

Article

A New Fuzzy Backstepping Control Based on RBF Neural Network for Vibration Suppression of Flexible Manipulator

Zhiyong Wei ^{1,2,3}, Qingchun Zheng ^{1,2}, Peihao Zhu ^{1,2,*}, Wenpeng Ma ^{1,2} and Jieyong Deng ⁴

¹ Tianjin Key Laboratory for Advanced Mechatronic System Design and Intelligent Control, Tianjin University of Technology, Tianjin 300384, China; weijyon990320@stud.tjut.edu.cn (Z.W.); zhengqingchun@tjut.edu.cn (Q.Z.); wenpengma@email.tjut.edu.cn (W.M.)

² National Demonstration Center for Experimental Mechanical and Electrical Engineering Education, Tianjin University of Technology, Tianjin 300384, China

³ School of Mechanical Engineering, Tianjin University of Technology, Tianjin 300384, China

⁴ Jiangxi Technical College of Manufacturing, Nanchang 330095, China; dengjieyong1@jxmtc.edu.cn

* Correspondence: zhupeihao_gp@163.com

Abstract: Flexible manipulators have been widely used in industrial production. However, due to the poor rigidity of the flexible manipulator, it is easy to generate vibration. This will reduce the working accuracy and service life of the flexible manipulator. It is necessary to suppress vibration during the operation of the flexible manipulator. Based on the energy method and the Hamilton principle, the partial differential equations of the manipulator were established. Secondly, an improved radial basis function (RBF) neural network was combined with the fuzzy backstepping method to identify and suppress random vibration during the operation of the flexible manipulator, and the Lyapunov function and control law were designed. Finally, Simulink was used to build a simulation platform, three different external disturbances were set up, and the effect of vibration suppression was observed through the change curves of the final velocity error and displacement error. Compared with the RBF neural network boundary control method and the RBF neural network inversion method, the simulation results show that the effect of the RBF neural network fuzzy inversion method is better than the previous two control methods, the system convergence is faster, and the equilibrium position error is smaller.

Keywords: fuzzy backstepping control; RBF neural network; flexible manipulator; vibration suppression



Citation: Wei, Z.; Zheng, Q.; Zhu, P.; Ma, W.; Deng, J. A New Fuzzy Backstepping Control Based on RBF Neural Network for Vibration Suppression of Flexible Manipulator. *Appl. Sci.* **2024**, *14*, 6054. <https://doi.org/10.3390/app14146054>

Academic Editor: Yutaka Ishibashi

Received: 10 June 2024

Revised: 3 July 2024

Accepted: 8 July 2024

Published: 11 July 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

With lower power consumption, lighter weight and higher flexibility, flexible manipulators have been largely used in industrial production, and aerospace, search in some complex environments, thereby expanding the use of manipulators [1,2]. However, because flexible manipulators use flexible materials, the rigidity of the whole system is poor, and it is easy to produce vibration. Therefore, how to suppress the vibration of flexible manipulators has become one of the hottest issues now.

The vibration suppression of the flexible manipulator is divided into passive suppression in the early stage and active suppression in the present stage. In early passive suppression, accurate dynamic models were established, and the structure of the manipulator was optimized using the finite element method [3,4]. This method has a long development cycle and cannot guarantee a good vibration suppression effect.

Active suppression is to control manipulators through the control algorithm [5,6] to achieve vibration suppression of flexible manipulators. With the emergence of intelligent control algorithms, neural networks have also been applied to the vibration suppression of flexible manipulators. Li et al. [7] used the Particle Swarm Optimization (PSO) intelligent search algorithm and back propagation neural networks (BPNNs) to suppress the vibration of flexible manipulators. Jia et al. [8] proposed a neural network based adaptive integral

sliding mode observer to suppress the vibration of flexible space manipulators. Based on rigid-flexible manipulators, Liu et al. [9] proposed an adaptive neural network sliding mode control strategy to estimate unknown disturbances and uncertain parameters. Mei et al. [10] introduced radial basis neural network functions (RBNNFs) to cope with system parameter uncertainties and input saturations.

As a common control method, the boundary control method is often used to suppress the vibration of manipulators. Liu et al. [11] devised a boundary control approach to suppress distributed elastic deformation and vibration. However, there are still large errors in angle tracking. Zhou et al. [12] proposed an adaptive boundary iterative learning vibration control for a class of rigid-flexible manipulator systems under distributed disturbances and input constraints. However, the design of the control law is complicated. Li et al. [13] proposed a novel boundary control strategy for a vibrating single-link flexible manipulator system modeled using partial differential equations. In the design of the control law, the expression is still complicated, and there are some small high-frequency vibrations in the control input of the simulation results. Liu et al. [14] proposed two boundary control laws to manage vibration suppression and angular position tracking of manipulator systems. However, there are some problems in that the suppression effect of some parameters is not good.

As an intelligent control method, the fuzzy backstepping control method does not need an accurate mathematical model and has good adaptability to complex systems. Min et al. [15] proposed an adaptive fuzzy backstepping method that reflected the effect of vibration suppression mainly through the parameters of velocity and displacement tracking. However, the specific type of external interference is not clearly pointed out in the paper, and the parameters and state of the motor when the manipulator is running are not considered. Wan et al. [16] proposed an adaptive fuzzy backstepping control method for uncertain nonlinear systems. The paper still fails to consider the influence of specific external interference on the system in the simulation and does not consider the parameter setting of the motor. If the parameters of the motor are not considered, the reliability and authenticity of the simulation will be slightly reduced.

To sum up, the hybrid control method combining intelligent control and the traditional control method has become the main method for vibration suppression of manipulators at present. The boundary control method is also widely used in vibration suppression, but the design control law is complicated and sometimes the vibration suppression effect is not good. Most studies do not consider the systematic impact of the motor on the whole system. Based on these, the main contributions of this paper are as follows:

(1) A fuzzy backstepping control method based on an RBF neural network is proposed. An RBF neural network is combined with a backstepping control method, and unknown parameters are compensated for by the RBF neural network.

(2) A controller is designed, and nonlinear dynamic equations are established. The simulated platform was built using Simulink, and the vibration suppression effect was analyzed using two parameters: the error of displacement and the error of velocity.

(3) The dynamic equation of the flexible manipulator considering the motor is established, the specific parameters of the motor are established, and the influencing factors and dynamic characteristics of the system are considered and compared with the RBF neural network boundary control method and RBF neural network backstepping method.

The rest of the paper is as follows: In the second part, with a model approach for the identification of joint and link compliance of an industrial manipulator, partial differential equations of the flexible manipulator are established based on the energy method and Hamilton principle. In the third part, an improved RBF neural network is designed, a new fuzzy backstepping control method based on an RBF neural network is proposed, and Lyapunov function and control law are designed. In the fourth part, the simulation platform is established using Simulink. The effect of vibration suppression is measured by displacement error, velocity error, stable frequency, and stable point, compared with

the RBF neural network boundary control method and RBF neural network backstepping control. The fifth part summarizes the work and looks forward to the future.

2. Dynamic Model

The system studied in this paper is a flexible single-link manipulator at the end of the arm in a plane. A schematic diagram of the system is shown in Figure 1. In the natural state of the flexible manipulator, with the central axis of the flexible manipulator as the axis and the rotation axis of the motor and flexible manipulator as the origin, a world coordinate system is established. After the deformation, the direction of tangent line of the central axis of the flexible manipulator is the y axis, and a fixed coordinate system is established with the origin of world coordinate system. Some basic parameters and units of the flexible manipulator defined below are shown in Table 1:

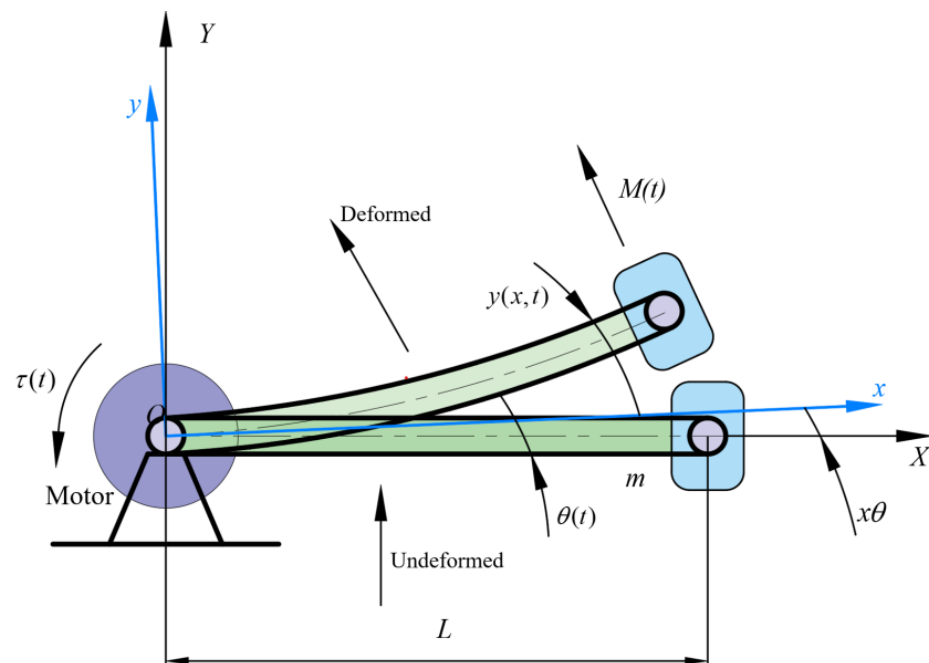


Figure 1. Structure diagram of the flexible manipulator.

Table 1. The basic parameters and units of the flexible manipulator.

Symbol	Physical Meaning	Unit
L	Length of the flexible manipulator in natural state	m
EI	The bending stiffness	$\text{N}\cdot\text{m}^2$
m	Load mass at the end of the flexible manipulator	kg
I	Moment of inertia of the flexible manipulator	$\text{kg}\cdot\text{m}^2$
$\tau(t)$	Input torque of the initial end motor	$\text{N}\cdot\text{m}$
$M(t)$	End load motor input control torque	$\text{N}\cdot\text{m}$
$\theta(t)$	Rotation angle of joint of the flexible manipulator	rad
ρ	Mass per unit length of the flexible manipulator	kg/m
$y(x, t)$	Elastic deformation of the flexible manipulator at point x	m

According to Hamilton's principle [17–20], the relationship between the kinetic energy of the system and the potential energy, as well as the non-conservative force, is expressed during the time interval t_1 to t_2 :

$$\int_{t_1}^{t_2} (\varepsilon T - \varepsilon U - \varepsilon W_{nc}) dt = 0 \quad (1)$$

Here, T is the kinetic energy of the flexible manipulator, U is the potential energy of the flexible manipulator, W_{nc} is the power by non-conservative force of the flexible manipulator, and $\varepsilon(A)$ means the variation of (A) . The kinetic energy of the system is expressed as the following equation:

$$T = \frac{1}{2}I\dot{\theta}^2(t) + \frac{1}{2}\int_0^L \rho \dot{u}^2(y, t)dy + \frac{1}{2}m\dot{u}^2(L, t) \quad (2)$$

where $u(x, t)$ is the offset of the flexible manipulator at any time t . The potential energy of the system is expressed as following equation:

$$U = \frac{1}{2}\int_0^L EI x''^2(y, t)dy \quad (3)$$

The power by non-conservative force of the flexible manipulator is expressed as the following equation:

$$W_{nc} = \tau(t)\theta(t) + F(t)u(L, t) \quad (4)$$

In the case of initial position 0, the boundary conditions are $u(0, t) = 0$, $u'(0, t) = \theta$. Bringing Equations (2)–(4) into Equation (1), the following equation can be obtained:

$$\int_{t_1}^{t_2} (\varepsilon T - \varepsilon U + \varepsilon W_{nc})dt = -\int_{t_1}^{t_2} \int_0^L A \varepsilon u(x, t)dydt - \int_{t_1}^{t_2} B \varepsilon u'(0, t)dt - \int_{t_1}^{t_2} C \varepsilon u(L, t)dt - \int_{t_1}^{t_2} D \varepsilon u'(L, t)dt = 0 \quad (5)$$

Parameters of A , B , C , and D are respectively expressed as: $A = \rho \ddot{u}(x, t) + EI u^{(4)}(L, t)$, $B = I \ddot{u}'(0, t) - EI u''(0, t) - \tau(t)$, $C = m \ddot{u}(L, t) - EI u^{(3)}(L, t) - M(t)$, $D = EI u''(L, t)$. As $\varepsilon u(x, t)$, $\varepsilon u'(0, t)$, $\varepsilon u(L, t)$ and $\varepsilon u'(L, t)$ are independent variables, all of these equations are linearly independent, $A = B = C = D = 0$. Thus, these partial differential equations (PDEs) [21,22] of the flexible manipulator are established as follows:

$$\rho \ddot{u}(x, t) = -EI u^{(4)}(x, t) \quad (6)$$

$$\tau(t) = I \ddot{u}'(0, t) - EI u''(0, t) \quad (7)$$

$$M(t) = m \ddot{u}(L, t) - EI u^{(3)}(L, t) \quad (8)$$

$$u''(L, t) = 0 \quad (9)$$

3. Control Law and Stability

Neural networks [23–25] have good adaptability for unknown parameters; they do not need very accurate mathematical models and can be suitable for nonlinear, strongly coupled systems with many unknown parameters.

The backstepping control method is a step-by-step recursive design method. The introduction of virtual control in the design of this method is essentially a static compensation idea, the front subsystem must pass the virtual control of the back subsystem to achieve the purpose of stabilization. Therefore, this method requires that the structure of the system must design a robust controller or adaptive controller of the uncertain system (especially when the interference or uncertainty does not meet the matching conditions), which has shown its advantages. In this paper, a fuzzy backstepping control method based on an RBF neural network is proposed to control flexible manipulators.

3.1. Description of the System

According to the partial differential equations established above, the flexible manipulator is a nonlinear multiple-input multiple-output system. For this system, the flexible manipulator can be written as a nonlinear state space:

$$\begin{cases} \dot{x}_i(t) = b_i x_{i+1}(t) + f_i(x_1(t), x_2(t), \dots, x_i(t)) + w_i(t), & i \leq n-1 \\ \dot{x}_n(t) = b_n u(t) + f_n(x_1(t), x_2(t), \dots, x_n(t)) + w_n(t) \\ y(t) = x_1(t) \end{cases} \quad (10)$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u \in R$, and $y \in R$ are the state variables of the system, control input and object output; $w_i(t)$ is external interference; f_i is unknown nonlinear function; and b_i is an unknown constant.

Hypothesis 1. The positive constants b_{im} and b_{iM} satisfy the inequality: $0 < b_{im} \leq |b_i| \leq b_{iM}$, $i = 1, 2, \dots, n$.

Hypothesis 2. For smooth functions $f(x)$ and fuzzy systems, there is an optimal constant parameter $\vec{\theta}^*$ that minimizes the approximation error. Here the optimal parameter constant is defined as $\vec{\theta}^* = \underset{\theta \in \Omega_0}{\operatorname{argmin}} \left[\sup_{x \in \Omega} \left| f(x) - \vec{\theta}^T \zeta(x) \right| \right]$, Ω_0 and Ω are the bounded sets of $\vec{\theta}$ and x .

The control goal is to make the output $y(t)$ of the system well clear of vibration errors, and all signals are bounded. For simplicity, the symbols $\bar{x}_i$ and y_{di} are introduced. Where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$ ($i = 1, 2, \dots, n$) is the input of the system, $y_{di} = [y_d, \dot{y}_d, \dots, y_d^{(i)}]^T \in R^i$ ($i = 1, 2, \dots, n-1$) is the expected trajectory value of the system.

Using single-value fuzzers, product reasoners and barycentric average defuzzers, the fuzzy rules can be expressed as follows: IF x_1 is F_1^j and ... and x_n is F_n^j then y is B^j ($j = 1, 2, \dots, N$). The fuzzy output is:

$$y(x) = \frac{\sum_{j=1}^N \theta_j \prod_{i=1}^n \mu_i^j(x_i)}{\sum_{j=1}^N \prod_{i=1}^n \mu_i^j(x_i)} \quad (11)$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ and $\mu_i^j(x_i)$ are the membership function μ_i^j , $\theta_j = \max_{y \in R} B^j(y)$. Defining $\zeta(x) = [\zeta_1(x), \zeta_2(x), \dots, \zeta_N(x)]^T$, $\vec{\theta} = [\theta_1, \theta_2, \dots, \theta_N]^T$, then the equation can be obtained: $y(x) = \zeta^T(x) \vec{\theta}$.

According to the fuzzy universal approximation theorem, if $f(x)$ is a continuous function defined on a compact set Ω , then for any constant $\varepsilon > 0$, there exists a fuzzy system, as shown above, that satisfies $\sup_{y \in \Omega} |f(x) - y(x)| \leq \varepsilon$.

3.2. The Design of the Backstepping Controller

The design process of the backstepping method [26–29] is completed by constructing the intermediate quantity $e_i = x_i - \alpha_{i-1}$, where α_i is the virtual control quantity of step i . The final virtual control quantity α_n is part of the actual control quantity $u(t)$ applied to the system.

The backstepping method is adopted to reconstruct the system Formula (10) as follows:

Step 1. For the first subsystem $\dot{x}_1(t) = b_1x_2(t) + f_1(x_1) + w_1(t)$, defining $e_1 = y - y_d$, where y_d is the expected tracking trajectory, and virtual control signal α_1 is introduced, an equation can be obtained as follows:

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_d = b_1e_2 - b_1\alpha_1 + f_1(x_1) + w_1(t) - \dot{y}_d \quad (12)$$

where $e_2 = x_2 - \alpha_1$.

Step 2. By introducing the virtual control quantity α_2 into the second subsystem, the following equation can be obtained:

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = b_2e_3 + b_2\alpha_2 - \dot{\alpha}_1 + f_2(\bar{x}_2) + w_2 \quad (13)$$

where $e_3 = x_3 - \alpha_2$.

Step k. By analogy, introducing a virtual control quantity α_k is introduced into the k subsystem, defining $e_{k+1} = x_{k+1} - \alpha_k$, the following equation can be obtained:

$$\dot{e}_k = \dot{x}_k - \dot{\alpha}_{k-1} = b_ke_{k+1} + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\bar{x}_k) + w_k \quad (14)$$

let $e_n = x_n - \alpha_{n-1}$; then, for the last subsystem:

$$\dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} = b_nu + f_n(\bar{x}_n) + w_n - \dot{\alpha}_{n-1} \quad (15)$$

therefore, the system Equation (10) can be converted to these equations:

$$\begin{cases} \dot{e}_k = b_ke_{k+1} + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\bar{x}_k) + w_k, & 1 \leq k \leq n-1 \\ \dot{e}_n = b_nu - \dot{\alpha}_{n-1} + f_n(\bar{x}_n) + w_n \end{cases} \quad (16)$$

where $\alpha_0 = y_d$.

By determining the virtual control quantity α_k and u , the system is stable and can reach the desired tracking trajectory, and the vibration error is eliminated.

When $k = 1$, choosing the Lyapunov function gives the following equation:

$$V_1 = \frac{1}{2b_1}e_1^2 \quad (17)$$

taking the derivative of V_1 gives the following equation:

$$\dot{V}_1 = e_1(e_2 + \alpha_1 + \hat{f}_1) + \frac{1}{b_1}e_1w_1 \quad (18)$$

where $\hat{f}_1 = \frac{f_1(\bar{x}_1) - \dot{y}_d}{b_1}$.

Defining $\alpha_1 = -\lambda_1e_1 - \varphi_1$, $\lambda_1 > 0$, φ_1 is used to approximate a fuzzy system with nonlinear function $\hat{f}_1$. An equation can be obtained as follows:

$$\dot{V}_1 = -\lambda_1e_1^2 + e_1e_2 + e_1(\hat{f}_1 - \varphi_1) + \frac{1}{b_1}e_1w_1 \quad (19)$$

When $k = 2$, designing the Lyapunov function gives the following equation:

$$V_2 = V_1 + \frac{1}{2b_2}e_2^2 \quad (20)$$

defining $\alpha_2 = -\lambda_2e_2 - e_1 - \varphi_2$, $\lambda_2 > 0$, φ_2 is used to approximate a fuzzy system with nonlinear function $\hat{f}_2$. An equation can be obtained as follows:

$$\dot{V}_2 = -\sum_{i=1}^2 \lambda_i e_i^2 + \sum_{i=1}^2 e_i(\hat{f}_i - \varphi_i) + e_2e_3 + \sum_{i=1}^2 e_iw_i \quad (21)$$

where $\hat{f}_2 = \frac{f_2 - \dot{\alpha}_1}{b_2}$.

Defining $\alpha_{k-1} = -\lambda_{k-1}e_{k-1} - e_{k-2} - \varphi_{k-1}$, $\lambda_{k-1} > 0$, φ_{k-1} is used to approximate a fuzzy system with nonlinear function $\hat{f}_{k-1}$, then the derivative of the $k - 1$ Lyapunov function can be recursively obtained as:

$$\dot{V}_{k-1} = -\sum_{i=1}^{k-1} \lambda_i e_i^2 + \sum_{i=1}^{k-1} e_i(\hat{f}_i - \varphi_i) + e_{k-1}e_k + \sum_{i=1}^{k-1} e_i w_i \quad (22)$$

Designing the Lyapunov function at step k , an equation can be obtained as follows:

$$V_k = V_{k-1} + \frac{1}{2b_k} e_k^2 \quad (23)$$

taking the derivative of V_k gives the following equation:

$$\dot{V}_k = -\sum_{i=1}^{k-1} \lambda_i e_i^2 + \sum_{i=1}^{k-1} e_i(\hat{f}_i - \varphi_i) + e_k(e_{k+1} + \alpha_k + e_{k-1} + \hat{f}_k) + \sum_{i=1}^k e_i w_i \quad (24)$$

For $k = n$, designing the Lyapunov function, an equation can be obtained as follows:

$$V_n = V_{n-1} + \frac{1}{2b_n} e_n^2 \quad (25)$$

taking the derivative of V_n , the following equation can be obtained:

$$\dot{V}_n = -\sum_{i=1}^{n-1} \lambda_i e_i^2 + \sum_{i=1}^{n-1} e_i(\hat{f}_i - \varphi_i) + e_n(u + e_{n-1} + \frac{1}{b_n}(f_n - \dot{\alpha}_{n-1})) + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \quad (26)$$

where $\hat{f}_n = \frac{1}{b_n}(f_n - \dot{\alpha}_{n-1})$.

Defining φ_n , which is used to approximate a fuzzy system with nonlinear function $\hat{f}_n$, the design control law is expressed as follows:

$$u = -\lambda_n e_n - e_{n-1} - \varphi_n \quad (\lambda_n > 0) \quad (27)$$

putting in $\dot{V}_n$, the following equation can be obtained:

$$\dot{V}_n = -\sum_{i=1}^{n-1} \lambda_i e_i^2 + \sum_{i=1}^{n-1} e_i(\hat{f}_i - \varphi_i) + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \quad (28)$$

3.3. The Design of the RBF Neural Network

As an intelligent control, neural networks can compensate for unknown parameters and uncertain disturbances greatly. In this paper, the unknown disturbances and random vibration of a system are estimated and compensated based on RBF neural network. The Gaussian function expression of the RBF neural network is as follows:

$$\sigma_i = \frac{2b_i^2}{1 + e^{-2V_i \hat{Y}_i}} \quad (29)$$

where $V_i = [v_1, v_2, \dots, v_n]^T$ is the input vector of the RBF neural network. n is the number of input layers of the neural network, s is defined as the number of hidden layers of the neural network, $\hat{Y}_i$ is the input vector in the world coordinate system. And b_i is the width of the Gaussian function of the i -th neuron in the hidden layer of the neural network. Defining the ideal weight of the neural network as $w = [w_1, w_2, \dots, w_s]^T$, the output of the RBF neural network is $\hat{X}_{is} = w^T \sigma (i = 1, 2, 3, 4)$. The structure of this neural network is shown in Figure 2.

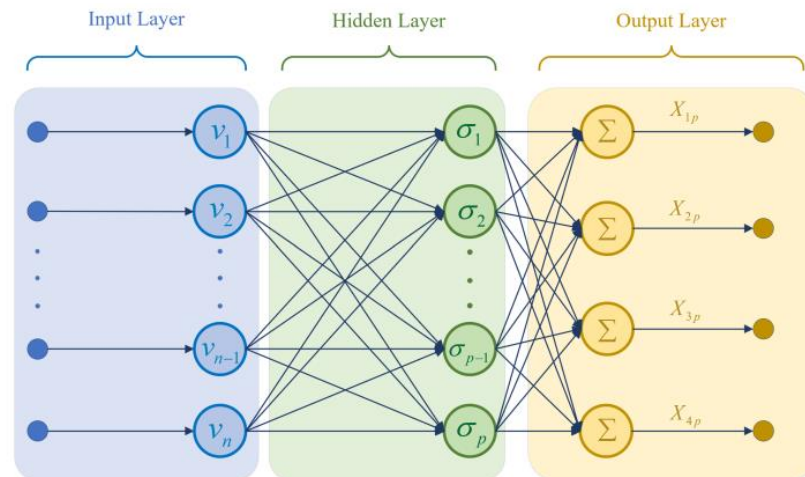


Figure 2. RBF neural network structure diagram.

In the whole neural network, the weighted gain is entered into the RBF neural network for calculation. Therefore, the factors affecting the input variables are complicated and difficult to control. Moreover, external interference leads to many uncertainties. Through the good adaptability of the RBF neural network, unknown disturbances and the multi-order differential coupling terms in the system can be compensated for.

3.4. The Design of Adaptive Fuzzy Control Based on the Backstepping Method

f_k is an unknown function approximated by $\varphi_k = \zeta^T(\bar{x})\vec{\theta}$. There is an optimal approximation vector $\vec{\theta}_k^*$ for a given arbitrarily small constant $\varepsilon_k > 0$, $\left| \hat{f}_k - \vec{\theta}_k^* \zeta(\bar{x}_k) \right| \leq \varepsilon_k$ ($k = 1, 2, \dots, n$).

Defining $\tilde{\vec{\theta}}_k = \vec{\theta}_k^* - \vec{\theta}_k$, $\tilde{\vec{\theta}}_k$ as the error of $\vec{\theta}_k$, the control law designed by Equation (27) above is adopted, and the adaptive control law is designed as follows:

$$\dot{\vec{\theta}}_i = r_i e_i \zeta_i(\bar{x}_i) - 2k_i \vec{\theta}_i, \quad (i = 1, 2, \dots, n) \quad (30)$$

where r_i is the correction factor and k_i is the gain factor. The signals of all closed-loop systems are bounded, and for a given attenuation coefficient $\rho > 0$, the tracking performance index satisfies the following expression:

$$\sum_{i=1}^n \int_0^T e_i^2(s) ds \leq \frac{1}{d_0} V(0) + \frac{1}{d_0} T b_0 + \sum_{i=1}^n \frac{1}{2d_0} \int_0^T \rho^2 w_i^2 dt, \quad T \in [0, \infty] \quad (31)$$

where d_0 and b_0 are constants.

Stability certificate: Designing the Lyapunov function is the following equation:

$$V = V_n + \sum_{i=1}^n \frac{1}{2r_i} \tilde{\vec{\theta}}_i^T \tilde{\vec{\theta}}_i, \quad \tilde{\vec{\theta}}_i = \vec{\theta}_i^* - \vec{\theta}_i \quad (32)$$

taking the derivative of V the following equation can be obtained:

$$\dot{V} = \dot{V}_n + \sum_{i=1}^n \frac{1}{r_i} \tilde{\vec{\theta}}_i^T \dot{\tilde{\vec{\theta}}}_i = -\sum_{i=1}^n \lambda_i e_i^2 + \sum_{i=1}^n e_i (\hat{f}_i - \vec{\theta}_i^{*T} \zeta_i(\bar{x}_i)) + \sum_{i=1}^n e_i \tilde{\vec{\theta}}_i^T \zeta_i(\bar{x}_i) - \sum_{i=1}^n \frac{1}{r_i} \tilde{\vec{\theta}}_i^T \dot{\tilde{\vec{\theta}}}_i + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \quad (33)$$

by reducing the above equation, the following inequality can be obtained:

$$\dot{V} \leq -\sum_{i=1}^n \lambda_i e_i^2 + \sum_{i=1}^n \tilde{\theta}_i^T \left(e_i \xi_i(\bar{x}_i) - \frac{1}{r_i} \dot{\tilde{\theta}}_i \right) + \sum_{i=1}^n |e_i \varepsilon_i| + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \quad (34)$$

defining the equation $S = -\sum_{i=1}^n \lambda_i e_i^2 + \sum_{i=1}^n \tilde{\theta}_i^T \left(e_i \xi_i(\bar{x}_i) - \frac{1}{r_i} \dot{\tilde{\theta}}_i \right) + \sum_{i=1}^n |e_i \varepsilon_i| + \sum_{i=1}^n \frac{1}{b_i} e_i w_i$ and, at the same time, defining $a_i = \lambda_i - \frac{1}{2} - \frac{1}{2\rho^2 b_i^2}$, solving for λ_i , and putting it into S gives the following equation:

$$S = -\sum_{i=1}^n a_i e_i^2 - \frac{1}{2} \sum_{i=1}^n e_i^2 - \sum_{i=1}^n \frac{1}{2\rho^2 b_i^2} e_i^2 + \sum_{i=1}^n \tilde{\theta}_i^T \left(e_i \xi_i(\bar{x}_i) - \frac{1}{r_i} \dot{\tilde{\theta}}_i \right) + \sum_{i=1}^n |e_i \varepsilon_i| + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \quad (35)$$

shrinking the above equation, because of inequalities $-\frac{1}{2} \sum_{i=1}^n e_i^2 + \sum_{i=1}^n |e_i \varepsilon_i| \leq \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2$ and $-\sum_{i=1}^n \frac{1}{2\rho^2 b_i^2} e_i^2 + \sum_{i=1}^n \frac{1}{b_i} e_i w_i \leq \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2$, and considering the adaptive Equation (30), the following inequalities are obtained:

$$S \leq -\sum_{i=1}^n a_i e_i^2 + \sum_{i=1}^n \frac{k_i}{r_i} \left(\overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i - 2 \overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i \right) + \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2 \quad (36)$$

According to inequality $\overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i + \overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i \geq 2 \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i$, the inequality $\overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i - 2 \overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i \leq \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i - \overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i$ is obtained. If the original expression is reduced, the following expression can be obtained:

$$\dot{V} \leq -\sum_{i=1}^n a_i e_i^2 + \sum_{i=1}^n \frac{k_i}{r_i} \left(-\overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i - \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i \right) + \sum_{i=1}^n \frac{2k_i}{r_i} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i + \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2 \quad (37)$$

According to inequality $-\frac{1}{2} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i \geq -\overset{\rightarrow}{\theta}_i^T \overset{\rightarrow}{\theta}_i - \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i$, shrinking the above expression again gives the following expression:

$$\dot{V} \leq -\sum_{i=1}^n a_i \frac{2b_{im}}{2b_i} e_i^2 - \sum_{i=1}^n \frac{k_i}{2r_i} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i + \sum_{i=1}^n \frac{2k_i}{r_i} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i + \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2 \quad (38)$$

defining $\lambda_i \geq \frac{1}{2} + \frac{1}{2\rho^2 b_i^2}$ and letting $a_i > 0$, then defining $a_0 = \min\{2b_{im}a_i, k_i, i = 1, 2, \dots, n\}$ and $b_0 = \sum_{i=1}^n \frac{2k_i}{r_i} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i + \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2$, the primitive can be written as the following expression:

$$\dot{V} \leq -a_0 \left(\sum_{i=1}^n \frac{1}{2b_i} e_i^2 + \sum_{i=1}^n \frac{1}{2r_i} \overset{\rightarrow}{\theta}_i^{*T} \overset{\rightarrow}{\theta}_i \right) + b_0 + \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2 = -a_0 V + b_0 + c_0 \quad (39)$$

where $w_i^2 \leq c_i$, $c_0 = \sum_{i=1}^n \frac{1}{2} \rho^2 c_i$.

To solve the above first-order linear differential equation, the solution of Equation (39) is obtained:

$$V(t) \leq \left(V(0) - \frac{b_0 + c_0}{a_0} \right) e^{-a_0 t} + \frac{b_0 + c_0}{a_0} \leq V(0) e^{-a_0 t} + \frac{b_0 + c_0}{a_0} \leq V(0) + \frac{b_0 + c_0}{a_0}, t \geq 0 \quad (40)$$

Defining the compact set $\Omega_0 = \{X|V(X) \leq C_0\}$, $C_0 = V(0) + \frac{b_0+c_0}{a_0}$, from the definition of Equation (31), it can be seen that all signals of the closed-loop system are bounded: $(e_1, e_2, \dots, e_n, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n)^T \in \Omega_0$.

Let $d_0 = \min\{a_i; 1, 2, \dots, n\}$. Then the following equation can be obtained from Equation (38):

$$\dot{V} \leq -\sum_{i=1}^n a_i e_i^2 - \sum_{i=1}^n \frac{k_i}{2r_i} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^n \frac{2k_i}{r_i} \tilde{\theta}_i^{T*} \tilde{\theta}_i^{*} + \frac{1}{2} \sum_{i=1}^n \varepsilon_i^2 + \sum_{i=1}^n \frac{1}{2} \rho^2 w_i^2 = -d_0 \sum_{i=1}^n e_i^2 + b_0 + \frac{1}{2} \rho^2 w_i^2 \quad (41)$$

Integrating the above equation in $[0, T]$, because of $-\frac{1}{d_0} V(T) \leq 0$, the convergence result of the system is as follows:

$$\sum_{i=1}^n \int_0^T e_i^2(s) ds \leq \frac{1}{d_0} V(0) + \frac{1}{d_0} b_0 T + \sum_{i=1}^n \frac{1}{2d_0} \int_0^T \rho^2 w_i^2 dt \quad (42)$$

According to the above equation, the system converges, and the convergence accuracy of the final error depends on the upper bounds of the perturbation and approximation errors.

4. Simulation

4.1. Simulation Platform and Parameter

According to the dynamic model and control law established in the above section, this section simulates the nonlinear space state equation of the flexible manipulator by designing a controller. This simulation compares the dynamic surface method of the backstepping space state equation with the boundary control method of the linear state equation. Generally, displacement, velocity, angle, angular velocity, force, torque, and other variables are used to measure the quality of vibration suppression. In this paper, the error analysis method is used to compare the two variables of displacement and velocity. The Simulink platform is used to build the controller. The designed controller structure is shown in Figure 3, where C is the fourth-order unit matrix.

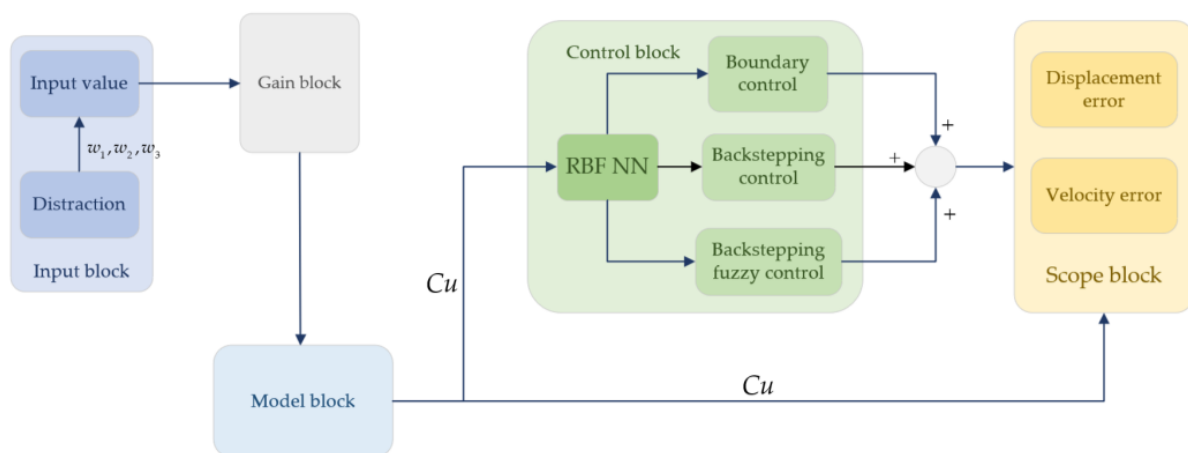


Figure 3. Structure diagram of the controller.

In this simulation, because the external disturbance of the manipulator is random and uncertain, the type of vibration is also very complex and changeable, so it cannot be simulated one by one. Therefore, this paper sets three different vibration types to interfere with the vibration of the manipulator to analyze the vibration suppression effect of the proposed algorithm. The selected vibration interference is shown as follows:

$$\begin{cases} w_1 = 0.06 \sin(t) \\ w_2 = 0.06 \sin(t) + 0.04 \sin(3t) \\ w_3 = 0.06 \sin(t) + 0.04 \sin(3t) + 0.06 \sin(3t) \end{cases} \quad (43)$$

where is the simplest harmonic vibration wave, the amplitude is 0.06 mm, and the frequency is 0.159 Hz. For easy comparison, it is based on adding a low amplitude, high-frequency harmonic vibration. When the frequency of the simple harmonic vibration is increased and the amplitude is superimposed, the vibration suppression effect of the proposed control algorithm on the manipulator is observed. The maximum amplitude is 0.1mm and the maximum frequency is 0.4774 Hz based on adding a simple harmonic vibration with a higher relative amplitude and higher frequency. Three different simple harmonic vibrations are superimposed on each other to simulate the external interference of the manipulator in a complex environment with a maximum frequency of 0.4774 Hz and a maximum amplitude of 0.16 mm.

A flexible manipulator driven by a motor is used for the simulation. The dynamic equation of the system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{B}{M_t}x_2 + \frac{N}{M_t}f(x_1, x_2) + \frac{K_t}{M_t}x_3 \\ \dot{x}_3 = -\frac{R}{L}x_3 - \frac{K_b}{L}x_2 + \frac{1}{L}u - w \\ y = x_1 \end{cases} \quad (44)$$

where the parameters are $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = I$, $M_t = J + \frac{1}{3}ml^2 + \frac{1}{10}Ml^2D$, and $N = mgl + Mgl$; f is an unknown nonlinear function; θ is connecting rod angle; I is the electric current and u is the control voltage of the motor. The physical meanings and parameter settings of the variables in the above expression are as Table 2.

Table 2. The physical meaning of parameters in dynamic equations.

Symbol	Physical Meaning	Unit and Size
B	Bearing viscous friction coefficient	0.015
L	Reactance	0.0008 Ω
D	Load diameter	0.05 m
R	Resistance	0.075 Ω
m	Connecting rod mass	0.5 kg
J	Actuator torque	0.05 N·m
l	Connecting rod length	0.6 m
K_b	Back electromotive force coefficient	0.085
M	Load mass	0.05 kg
K_t	Torque constant	1
g	Acceleration of gravity	9.8 m/s ²

Taking the expected trajectory as $y_d = \sin(t)$, the nonlinear function is $f = \sin(\theta)$, and the controller parameters are $\rho = 1$, $\lambda_1 = 3$, $\lambda_2 = 8.5$, $\lambda_3 = 8.5$, $k_1 = k_2 = k_3 = 1.5$, and $r_1 = r_2 = r_3 = 2$. The initial system status is $x(0) = [0.5\pi \ 0 \ 0]^T$. The initial value $\theta_1(0)$ and $\theta_2(0)$ is zero.

The first set of the boundary control method based on linear state space equations based on an RBF neural network is designed for comparison. The dynamic equations established are as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{I_L}[m_L g L_L x_2 + EI(x_1 - x_3)] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\frac{1}{I_m}[m_m g L_m - EI(x_1 - x_3) - u] \end{cases} \quad (45)$$

where x_1 and x_2 are, respectively, the displacement error and velocity error of the flexible manipulator; x_3 and x_4 are, respectively, the displacement error and velocity error of motor shaft; I_L is the moment of inertia of the flexible manipulator; I_m is the moment of inertia of the motor; m_L is the mass of the flexible manipulator. m_m is the mass of the motor shaft;

and g is the gravitational acceleration. The variable parameters of the linear space state equation are shown in Table 3.

Table 3. Parameters of the linear state space equations for the flexible manipulator.

Symbol	Physical Meaning	Unit and Size
m_m	Mass of the motor shaft	1.5 kg
L_m	Length of the motor shaft	10 mm
I_m	Moment of inertia of the motor	0.4 kg·m
m_L	Mass of the flexible manipulator	500 g
L_L	Moment of inertia of the flexible manipulator	0.4 kg·m
g	Acceleration of gravity	9.8 m/s ²
EI	Stiffness coefficient	0.8 N·m

The second set of the backstepping control method based on nonlinear state space equations based on an RBF neural network is designed for comparison. The dynamic equations established are as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{EI_s}{I_h}x_1 + \frac{EI_m^2EI_g^2}{R_mI_h}x_4 - \frac{EI_mEI_g}{R_mI_h}u - \frac{m_mgL_L}{I_L}\sin(x_1 + x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{EI_s}{I_h}x_1 - \frac{EI_m^2EI_g^2}{R_mI_h}x_4 + \frac{EI_mEI_g}{R_mI_h}u \end{cases} \quad (46)$$

The parameters of the nonlinear state equations are shown in Table 4:

Table 4. Nonlinear state space equation parameters of the flexible manipulator.

Symbol	Physical Meaning	Unit and Size
EI_s	Bottom stiffness of the flexible manipulator	1.61 N·m
I_h	Moment of inertia at bottom of the flexible manipulator	0.0021 kg·m
EI_m	Stiffness coefficient of the motor	70 N·m
EI_g	Stiffness coefficient at the top of the flexible manipulator	0.00767 N·m
I_L	Moment of inertia at the top of the flexible manipulator	0.0059 kg·m
R_m	Damping coefficient of the motor	2.6 N·s/m

In above equation, u is the input of the controller, $u = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4$. In this equation, $k_1 = 1$, $k_2 = 0.02$, $k_3 = 10$ and $k_4 = 0.01$ are the gain coefficients of each item. The simulation time is set to 1.3 s. Since the flexible manipulator is the end of the whole rigid–flexible manipulator system, the whole system is not completely flexible, so the natural frequency of the flexible manipulator is not considered. The natural frequency of the whole system is set to 0.

4.2. Analysis of Simulation Results

4.2.1. Analysis of Curves under External Disturbance w_1

Figures 4 and 5 show the displacement error and velocity error curves of each system when the manipulator is subjected to external interference w_1 , in which the red curve is the error curve when the vibration suppression of the system is not applied, the black curve is the error curve using the RBF neural network boundary control method, and the blue curve is the error curve using the RBF neural network inversion control method. The green curve is the error curve of the RBF neural network fuzzy backstepping control method. As can be seen from the displacement error of the manipulator in Figure 4, there is an unusual displacement error of the manipulator at the start of the motor, which is mainly due to the sudden change of force and the change of motion state at the start of the motor. Then it fluctuates within a smaller range. Under the interference of simple harmonic vibration,

the error fluctuation is not obvious. However, the small fluctuation frequency in the local range is very high. It is about 1000 Hz. Long-term and high-frequency vibration affects the working accuracy and service life of the manipulator, so vibration suppression is needed. Under the vibration suppression of the RBF neural network boundary control method and the RBF neural network backstepping control method, the vibration frequency of the flexible arm can be well suppressed, and the vibration displacement error is suppressed to about 0.05 mm. The RBF neural network fuzzy backstepping control method not only restrains the vibration frequency well, but also restrains the displacement error of the manipulator, so that the final convergence result is close to 0. The velocity error curve is similar to the displacement error curve when vibration is not suppressed. After the three control methods start vibration suppression, there is a certain fluctuation frequency in the initial operation, which is the largest for the RBF neural network backstepping control method, the second-largest for the RBF neural network boundary control method, and the least for the RBF neural network fuzzy backstepping control method. According to the final convergence results, RBF neural network fuzzy inversion control method is still the best, and the velocity error and vibration frequency error are almost eliminated. Table 5 provides a detailed analysis of the data.

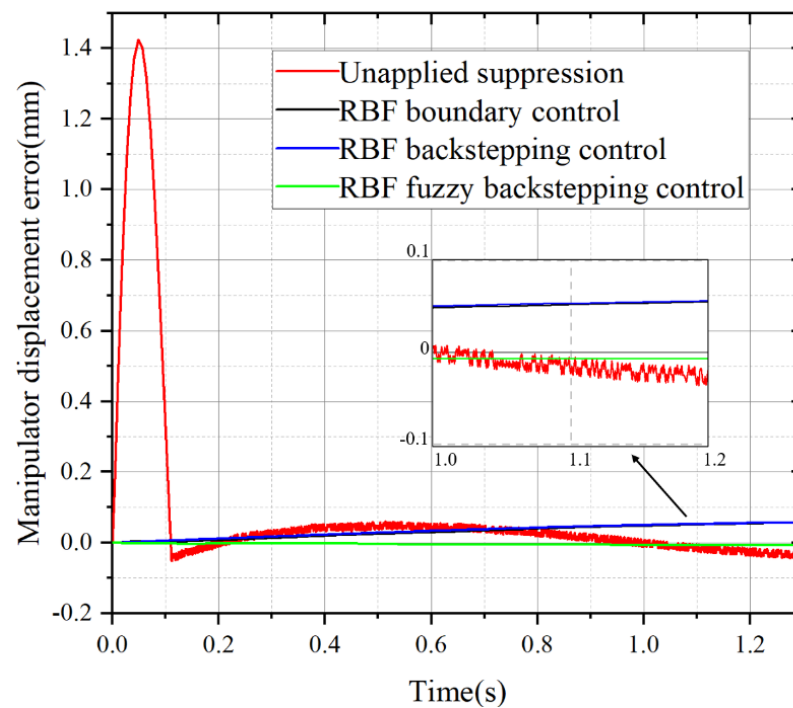


Figure 4. Displacement error of manipulator under the w_1 interference.

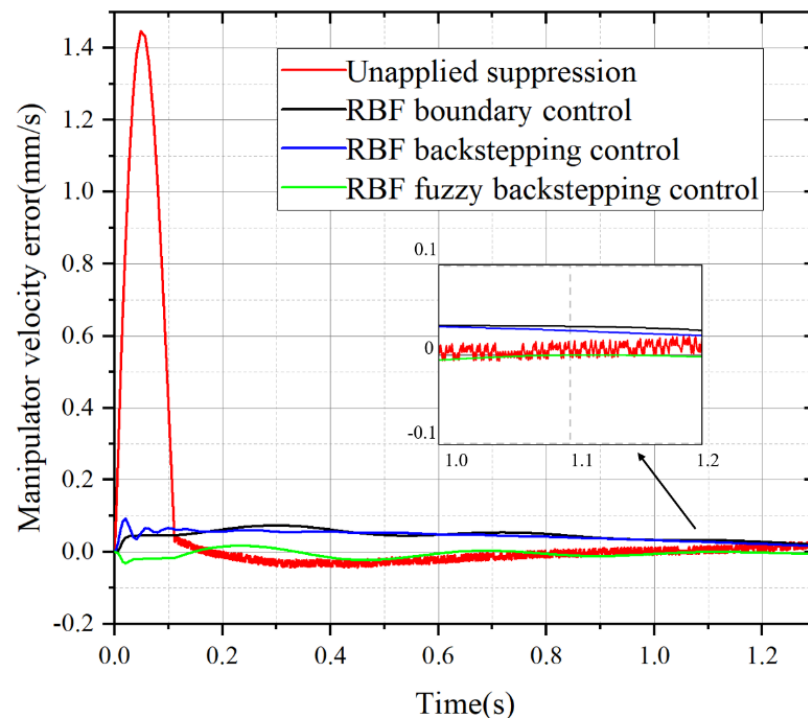


Figure 5. Velocity error of manipulator under the w_1 interference.

Table 5. Comparison of the data when the manipulator is under the w_1 interference.

Control Method	Displacement Error		Velocity Error	
	Steady Displacement Error (mm)	Stable Frequency (Hz)	Steady Velocity Error (mm/s)	Stable Frequency (Hz)
RBF boundary control	0.0578	Approximately equal to 0	0.0162	Approximately equal to 0
RBF backstepping control	0.0574	Approximately equal to 0	0.02	Approximately equal to 0
RBF fuzzy backstepping control	0.0071	0	0.0049	0

4.2.2. Analysis of Curves under the External Disturbance w_2

Figures 6 and 7 show the variation curves of the displacement error and velocity error of the flexible manipulator when it is subjected to external interference w_1 during operation. As shown in Figure 6, in the displacement error curve, due to the superposition of two simple harmonic motions, the overall displacement error fluctuates more, and so does the velocity error. However, in the wave frequency, due to the combination of two simple harmonic motions, when the two simple harmonic motions have the same direction, is a superposition. When the two harmonic motion directions are opposite, the frequency and amplitude cancel each other. In this simulation, the two sets of simple harmonic motion cancel each other, making the maximum frequency about 400 Hz. However, in terms of fluctuation error, if the instantaneous error at start-up is not considered, the maximum fluctuation error reaches 0.1448 mm, so vibration suppression is required. Both the RBF neural network boundary control method and the RBF neural network backstepping control method can restrain the vibration frequency well and limit the displacement error to less than 0.1 mm. The RBF neural network fuzzy inversion control method can eliminate the vibration frequency and displacement error well under external interference w_2 , and almost eliminate the vibration error and displacement error. Although the velocity error is not large in the velocity error curve in Figure 7, the fluctuation frequency is 500 Hz. Although the RBF neural network boundary control method and RBF neural network backstepping control method have a certain fluctuation frequency at the initial stage of operation, they both suppress the fluctuation frequency well in the end, and the final convergence equilibrium position is -0.1 mm/s. The RBF neural network fuzzy backstepping control method also

has certain fluctuation frequency and velocity errors when the system starts running, but in the final phase of operation, the fluctuation frequency and velocity error balance point is better than for the first two control methods, and the speed error is better suppressed. Table 6 provides a detailed analysis of the data.

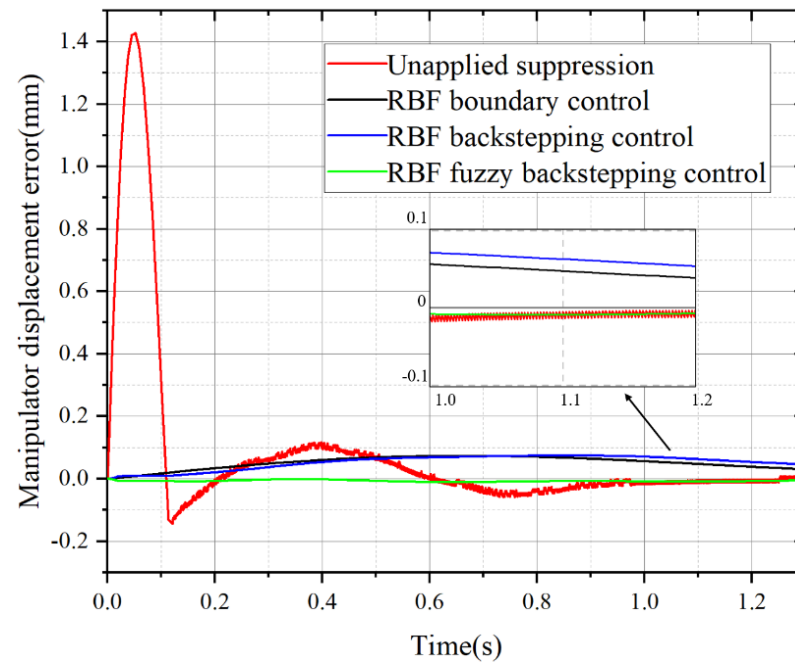


Figure 6. Displacement error of the manipulator under the w_2 interference.

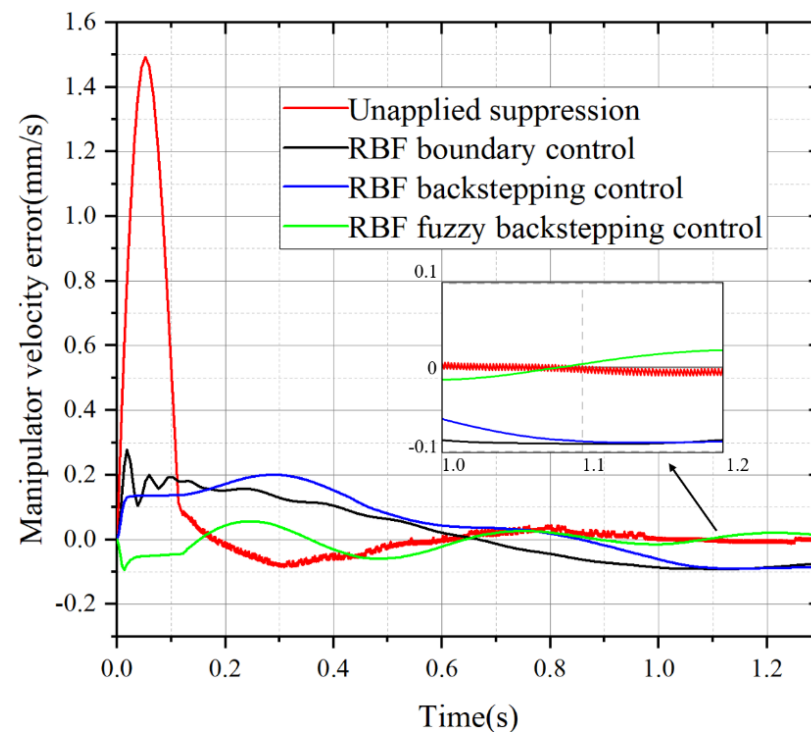


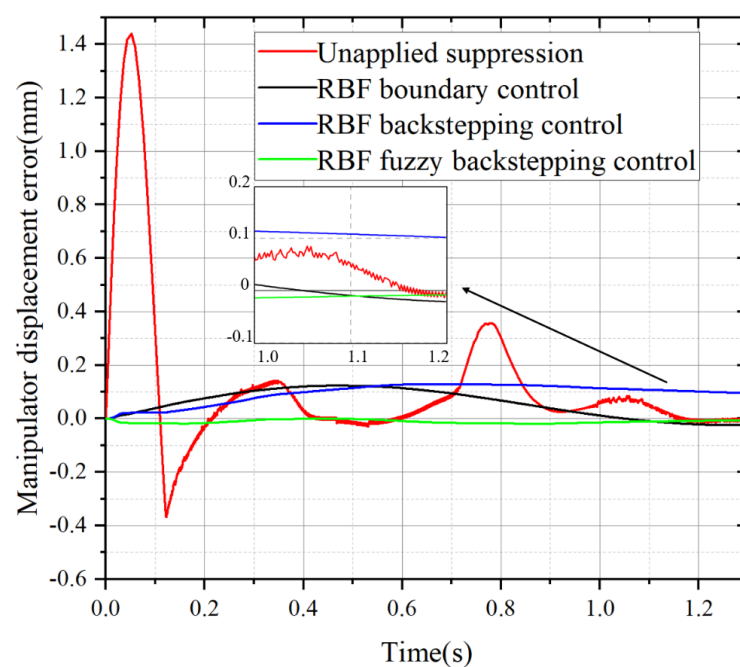
Figure 7. Velocity error of the manipulator under the w_2 interference.

Table 6. Comparison of the data when the manipulator is under the w_2 interference.

Control Method	Displacement Error		Velocity Error	
	Steady Displacement Error (mm)	Stable Frequency (Hz)	Steady Velocity Error (mm/s)	Stable Frequency (Hz)
RBF boundary control	0.04536	Approximately equal to 0	0.0867	1
RBF backstepping control	0.03059	Approximately equal to 0	0.0721	0.714
RBF fuzzy backstepping control	0.00533	0	0.0133	0.333

4.2.3. Analysis of Curves under External Disturbance w_3

Figures 8 and 9 show the variation curves of the displacement error and velocity error of the flexible manipulator when it is subjected to external interference w_3 during operation. As shown in Figure 8, with the combination of three simple harmonic motions, the superposition and canceling effects of displacement fluctuations are further enhanced. Sometimes the amplitude is very large, and sometimes it is at the equilibrium point; the maximum frequency is 500 Hz, and the amplitude of the wave is 0.4 mm. The three control methods also suppress the fluctuation frequency well. In terms of final convergence, the RBF neural network backstepping control method finally converges at the equilibrium position where the displacement error is 0.1 mm. The RBF neural network boundary control method and RBF neural network fuzzy backstepping control method have a smaller error at the final convergence point, but RBF neural network fuzzy backstepping control method has a better inhibition effect and shorter convergence time. In the velocity error curve shown in Figure 9, the three simple harmonics are superimposed on each other, and both the fluctuation frequency and the velocity error of the unsuppressed curve are large. Not only the fluctuation frequency but also the velocity error fluctuation frequency is small. The fluctuation frequency is 300 Hz, and the maximum velocity error fluctuation frequency is 2.5 Hz. The maximum velocity error is 1.5 mm/s. The RBF neural network boundary control method and the inversion control method cannot suppress vibration well. The RBF neural network backstepping method has a large convergence equilibrium position error, and the RBF neural network boundary control method has a high vibration frequency at the beginning of operation. According to the image, the RBF neural network fuzzy backstepping control method has the best effect on vibration frequency and velocity error suppression. Table 7 provides a detailed analysis of the data.

**Figure 8.** Displacement error of the manipulator under the w_3 interference.

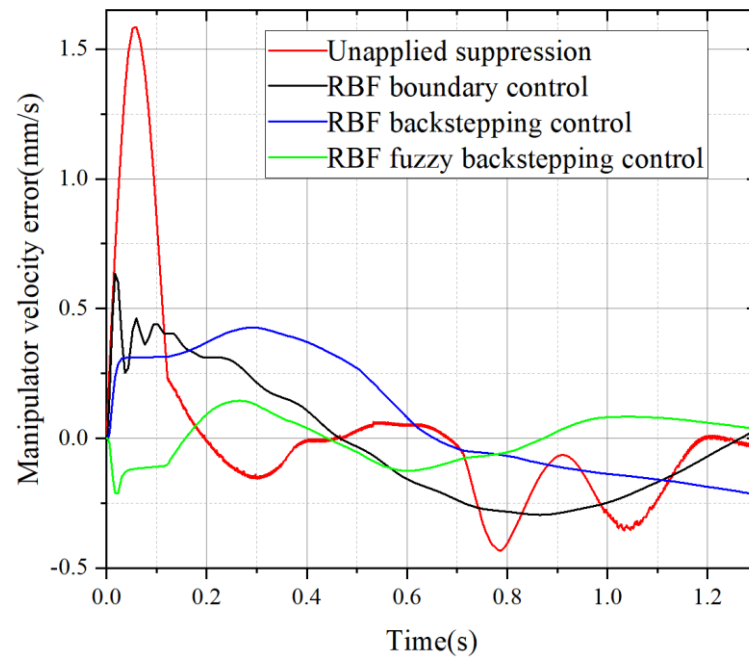


Figure 9. Velocity error of the manipulator under the w_3 interference.

Table 7. Comparison of the data when the manipulator is under the w_3 interference.

Control Method	Displacement Error		Velocity Error	
	Steady Displacement Error (mm)	Stable Frequency (Hz)	Steady Velocity Error (mm/s)	Stable Frequency (Hz)
RBF boundary control	0.02289	Approximately equal to 0	0.0232	0.56
RBF backstepping control	0.09486	Approximately equal to 0	0.2173	0.714
RBF fuzzy backstepping control	0.00646	0	0.0354	0.83

Based on the analysis of the above simulation results, the RBF neural network fuzzy backstepping control method has better vibration suppression effect than the RBF neural network boundary control method and the RBF neural network backstepping control method, regardless of whether there is simple harmonic motion or multiple harmonic motion phase superposition. In most cases, the vibration frequency and vibration error can be quickly suppressed, and the final convergence equilibrium position error is small.

5. Conclusions

In this paper, a single-link flexible manipulator was studied. The partial differential equations of the flexible manipulator were established first, and then the fuzzy backstepping control algorithm was designed, and the algorithm was combined with the RBF neural network. The Lyapunov function was designed, and the stability of the system was proved. Finally, Simulink was used to build a simulation platform, and three different groups of external interference were simulated using the proposed algorithm and compared with the RBF neural network boundary control method and the RBF neural network inversion control method. From the two dimensions of displacement error and velocity error, the two parameters of stable convergence equilibrium position and stable frequency were quantified and compared. The effect of vibration suppression was roughly analyzed by the trend of the curves. The simulation results show that the RBF neural network fuzzy backstepping control method is superior to the two control algorithms the vibration is well suppressed, and the whole system converges to an equilibrium position with less error.

Future research can consider expanding to three dimension to study the vibration suppression of the manipulators, and adding practical experiments to verify the algo-

rhythm, besides adding a variety of complex vibration types, to make the simulation and experimental environment more realistic.

Author Contributions: Conceptualization, Q.Z.; methodology, Z.W.; validation, W.M.; investigation, P.Z.; writing Z.W. and J.D. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Natural Science Foundation of China (62073239).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding authors.

Conflicts of Interest: All authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

- Kim, W.; Tendick, F.; Ellis, S.; Stark, L. A comparison of Position and Rate Control for Telemanipulations with Consideration of Manipulator System Dynamics. *IEEE J. Robot. Autom.* **1987**, *3*, 426–436. [\[CrossRef\]](#)
- Yun, A.; Moon, D.; Ha, J.; Kang, S.; Lee, W. Modman: An Advanced Reconfigurable Manipulator System with Genderless Connector and Automatic Kinematic Modeling Algorithm. *IEEE Robot. Autom. Lett.* **2020**, *5*, 4225–4232. [\[CrossRef\]](#)
- Mohamed, Z.; Tokhi, M.O. Hybrid Control Schemes for Input Tracking and Vibration Suppression of a Flexible Manipulator. *Proc. Inst. Mech. Eng. Part I J. Syst. Control Eng.* **2003**, *217*, 23–34. [\[CrossRef\]](#)
- Uyar, M. Comparison of Classical and Newly Designed Motion Profiles for Motion-Based Control of Flexible Composite Manipulator. *J. Braz. Soc. Mech. Sci. Eng.* **2022**, *44*, 414. [\[CrossRef\]](#)
- Shi, M.; Cheng, Y.; Rong, B.; Zhao, W.; Yao, Z. Research on Vibration Suppression and Trajectory Tracking Control Strategy of a Flexible Link Manipulator. *Appl. Math. Model.* **2022**, *110*, 78–98. [\[CrossRef\]](#)
- Alandoli, E.A.; Lee, T.S.; Lin, T.S.; Mohammed, M.Q. Dynamic Modelling and Intelligent Hybrid Optimal Controller of Hybrid Manipulator for Vibration Suppression and Tracking Control. *Arab. J. Sci. Eng.* **2023**, *49*, 1–17. [\[CrossRef\]](#)
- Li, Y.; Ge, S.S.; Wei, Q.; Gan, T.; Tao, X. An Online Trajectory Planning Method of a Flexible-Link Manipulator Aiming at Vibration Suppression. *IEEE Access* **2020**, *8*, 130616–130632. [\[CrossRef\]](#)
- Jia, S.; Shan, J. Velocity-Free Trajectory Tracking and Active Vibration Control of Flexible Space Manipulator. *IEEE Trans. Aerosp. Electron. Syst.* **2021**, *58*, 435–450. [\[CrossRef\]](#)
- Liu, S.; Yang, H.; Liu, Z.; Langari, R. Adaptive Neural Network Independent Joint-Based Control for an ODE-PDE Rigid-Flexible Manipulator with Multiple Constraints. *J. Vib. Control* **2024**, *30*, 1647–1658. [\[CrossRef\]](#)
- Mei, Y.; Liu, Y. ILC-RBNNF-Based Vibration Control of a Rotatable Manipulator with Time-Varying Output Constraints. *IEEE Trans. Syst. Man Cybern. Syst.* **2023**, *53*, 6416–6425. [\[CrossRef\]](#)
- Wang, J.; Liu, J.; Ji, B.; He, Y.; Xia, S.; Zhou, Y. Vibration Suppression and Boundary Control for Nonlinear Flexible Rotating Manipulator in Three-Dimensional Space Subject to Output Restrictions. *Commun. Nonlinear Sci. Numer. Simul.* **2023**, *120*, 107151. [\[CrossRef\]](#)
- Zhou, X.; Wang, H.; Tian, Y. Adaptive Boundary Iterative Learning Vibration Control Using Disturbance Observers for a Rigid-Flexible Manipulator System with Distributed Disturbances and Input Constraints. *J. Vib. Control* **2022**, *28*, 1324–1340. [\[CrossRef\]](#)
- Li, L.; Liu, J. Nussbaum Function-Based Adaptive Boundary Control for Flexible Manipulator with Unknown Control Directions and Nonlinear Time-Varying Actuator Faults. *Int. J. Robust Nonlinear Control* **2023**, *33*, 6778–6798. [\[CrossRef\]](#)
- Liu, Y.; Zhan, W.; Xing, M.; Wu, Y.; Xu, R.; Wu, X. Boundary Control of a Rotating and Length-Varying Flexible Robotic Manipulator System. *IEEE Trans. Syst. Man Cybern. -Syst.* **2020**, *52*, 377–386. [\[CrossRef\]](#)
- Min, W.; Liu, Q. An improved adaptive fuzzy backstepping control for nonlinear mechanical systems with mismatched uncertainties. *Automatika* **2019**, *60*, 1–10. [\[CrossRef\]](#)
- Wan, M.; Liu, Q. Adaptive fuzzy backstepping control for uncertain nonlinear systems with tracking error constraints. *Adv. Mech. Eng.* **2019**, *11*, 1687814019851309. [\[CrossRef\]](#)
- Liu, C.; Gao, W.; Gao, P. Vibration Suppression Method for a Two-Link Flexible Manipulator Based on Adaptive Iterative Learning Algorithm. *J. Vib. Eng. Technol.* **2024**, *12*, 587–599. [\[CrossRef\]](#)
- Li, L.; Liu, J. Consensus Tracking Control and Vibration Suppression for Nonlinear Mobile Flexible Manipulator Multi-Agent Systems Based on PDE Model. *Nonlinear Dyn.* **2023**, *111*, 3345–3359. [\[CrossRef\]](#)
- Li, L.; Cao, F.; Liu, J. Adaptive Vibration Control for Constrained Moving Vehicle-Mounted Nonlinear 3D Rigid-Flexible Manipulator System Subject to Actuator Failures. *J. Vib. Control* **2023**, *29*, 4155–4171. [\[CrossRef\]](#)

20. Zhu, J.; Zhang, J.; Tang, X.; Pi, Y. Adaptive Boundary Control of a Flexible-Link Flexible-Joint Manipulator Under Uncertainties and Unknown Disturbances. *J. Vib. Control* **2023**, *29*, 169–184. [\[CrossRef\]](#)
21. Han, F.; Jia, Y. Bipartite Consensus and Distributed Boundary Control for Multiple Flexible Manipulators Associated with Signed Digraph. *IEEE Trans. Syst. Man Cybern. Syst.* **2021**, *52*, 3005–3014. [\[CrossRef\]](#)
22. Wang, T.; Umemoto, K.; Endo, T.; Matsuno, F. Modeling and Control of a Quadrotor UAV Equipped with a Flexible Arm in Vertical Plane. *IEEE Access* **2021**, *9*, 98476–98489. [\[CrossRef\]](#)
23. Tao, W.; Jiang, F.; Li, L.; Zhang, D.; Guo, X. Dynamical Analysis and Vibration Estimation of a Flexible Plate with Enhanced Active Constrained Layer Damping Treatment by Combinatorial Neural Networks of Surrogates. *Aerosp. Sci. Technol.* **2023**, *133*, 108136. [\[CrossRef\]](#)
24. Shang, D.; Li, X.; Yin, M.; Liu, J. Rotation tracking control strategy for space flexible structures based on neural network compensation. *Adv. Space Res.* **2024**, *73*, 2004–2023. [\[CrossRef\]](#)
25. Song, H.; Shan, X.; Zhang, L.; Wang, G.; Fan, J. Research on Identification and Active Vibration Control of Cantilever Structure Based on NARX Neural Network. *Mech. Syst. Signal Process.* **2022**, *171*, 108872. [\[CrossRef\]](#)
26. He, W.; Kang, F.; Kong, L.; Feng, Y.; Feng, Y.; Cheng, G.; Sun, C. Vibration Control of a Constrained Two-Link Flexible Robotic Manipulator with Fixed-Time Convergence. *IEEE Trans. Cybern.* **2021**, *52*, 5973–5983. [\[CrossRef\]](#)
27. Zhao, Z.; Cai, S.; Ma, G.; Yu, F.R. Vibration Control of an Experimental Flexible Manipulator Against Input Saturation. *IEEE/CAA J. Autom. Sin.* **2023**, *10*, 1340–1342. [\[CrossRef\]](#)
28. Wang, T.; Chen, Y. Event-Triggered Control of Flexible Manipulator Constraint System Modeled by PDE. *Math. Biosci. Eng.* **2023**, *20*, 10043–10062. [\[CrossRef\]](#)
29. Esmaeilzadeh, S.M.; Zeyghami, M.S. Nonlinear Finite Time Attitude Control of Flexible Spacecraft Based on a Novel Output Redefinition Method. *Chin. J. Aeronaut.* **2023**, *36*, 373–385. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.