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A Fuzzy Scheduling Method for Pipeline Processing in Shipyards Incorporating the Black Widow Optimization Algorithm

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Abstract: In the process of ship production, the production of pipeline occurs throughout the production process. Key issues to address in the pipeline-processing workshop of a shipyard include uneven production loads and weak flow rhythmicity. Additionally, uncertainties in processing time and other real-world factors can pose significant challenges. The fuzzy number method is used to describe processing and completion times, with the goal of minimizing the fuzzy maximum completion time. To achieve this, a scheduling method based on the Black Widow Optimization Algorithm (BWOA) is proposed for the pipeline-processing workshop in a shipyard. This algorithm aims to effectively reduce the blindness of the production plan, ensure the rationality and stability of the plan, shorten the production cycle of a ship pipeline, improve the production efficiency, and realize the lean shipbuilding mode. A simulation was conducted to evaluate the BWOA algorithm. The final simulation test results show that the algorithm provided a better scheduling plan and a stabler average value than comparable methods, which proves its effectiveness in the scheduling of pipeline processing in shipyards.

Keywords: ship pipeline; fuzzy flexible workshop; triangular fuzzy number; Black Widow Optimization Algorithm



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1. Introduction

During the construction of ships and offshore equipment, pipeline processing in the workshop is an important part of the whole construction project. The pipeline manufacturing and installation workload accounts for about 23–30% of the workload of the whole ship, and its production progress and quality directly affect the smooth advancement of the major nodes of the ship construction process [1].

Under the modern shipbuilding mode, with a reasonable arrangement of the production plan, reducing the waiting time is the top priority of the whole production scheduling. For the shipyard pipeline-processing workshop scheduling problem, it can be regarded as a flexible job-shop scheduling problem (FJSP) [2]. The FJSP was first proposed by Brucker and Schlie in 1990. Currently, we have several production-scheduling methods, including mathematical planning, rule scheduling, simulation scheduling, and group intelligent optimization. Among these, the group intelligence algorithm stands out for its accuracy and efficiency. By imitating the behavioral patterns of nature or living organisms, it can obtain an ideal scheduling plan. Common examples of this approach include the genetic algorithm, artificial fish swarm algorithm, ant colony algorithm, and particle swarm algorithm. For the FJSP problem, a variety of different optimization algorithms have been used to solve this type of problem. For instance, Jie [3] proposed a new objective function using the improved firefly algorithm to solve FJSP with the objective of minimizing the

maximum completion time and minimizing the total machine load. Shuning [4] used a multi-objective sparrow search algorithm based on non-dominated sorting to solve the single objective FJSP. Hua [5] used the bat algorithm to solve the FJSP by analyzing the relationship between the initially selected machine and the scheduled completion time of each process. Alzaqebah [6] introduced an adaptive mechanism in the swarm algorithm to enhance the local intensification of the algorithm capability, providing a new idea for solving FJSP.

All of the above studies were based on deterministic process processing times; however, the actual production environment is dynamic, and accurate process processing times are often difficult to obtain in advance, which impacts the actual utility of scheduling plans. Therefore, the study of the fuzzy flexible job-shop scheduling problem (fFJSP) with an uncertain processing time is of greater practical significance. At present, the use of triangular fuzzy numbers to describe the processing time of the workpiece has become the main research direction to solve the fFJSP, and scholars worldwide have carried out extensive research and exploration in this regard. For example, Xu [7] expressed the processing time, processing cost, and raw-material cost in terms of a triangular fuzzy number and solved the fFJSP by using an adaptive discrete flower-pollination algorithm. Furthermore, Zheng [8] used a decomposition-based multi-objective evolutionary algorithm that is a variable neighborhood search based on four neighborhood actions to solve fFJSP. Moreover, Gang [9] constructed a mechanism for interconverting scheduling solutions and whale individuals for the fFJSP scheduling problem to innovate the solution of the fFJSP scheduling problem. Wang et al. [10] applied the teach-and-learn algorithm to the fFJSP by adding a search framework based on the teach-and-learn mechanism and a special local search operator to enhance the algorithm. A two-stage crossover scheme is added to the search framework based on the teach-and-learn mechanism and special local search operators to enhance the local search capability of the algorithm. Many researchers have also carried out related work in this area and have made some important advances [11–13].

The processing time of different pipelines varies widely, ranging from a few minutes to several hours. Therefore, under the constraints of process flow and production resources in the production plant, it becomes particularly important to rationally allocate the processing sequence and machine selection of pipelines and to adopt scientific production-scheduling schemes to optimize the production process to shorten the maximum completion time, reduce the flow time of pipelines in the production system, and increase the machine utilization rate and other scheduling objectives.

Many uncertainties exist in actual production, such as equipment failure, changes in the material supply, and the physical condition of workers, all of which affect the actual processing time of the pipeline. By using the triangular fuzzy number in fuzzy set theory, we allow the processing time to be a fuzzy variable, which is more in line with the actual production situation.

The Black Widow Optimization Algorithm (BWOA) simulates the special reproductive behavior of black widow spiders and continuously adjusts the search strategy of the solution space through iterative and adaptive mechanisms to better approximate the optimal solution. In this paper, considering the uncertainty of processing time in the actual processing workshop, we use the improved BWOA to solve the scheduling problem of a pipeline-production workshop in shipyards. The obtained results were compared with the results of the Genetic Algorithm (GA) and the program given by the scheduler's calculation. The effectiveness of the algorithm was then verified in the scheduling of a pipeline-processing workshop in shipyards, and a reasonable scheduling program was generated to save production costs and improve production efficiency.

2. Mathematical Model for Scheduling of a Pipeline-Processing Workshop in a Shipyard

2.1. Shipyard Pipeline-Processing Workshop Analysis

Shipbuilding is a process with a long cycle and tedious tasks, and this process requires the cooperative production of various workshops and departments. Among them, the

shipyard pipeline-processing workshop is an important workshop in the shipbuilding process, and its task is to process various kinds of marine pipes with different specifications. The process of shipyard pipeline processing usually includes six main processes, i.e., undercutting, bending, pipe calibration, welding, cleaning and grinding, and pump-pressure testing. According to the specific application requirements of the pipeline, some pipelines need to bend, while others do not. Figure 1 shows the typical process of shipyard pipeline processing.

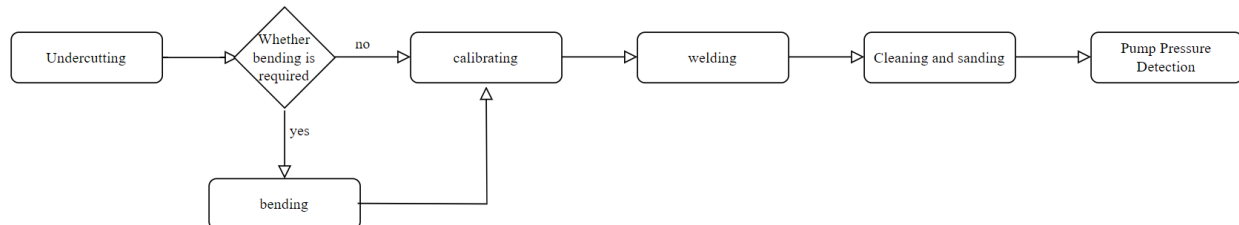


Figure 1. Shipyard pipeline-processing process.

The operation status of the ship pipeline-processing workshop directly affects the overall production efficiency and quality of the ship. In the actual production process, some problems arise in the pipeline-processing workshop, which can have adverse consequences such as decreased production efficiency and increased costs. Through research on the shipyard, the following issues were found:

- **Low production efficiency:** The speed of technological updates is fast, but shipyards have failed to keep up with the latest pipeline-processing technologies and methods in a timely manner. The backward level of technology has led to the inability to adopt more efficient production processes, and processing methods still remain in traditional and time-consuming operations.
- **Improper production organization and management:** The production-plan arrangement is unreasonable and lacks flexible production-scheduling capabilities. The current shipyard pipeline-processing workshop lacks scientific and effective methods for formulating plans. Planners usually arrange work plans based on past experience and only on the number of pipelines or segments. Insufficient coordination occurs between processes, often resulting in material waiting and idle equipment, greatly extending the production cycle.
- **Low level of informatization:** Owing to insufficient investment in information technology construction, workshops overly rely on traditional manual operations and experiential judgment in the formulation, execution, and adjustment of production plans. This approach is not only inefficient but also difficult to quickly and accurately respond to emergency changes or abnormal situations, resulting in production progress being affected and even production bottlenecks. The low level of informatization is also reflected in the lack of effective decision-support systems. The management lacks sufficient data support when making key decisions such as production scheduling, resource allocation, and cost control, resulting in strong subjectivity and high risk in the decision-making process.

Under the constraints of the process flow and production resources in the production workshop, it is particularly important to allocate the processing order and machine selection of pipelines reasonably, adopt scientific production-scheduling schemes to optimize the production process, shorten the maximum completion time, reduce the flow time of pipelines in the manufacturing system, and improve machine utilization, among other scheduling objectives.

2.2. Fuzzy Scheduling Model for a Pipeline-Processing Workshop in a Shipyard

Based on the description and reasonable assumptions of the scheduling problem in shipyard pipeline-processing workshops, it can be described as a flexible job-shop

scheduling problem. Many uncertain factors often exist in actual production, such as equipment failures, changes in the material supply, and physical conditions of workers, all of which can affect the actual processing time of pipelines. Thus, the triangular fuzzy number in fuzzy set theory is introduced to allow the processing time to be a fuzzy variable, making it more in line with the actual production situation. This scheduling problem can ultimately be reduced to a fuzzy flexible job-shop scheduling problem.

For fFJSP, the key is to determine the processing sequence of each workpiece and allocate machines for each process. After the completion of all processing tasks, the optimal scheduling solution is found by minimizing the maximum fuzzy completion time as the indicator, and the final production-scheduling plan is generated. The specific description can be as follows: There are n independent pipelines $J = \{J_1, J_2, \dots, J_n\}$ to be processed on m machines $M = \{M_1, M_2, \dots, M_m\}$. The total number of pipeline processes J_i is represented as n_i , J_i , and the j process is represented as O_{ij} . O_{ij} means that only a portion of the machines can be selected for processing in the machine set, that the processing time on each machine varies, and that the processing time is not a fixed value.

For the fFJSP, the problem needs to determine the processing order of all processes and allocate the processing machine for each process. After the completion of all the process processing tasks, the optimal scheduling solution is found by minimizing the maximum fuzzy completion time, thus generating the final production-scheduling plan. Specifically, it can be described as follows: there are n independent pipelines $J = \{J_1, J_2, \dots, J_n\}$ that need to be processed on M machines $M = \{M_1, M_2, \dots, M_m\}$. The total number of processes for pipeline J_i is represented as n_i , and the j -th process of J_i is represented as O_{ij} . O_{ij} can only select some machines from machine set M_{ij} for processing—that is $\exists i, j, M_{i,j} \subsetneq M$; the processing time on each machine is different, while the processing time is not a fixed value.

The membership function image of triangular fuzzy numbers $\tilde{T} = (t_1, t_2, t_3)$ in a two-dimensional coordinate system is a triangle, as shown in Figure 2. Compared with traditional fixed processing times, using triangular fuzzy numbers to represent uncertain processing times has greater practical significance. Its membership function is shown in Figure 2, corresponding to the following:

$$\mu_{\tilde{T}}(x) = \begin{cases} \frac{x-t_1}{t_2-t_1}, & t_1 \leq x < t_2, \\ \frac{t_3-x}{t_3-t_2}, & t_2 \leq x < t_3, \\ 0, & x < t_1 \text{ or } x \geq t_3. \end{cases} \quad (1)$$

The processing time of the J -th operation of the pipeline J_i on machine M_k is represented by the triangular fuzzy number $\tilde{P}_{ijk} = (P_{ijk}^1, P_{ijk}^2, P_{ijk}^3)$, where P_{ijk}^1 represents the minimum processing time of the pipeline, P_{ijk}^2 represents the most likely processing time of the pipeline, P_{ijk}^2 corresponds to a membership function of 1, and P_{ijk}^3 represents the maximum processing time of the pipeline.

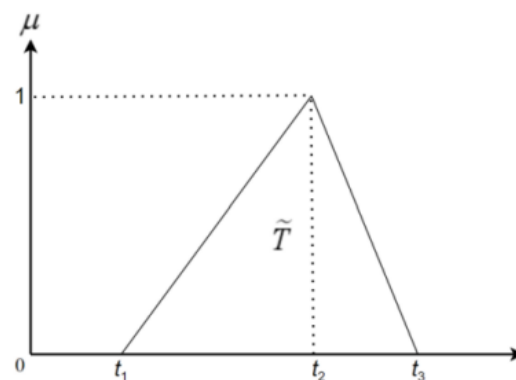


Figure 2. Fuzzy number membership function.

The process of solving fFJSP can be briefly described as follows: under the premise of meeting production constraints, by reasonably arranging the processing sequence of workpieces and assigning corresponding machines to each process, we can ensure that the final scheduling plan meets specific indicators. This article uses $\tilde{C}_{\max}$ to represent the maximum fuzzy completion time of each scheduling task, and the corresponding single-objective fFJSP optimization model is as follows:

$$\begin{cases} f = \min \{ \tilde{C}_{\max} \} \\ \tilde{C}_{\max} = \max_{i=1}^n \{ \tilde{C}_i \} \\ i = 1, 2, \dots, n \end{cases} \quad (2)$$

The solving process of single objective fFJSP can be expressed as follows: Under the condition of meeting actual production needs, solving the two subproblems of process sorting and machine allocation, $\tilde{C}_{\max}$ is minimized, and the scheduling solution that minimizes $\tilde{C}_{\max}$ is the optimal scheduling solution.

In the fuzzy scheduling model of flexible job workshops, pipelines and processing machines must simultaneously follow the following constraints:

1. The different processes of the same workpiece must be processed in the following sequence:

$$\begin{cases} \tilde{S}_{ij} + \tilde{P}_{ijk} \cdot x_{ijk} \leq \tilde{C}_{ij} \\ \tilde{C}_{ij} \leq \tilde{S}_{i(j+1)} \end{cases} \quad (3)$$

2. The fuzzy completion time of the last process of each workpiece should not exceed the fuzzy maximum completion time of the entire scheduling process as follows:

$$\tilde{C}_{in_i} \leq \tilde{C}_{\max} \quad (4)$$

3. The same machine cannot process multiple processes at the same time, where L is a sufficiently large positive integer, as seen below.

$$\begin{cases} \tilde{S}_{ij} + \tilde{P}_{ijk} \leq \tilde{S}_{lh} + L(1 - y_{ijlhk}) \\ \tilde{C}_{ij} \leq \tilde{S}_{i(j+1)k} + L(1 - y_{lhi(j+1)k}) \end{cases} \quad (5)$$

4. Each process cannot be processed by multiple machines at the same time, as follows:

$$\sum_{k=1}^{M_{ij}} x_{ijk} = 1 \quad (6)$$

5. Each machine is allowed to operate in a loop on the same machine while meeting the conditions of an optional set of machines as shown below.

$$\begin{cases} \sum_{i=1}^n \sum_{j=1}^{n_i} x_{ijlhk} = x_{lhk} \\ \sum_{l=1}^n \sum_{h=1}^{n_l} x_{ijlhk} = x_{ijk} \end{cases} \quad (7)$$

6. A process that is currently in processing status cannot be terminated midway.

$$\tilde{C}_{ij} - \tilde{S}_{ij} = \sum_{k=1}^m \tilde{P}_{ijk} \cdot x_{ijk} \quad (8)$$

7. All machines exist independently of each other, and all machines are in a machinable state at the initial moment of processing.
8. There is no strict processing sequence between different workpieces.

Different processes of the same workpiece must be processed in sequence in fFJSP. Owing to the consideration of uncertain processing time, triangular fuzzy numbers are used to represent uncertain processing time and completion time parameters, and a flexible job-shop fuzzy scheduling model is established. The relevant parameters and the meanings involved in the model are shown in Table 1.

Table 1. Related symbols and meanings of fuzzy scheduling models for flexible job workshops.

Variables	Meanings
n	The total number of workpieces
m	The total number of machines
J	The set of workpieces to be processed
M	The set of available machines
i	The number of the workpiece
j	The number of the process
k	The number of the machine
N	The total number of processes for all workpieces
n_i	The number of processes for workpiece i
O_{ij}	The j -th process of workpiece i
M_{ij}	The set of machinable machines for process O_{ij}
$\tilde{P}_{ijk}$	Fuzzy processing time of process O_{ij} on machine M_k
$\tilde{C}_i$	Fuzzy completion time of workpiece i
$\tilde{S}_i$	Fuzzy start time of workpiece i
$\tilde{C}_{ij}$	Fuzzy completion time of process O_{ij}
$\tilde{C}_{\max}$	Fuzzy maximum completion time
$\tilde{S}_{ij}$	Fuzzy start time of process O_{ij}
x_{ijk}	When it is 1, it indicates that process O_{ij} is being processed on machine M_k
y_{ijlhk}	When it is 1, it indicates that process O_{ij} is processed before process O_{lh} on machine M_k

2.3. Operations of Triangular Fuzzy Numbers

When solving the fFJSP, all processing times are represented by triangular fuzzy numbers, so it is necessary to determine the basic operational rules of triangular fuzzy numbers, including addition, maximum operation, and comparison operations. For two arbitrary triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, the following operational rules apply:

- Four operations: This is used to calculate the fuzzy completion time of each process.
 Addition operation: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
 Subtraction operation: $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.
 Multiplication operation: $\tilde{A} \times \tilde{B} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$.
 Division operation: $\tilde{A} \div \tilde{B} = (a_1 \div b_1, a_2 \div b_2, a_3 \div b_3)$.
- Maximum operation: This is used to determine the fuzzy start time of each process. The currently used maximum operation strategies include Sakawa's criterion [14] and Lei's criterion [15]. To ensure that the fuzzy number always has a triangular characteristic and to facilitate subsequent calculations, this paper adopts a combination of Lei's criterion and comparison operation rules for fuzzy maximum operation. If $\tilde{A} > \tilde{B}$, then $\tilde{A} \vee \tilde{B} = \tilde{A}$; otherwise, $\tilde{A} \vee \tilde{B} = \tilde{B}$, where the symbol $\vee$ represents the maximum operation operator.
- Comparison operation: This is used to compare the fuzzy end times of processes. The purpose is to obtain the maximum fuzzy completion time, which is compared by the following three criteria for two triangular fuzzy numbers:

Criterion one: first, calculate the value of $C(\tilde{A})$ and $C(\tilde{B})$ according to Equation (9). Here, the weight of the most likely time is set to 2, and the weights of the minimum and maximum processing times are 1. If $C(\tilde{A}) > (<)C(\tilde{B})$, then $\tilde{A} > (<)\tilde{B}$:

$$\begin{cases} C(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}, \\ C(\tilde{B}) = \frac{b_1 + 2b_2 + b_3}{4}. \end{cases} \quad (9)$$

Criterion two: when $C(\tilde{A}) = C(\tilde{B})$, compare the cores of the two fuzzy numbers. If $a_2 > (<)b_2$, then $\tilde{A} > (<)\tilde{B}$.

Criterion three: if $a_2 = b_2$, compare the spans of the two triangular fuzzy numbers. If $a_3 - a_1 > (<)b_3 - b_1$, then $\tilde{A} > (<)\tilde{B}$.

3. BWOA

The black widow spider is a medium-sized spider, mainly distributed in European countries along the Mediterranean coast [16]. Female spiders need to find males to mate and produce the next generation for the spider population to develop. During or after mating, the female consumes the male and transfers the eggs to the egg sac, where the offspring engage in cannibalism after hatching. The offspring stay on the mother's web for a short time, during which they may even eat the mother. This special developmental mechanism ensures that the stronger spiders survive, which can be seen as a search process for the optimal solution. This is the inspiration for the BWOA, which simulates the life cycle of the black widow spider. Each black widow spider in the swarm is considered a candidate solution.

Let the single-objective optimization model with D variables be represented as follows:

$$\begin{cases} \min f(x) \\ \text{s.t. } lb < x < ub \end{cases} \quad (10)$$

where $f(x)$ is the objective function; ub and lb are the upper and lower bounds of the variable $x \in R^D$.

Similarly to other evolutionary algorithms, BWOA mainly includes four steps:

4. Initialization of the swarm

The initial black widow spider population with N individuals is an $N \times D$, $X = [x_1, x_2, \dots, x_N]$ matrix. Each individual x_i ($1 \leq i \leq N$) in the group has D elements, $x_i = [x_{i,1}, x_{i,2}, \dots, x_{i,D}]$, initialized by Equation (11).

$$x_i = lb + \text{rand}(0,1)(ub - lb) \quad 1 \leq i \leq N. \quad (11)$$

The fitness function in Equation (12) is used to evaluate the fitness of each black widow spider in the population as follows:

$$f(x_i), \quad 1 \leq i \leq N. \quad (12)$$

For variable x_i , this structure is referred to as a "chromosome" or "particle position" in Genetic Algorithm and Particle Swarm Optimization, and here, it is referred to as a "black widow". That is, the potential solution to each problem is considered the black widow spider.

5. Reproduction

The new generation of spiders reproduces through the mating behavior of black widows. At the beginning of mating, select one group of spiders labeled as mother and parent spiders based on their reproductive rate P_p for mating. The generation of offspring is determined by Equation (13).

$$\begin{cases} y_i = \alpha x_i + (1 - \alpha)x_j \\ y_j = \alpha x_j + (1 - \alpha)x_i \end{cases} \quad (13)$$

In the formula, x_i and x_j represent the mother spider and the parent spider, respectively; y_i and y_j are the offspring of mating; α means that for a random number vector within the range of 0–1, each pair of parents needs to repeat this reproductive process $N/2$ times.

6. Cannibalism

The black widow spider got its name based on the fact that female eats the male after mating. The BWOA mimics the process of sexual cannibalism and increases the cannibalism of brothers and sisters.

- A female black widow eats her husband during or after mating, and the remaining female spiders are preserved for the next generation.
- Owing to limited food resources, strong spiders eat their brothers and sisters. The fitness value of a spider is considered to be its strength. In this algorithm, a cannibalistic rating (CR) is set based on which some descendants with the worst fitness values are destroyed to determine the number of survivors.
- Some very strong offspring spiders even eat their mothers. That is, if an offspring spider with high fitness is produced, the offspring spider will replace its mother and enter the next generation. After the above process of cannibalism, weak spiders will be eliminated, while excellent individuals will be preserved.

7. Variation

The number of mutated populations is determined by the mutation rate P_m . For the selected individual x_i ($1 \leq i \leq N$), randomly select two elements from the arrays x_{im} , and x_{in} ($1 \leq m, n \leq D$), and then exchange them.

4. Improved BWOA for Solving

4.1. Encoding and Decoding

A good encoding and decoding design not only affects the effective extraction and expression of information but also determines the algorithm's ability to recover information and the final application effect. fFJSP needs to determine the processing order of all workpieces and allocate processing machines for each operation. Therefore, this article uses the OSMS method to code the problem. Each black widow spider consists of two parts, i.e., operations sequencing (OS) and machine selection (MS). For the part of workpiece process arrangement, the workpiece number is directly encoded. The order in which the workpiece number appears represents the processing sequence between the workpiece processes, which is interpreted in order from left to right. T_0 = total number of processes, representing the length of the workpiece sorting coding part, which is half of the total coding length. The process number represents a specific process number for each workpiece. For the i -th occurrence of workpiece number j , it represents the i -th process of the workpiece j , and the total number of occurrences of the workpiece number is equal to the total number of processes of the workpiece. For the machine-allocation part, each element represents the selected machine corresponding to each process, so the length of the machine selection part code is also T_0 . Figure 3 illustrates the method.

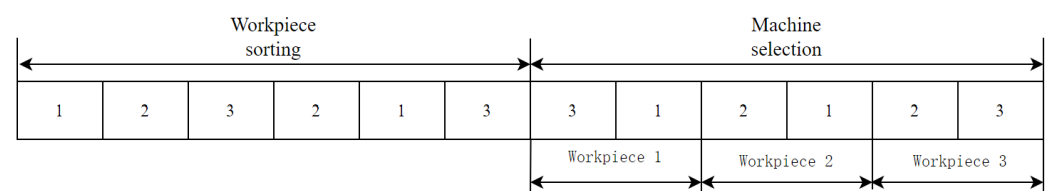


Figure 3. OSMS encoding.

For the convenience of explaining this representation, an example of a problem with three workpieces and three machines is shown in Table 2. According to the varying degrees of flexibility of the optional machines in the workshop for each process (i.e., whether M_{ij} equals M), fFJSP can also be divided into total fFJSP (T-fFJSP) and partial fFJSP (P-fFJSP). In this chapter, T-fFJSP is used as a test case to verify the algorithm's performance. In practical application, the shipyard example is P-fFJSP.

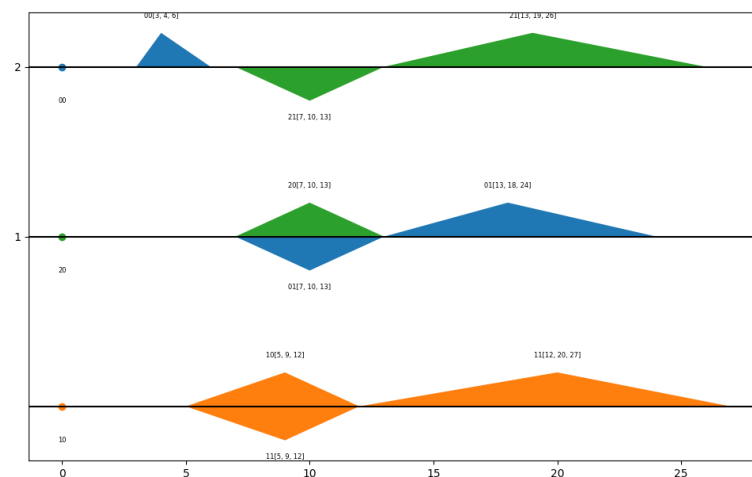
Table 2. Process and fuzzy processing time.

Workpiece	Working Procedure	Machine		
		M_1	M_2	M_3
Workpiece1	$O_{1,1}$	(5,6,8)	(4,5,7)	(3,4,6)
	$O_{1,2}$	(5,7,10)	(6,8,11)	(4,6,10)
Workpiece2	$O_{2,1}$	(5,9,12)	(6,9,11)	(7,10,11)
	$O_{2,2}$	(7,11,15)	(9,11,14)	(6,10,13)
Workpiece3	$O_{3,1}$	(6,9,12)	(7,10,13)	(9,12,16)
	$O_{3,2}$	(10,14,17)	(8,11,15)	(6,9,13)

Use BWOA to compute Table 2 and select one of the feasible solutions, which has the following sequence of operations and machine assignments:

$$(O_{11}, M_3), (O_{21}, M_1), (O_{31}, M_2), (O_{22}, M_1), (O_{12}, M_2), (O_{32}, M_3)$$

The fuzzy Gantt chart of this solution is shown in Figure 4. Among them, the offline TFN is the blur start time of the operation, and the online TFN is the blur completion time of the operation.

**Figure 4.** Fuzzy Gantt chart of the solution $(O_{11}, M_3), (O_{21}, M_1), (O_{31}, M_2), (O_{22}, M_1), (O_{12}, M_2), (O_{32}, M_3)$.

4.2. Design of Fitness Function

When using BWOA to solve the fuzzy flexible job-shop scheduling problem, the objective function is to minimize the fuzzy maximum completion time. Fuzzy maximum completion time refers to the shortest, most likely, and longest time required to complete all pipelines, taking into account the uncertainty of pipeline processing time and machine allocation. This goal reflects the overall efficiency of the scheduling plan.

In this context, the design of the fitness function needs to comprehensively consider the processing time of the pipeline, machine allocation, and possible delays. Specifically, the fitness value of each candidate solution (scheduling scheme) is determined by its corresponding fuzzy maximum completion time. First, it is necessary to perform fuzzy calculations on the start and end times of each pipeline. Then, by aggregating the fuzzy completion times of all pipelines, the fuzzy maximum completion time of the entire workshop is calculated.

The pipeline-processing workshop solved in this chapter aims to minimize the fuzzy maximum completion time as the optimization objective. The optimization objective

$f = \min\{\tilde{C}_{\max}\}$ can be obtained from Equation (14), and the fitness function can be set as follows:

$$fit = \frac{1}{f}. \quad (14)$$

In the iteration process of BWOA, the algorithm generates a new scheduling scheme by simulating the reproductive behavior of black widow spiders (including mate selection, mating, and mutation), and it evaluates it using the above fitness function. The higher the fitness of the scheme (i.e., the shorter the fuzzy maximum completion time), the higher the probability of it being selected as the next-generation population. This selection process ensures that the algorithm gradually tends to discover efficient scheduling solutions that minimize the fuzzy maximum completion time. Through this method, BWOA can effectively handle the uncertainty and fuzziness in job scheduling while finding reasonable and effective scheduling solutions. This is of great importance for improving the operational efficiency and flexibility of the workshop, especially when facing complex and dynamic production environments.

4.3. Breeding Crossover Operator

Because fFJSP needs to consider both the machine allocation and operation sequence, two crossover operators are used for operation sequence and machine allocation to better solve this problem.

Partial mapped crossover (PMX) is used in the operation sequence to ensure the feasibility of process selection; that is, there will not be more genes than the number of workpieces, avoiding infeasible solutions. The process is as follows:

8. Randomly select two individuals from the parents as parent individuals.
9. Randomly select two position indices, which define a partially intersecting area. Typically, this area selects a continuous segment between two positions.
10. In the cross region, map the corresponding substring from the first parent individual to the second parent individual, and map the corresponding substring from the second parent individual to the first parent individual. This process is completed by establishing a mapping table.
11. During the process of establishing a mapping table, conflict resolution is necessary. If there is a conflict in the mapping table (i.e., one position is mapped to multiple positions or multiple positions are mapped to the same position), conflict resolution is required.
12. After establishing the mapping table, the genes outside the crossover region are transferred by the mapping table. As a result, the genes within the crossover region are partially crossed, and their positions are changed, while the genes outside the crossover region remain unchanged.
13. Take the two new individuals obtained as offspring and add them to the next generation.

Partial mapping crossover is a relatively simple and effective crossover operation, which can introduce new mutation combinations for the next generation of individuals by exchanging some gene information in the parent individual. This can increase the search space of the algorithm and help it better explore the solution space of the problem. The process is shown in Figure 5.

In machine selection, priority-preserving crossover (GPPX) is used, with two feasible parents. The solution generated by this crossover operator is still feasible. First, for each individual in the population, another individual is randomly selected as the parent, and then a binary string consisting of 0 and 1 is uniformly generated, with the same length as the machine assignment vector. During the crossover process, each element of the binary string is checked. If it is 0, the corresponding gene of the second parent will be selected; if it is 1, the corresponding gene of the first parent will be selected. This design preserves the sequence of genes, ensuring that the chromosome structure of individuals after crossing is similar to that of one of their parents. Finally, the new individuals generated

by the crossover operation are added to a new population, which can be used in subsequent algorithm iterations to generate better solutions. This approach helps preserve the beneficial characteristics of the parent individuals and promotes the transmission of excellent genes, thereby improving the adaptability of the entire population. The process is shown in Figure 6.

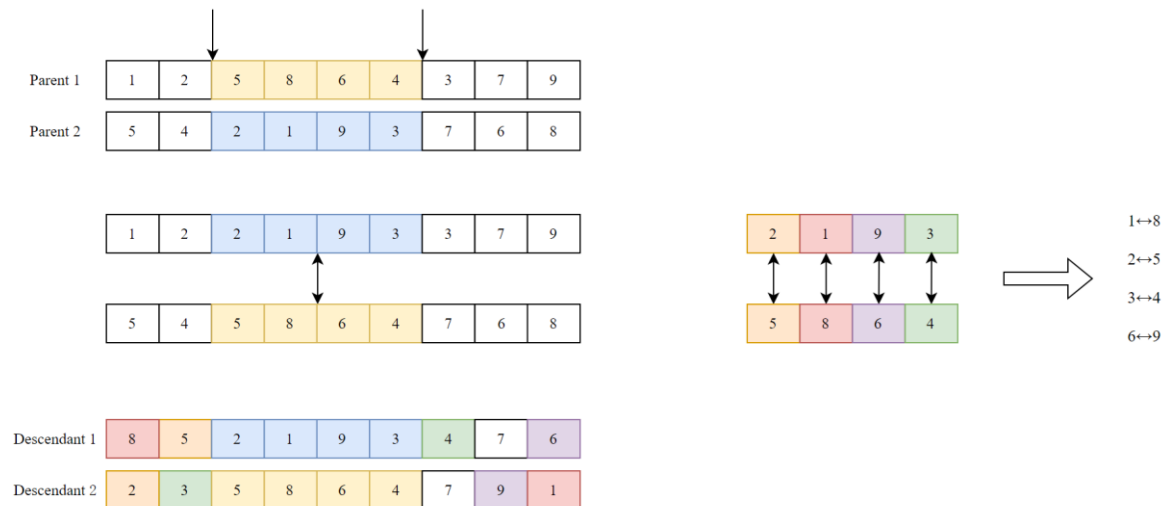


Figure 5. Partial mapping cross schematic diagram of the algorithm.

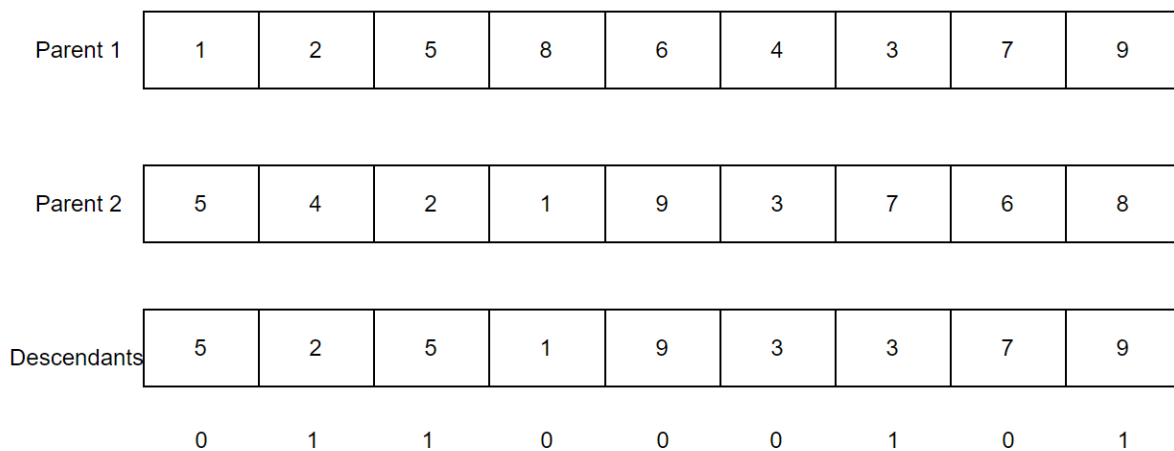


Figure 6. Priority protection cross schematic diagram.

5. Instance Calculation

To verify the practical application value of the BWOA based on the investigation of the pipeline processing situation in the ship pipeline-processing workshop of a large shipyard in Guangxi, Table 3 was appropriately scaled and generated to solve the production-scheduling problem of the company's pipeline-processing workshop. The actual description was as follows: multiple sets of pipelines had to go through [5,6] processing steps (i.e., namely cutting, bending, calibration, welding, cleaning and polishing, and pump-pressure testing). The number of parallel machines for each step was 2; that is, the machine sequence numbers were 1 to 12 in sequence. The total number of pipe fittings was $N \in [10, 200]$. For each workpiece's process, the fuzzy processing time $\tilde{P}_{ijk} = (P_{ijk}^1, P_{ijk}^2, P_{ijk}^3)$ of the corresponding equipment indicates that the machine could not process the corresponding process. The fuzzy processing schedule was determined by the factory technical personnel based on simulation data and the actual operating experience of the workshop operator.

Table 3. Six sets of pipe fittings and twelve machines with fuzzy processing time.

Pipe Fitting Serial Number	Working Procedure	Machine											
		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}
Pipe fittings1	O_{11}	(88,99,106)	(73,82,97)	—	—	—	—	—	—	—	—	—	—
	O_{12}	—	—	(60,69,80)	(72,84,98)	—	—	—	—	—	—	—	—
	O_{13}	—	—	—	—	(50,54,65)	(53,58,64)	—	—	—	—	—	—
	O_{14}	—	—	—	—	—	—	(24,26,35)	(32,39,46)	—	—	—	—
	O_{15}	—	—	—	—	—	—	—	—	(75,84,92)	(67,74,86)	—	—
	O_{16}	—	—	—	—	—	—	—	—	—	—	(25,28,30)	(22,26,29)
Pipe fittings2	O_{21}	(77,87,98)	(75,85,95)	—	—	—	—	—	—	—	—	—	—
	O_{22}	—	—	(41,48,57)	(37,44,53)	—	—	—	—	—	—	—	—
	O_{23}	—	—	—	—	(72,81,89)	(70,78,86)	—	—	—	—	—	—
	O_{24}	—	—	—	—	—	—	(71,80,95)	(66,78,85)	—	—	—	—
	O_{25}	—	—	—	—	—	—	—	—	(70,76,83)	(67,75,82)	—	—
	O_{26}	—	—	—	—	—	—	—	—	—	—	(91,96,104)	(100,112,120)
Pipe fittings3	O_{31}	(4,5,7)	(3,5,6)	—	—	—	—	—	—	—	—	—	—
	O_{32}	—	—	—	—	(25,31,40)	(28,37,43)	—	—	—	—	—	—
	O_{33}	—	—	—	—	—	—	(8,10,13)	(10,14,18)	—	—	—	—
	O_{34}	—	—	—	—	—	—	—	—	(81,87,94)	(75,83,90)	—	—
	O_{35}	—	—	—	—	—	—	—	—	—	—	(9,10,12)	(11,15,17)
Pipe fittings4	O_{41}	(42,46,55)	(45,48,58)	—	—	—	—	—	—	—	—	—	—
	O_{42}	—	—	—	—	(80,87,99)	(77,83,88)	—	—	—	—	—	—
	O_{43}	—	—	—	—	—	—	(24,28,35)	(21,25,31)	—	—	—	—
	O_{44}	—	—	—	—	—	—	—	—	(50,56,60)	(55,59,66)	—	—
	O_{45}	—	—	—	—	—	—	—	—	—	—	(39,41,46)	(38,41,45)
Pipe fittings5	O_{51}	(70,80,92)	(73,88,97)	—	—	—	—	—	—	—	—	—	—
	O_{52}	—	—	(82,88,95)	(79,86,92)	—	—	—	—	—	—	—	—
	O_{53}	—	—	—	—	(24,26,37)	(21,25,34)	—	—	—	—	—	—
	O_{54}	—	—	—	—	—	—	(23,26,34)	(26,29,36)	—	—	—	—
	O_{55}	—	—	—	—	—	—	—	—	(80,88,96)	(83,90,98)	—	—
	O_{56}	—	—	—	—	—	—	—	—	—	—	(11,12,15)	(13,16,19)
Pipe fittings6	O_{61}	(65,71,80)	(62,69,76)	—	—	—	—	—	—	—	—	—	—
	O_{62}	—	—	—	—	(21,25,35)	(23,27,32)	—	—	—	—	—	—
	O_{63}	—	—	—	—	—	—	(22,25,33)	(21,24,29)	—	—	—	—
	O_{64}	—	—	—	—	—	—	—	—	(5,7,8)	(6,8,9)	—	—
	O_{65}	—	—	—	—	—	—	—	—	—	—	(72,76,83)	(67,72,76)

The set parameters for BWOA were as follows: The population size was set to 100; the maximum iteration count, 3000; the reproduction rate, 0.8; the mutation rate, 0.2; and the cannibalism rate, 0.2. Set parameters for the GA were as follows: The population size was set to 100; the maximum number of iterations, 3000; the reproduction rate, 0.8; and the mutation rate, 0.2. We ran each 20 times, recorded the five better feasible solutions, and then compared them with the solutions provided by experienced dispatchers in the factory. Table 4 shows the results of BWOA and GA, as well as the comparison of the solutions provided by dispatchers.

Table 4. Comparison of results.

BWOA results	(418, 470, 530)	(419, 470, 523)	(411, 466, 518)	(420, 478, 531)	(417, 468, 520)
GA results	(528, 586, 670)	(492, 552, 613)	(514, 566, 654)	(534, 588, 658)	(565, 632, 726)
Dispatcher Plan	(628, 709, 791)	(628, 709, 791)	(628, 709, 791)	(628, 709, 791)	(628, 709, 791)

The improved BWOA iteration process is shown in Figure 7.

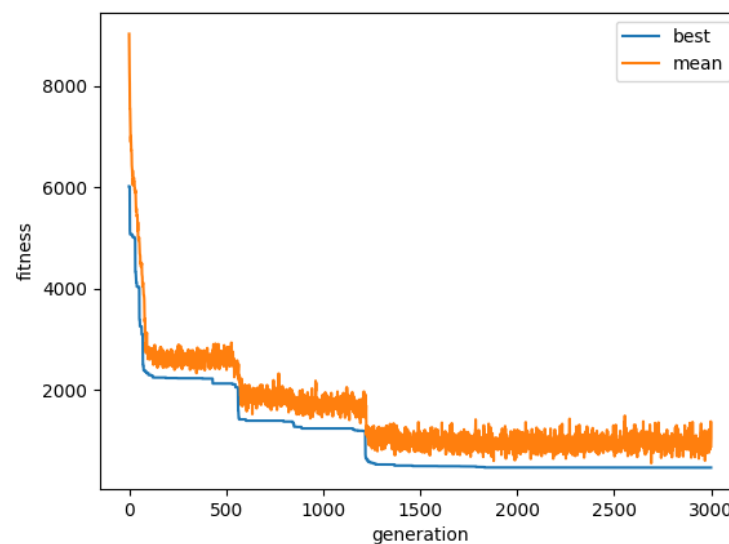


Figure 7. The improved iterative process of BWOA.

Decoding the optimal feasible solution (411, 466, 518) obtained from BWOA in the table, we present the fuzzy Gantt chart of the scheduling scheme in Figure 8.

The optimal pipeline production scheduling scheme after BWOA calculation can be obtained from Figure 8, and the scheduling results are explained here due to the excellent scheduling scheme, high utilisation of processing machines and dense data. In the pipeline processing workshop of this shipyard, take the processing flow of pipe fittings 1 (blue) as an example: firstly, the fuzzy start time of the first process of pipe fittings 1 is moment 0, that is, the process can be scheduled to be processed on M_1 at moment 0, and the fuzzy completion time of the first process on M_1 is (88, 99, 106). After the first process is completed, the second process of tubing 1 is processed on M_4 , which has a fuzzy start time of (112, 129, 148) and a fuzzy finish time of (184, 213, 246), the third process is processed on M_5 , which has a fuzzy start time of (203, 225, 258) and a fuzzy finish time of (253, 279, 323), and the fourth process is processed on the process is machined on M_8 with a fuzzy start time of (248, 285, 319) and a fuzzy finish time of (280, 324, 365), the fifth process is machined on M_9 with a fuzzy start time of (280, 324, 365) and a fuzzy finish time of (355, 408, 457), the sixth process is machined on M_{12} with a fuzzy start time of (355, 408, 457) and fuzzy completion time is (377, 434, 486). It can be seen that this allocation scheme fully utilized the production resources of each machine, reduced the fuzzy completion time from (628, 709, 791) to the optimal value (411, 466, 518), and significantly improved production efficiency. Experiments also showed

that the optimized scheduling plan objective (minimum fuzzy maximum completion time) using improved BWOA increased by an average of 28% compared to GA. BWOA had better optimization ability and stability compared with GA, and the improved BWOA could effectively solve fFJSP. In a fuzzy time processing environment, a reasonable scheduling scheme could effectively save production resources for enterprises. For the processing of shipyard pipelines, it could greatly improve production efficiency and reduce costs.

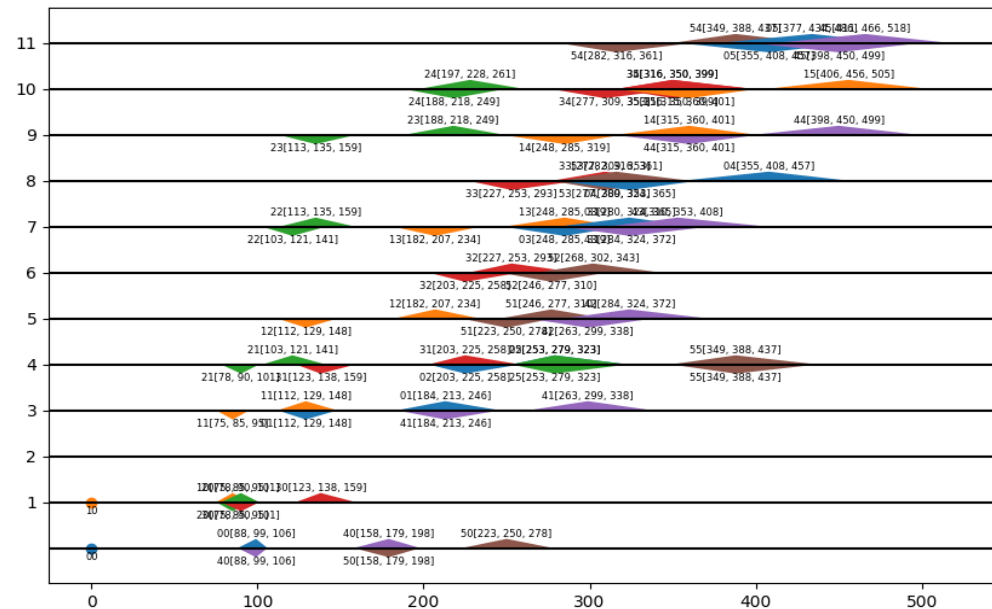


Figure 8. Optimal scheduling scheme fuzzy Gantt chart.

6. Conclusions

In this paper, we addressed the production-scheduling problems existing in the pipeline production workshop of shipyards and innovatively used triangular fuzzy numbers to represent the uncertain processing time in the shipyard pipeline-processing workshop of shipyards. An improved BWOA algorithm was to handle triangular fuzzy numbers. BWOA was applied for the first time to the fuzzy scheduling optimization problem in a shipyard pipeline-processing workshop, which effectively reduced the blindness of the production plan and ensured the rationality and stability of the plan.

The optimal scheduling plan scheme was given with the objective of minimizing the maximum fuzzy time, such as the workpiece machining sequence, the process sequence order, and the selection of the machining machine serial number. The final simulation test results show that the BWOA algorithm could provide a better scheduling plan with a stabler average value, proving its effectiveness in the scheduling of pipeline processing in shipyards. The algorithm is thus suitable for scheduling in a ship pipeline-processing workshop, which can improve the productivity of a ship pipeline-processing workshop to a great extent. Our subsequent research work will aim to extend the proposed algorithm to solve the multi-objective fFJSP shipyard workshop scheduling problem and to integrate other improvement strategies, which can help further improve the performance of the algorithm so that it is more in line with the actual production situation.

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