.14	−0.01	0.054	−0.04	0.02	0.052	0.10	0.05
g09	0.056	−0.13	−0.02	0.052	−0.05	0.02	0.050	0.03	0.05
g10	0.064	−0.07	−0.01	0.062	−0.01	0.02	0.059	0.07	0.05
g11	0.064	−0.13	0.00	0.064	−0.02	0.03	0.054	0.14	0.06
g12	0.057	−0.13	−0.01	0.055	−0.04	0.02	0.054	0.08	0.06
g13	0.069	−0.08	−0.01	0.067	0.00	0.02	0.064	0.10	0.05
g14	0.047	−0.16	−0.02	0.047	−0.08	0.02	0.046	0.02	0.05
g15	0.059	−0.12	−0.01	0.058	−0.03	0.02	0.055	0.07	0.05

In Figure 11, the graphical representation of the surrogate model for the drag coefficient is depicted as a matrix of charts over the range of C/C_{ref} and D/D_{ref} considered in this study, while constant values are adopted for the angle of attack and G/D_{ref} inputs, defining the rows and columns of the matrix representation. It is to note that this criterium is adopted to aim to efficiently represent this surrogate model comprising four input variables. It can also be noted that larger gap-to-depth ratios provide higher drag coefficient, as a longer slot facilitates the flow through the gap, increasing the drag contribution of the downwind box panel facing the gap. Furthermore, narrower boxes (smaller C/C_{ref}) are linked to larger drag coefficients, although this is mainly related to the smaller value adopted as the reference dimension in Equation (1) for the calculation of the drag coefficient, while the overall drag action is roughly unchanged. It is noted that, in [23], a similar trend was identified for short gap twin-box decks. As expected, smaller box depths produce lower drag coefficients, as the exposed deck area facing the incoming flow is decreased. The impact of the angle of attack in the drag coefficient is minor for the studied narrow range of AoA around 0° .

Similarly to the drag coefficient, the graphical representation of the lift coefficient provided by the RBF surrogate model over the considered design domain is depicted in Figure 12. In this case, for G/D_{ref} values between 2.5 and 4.5, a minimum in the lift coefficient can be identified for values of $C/C_{\text{ref}} \cong -0.015$ and $D/D_{\text{ref}} \cong -0.05$. In general, shorter gaps show stronger changes in the lift coefficient, depending on the geometry of the boxes. The overall lift coefficient pattern changes for larger G/D_{ref} ratios, as for $G/D_{\text{ref}} \cong 5.5$ only minor changes may be identified over the full range of C/C_{ref} and D/D_{ref} values (box geometry), with the minimum of the lift coefficient in the vicinity of $C/C_{\text{ref}} = 0.05$ and $D/D_{\text{ref}} = 0.05$.

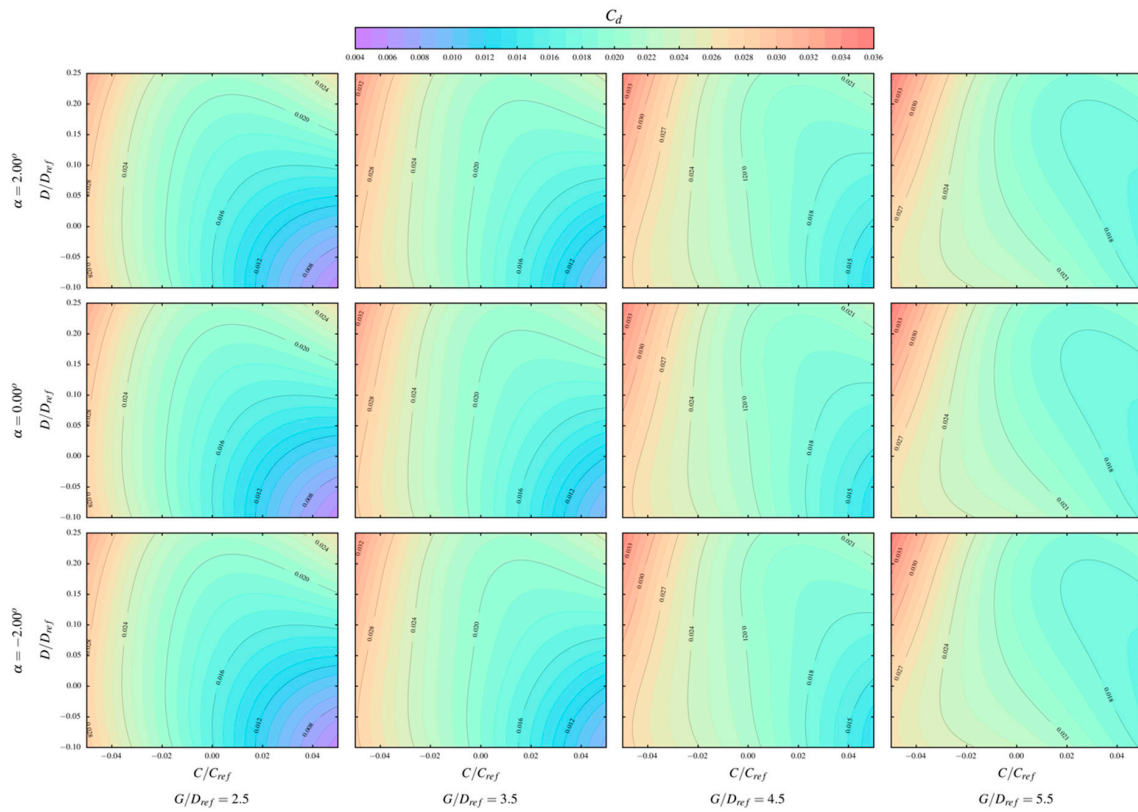


Figure 11. Graphical representation of the surrogate model of the drag coefficient over the considered design domain for the twin-box deck.

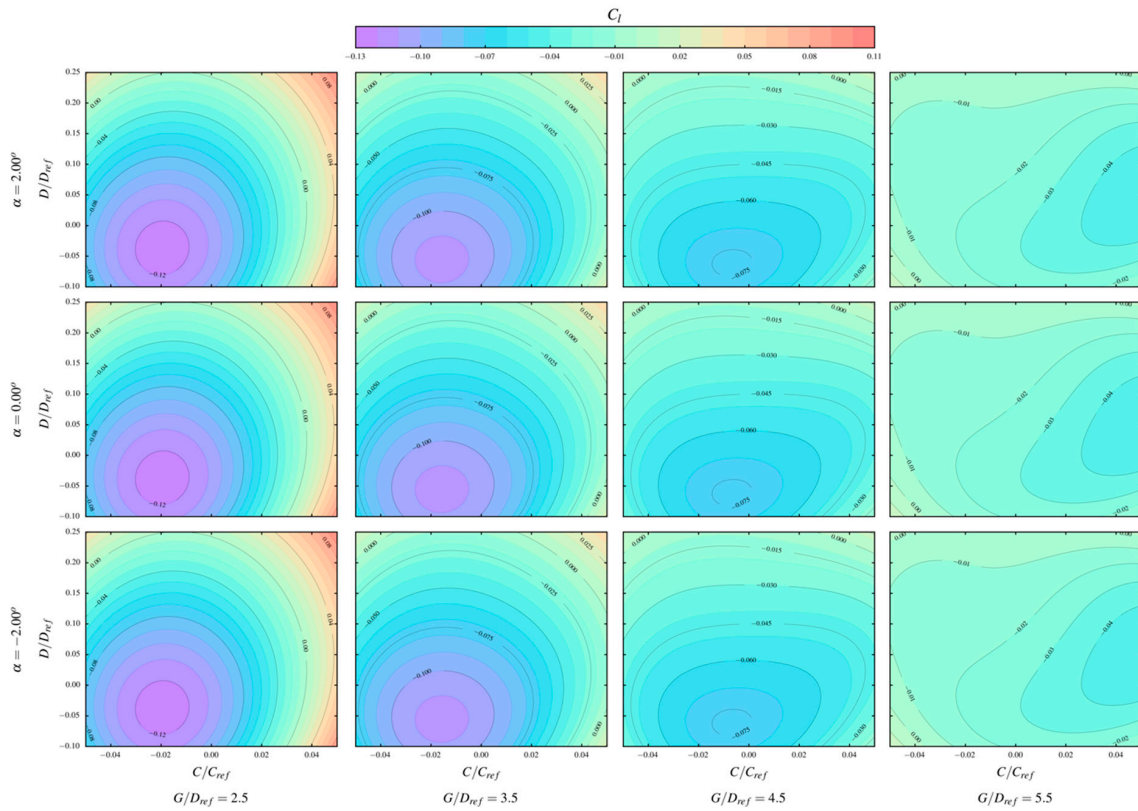


Figure 12. Graphical representation of the surrogate model of the lift coefficient over the considered design domain for the twin-box deck.

Finally, in Figure 13, the output of the surrogate model for the moment coefficient is presented, adopting the same criteria as in the previous figures in this section. For the moment coefficient, there is a clear trend associated with high D/D_{ref} and low C/C_{ref} values featuring relatively high values of moment coefficients, with the minimum values obtained for low D/D_{ref} and high C/C_{ref} , within the studied design domain. A similar pattern relating moment coefficient and box geometry was also identified for short-gap twin-box decks in [23].

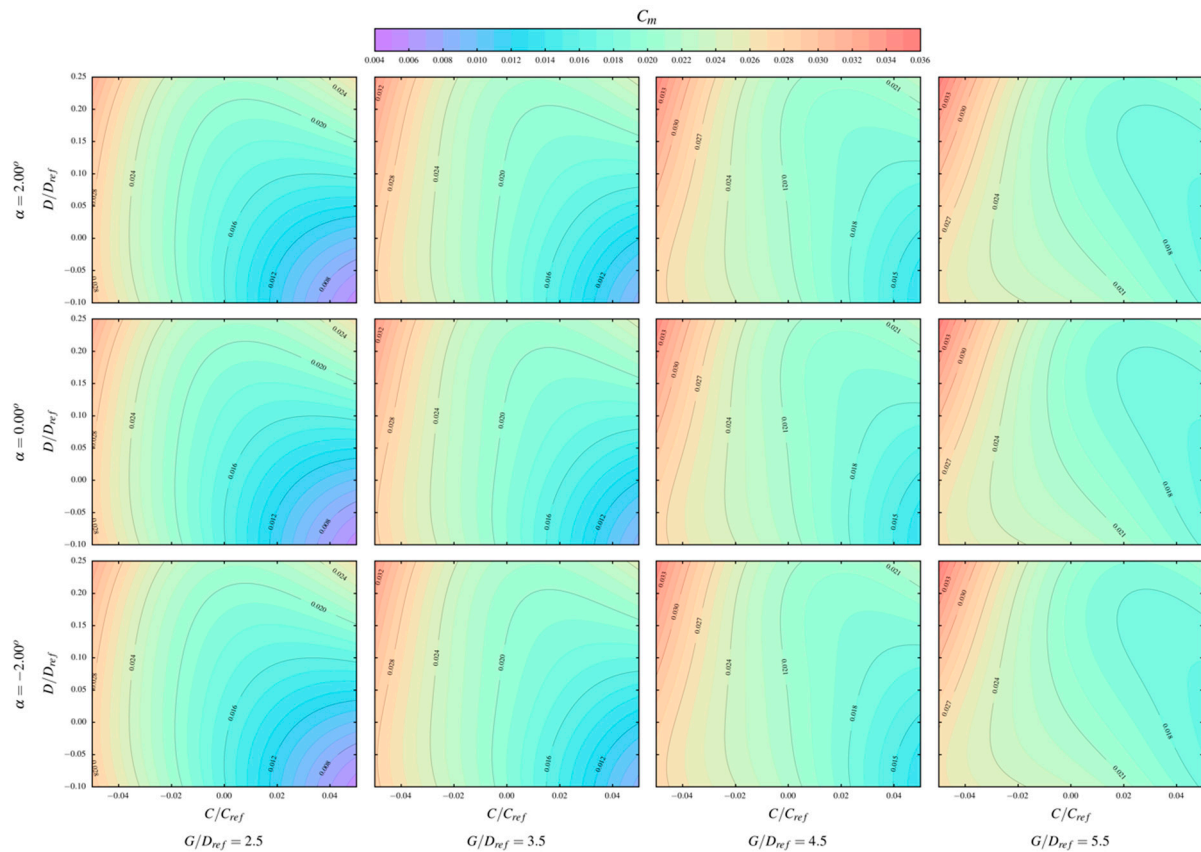


Figure 13. Graphical representation of the surrogate model of the moment coefficient over the considered design domain for the twin-box deck.

At a general level, the charts in Figures 11–13 show that, for a given gap distance (G/D_{ref}), the angle of attack plays a minor role in the relationship between the boxes' width and depth (D/D_{ref} and C/C_{ref}) and the force coefficients. On the contrary, it can be appreciated that the gap distance (G/D_{ref}) strongly modifies the linkage between the box geometry (D/D_{ref} and C/C_{ref}) and the force coefficients. The graphical representation of the RBF surrogate model deciphers an evident non-linear trend in the aerodynamic responses as a function of the geometry-related deck design variables. The surrogate model provides a quantitative assessment of such behaviour, previously only conjectured based on limited-scope parametric studies for large gap distances, as remarked in the introduction.

7.2. Surrogate Model of the Flutter Derivatives

The definition, representation and interpretation of a surrogate model capable of approximating the flutter derivatives for the considered application case is challenging, as four inputs should be considered: the three geometric variables defining the geometry of the deck and the reduced velocity that enables the identification of the six considered outputs, that is, the six main flutter derivatives H_i^* and A_i^* ($i = 1, 2, 3$). In this case, the nonlinearity of the linkage between input and output should be remarked on, as well as the multidimensionality of the problem addressed, comprising four inputs and six outputs.

The need for the definition of a surrogate model enabling the identification of the impacts in the flutter derivatives caused by changes in the deck geometry is of utmost importance, as the naked eye analysis of the flutter derivatives obtained for the set of 15 samples defined over the design domain does not allow the extraction of founded conclusions. In Figure 14, the flutter derivatives H_i^* and A_i^* ($i = 1, 2, 3$) obtained by means of 2D URANS forced-oscillation simulations for the set of 15 samples are reported. Notably, there is a large amplitude of the values obtained for different reduced velocities for a given flutter derivative, remarkably for H_i^* and A_i^* ($i = 1, 2$), along with the absence of a clear pattern in the aeroelastic response.

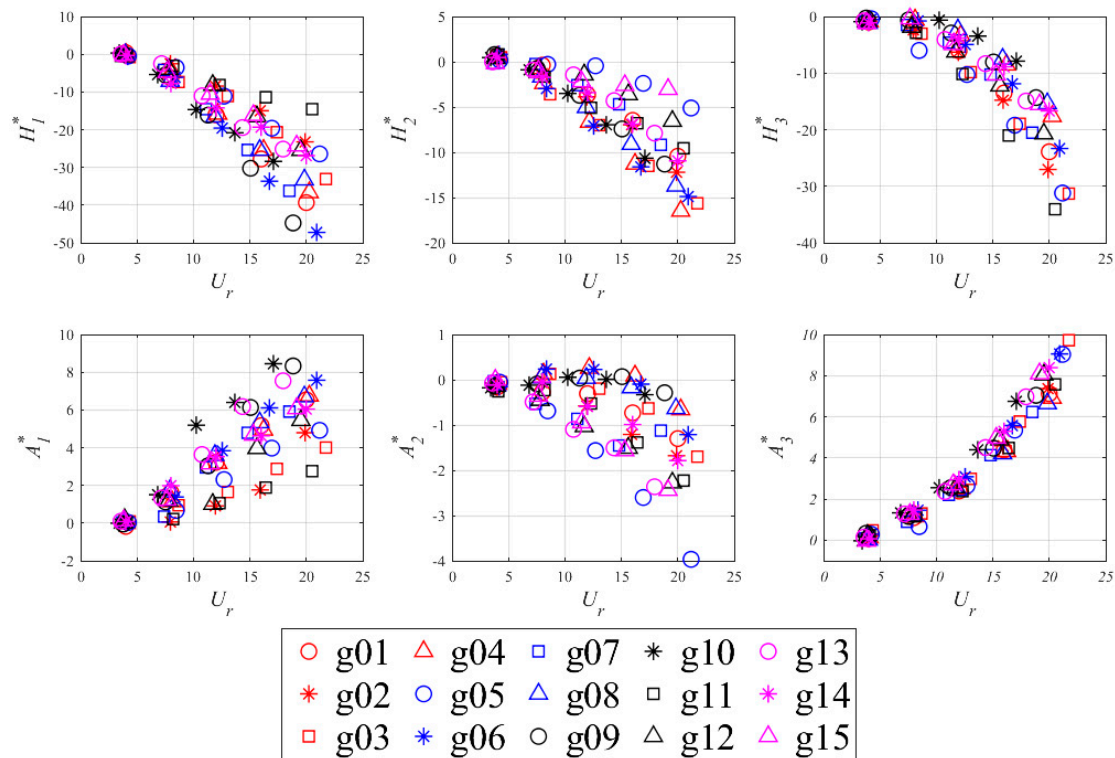


Figure 14. CFD-based flutter derivatives for the set of 15 samples considered in the training of the RBF surrogate model.

Although the close relationship between the gap-to-depth ratio and the flutter derivatives in twin-box decks has been previously studied by other researchers based upon parametric experimental studies, such as in [9], here, a global quantitative assessment is provided, along with additional interplay with the geometric variables of the boxes.

The Radial Basis Function has been adopted for the definition of the surrogate model, imposing, after several trials, a spread value of one and a maximum number of neurons of 15 for the surrogate of the flutter derivatives. The resultant overall mean square error of this surrogate model is 0.24, which shows the ability of the RBF surrogate to faithfully approximate the CFD realizations for the considered set of samples.

The surrogate model linking the geometry of the twin-box deck and the flutter derivatives H_i^* and A_i^* ($i = 1, 2, 3$) is graphically represented in Figures 15–20 for each individual flutter derivative. Each figure depicts a matrix of charts where the rows represent specific values of the gap-to-depth ratio G/D_{ref} , and the columns correspond to specific values of the reduced velocity $U/(fB)$; furthermore, each individual chart represents the value of the flutter derivative as a function of the box geometry design variables, depth (D/D_{ref}) and width (C/C_{ref}).

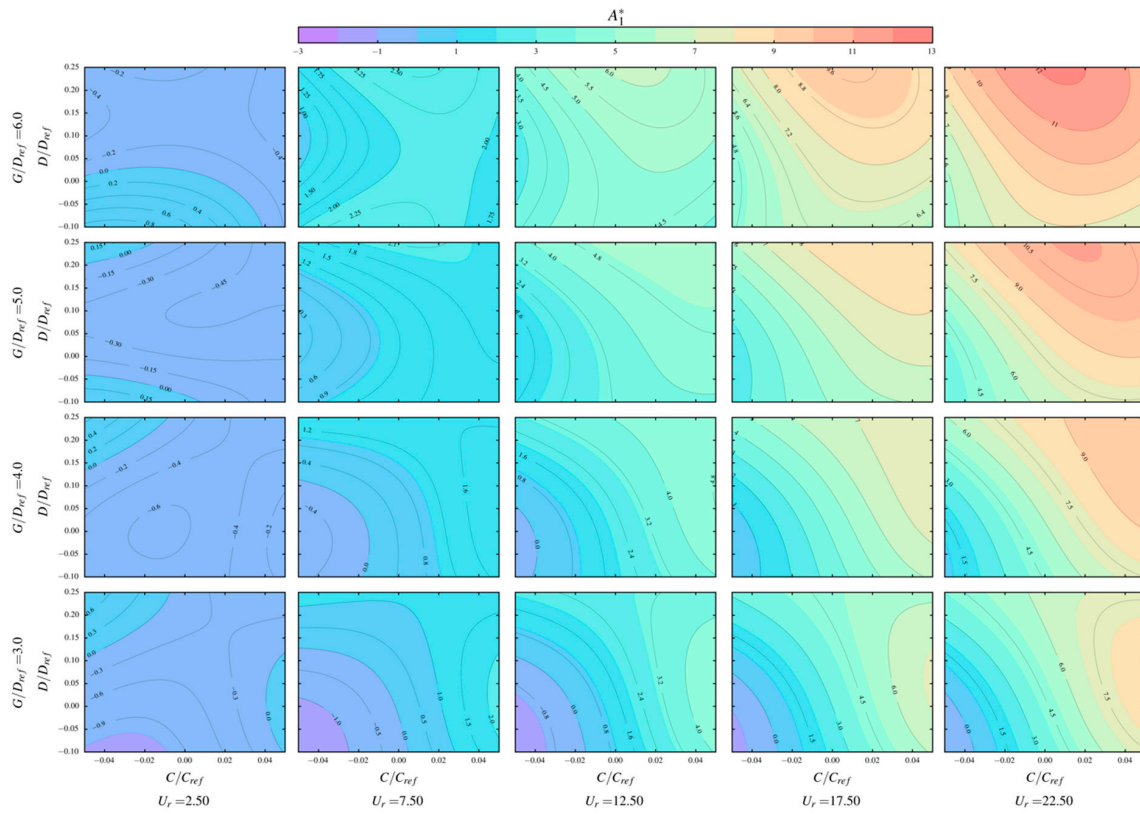


Figure 15. Surrogate model for the A_1^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

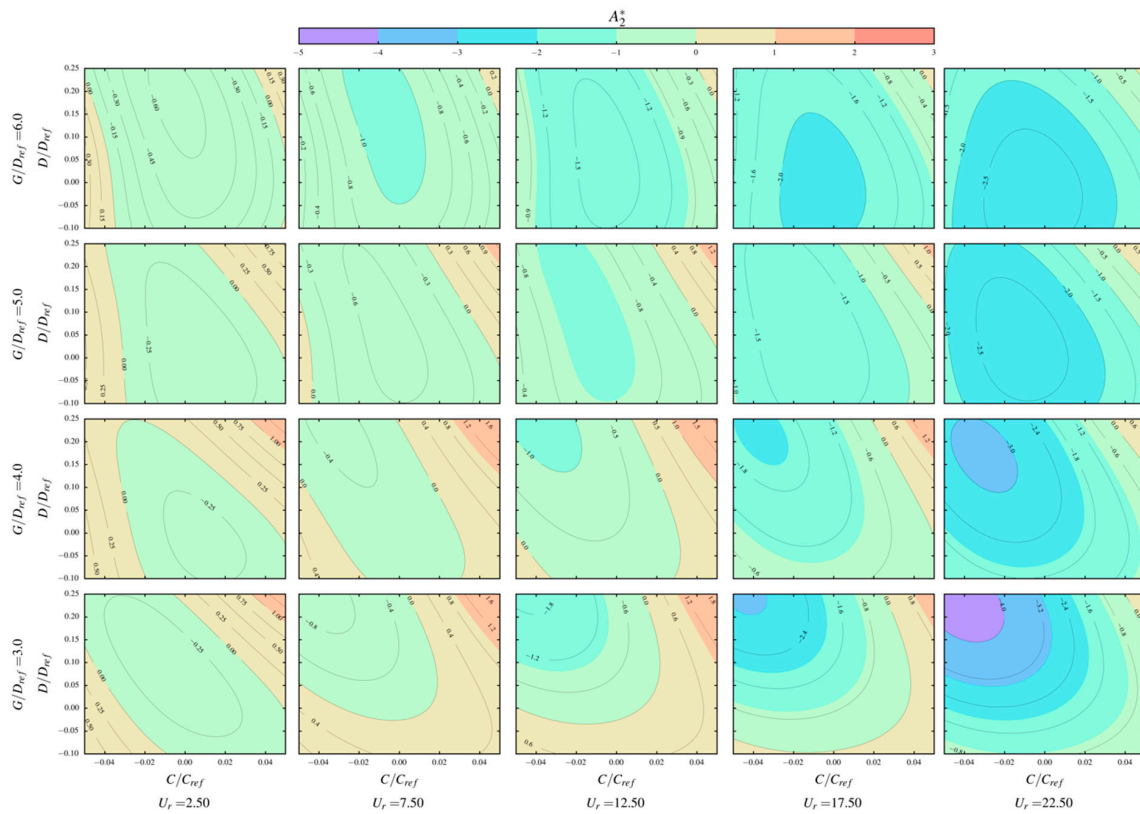


Figure 16. Surrogate model for the A_2^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

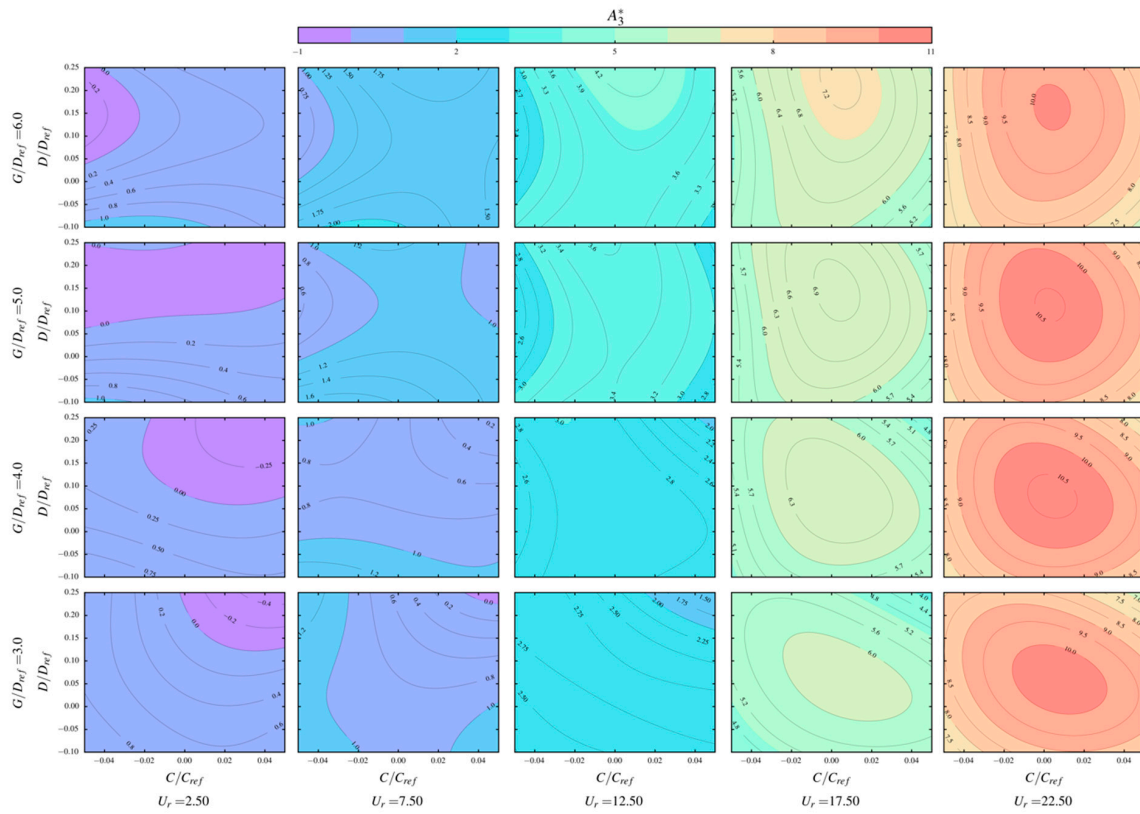


Figure 17. Surrogate model for the A_3^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

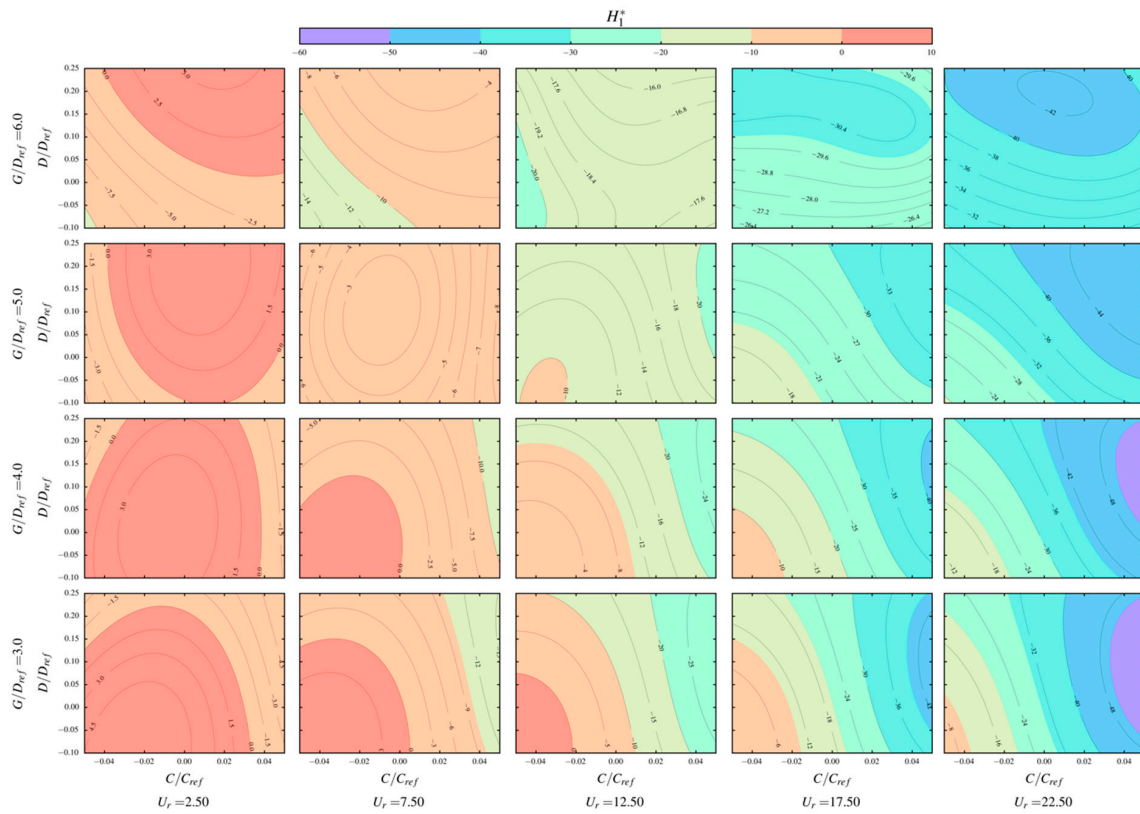


Figure 18. Surrogate model for the H_1^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

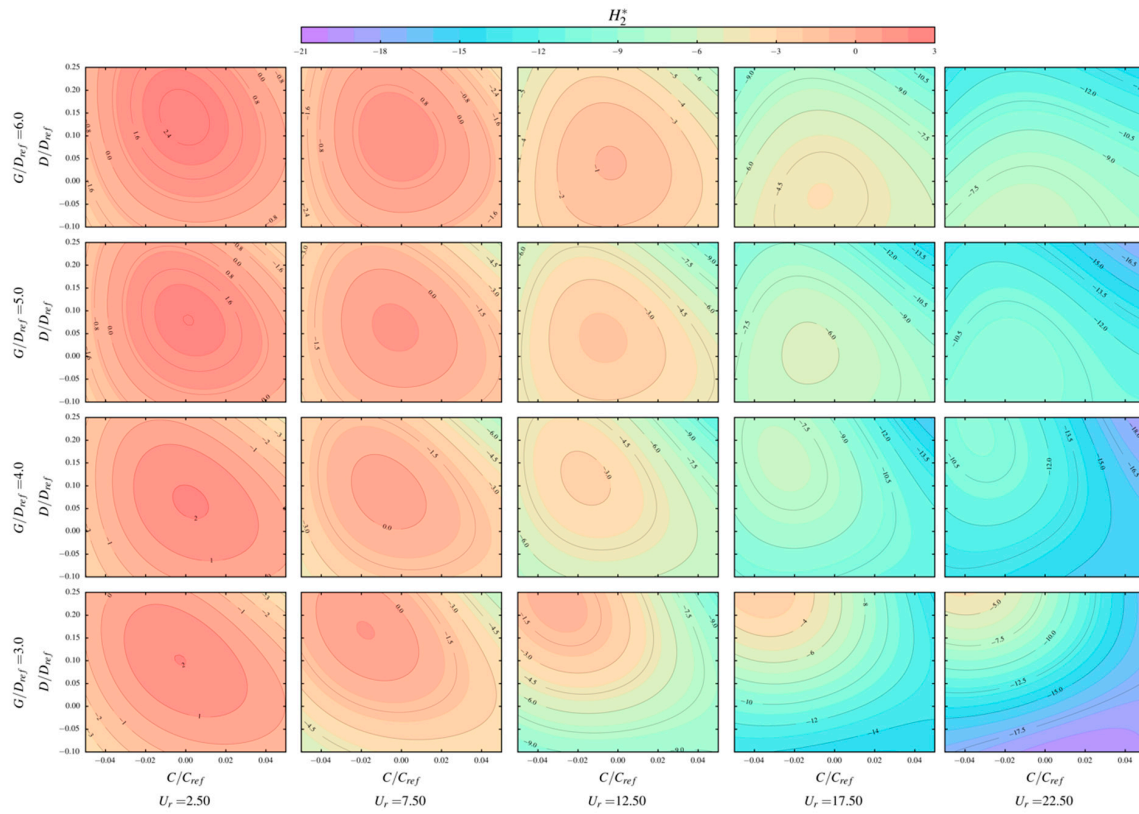


Figure 19. Surrogate model for the H_2^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

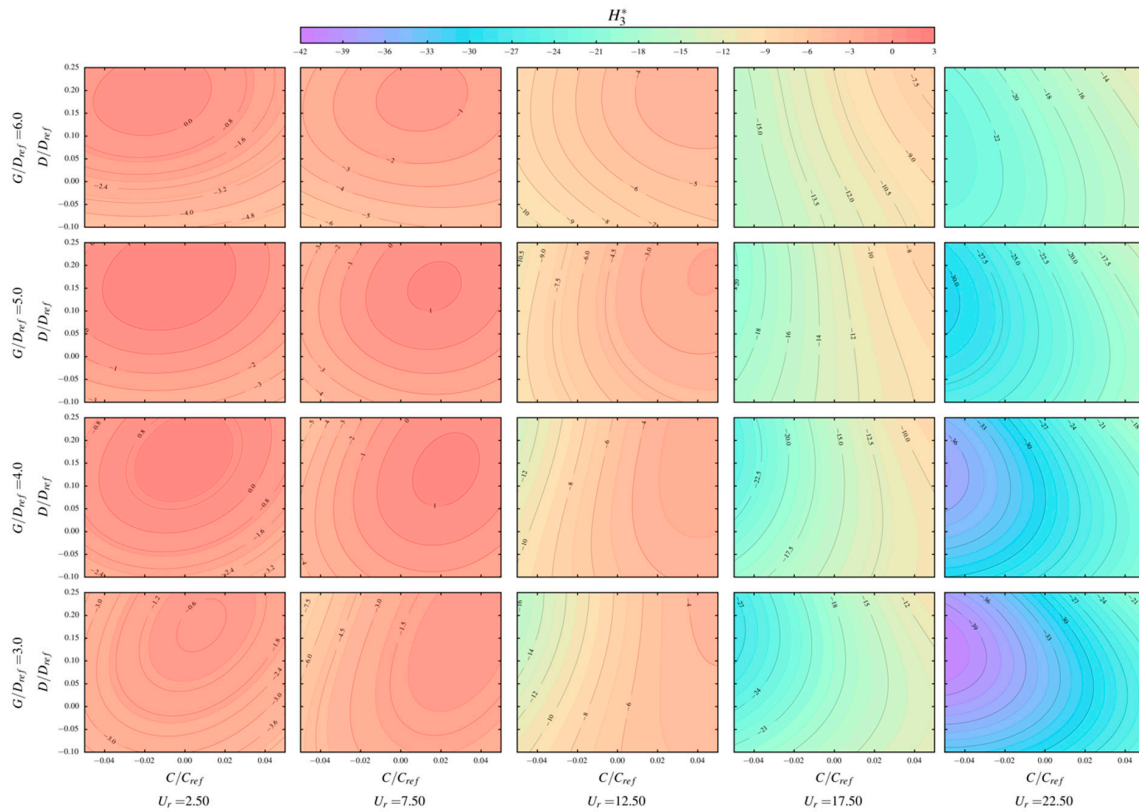


Figure 20. Surrogate model for the H_3^* flutter derivative as a function of the geometric design variables C/C_{ref} , D/D_{ref} and G/D_{ref} and the reduced velocity U_r .

In [38], it is stated that coupled flutter stability might happen in slender deck cross-sections featuring flutter derivatives $H_1^* < 0$ and $A_2^* < 0$, which corresponds with the values for the flutter derivatives in Figure 14. In the next paragraphs, the impact of the changes in the deck geometry in the flutter performance will be explained, based on the role played by each flutter derivative in the stabilization or destabilization in coupled flutter, as proposed in [38].

Focusing on how the flutter derivatives related to the aeroelastic moment A_i^* ($i = 1, 2, 3$) respond to changes in the deck geometry, the surrogate model for the A_1^* flutter derivative, in Figure 15, shows a trend for higher values of A_1^* as the G/D_{ref} ratio increases; furthermore, for a given gap-to-depth ratio and reduced velocity, the flutter derivative A_1^* increases with higher C/C_{ref} and D/D_{ref} ratios. According to Train and Shiroto (2011) [8], low absolute values in A_1^* are linked with coupled flutter stabilization; thus, low values for the box width and depth (C/C_{ref} and D/D_{ref} ratios) should be beneficial for flutter performance. Larger slots are linked with higher values for A_1^* , explaining the lower critical flutter speed obtained, for instance, in Yang et al. (2015) [9] in the upper range of the studied gap distances in that reference.

The A_2^* flutter derivative follows the opposite trend (see Figure 16), as it increases with shorter slots between boxes. Similarly, A_2^* increases as the D/D_{ref} ratio decreases and the C/C_{ref} ratio increases. Negative values for A_2^* show a stabilizing effect, thus larger gaps, and values for the box width and depth close to the ones adopted as reference for this study are related to a better flutter performance. The impact of the A_2^* flutter derivative in the coupled flutter stabilization has been further stressed in [8].

The A_3^* flutter derivative is relatively insensitive to changes in the twin-box deck geometry within the considered design domain (Figure 17), as can be also noticed in Figure 14. For medium and high reduced velocities, A_3^* tends to reach maximum values for box widths close to C_{ref} and depths slightly higher than D_{ref} .

For the aeroelastic forces related to the aeroelastic lift force, the surrogate model for the H_1^* flutter derivative, depicted in Figure 18, shows a strong sensitivity to the gap distance, with lower values of H_1^* as the gap-to-depth ratio increases, therefore improving the coupled flutter stability of the bridge. On the other hand, for medium and high reduced velocities, shorter box widths C/C_{ref} are associated with higher values of H_1^* , with destabilizing effect on flutter performance. For this flutter derivative, the impact of changes in the box depth is minor.

In Figure 19, the H_2^* flutter derivative shows a strong sensitivity to the gap to depth ratio G/D_{ref} , as larger gaps produce higher values for the flutter derivatives. Additionally, for small reduced velocities, the maximum in H_2^* is located in the vicinity of the reference box design ($C_{\text{ref}}, D_{\text{ref}}$). Nevertheless, as the reduced velocity increases, the depth plays a more evident role, with opposite trend depending on the G/D_{ref} ratio: for short gaps, H_2^* decreases with D/D_{ref} ; however, for larger gaps, H_2^* increases as D/D_{ref} increases. This is remarkable evidence of the complex interplay between deck geometry and aeroelastic response, which can only be deciphered with this clarity by exploiting the capabilities of the surrogate model. However, it should be noted that the relative importance of the H_2^* flutter derivative is minor when compared with the H_1^* , A_2^* and A_3^* flutter derivatives in long-span bridges [39].

Finally, the surrogate for the H_3^* flutter derivative, as depicted in Figure 20, shows a qualitatively similar trend for different gap-to-depth ratios G/D_{ref} , with a clear sensitivity to changes in the box width C/C_{ref} for medium and high reduced velocities. In the range of high reduced velocities, the box depth D/D_{ref} only plays a significant role in the lower range of gap-to-depth ratios, with minimum values for H_3^* around $D/D_{\text{ref}} \cong 0.1$. In this case, the negative values resultant from the product of $A_1^* \times H_3^*$ are indicative of a stabilizing effect for coupled flutter.

8. Concluding Remarks

This work extends previous research on the development of surrogate models for the aerodynamic and aeroelastic characterization of long span bridges. The fundamental challenge addressed herein has been the multidimensionality in the inputs of the surrogate model, as four input design variables were considered for the definition of the force coefficients surrogate model: the width and depth of the individual boxes, the gap distance between boxes and the angle of attack. Similarly, for the flutter derivatives surrogate model, the input variables adopted were the width and depth of the individual boxes, the gap distance between boxes and the reduced velocity.

Another challenge has been the computation of the flutter derivatives for the 15 samples considered in the definition of the surrogate by means of 2D URANS forced-oscillation simulations in the pitch and heave degrees of freedom at five different reduced velocities. This more burdensome approach was required, as the consideration of large gap distances in the twin-box deck arrangements does not allow for the application of the quasi-steady theory for the approximation of the flutter derivatives adopted in other cases by the authors. In addition to the implicit computational burden, spatial and temporal verification studies have been completed, and the numerical data have been validated with the experimental data for a subset of three geometries within the set of samples resulting from the application of the LHS method.

The surrogate model of the force coefficients enables the identification of larger sensitivity in the lift and moment coefficient with the box width and depth, while the gap-to-depth ratio plays a decisive role only in the lift coefficient.

For the flutter derivatives, the surrogate model shows a strong dependence in the values with the gap-to-depth ratio for any given reduced velocity for H_1^* , H_2^* , A_1^* and A_2^* but a minor influence for H_3^* and A_3^* . This last flutter derivative is, however, more affected by changes in the box width C . Furthermore, it is also interesting to note how, for a given flutter derivative and gap to depth ratio G/D_{ref} , the qualitative influence of the deck depth D and box width C is also dependent on the reduced velocity. For instance, flutter derivatives H_1^* and H_3^* show different trends with C/C_{ref} and D/D_{ref} , as the reduced velocity increases for a given gap distance. On the other hand, flutter derivatives such as H_2^* and A_2^* show similar patterns with C/C_{ref} and D/D_{ref} , independently of the reduced velocity for a given G/D_{ref} ratio. The impact of the changes in the twin-box deck geometry in the flutter response has been analysed based on the stabilizing or destabilization effect, depending on the values of each individual flutter derivative, following [38].

In addition to the qualitative and quantitative interpretation of the aerodynamic and aeroelastic parameters, the surrogate model is ready to be applied in the application of surrogate-based design optimization problems considering the flutter, buffeting and aerostatic stability, following the methodology outlined in [2].

As surrogate modelling techniques gain maturity, further advances would improve our understanding of the aeroelastic responses of long-span bridges. In this regard, adding additional geometric input variables, such as the length and angle of a recessed lower internal corner or the fairing corner angle, would enable a more precise tailoring of the desired flutter response, although at the expense of higher dimensionality in the model and, therefore, larger computational demands. Moreover, it was mentioned in the introduction that twin-box decks are prone to vortex-induced vibration; hence, the development of a surrogate model linking the deck geometry with the VIV response would be of utmost importance for practical long-span bridge design, wherein deck geometries showing a low excitation risk should be favoured, as they also maintain enhanced flutter performance.

Author Contributions: Conceptualization, S.H.; methodology, R.B., A.J.Á., F.N., J.Á.J. and S.H.; software, A.J.Á.; validation, R.B., J.Á.J. and A.J.Á.; investigation, R.B., A.J.Á., F.N. and J.Á.J.; resources, S.H.; data curation, R.B.; writing—original draft preparation, R.B. and F.N.; writing—review and editing, A.J.Á., F.N., J.Á.J. and S.H.; visualization, R.B. and A.J.Á.; supervision, F.N. and J.Á.J.; project administration: J.Á.J.; funding acquisition, F.N. and J.Á.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research has been supported by the grant PID2019-110786GB-I00 and the grant PID2022-138069OB-I00, funded by Ministry for Science and Innovation of Spain MICIU/AEI/10.13039/501100011033 and, by “ERDF A way of making Europe”, along with the grant ED431C 2021/33 funded by the Galician Regional Government, including FEDER funds.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments: The computations have been carried out in the computer cluster Breogán and in the Galicia Supercomputing Center (CESGA). The authors gratefully acknowledge the access granted to the HPC infrastructure.

Conflicts of Interest: The authors declare no conflict of interest.

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