

Article

Train Service Design for Rail Transit Cross-Line Operation Applying Virtual Coupling

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Abstract: The cross-line operation (CO) of trains in urban rail transit is an effective method to efficiently satisfy transfer passenger travel demand as well as relieve the pressure of transfer stations. The primary problem of CO is designing train services to satisfy travel demand with an uneven spatial distribution of passengers. This study constructs a nonlinear integer programming model with a novel train operation scheme, i.e., virtual coupling (VC) technology, which allows the coupling/decoupling of trains on different lines at both ends of each operation zone. This scheme makes the train capacity equitably distributed in each operation zone, thereby balancing train capacity utilization over the whole CO system. Regarding the nonlinear characteristics of the proposed model, an adaptive simulated annealing genetic algorithm (ASA-GA) was designed to quickly generate high-quality solutions. Based on real-world data from the Beijing Changping Line and Line 13, the effectiveness of the proposed model and algorithm were verified. The computation results show that in comparison to a single grouping train composition scheme without CO, a VC scheme with CO would reduce operation costs by 46.8%, with 80.6% savings of train capacity equity. Furthermore, the average passenger residence time would be reduced by 25.9%.

Keywords: cross-line operation; virtual coupling; train service design; train capacity equity



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1. Introduction

Nowadays, the transfer demand of passengers has increased dramatically due to the rapid development of urban rail transit (URT). Different from single-line operation, cross-line operation (CO) is mainly used to save transfer time and improve the accessibility of travel, which is widely regarded as an effective way to reduce passenger transfer behavior (Drechsler et al. [1]; Zhan et al. [2]). With CO, the spatial distribution of passenger flow has obvious uneven characteristics (Burdett [3]), such as a large passenger flow in the collinear zones (the overlapping zones of local lines and cross-lines) and a small passenger flow in the non-collinear zones (the independent zones of local lines). Further, in the collinear zones, cross-line trains should be increased as they can meet passengers' travel demand on both local and cross-lines (Zheng et al. [4]). Hence, optimizing train service design for CO has been a hot topic in recent years.

As the main method to save transfer time in URT systems, CO has attracted much attention recently. Novales et al. [5] studied the technical solutions for wheel-track design, traction power systems, and communication systems when state-owned railroads cross into tram lines. Wang et al. [6] emphasized the importance of considering networked operations in the planning and construction stage of URT. In addition, Yang et al. [7] considered the use of a jump-stop scheme for cross-line trains and established a capacity allocation model for CO to improve efficiency. To meet the challenge of uneven passenger flow, there has been extensive research on equity in train services. Li et al. [8] analyzed min-max equity and α -equity to establish a train schedule optimization scheduling model.

Wu et al. [9] considered the equity of waiting times at each transfer station and adopted the min-max equity function to balance URT schedules. Guo et al. [10] established a nonlinear integer programming model to determine the number of reserved trains for each train, thus ensuring the equity of train resources for downstream passengers. Shao et al. [11] proposed a train schedule optimization method that measures equity in terms of differences in passenger travel utility. Based on the concept of α -equity, Sun et al. [12] constructed a CO train service scheme aiming at minimizing the passenger waiting time, fairness, and operating cost, giving an example to verify that the optimal train service scheme can significantly improve the fairness performance of passenger travel.

In order to allocate train capacity properly according to passenger demand, many researchers have attempted to develop train service design using multiple grouping (MG) (see Kozachenko et al. [13], Yang et al. [14], and Cacchiani et al. [15]). However, due to the limitations of the traditional fixed block system, this pattern still leads to serious congestion in some busy zones. In recent years, virtual coupling (VC) technology has attracted the attention of many researchers. Many studies have discussed the basic functions of VC and verified that VC trains can significantly improve the overall line passing capacity (e.g., Stander et al. [16], Schumann et al. [17], and Jesús Félez et al. [18]). Zhou et al. [19] applied VC to full-length and short-turn routes and investigated the efficiency benefit for train operations. Lu et al. [20] further considered the constraints of train operation, rolling stock circulation, etc., in order to achieve the goal of minimizing passenger travel and operating costs. The results showed that VC can effectively adapt to the uneven distribution of passenger flow. Flammini et al. [21] verified that VC is one of the most innovative solutions to increase railroad capacity by significantly reducing train headway. Wang et al. [22] established a URT scheduling optimization model based on VC technology. By combining the train schedule with the train scheduling problem, a train service scheme with a minimum passenger waiting time was designed. Wang et al. [23] verified that VC is more suitable for passenger flows with uneven spatial and temporal distributions.

VC is a train-coupling technology that enables multiple trains to form a logical whole through vehicle-vehicle communication. In addition, VC allows for more flexible adjustment of coupling schemes to match dynamic passenger demand (Ning et al. [24]; Aoun et al. [25]; Di et al. [26]; Lu et al. [27]). At present, the number of train carriages in CO systems is fixed. With VC, the number of train carriages can be changed during operation. Thus, we attempted to apply VC between cross-line trains and local-line trains, with virtual coupling in the collinear zones to serve passengers in large groups and virtual decoupling in the non-collinear zones to serve passengers in small groups. In this study, we considered train service design in CO during a given period, where the train departure frequency, train composition of each line, and the turn-back stations of the cross-line have to be determined simultaneously. A mathematical model was constructed to minimize the passenger travel cost and the operation cost while ensuring train capacity allocation equity. Meanwhile, using VC, the trains of different lines were coupled in the collinear zone and decoupled in the single-line zone. In this way, it was possible to balance the accumulation of passenger flow in each zone of the CO, reduce the operational risk in some busy zones, and reasonably allocate the train capacity on each line. To sum up, previous studies have the following characteristics: (1) The research on line design mainly focuses on single lines, and trains can only operate on fixed lines, while research on CO is scarce. (2) The research on train-grouping methods in CO mainly focuses on the traditional SG and MG. There is little research on advanced VC. (3) The equity study of URT systems mainly considers passenger travel equity; i.e., the waiting time of passengers is proportional to the time on the train, ignoring the equity of the train capacity. There is no in-depth research on the travel equity gap of train capacity in URT systems. In Table 1, some related published papers are compared.

Table 1. Comparison of this paper with previous papers.

Literature	Line Type	Group Type	Equity	Objective Function
Shao et al. [11]	S	SG	TTE	Min. PTC, OC, TTE
Wu et al. [9]	S	SG	TTE	Min. PTC, TTE
Zhou et al. [19]	S	VC	TCE	Min. PTC, OC, TCE
Sun et al. [12]	Y	SG	TTE	Min. PTC, OC
Zheng et al. [4]	Y	MG	TCE	Min. PTC, OC, TCE
This paper	Y	VC	TCE	Min. PTC, OC, TCE

Note: S: Single line; Y: “Y”-type; SG: Single grouping; MG: Multiple grouping; VC: Virtual Coupling; TTE: Travel time equity; TCE: Train capacity equity; PTC: Passenger travel cost; OC: Operation cost.

The contributions of this study are summarized as follows.

- (1) A train service design method for CO to address uneven passenger flow is proposed; in particular, VC is applied. This approach enables trains from different lines to couple in collinear zones and decouple in non-collinear zones.
- (2) To better measure train capacity equity, this study precisely determines the train capacity utility for all zones by matching passenger demand with allocated train capacity, thereby enhancing the train capacity equity on the whole.
- (3) An improved adaptive simulated annealing genetic algorithm (ASA-GA) is designed to obtain a near-optimal solution. We test the performance of the proposed scheme using realistic instances from Beijing URT Changping Line and Line 13.

The remainder of this paper is organized as follows. In Section 2, we begin by providing a detailed description of the problem and discuss a simple case. In Section 3, a nonlinear integer programming model is built. In Section 4, we design an ASA-GA to solve the proposed problem. After that, a set of numerical experiments with the real-world operation data are implemented in Section 5. In the last section, some conclusions and further research works are presented.

2. Problem Description

2.1. Problem Statement

This study examines the train service design of CO systems during a specified study period. The system comprises two tracks and three lines, as illustrated in Figure 1. We use line l to denote local lines and cross-line to represent inter-line connections. Line 1 includes n_1 stations, and line 2 includes n_2 stations. We use l, i to represent the i -th station on line l . In addition, $1, b$, and $2, a$ are turn-back stations for cross-line trains, respectively. $1, p/2, n_2$ indicates a transfer station of three lines. There are three train types on line 1, line 2, and the cross-line, namely, train 1, train 2, and train 3, respectively. The spatial distribution of three lines divides two tracks into five zones, denoted by set $Z = \{I, II, III, IV, V\}$. In the CO system, zones I, III, and V are non-collinear zones; zones II and IV are collinear zones. Among them, line 1 and the cross-line are collinear in zone II, and line 2 and the cross-line are collinear in zone IV.

Train service design represents a social transportation resource-allocation problem. Focusing solely on minimizing passenger travel costs and operation costs may result in an inequitable distribution of train capacity. Especially in the oversaturation state of CO systems, inequitable distribution primarily exists in two aspects: (1) The passenger travel demand is high in collinear zones but relatively low in non-collinear zones. Consequently, train capacity in non-collinear zones may be underutilized when using uniform train compositions. (2) In collinear zones, local line passengers can choose between local and cross-line trains. During peak hours, local line passengers may occupy cross-line train resources, resulting in insufficient capacity on cross-line trains and a crowded travel environment. Therefore, this study aims to address the inequitable distribution of train capacity in CO systems.

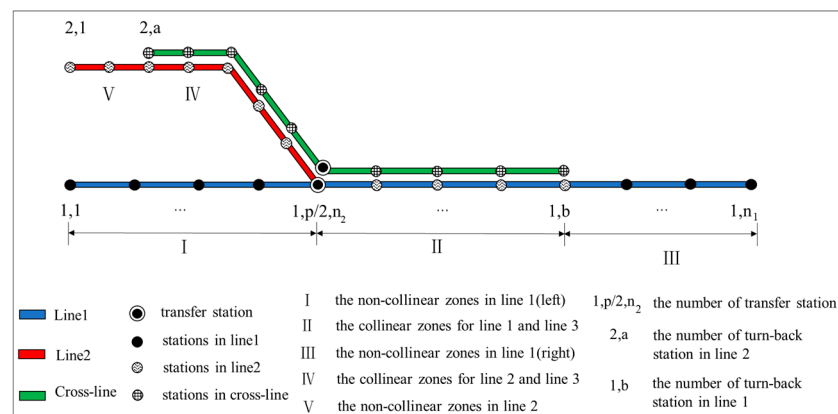


Figure 1. Cross-line operation scenario.

Figure 2 illustrates the VC process considered in this study. The blue, red, and green trains represent train 1, train 2, and train 3, respectively. The number of train groups can be flexibly changed during operation by coupling/decoupling activities on different lines. Generally, train 2 and train 3 operate in VC configuration in zone IV, while train 1 and train 3 operate in VC configuration in zone II. Taking direction as an example, train 2, after completing service in zone V, is required to operate in zone IV in VC configuration with train 3 and perform a virtual decoupling operation at the terminal of zone IV. Train 1, after completing service in zone I, is required to operate in zone II in VC configuration with train 3, and virtual decoupled at the terminal of zone II. The downward direction follows a similar pattern, with train 1 operating in zone II in VC configuration with train 3, and train 2 operating in zone IV in VC configuration with train 3.

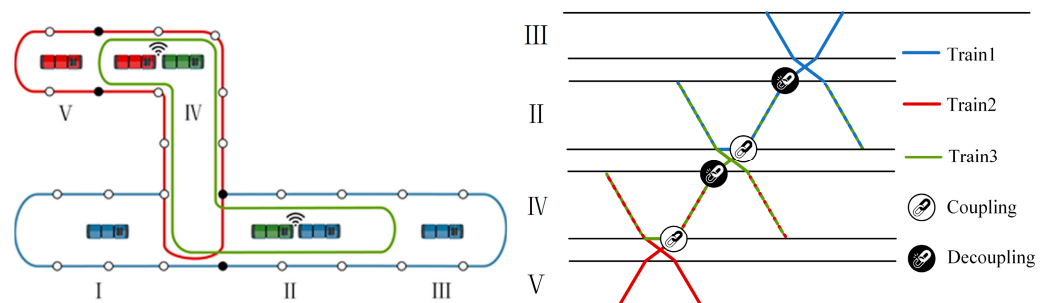


Figure 2. Illustration of rolling stock circulation with VC in the up direction.

VC technology is used in CO systems. While the number of carriages for each line remains fixed, trains from different lines can VC in collinear zones and virtually decouple in non-collinear zones. This approach better aligns the train capacity with passenger travel demand, potentially alleviating congestion on cross-line trains, reducing capacity waste in non-collinear zones, and improving overall system equity in the train capacity distribution.

2.2. Notations

The notations in Table 2 are used in this paper.

Table 2. Notations.

Notations	Definition
Sets and Indices	
L	The set of line types, $L = \{1, 2, 3\}$
l	Line index, $l \in L$; if $l = 3$, it is cross-line
k	Train type index, $k \in \{1, 2, 3\}$
Z	The set of operating zones, $Z = \{I, II, III, IV, V\}$
z	Operating zones index, $z \in Z$
P	The set of passenger types, $P = \{1a, 1b, 1c, 1d, 1e, 2a, 2b, 2c\}$
p	Passenger types index, $p \in P$
S	The set of stations, $S = \{1, 2, 3, \dots, S\}$
S_z	The set of stations in zone z , $z \in Z$, $S_z \in S$
m, n	The metro station index, $m, n \in S$
Parameters	
T_l	The full cycle time of line l
$q^{m,n}$	The number of passengers from m to n
$t^{m,n}$	The running time of the train from m to n
t_{walk}	Average transfer time
t^{vc}	Average additional VC time
W	Passenger travel time costs
RT_p	The residence time for type- p passengers
TT_p	The travel time for type- p passengers
VT_p	The additional VC time for type- p passengers
γ	The passenger unit time cost
Q_z	The maximum demand of zone z
$\eta_{l,z}$	The travel value of zone z in line l
f_{equity}	The value of equity performance
C_1	Train operating cost
C_2	Train usage cost
C	Train total operational costs
λ	Train operational cost per kilometer
D_l	The operating distance of line l
ω	Train depreciation cost per min
Cap	Train capacity
η_{max}	The minimum/maximum load rates
$H_{min}^{vc}/H_{max}^{vc}$	The minimum/maximum headway under VC
num_l	The number of carriages operating in line l
Decision variables	
f_l	The operation frequency of line l
n_l	The train composition of line l
$x_{l[i]}$	Binary variables; if the station i in line l is a cross-line terminal station, it is 1. Otherwise, it is 0.

3. Mathematical Model

The assumptions made to construct the mathematical model are as follows:

Assumption 1. Passenger arrival times at each station follow a uniform distribution. Passengers are expected to select direct trains to their destinations. To prevent passengers from boarding incorrect trains in VC, each train should be equipped with a destination indicator board displaying the terminal station of each train through the indicator board.

Assumption 2. The train control system has been upgraded to meet the functional requirements of VC. Specifically, the upgrade enables vehicle-to-vehicle (V2V) communication between trains, allowing the following train to establish direct communication with the leading train, acquire real-time data from the leading train, and make informed decisions regarding coupling conditions.

Assumption 3. Considering the inconvenience of transfer, it is assumed that passengers can transfer at most once.

3.1. Objective Function

This study considers passenger travel costs, operation costs, and train capacity equity in VC. There are constraints on available carriages, load rates, and other factors to determine the optimal combination of train departure frequency, train composition, and turn-back stations.

3.1.1. Passenger Travel Costs

To accurately calculate passenger travel costs, we divide passengers in Table 3. Class 1 represents passengers who can reach their destination directly without transfer, while Class 2 represents passengers who need to transfer to reach their destination. Additionally, passengers are further subdivided based on their origin-destination (OD) pairs, accounting for varying train frequencies in different zones. For instance, in Class 1, passengers traveling online 1 with their OD not entirely within zone II are categorized as Case 1a. Consequently, we further classify Class 1 into five cases, denoted as $P_1 = \{1a, 1b, 1c, 1d, 1e\}$, as illustrated in Figure 3. Similarly, Class 2 is subdivided into three cases, denoted as $P_2 = \{2a, 2b, 2c\}$, as illustrated in Figure 4.

Table 3. The table of passenger classification.

Passenger Class		OD Distribution	Frequencies
Class 1	Case 1a	OD is within <i>line</i> 1, without OD in zone II	f_1
	Case 1a	OD in zone II	f_1, f_3
	Case 1a	OD is zone II and IV	f_3
	Case 1a	OD is within <i>line</i> 2, without OD in zone IV	f_2
	Case 1a	OD in zone IV	f_2, f_3
Class 2	Case 2a	OD in zone I, III, and IV	f_1, f_2, f_3
	Case 2b	OD in zone I, III, and V	f_1, f_2
	Case 2c	OD in zone II and V	f_1, f_2, f_3

Generally, passenger travel costs mainly include the resistance time for type- p passengers RT_p , in-vehicle time for type- p passengers TT_p , and additional VC time for type- p passengers VT_p . Among them, RT_p includes the waiting time and walking transfer time; the waiting time is set as half of the train departure interval as the passenger arrival is uniformly distributed, i.e., $30/f^{m,n}$. Notably, when trains from different lines enter collinear zones, VC is performed, wherein two trains operate in the form of one train. Consequently, for passengers who can travel on both local and cross-line trains, the passenger waiting time is calculated using the greater of the two trains' departure frequencies. TT_p is mainly influenced by the origin and destination stations of passengers. In addition, in CO, the uncertainty of the train's arrival time may lead to additional VC time for the trains of different lines during the VC process. Thus, VT_p represents additional VC time for type- p passengers. The critical point will be the first station where the VC occurs (including cross-line turn-back station 2, $a, 1, b$; transfer station $1p$, which is represented by a, b, tra). Additional VC time applies to passengers who either originate from or pass critical points. Additional VC time is proportional to the number of VC and is determined by lower train departure frequency between the local line and the cross-line services. The passenger travel cost is calculated using Equation (1).

$$W = \gamma \cdot \sum_{p \in P} (RT_p + TT_p + VT_p), \forall p \in P \quad (1)$$

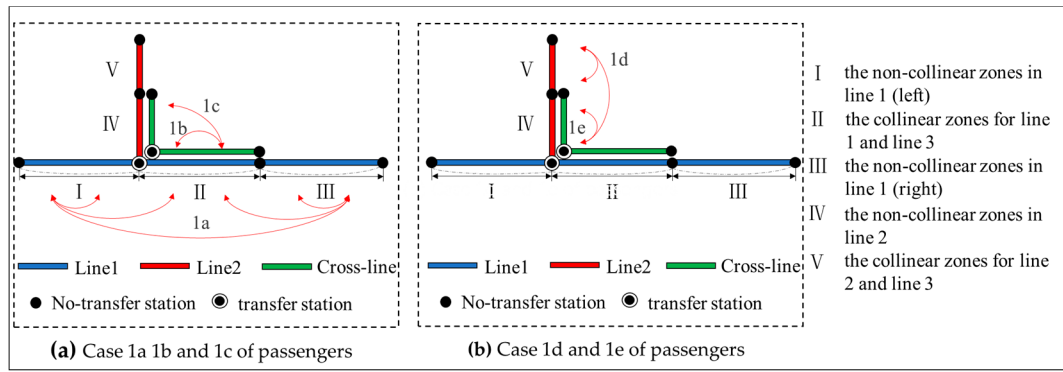


Figure 3. Illustration of trips for different cases without transfer.

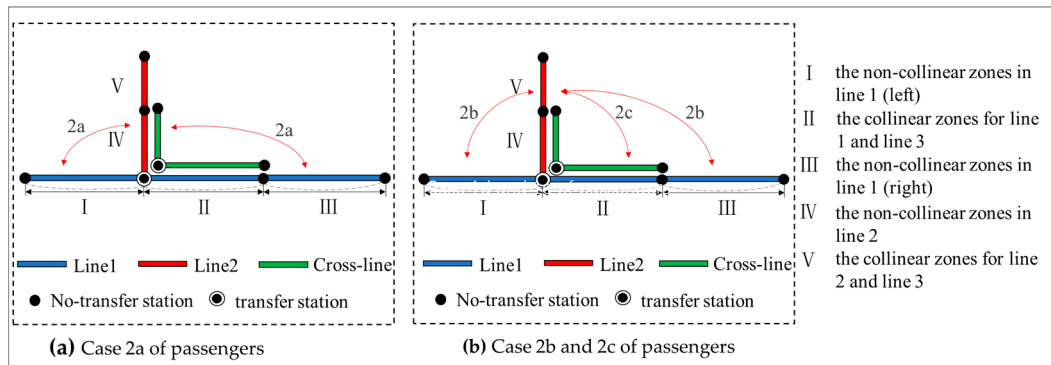


Figure 4. Illustration of trips for different cases with transfer.

The detailed calculation process for each type of passenger is as follows.

(1) The travel costs of the direct passengers

Case 1a: The passengers in line 1, excluding those whose ODs are both within zone II (see Figure 3a). The resistance time for type-1a passengers RT_{1a} , in-vehicle time for type-1a passengers TT_{1a} , and additional VC time for type-1a passengers VT_{1a} can be calculated as follows. In addition, $q^{m,n}$ represents the number of passengers from m to n , and $t^{m,n}$ represents the running time of the train from m to n .

$$RT_{1a} = \left(\sum_{m,n \in S_I \cup S_{II} \cup S_{III}} q^{m,n} - \sum_{m,n \in S_{II}} q^{m,n} \right) \cdot \frac{30}{f_1} \quad \forall m \neq n \quad (2)$$

$$TT_{1a} = \left(\sum_{m,n \in S_I \cup S_{II} \cup S_{III}} q^{m,n} - \sum_{m,n \in S_{II}} q^{m,n} \right) \cdot t^{m,n} \quad \forall m \neq n \quad (3)$$

$$VT_{1a} = \begin{cases} \left(\sum_{m \in S_I} \sum_{n \in S_{II} \cup S_{III}} q^{m,n} + \sum_{m \in S_{III}} \sum_{n \in S_I \cup S_{II}} q^{m,n} - \sum_{m \in S_I} q^{m,tra} - \sum_{m \in S_{III}} q^{m,b} \right) \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} f_1 \geq f_3 \\ \left(\sum_{m \in S_I} \sum_{n \in S_{II} \cup S_{III}} q^{m,n} + \sum_{m \in S_{III}} \sum_{n \in S_I \cup S_{II}} q^{m,n} - \sum_{m \in S_I} q^{m,tra} - \sum_{m \in S_{III}} q^{m,b} \right) \cdot t^{vc} \cdot \frac{f_1}{f_1 + f_3} f_1 < f_3 \end{cases}, \forall m \neq n \quad (4)$$

Case 1b: The OD of these passengers both within zone II. The resistance time for type-1b passengers RT_{1b} , in-vehicle time for type-1b passengers TT_{1b} , and additional VC time for type-1b passengers VT_{1b} can be calculated by Equations (5)–(7).

$$\begin{cases} RT_{1b} = \sum_{m,n \in S_{II}} q^{m,n} \cdot \frac{30}{f_1} & f_1 \geq f_3 \\ RT_{1b} = \sum_{m,n \in S_{II}} q^{m,n} \cdot \frac{30}{f_3} & f_1 < f_3 \end{cases} \quad \forall m \neq n \quad (5)$$

$$TT_{1b} = \sum_{m,n \in S_{II}} q^{m,n} \cdot t^{m,n} \forall m \neq n \quad (6)$$

$$VT_{1b} = \begin{cases} (\sum_{n \in S_{II}} q^{tra,n} + \sum_{n \in S_{II}} q^{b,n}) \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} f_1 \geq f_3 \\ (\sum_{n \in S_{II}} q^{tra,n} + \sum_{n \in S_{II}} q^{b,n}) \cdot t^{vc} \cdot \frac{f_1}{f_1 + f_3} f_1 < f_3 \end{cases} \forall m \neq n \quad (7)$$

Case 1c: Passengers traveling on the cross-line and OD within zones II and IV. resistance time for type-1c passengers RT_{1c} , in-vehicle time for type-1c passengers TT_{1c} , and additional VC time for type-1c passengers VT_{1c} can be calculated by Equations (8)–(12). Among them, type-1c passengers not only need additional VC time at the transfer station *tra* for train 1 and train 3. Part passengers also need additional VC time at station *a* for train 2 and train 3. Therefore, the calculation is divided into two parts, shown in Equations (11) and (12).

$$RT_{1c} = (\sum_{m \in S_{II}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{II}} q^{m,n}) \cdot \frac{30}{f_3} \quad \forall m \neq n, \quad (8)$$

$$TT_{1c} = (\sum_{m \in S_{II}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{II}} q^{m,n}) \cdot (t^{m,n} + t^{vc}) + (\sum_{n \in S_{II}} q^{a,n} + \sum_{n \in S_{IV}} q^{b,n}) \cdot t^{vc} \quad \forall m \neq n \quad (9)$$

$$VT_{1c} = VT_{1c1} + VT_{1c2} \quad (10)$$

$$VT_{1c1} = \begin{cases} (\sum_{n \in S_{IV}} q^{b,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{II}} q^{m,n}) \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} & f_1 \geq f_3 \\ (\sum_{n \in S_{IV}} q^{b,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{II}} q^{m,n}) \cdot t^{vc} \cdot \frac{f_2}{f_1 + f_3} & f_1 < f_3 \end{cases} \forall m \neq n \quad (11)$$

$$VT_{1c2} = \begin{cases} (\sum_{n \in S_{II}} q^{a,n} + \sum_{m \in S_{II}} \sum_{n \in S_{IV}} q^{m,n}) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_2 \geq f_3 \\ (\sum_{n \in S_{II}} q^{a,n} + \sum_{m \in S_{II}} \sum_{n \in S_{IV}} q^{m,n}) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 < f_3 \end{cases} \forall m \neq n \quad (12)$$

Case 1d: Passengers traveling on line 2, excluding those whose OD is within zone V (see Figure 3b). The resistance time for type-1d passengers RT_{1d} , in-vehicle time for type-1d passengers TT_{1d} , and additional VC time for type-1d passengers VT_{1d} can be calculated as follows:

$$RT_{1d} = (\sum_{m \in S_V} \sum_{n \in S_{IV \cup V}} q^{m,n} + \sum_{m \in S_{IV \cup V}} \sum_{n \in S_V} q^{m,n}) \cdot \frac{30}{f_2} \quad \forall m \neq n \quad (13)$$

$$TT_{1d} = (\sum_{m \in S_V} \sum_{n \in S_{IV \cup V}} q^{m,n} + \sum_{m \in S_{IV \cup V}} \sum_{n \in S_V} q^{m,n}) \cdot t^{m,n} \quad \forall m \neq n \quad (14)$$

$$VT_{1d} = \begin{cases} (\sum_{m \in S_V} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_V} q^{m,n} - \sum_{m \in S_V} q^{m,a} - \sum_{n \in S_V} q^{a,n}) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_2 \geq f_3 \\ (\sum_{m \in S_V} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_V} q^{m,n} - \sum_{m \in S_V} q^{m,a} - \sum_{n \in S_V} q^{a,n}) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 < f_3 \end{cases} \forall m \neq n \quad (15)$$

Case 1e: The OD of these passengers both in zone IV. The resistance time for type-1e passengers RT_{1e} , in-vehicle time for type-1e passengers TT_{1e} , and additional VC time for type-1e passengers VT_{1e} can be calculated as follows:

$$\begin{cases} RT_{1e} = \sum_{m,n \in S_{IV}} q^{m,n} \cdot \frac{30}{f_2} & f_2 \geq f_3 \\ RT_{1e} = \sum_{m,n \in S_{IV}} q^{m,n} \cdot \frac{30}{f_3} & f_2 < f_3 \end{cases} \forall m \neq n \quad (16)$$

$$TT_{1e} = \sum_{m,n \in S_{IV}} q^{m,n} \cdot t^{m,n} \quad \forall m \neq n \quad (17)$$

$$VT_{1e} = \begin{cases} \left(\sum_{n \in S_{IV}} q^{a,n} + \sum_{n \in S_{IV}} q^{tra,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_2 \geq f_3 \\ \left(\sum_{n \in S_{IV}} q^{a,n} + \sum_{n \in S_{IV}} q^{tra,n} \right) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 < f_3 \end{cases} \quad \forall m \neq n \quad (18)$$

(2) The travel costs of the transfer passengers

Case 2a: Passengers that need to transfer and the OD within zones I, III, and IV (see Figure 4a). For instance, consider passengers traveling from zone I to zone IV: these passengers must first take train 1 in zone I to reach the transfer station, then board either train 2 or train 3 to complete their travel to zone IV. $RT_{2a}^{m,n}$ of Case 2a passengers is affected by frequency of train 1, train 2, and train 3, i.e., f_1, f_2, f_3 . Thus, the resistance time for type-2a passengers RT_{2a} , in-vehicle time for type-2a passengers TT_{2a} , and additional VC time for type-2a passengers VT_{2a} can be calculated as follows. For type-2a passengers, additional VC time is required at the transfer station for train 1 and train 3. Furthermore, some passengers require additional VC time at station a for train 2 and train 3. Consequently, the calculation is divided into two components, as represented in Equations (22) and (23).

$$\begin{cases} RT_{2a} = \left(\sum_{m \in S_{I \cup III}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot \left(\frac{30}{f_1} + \frac{30}{f_2} \right) + t_{walk} & f_2 \geq f_3 \\ RT_{2a} = \left(\sum_{m \in S_{I \cup III}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot \left(\frac{30}{f_1} + \frac{30}{f_3} \right) + t_{walk} & f_2 < f_3 \end{cases} \quad \forall m \neq n \quad (19)$$

$$TT_{2a} = \left(\sum_{m \in S_{I \cup III}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot t^{m,n} \quad \forall m \neq n. \quad (20)$$

$$VT_{2a} = VT_{2a1} + VT_{2a2} \quad (21)$$

$$VT_{2a1} = \begin{cases} \left(\sum_{m \in S_{III}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} & f_1 \geq f_3 \\ \left(\sum_{m \in S_{III}} \sum_{n \in S_{IV}} q^{m,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_1}{f_1 + f_3} \cdot t^{vc} & f_1 < f_3 \end{cases} \quad \forall m \neq n \quad (22)$$

$$VT_{2a2} = \begin{cases} \left(\sum_{n \in S_{I \cup III}} q^{a,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 \geq f_3 \\ \left(\sum_{n \in S_{I \cup III}} q^{a,n} + \sum_{m \in S_{IV}} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_3 < f_2 \end{cases} \quad \forall m \neq n \quad (23)$$

Case 2b: Passengers are in zones I, III, and V (see Figure 4b). The resistance time for type-2b passengers RT_{2b} , in-vehicle time for type-2b passengers TT_{2b} , and additional VC time for type-2b passengers VT_{2b} can be calculated as follows. For type-2b passengers, additional VC time is required at the transfer station for train 1 and train 3. Furthermore, some passengers require additional VC time at station a for train 2 and train 3. Consequently, the calculation is divided into two components, as represented in Equations (27) and (28).

$$RT_{2b} = \left(\sum_{m \in S_{I \cup III}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot \left(\frac{30}{f_1} + \frac{30}{f_2} \right) + t_{walk} \quad \forall m \neq n \quad (24)$$

$$TT_{2b} = \left(\sum_{m \in S_{I \cup III}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{I \cup III}} q^{m,n} \right) \cdot t^{m,n}, \forall m \neq n \quad (25)$$

$$VT_{2b} = VT_{2b1} + VT_{2b2} \quad (26)$$

$$VT_{2b1} = \begin{cases} \left(\sum_{m \in S_{III}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} & f_1 \geq f_3 \\ \left(\sum_{m \in S_{III}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{III}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_1}{f_1 + f_3} \cdot t^{vc} & f_1 < f_3 \end{cases} \quad \forall m \neq n \quad (27)$$

$$VT_{2b2} = \begin{cases} \left(\sum_{m \in S_{I \cup S_{III}}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{I \cup S_{III}}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 \geq f_3 \\ \left(\sum_{m \in S_{I \cup S_{III}}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{I \cup S_{III}}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_3 < f_2 \end{cases} \quad \forall m \neq n \quad (28)$$

Case 2c: Passengers requiring transfers and the OD within zones II and V. The resistance time for the type-2c of passengers RT_{2c} , in-vehicle time for type-2c passengers TT_{2c} , and additional VC time for type-2c passengers VT_{2c} can be calculated as follows. For type-2c passengers, additional VC time is required at the transfer station for train 1 and train 3. Furthermore, some passengers require additional VC time at station a for train 2 and train 3. Consequently, the calculation is divided into two components, as represented in Equations (32) and (33).

$$\begin{cases} RT_{2c} = \left(\sum_{m \in S_{II}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{II}} q^{m,n} \right) \cdot \left(\frac{30}{f_1} + \frac{30}{f_2} \right) + t_{walk} & f_1 \geq f_3 \\ RT_{2c} = \left(\sum_{m \in S_{II}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{II}} q^{m,n} \right) \cdot \left(\frac{30}{f_2} + \frac{30}{f_3} \right) + t_{walk} & f_1 < f_3 \end{cases} \quad \forall m \neq n \quad (29)$$

$$TT_{2c} = \left(\sum_{m \in S_{II}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{II}} q^{m,n} \right) \cdot (t^{m,n} + t^{vc}) \quad \forall m \neq n \quad (30)$$

$$VT_{2c} = VT_{2c1} + VT_{2c2} \quad (31)$$

$$VT_{2c1} = \begin{cases} \sum_{n \in S_V} q^{b,n} \cdot t^{vc} \cdot \frac{f_3}{f_1 + f_3} & f_1 \geq f_3 \\ \sum_{n \in S_V} q^{b,n} \cdot t^{vc} \cdot \frac{f_1}{f_1 + f_3} \cdot t^{vc} & f_1 < f_3 \end{cases} \quad \forall m \neq n \quad (32)$$

$$VT_{2c2} = \begin{cases} \left(\sum_{m \in S_{II}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{II}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_2}{f_2 + f_3} & f_2 \geq f_3 \\ \left(\sum_{m \in S_{II}} \sum_{n \in S_V} q^{m,n} + \sum_{m \in S_V} \sum_{n \in S_{II}} q^{m,n} \right) \cdot t^{vc} \cdot \frac{f_3}{f_2 + f_3} & f_3 < f_2 \end{cases} \quad \forall m \neq n \quad (33)$$

3.1.2. Train Capacity Equity

Train capacity equity refers to the provision of equitable passenger experiences in transportation activities within the constraints of limited social transportation resources. This study examines the degree of alignment between passenger flow demand and train capacity across various zones of each line, quantified as a ratio. The concept of train capacity equity, as considered in this study, aims to optimize the alignment between passenger demand and train capacity across different zones and lines, minimizing disparities. This optimization can be achieved through adjustments to cross-line turn-back stations, train composition, and train frequency within the operational planning process.

The travel value of zone z in line l $\eta_{l,z}$ is computed using Equations (34) and (35), where $q^{m,n}$ represents the number of passengers from m to n , Cap represents the train capacity, f_l represents the operation frequency of line l , and n_l represents the train composition of line l .

$$\eta_{l,z} = \begin{cases} \frac{\sum_{m \in S_z} \sum_{n \in S} q^{m,n}}{Cap \cdot f_l \cdot n_l}, & \text{if } l = \{1, 2\}, z = \{I, III, V\} \\ \frac{\sum_{m \in S_z} \sum_{n \in S} q^{m,n}}{Cap \cdot (f_l \cdot n_l + f_3 \cdot n_3)}, & \text{if } l = \{1, 2\}, z = \{II, IV\} \end{cases} \quad (34)$$

The value of equity performance f_{equity} can be expressed by Equation (35). Especially, $\bar{\eta}$ is the average value of train capacity equity.

$$f_{equity} = \sum_{l \in L} \sum_{z \in Z} (\eta_{l,z} - \bar{\eta})^2 \quad (35)$$

3.1.3. Operation Costs

This study examines two primary components of operation costs, the train operating cost and train usage cost, as represented in Equation (36), where C_1 represents the train operating cost, and C_2 represents the train usage cost.

$$C = C_1 + C_2 \quad (36)$$

The train operating cost C_1 represents the expenses incurred for the distance traveled during train service. In CO systems, the train composition of line l n_l , operation frequency of line l f_l , operating distance of line l D_l , and train operation cost per kilometer λ can be calculated using Equation (37).

$$C_1 = \lambda \cdot \sum_{l=1}^3 D_l \cdot f_l \cdot n_l \quad (37)$$

The train usage cost C_2 consist of wear and tear, depreciation, etc. The train usage cost is related to the number of carriages in service and can be calculated by the train composition of line l n_l , operation frequency of line l f_l , the full cycle time of line l T_l , and the train depreciation cost per min ω . Unlike traditional trains, VC trains are equipped with multiple sensors, communication equipment, and other technologies to facilitate coupling operations. The train usage cost is calculated using Equation (38).

$$C_2 = \omega \cdot \sum_{l=1}^3 \left[\frac{f_l \cdot n_l \cdot T_l}{60} \right] \quad (38)$$

3.2. Constraints

3.2.1. Train Operation Plan Constraints

To optimize the alignment with transfer passenger travel demand, the locations of two turn-back stations on cross-line are designated as decision variables. This can be expressed as follows:

$$\begin{cases} \sum_{i \in S_{IV}} x_{2[i]} = 1, \\ \sum_{i \in S_{II}} x_{1[i]} = 1. \end{cases} \quad (39)$$

The load rate can reflect the resource utilization rate. Therefore, we limit the maximum load rate for each zone within a reasonable range, expressed as $\eta_{\min}$ and $\eta_{\max}$, to avoid overcrowding and the waste of resources. In this study, Q_z represents the maximum passenger demand in zone $z, z \in Z$, and the available transportation capacity in each zone is determined by frequency f_l and train composition n_l . Thus, the load rate of zones I, II, III, IV, and V needs to satisfy Equation (40).

$$\begin{cases} \frac{Q_I}{Cap \cdot f_1 \cdot n_1} \leq \eta_{\max} \\ \frac{Q_{II}}{Cap \cdot (f_1 n_1 + f_3 n_3)} \leq \eta_{\max} \\ \frac{Q_{III}}{Cap \cdot f_1 \cdot n_1} \leq \eta_{\max} \\ \frac{Q_{IV}}{Cap \cdot (f_2 n_2 + f_3 n_3)} \leq \eta_{\max} \\ \frac{Q_V}{Cap \cdot f_2 n_2} \leq \eta_{\max} \end{cases} \quad (40)$$

The train departure frequency and train composition of all lines are positive, so the train frequency and train composition should satisfy Equation (41).

$$\begin{cases} f_l \in Z^+ \\ n_l \in Z^+, l = 1, 2, 3 \end{cases} \quad (41)$$

3.2.2. Headway Constraints

To ensure the safe operation of trains, two adjacent trains must maintain a minimum departure interval. As illustrated in Figure 5, a relative braking distance and safety margin distance should be maintained between two adjacent trains. The time interval between these trains should be no less than the minimum departure time interval (i.e., $H_{\min}^{vc}$) under the VC state. Simultaneously, the distance between the two trains must remain within communication range while not exceeding the maximum departure time interval (i.e., $H_{\max}^{vc}$). Consequently, train departure intervals of train 1 and train 2 in zones I, III, and V should satisfy Equations (42) and (43).

$$H_{\min}^{vc} \leq \frac{60}{f_1} \leq H_{\max}^{vc} \quad (42)$$

$$H_{\min}^{vc} \leq \frac{60}{f_2} \leq H_{\max}^{vc} \quad (43)$$

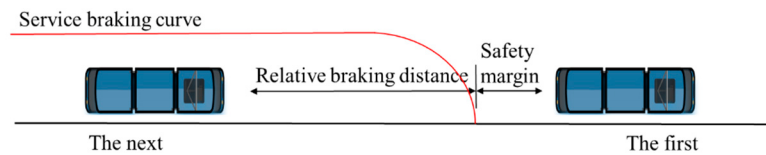


Figure 5. Zone I, III, and V head spacing indicators.

As shown in Figure 6, train 1 and train 3 operate in zone II, and train 2 and train 3 operate in zone IV. After VC, two trains can be considered the same train. Therefore, in collinear zones, train departure intervals operating at higher frequencies should satisfy Equations (44) and (45).

$$H_{\min}^{vc} \leq \frac{60}{\max\{f_1, f_3\}} \leq H_{\max}^{vc} \quad (44)$$

$$H_{\min}^{vc} \leq \frac{60}{\max\{f_2, f_3\}} \leq H_{\max}^{vc} \quad (45)$$

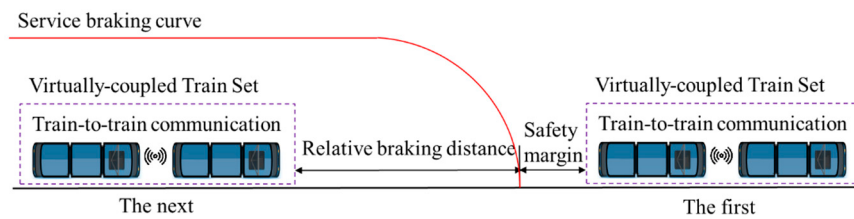


Figure 6. Zone II and IV head spacing indicators.

3.2.3. Train Carriages Constraints

Due to platform-length limitations, the threshold value of train carriages is given as 8. In this study, we assume that the minimum number of VC formation carriages is 2. Therefore, the carriage size should satisfy Equation (46).

$$2 \leq n_l \leq 8 \quad l = 1, 2, 3 \quad (46)$$

Carriages resources in URT systems are limited. The carriages used in the train operation scheme of three lines should not exceed the total number of available carriages provided by the operator.

$$\sum_{l=1}^3 \left[\frac{f_l \cdot n_l \cdot T_l}{60} \right] \leq \sum_{l=1}^3 num_l \quad (47)$$

4. Adaptive Simulated Annealing Genetic Algorithm (ASA-GA)

As the optimization model of the train service scheme established in this study is a nonlinear integer programming model, it is mostly solved by a heuristic algorithm. The genetic algorithm (GA) is widely used in URT system optimization problems, such as the study of integrated optimization of express trains and cross-line trains by Tang et al. [28] and the study of energy consumption and timetable operation of trains by Aredah et al. [29]. Although the GA has a strong global search ability and is suitable for large-scale optimization problems, it is prone to fall into local optimality, while the simulated annealing algorithm (SAA) simulates the cooling process and continuously adds disturbance to the generated solution to find the optimal solution, which has a strong local search ability. Combined with GA, it can effectively avoid falling into the local optimal situation and achieve the purpose of complementary advantages. Moreover, to improve the search performance of SA-GA, an adaptive selection mechanism, namely, an adaptive simulated annealing genetic algorithm (ASA-GA), is designed to solve the proposed model in this study, and its main steps are as follows.

(1) Population Initialization

Train frequency, train composition, and turn-back stations are selected as genes on each chromosome. In this study, we use decimal coding, and one chromosome represents one train service scheme. As shown in Figure 7, a chromosome is divided into three parts, where part A indicates the train frequency of three types of trains (including train 1, train 2, and train 3), part B indicates the train composition of three types of trains, and part C indicates the turn-back stations of the cross-line.

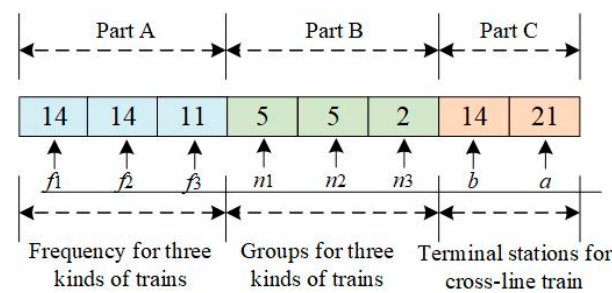


Figure 7. Illustration of the composition of chromosome.

Step 1: Set the initial value parameters of GA and SA, including the population size, initial generation, initial temperature, final temperature, cooling coefficient, maximum number of generations, and number of cycles per temperature level.

Step 2: Input boundaries of genes in parts A, B, and C of chromosomes, which facilitates the generation of feasible solutions and faster convergence.

Step 3: Perform random generation of parental chromosome genes: this includes the three parts.

Step 4: Check the viability of chromosomes. If this is not feasible, return to step 2 until feasible populations are obtained.

(2) Adaptive crossover and mutation operator

Crossover is one of the main operations of GA, and the process is shown in Figure 8. To avoid that, a large crossover probability will destroy the optimal solution, and a small crossover probability will affect the diversity of individuals. In this study, the adaptive annealing temperature crossover probability p_c is expressed as in Equation (48):

$$p_c = \begin{cases} \frac{p_{c1} + p_{c2}}{2} \cdot \frac{f_{\max} - f'}{f_{\max} - f_{\text{avg}}} - \frac{\beta N}{T_{\text{now}}}, & f \geq f_{\text{avg}}, \\ \frac{p_{c1} + p_{c2}}{2} - \frac{\beta N}{T_{\text{now}}}, & f \leq f_{\text{avg}}. \end{cases} \quad (48)$$

where p_{c1} and p_{c2} are the maximum and minimum values of crossover probability, respectively. f_{max} and f_{avg} are the highest adaptation values and the average adaptation values, respectively. f'' is the larger adaptation value of the two crossed chromosomes.

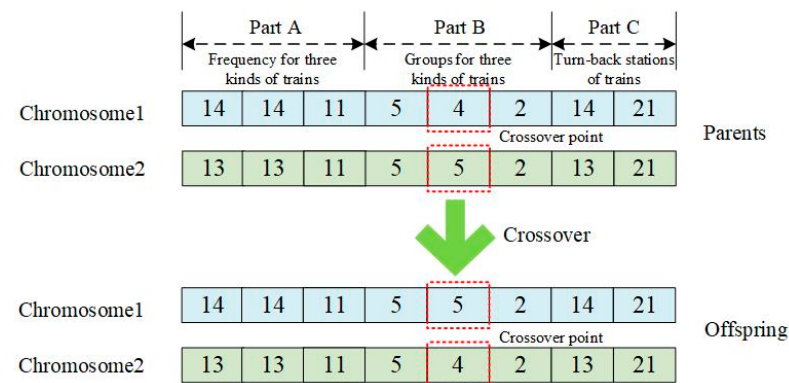


Figure 8. Illustration of a single-point crossover.

Mutation operation is the replacement of a specific gene in an individual's chromosome with an allele to form a new individual. Some chromosomes are randomly selected to be the parents of mutation. The adaptive annealing temperature mutation probability p_m is shown in Equation (49):

$$p_m = \begin{cases} \frac{p_{m1} + p_{m2}}{2} \cdot \frac{f_{max} - f}{f_{max} - f_{avg}} - \frac{\beta N}{T_{now}}, & f \geq f_{avg}, \\ \frac{p_{m1} + p_{m2}}{2} - \frac{\beta N}{T_{now}}, & f \leq f_{avg}. \end{cases} \quad (49)$$

where p_{m1} and p_{m2} are the maximum and minimum values of mutation probability, respectively.

(3) Simulated annealing operator

Step 1: Construct an initial solution and record optimal solutions. Next, a neighborhood solution is generated based on two-point exchange operations.

Step 2: Compare a new neighborhood solution with an old initial solution. The metropolis criterion is used to allow the algorithm to accept some low-quality solutions with a small probability, thereby preventing the algorithm from falling into local optimality. The acceptance probability is as follows:

$$p_a = \begin{cases} 1, & x_{cur} < x_{new}, \\ \exp\left(-\frac{x_{cur} - x_{new}}{T_{now}}\right), & x_{cur} > x_{new}. \end{cases} \quad (50)$$

Step 3: If the current number of cycles does not reach the maximum number of cycles, GA selection, crossover, and mutation operations are repeated.

Step 4: If $T_{now} > T_{end}$, the temperature is reduced according to the cooling coefficient, and an iterative loop is started at a new temperature. When $T_{now} < T_{end}$, the optimal solution is output.

The algorithm process is shown in Figure 9.

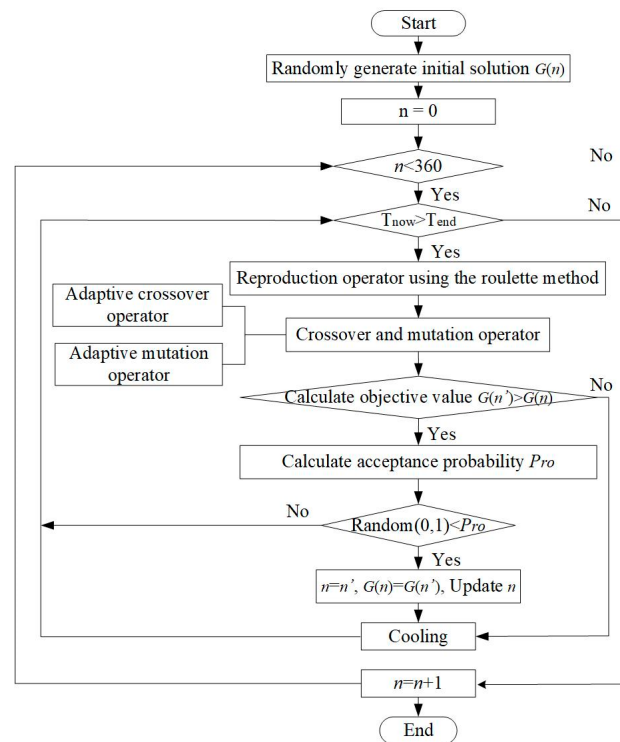


Figure 9. Algorithm process.

5. Case Study

In this section, small-scale and real-world numerical cases are conducted to verify the proposed method. In the small-scale experiment, the performance of GA, Gurobi 11.0.2 solver, and ASA-GA are compared. For real-world scenarios, only ASA-GA is employed to solve the model.

5.1. A Small-Scale Case Study

5.1.1. Description of Small-Scale Case

In a small-scale case, two subway lines, each comprising five stations, designated as *line k* (stations numbered 1–6) and *line k'* (stations numbered 7–10), are examined. Station 2 on *line k* intersects with station 7 on *line k'*, forming a “Y”-shape configuration. Figure 10 illustrates the time-dependent passenger demand in a small-scale case, spanning a time range of [0, 20 min], with a cross-line direction from station 10 to station 6 via the interchange at station 7/station 2—station 6. To evaluate the effectiveness of CO, the cross-line passenger flow is set to account for 18% of the total passenger flow.

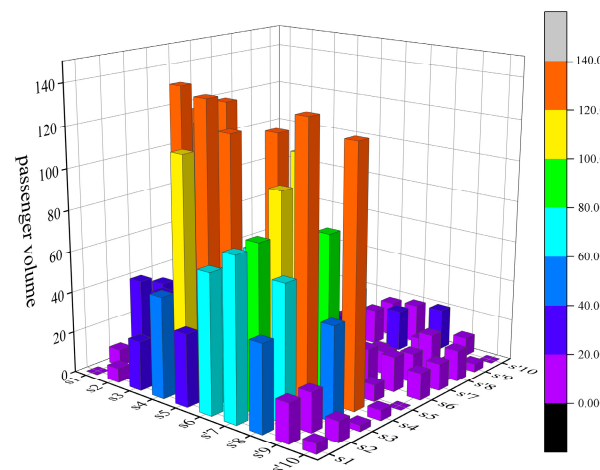


Figure 10. Passenger OD matrix.

5.1.2. Comparison of the Performance of Algorithm

To evaluate the effectiveness and efficiency of the proposed ASA-GA, its performance is compared with that of GA and Gurobi 11.0.2 solver. For this experiment, GA parameters are set as follows: $N = 100$, $P_c = 0.75$, $P_m = 0.1$. ASA-GA parameters are set as $K = 10$, maximum number of iterations $K_{\max} = 100$, cooling coefficient $\theta = 0.7$, initial temperature $T_0 = 800$, and final temperature $T_{\text{end}} = 1$. The encoding and decoding techniques for both ASA-GA and GA are identical. The optimal solution is obtained using the Gurobi 11.0.2 solver on the transformed linear model. Based on the same arithmetic example, the GA, Gurobi 11.0.2 solver, and ASA-GA are carried out 20 times, respectively. Table 4 presents the experimental results, while Figure 11 illustrates the convergence tendency of the objective value.

Table 4. Experimental results when using different algorithms.

Algorithms	Optimal Value	Maximum Value	Average Value	CPU (s)
GA	6639	8578	7127	406
Gurobi	5831	7954	6308	560
ASA-GA	5870	8025	6248	424

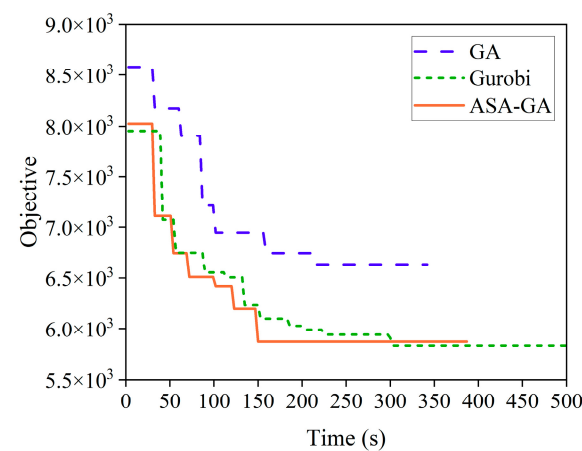


Figure 11. The convergence tendency of the objective value.

As shown in Table 4, the Gurobi 11.0.2 solver outperforms the GA in terms of optimal value, maximum value, and average value by 12.2%, 7.3%, and 11.5%, respectively. Furthermore, the Gurobi 11.0.2 solver achieves a better optimal value and maximum value

than the ASA-GA algorithm by 0.7% and 0.9%, respectively. The optimal value obtained by the ASA-GA algorithm closely approximates the exact solution of the Gurobi 11.0.2 solver. Figure 11 shows that the Gurobi 11.0.2 solver obtains the optimal value at 560 s, while ASA-GA obtains the optimal solution at 424 s. Consequently, ASA-GA significantly enhances the efficiency of solving the model and obtains a near-optimal solution approximating the exact solution. Therefore, in the following real-world case, we can adopt ASA-GA to produce near-optimal solutions.

5.2. A Real-World Case Study

5.2.1. Description of Real-World Case

Figure 12 depicts the basic conditions of the Beijing Changping Line and Line 13. Among them, the intersection of two lines is at XEQ Station. Changping Line extends from CPX Station in the north to XEQ Station in the south, spanning a total length of 31.9 km and comprising 12 stations. Line 13 spans 40.5 km long and consists of 16 stations. According to the method of zone division explained in Section 2, lines are divided into zones I, II, III, IV, and V.

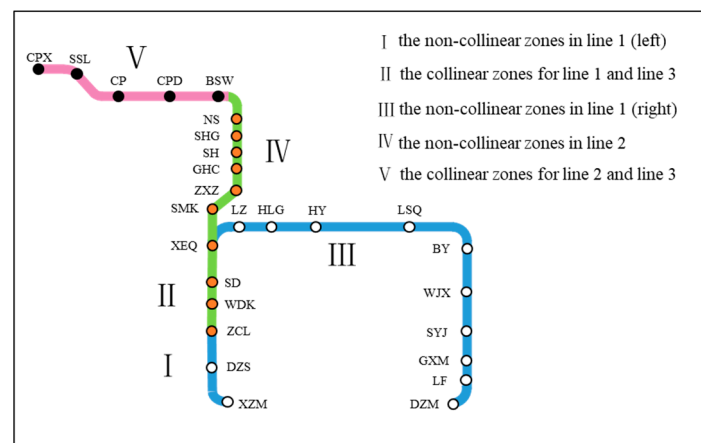


Figure 12. Beijing Rail Transit Changping Line and Line 13.

Based on AFC data of the Beijing subway on a certain day, this study selects the morning peak (8:00–9:00 a.m.) as the study period. The statistics of passenger flow, passenger flow of different lines, and their proportions are shown in Table 5. Among them, the ratio of total direct passengers to transfer passengers is 2.07:1. In addition, transfer passengers in the CPX-XZM direction account for 79.1% of total transfer passengers, so transfer passengers of CPX-XZM are mainly considered. By referring to relevant literature, the settings of parameters related to the train service scheme in this example are shown in Table 6 (Yang et al. [7]).

Table 5. Parameters of lines.

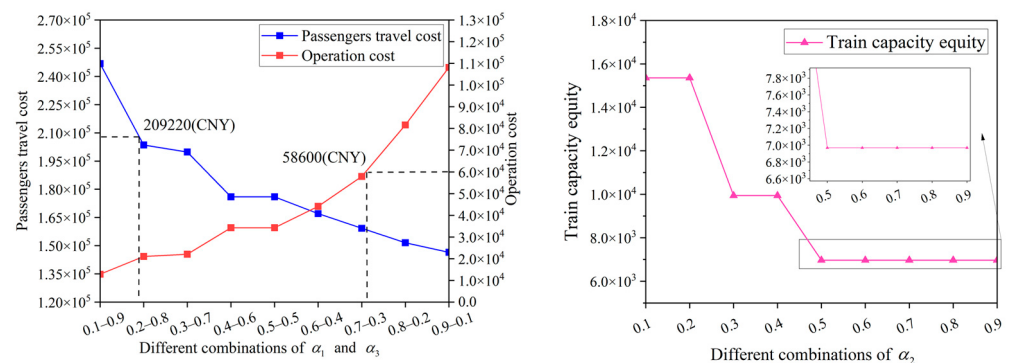
Line	Passenger Volume (prs)	Length (km)	Proportion (%)
XZM-DZM	11,076	40.5	-
CPX-XEQ	8702	31.9	-
CPX-DZM	1994	60.4	20.9%
CPX-XZM	7542	39.9	79.1%

Table 6. The parameter values of the train operation plan.

Parameter	Value	Parameter	Value
Cap	245 prs	num_l	180
t_{walk}	5 min	γ	0.45 CNY/min
λ	16.7 CNY/km	ϕ	4×10^6
ω	40 CNY/min	$H_{min}^{vc}/H_{max}^{vc}$	3 min/15 min
η_{min}/η_{max}	0.2–1.0	t^{vc}	30 s

5.2.2. Weight Coefficient

Our problem consists of passenger travel costs, train capacity equity, and operation costs, which are denoted by α_1 , α_2 , and α_3 , respectively. The train capacity equity weighting factor is less stable as the weight increases. Therefore, our first concern is to determine the value of α_1 , α_3 . At the same time, to avoid affecting the objective value, we set α_2 to 0. In this study, α_1 is set from 0.1 to 0.9 (correspondingly, α_3 is from 0.9 to 0.1), and the results are shown in Figure 13a. With the increase in α_1 , operational costs gradually increase, and passenger travel costs gradually decrease. Operational costs of the SG scheme without CO amount to CNY 58,600, and passenger travel costs amount to CNY 209,220. When α_1 is in interval of 0.2–0.7, the optimized train service scheme can reduce both of them at the same time. Thus, we consider both objective values equally important and set α_1 to 0.5 and α_3 to 0.5.



(a) Variation curves for the parameter α_1, α_3 . (b) Variation curves for the parameter α_2 .

Figure 13. Tuning results of weight factors.

We further calibrate α_2 by going from 0.1 to 0.9. As shown in Figure 13b, as α_2 increases, train capacity equity decreases and remains in the interval of 0.5–0.9. Therefore, we chose 0.5 as the value of α_2 weights. The three weight coefficients are normalized, such that α_1 , α_2 , and α_3 are each assigned a value of 1/3.

5.2.3. The Comparison of Optimized Performance Results

In order to verify the quality of the proposed scheme, we applied the following four different schemes.

- SG-No-CO: single grouping service scheme without cross-line operation.
- SG-CO: single grouping service scheme under the cross-line operation scheme.
- MG-CO: multiple grouping service scheme under the cross-line operation scheme.
- VC-CO: virtual coupling service scheme under the cross-line operation scheme.

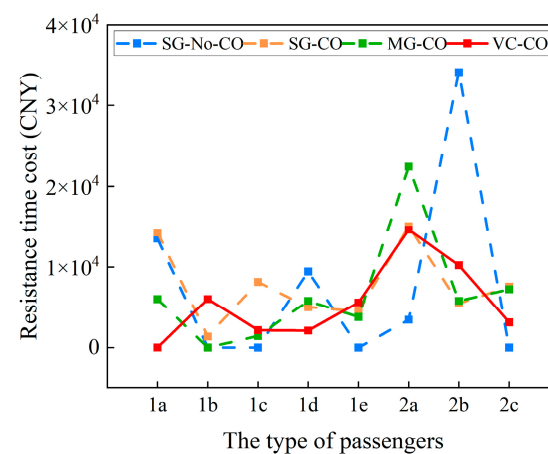
The comparison results are tabulated in Table 7.

Table 7. The table of passenger classification.

Objective (CNY)	SG-No-CO	SG-CO	MG-CO	VC-CO
(1) Passenger travel cost	184,752	186,987	178,017	171,679
residence time cost	59,127	61,362	52,392	43,824
in-vehicle cost	125,625	125,625	125,625	125,625
additional VC time cost	0	0	0	2230
(2) Train capacity equity	45,230	52,680	9934	8778
(3) Operation cost	116,671	134,600	57,005	62,074
usage cost	6720	6840	2452	2794
operating cost	109,951	127,760	54,553	59,280
Objective value	112,895	109,594	81,652	80,843

(1) In terms of passenger travel cost

As shown in Figure 14, compared with SG-No-CO, SG-CO can effectively reduce the passenger residence time cost of case 1d and 2b passengers. Although the resistance time cost of other passengers has increased, the total resistance time cost for all types of passengers has increased by 3.8%. Because the addition of cross-line trains can reduce the passenger transfer time, the SG method is not able to adapt to passenger flow. Further, compared to SG-CO, MG-CO effectively reduces the total resistance time cost by 14.6%. The MG method can reasonably allocate train capacity according to the needs of various passenger types and meet passenger flow demand. Furthermore, under the VC-CO scheme, the total resistance time cost is reduced by 16.4%. This reduction can be attributed to the increase in the maximum number of trains and more flexible train composition and frequency, resulting in a further decrease in the overall resistance time cost.

**Figure 14.** The resistance time cost of different case passengers.

(2) In terms of additional VC time cost

In CO, the uncertainty of the train arrival time can result in an average additional VC time cost for trains on different lines during the coupling process. In terms of the additional VC time cost, SG-No-CO, SG-CO, and MG-CO schemes incur no costs, while only the VC-CO scheme requires CNY 2230 (assuming an average VC time is 30 s). To analyze the variation in objective values under different average VC times, we examined scenarios with average VC times ranging from 10 s to 150 s. Figure 15 illustrates the changes in objective values as average VC times change.

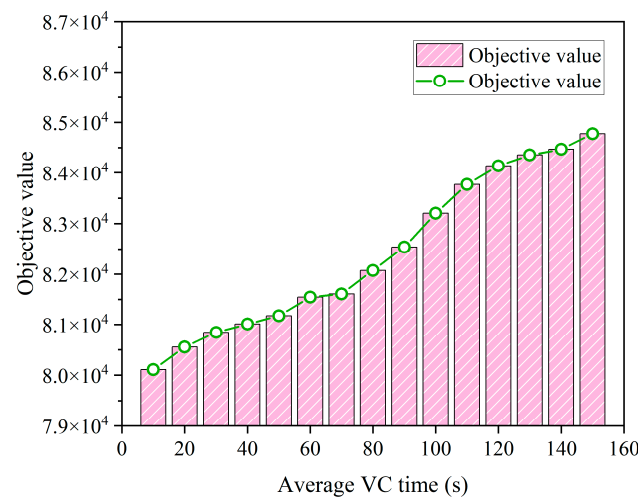


Figure 15. The average VC time.

With the increase in average VC times, objective values also increase gradually. This is because average VC times affect the coupling between different lines. As the average VC times increase, the coupling between different lines will result in higher train travel costs and affect the objective value. The objective value in MG-CO is CNY 81,652. As can be seen in Figure 15, when the average VC time is equal to 80 s, the objective value is 82,074, which is higher than that in MG-CO. Thus, when the average VC time is less than 80 s, VC can play an effective role and improve operational efficiency.

(3) In terms of train capacity equity

Compared with SG-No-CO, train capacity equity under SG-CO has increased by 16.5%, as shown in Figure 16. Regarding CO, using MG-CO and VC-CO can further balance the problem of capacity inequity between different lines by changing the train compositions. Among them, train capacity equity is reduced by 78.0% and 80.1% under MG-CO and VC-CO, respectively. MG-CO can determine the corresponding number of train compositions according to passenger demand between different lines, but the optimization effect needs to be improved due to the limitation of train departure frequency. VC-CO can flexibly couple/decouple trains of different lines, which can more flexibly adapt to passenger flow, thus making train capacity match more with passenger demand and providing a more equitable train service.

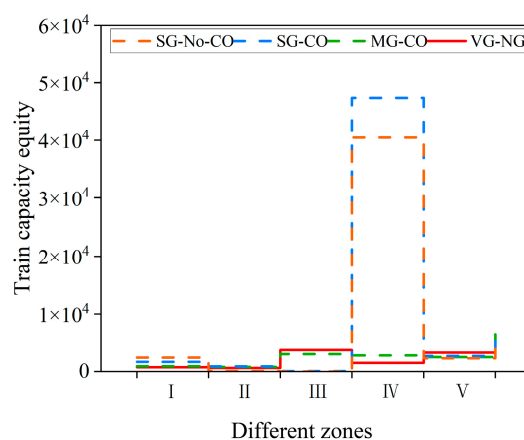
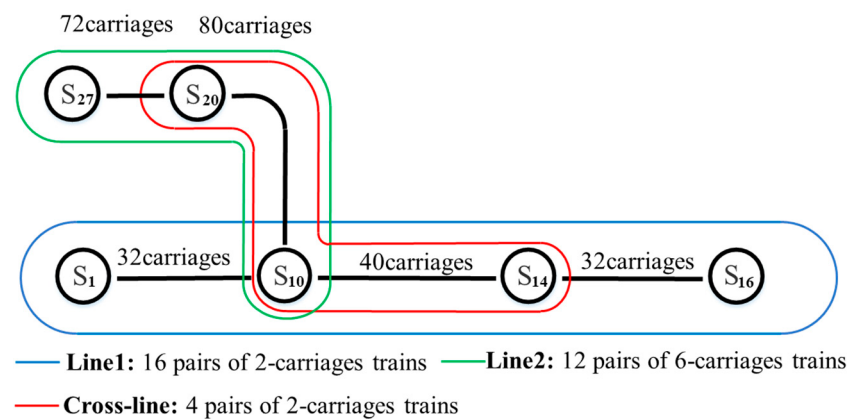


Figure 16. The train capacity equity in different zones.

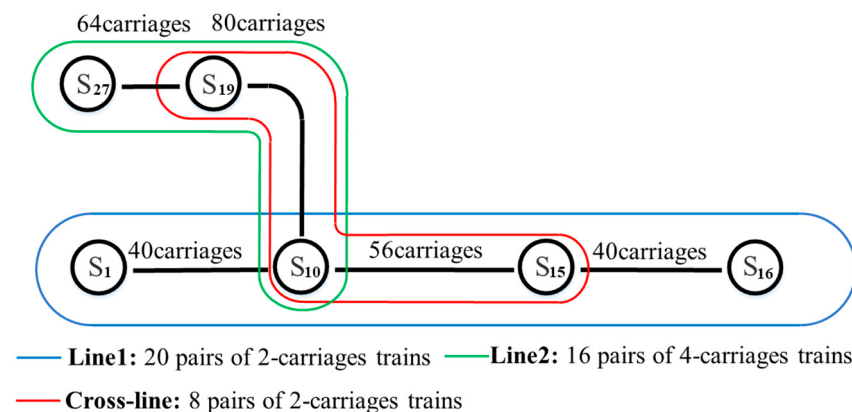
(4) In terms of operation cost

Figure 17 shows the train departure frequency on different lines, the number of train compositions, and turn-back stations under MG-CO and VC-CO. In addition, the minimum

headway is set as 3 min, so the maximum number of trains on one line under MG is 20. However, since VC technology can allow trains to be virtually reconnected into a virtual coupled formation, up to 40 trains can be run on one line. In MG-CO, train 1 and train 2 are operated with two and six carriages, respectively, as shown in Figure 17a. Train 1 and train 2 are operated with 16 pairs and 12 pairs in line 1 and line 2, respectively. Due to the constraints of traditional operating, cross-line trains operate in groups of two carriages per hour with four pairs. As shown in Figure 17b, in VC-CO, train 1 operates in 2 e in the S_1 – S_{10} interval and S_{15} – S_{16} interval, train 1 coupled with train 3 operates with 2 + 2 carriages in the S_{10} – S_{15} interval, train 2 operates with 4 carriages in the S_{19} – S_{27} interval, and train 2 coupled with train 3 operates with 2 + 4 carriages in the S_{10} – S_{20} interval. The VC increases the train departure frequency and improves the traffic capacity of each zone to meet passenger flow demand.



(a) The MG-CO service scheme.



(b) The VC-CO service scheme.

Figure 17. Comparison of train service scheme.

As shown in Table 7, SG-CO increased operation costs by 15.4% compared to SG-No-CO, which is because SG-CO adds cross-line trains. Under MG-CO and VC-CO, cost reductions of 51.1% and 46.8% are observed, respectively. This is because, under MG and VC, the train composition is allocated according to the passenger flow demand. However, compared to MG-CO, VC-CO increased by 8.2%. In Figure 18, it can be seen that the train usage cost and operating cost of VC-CO are increased in line 1 and the cross-line compared to MG-CO; the train usage cost and operating cost of VC-CO are decreased in line 2 compared to MG-CO. This is because, in the case of VC, the train departure frequency on each line can be increased, and the number of carriages on line 2 can be reduced to meet the needs of more passengers.

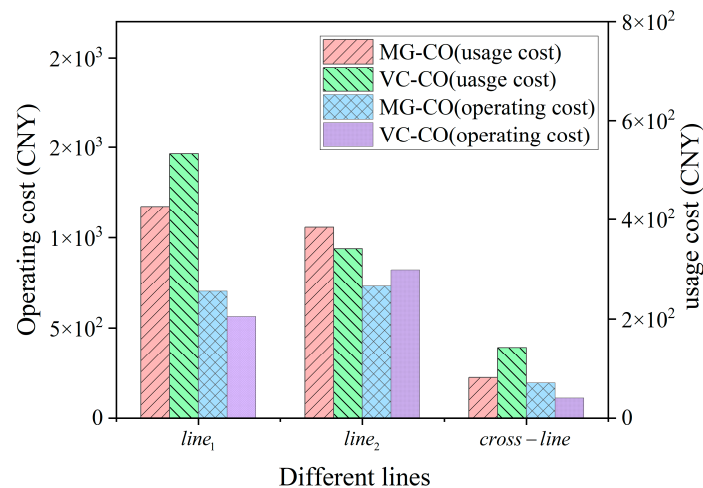


Figure 18. The operational cost of different lines.

Through the above comparison, we conclude that CO can effectively reduce passenger residence time costs. With CO, both MG and VC can further achieve a more balanced capacity allocation, but VG will slightly increase operation costs. Compared with SG and MG, VC technology can operate multiple-line trains in a more flexible manner. Namely, VC can better match the train capacity with passenger flow, increase the frequency of trains as much as possible, and design the number of train compositions in a reasonable way.

6. Conclusions and Future Work

To address non-equilibrium passenger demand more economically, this study investigates the optimization of train service design with CO utilizing VC, whereby the train composition can be changed by coupling/decoupling operations at both ends of the zones. The proposed nonlinear integer programming model aims to minimize passenger travel costs and operation costs while improving train capacity equity. It sets the train departure frequency, train composition, and turn-back stations as decision variables constrained by passenger travel demand, headway, etc. This study proposes an equity index, based on the variance of travel utility for all OD pairs passengers, to measure overall train capacity equity. Furthermore, an improved ASA-GA is proposed to effectively solve the practical case. A set of numerical experiments is conducted on Beijing Changping Line and Line 13 to verify the effectiveness of the proposed model and algorithm. The results demonstrate that the proposed ASA-GA can find a near-optimal solution approximating the exact solution in a shorter computation time, enhancing the algorithm's applicability for model-solving.

A comparison of the train service scheme under the SG-CO scheme with the SG-No-CO scheme reveals that CO can effectively reduce passenger residence time costs. Furthermore, a comparison of the train service scheme under the VC-CO scheme with that of the MG-CO scheme yields the following results: (1) In terms of passenger travel costs, the average passenger residence time decreases by 16.3%. (2) In terms of equity, the passenger travel utility variance decreases by 11.6%, resulting in a more even distribution of train capacity. (3) In terms of operation costs, the number of carriages used increases by 24, effectively meeting passenger flow demands. Based on these findings, this study concludes that VC effectively balances passenger demand and train capacity, providing an effective method to improve equity performance and reduce passenger travel time.

This study identifies three directions for future research. (1) VC can be applied to larger network subway networks, such as "X"-type lines or systems with three or more lines. In such networks, VC implementation offers greater flexibility. However, this expansion necessitates stricter safety requirements, including the determination of VC locations between lines, ensuring operational safety at transfer stations, and optimizing train turnaround. (2) ASA-GA is a heuristic algorithm that can only obtain approximate solutions, suggesting potential for refinement to improve solution accuracy. (3) This

study preliminarily explored a train service scheme using VC in CO. In contrast, the train-scheduling problem is more practical and attractive for URT operation, so it is a promising direction to generate feasible timetables based on the obtained train service scheme.

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