

Article

Numerical Calculation Method of Key Performance Parameters of Proppant Based on 2D Computer Simulation

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Abstract: The key performance parameters of proppant are mainly the crushing rate and fracture conductivity, which are usually evaluated using physical experimental methods. However, the testing method for fracture conductivity has limitations, such as its long time-consumption, high testing costs, instability, and even the presence of large errors in testing results under the same conditions. The purpose of this paper is to propose a calculation method that can replace physical experiments. Firstly, we analyze the random and deterministic phenomena in the contact relationship between proppant particles from a microscopic perspective. Subsequently, we develop a physical model of the microscopic arrangement of these particles, enabling us to conduct further computer simulations of their microscopic configuration. Secondly, we conduct a microscopic mechanical analysis of the contact between proppant particles and between particles and boundaries and establish a corresponding mathematical model. Then, utilizing the simulation and mechanical analysis results of the proppant, we calculate the crushing rate. Considering the crushing rate of proppant, we improve the Kozeny–Carmen equation to determine the fracture permeability, and subsequently calculate the fracture conductivity. Finally, the calculated results are compared with the experimental results. The results show that the calculated values for the proppant crushing rate and fracture conductivity matched well with experimental data, and that the model's calculation values were more accurate. As the number of simulations increased, the accuracy of the calculation results became higher. Research shows that the fracture conductivity is influenced by factors such as the particle size, microstructure, and crushing rate. Numerical calculation methods can replace physical experiments and provide theoretical support for engineering applications of hydraulic fracturing proppant materials.

Keywords: numerical calculation; proppant; crushing rate; fracture conductivity



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1. Introduction

Hydraulic fracturing is the main method for increasing oil and gas reservoir production, and proppant is one of the key materials for hydraulic fracturing. Its main function is to support fractures and create channels with high conductivity for fluid flow inside the fractures, achieving high and stable production in oil and gas wells [1–3]. The crushing rate and fracture conductivity of proppant are key parameters for evaluating its performance, as well as the necessary parameters for proppant selection, fracturing design, and post-fracturing evaluation.

The proppant crushing rate is usually obtained using physical experimental methods. Bandara et al. [4] investigated the effect of the proppant crushing rate on porosity and permeability by studying different proppant types, particle sizes, and placement concentrations. After experimental comparison, under the same closure pressure, quartz sand with the same particle size had a higher crushing rate than resin sand and ceramic, producing a large amount of fine powder. The more layers that were laid, the smaller the proppant

crushing rate and the larger the porosity. Man et al. [5] tested and analyzed the crushing rate of four types of proppant; through experiments, it was found that the proppant exhibited creep deformation behavior after being subjected to closing pressure, which is caused by the gradual fragmentation of the surface roughness of the proppant particles. Under long-term closure pressure, the proppant was more prone to breakage. Zheng et al. [6] investigated the influence of the geometric shape and fragmentation characteristics of four different types of proppant on the fracture conductivity. In addition, they analyzed the relationship between the particle size, properties, and crushing rate of the proppant, as well as the permeability and fracture conductivity.

The study of fracture conductivity is mainly based on physical experiments or physical experiments combined with numerical simulations. Osipov [7] analyzed permeability and fracture conductivity by combining physical experiments with numerical simulations for spheres, ellipsoids, cylinders, and mixtures of spheres and cylinders at different volume concentrations. Similarly, McDaniel et al. [8] experimentally verified that cylindrical proppant can provide a higher porosity than spherical proppant, realizing a significant increase in fracture conductivity. Jiang et al. [9] established a fracture width model for sandstone and shale reservoirs based on the mechanical properties of proppant in hydraulic fracturing, comprehensively considering the influence of closure pressure on the mechanical response of the proppant and surrounding rock. Taking into account the changes in the fracture width and proppant permeability, they further established a fracture conductivity model. Meng et al. [10] assumed that proppant is a rigid sphere and that proppant particles are arranged in a diamond shape, and established a fracture inflow calculation model with different proppant sizes based on the Carman–Kozeny formula, taking into account the closure pressure, proppant particle size, number of layers, and proppant embedment. Xu et al. [11] established a fracture conductivity calculation model considering the geometric shape and embedding of proppant columns. Guo et al. [12] studied the distribution and placement shape of proppants in channel fracturing technology through experiments, and established a calculation model for fracture conductivity using the lattice Boltzmann method. Zheng et al. [13] analyzed the influence of proppant distribution density and proppant columnar radius on fracture conductivity. Furthermore, based on the Hertz contact theory and proppant embedding, they established a permeability and fracture conductivity model for proppant columnar placement. Zhu et al. [14] established a mathematical model for fracture width and fracture conductivity by studying the columnar placement of proppants and the mechanical properties of formation rocks, taking into account the elastic deformation of proppant and fracture walls. Some scholars have introduced auxiliary software, including Computational Fluid Dynamics (CFD), the Finite Element Method (FEM), the Discrete Element Method (DEM), and Particle Flow Code (PFC), among others [15–25], to investigate the transport process of proppant and the impact of proppant particles on fracture conductivity. Zhu et al. [16] used a DEM–CFD coupled model to establish a calculation model for fracture conductivity by considering parameters such as the fracture closure pressure, sand concentration, and elastic modulus.

In summary, preliminary indoor experiments indicated that physical experimental methods suffer from prolonged testing times, high costs (particularly for fracture conductivity experiments), and exhibit unstable or even significant errors in test results under identical conditions. The phenomenon of repeatedly conducting experiments with different results is common, and the reasons behind these discrepancies and the underlying laws governing them are not clearly understood. The methods of physical experiments and numerical simulations mainly use empirical and semi-empirical models with poor generalizability, and some mathematical models also ignore some key parameters (such as the crushing rate). If there is also the need for rock triaxial mechanical testing to obtain parameters such as the elastic modulus and Poisson's ratio of rock samples and proppant, this will greatly increase time and economic costs.

According to the limitations of the above research methods, this paper proposes a theoretical calculation method that can replace repetitive physical experiments to obtain

the key performance parameters of proppant. By studying the micro-arrangement of proppant particles through computer simulation, both the randomness and deterministic patterns of the micro-arrangement are revealed. On this basis, a mathematical model of the mechanical relationship between proppant particles is established to grasp the force situation of each proppant particle in the fracture under a given closure pressure. Starting from the micro-arrangement simulation and mechanical analysis of proppant particles, we combine micro and macro perspectives, numerical calculations, and simple proppant crushing experiments to obtain the key performance parameters of the proppant and reveal the possible reasons and patterns behind different repeated experimental results. This approach provides theoretical support for the engineering application of hydraulic fracturing proppant materials.

2. Simulation of Proppant Particle Arrangement and Micromechanical Analysis

2.1. Physical Modeling and Computer Simulation

A physical model of proppant particles with different diameters was established based on the arrangement pattern of hydraulic fracturing proppant particles and their arrangement in a proppant performance testing device, as shown in Figure 1. In the x direction is the fracture width w_f , and in the y direction is the fracture length L_f . The pressure F_c was applied at the left and right boundaries of the fracture, while the upper and lower boundaries were closed. The magnitude of the pressure on each proppant particle was determined through mathematical modeling of the mechanical relationships between the proppant particles. When the pressure F_c was applied at the upper and lower boundaries of the fracture, the left and right boundaries were closed and could also be modeled.

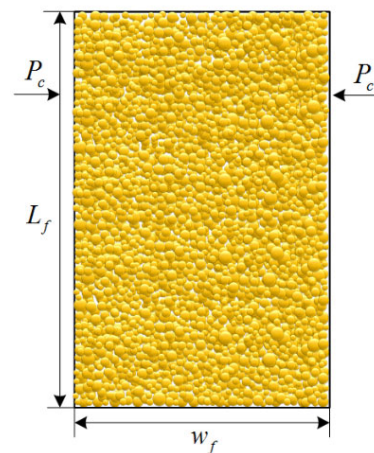


Figure 1. Physical model of fracture and proppant.

In mathematical modeling and 2D simulations, it is assumed that the proppant particles are circular. Under closed pressure, the fractures propagate parallel to the direction of force, while remaining unchanged perpendicular to the direction of force. The arrangement rule of proppant particles is divided into three steps [25,26]:

1. Proppant particles are arranged one by one, layer by layer, from the bottom up.
 2. The location of each proppant particle placement is determined to be at the shortest distance between the center of the proppant particle circle and the bottom of the fracture.
 3. When the shortest distance between the center of the proppant particle and the bottom of the fracture is unique, it is a deterministic placement; otherwise, if this position is not unique, it is a random placement.
- (1) Random phenomena of multi-particle-size proppant and its treatment

The boundary conditions of the 2D graph have a width of w_f and a length of L_f . The fracture width in the x direction is w_f and the fracture length in the y direction is L_f . The

range of the proppant particle sizes is $[R_{\min}, R_{\max}]$ and $R_{\min} < R_{\max}$. The Monte Carlo method is used to generate the random numbers ζ_j ; the random number range of ζ_j is $[0, 1]$, which represents the random numbers of the proppant particles' positions that are in contact with the lower boundary. The random particle size (circle diameter) of the proppant particle can be determined according to Formula (1). The circle center of the first proppant particle is (x_1, y_1) .

$$\begin{cases} x_1 = \frac{1}{2}R_1, \\ y_1 = \frac{1}{2}R_1. \end{cases} \quad (1)$$

Then, the coordinates of the circle center of the proppant particle that are in contact with the lower boundary (x_i, y_i) are calculated by the following equation:

$$\begin{cases} x_i = x_{i-1} + \frac{1}{2}R_{i-1} + \lambda_j + \frac{1}{2}R_i, \\ y_i = \frac{R_i}{2}. \end{cases} \quad (1 < j < n_x, j = i - 1) \quad (2)$$

where $R_i = R_{\min} + \zeta (R_{\max} - R_{\min})$, ζ is a random number, and $\zeta \in [0, 1]$; λ_j is a random assignment coefficient, $\lambda_j = \zeta_j + 0.5 R_{\min}$, and ζ_j is a random number for the position of the proppant particle in contact with the lower boundary.

The total number of proppant particles in contact with the lower boundary is n_x . As shown in Figure 2a, due to the uncertainty of the particle size R_i in the fracture width w_f , the total number of n_x is also uncertain. However, the interval of n_x can be determined as:

$$\text{int}\left(\frac{w_f}{R_{\max}}\right) \leq n_x \leq \text{int}\left(\frac{w_f}{R_{\min}}\right). \quad (3)$$

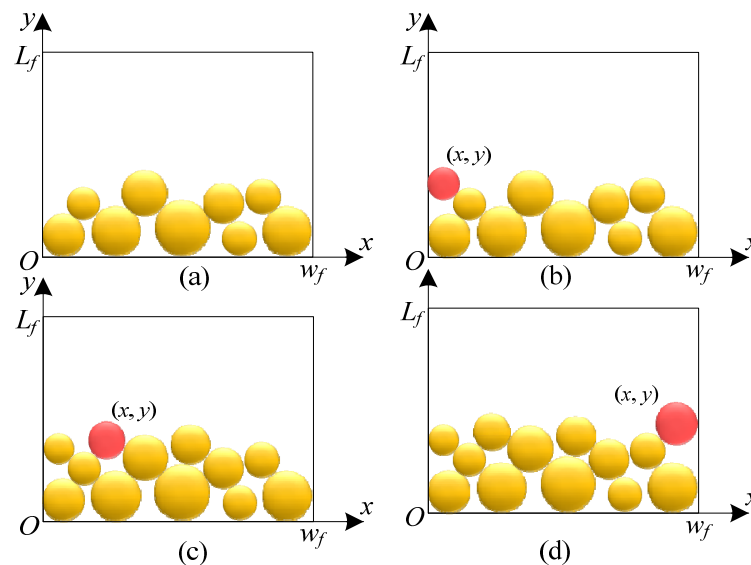


Figure 2. Schematic diagram for modeling the arrangement of proppant particles.

The set of coordinates for the circle centers that are in contact with the lower boundary is denoted as:

$$Q = \begin{bmatrix} x_1 & x_2 & \cdots & x_{n_x} \\ y_1 & y_2 & \cdots & y_{n_x} \\ R_1 & R_2 & \cdots & R_{n_x} \end{bmatrix}. \quad (4)$$

where R_1 is the proppant diameter, $R_1 \in [R_{\min}, R_{\max}]$, mm; n_x is the total number of proppant particles in contact with the lower boundary, dimensionless; $\text{int}(x_0)$ represents the rounding down of any real number x_0 ; x_{n_x} is the horizontal coordinate; y_{n_x} is the vertical coordinate; and Q is the set of all the circle centers.

(2) Deterministic phenomena of multi-particle-size proppant and its treatment

Set a temporary dynamic matrix T to be the set of the circle center of the highest (outermost) layer on the 2D plane space. When the total number of proppant particles n_p is less than or equal to the maximum number of the first layer in the x -axis direction n_x ($n_p \leq n_x$), then the temporary dynamic matrix is $T = Q$ for the circle centers of the highest layer of proppant particles.

When the total number of proppant particles n_p is greater than the maximum number n_x of the first layer in the x -axis direction, arrange the particles one by one along the longitudinal direction (y -axis direction), ensuring that the distance between the central axis of the particle arrangement and the bottom boundary is minimized. Then, the coordinates Q of the circle center of all particles can be expressed as:

$$Q = \begin{bmatrix} x_1 & x_2 & \cdots & x_{n_p} \\ y_1 & y_2 & \cdots & y_{n_p} \\ R_1 & R_2 & \cdots & R_{n_p} \end{bmatrix}. \quad (5)$$

where n_p is the total number of proppant particles, dimensionless and R_{np} is the diameter of the circle center, $R_{np} \in [R_{\min}, R_{\max}]$, mm.

Assuming that the coordinates of the new circle center of the proppant particles are (x'_i, y'_i) , and that the particle size of the new circle is R_i , we can classify the arrangement of this new center relative to other centers into three main situations: tangential arrangement with the left boundary, tangential arrangement with the right boundary, and middle arrangement. These three situations will be modeled and analyzed separately.

- (1) When the proppant particles are tangent to the left boundary, as shown in Figure 2b, the coordinates of the new circle center of the particles that may be laid are calculated by the following formula:

$$\begin{cases} x'_i = R_i/2, \\ \sqrt{(x'_i - x_k)^2 + (y'_i - y_k)^2} = R_i/2 + R_k/2. \end{cases} \quad (6)$$

- (2) When the proppant particles are not tangential to both the left and right boundaries, they are considered to be in a middle position, as illustrated in Figure 2c. In such cases, where multiple particles occupy the middle position, we identify the particle with the lowest center among them. The coordinates of this lowest central particle can then be calculated using the following formula:

$$\begin{cases} \sqrt{(x'_i - x_k)^2 + (y'_i - y_k)^2} = R_i/2 + R_k/2, \\ \sqrt{(x'_i - x_l)^2 + (y'_i - y_l)^2} = R_i/2 + R_l/2. \end{cases} \quad (7)$$

- (3) When the proppant particles are tangential to the right boundary, as shown in Figure 2d, the coordinates of the new circle center of the particles that may be laid are calculated by the following formula:

$$\begin{cases} x'_i = w_f - R_i/2, \\ \sqrt{(x'_i - x_k)^2 + (y'_i - y_k)^2} = R_i/2 + R_k/2. \end{cases} \quad (8)$$

In summary, according to the three cases of micro-arrangements of proppant particles, the objective function is to find the lowest position of the new circle center:

$$y_i = \min\{y'_i\}. \quad (9)$$

where (x'_i, y'_i) are the new circle center coordinates of the proppant particle; (x_k, y_k) , (x_l, y_l) are the existing circle centers in the set of Q ; R_i is the new circle diameter, $R_i \in [R_{\min}, R_{\max}]$, mm; and R_k and R_l are the existing circles diameters in the set of Q , mm.

The simulation process for proppant particles is comprised of 7 distinct steps:

- ① Given the maximum boundary values along the x -axis and y -axis, denoted as w_f and L_f , respectively, generate random numbers within a specified radius range, and subsequently create circles randomly based on a distribution range of particle sizes corresponding to the proppant material.
- ② Generate the first circle center (x_1, y_1) as the initial circle center (R_1, R_1) in the first row (tangent to the x -axis $(0, 0) \sim (w_f, 0)$). The interval between circles in the first row is calculated by multiplying the minimum specified radius by a random number λ_j . The position of each subsequent circle is determined based on this random interval.
- ③ If $x_i > w_f - R_i/2$ of the last circle center (x_i, y_i) in the first row, then the last circle center in the first row is adjusted to $(w_f - R_i/2, y_i)$. If $x_i < w_f - R_i$, then the circle in the first row fills the space $(0, 0) \sim (w_f, 0)$ in the x -axis direction.
- ④ Based on the existing circle, a traversal search algorithm is used to find the new circle with the minimum y_i in the temporary dynamic matrix T ; that is, using $\min(y_i)$ as the criterion, a new center $(x_i, \min(y_i))$ is generated.
- ⑤ Determine whether the new circle is crossed, overlapped, or suspended with other filled circles. If the above phenomenon exists, go to step ④ and readjust the circle. If this phenomenon does not exist, the new circle center (x_i, y_i) is determined.
- ⑥ Determine whether the new circle exceeds the maximum boundary value L_f in the y -axis direction—specifically, whether $y_i > L_f - R_i/2$. If not, this means that the two-dimensional space has not been filled yet, and we will proceed to step ④ to continue filling the new circle. If so, it can be determined that the entire 2D space has been filled.
- ⑦ Select a smaller R_i and fill the existing circle with a new circle (x_i, y_i) . When $y_i < L_f - R_i/2$, it is considered that the current 2D space is not fully filled (Figure 3a). If so, proceed to step ④ and continue filling the 2D space with a new small circle with a diameter of R_i . When $y_i > L_f - R_i/2$, select a smaller diameter R_i again. If the diameter R_i is selected in a decreasing manner three times in a row, always $y_i > L_f - R_i/2$; in such cases, it is considered that the current bottom space has been filled up (Figure 3b), the loop is exited, and the operation ends.

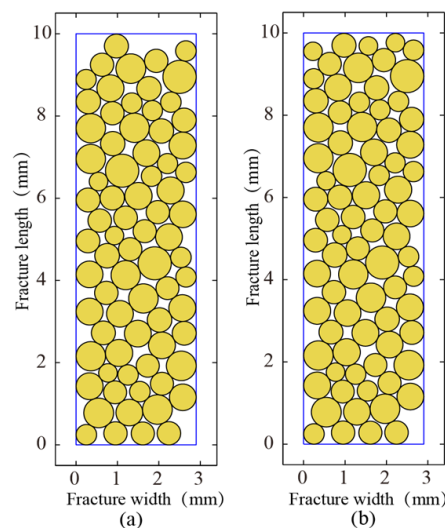


Figure 3. Example diagram of 2D simulation of proppant particles ((a) before top layer optimization, (b) after top layer optimization).

2.2. Mathematical Modeling of the Mechanical Relationships of Proppant

The microscopic arrangement of two or more proppant particles with different particle sizes is extremely complex. When conducting mechanical analysis and modeling between proppant particles, it is imperative to consider and distinguish the stress conditions not

only between particles but also between particles and boundaries, subsequently modeling each condition separately to ensure accuracy.

(1) Mathematical expression of proppant particle contact

Any particle may or may not be in contact with other particles. For the convenience of expression, a contact coefficient δ_{ix} , δ_{iy} , δ_{ij} is introduced to represent the state of contact.

(1) When the proppant particles contact the boundaries along the x-axis (either the left or right side), the contact is noted.

$$\delta_{ix} = \begin{cases} 1, & x_i = R_i/2 \text{ or } x_i = w_f - R_i/2, \\ 0, & x_i \neq R_i/2 \text{ and } x_i \neq w_f - R_i/2. \end{cases} \quad (10)$$

(2) When the proppant particles contact the boundaries along the y-axis (either the upper or lower side), the contact is noted.

$$\delta_{iy} = \begin{cases} 1, & y_i = R_i/2 \text{ or } y_i = L_f - R_i/2, \\ 0, & y_i \neq R_i/2 \text{ and } y_i \neq L_f - R_i/2. \end{cases} \quad (11)$$

(3) When the proppant particles are not in contact on the boundaries along the x-axis and y-axis, the non-contact condition is noted.

$$\delta_{ij} = \begin{cases} 1, & \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} = R_i/2 + R_j/2, \\ 0, & \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \neq R_i/2 + R_j/2. \end{cases} \quad (12)$$

where $i = 1, 2, 3, \dots, n_p$; $j = 1, 2, 3, \dots, n_p$, and $i \neq j$.

(2) Mathematical modeling of pressures on proppant particles

The mechanical relationship between proppant particles and between particles and boundaries includes three aspects:

(1) The pressure on the proppant particle

The arrangement of proppants becomes significantly more complex due to their varying particle sizes, resulting in a differing number of contacts between each particle and others. Any proppant particle may or may not come into contact with other particles. Similarly, any proppant particle may or may not be in contact with the upper, lower, left, and right boundaries. Therefore, the mathematical model of mutual contact between proppant particle can be expressed as:

$$\left\{ \begin{array}{l} \sum_{j=1}^{n_p} \delta_{1j} \mathbf{F}_1^j + \delta_{1x=0} \mathbf{F}_1^{x=0} + \delta_{1x=w_f} \mathbf{F}_1^{x=w_f} + \delta_{1y=0} \mathbf{F}_1^{y=0} + \delta_{1y=L_f} \mathbf{F}_1^{y=L_f} = 0, \\ \sum_{j=1}^{n_p} \delta_{2j} \mathbf{F}_2^j + \delta_{2x=0} \mathbf{F}_2^{x=0} + \delta_{2x=w_f} \mathbf{F}_2^{x=w_f} + \delta_{2y=0} \mathbf{F}_2^{y=0} + \delta_{2y=L_f} \mathbf{F}_2^{y=L_f} = 0, \\ \dots \quad \dots \quad \dots \\ \sum_{j=1}^{n_p} \delta_{ij} \mathbf{F}_i^j + \delta_{ix=0} \mathbf{F}_i^{x=0} + \delta_{ix=w_f} \mathbf{F}_i^{x=w_f} + \delta_{iy=0} \mathbf{F}_i^{y=0} + \delta_{iy=L_f} \mathbf{F}_i^{y=L_f} = 0, \\ \dots \quad \dots \quad \dots \\ \sum_{j=1}^{n_p} \delta_{(n_p-1)j} \mathbf{F}_{n_p-1}^j + \delta_{(n_p-1)x=0} \mathbf{F}_{n_p-1}^{x=0} + \delta_{(n_p-1)x=w_f} \mathbf{F}_{n_p-1}^{x=w_f} + \delta_{(n_p-1)y=0} \mathbf{F}_{n_p-1}^{y=0} + \delta_{(n_p-1)y=L_f} \mathbf{F}_{n_p-1}^{y=L_f} = 0, \\ \sum_{j=1}^{n_p} \delta_{nj} \mathbf{F}_{n_p}^j + \delta_{nx=0} \mathbf{F}_{n_p}^{x=0} + \delta_{nx=w_f} \mathbf{F}_{n_p}^{x=w_f} + \delta_{ny=0} \mathbf{F}_{n_p}^{y=0} + \delta_{ny=L_f} \mathbf{F}_{n_p}^{y=L_f} = 0. \end{array} \right. \quad (13)$$

where $\mathbf{F}_{np}^j$ represents the pressure between the n_p -th proppant particle and the j -th proppant particle, N; n_p is the total number of proppant particles, dimensionless; i is a subscript,

$i = 1, 2, \dots, n_p$; j is the subscript, $j = 1, 2, \dots, n_p$; w_f is the fracture width, mm; and L_f is the fracture length, mm.

(2) The interaction and reaction force between proppant particles

For any two particles that come into contact with each other, there is a relationship between the action force and the reaction force. The expression that describes the relationship between the force exerted by each particle on another particle and the resulting reaction force is:

$$\begin{cases} \mathbf{F}_1^j - \delta_{1j} \mathbf{F}_j^1 = 0, \\ \mathbf{F}_2^j - \delta_{2j} \mathbf{F}_j^2 = 0, \\ \dots \quad \dots \\ \mathbf{F}_i^j - \delta_{ij} \mathbf{F}_j^i = 0, \\ \dots \quad \dots \\ \mathbf{F}_{n-1}^j - \delta_{(n-1)j} \mathbf{F}_j^{n-1} = 0, \\ \mathbf{F}_{n_p}^j - \delta_{nj} \mathbf{F}_j^{n_p} = 0. \end{cases} \quad (14)$$

where $i = 1, 2, 3, \dots, n_p$; $j = 1, 2, 3, \dots, n_p$, and $i \neq j$.

(3) Fracture boundary pressures

When pressure P_c is applied to the fracture length (left and right boundaries)—that is, the boundary pressures applied to the left boundary $x = 0$ and on the right boundary $x = w_f$ are $\mathbf{F}_c^{x=0}$ and $\mathbf{F}_c^{x=w_f}$, respectively—considering the characteristics of the 2D simulation, the magnitude of the pressures on the left and right boundaries is:

$$|\mathbf{F}_c^{x=0}| = |\mathbf{F}_c^{x=w_f}| = P_c L_f \frac{1}{n_p} \sum_{i=1}^{n_p} R_i. \quad (15)$$

where n_p is the number of different particle sizes of the proppant, dimensionless; P_c is the closure pressure, MPa; and R_i is the particle size of the proppant, $R_i \in [R_{\min}, R_{\max}]$.

The pressure equations for the left and right boundaries and the upper and lower boundaries are expressed as:

$$\begin{cases} \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{x=0} = \mathbf{F}_c^{x=0}, \\ \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{x=0} = \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{x=w_f}, \\ \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{y=0} - \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{y=L_f} = 0. \end{cases} \quad (16)$$

When the pressure P_c is in the direction of the upper and lower boundaries—that is, the boundary pressures applied to the lower boundary $y = 0$ and the upper boundary $y = L_f$ are $\mathbf{F}_c^{y=0}$ and $\mathbf{F}_c^{y=L_f}$, respectively—considering the characteristics of the 2D simulation, the magnitudes of the pressures on the lower and upper boundaries are:

$$|\mathbf{F}_c^{y=0}| = |\mathbf{F}_c^{y=L_f}| = P_c w_f \frac{1}{n_p} \sum_{i=1}^{n_p} R_i. \quad (17)$$

The pressure equations for the lower and upper boundaries and the left and right boundaries are expressed as:

$$\begin{cases} \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{y=0} = \mathbf{F}_c^{y=0}, \\ \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{y=0} = \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{y=L_f}, \\ \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{x=0} - \sum_{i=1}^{\text{sum}(i)} \mathbf{F}_i^{x=w_f} = 0. \end{cases} \quad (18)$$

According to the three types of mechanical relationships among particles and between particles and boundaries, the mathematical model can be formulated as the following system of linear equations:

$$Ax = b \quad (19)$$

where A is a matrix of coefficients of a system of linear equations with sparse, pathological, and asymmetric characteristics, and x and b are vectors consisting of unknowns and constant terms at the right end, respectively.

Based on the established mathematical model for the micromechanical analysis of the proppant particle, the coefficient matrix of the model exhibits obvious sparse and pathological characteristics. The solution method for this linear method group adopts singular value decomposition, and the specific method is described in references [27–29].

3. Methods for Calculating the Proppant Crushing Rate and Fracture Conductivity

3.1. Crushing Rate

For a proppant particle, its arrangement structure is extremely chaotic, with irregular and irregular characteristics. Therefore, to calculate the proppant particle crushing rate, there are three steps involved: Firstly, the 2D proppant simulation results are required for mechanical analysis and linear equation solving to obtain the force values of each proppant particle in the crack under a given closure pressure. Secondly, follow up with sample curves and numerical algorithms to get the crushing rate η_i of each proppant particle [30,31]. Finally, the average value is taken as the crushing rate of the proppant under a given closure pressure.

$$\bar{\eta} = \frac{\sum_{i=1}^{n_p} \eta_i}{n_p}. \quad (20)$$

where η is the average crushing rate, dimensionless; η_i is the crushing rate; n_p is the total number of proppant particle, dimensionless; and i is the number of proppant particle, $i = 1, 2, \dots, n_p$.

3.2. Fracture Conductivity

Fracture conductivity is the product of fracture permeability and fracture width. Considering the dimensional relationship, the fracture conductivity can be expressed as:

$$C_f = 0.1K_f w_f. \quad (21)$$

where C_f is the fracture conductivity, $\mu\text{m}^2 \cdot \text{cm}$; K is the fracture permeability, μm^2 ; and w_f is the fracture width, mm.

According to the Kozeny–Garman equation [7,13,32], the fracture permeability becomes:

$$K = \frac{\bar{R}_p^2}{180} \frac{\phi^3}{(1 - \phi)^2}. \quad (22)$$

Among them:

$$\bar{R}_p = \frac{\sum_{i=1}^{n_p} R_i}{n_p}. \quad (23)$$

where K is the fracture permeability, μm^2 ; ϕ is the porosity, dimensionless; R_i is the proppant particle size (diameter), $R_i \in [R_{\min}, R_{\max}]$, mm; $\bar{R}_p$ is the average size of the proppant, mm; n_p is the total number of proppant particle, dimensionless; and i is the number of proppant particles, $i = 1, 2, \dots, n_p$.

The porosity is related to the total area of the fracture and the area occupied by the proppant particle. In the next section, we discuss the two situations of the non-breakage and breakage of proppant particles separately:

(1) Situations of non-breakage ($\eta = 0$)

When the fracture closure pressure is not applied, the proppant particles in the fracture do not break. For this situation, the porosity is the ratio of the pore area in the fracture to the total area of the fracture. Also being based on the characteristics of the proppant particles, the porosity is calculated as:

$$\phi = \frac{w_f L_f - \sum_{i=1}^{n_p} \left(\frac{1}{4} \pi R_i^2 \right)}{w_f L_f}. \quad (24)$$

where ϕ is the porosity; w_f is the fracture width, mm; L_f is the fracture length, mm; R_i is the proppant particle size (diameter), $R_i \in [R_{\min}, R_{\max}]$, mm; n_p is the total number of proppant particles; and i is the number of proppant particle, $i = 1, 2, \dots, n_p$.

(2) Situations of breakage ($\eta \neq 0$)

When the closure pressure is applied, the proppant particles in the fracture will break. The broken proppant particles fill the pores between the unbroken proppant particles, and the original area occupied by them causes the fracture to shrink and the width of the fracture to become smaller under the fracture closure pressure. Based on the principle of the conservation of mass, the area occupied by the proppant particles remains unchanged regardless of changes in the microscopic shape and size of the broken proppant particles.

Therefore, the formula for the porosity is changed from (24) to:

$$\phi' = \frac{(w_f L_f - A_c) - \sum_{i=1}^{n_p} \left(\frac{1}{4} \pi R_i^2 \right)}{w_f L_f - A_c}, \quad (25)$$

where A_c is the area of the broken proppant particles, that is:

$$A_c = \sum_{i=1}^{n_p} \left(\frac{1}{4} \pi R_i^2 \eta_i \right), \quad (26)$$

Substituting Equation (26) into Equation (25), the porosity is obtained as:

$$\phi' = 1 - \frac{\sum_{i=1}^{n_p} (\pi R_i^2)}{4w_f L_f - \sum_{i=1}^{n_p} (\pi R_i^2 \eta_i)}. \quad (27)$$

At the same time, the fracture width becomes smaller under the fracture closure pressure. The formula for calculating the fracture width is:

$$w'_f = w_f - \frac{\sum_{i=1}^{n_p} (\pi R_i^2 \eta_i)}{4L_f}. \quad (28)$$

where w'_f is the fracture width of the proppant after breakage, mm; w_f is the fracture width of the proppant before breakage, mm; L_f is the fracture length, mm; R_i is the proppant particle size (diameter), $R_i \in [R_{\min}, R_{\max}]$, mm; n_p is the total number of proppant particles; η_i is the crushing coefficient of the proppant; and i is the number of proppant particles, $i = 1, 2, \dots, n_p$.

Substituting Equation (27) into Equation (22) to obtain the permeability, and then combining it with Equation (28), we get the formula for the fracture conductivity as:

$$C_f = \frac{\sum_{i=1}^{n_p} R_i^2}{1800n_p^2} \frac{\left(4w_f L_f - \sum_{i=1}^{n_p} (\pi R_i^2 \eta_i) - \sum_{i=1}^{n_p} (\pi R_i^2) \right)^3}{4L_f \left(\sum_{i=1}^{n_p} (\pi R_i^2) \right)^2}. \quad (29)$$

where C_f is the fracture conductivity, $\mu\text{m}^2 \cdot \text{cm}$.

4. Results and Discussion

We selected a 20/40 mesh LZ sand and 20/40 mesh XY ceramic, which are widely used in China, as experimental materials. The basic properties of LZ sand are similar to those of Lanzhou quartz sand in China, with an apparent density of 2.91 g/cm^3 , a bulk density of 1.57 g/cm^3 , a particle size range of 0.9–0.45 mm, and a crushing rate of 40.50% under a closing pressure of 52 MPa. The basic properties of XY ceramic are similar to those of Zhengzhou ceramic in China, with an apparent density of 3.11 g/cm^3 , a bulk density of 1.71 g/cm^3 , a particle size range of 0.9–0.45 mm, and a crushing rate of 3.75% under a closing pressure of 52 MPa. The crushing rate and fracture conductivity of the proppant were tested, according to the standard testing procedures recommended by SY/T 5108-2006 [33] and API RP56 [34].

According to the range of the particle size distribution of the LZ sand and the range of the particle size distribution of the XY ceramic [30,31], the LZ sand and XY ceramic particles were subjected to computer simulations and mechanical analyses, respectively. The crushing rate and the fracture conductivity were obtained by numerical calculations.

4.1. Results

Based on the geometrical dimensions of the proppant crushing chamber and fracture conductivity chamber, the diameter of the proppant crushing chamber was 40.5 mm, the length of the proppant conductivity chamber was 177.8 mm, and the proppant placement concentration was set at 5 kg/m^2 (the fracture width was about 2.9 mm). The LZ sand crushing rate and the fracture conductivity were simulated and calculated. Figure 4 shows the simulation and pressure heat map of the fracture conductivity with the LZ sand. Figure 4a presents the results prior to the optimization of the LZ sand simulation, revealing that LZ sand particles did not fully occupy the entire fracture interval. Figure 4b showcases the optimized outcomes of the LZ sand simulation, where the entire fracture space was filled with proppant particles as much as possible through optimization within the fracture interval. Figure 4c displays the stress distribution heatmap of the LZ sand under fracture pressure.

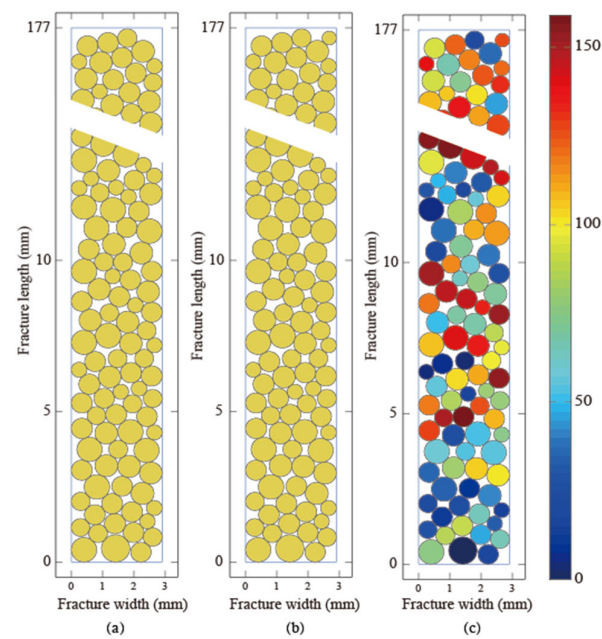


Figure 4. LZ sand simulation and mechanical relationship heat map.

The average crushing rate and fracture conductivity of LZ sand were derived from multiple computer simulations of the proppant particle placement morphology at closure pressures of 10 to 60 MPa. The model values of the LZ sand model were compared with the experimental values, as shown in Figures 5 and 6.

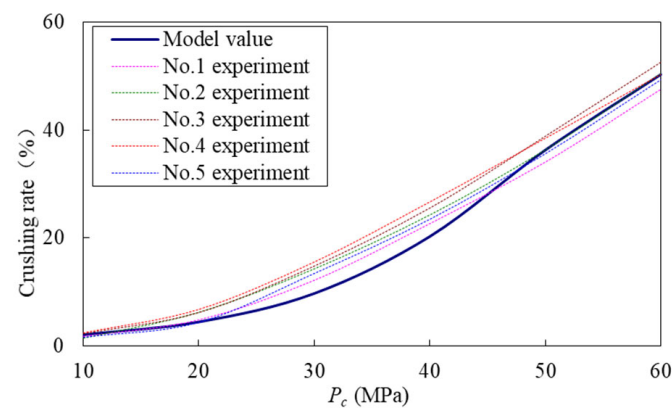


Figure 5. Crushing rate of LZ sand.

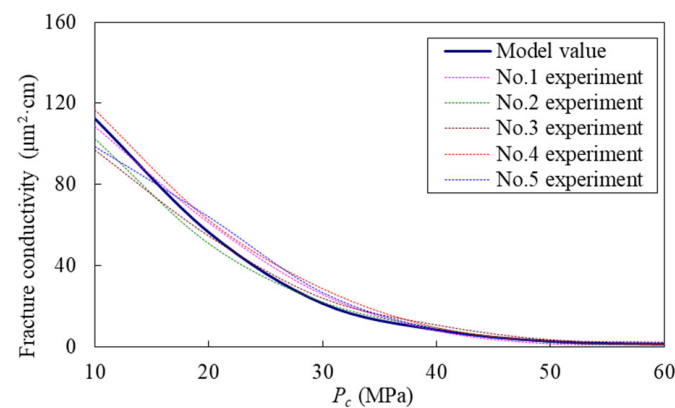


Figure 6. Fracture conductivity of LZ sand.

The crushing rate of the LZ sand continuously increased with increases in the closure pressure. The average error between the model values and the experimental values of the LZ sand crushing rate was 13.68%. The fracture conductivity of the LZ sand was inversely proportional to the closure pressure, and decreased with increases in the closure pressure. The average error between the model values and the experimental values of the LZ sand fracture conductivity was 11.39%.

Following the same approach as for the LZ sand, simulations and calculations were conducted to assess the crushing rate and fracture conductivity of the XY ceramic, as illustrated in Figure 7.

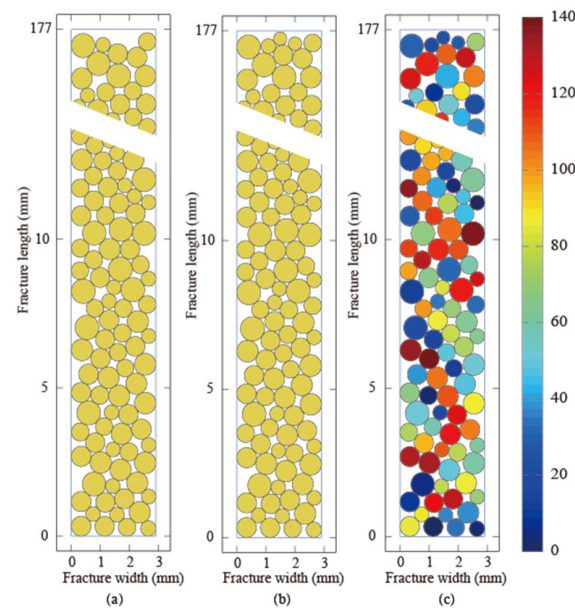


Figure 7. XY ceramic simulation and mechanical relationship heat map ((a) presents the results before the optimization of the XY ceramic simulation, (b) displays the optimized outcomes of the XY ceramic simulation, (c) shows the stress distribution heatmap of XY ceramic under fracture pressure).

The average crushing rate and fracture conductivity of the XY ceramic were derived from multiple computer simulations of the proppant particle placement morphology at closure pressures of 10 to 60 MPa. The model values of the XY ceramic model were compared with the experimental values, as shown in Figures 8 and 9.

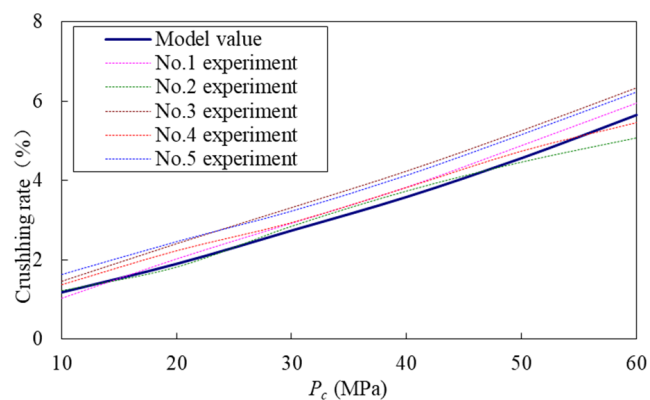


Figure 8. Crushing rate of XY ceramic.

The crushing rate of the XY ceramic continuously increased with increases in the closure pressure. The average error between the model values and the experimental values of XY ceramic crushing rate was 9.26%. The fracture conductivity of the XY ceramic was

inversely proportional to the closure pressure, and decreased with increases in the closure pressure. The average error between the model values and the experimental values of the XY ceramic fracture conductivity was 9.08%.

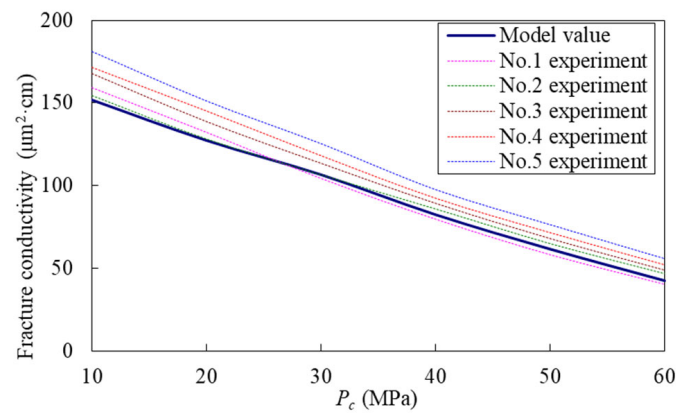


Figure 9. Fracture conductivity of XY ceramic.

4.2. Discussion

For the 20/40 mesh LZ sand, the multi-particle size calculation model presented in this paper was compared with the single particle size model developed by Guo et al. [30], as shown in Figures 10 and 11.

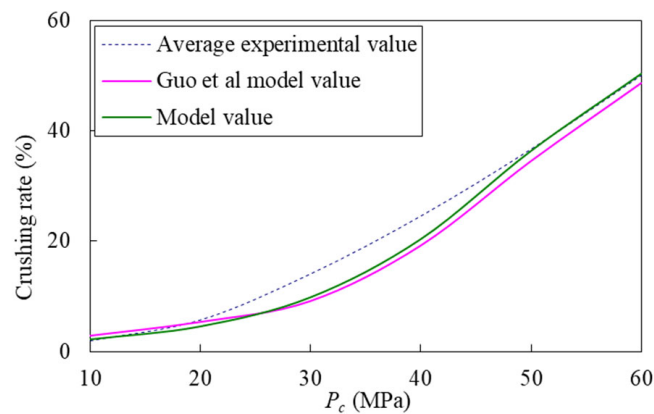


Figure 10. Comparison of the crushing rate of LZ sand [30].

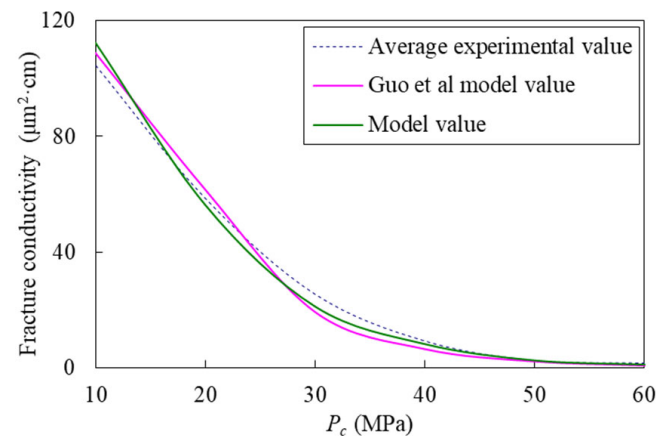


Figure 11. Comparison of the fracture conductivity of LZ sand [30].

The average error between the model values and the experimental values of the single particle-size LZ sand crushing rate was 20.16%. The average error between the model values and the experiment values of the multi-particle LZ sand crushing rate was 13.68%. The average error between the model values and the experimental values of the single particle-size LZ sand fracture conductivity was 21.97%. The average error between the model values and the experimental values of the multi-particle LZ sand fracture conductivity was 11.39%. The particle size distribution of the multi-particle LZ sand was more in-line with the actual particle size distribution of the LZ sand, and the error between the model values and the average experimental values was smaller.

For the 20/40 mesh XY ceramic, the multi-particle-size calculation model presented in this paper was compared with the single particle-size model developed by Guo et al. [30], as shown in Figures 12 and 13.

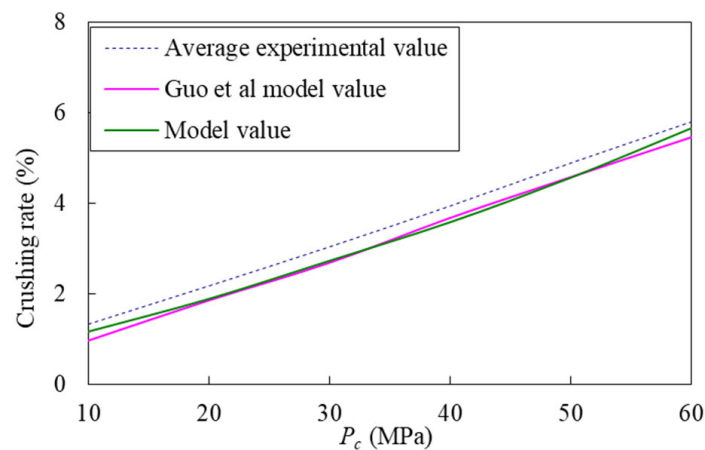


Figure 12. Comparison of the crushing rate of XY ceramic [30].

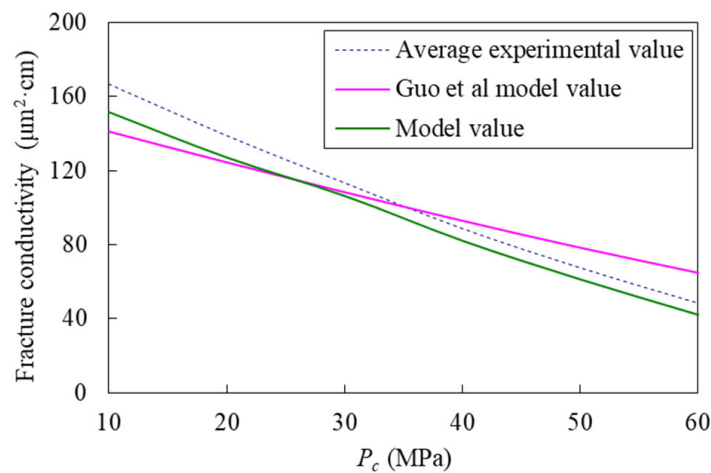


Figure 13. Comparison of the fracture conductivity of XY ceramic [30].

The average error between the model values and the experiment values of the single particle-size XY ceramic crushing rate was 12.61%. The average error between the model values and the experimental values of the multi-particle XY ceramic crushing rate was 9.26%. The average error between the model values and the experimental values of the single particle-size XY ceramic fracture conductivity was 13.90%. The average error between the model values and the experimental values of the multi-particle XY ceramic fracture conductivity was 9.08%. The particle size distribution of the multi-particle XY ceramic was more in-line with the actual particle size distribution of the XY ceramic, and the error between the model values and the average experimental values was smaller.

Each calculation of fracture conductivity requires several computer simulations and calculations, and it is not known how many times the computer simulation converges to a stable result, which consumes a lot of time and effort. The convergence of the number of computer simulations and calculations was investigated by studying the error between the cumulative average and the total average. In Figure 14, the relationship between the cumulative average and total average of the LZ sand fracture conductivity is presented. When the number of simulations was small, the error of the LZ sand calculation results was large. When the number of simulations reached 10 times, the error between the cumulative average and the total average was less than 3%. When the number of simulations reached 27 times, the error between the cumulative average and the total average was less than 1%. Furthermore, as the number of simulations increased, the error tended towards zero. Therefore, setting the number of times for the simulations and calculations for the LZ sand at 27 was able to meet the accuracy requirements.

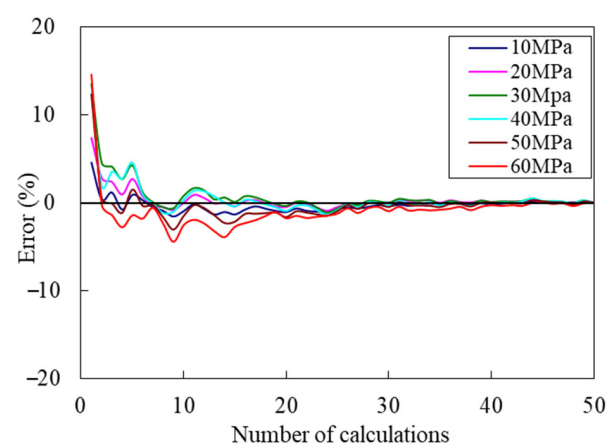


Figure 14. Relationship between the cumulative average and total average of LZ sand.

In Figure 15, the relationship between the cumulative average and total average of the XY ceramic fracture conductivity is presented. The more computer simulations conducted, the more accurate the calculated results. When the number of simulations reached eight times, the error between the cumulative average and the total average was less than 3%. When the number of simulations reached 25 times, the error between the cumulative average and the total average was less than 1%. Furthermore, as the number of simulations increased, the error tended towards zero. Therefore, setting the number of times for simulations and calculations for the LZ sand at 25 was able to meet the accuracy requirements.

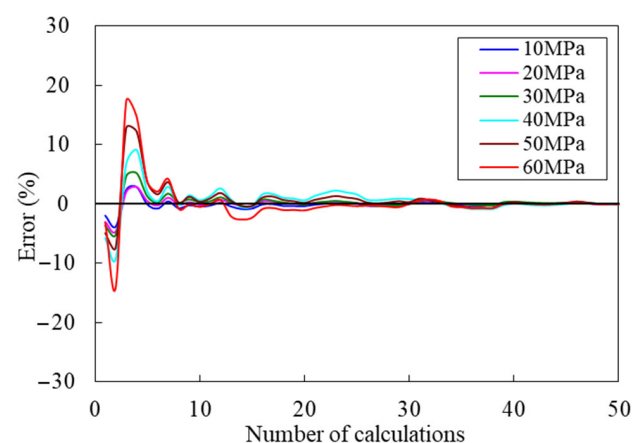


Figure 15. Relationship between the cumulative average and total average of XY ceramic.

5. Conclusions

Based on previous research [30,31], the results obtained in our study were as follows:

- (1) The proppant crushing rate was affected by the distribution of proppant particle size, and the microscopic arrangement of proppant particle varied greatly. The particle sizes of single-sized proppant and actual proppant with a range of particle sizes were very different, resulting in a relatively large error between the calculated crushing rate of the single-sized proppant and the actual crushing rate of the mixed-sized proppant. The calculated value of the multi-particle-size proppant crushing rate was based on multiple simulations and calculations of the actual particle size distribution range of the proppant particles. The error between the calculated value of the proppant crushing rate and the actual crushing rate of the proppant particles was relatively small.
- (2) The fracture conductivity of the proppant was affected by the proppant particle size, micro-arrangement structure, and crushing rate, etc., and the fracture conductivity gradually decreased with increases in the closure pressure. The calculation results of the crushing rate and fracture conductivity of single particle-size proppants had relatively large errors compared to the experimental test results. The model values of the fracture conductivity of proppant particles matched well with the experimental values, and the model calculation values were relatively accurate.
- (3) As the number of simulations increased, the accuracy of the calculation results improved. For different proppant types, the number of simulations should be set according to the demand to meet a required calculation accuracy. When the LZ sand simulation had been run 27 times and the XY ceramic simulation had been run 25 times, the error between the cumulative average value and the total average value of the crack conductivity was less than 1%.

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References

1. Michael, J.; Zhang, B. *Reservoir Stimulation*, 3rd ed.; Petroleum Industry Press: Beijing, China, 2002.
2. Luo, Y.; Wan, R. *Handbook of Petroleum Production Technology*; Petroleum Industry Press: Beijing, China, 2005.
3. Britt, L.K.; Smith, M.B.; Haddad, Z.; Lawrence, P.; Chipperfield, S.; Hellman, T. Waterfracs: We do need proppant after all. In Proceedings of the SPE Annual Technical Conference and Exhibition, Society of Petroleum Engineers, San Antonio, TX, USA, 24–27 September 2006.
4. Bandara, K.M.A.S.; Ranjith, P.G.; Rathnaweera, T.D.; Wanniarachchi, W.A.M.; Yang, S.Q. Crushing and embedment of proppant packs under cyclic loading: An insight to enhanced unconventional oil/gas recovery. *Geosci. Front.* **2020**, *12*, 100970. [[CrossRef](#)]
5. Man, S.; Wong, R. Compression and crushing behavior of ceramic proppants and sand under high stresses. *J. Pet. Sci. Eng.* **2017**, *158*, 268–283. [[CrossRef](#)]
6. Zheng, W.; Silva, S.; Tannant, D. Crushing characteristics of four different proppants and implications for fracture conductivity. *J. Nat. Gas Sci. Eng.* **2018**, *53*, 125–138. [[CrossRef](#)]
7. Osipov, A. Hydraulic fracture conductivity: Effects of rod-shaped proppant from lattice-Boltzmann simulations and lab tests. *Adv. Water Resour.* **2017**, *104*, 293–303. [[CrossRef](#)]

8. McDaniel, G.; Abbott, J.; Mueller, F.; Mokhtar, A.; Pavlova, S.; Nevvonen, O.; Parias, T.; Alary, J.A. Changing the shape of fracturing: New proppant improves fracture conductivity. In Proceedings of the SPE Annual Technical Conference and Exhibition, Society of Petroleum Engineers, Florence, Italy, 19 September 2010.
9. Jiang, S.; Chen, P.; Yan, M.; Liu, B.; Liu, H.; Wang, H. Model of effective width and fracture conductivity for hydraulic fractures in tight reservoirs. *Arab. J. Sci. Eng.* **2020**, *45*, 7821–7834. [\[CrossRef\]](#)
10. Meng, Y.; Li, Z.; Guo, Z. Calculation model of fracture conductivity in coal reservoir and its application. *J. China Coal Soc.* **2014**, *39*, 1852–1856.
11. Xu, G.; Pei, X.; Shi, Y.; Jiang, Y.; Yang, X. Theoretical analysis of characteristics and influencing factors for channel fracturing conductivity. *J. Nat. Gas Sci. Eng.* **2018**, *57*, 166–177. [\[CrossRef\]](#)
12. Guo, J.; Yang, R.; Zhang, T.; Lu, Q.; Mu, K. Experimental and numerical investigations of proppant pack effect on fracture conductivity of channel fracturing. *Energy Sci. Eng.* **2020**, *8*, 3995–4013. [\[CrossRef\]](#)
13. Zheng, X.; Chen, M.; Hou, B.; Ye, Z.; Wang, W.; Yin, C.; Chen, X. Effect of proppant distribution pattern on fracture conductivity and permeability in channel fracturing. *J. Pet. Sci. Eng.* **2017**, *149*, 98–106. [\[CrossRef\]](#)
14. Zhu, H.; Zhao, Y.P.; Feng, Y.; Wang, H.; Zhang, L.; McLennan, J.D. Modeling of fracture width and conductivity in channel fracturing with nonlinear proppant-pillar deformation. *SPE J.* **2019**, *24*, 1288–1308. [\[CrossRef\]](#)
15. Zhang, G.; Li, M.; Gutierrez, M. Numerical simulation of proppant distribution in hydraulic fractures in horizontal wells. *J. Nat. Gas Sci. Eng.* **2017**, *48*, 157–168. [\[CrossRef\]](#)
16. Zhu, H.; Shen, J.; Zhou, H. Numerical simulation on propped fracture conductivity. *Acta Pet. Sin.* **2018**, *39*, 1410–1420.
17. Zhu, H.; Shen, J.; Zhang, F. A fracture conductivity model for channel fracturing and its implementation with Discrete Element Method. *J. Pet. Sci. Eng.* **2019**, *172*, 149–161. [\[CrossRef\]](#)
18. Fan, M.; Han, Y.; Gu, M.; McClure, J.; Ripepi, N.; Westman, E.; Chen, C. Investigation of the conductivity of a proppant mixture using an experiment/simulation-integrated approach. *J. Nat. Gas Sci. Eng.* **2020**, *78*, 103234. [\[CrossRef\]](#)
19. Yue, M.; Zhang, Q.; Zhu, W.; Zhang, L.; Song, H.; Li, J. Effects of proppant distribution in fracture networks on horizontal well performance. *J. Pet. Sci. Eng.* **2020**, *187*, 106816. [\[CrossRef\]](#)
20. Roostaie, M.; Nouri, A.; Fattahpour, V.; Chan, D. Coupled hydraulic fracture and proppant transport simulation. *Energies* **2020**, *13*, 2822. [\[CrossRef\]](#)
21. Suri, Y.; Islam, S.; Hossain, M. Proppant transport in dynamically propagating hydraulic fractures using CFD-XFEM approach. *Int. J. Rock Mech. Min. Sci.* **2020**, *131*, 104356. [\[CrossRef\]](#)
22. Gong, Y.; Mehana, M.; El-Monier, I.; Viswanathan, H. Proppant placement in complex fracture geometries: A computational fluid dynamics study. *J. Nat. Gas Sci. Eng.* **2020**, *79*, 103295. [\[CrossRef\]](#)
23. Hosseini, N.; Khoei, A. Numerical simulation of proppant transport and tip screen-out in hydraulic fracturing with the extended finite element method. *Int. J. Rock Mech. Min. Sci.* **2020**, *128*, 104247. [\[CrossRef\]](#)
24. Zheng, W.; Tannant, D.D.; Cui, X.; Xu, C.; Hu, X. Improved discrete element modeling for proppant embedment into rock surfaces. *Acta Geotech.* **2020**, *15*, 347–364. [\[CrossRef\]](#)
25. Zuo, H.; Deng, S.; Huang, Z.; Xiao, C.; Zeng, Y.; Jiang, J. Conductivity performance evaluation of fractures filled with proppant of different sizes in shale with LBM-DEM. *Int. J. Oil Gas Coal Technol.* **2020**, *23*, 44–75. [\[CrossRef\]](#)
26. Li, X.; Wang, Z.; Zhang, S.; Duan, Q. Multiscale modeling and characterization of coupled damage-healing-plasticity for granular materials in concurrent computational homogenization approach. *Comput. Methods Appl. Mech. Eng.* **2018**, *342*, 354–383. [\[CrossRef\]](#)
27. Dongarra, J.; Gates, M.; Haidar, A.; Kurzak, J.; Luszczek, P.; Tomov, S.; Yamazaki, I. The singular value decomposition: Anatomy of optimizing an algorithm for extreme scale. *SIAM Rev.* **2018**, *60*, 808–865. [\[CrossRef\]](#)
28. Fang, Z.; Yang, X.; Han, L.; Liu, X. A sequentially truncated higher order singular value decomposition-based algorithm for tensor completion. *IEEE Trans. Cybern.* **2019**, *49*, 1956–1967. [\[CrossRef\]](#) [\[PubMed\]](#)
29. Kern, M. An Analysis of the Combination and Downward Continuation of Satellite, Airborne and Terrestrial Gravity Data. Master's Thesis, Department of Geomatics Engineering, University of Calgary, Calgary, AB, Canada, 2003; pp. 46–78.
30. Guo, D.; Zhao, Y.; Guo, Z.; Cui, X.; Huang, B. Theoretical and experimental determination of proppant crushing rate and fracture conductivity. *J. Energy Resour. Technol.* **2020**, *142*, 103005. [\[CrossRef\]](#)
31. Guo, Z.; Zhao, J.; Li, Y.; Zhao, Y.; Hu, H. Theoretical and experimental determination of proppant-crushing ratio and fracture conductivity with different particle sizes. *Energy Sci. Eng.* **2022**, *10*, 177–193. [\[CrossRef\]](#)
32. Bolintineanu, D.S.; Rao, R.R.; Lechman, J.B.; Romero, J.A.; Jove-Colon, C.F.; Quintana, E.C.; Bauer, S.J.; Ingraham, M.D. Simulations of the effects of proppant placement on the conductivity and mechanical stability of hydraulic fractures. *Int. J. Rock Mech. Min. Sci.* **2017**, *100*, 188–198. [\[CrossRef\]](#)
33. SY/T 6302-2009; Standard for the Oil and Gas Industry of the People's Republic of China. Recommended Practice for Evaluating Short Term Proppant Pack Conductivity. National Energy Administration: Beijing, China, 2009.
34. RP56 API; Recommended Practices for Testing Sand Used in Hydraulic Fracturing Operations. American Petroleum Institute: Washington, DC, USA, 1983.

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