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Analytical Investigation of Vertical Force Control in In-Wheel Motors for Enhanced Ride Comfort

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Abstract: This study explores the effectiveness of vertical force control in in-wheel motors (IWMs) to enhance ride comfort in electric vehicles (EVs). A dynamic vehicle model and a proportional ride-blending controller were used to reduce vertical vibrations of the sprung mass. By converting the state-space model into a transfer function, the system's frequency response was evaluated using road profiles generated according to ISO 8608 standards and converted into Power Spectral Density (PSD) inputs. The frequency-weighted acceleration (a_w) was calculated based on ISO 2631 standards to measure ride comfort improvements. The results showed that increasing the proportional gain (K_p) effectively reduced the frequency-weighted acceleration and the RMS of the vertical acceleration of the sprung mass. However, the proportional gain could not be increased indefinitely due to the torque limitations of the IWMs. Optimal proportional gains for various road profiles demonstrated significant improvements in ride comfort. This study concludes that advanced suspension technologies, including the proportional ride-blending controller, can effectively mitigate the challenges of increased unsprung mass in IWM vehicles, thereby enhancing ride quality and vehicle dynamics.

Keywords: in-wheel motor; ride comfort; vehicle control; vehicle dynamics; frequency-weighted acceleration; power spectral density; road profile



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1. Introduction

The rapid rise of electric vehicles (EVs) is transforming the automotive industry. Central to this transformation is the use of in-wheel motors (IWMs), a technology first proposed in 1900. Today, with the growing market potential of EVs, IWMs have emerged as a promising approach to revolutionize vehicle design and performance [1,2]. Relocating the motor to the wheel brings numerous advantages. It reduces the overall vehicle weight, thereby increasing driving range, reducing running costs, and minimizing the frequency of battery charging. Additionally, IWMs improve handling due to precise torque control on each wheel, resulting in enhanced stability, traction control, and shorter stopping distances. This technology also offers more efficient production and flexible vehicle design, making it economically attractive for manufacturers.

However, integrating IWMs into vehicle wheels increases unsprung mass, adversely affecting ride comfort and handling, especially at high speeds [3]. Each IWM adds 30–52 kg per wheel, significantly boosting unsprung mass and reducing the body-to-wheel mass ratio [4]. This imbalance compromises ride smoothness, particularly at high frequencies where dynamic load changes are more pronounced. The increased unsprung mass causes the wheels to react more intensely to road irregularities, resulting in greater vertical vibrations. When the dynamic load and dead load are equal but opposite, the vertical load drops to zero, causing the wheel to lose contact with the road. This loss of adhesion can make driving dangerous during high-speed driving or sudden maneuvers [3]. Furthermore,

this situation stresses suspension components, increasing wear and potential damage. Maintaining tire contact with the road is crucial for safety and performance, as any loss of contact reduces stability and control [3,5].

To address these challenges, vehicle suspension systems play a critical role in transmitting forces between the wheels and the vehicle frame, reducing shocks from rough roads, and ensuring driving smoothness and stability. Extensive research has focused on developing both active and passive suspension systems to enhance vehicle performance. One common solution involves applying passive suspensions with springs and dampers between the IWM and the wheel to improve comfort [1,6–15]. However, these systems often fail to address the relative vertical displacement or vibration of the IWM, necessitating more advanced solutions.

These challenges require advanced suspension systems, such as semi-active or active systems equipped with magneto-rheological (MR) [7,9,16], hydraulic (HA) [17], or electromagnetic (EMA) absorbers [18]. Numerous control techniques have been developed to enhance vehicle ride and handling, ranging from comfort-oriented sky-hook to handling-oriented ground-hook and multi-objective linear optimal control techniques [6]. Despite these advancements, traditional suspension systems cannot fully eliminate vibrations across all frequency ranges, necessitating a compromise between ride comfort and handling performance. This compromise is further exacerbated in IWM vehicles due to the increased unsprung mass, highlighting the need for more sophisticated suspension technologies to maintain optimal ride quality and vehicle control.

Configurations using MR dampers showed low efficiency [9], with unchanged trade-offs between low- and high-frequency responses. Dynamic dampers like tuned mass dampers (TMDs) can partially reduce this issue. Ref. [8] reported a 31.2% improvement in ride comfort and a 2.2% increase in road holding. Ref. [19] introduced a compact electromagnetic TMD, achieving a 21.7% increase in comfort and a 5.7% improvement in handling. Refs. [5,10,13] demonstrated a two-stage optimization control reducing body acceleration by 11.34% and impulse force on the IWM by 88.57%. Despite these advances, implementation remains challenging due to limited wheel space and system robustness.

Then, to address these challenges, researchers have explored advanced control strategies for active suspension systems in in-wheel motor-driven electric vehicles (IWMD-EVs). One notable approach is the mixed H_2/H_∞ -based robust guaranteed cost control system, which integrates dynamic vibration absorbers into the active suspension model to significantly improve ride comfort and handling stability. Ref. [20] demonstrated through simulations that this controller can outperform traditional passive suspensions and standalone H_∞ controllers, offering a marked improvement in overall vehicle dynamics.

Additionally, decentralized dynamic event-triggered communication mechanisms have been developed to enhance control efficiency in IWMD-EVs. Ref. [21] has proposed a decentralized dynamic event-triggered control approach that not only improves ride comfort but also reduces the communication load. Adaptive control strategies, such as sliding mode control and fuzzy logic systems, also have shown significant effectiveness in addressing the nonlinearity and time-varying dynamics inherent in active suspensions. Refs. [14,22] have developed robust adaptive controllers that ensure consistent tire–road contact and vehicle stability under varying load conditions and road profiles.

Furthermore, regenerative active suspension systems have been explored to harness residual energy from suspension movements. Ref. [23] demonstrated that such systems can improve energy efficiency while maintaining or enhancing ride comfort and handling. Model predictive control (MPC) has emerged as a promising approach for real-time optimization of suspension performance. Ref. [24] showed that MPC can effectively balance ride comfort and handling by predicting future states and optimizing control inputs accordingly. Despite these advancements, practical implementation challenges persist, particularly due to limited wheel space and the need for robust control systems [25–27]. Further research and development are required to fully realize the potential of these advanced suspension technologies in IWMD-EVs.

One promising solution to the suspension issue is the ride-blending control method. Simulations by Ref. [3] showed that this technique effectively reduces the pitch response of the vehicle body by leveraging the vertical component of the driving force, all while keeping the tire dynamic force within safe limits. Ride-blending control works by integrating vehicle dynamics through electronic brake distribution and active suspension systems, coordinated in a decentralized manner [28]. Essentially, when one system is active, the other minimizes its force, ensuring a balanced and efficient performance. This method offers a more refined approach to handling the challenges posed by increased unsprung mass in IWM vehicles.

Ref. [1] demonstrates the development of ride-blending control, a strategy that utilizes the vertical component of IWM driving force to enhance ride comfort and handling. Simulation tests on a vehicle model with coupled dynamics showed a 31.8% improvement in ride comfort without impacting tire load or longitudinal dynamics. The method also confirmed stable lateral dynamics.

It seems that ride-blending control has significant potential to improve ride comfort, as demonstrated by various studies [1,3,28]. However, most of these studies present results in the time domain and in terms of sprung mass acceleration. This research extends the findings of Ref. [1] and aims to develop a proportional ride-blending controller for a vehicle and analyze the results in the frequency domain to better understand how well ride-blending control can improve suspension performance. Additionally, frequency-weighted acceleration will be calculated to compare ride comfort. The contribution of this research lies in the investigation of proportional ride-blending control in the frequency domain and its impact on enhancing ride comfort for EVs with IWMs.

2. Dynamics Models and Methods

To investigate how vertical force control in IWMs can enhance ride comfort, a dynamic vehicle model in state-space form is developed. A proportional ride-blending controller is then implemented, transforming the model into a multiple-input single-output (MISO) system. The state-space model is converted into a transfer function to analyze the system's frequency response. To evaluate ride comfort, road profiles are generated according to ISO 8608 standards [29] and converted into Power Spectral Density (PSD) inputs for the vehicle model. Both the open-loop and closed-loop systems of the vehicle are simulated, and the results are collected. Finally, frequency-weighted acceleration (a_w) is calculated according to ISO 2631 standards [30] to assess ride comfort improvements.

2.1. Vehicle Dynamics Model

The ride-blending controller leverages the construction constraints of typical suspensions, which have rotational centers for both roll and pitch movements in the longitudinal direction. This rotational movement allows the suspension to transfer some of the tire's driving force into a vertical direction. The vertical component of this driving force can then be utilized to control the dynamics of the vehicle's sprung mass. To examine the impact of controlling the vertical component of the driving force on the vehicle's dynamics, a half-vehicle model is introduced. This model includes longitudinal and vertical dynamics, as illustrated in Figure 1.

Based on Figure 1, point G represents the center of gravity of the sprung mass, while points A and B are the positions of the shafts connecting the wheels to the sprung mass. The forces from the wheels can be transmitted to the sprung mass through these points. Additionally, the sprung mass is supported by the suspension system at both the front and rear wheels. For simplicity, the wheels are assumed to be rigid bodies, meaning their positions, z_f and z_r , equal the elevation of the road profile.

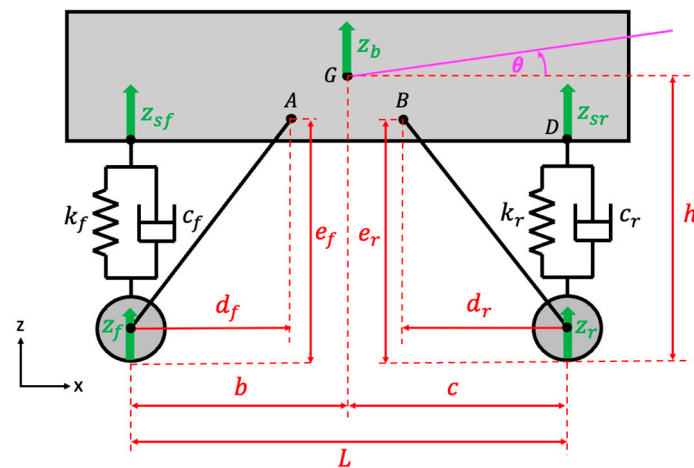


Figure 1. A half-vehicle dynamics model.

The free body diagram in Figure 2a shows that

$$A_x = -F_{xf} \quad (1)$$

$$B_x = F_{xr} \quad (2)$$

where F_{xf} is additional front longitudinal drive force and F_{xr} is additional rear longitudinal drive force.

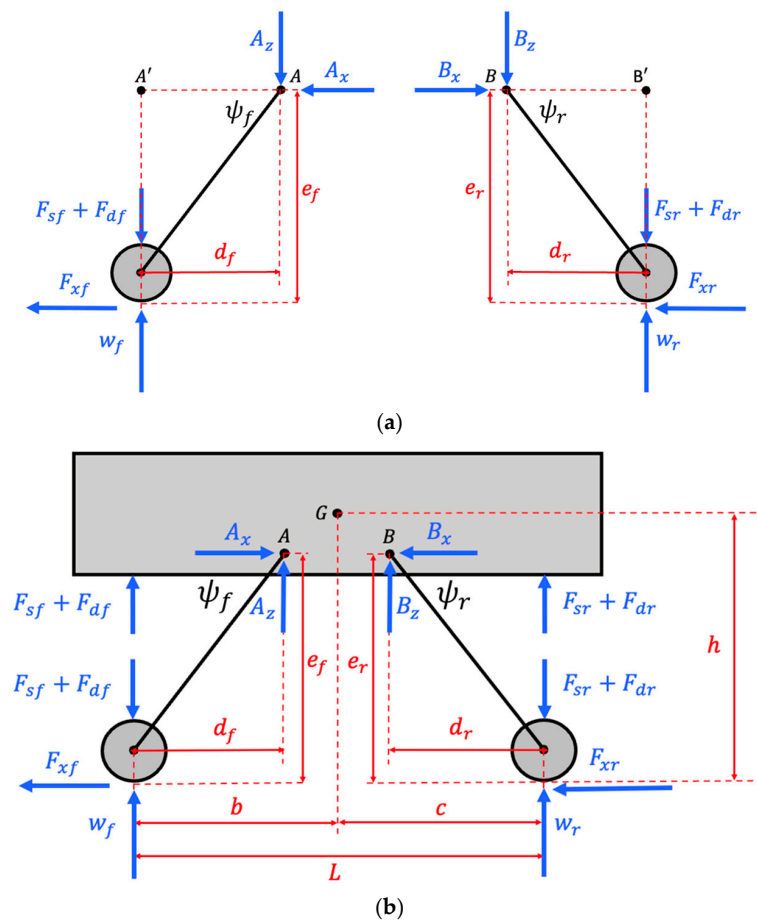


Figure 2. Free body diagram of the half-vehicle model: (a) free body diagram of front and rear wheels; (b) free body diagram of sprung mass.

Also, based on Figure 2a, the vertical forces in terms of the tire's driving forces can be presented as follows [1,3,31,32]:

$$A_z = -F_{xf} \tan \psi_f = -F_{xf} \frac{e_f}{d_f} \quad (3)$$

$$B_z = F_{xr} \tan \psi_r = F_{xr} \frac{e_r}{d_r} \quad (4)$$

where A_z is front vertical force from the front longitudinal drive force, B_z is rear vertical force from the rear longitudinal drive force, e_f is the vertical distance from the ground to point A, e_r is the vertical distance from the ground to point B, d_f is the horizontal distance from the front wheel to point A, and d_r is the horizontal distance from the rear wheel to point B.

Based on Figure 2b, the vehicle dynamics of the sprung mass are

$$m_s \ddot{z}_b = F_{sf} + F_{sr} + F_{df} + F_{dr} + A_z + B_z \quad (5)$$

$$I_z \ddot{\theta} = \left(-b(F_{sf} + F_{df}) + c(F_{sr} + F_{dr}) \right) + (h - e_f)A_x - (b - d_f)A_z - (h - e_r)B_x + (c - d_r)B_z \quad (6)$$

where z_b is the vertical displacement of sprung mass, θ is the pitch angel of sprung mass, m_s is the sprung mass, I_z is the pitch inertia of the sprung mass, F_{sf} and F_{sr} are front and rear spring forces from the suspension, F_{df} and F_{dr} are front and rear damping forces from the suspension, and b, c, d_f, d_r , and h are the distance according to Figure 2b.

The suspension forces are defined as follows:

$$F_{sf} = k_f(z_f - z_{sf}) = k_f(z_f - (z_b - b\theta)) \quad (7)$$

$$F_{sr} = k_r(z_r - z_{sr}) = k_r(z_r - (z_b + c\theta)) \quad (8)$$

$$F_{df} = c_f(\dot{z}_f - \dot{z}_{sf}) = c_f(\dot{z}_f - (\dot{z}_b - b\dot{\theta})) \quad (9)$$

$$F_{dr} = c_r(\dot{z}_r - \dot{z}_{sr}) = c_r(\dot{z}_r - (\dot{z}_b + c\dot{\theta})) \quad (10)$$

where k_f and k_r are the front and rear spring stiffness, and c_f and c_r are the front and rear damping coefficients.

For the case where there is no additional front and rear longitudinal drive force, $F_{xf} = F_{xr} = 0$, there is no vertical component of the driving force; $A_z = B_z = 0$. This system is an uncontrolled or open-loop system. The dynamics model of the system is determined as follows: substitute Equations (1)–(4) and (7)–(10) into Equations (5) and (6), and then the vehicle dynamics of the sprung mass are defined by

$$\ddot{z}_b = -\frac{k_f+k_r}{m_s}z_b - \frac{c_f+c_r}{m_s}\dot{z}_b + \frac{bk_f-ck_r}{m_s}\theta + \frac{bc_f-cc_r}{m_s}\dot{\theta} + \frac{k_f}{m_s}z_f + \frac{k_r}{m_s}z_r + \frac{c_f}{m_s}\dot{z}_f + \frac{c_r}{m_s}\dot{z}_r \quad (11)$$

$$\ddot{\theta} = \frac{bk_f-ck_r}{I_z}z_b + \frac{bc_f-cc_r}{I_z}\dot{z}_b - \frac{b^2k_f+c^2k_r}{I_z}\theta - \frac{b^2c_f+c^2c_r}{I_z}\dot{\theta} - \frac{bk_f}{I_z}z_f + \frac{ck_r}{I_z}z_r - \frac{bc_f}{I_z}\dot{z}_f + \frac{cc_r}{I_z}\dot{z}_r \quad (12)$$

Then, the state-space representation of the system is shown as follows:

$$\begin{aligned} \dot{x} &= A_{ol}x + B_{ol}u \\ y &= C_{ol}x + D_{ol}u \end{aligned} \quad (13)$$

where A_{ol} , B_{ol} , C_{ol} , and D_{ol} are defined in Appendix A, Equation (A2), and x and u are defined as follows:

$$\begin{aligned} x &= \begin{bmatrix} z_b & \dot{z}_b & \theta & \dot{\theta} \end{bmatrix}^T, \\ u &= \begin{bmatrix} z_f & z_r & \dot{z}_f & \dot{z}_r \end{bmatrix}^T \end{aligned} \quad (14)$$

The system is a multiple-input single-output (MISO) system. Determining the transfer function of the input or road input to the output or the vertical acceleration of the sprung mass is not easy in this case due to the MISO nature of the system. However, given the characteristics of the system and its linearity, the transfer function of the road input to the vertical acceleration of the sprung mass can be determined. The inputs z_f , z_r , $\dot{z}_f$, and $\dot{z}_r$ have the relationships shown in Figure 3. The inputs $\dot{z}_f$ and $\dot{z}_r$ are the derivatives of the inputs, z_f and z_r and the input z_r is the input z_f with a delay time. Moreover, the block diagram of the vehicle system based on the MISO system principle, as shown in Figure 4a, can be rewritten using the linear combination principle, as shown in Figure 4b. Thus, the transfer function of the road input to the vertical acceleration of the sprung mass is given in Equations (15)–(18).

$$\frac{\dot{Z}_{b,ol}(s)}{Road(s)} = G_{1,ol}(s)_{z_f \rightarrow \dot{z}_b} + G_{2,ol}(s)_{z_r \rightarrow \dot{z}_b} + G_{3,ol}(s)_{\dot{z}_f \rightarrow \dot{z}_b} + G_{4,ol}(s)_{\dot{z}_r \rightarrow \dot{z}_b} \quad (15)$$

$$\frac{\dot{Z}_{b,ol}(s)}{Road(s)} = \frac{\dot{Z}_{b,ol}(s)}{Z_f(s)} + \frac{\dot{Z}_{b,ol}(s)}{Z_r(s)} + \frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_f(s)} + \frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_r(s)} \quad (16)$$

$$\frac{\dot{Z}_{b,ol}(s)}{Road(s)} = \frac{\dot{Z}_{b,ol}(s)}{Z_f(s)} + \frac{e^{-\tau s} \dot{Z}_{b,ol}(s)}{Z_r(s)} + \frac{s \dot{Z}_{b,ol}(s)}{\dot{Z}_f(s)} + \frac{e^{-\tau s} s \dot{Z}_{b,ol}(s)}{\dot{Z}_r(s)} \quad (17)$$

$$\frac{\ddot{Z}_{b,ol}(s)}{Road(s)} = \frac{s \dot{Z}_{b,ol}(s)}{Road(s)} = \frac{s \dot{Z}_{b,ol}(s)}{Z_f(s)} + \frac{e^{-\tau s} s \dot{Z}_{b,ol}(s)}{Z_r(s)} + \frac{s^2 \dot{Z}_{b,ol}(s)}{\dot{Z}_f(s)} + \frac{e^{-\tau s} s^2 \dot{Z}_{b,ol}(s)}{\dot{Z}_r(s)} \quad (18)$$

where $Road(s)$ is the Laplace transform of road input, $e^{-\tau s}$ is the Laplace transform of exact delay, and τ is the delay time. Moreover, each transfer function of Equation (15) can be determined by

$$\frac{\dot{Z}_{b,ol}(s)}{Z_f(s)} = C(sI - A_{ol})^{-1} B_{ol}(:, 1) + D(:, 1) \quad (19)$$

$$\frac{\dot{Z}_{b,ol}(s)}{Z_r(s)} = C(sI - A_{ol})^{-1} B_{ol}(:, 2) + D(:, 2) \quad (20)$$

$$\frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_f(s)} = C(sI - A_{ol})^{-1} B_{ol}(:, 3) + D(:, 3) \quad (21)$$

$$\frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_r(s)} = C(sI - A_{ol})^{-1} B_{ol}(:, 4) + D(:, 4) \quad (22)$$

Using the parameters in Table 1, substitute them into Equations (14) and (19)–(22). Then, Equations (19)–(22) become

$$\frac{\dot{Z}_{b,ol}(s)}{Z_f(s)} = \frac{50.51s^3 + 597.3s^2 + 5921s - 2.777 \times 10^{-11}}{s^4 + 17.81s^3 + 308.2s^2 + 2067s + 1.301 \times 10^4} \quad (23)$$

$$\frac{\dot{Z}_{b,ol}(s)}{Z_r(s)} = \frac{48.27s^3 + 410.7s^2 + 7094s + 2.892 \times 10^{-11}}{s^4 + 17.81s^3 + 308.2s^2 + 2067s + 1.301 \times 10^4} \quad (24)$$

$$\frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_f(s)} = \frac{2.924s^3 + 34.59s^2 + 342.8s - 1.608 \times 10^{-12}}{s^4 + 17.81s^3 + 308.2s^2 + 2067s + 1.301 \times 10^4} \quad (25)$$

$$\frac{\dot{Z}_{b,ol}(s)}{\dot{Z}_r(s)} = \frac{4.87s^3 + 41.44s^2 + 715.7s + 2.918 \times 10^{-12}}{s^4 + 17.81s^3 + 308.2s^2 + 2067s + 1.301 \times 10^4} \quad (26)$$

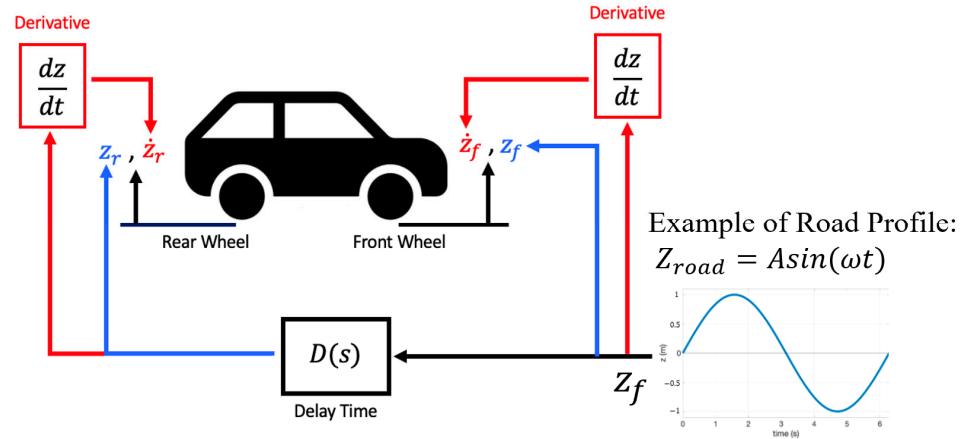


Figure 3. Schematic diagram of the input for the vehicle system.

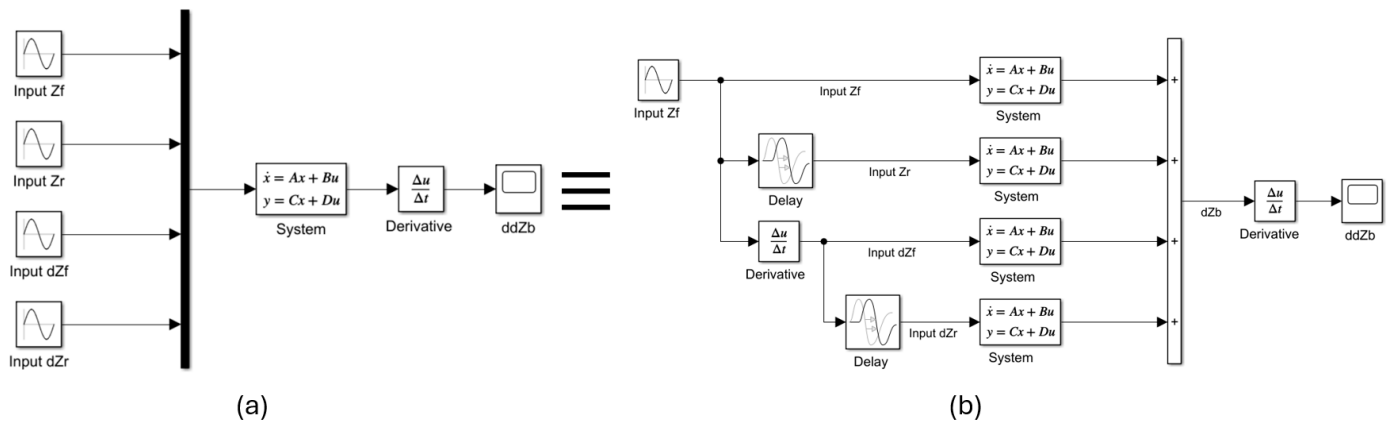


Figure 4. Block diagram of the vehicle system: (a) block diagram with MISO system principle; (b) block diagram with linear combination principle.

Table 1. The vehicle parameters [1].

Parameters	Symbol	Value	Units
Sprung mass	m_s	2055.2	kg
Pitch inertia	I_z	3404.9	kg·m ²
Horizontal distance from front wheel to point A	d_f	0.51	m
Horizontal distance from rear wheel to point B	d_r	0.53	m
Horizontal distance from front wheel to rear wheel	L	2.974	m
Horizontal distance from front wheel to point G	b	1.621	m
Vertical distance from rear wheel to point G	c	1.353	m
Vertical distance from ground to point A	e_f	0.16	m
Vertical distance from ground to point B	e_r	0.15	m
Vertical distance from ground to point G	h	0.418	m
Spring stiffness of front suspension	k_f	103,800	N/m
Spring stiffness of rear suspension	k_r	99,200	N/m

Table 1. Cont.

Parameters	Symbol	Value	Units
Damping coefficient of front suspension	c_f	6010	N·s/m
Damping coefficient of rear suspension	c_r	10,008	N·s/m
Unloaded tire radius	R_w	0.364	m

Then, the transfer function of the road input to the vertical acceleration of the sprung mass can be calculated from Equation (18).

2.2. Vehicle Dynamics Model with Proportional Ride-Blending Controller

For the closed-loop system of the vehicle, the proportional ride-blending controller is implemented. The control law is defined as follows:

$$\begin{aligned} F_{xf} &= K_p (\ddot{z}_b - \ddot{z}_{b,ref}) \\ F_{xr} &= -F_{xf} \end{aligned} \quad (27)$$

where K_p is proportional gain. To improve the ride comfort, the vertical acceleration of the sprung mass should be minimized; therefore, the reference vertical acceleration of the sprung mass, $\ddot{z}_{b,ref}$, is set to zero. Additionally, $F_{xr} = -F_{xf}$ will not affect the velocity of the vehicle. Equation (27) will be

$$\begin{aligned} F_{xf} &= K_p \ddot{z}_b \\ F_{xr} &= -F_{xf} \end{aligned} \quad (28)$$

Once F_{xf} and F_{xr} are known, the front and rear vertical forces, A_z and B_z , can be computed and used to control the vibration of the sprung mass. Then, the system is controlled by closed-loop control. The dynamics model of the closed-loop system is determined as follows: substitute Equations (1)–(4), (7)–(10), and (28) into Equations (5) and (6), and then the vehicle dynamics of the sprung mass are defined by

$$\begin{aligned} \ddot{z}_b &= -\frac{k_f+k_r}{M} z_b - \frac{c_f+c_r}{M} \dot{z}_b + \frac{bk_f-ck_r}{M} \theta + \frac{bc_f-cc_r}{M} \dot{\theta} \\ &\quad + \frac{k_f}{M} z_f + \frac{k_r}{M} z_r + \frac{c_f}{M} \dot{z}_f + \frac{c_r}{M} \dot{z}_r \end{aligned} \quad (29)$$

where $M = m + K_p \left(\frac{e_f}{d_f} + \frac{e_r}{d_r} \right)$.

$$\begin{aligned} \ddot{\theta} &= \frac{bk_f-ck_r+N(k_f+k_r)}{I_z} z_b + \frac{bc_f-cc_r+N(c_f+c_r)}{I_z} \dot{z}_b \\ &\quad - \frac{b^2k_f+c^2k_r+N(bk_f-ck_r)}{I_z} \theta - \frac{b^2c_f+c^2c_r+N(bc_f-cc_r)}{I_z} \dot{\theta} \\ &\quad - \frac{bk_f+Nk_f}{I_z} z_f + \frac{ck_r-Nk_r}{I_z} z_r - \frac{bc_f+Nc_f}{I_z} \dot{z}_f + \frac{cc_r-Nc_r}{I_z} \dot{z}_r \end{aligned} \quad (30)$$

where $N = \frac{\left[\left(h - \frac{be_f}{d_f} \right) - \left(h - \frac{ce_r}{d_r} \right) \right]}{M} K_p$.

Then, the state-space representation of the system is shown as follows:

$$\begin{aligned} \dot{x} &= A_{cl}x + B_{cl}u \\ y &= C_{cl}x + D_{cl}u \end{aligned} \quad (31)$$

where A_{cl} , B_{cl} , C_{cl} , and D_{cl} are defined in Appendix A, Equation (A4), and x and u are defined as follows:

$$\begin{aligned} x &= \begin{bmatrix} z_b & \dot{z}_b & \theta & \dot{\theta} \end{bmatrix}^T, \\ u &= \begin{bmatrix} z_f & z_r & \dot{z}_f & \dot{z}_r \end{bmatrix}^T \end{aligned} \quad (32)$$

Since the format of the state-space representation of the closed-loop system in Equation (32) is the same as the format of the state-space representation of the open-loop system

in Equation (14), the transfer function of the road input to the vertical acceleration of the sprung mass of the closed-loop system can be determined by the same steps as for the open-loop system in Equations (15)–(18). It is defined by Equation (33).

$$\frac{\dot{Z}_{b,cl}(s)}{Road(s)} = G_{1,cl}(s)_{z_f \rightarrow \dot{z}_b} + G_{2,cl}(s)_{z_r \rightarrow \dot{z}_b} + G_{3,cl}(s)_{\dot{z}_f \rightarrow \dot{z}_b} + G_{4,cl}(s)_{\dot{z}_r \rightarrow \dot{z}_b} \quad (33)$$

$$\frac{\ddot{Z}_{b,cl}(s)}{Road(s)} = \frac{s\dot{Z}_{b,cl}(s)}{Road(s)} = \frac{s\dot{Z}_{b,cl}(s)}{Z_f(s)} + \frac{e^{-\tau s}s\dot{Z}_{b,cl}(s)}{Z_r(s)} + \frac{s^2\dot{Z}_{b,cl}(s)}{\dot{Z}_f(s)} + \frac{e^{-\tau s}s^2\dot{Z}_{b,cl}(s)}{\dot{Z}_r(s)} \quad (34)$$

where

$$\frac{\dot{Z}_{b,cl}(s)}{Z_f(s)} = C(sI - A_{cl})^{-1}B_{cl}(:,1) + D(:,1) \quad (35)$$

$$\frac{\dot{Z}_{b,cl}(s)}{Z_r(s)} = C(sI - A_{cl})^{-1}B_{cl}(:,2) + D(:,2) \quad (36)$$

$$\frac{\dot{Z}_{b,cl}(s)}{\dot{Z}_f(s)} = C(sI - A_{cl})^{-1}B_{cl}(:,3) + D(:,3) \quad (37)$$

$$\frac{\dot{Z}_{b,cl}(s)}{\dot{Z}_r(s)} = C(sI - A_{cl})^{-1}B_{cl}(:,4) + D(:,4) \quad (38)$$

Using the parameters in Table 1, substitute them into Equations (32) and (35)–(38). Then, the transfer function of the road input to the vertical acceleration of the sprung mass can be calculated from Equation (34).

2.3. Frequency Response of the Vehicle System

Based on the transfer function of the road input to the vertical acceleration of the open-loop and closed-loop systems, as shown in Equations (18) and (34), the frequency response of both systems can be compared. To evaluate the frequency response, the velocity of the vehicle is assumed to be constant at 30 m/s. Given that the distance between the front and rear wheels is $b + c$, the delay time, τ , of exact delay is 0.0991. The frequency response will be computed using MATLAB R2024a software [33]. An example of the frequency response of the open-loop system is shown in Figure 5.

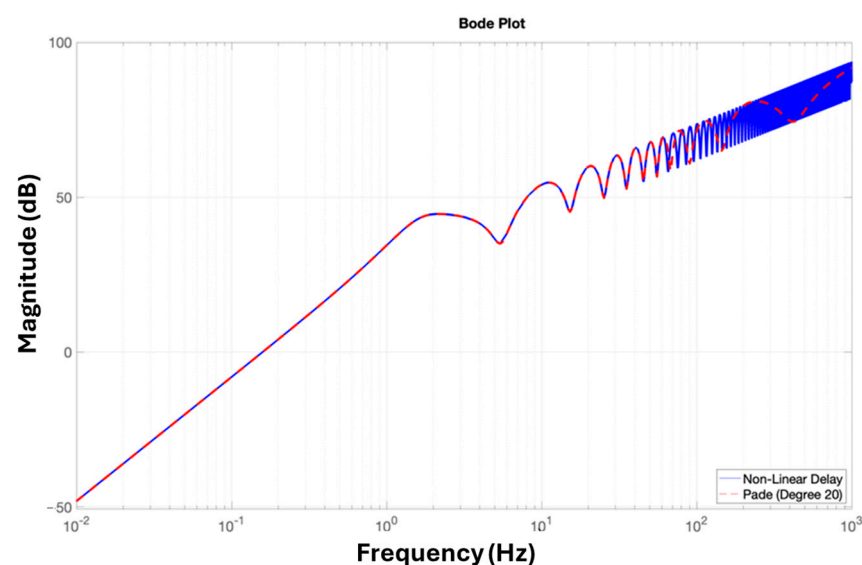


Figure 5. The frequency response of open-loop system by the exact delay and Pade approximation methods.

Figure 5 compares the frequency response of the open-loop system when the delay time is modeled using the exact delay as in Equation (39) and approximated with the Pade approximation [34] with a Pade degree of 20. The figure shows that the frequency responses from both methods are the same up to a frequency of 60 Hz. Beyond 60 Hz, the frequency response from the Pade approximation cannot match the response from the exact delay. Therefore, the exact delay will be used for further evaluations in this case, even though it may be more complicated. The frequency response of the vehicle system is used to evaluate the system's response at each frequency and to calculate the ride comfort for both systems.

Figure 6 shows the frequency response of both the open-loop and closed-loop systems. The proportional gain, K_p , is varied from 100 to 500 to present the frequency response of the closed-loop system. Figure 6a compares the frequency responses of the open-loop and closed-loop systems. Figure 6b,c provide zoomed-in views focusing on the frequency ranges of 5 to 20 Hz and 0.1 to 5 Hz, respectively. It can be seen from Figure 6 that as the proportional gain, K_p , increases, the amplitude response of the closed-loop system decreases compared to that of the open-loop system. Therefore, it can be concluded that the proportional ride-blending controller effectively reduces the vibration of the vehicle's sprung mass.

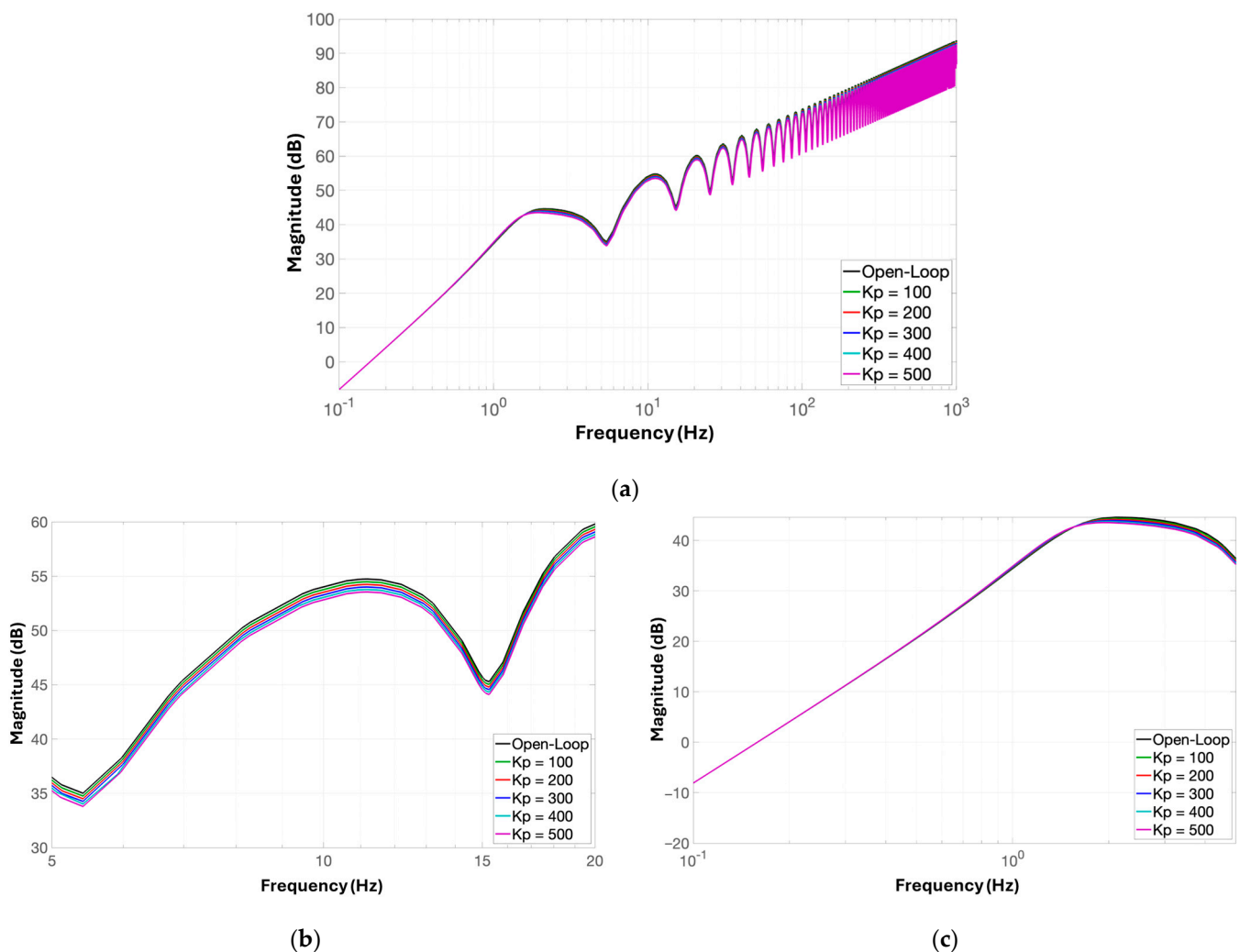


Figure 6. The frequency response of open-loop and closed-loop system: (a) the frequency response from 0.1 to 1000 Hz, (b) the frequency response from 5 to 20 Hz, and (c) the frequency response from 0.1 to 5 Hz.

2.4. Ride Comfort (ISO 2631)

According to ISO 2631 [30], ride comfort is the comfort reaction evaluated by examining the frequency-weighted acceleration, a_w . The frequency-weighted acceleration, a_w , is computed from the Power Spectral Density (PSD) of the sprung mass vertical acceleration, $\ddot{z}_b$. The equation for the frequency-weighted acceleration, a_w , is shown in Equation (39).

$$a_w = \sqrt{\sum_i (W_i a_i)^2} \quad (39)$$

where W_i is frequency weighting in one-third-octaves for the frequency i according to the Principal Frequency Weighting in One-Third-Octaves Table (parameter: W_k) [35] and a_i is root mean square (RMS) acceleration for the frequency, i , as shown in Equation (40).

$$a_i = \sqrt{P_i} \quad (40)$$

where P_i is the area under the plot of Power Spectral Density (PSD) of the sprung mass vertical acceleration, $\ddot{z}_b$, for frequency i . Frequency i is the range between $f_{i-\frac{1}{2}}$ and $f_{i+\frac{1}{2}}$ where $f_{i-\frac{1}{2}} = 2^{(-\frac{1}{6})} f_i$ and $f_{i+\frac{1}{2}} = 2^{(\frac{1}{6})} f_i$. (Note: the frequency according to one-third-octaves is defined by $\frac{f_{n+1}}{f_n} = 2^{\frac{1}{3}}$.) The explanation of P_i is shown in Figure 7.

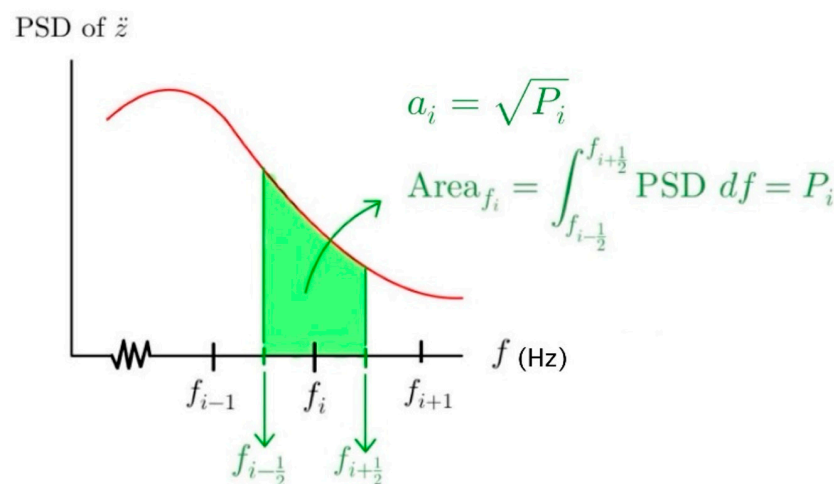


Figure 7. The calculation of P_i .

2.4.1. Power Spectral Density (PSD)

PSD is a measure of the power distribution of a signal over different frequencies. It shows how the power (or variance) of a time series is distributed with frequency, providing insights into the dominant frequencies present in the signal. To compute the PSD of the sprung mass vertical acceleration, $\ddot{z}_b$, the road profile input is supplied to the vehicle system or the transfer function as described in Equations (18) and (34). The system will generate an output, which will then be used to calculate the PSD of the vertical acceleration of the sprung mass. The PSD of the vertical acceleration of the sprung mass at frequency i can be computed by Equation (41). An example of calculating the PSD of the vertical acceleration of the sprung mass is shown in Figure 8. The PSD of acceleration at frequency, f , is calculated from the PSD of road profile and the frequency response of the vehicle as shown by the red line in Figure 8.

$$P_{avg,iout} = \int_{-\frac{T}{2}}^{\frac{T}{2}} [A_i |G(\omega_i i)| \sin(\omega_i t + \phi_i + \angle G(\omega_i i))]^2 dt = \frac{(A_i |G(\omega_i i)|)^2}{2} \quad (41)$$

where i is frequency, $G(s)$ is transfer function as described in Equations (18) and (34), and the road input at frequency i is

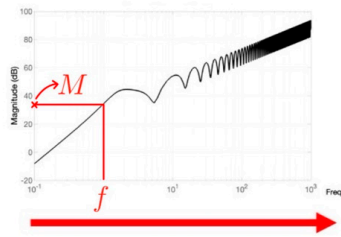
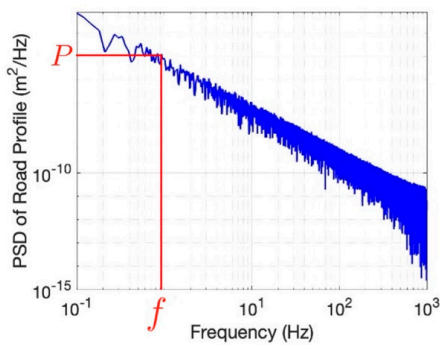
$$z_i(t) = A_i \sin(2\pi f_i t + \phi_i) \quad (42)$$

The instantaneous power of road profile, P_{avg} , is

$$P_{avg} = \int_{-\frac{T}{2}}^{\frac{T}{2}} P(t) dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} z_i^2(t) dt = \frac{A_i^2}{2} \quad (43)$$

Equation (43) shows that P_{avg} depends on the amplitude of the road profile as described in Equation (42).

“Power Spectral Density of Road Profile”



“Power Spectral Density of Acceleration”

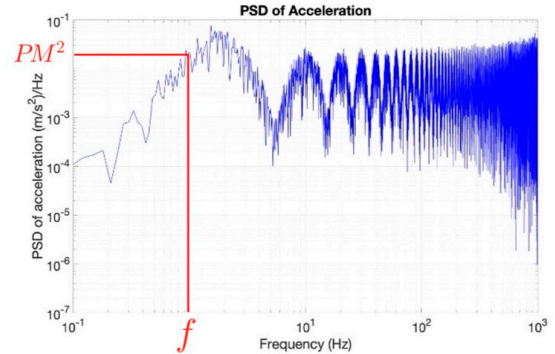


Figure 8. The example of PSD conversion via Bode plot.

2.4.2. Classification of Roads (ISO 8608)

According to ISO 8608 [36,37], road profiles are classified based on the PSD of the road profile elevation, as shown in Figure 9. There are eight classes of road profiles: A, B, C, D, E, F, G, and H. The road profiles can be created by Equation (44) [38].

$$z(x) = \sum_{i=1}^N \sqrt{\Delta n} \cdot 2^k \cdot 10^{-3} \cdot \left(\frac{n_0}{i \cdot \Delta n} \right) \cdot \cos(2\pi i \cdot \Delta n \cdot x + \phi_i) \quad (44)$$

where x is the horizontal displacement (0 to L), $\Delta n = \frac{1}{L}$, $n_0 = 0.1 \text{ rad/s}$, N is number of the total data points, ϕ_i is the random phase angle in the interval of 0 to 2π , and k is the constant to classify the road profile according to Table 2. Examples of road profiles for classes A to D are shown in Figure 10. Ref. [29] indicates that class A represents a very good road, class B a good road, class C an average road, and class D a poor road. For this research, only classes A to D will be used to evaluate the control technique.

Table 2. The relationship between k and the class of road profile.

Class of Road Profile	k
A to B	3
B to C	4
C to D	5
D to E	6
E to F	7
F to G	8
G to H	9

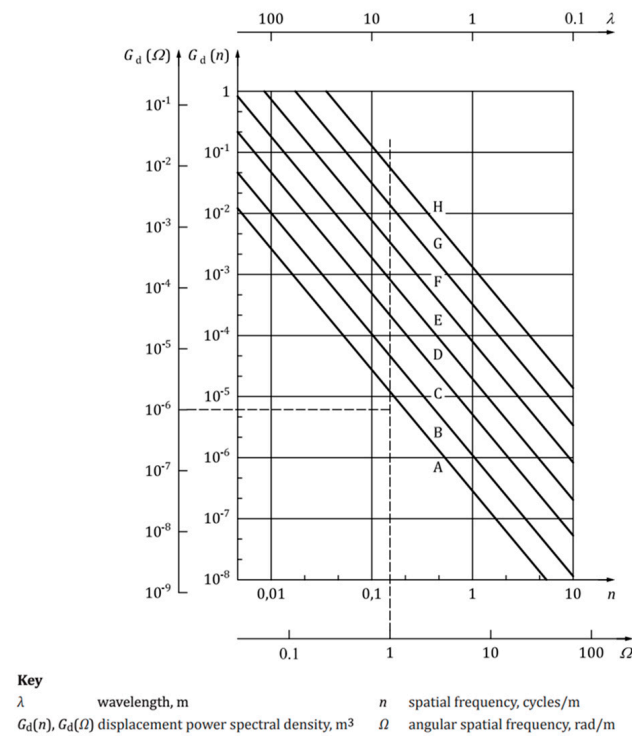


Figure 9. Road profile classification according to ISO 8608 [36,37].



Figure 10. Examples of road profiles for classes A to D: (a) class A, (b) class B, (c) class C, and (d) class D.

2.4.3. PSD of the Road Profiles

Three types of road profiles are created by Equation (44): (1) $k = 3$: road profile class A to B; (2) $k = 4$: road profile class B to C; and (3) $k = 5$: road profile class C to D. Figure 11a shows the elevation of these three types of road profiles versus distance (x). Assuming that the vehicle velocity is constant at 30 m/s, Figure 11b presents the elevation of the three types of road profiles versus time over a 30 s period. This conversion is carried out using Equations (45) and (46). Equation (45) shows the relationship between spectral frequency and temporal frequency, while Equation (46) outlines the method for converting elevation from spectral frequency to temporal frequency.

$$f_{Spectral} = v f_{Temporal} \quad (45)$$

where $f_{Spectral}$ is spectral frequency (cycle/m), $f_{Temporal}$ is temporal frequency (cycle/s or Hz), and v is velocity.

$$z_i(x) = A_i \sin(2\pi f_i x + \phi_i) \rightarrow z_i(t) = A_i \sin(2\pi f_i t + \phi_i) \quad (46)$$

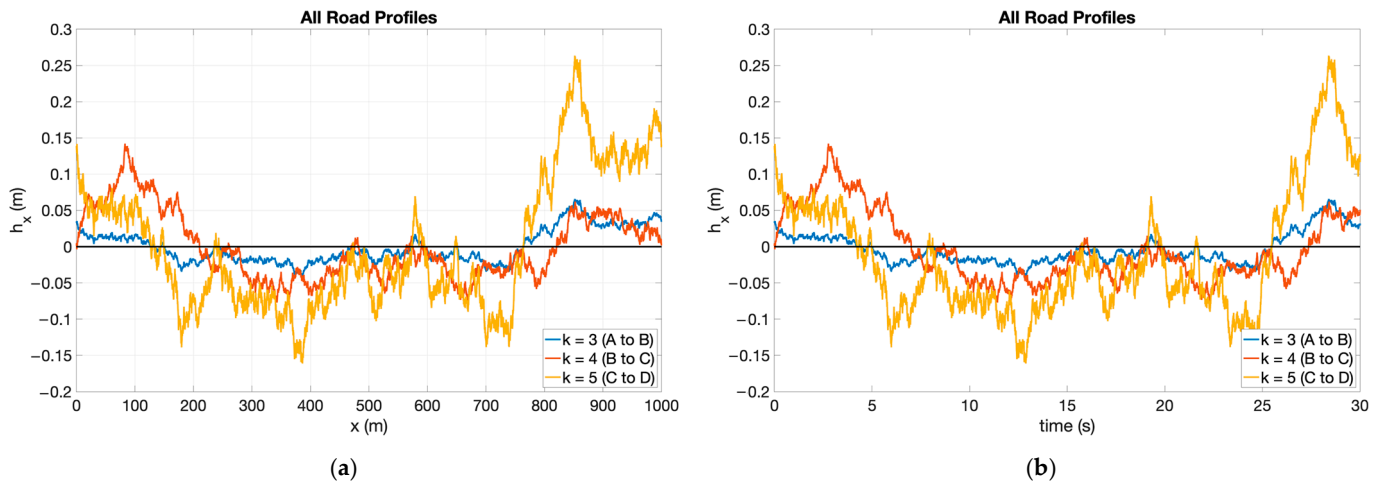


Figure 11. The road profiles created by Equation (44): (a) the road profiles' elevation versus distance (x); (b) the road profiles' elevation versus time.

Then, Figure 11 is used to calculate the PSD of the road profiles.

Figure 12 shows the PSD of the road profiles created by Figure 11. Figure 12a presents the PSD with the spatial frequency domain, while Figure 12b shows the PSD with the temporal frequency domain. The PSD is computed by the MATLAB software using the `psd_1D` function [39]. Figure 11b will be used to calculate the PSD of the sprung mass vertical acceleration, $\ddot{z}_b$, according to the method explained in Figure 8.

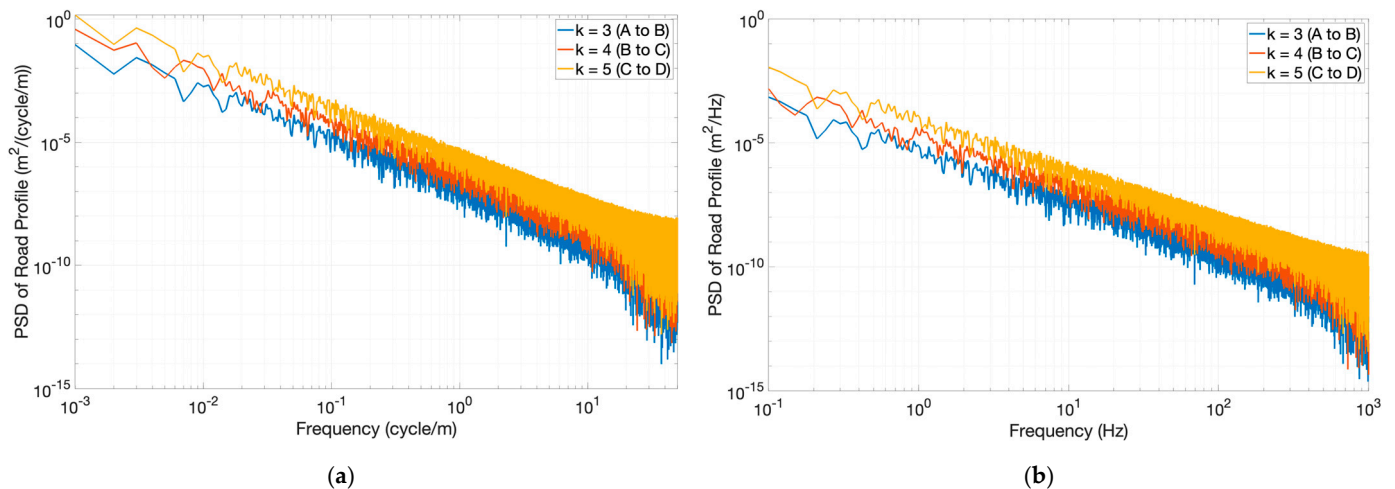


Figure 12. PSD of the road profiles: (a) PSD with spatial frequency domain; (b) PSD with temporal frequency domain.

2.4.4. PSD of the Sprung Mass Vertical Acceleration

The PSD of the sprung mass vertical acceleration is calculated based on Equation (41) and Figure 8. The results are shown in Figure 13. Figure 13a presents the PSD of the open-loop system for road profile classes A to B, B to C, and C to D. Figure 13b compares the PSD of the open-loop and closed-loop systems for road profile classes A to B. Figure 13c compares the PSD of the open-loop and closed-loop systems for road profile classes B to C. Finally, Figure 13d compares the PSD of the open-loop and closed-loop systems for road profile classes C to D.

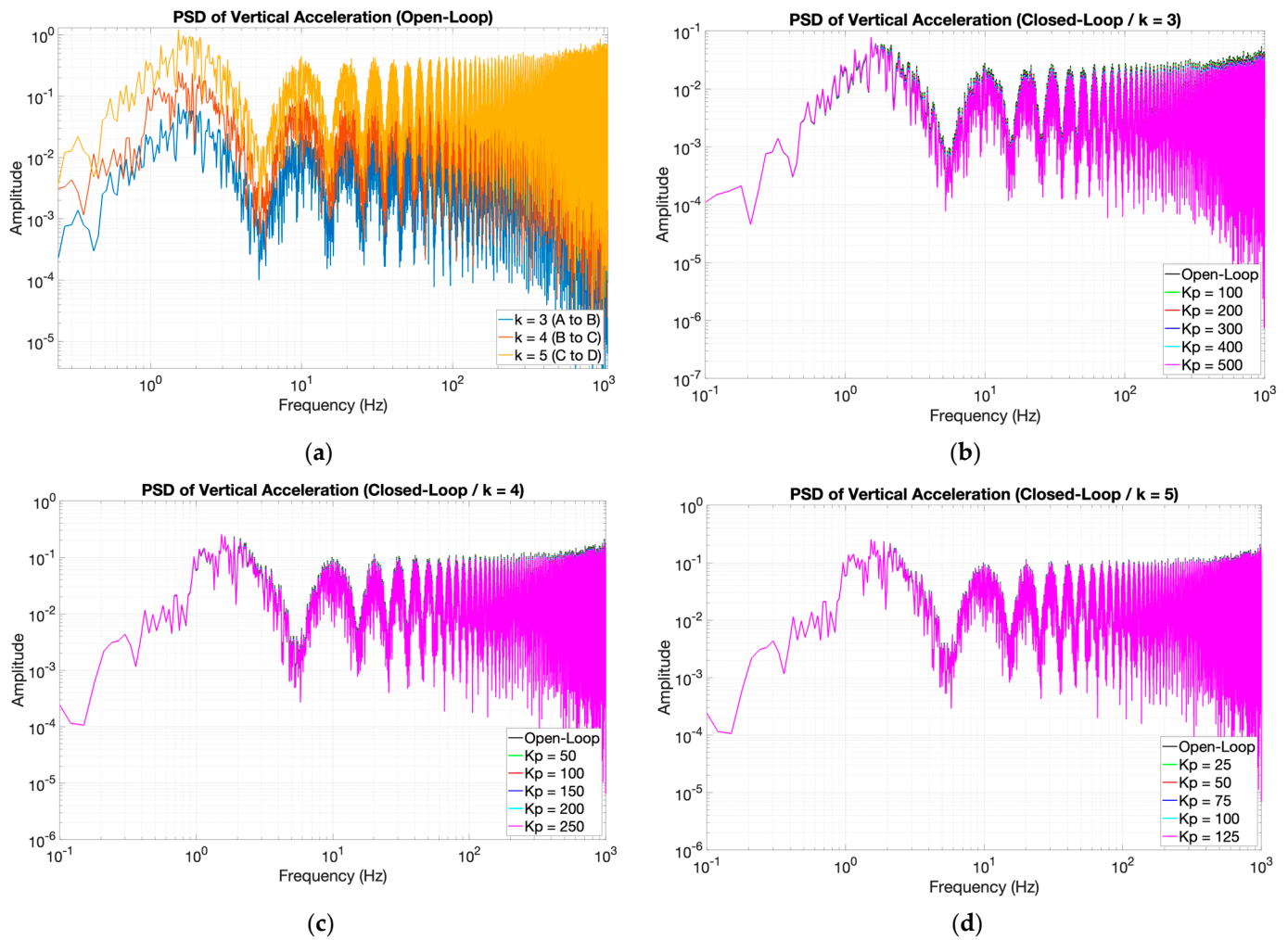


Figure 13. PSD of the sprung mass vertical acceleration: (a) PSD of the open-loop system, (b) PSD of the closed-loop system for road profile classes A to B, (c) PSD of the closed-loop system for road profile classes B to C, and (d) PSD of the closed-loop system for road profile classes C to D.

In the next section, the results of the frequency-weighted acceleration, a_w , are presented to compare the ride comfort.

3. Results and Discussion

3.1. Frequency Response of the Open-Loop and Closed-Loop System

The transfer functions of the open-loop and closed-loop systems, shown in Equations (18) and (34), are used to determine the frequency responses in Figure 6. The figure compares the frequency responses for both systems and demonstrates that increasing K_p results in a reduced amplitude response in the closed-loop system compared to the open-loop system. This indicates that the proportional ride-blending controller effectively reduces the vibration of the vehicle's sprung mass.

3.2. Frequency-Weighted Acceleration

The frequency-weighted acceleration, a_w , is an indicator used to evaluate ride comfort; lower values of the frequency-weighted acceleration correspond to better ride comfort. The frequency-weighted acceleration is calculated from Equations (39) and (40), based on the Power Spectral Density (PSD) of the sprung mass vertical acceleration, $\ddot{z}_b$, shown in Figure 13. Comparisons of the frequency-weighted acceleration for each type of road profile are presented in Tables 3–5. Table 3 shows the frequency-weighted acceleration

for road profile classes A to B, Table 4 for road profile classes B to C, and Table 5 for road profile classes C to D.

Table 3. Proportional gain and frequency-weighted acceleration for road profile classes A to B.

Proportional Gain, K_p	Frequency-Weighted Acceleration ($a_w, \text{m/s}^2$)	Percentage Decrease (Compared with Open-Loop)
0 (Open-Loop)	0.3721	0%
100	0.3622	2.6641%
200	0.3528	5.1856%
300	0.3439	7.5756%
400	0.3355	9.8441%
500	0.3275	12.000%

Table 4. Proportional gain and frequency-weighted acceleration for road profile classes B to C.

Proportional Gain, K_p	Frequency-Weighted Acceleration ($a_w, \text{m/s}^2$)	Percentage Decrease (Compared with Open-Loop)
0 (Open-Loop)	0.7349	0%
50	0.7250	1.3583%
100	0.7153	2.6789%
150	0.7058	3.9632%
200	0.6966	5.2127%
250	0.6877	6.4288%

Table 5. Proportional gain and frequency-weighted acceleration for road profile classes C to D.

Proportional Gain, K_p	Frequency-Weighted Acceleration ($a_w, \text{m/s}^2$)	Percentage Decrease (Compared with Open-Loop)
0 (Open-Loop)	1.4884	0%
25	1.4783	0.6800%
50	1.4683	1.3506%
75	1.4585	2.0119%
100	1.4488	2.6641%
125	1.4392	3.3073%

Tables 6–8 show that as the proportional gain (K_p) increases, the frequency-weighted acceleration (a_w) decreases, leading to enhanced ride comfort. This indicates that the method of controlling the driving force from the IWM using the proportional ride-blending controller, as discussed in Section 2.2, effectively reduces the vehicle's vibration. The decrease in a_w is directly correlated with the increase in K_p .

Table 6. The root mean square (RMS) of the vertical acceleration of the sprung mass for the road profile classes A to B.

Proportional Gain, K_p	RMS of $\ddot{z}_b$ (m/s^2)
0 (Open-Loop)	4.130
100	4.014
200	3.904
300	3.800
400	3.702
500	3.609

Table 7. The root mean square (RMS) of the vertical acceleration of the sprung mass for the road profile classes B to C.

Proportional Gain, K_p	RMS of $\ddot{z}_b$ (m/s ²)
0 (Open-Loop)	8.269
50	8.151
100	8.037
150	7.925
200	7.817
250	7.712

Table 8. The root mean square (RMS) of the vertical acceleration of the sprung mass for the road profile classes C to D.

Proportional Gain, K_p	RMS of $\ddot{z}_b$ (m/s ²)
0 (Open-Loop)	16.52
25	16.40
50	16.28
75	16.17
100	16.05
125	15.94

3.3. Frequency-Weighted Acceleration

Based on the IWM L1500 information in Figure 14, the wheel forces are limited. Figures 15–17 illustrate examples of the front wheel forces based on the proportional ride-blending controller as defined in Equation (28). These figures show the time response of the front wheel forces over the first 30 s for each type of road profile (classes A to B, B to C, and C to D) and for a variety of proportional gains. The black dashed lines represent the maximum continuous force limit of 3570 N, while the black dotted lines represent the maximum peak force limit of 8240 N, according to the parameters in Figure 14 (The yellow line in Figures 15–17 is the reference force, zero newtons). Moreover, the root mean square (RMS) of the vertical acceleration of the sprung mass, corresponding to various proportional gains and types of road profiles, is shown in Tables 6–8.

Tables 6–8 demonstrate that increasing the proportional gains reduces the RMS of the vertical acceleration of the sprung mass, or the frequency-weighted acceleration. Conversely, Figures 15–17 illustrate that higher proportional gains lead to an increase in the front wheel driving forces of the IWM.

370V nominal	
Added weight	34.8 kg
Peak torque	1500 Nm
Continuous torque	650 Nm
Top speed [at nominal voltage]	1480 rpm
Peak power [at nominal voltage]	110 kW
Continuous power [at nominal voltage]	77 kW (liquid cooling)

**Figure 14.** Parameters of the IWM L1500 [1].

Considering the time response of the front wheel driving forces for the road profile classes A to B in Figure 15, with $K_p = 300$, the blue line indicates that most of the driving forces are within the continuous force limit, and the peak forces are within the peak force limit. Thus, $K_p = 300$ is suitable for this case, resulting in an improvement of up to 7.58% in frequency-weighted acceleration compared to the open-loop system.

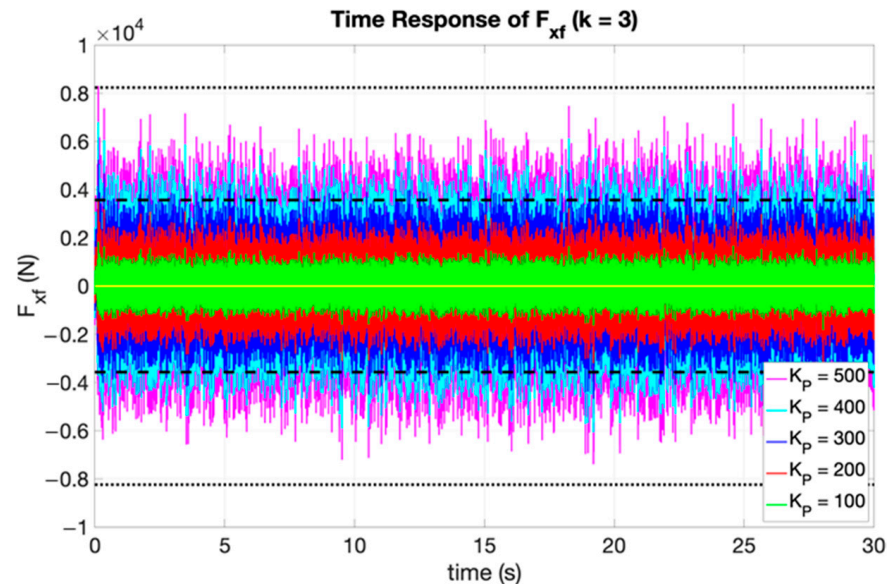


Figure 15. Time response of the front wheel force for the road profile classes A to B.

Similarly, for the road profile classes B to C in Figure 16, with $K_p = 150$, the blue line shows that most of the driving forces are within the continuous force limit, and the peak forces are within the peak force limit. Therefore, $K_p = 150$ is appropriate for this case, resulting in an improvement of up to 3.96% in frequency-weighted acceleration compared to the open-loop system.

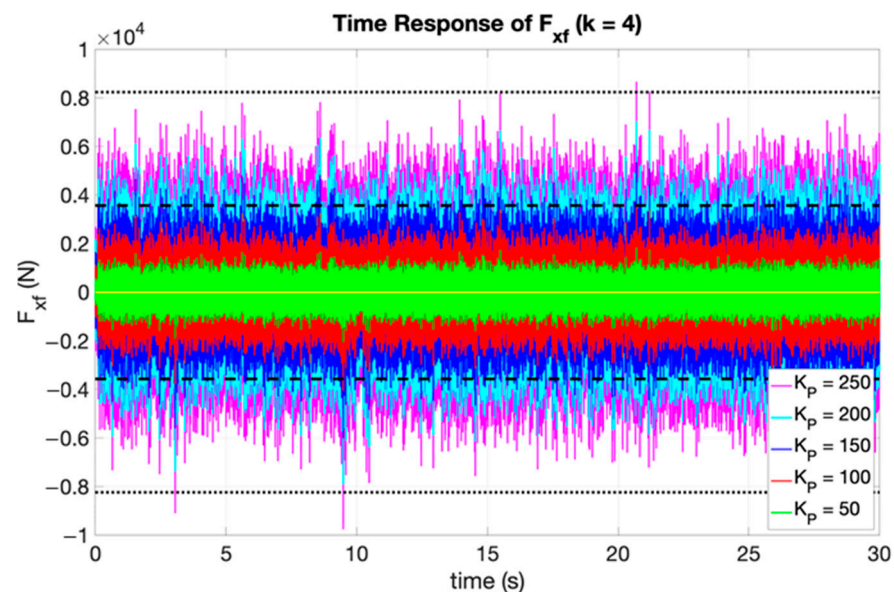


Figure 16. Time response of the front wheel force for the road profile classes B to C.

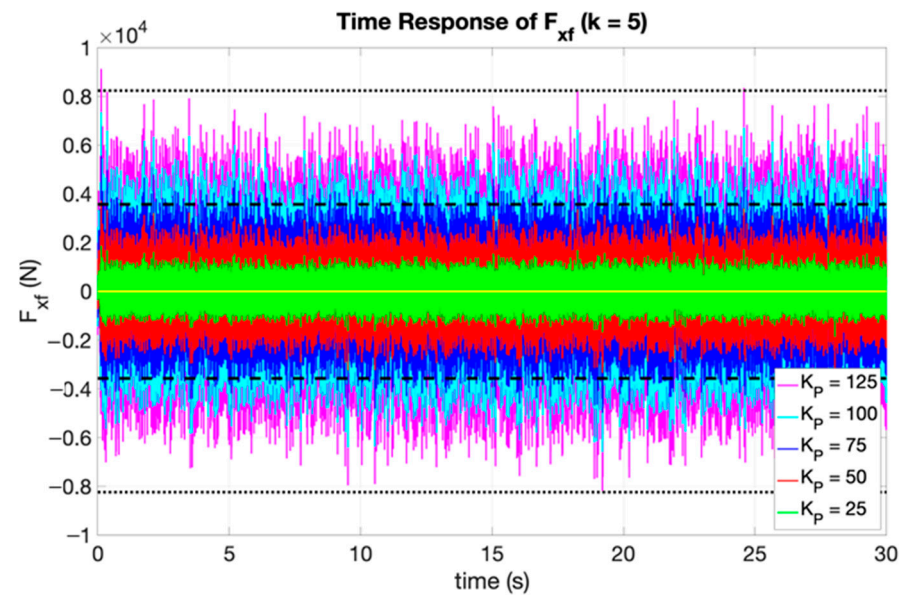


Figure 17. Time response of the front wheel force for the road profile classes C to D.

Additionally, considering the time response of the front wheel driving forces for the road profile classes C to D in Figure 17, with $K_p = 75$, the blue line indicates that most of the driving forces are within the continuous force limit, and the peak forces are within the peak force limit. Thus, $K_p = 75$ is suitable for this situation, resulting in an improvement of up to 2.01% in frequency-weighted acceleration compared to the open-loop system.

In summary, the closed-loop systems with the proportional ride-blending controller for the IWM effectively reduce the frequency-weighted acceleration, as per ISO 2631, and the RMS of the vertical acceleration of the sprung mass. The results demonstrate that increasing the proportional gain decreases both the frequency-weighted acceleration and the RMS of the vertical acceleration of the sprung mass. However, due to the torque limitations of the IWMs, excessively high proportional gains may prevent the motors from generating sufficient torque and force to reduce the vertical acceleration effectively. Table 9 summarizes the suitable proportional gains for each type of road profile and shows the corresponding percentage reduction in frequency-weighted acceleration. To achieve optimal reduction, an adaptive proportional gain controller based on the values in Table 9 may be implemented to maximize the decrease in frequency-weighted acceleration.

Table 9. The summary of the appropriate value of K_p for each type of road profile.

Type of Road Profile	The Most Appropriate Value of K_p	Percentage Decrease in Frequency-Weighted Acceleration (Compared with Open-Loop)
$k = 3$ (Class A to B)	300	7.5756%
$k = 4$ (Class B to C)	150	3.9632%
$k = 5$ (Class C to D)	75	2.0119%

4. Conclusions

This study investigated the impact of vertical force control in IWMs on enhancing ride comfort in electric vehicles (EVs) using a dynamic vehicle model and a proportional ride-blending controller. The implementation and analysis of the system's frequency response demonstrated that the proportional ride-blending controller effectively reduces vertical vibrations of the sprung mass, thereby improving ride comfort.

This research converted the state-space model into a transfer function to evaluate the system's response. Road profiles were generated according to ISO 8608 standards

and converted into Power Spectral Density (PSD) inputs for simulation. The frequency-weighted acceleration (a_w) was calculated based on ISO 2631 standards to assess ride comfort improvements. The findings revealed that increasing the proportional gain (K_p) reduced both the frequency-weighted acceleration and the RMS of the vertical acceleration of the sprung mass. However, due to the torque limitations of the IWMs, the gain could not be increased indefinitely.

In summary, this study provides a comprehensive analysis of using vertical force control in IWMs to enhance ride comfort in EVs. The results suggest that advanced suspension technologies, including the proportional ride-blending controller, can effectively address the challenges posed by increased unsprung mass in IWM vehicles, offering a promising approach to improve ride quality and vehicle dynamics.

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Appendix A

The state-space representation of the open-loop system according to Equation (13) is shown as follows:

$$\begin{aligned}\dot{x} &= A_{ol}x + B_{ol}u \\ y &= C_{ol}x + D_{ol}u\end{aligned}\quad (A1)$$

where

$$\begin{aligned}x &= \begin{bmatrix} z_b & \dot{z}_b & \theta & \dot{\theta} \end{bmatrix}^T, \\ u &= \begin{bmatrix} z_f & z_r & \dot{z}_f & \dot{z}_r \end{bmatrix}^T, \\ A_{ol} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_f+k_r}{m} & -\frac{c_f+c_r}{m} & \frac{bk_f-ck_r}{m} & \frac{bc_f-cc_r}{m} \\ 0 & 0 & 0 & 1 \\ \frac{bk_f-ck_r}{I_z} & \frac{bc_f-cc_r}{I_z} & -\frac{b^2k_f+c^2k_r}{I_z} & -\frac{b^2c_f+c^2c_r}{I_z} \end{bmatrix}, \\ B_{ol} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k_f}{m} & \frac{k_r}{m} & \frac{c_f}{m} & \frac{c_r}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{bk_f}{I_z} & \frac{ck_r}{I_z} & -\frac{bc_f}{I_z} & \frac{cc_r}{I_z} \end{bmatrix}, \\ C_{ol} &= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \\ D_{ol} &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}\end{aligned}\quad (A2)$$

The state-space representation of the closed-loop system is shown as follows:

$$\begin{aligned}\dot{x} &= A_{cl}x + B_{cl}u \\ y &= C_{cl}x + D_{cl}u\end{aligned}\quad (A3)$$

where

$$\begin{aligned}
 x &= \begin{bmatrix} z_b & \dot{z}_b & \theta & \dot{\theta} \end{bmatrix}^T, \\
 u &= \begin{bmatrix} z_f & z_r & \dot{z}_f & \dot{z}_r \end{bmatrix}^T, \\
 A_{cl} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_f+k_r}{M} & -\frac{c_f+c_r}{M} & \frac{bk_f-ck_r}{M} & \frac{bc_f-cc_r}{M} \\ 0 & 0 & 0 & 1 \\ \frac{bk_f-ck_r+N(k_f+k_r)}{I_z} & \frac{bc_f-cc_r+N(c_f+c_r)}{I_z} & -\frac{b^2k_f+c^2k_r+N(bk_f-ck_r)}{I_z} & -\frac{b^2c_f+c^2c_r+N(bc_f-cc_r)}{I_z} \end{bmatrix}, \\
 B_{cl} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{k_f}{M} & \frac{k_r}{M} & \frac{c_f}{M} & \frac{c_r}{M} \\ 0 & 0 & 0 & 0 \\ -\frac{(b+N)k_f}{I_z} & -\frac{(c-N)k_r}{I_z} & -\frac{(b+N)c_f}{I_z} & -\frac{(c-N)c_r}{I_z} \end{bmatrix}, \\
 C_{cl} &= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \\
 D_{cl} &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{A4}$$

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