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A Standard Penetration Test-Based Step-by-Step Inverse Method for the Constitutive Model Parameters of the Numerical Simulation of Braced Excavation

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Abstract: Numerical simulation is an essential method for predicting soil deformation caused by foundation pit excavation. Its accuracy relies on the constitutive model and its parameters. However, obtaining these parameters through lab tests has limitations like long durations, high costs, and potential errors. To improve the simplicity and accuracy of the selection of constitutive model parameters, this study proposes a step-by-step inverse analytical method. Using the braced excavation project in Hangzhou City, China as a case study, the proposed method firstly determines the ratio of the important constitutive parameters then the values of the key constitutive parameters. The results show that the reference secant stiffness (E_{50}^{ref}) and shear strain ($\gamma_{0.7}$) of the hardening soil (HS) and hardening soil–small soil (HSS) models are the key constitutive parameters. The step-by-step inverse method not only reduces the number of parameters, but also improves the predicting accuracy. The established empirical relationship between the E_{50}^{ref} and standard penetration test (SPT) blow counts exhibits a good linear correlation. The parameter selection method proposed in this study is an accurate, practical, and efficient method, which can effectively predict the horizontal displacement and surface settlement of the retaining structure in multiple excavation stages.

Keywords: braced excavation; constitutive model; parameter selection method; numerical simulation; standard penetration test (SPT)



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1. Introduction

The demand for underground structures is increasing with the rapid growth of the economy and population, especially for projects such as subways, underpasses, underground shopping streets, etc. However, due to limited space within cities, many underground projects are adjacent to the operating metro structures. Therefore, soil deformation induced by the braced excavation should be carefully controlled so as to minimize the impact on the existing metro system.

Numerical simulation analysis is an important method for assessing and predicting the soil deformation in foundation pit engineering. Finite element modeling (FEM) is widely used to assess foundation pits in the design stage [1–7]. The selection of the constitutive model and its parameters play a key role in the accuracy of numerical simulation. Currently, the commonly used constitutive models for braced excavation include the Mohr–Coulomb (MC) model, the hardening soil (HS) model, the hardening soil–small (HSS) model, etc. Among them, the HSS model contains all the model parameters of the HS model, as well as two parameters to control the stiffness of the soil in a small-strain state, i.e., the initial shear modulus G_0^{ref} and shear strain $\gamma_{0.7}$. The HS and HSS models have been widely adopted due to their ability to accurately simulate the stress–strain relationship of soil [8–13]. However, to accurately predict the soil deformation caused by braced excavation during the design

stage of pit engineering, so as to carry out a safe and economical design that retains the structure of the pit, the selection of constitutive model parameters appears to be crucial.

The values of the constitutive parameters of a soil are usually determined by laboratory tests. A large number of scholars have carried out experimental studies to determine the parameters of constitutive models and the proportional relationships among the parameters. For example, Liang et al. [14] and Wang et al. [15,16] obtained the parameters and the proportional relationship of the HSS model and HS model for typical soft soil layers in Shanghai City through indoor experiments. However, the significant number of experiments required to determine the constitutive parameters of the HS and HSS models, coupled with the lengthy test period and substantial uncertainties involved, poses a challenge for achieving an effective and accurate deformation assessment of foundation pits.

In contrast, the inverse analysis method can accurately invert soil parameters by using only the field-measured data without the need to conduct complicated indoor tests. This approach is widely recognized as an effective technique for determining the parameters of soil constitutive models [17,18]. Many scholars have used inverse analysis to determine soil parameters, such as Wang et al. [19], who proposed an inverse analysis method to quantify the three-dimensional effect of excavation, and corrected the prediction based on plane strain. Mu et al. [20] proposed an inverse analysis method that could predict the three-dimensional displacement of soil caused by braced excavation. Kim et al. [21,22] used the HSS and HC models to invert the constitutive parameters for the sidewall deformation of a deep foundation pit in Chicago, and analyzed the stress–strain characteristics of the soil layer during the excavation of the foundation pit. Hashash et al. [23] proposed a three-dimensional inverse analysis method for foundation excavation and successfully predicted the deformation of the sidewalls. Li et al. [24] used a general selection method for the parameters of the HSS model using the inverse analysis method, and compared the applicability of the MC and the HSS models under different excavation depths. In addition, it has been shown that the compressive modulus of the soil has a good linear relationship with the standard penetration test (SPT) blow counts [25–27]. Overall, the inverse analysis can be used to obtain the constitutive parameters, and then the empirical relationship between the constitutive parameters and the field test parameters can be established to calibrate the soil constitutive parameters in a simpler way.

Although the inverse analysis method based on field-measured data is quite mature, there are drawbacks such as the cumbersome steps of parameter inversion, too many optimization parameters, and the established empirical relationships not being easy to obtain. Therefore, it is of great significance to propose a selection method for constitutive parameters with simple steps and high accuracy to satisfy the design requirements.

To solve these limitations, this study takes a foundation pit excavation project adjacent to a subway in Hangzhou as an example. A step-by-step inverse analysis method was proposed, initially determining the ratio of important constitutive parameters and subsequently calculating the key constitutive parameters. Then, the empirical relationship between the optimized key constitutive parameters and the blow counts (N) of the in situ standard penetration test (SPT) was established. The selection method for constitutive parameters of typical silt–sandy soil in Hangzhou City was obtained. This study provides an important reference and insights for the assessment and design of the braced excavation by numerical simulation.

2. SPT-Based Determination Method for Constitutive Parameters

In order to improve the accuracy, simplicity, and practicability of numerical modeling, this study proposes an SPT-based method for selecting the parameters of the constitutive model. Figure 1 is the computational flowchart of the method. Three main steps are involved: firstly, the key parameters of the constitutive model are obtained through the sensitivity and correlation analyses. Then, the optimal ratios between the key parameters and the rest of the important parameters are obtained by using the multi-parameter optimization method. The inverse analysis using the monitoring data of braced excavation is

then carried out to obtain the optimized constitutive parameters, and finally the optimized parameters are empirically related to the SPT blow counts. The proposed method in this study is a step-by-step inverse analysis method. The method is proposed to reduce the number of optimized parameters and improve the efficiency of parameter optimization.

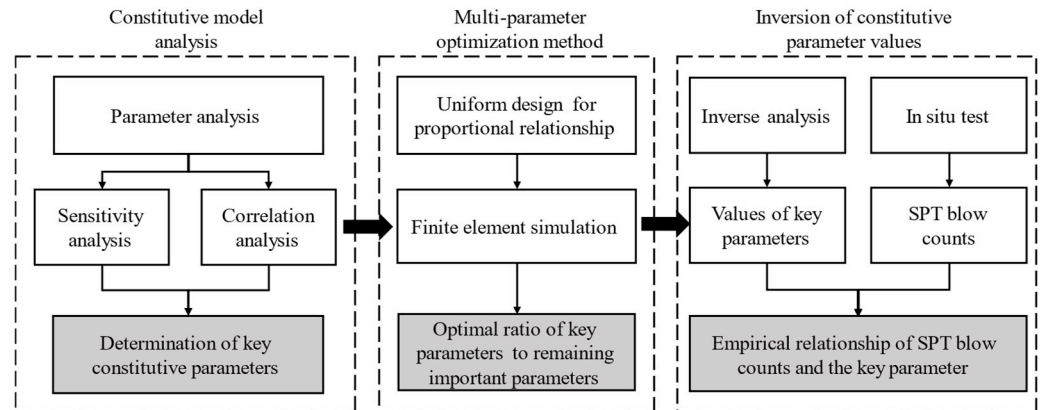


Figure 1. The flowchart for the SPT-based step-by-step inverse method for constitutive parameters.

2.1. Parameter Analysis of Constitutive Model

The HS and HSS constitutive models were adopted to simulate the braced excavation of soft soil, which contain 11 and 13 model parameters, respectively. According to the literature review [11,15,24,28], among the constitutive parameters of the HS and HSS models, the strength parameters cohesion c and internal friction angle φ have little effect on the deformation of the retaining structure, while the large strain parameters, namely reference secant stiffness E_{50}^{ref} , reference tangent stiffness E_{oed}^{ref} , and reference Young's modulus for unload–reload E_{ur}^{ref} , as well as the small strain parameters, namely initial shear modulus G_0^{ref} and shear strain $\gamma_{0.7}$, are important parameters. Therefore, this study focuses on these important parameters to predict the deformation induced by braced excavation. On the other hand, a proportional relationship exists among the important parameters of the HS and HSS models. Using inverse analysis for all the parameters will easily lead to multiple solutions and lack of intrinsic correlations among these important parameters. Therefore, a sensitivity analysis was conducted in this study to obtain the most sensitive parameters to the soil displacement. The correlation analyses of the constitutive parameters were then analyzed to obtain the key parameters that could accurately characterize the soil displacement caused by braced excavation.

In this study, a one-factor sensitivity analysis was used, in which only one of the corresponding parameters was varied and the rest of the parameters were kept unchanged during the sensitivity analysis. The parameters were assumed to be independent of each other in the one-factor sensitivity analysis. The correlation coefficients of the parameters reflect whether the estimated parameter values are unique and whether the parameters should be optimized simultaneously. The Pearson's correlation coefficient (PCC) was adopted to measure the strength and direction of the linear relationship between two variables. If the absolute value of the PCC between two parameters is greater than 0.9, these two parameters cannot be optimized at the same time. The definition formula for the PCC is:

$$PCC = cor(i, j) = \frac{cov(i, j)}{var(i)^{1/2} \bullet var(j)^{1/2}} \quad (1)$$

where cov is the covariance between the parameters, and var is the standard deviation of the parameters.

2.2. Multi-Parameter Optimization Method

In this study, the proportional relationship between the key parameters of the constitutive model and remaining important parameters is firstly determined that can effectively characterize the deformation characteristics of braced excavation, so as to realize the indirect optimization of other important parameters. The parameter analysis in this stage is based on the proportional relationship obtained from the laboratory test. The proportional relationship of the key parameters to the remaining important parameters is determined in the first step, and then the next step is the determination of the key parameters by the inverse analysis using the measured data (see Section 2.3).

Uniform design can effectively deal with multi-factor and multi-level tests and requires fewer tests [29]. The uniform design method was adopted to design a scheme for the proportional relationship between multiple sets of parameters. The coincidence index (W) was proposed in this study as the objective function, which characterizes the similarity between the numerical simulation curve and the measured curve. The coincidence index is defined as:

$$W = \frac{R^2}{M} \quad (2)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (3)$$

$$M = \sqrt{\Delta s^2 + \Delta d^2} \quad (4)$$

where R^2 is the goodness of curve fitting; y_i is the measured value of lateral displacement of the retaining wall; $\hat{y}_i$ is the simulated value; M is a parameter reflecting the shape similarity between y_i and $\hat{y}_i$; Δs is the value difference in the maximum lateral displacement between the measured and simulated value; and Δd is the position difference in the maximum lateral displacement between the measured and simulated value.

The coincidence index (W) is an index that can reflect the numerical size and shape similarity between the simulated and measured curve. A higher W value indicates a more suitable proportional relationship. Then, the optimal ratio of the key parameters to the other important constitutive parameters can be obtained, which provides the basis for the inverse analysis of the key parameters in the last step.

2.3. SPT-Based Inverse Method for Constitutive Parameters

In this study, the programming software coupled with Macro was adopted to call the finite element program for multiple inverse calculations for the key constitutive parameters. The numerical simulation was carried out and the output results were stored, i.e., the ground surface settlement and lateral displacement of the retaining wall. In the inverse analysis, the parameter input of the constitutive model was updated iteratively and stopped until the optimized model reached the best fit with the field measurement, namely minimizing the objective function $S(b)$ defined below:

$$S(b) = [y' - y'(b)]^T \omega [y' - y'(b)] = e^T \omega e \quad (5)$$

where b is the parameter vector to be optimized, y is the measured result vector, $y'(b)$ is the calculation result vector, ω is the weight matrix, and e is the residual vector.

Finally, the empirical relationship between the optimized parameters and the corresponding SPT blow counts in the same soil layer were established. The proposed SPT-based step-by-step inverse method for the constitutive model parameters is summarized in Figure 1.

3. Case Study and Numerical Modeling

3.1. Construction Site

This study takes a high-rise building in Hangzhou, Zhejiang Province as a case study. The nearest distance between the building under construction and the existing subway line is only 20 m. Therefore, the requirements for the construction and monitoring of the foundation pit are strict and of high quality. The layout and the monitoring scheme of the foundation pit are shown in Figure 2. The inclinometer monitors the deformation of the sidewall, and the total station monitors the surface settlement. There are four small foundation pits and one large foundation pit according to the braced excavation plan. The sequence for the excavation is shown in the figure, marked with 1 to 5. The high-rise building has a three-story basement, and the designed excavation depth of the foundation pit is 13.9 m. According to the design, the supporting system of the foundation pit is composed of a diaphragm wall with a thickness of 800 mm and a height of 44.1 m. Within the excavation depth, there are three concrete internal supports with a horizontal spacing of 8 m (Figure 3). Each concrete internal support is arranged 0.5 m above the excavation face.

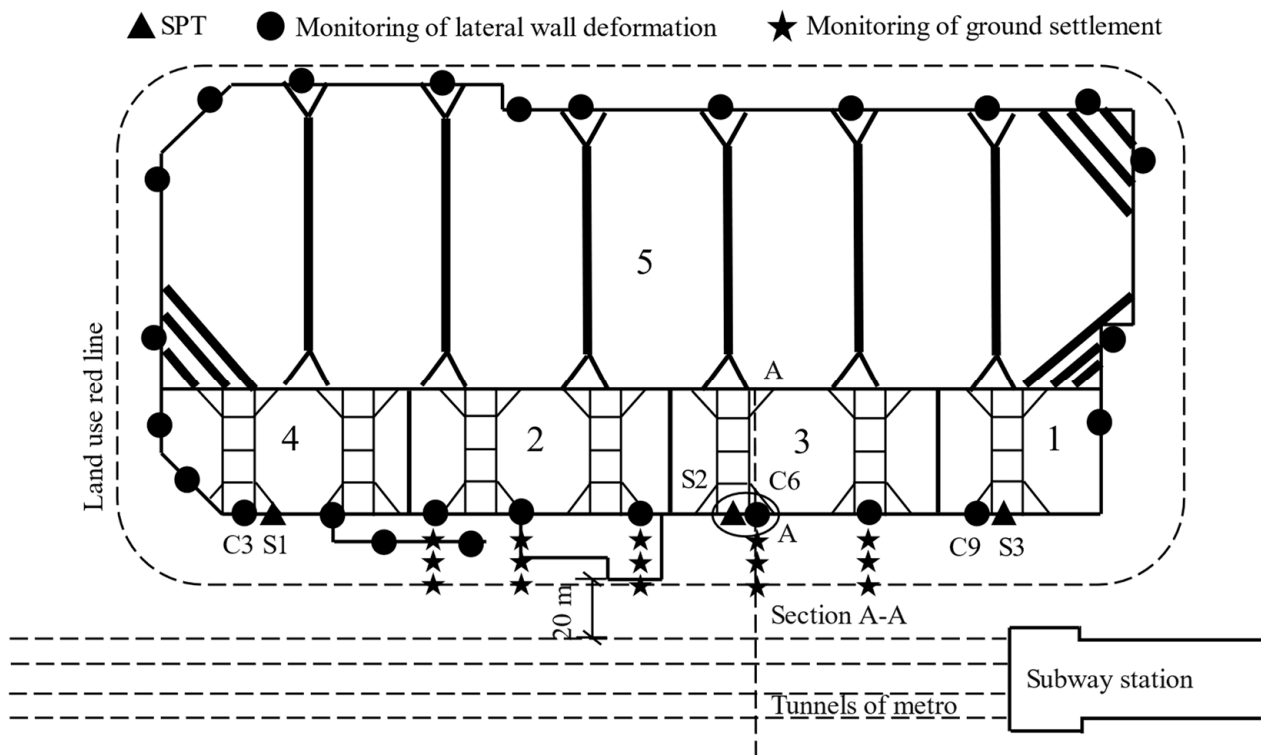


Figure 2. Layout of the foundation pit and its monitoring scheme.

The excavated soil layers from top to bottom are fill, clayey silt, sandy silt, sandy silt mixed with silt, muddy clay, silty clay, and rounded gravel (Figure 3). The groundwater level of the construction site is located 0.5 m below the ground surface. The groundwater level had been artificially reduced to the bottom of the foundation pit before the excavation.

In addition, the in situ SPTs were carried out at the stage of the field investigation (Figure 2). The blow counts of the soil layer at every 2 m were measured. The SPT results at borehole S2 are shown in Figure 4. In the same soil layer, the SPT blow count increases with the increase in buried depth. There are no data for the muddy clay layer and the rounded gravel layer as the SPT was not suitable.

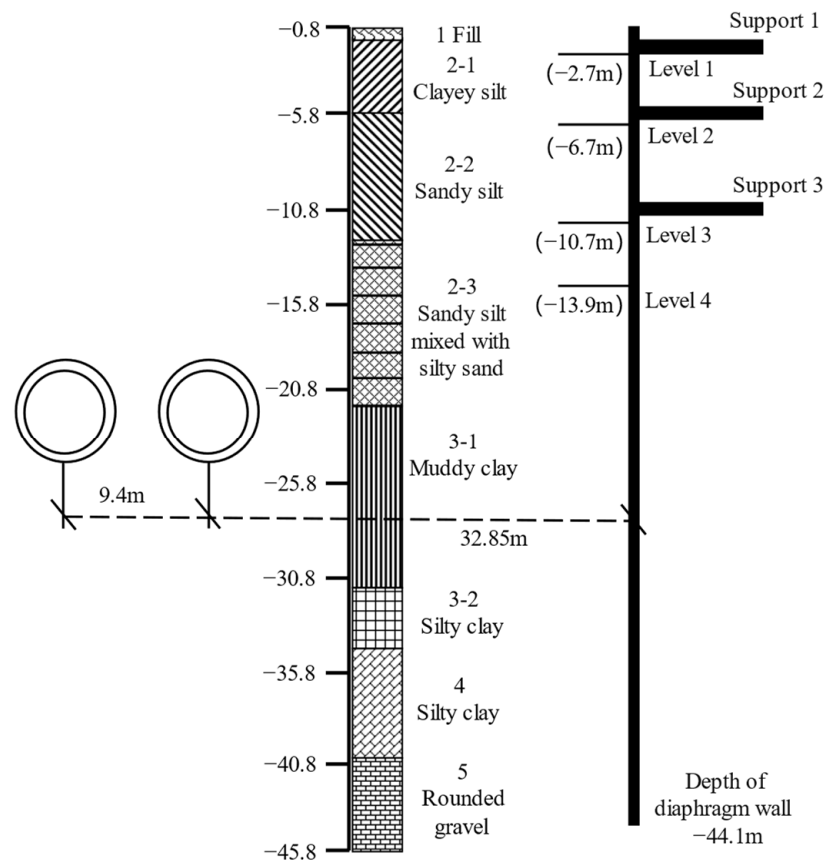


Figure 3. Profile of the foundation pit and the soil layers (section A-A).

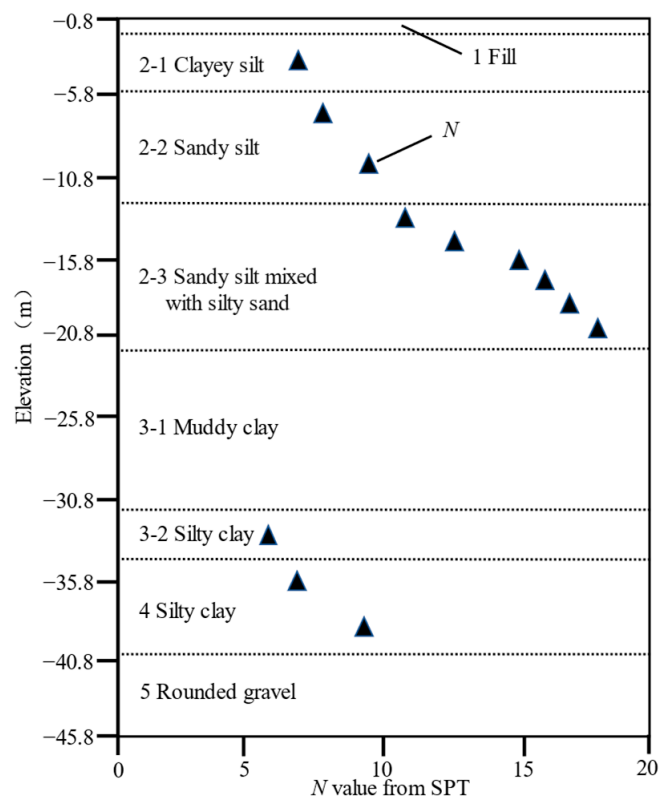


Figure 4. The soil layer and SPT results at borehole S2.

3.2. Finite Element Modeling

In this study, the section A-A of the foundation pit near the subway tunnel (Figure 2) was selected for finite element modeling by PLAXIS 2D. This section is located in the middle of the pit, and thus the deformation of the retaining structure and the soil at this section can be treated as plane strain. According to the literature review, the HS and HSS constitutive models can accurately characterize the stress–strain changes in soil caused by braced excavation. Therefore, these two constitutive models were selected. In the established finite element model, the HSS model was adopted to simulate muddy clay, silty clay, and miscellaneous fill, and the HS model was used for the remaining soil layers. The initial soil parameters are shown in Table 1, where the unit weight γ , cohesion c , and friction angle φ were selected from the soil survey report. The initial values of the constitutive parameters were determined according to the literature and engineer’s design experience [14–16,30].

Table 1. Initial values of the physical and mechanical parameters for the constitutive model.

Soil Layer No.	Constitutive Model	γ (kN/m ³)	c (kPa)	φ (°)	m^*	R_f^*	p^{ref}	ν^*	K_0^*	E_{oed}^{ref} (MPa)	E_{50}^{ref} (MPa)	E_{ur}^{ref} (MPa)	$\gamma_{0.7}$ ($\times 10^{-4}$)	G_0^{ref} (MPa)
1	HSS	16.1	14.6	8.3	0.7	0.9	100	0.2	0.826	3.0	3.0	12.0	2.0	36.0
2-1	HS	18.6	3.0	31.6	0.8	0.9	100	0.2	0.476	9.0	9.0	36.0	/	/
2-2	HS	18.6	1.9	32.0	0.5	0.9	100	0.2	0.478	9.5	13.3	40.0	/	/
2-3	HS	18.7	1.7	32.3	0.5	0.9	100	0.2	0.481	10.5	14.7	45.0	/	/
3-1	HSS	19.5	8.4	27.0	0.8	0.9	100	0.2	0.546	2.5	3.8	37.5	3.2	75.0
3-2	HSS	19.6	5.8	30.4	0.8	0.9	100	0.2	0.494	5.0	5.0	35.0	3.2	70.0
4	HSS	19.9	5.0	31.0	0.8	0.9	100	0.2	0.485	6.5	6.5	45.5	3.2	91.0
5	HS	21.0	3.0	35.0	0.5	0.9	100	0.2	0.426	20.0	20.0	80.0	/	/

* where m is the power for the stress-level dependency of stiffness; R_f is the failure ratio; p^{ref} is the reference stress for stiffnesses; ν is Poisson’s ratio; and K_0 is the K_0 -value for normal consolidation.

The schematic diagram of the finite element model is shown in Figure 3, where the retaining wall and the operating subway tunnel were simulated by plate elements. The three concrete supports in the pit were simulated by anchor elements. The material properties are listed in Table 2. There is contact between the underground structure (retaining structure and tunnel lining) and the soil. The contact between the structure and the soil was set by the reduction coefficient R_{inter} to reduce the strength and stiffness, as defined in Equation (6). In this study, the value of R_{inter} is 0.7.

$$\tan \varphi_i = R_{inter} \cdot \tan \varphi_{soil} \quad (6)$$

where φ_i is the friction angle between the structure and soil; and φ_{soil} is the friction angle of the soil.

According to the excavation sequence of the foundation pit, the numerical simulation was divided into 10 construction stages, as shown in Table 3. All calculations were carried out under the assumption of undrained conditions, without considering the time effect or material effect of pore water pressure changes.

Table 2. Material parameters for FEM.

Structural Parameters	Support Wall	Strut
Bending stiffness, EI (kN·m ² /m)	1,280,000	/
Axial stiffness, EA (kN/m)	24,000,000	19,200,000
Poisson’s ratio (ν)	0.2	/
Horizontal spacing (m)	/	8

Table 3. Construction stages.

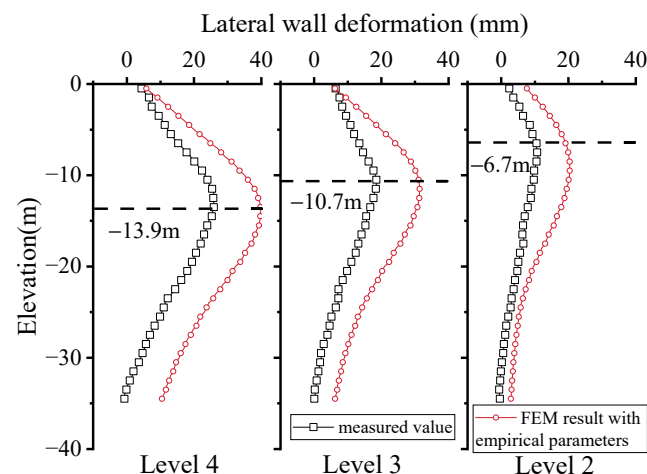
Calculation Phase	Construction Activity
1	Self-weight stress generation
2	Tunnel construction (displacement reset)
3	Construction of the diaphragm wall
4	Excavation level 1 (−1.9 m)
5	Installation of the first brace
6	Excavation level 2 (−5.9 m)
7	Installation of the second brace
8	Excavation level 3 (−9.9 m)
9	Installation of the third brace
10	Excavation of foundation pit 4 (−13.9 m)

4. Results and Discussion

4.1. Selection of Key Constitutive Parameters

Figure 5 is the result of the horizontal displacement of the retaining wall calculated by the empirical parameters of the constitutive model (Table 1). In the level 4, 3, and 2 excavation stages, the horizontal displacements of the retaining wall calculated by the empirical values of the constitutive model are roughly consistent with the measured value with respect to the curve shape. This indicates that the selected constitutive models for the soil layers can effectively characterize the deformation of the retaining wall caused by the braced excavation. The HS model is suitable for silt, and the HSS model is suitable for clay in this case.

However, the calculated values in each excavation stage are quite different from the measured values, and these differences are the largest at the excavation face. The calculated value overestimates the lateral displacement of the retaining wall, indicating that the value of the initial soil parameters is seriously underestimated. Specifically, in the level 4, 3, and 2 excavation stages, the relative error between the calculated value and the measured value of the maximum horizontal displacement is 52%, 65%, and 71%, respectively. According to the inherent soil variability, the predicting accuracy of the maximum horizontal displacement of the retaining wall should be within $\pm 15\%$ [31]. Therefore, the simulated result using the constitutive parameters determined by the soil survey report and the engineer's design experience overestimates the horizontal displacement of the retaining wall, which cannot meet the requirements of prediction accuracy.

**Figure 5.** Horizontal displacement of the retaining wall and the measured values.

In order to improve the accuracy of numerical simulation, the large strain parameters E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} shown in Table 1, which mainly control the deformation of soil, were selected as candidate important parameters to optimize the large strain response of the simulation. Meanwhile, the small strain parameters G_0^{ref} and $\gamma_{0.7}$, which control the shape of the shear stiffness degradation curve, were also selected as candidate important parameters. By the sensitivity and correlation analysis of these five important parameters, the key constitutive parameters controlling the deformation of the braced excavation can be determined.

Figure 6 shows the sensitivity analysis of five important parameters for the sandy silt layer, muddy clay layer, and silty clay layer. The y axis of Figure 6 represents the relative change rate, which is defined as $100\% \times | \text{the changed value} - \text{the original value} | / \text{the original value}$. The relative change rate reflects the impact of changing the constitutive parameters on the maximum horizontal displacement of the retaining wall during the fourth stage of braced excavation. The higher the relative change rate is, the more sensitive the parameter is. It can be seen that G_0^{ref} and $\gamma_{0.7}$ are the most sensitive parameters among all parameters. Among the three soil layers, E_{50}^{ref} is the most sensitive large strain parameter, which is consistent with the existing work [28]; E_{ur}^{ref} is the second-most sensitive large strain parameter, which is slightly more sensitive than E_{oed}^{ref} .

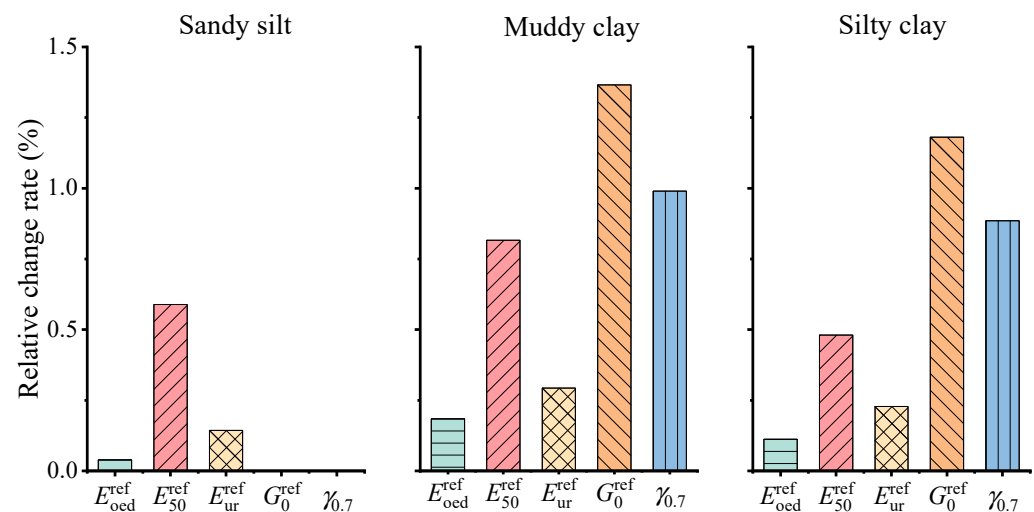


Figure 6. Sensitivity analysis of constitutive parameters.

Figure 7 shows a correlation analysis of the constitutive model parameters for the sandy silty soil layer, muddy clay layer, and silty clay layer in the fourth stage of the braced excavation. E_{50}^{ref} , G_0^{ref} , and $\gamma_{0.7}$ were selected for correlation analysis as they are the most sensitive parameters, as reflected in Figure 6. It is found that the PCC values between the large strain parameters in the sandy silt layer are less than 0.9, so these three parameters can be optimized at the same time. In the muddy clay layer, the correlation between E_{50}^{ref} and G_0^{ref} is high and cannot be optimized at the same time. In the silty clay layer, although the correlation between E_{50}^{ref} and G_0^{ref} is slightly lower than 0.9, it is still relatively close. The $\gamma_{0.7}$ can be optimized simultaneously with the other two parameters in any case. Therefore, the parameter E_{50}^{ref} that controls the large strain and the parameter $\gamma_{0.7}$ that controls the small strain are selected as the key parameters for subsequent parameter optimization and inverse analysis.

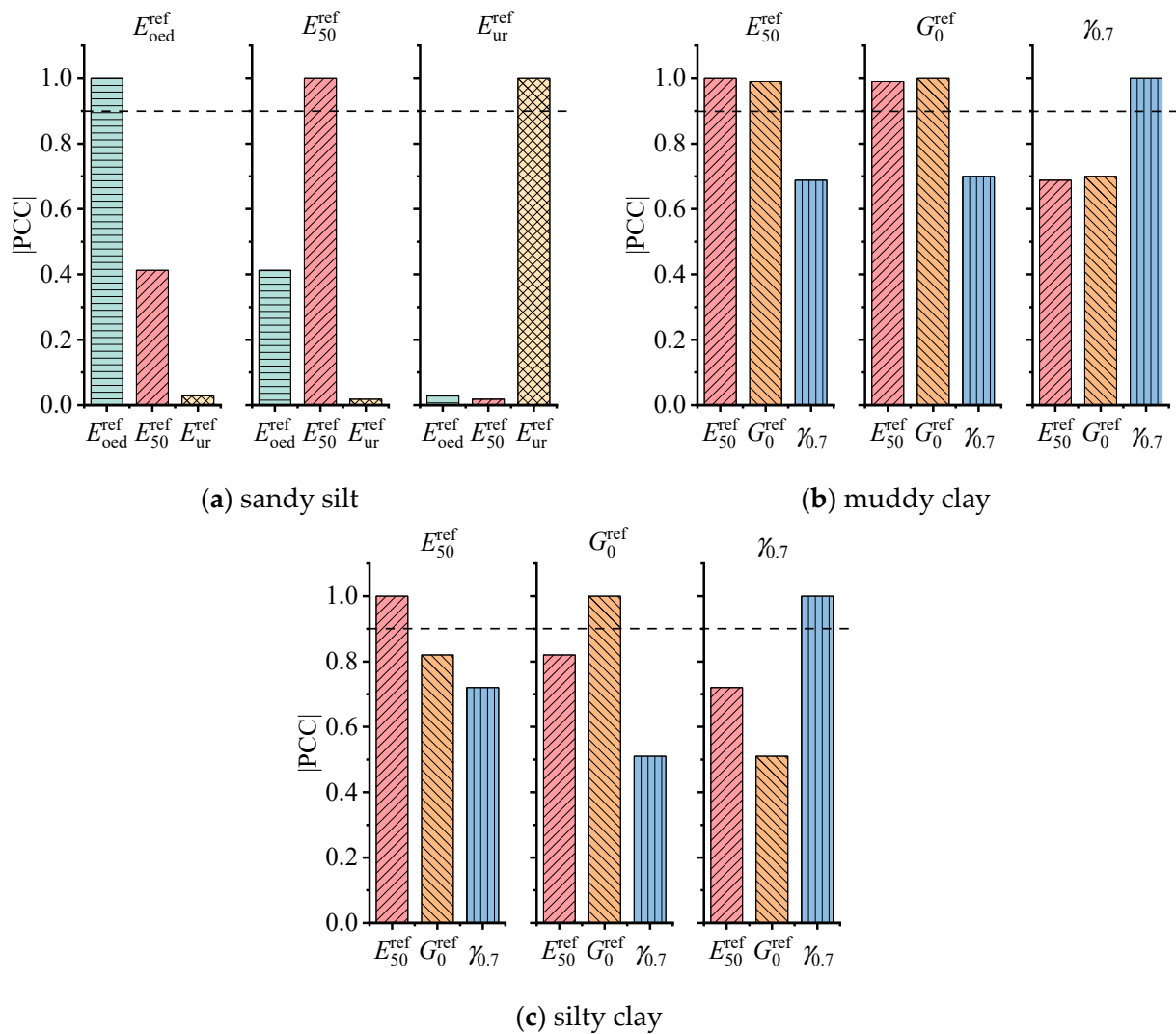


Figure 7. Correlation analysis of constitutive model parameters.

4.2. Inversion Results of Constitutive Model Parameters

4.2.1. Multi-Parameter Optimization Analysis

The proposed step-by-step inverse method is adopted to solve the constitutive model parameters. The first step is to determine the proportional relationship between the key parameters and the remaining important parameters of the constitutive model. The key constitutive parameters in this case study are E_{50}^{ref} and $\gamma_{0.7}$, and the remaining important parameters are E_{oed}^{ref} , E_{ur}^{ref} , and G_0^{ref} . According to an extensive indoor experimental literature review [14–16,30], a proportional relationship exists between the modulus of compression E_{s1-2} , E_{50}^{ref} , E_{oed}^{ref} , E_{ur}^{ref} , and G_0^{ref} , where the E_{s1-2} can be usually obtained from a soil survey report. As E_{50}^{ref} is the key parameter, the range of the ratio of E_{50}^{ref} to E_{s1-2} , E_{oed}^{ref} , E_{ur}^{ref} , and G_0^{ref} are listed in Table 4. The ranges of $E_{50}^{ref}:E_{s1-2}$ and $E_{50}^{ref}:E_{oed}^{ref}$ for mucky clay and silty clay are small, and thus the average is adopted in this study.

In order to reduce the number of numerical tests, the uniform design table $U_{12}^*(12^{10})$ was adopted to determine the optimal proportional relationship for the tested soil. The seven factors of the uniform design table were selected, and the uniformity deviation was 0.2768, which met the sampling requirements. The design schemes of different ratio combinations are shown in Table 4.

Table 4. The uniform design scheme for the ratio optimization.

Soil Type	Sandy Silt			Muddy Clay				Silty Clay			
Ratio	$E_{50}^{ref}:E_{s1-2}$	$E_{50}^{ref}:E_{oed}^{ref}$	$E_{ur}^{ref}:E_{50}^{ref}$	$E_{50}^{ref}:E_{s1-2}$	$E_{50}^{ref}:E_{oed}^{ref}$	$E_{ur}^{ref}:E_{50}^{ref}$	$G_0^{ref}:E_{50}^{ref}$	$E_{50}^{ref}:E_{s1-2}$	$E_{50}^{ref}:E_{oed}^{ref}$	$E_{ur}^{ref}:E_{50}^{ref}$	$G_0^{ref}:E_{50}^{ref}$
Range	0.9–2.8	0.9–1.4	3–5.6	0.82–1.34	1.08–1.38	6.6–11	6.6–22	0.92–1.1	0.93–1.4	5.8–8.4	5.8–16.8
1	0.9	0.945	4.20	1.08	1.2	9.4	17.8	1	1.15	7.96	16.8
2	1.07	1.035	5.64			7.4	12.2			7.24	15.8
3	1.24	1.125	3.96			10.6	6.6			6.52	14.8
4	1.41	1.215	5.40			8.6	19.2			5.80	13.8
5	1.58	1.305	3.72			6.6	13.6			8.20	12.8
6	1.75	1.400	5.16			9.8	8.0			7.48	11.8
7	1.92	0.900	3.48			7.8	20.6			6.76	10.8
8	2.09	0.990	4.92			11.0	15.0			6.04	9.8
9	2.26	1.080	3.24			9.0	9.4			8.44	8.8
10	2.43	1.170	4.68			7.0	22.0			7.72	7.8
11	2.6	1.260	3.00			10.2	16.4			7.00	6.8
12	2.8	1.350	4.44			8.2	10.8			6.28	5.8

According to the above 12 groups of proportional relationships, the finite element model was used to analyze the horizontal displacement at the fourth stage of braced excavation. At the same time, the calculated values were compared with the measured values, and the corresponding coincidence degrees (W) under various proportions were obtained. The results are shown in Table 5. As the inverse analysis at this stage is mainly to obtain the optimal ratio among the important parameters, the degree of fit between the simulated curve and the measured curve is not high, resulting in a negative value for the goodness of fit R^2 . Comparing the W values for the 12 tests in Table 4, the fourth group has the largest W value, indicating that the proportion of constitutive parameters best aligns with the measured value, considering both the shape and the maximum value of the deformation curve.

Table 5. Multi-parameter optimization results.

Parameter	1	2	3	4	5	6	7	8	9	10	11	12
R^2	−5.390	−5.413	−6.651	−0.752	−5.301	−4.557	−1.462	−1.602	−5.322	−0.252	−2.316	−2.203
M	23.302	20.362	23.098	6.996	19.297	17.916	8.513	8.762	19.355	2.008	10.146	10.440
W	−0.231	−0.266	−0.288	−0.107	−0.275	−0.254	−0.172	−0.183	−0.275	−0.126	−0.228	−0.211

4.2.2. Parameter Inversion of Constitutive Model

Existing studies have shown that the stiffness parameters of soil decrease with increasing strain and exhibit nonlinear changes, and a single and constant stiffness parameter value cannot describe the stress–strain response throughout the entire excavation phase [32]. In order to obtain a method to determine the constitutive parameters and initially assess and predict the deformation caused by foundation pit excavation during the design stage, this study conducts a inverse analysis of parameters using the measured deformation values of the foundation pit during the final stage of excavation, as the soil stiffness has basically evolved to its final state by this time.

Using the optimal ratio obtained above and the E_{s1-2} value derived from laboratory soil tests, the numerical model predicted the horizontal displacement of the retaining wall, which are shown as the blue hollow triangles in Figure 8, where the black squares

represent the measured value. At excavation stages 4, 3, and 2, the differences between the calculated and measured maximum horizontal displacements of the retaining wall above the excavation face are 1.59 mm, 4.22 mm, and 8.33 mm, respectively, accounting for 34.1%, 22.9%, and 13.1% of the measured values. Compared to the numerical results calculated using the empirical parameters (as shown in Figure 5), the prediction results have been significantly improved. However, below the excavation face, the calculated values are generally too large, failing to meet the prediction accuracy requirements. This indicates that relying solely on the proportional relationship between soil constitutive parameters cannot fully satisfy the prediction requirements, and the soil constitutive parameters are still underestimated.

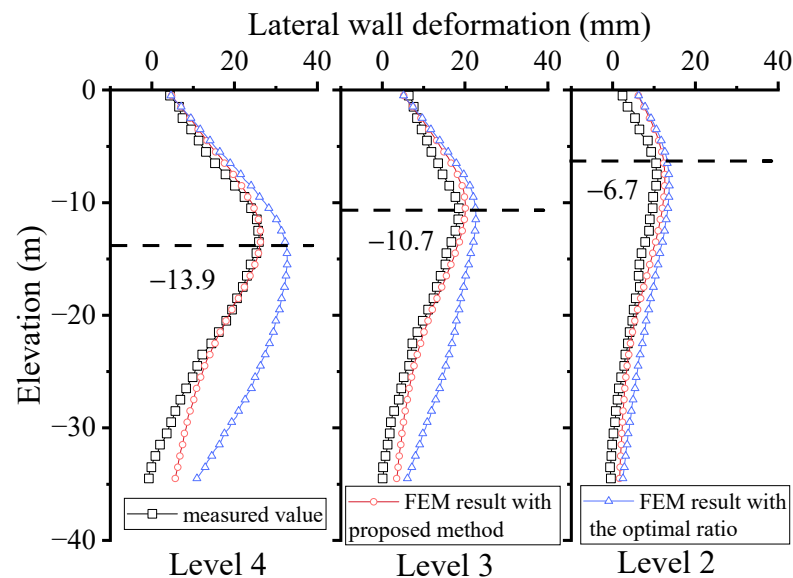


Figure 8. Horizontal displacement of retaining wall.

Based on the obtained optimal proportional relationship, this study further inverted the values of key constitutive parameters. The calculated horizontal displacement of the retaining wall by the numerical model is shown as red hollow circles in Figure 8. In the fourth stage of excavation, the calculated results on the excavation face align well with the measured results. However, below the excavation face, especially at depths exceeding 25 m, there is a discrepancy of approximately 5 mm between the calculated and measured results. This is due to the underestimation of passive resistance, which leads to an overestimation of the lateral displacement of the piles below the excavation face. The relative error between the calculated and measured values on the excavation face in the third and second stages of excavation are 8.5% and 14.1%, respectively. Although the degrees of agreement between the calculated and measured results in the first two excavation stages are not as good as in the fourth stage, they still fall within the $\pm 15\%$ range, meeting the prediction accuracy requirements. Therefore, in this case, the optimized constitutive parameters from the fourth stage of excavation can be adopted to predict the lateral displacement of the retaining wall in the third and second stages of excavation, indicating that the proposed step-by-step inverse method for the constitutive model parameters is feasible.

In addition, to establish the relationship between soil constitutive parameters and the SPT blow counts, this study conducted layered modeling of the sandy silt mixed with silty sand, which located at the bottom of the foundation pit in the aforementioned numerical model. The layer thickness was set to 2 m to maintain consistency with the sampling interval of SPT. Consequently, six sets of constitutive parameters were optimized for this soil layer. The optimized parameters using the proposed method are presented in Table 6.

Table 6. Optimization results of the constitutive parameters.

Soil Layer	E_{oed}^{ref} (MPa)	E_{50}^{ref} (MPa)	E_{ur}^{ref} (MPa)	G_0^{ref} (MPa)	$\gamma_{0.7}$
2-1 Clayey silt	9.4500	9.450	37.800	/	/
2-2 Sandy silt	12.345	15.000	81.000	/	/
2-3 Sandy silt mixed with silty sand 1	13.179	16.012	86.465	/	/
2-3 Sandy silt mixed with silty sand 2	13.251	16.100	86.940	/	/
2-3 Sandy silt mixed with silty sand 3	13.338	16.206	87.512	/	/
2-3 Sandy silt mixed with silty sand 4	13.419	16.304	88.041	/	/
2-3 Sandy silt mixed with silty sand 5	13.507	16.411	88.619	/	/
2-3 Sandy silt mixed with silty sand 6	13.589	16.507	89.138	/	/
3-1 Muddy clay	4.167	5.000	43.000	96.000	2.3×10^{-4}
3-2 Silty clay	6.087	7.000	40.600	96.600	2.7×10^{-4}
4 Silty clay	6.087	7.000	40.600	96.600	2.7×10^{-4}

4.3. Empirical Relationship with the SPT Blow Counts

For the sandy silt mixed with silty sand, six sets of key parameters E_{50}^{ref} of the soil layers were retrieved using the aforementioned method, and an empirical relationship was established between these parameters and the SPT blow counts (N value) of the corresponding soil layers. The results are presented in Figure 9. The R^2 value of the goodness of fit is 0.93, indicating a satisfactory curve fitting effect. It can be concluded that there is a good linear relationship between the compression modulus E_{s1-2} and the SPT blow count (N) of the soil layer, as represented by Equation (7). This linear relationship is consistent with the existing literature [25–27]. Additionally, since there is a certain proportional relationship between E_{50}^{ref} and E_{s1-2} (in Table 4), the constitutive parameters can be accurately determined. The E_{50}^{ref} and N values obtained through step-by-step inversion analysis can serve as an empirical formula for determining the constitutive parameters of the soil. The proposed method for determining the constitutive parameters can be applied to other soil layers, providing a basis for accurate numerical analysis during the design phase.

$$E_{50}^{ref} = 61.72N + 15341.15 \quad (7)$$

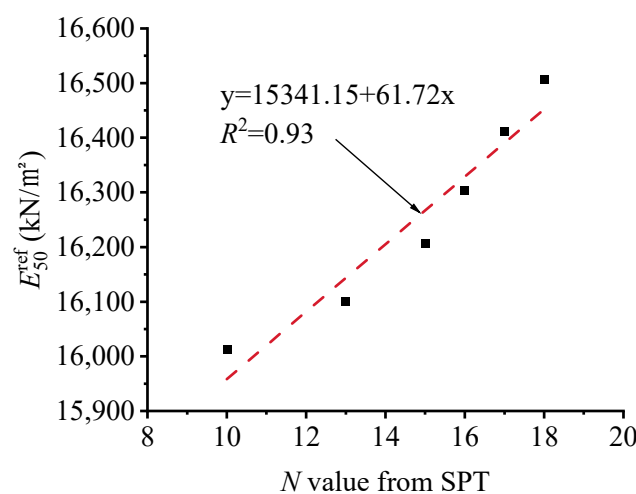


Figure 9. The empirical relationship between the inversion parameter E_{50}^{ref} and SPT blow counts (sandy silt mixed with silty sand).

The ground settlement measurements at section A-A were adopted to verify the effectiveness of the proposed SPT-based step-by-step inverse method for the constitutive parameters. As shown in Figure 10, the ground surface settlements predicted by the proposed method agree well with the measurements in the level 4, 3, and 2 excavation stages. The proposed method can not only predict the horizontal displacement of the retaining wall but also the ground settlement at different excavation stages. Overall, the parameter selection method of the constitutive model proposed in this paper is an accurate, practical, and efficient method.

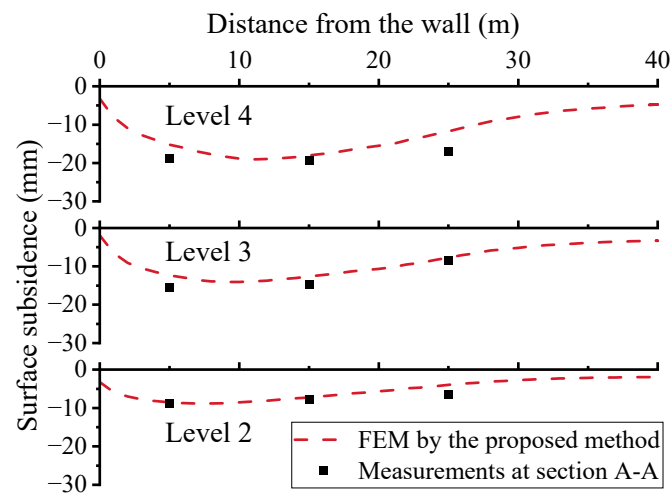


Figure 10. Measured ground surface settlement and simulated value based on the proposed method.

5. Conclusions

Based on the braced excavation project in Hangzhou, this study proposes an step-by-step inverse method for identifying the proportional relationships among the important constitutive parameters and then the values of key constitutive parameters. An empirical relationship between the inversion parameter and the SPT blow counts was established. The following conclusions can be drawn:

- (1). Through sensitivity and correlation analysis of the important parameters in the constitutive models, E_{50}^{ref} and $\gamma_{0.7}$ are identified as the key parameters in the HS and HSS constitutive models. Given the relatively small strain associated with foundation pit excavation, the sensitivity of small strain parameters is significantly higher than that of large strain parameters.
- (2). The step-by-step inverse method not only reduces the number of parameters, but also improves the predicting accuracy. The maximum horizontal displacement of the retaining wall predicted by the proposed method is within $\pm 15\%$ of the measured value.
- (3). For the soil layer composed of sandy silt mixed with silty sand, the goodness of fit R^2 between the key parameter E_{50}^{ref} of the HS model and the SPT blow counts was found to be 0.93, indicating a good linear correlation between them.
- (4). The proposed SPT-based step-by-step inverse method can effectively predict the horizontal displacement of the retaining wall and the ground settlement at multiple excavation stages, demonstrating that it is an accurate, practical, and efficient method for parameter determination.

The proposed method has proven to be effective and provides insights for the selection of the constitutive model parameters. The empirical relationship established in this study can be applied to the parameter selection of a similar soil layer around the construction site. Future work is suggested to investigate the influence of multiple soil layers, supporting system, and excavation method. Consequently, it is necessary to collect more foundation pit case studies and conduct parameter analysis to further enhance the current findings.

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