

Article

Modeling the Color Characteristics of Sapphires through the Statistical Method and Function Simulation Method

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Abstract: The aim of this study was to explore the feasibility of quantitatively evaluating a sapphire's color by analyzing gemstone images. The color characteristics of gemstones through photography and color extraction were accurately captured, and the key color parameters for different color regions of gemstones were extracted. Then, the color of sapphires was simulated by statistical method and function simulation method. The results indicate that the brightness value of the bright area was the highest and the chroma value of the "fire" area was the highest. The chroma (C) of the sapphires was linearly positively correlated with the absolute value of its **b** value, and the brightness (L) showed a quadratic correlation with its chromaticity parameters (**b** or **C**). The function simulation method with an average of **b** as the main independent variable had a larger standard deviation, which proves that this method has a stronger ability to distinguish the differences in a sapphire's color. This method can be extended to the color evaluation of other colored gemstones, and it can be used to form a gemstone color evaluation system.

Keywords: sapphire; color evaluation; statistical method; function simulation method



Citation: Liu, F.; Ying, G.; Zhao, B.; Liu, M. Modeling the Color

Characteristics of Sapphires through the Statistical Method and Function Simulation Method. *Appl. Sci.* **2024**, *14*, 8042. <https://doi.org/10.3390/app14178042>

Academic Editor: Nikolaos Koukouras

Received: 7 August 2024

Revised: 31 August 2024

Accepted: 5 September 2024

Published: 8 September 2024



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1. Introduction

As a kind of gem-quality corundum, sapphire is mainly produced in Myanmar, Thailand, Sri Lanka, the United States, Tanzania, China, Australia, etc. [1]. Pure corundum (Al_2O_3) is transparent and colorless or white, but it can appear in multiple colors such as red, blue, green, purple, or black when it contains other trace elements such as Cr^{3+} , Ti^{4+} , Fe^{2+} , Fe^{3+} , Mn^{2+} , V^{5+} , etc. [1,2]. The presence of Cr^{3+} causes a red hue [3–5], while the presence of Fe^{2+} , Fe^{3+} , and Ti^{4+} results in a blue hue [6–10]. The charge transfer between Fe^{3+} and O^{2-} in sapphire results in the absorption of visible light in the blue violet region [6,7]. Currently, the research on corundum gemstones often focuses on the chemical composition [11], the geological environment of their formation [12,13], the chromogenic origin [14], enhancement treatments [15–17], and synthetic methods [18].

The factors affecting the color of sapphire include the physicochemical properties of the gem itself, inclusions, cuts, optical effects [19], cracks, and color zoning [20]. And optical effects include the brilliance, fire, scintillation, and darkness of the gem. Brilliance refers to the light reflected back into the observer's eyes when white light falls on the front of a transparent faceted gem, including the total internal reflection from the pavilion and a small portion of the surface reflection [21]. Fire is the phenomenon where a transparent faceted gem shows spectral colors due to the dispersion that occurs when gem is illuminated by white light. Scintillation refers to the changes in the white and colored light we observe when the gem, light source, or observer moves [22]. Research on the impact of the optical effects on the color of colored gemstones is limited. In colored gemstones, the dispersion of colors other than the body color is suppressed, allowing the observer to see monochromatic fire colors [23]. The optical effects may blend with the gem's body color and affect the

overall color. Jose Sasián believes that, due to the reflection, refraction, and other effects of light, the human eye observes more facets than the actual facets of a gemstone, and these extra facets are called “virtual” facets [19]. These virtual facets present different colors and form “invisible boundary patterns” and “brightness contrast”, which affect the evaluation of a gemstone’s color [24].

The color of the table is observed when evaluating the color of colored gemstones. Transparent, faceted, and colored gemstones can exhibit different colors on each facet due to the reflection and refraction of light. The quantitative color evaluation of faceted, colored gemstones needs to consider the influence of each facet’s color on the overall color. The color of faceted colored gemstones is related not only to the color of the original stone, but also optical effects such as brightness, fire, and darkness. These factors affect the human visual perception system, ultimately influencing the color presented in our brain. At present, the research on the color evaluation of colored diamonds is relatively comprehensive, and some difficulties have also been found. For example, pink diamonds with low saturation in brown or brown shades are often confused with pink diamonds that have orange tones [25]. This indicates that the human eye cannot accurately distinguish the hue of gemstone colors. Two blue diamonds with the same color difference are more likely to be in different color grades than two yellow diamonds with the same color difference [26], and the K-M, N-R, and S-Z color grades of the D-to-Z series diamonds overlap with the Faint, Very Light, and Light levels of yellow diamonds [27].

The colorimetric method is widely used for evaluating the color of gemstone, which operates by comparing the color with color cards or standard gemstones. Han Jiayang et al. demonstrated the feasibility of using the Gemdialogue™ color card to evaluate sapphire colors [28]. Guo Ying et al. conducted semi-quantitative or quantitative evaluations of the colors of gemstones such as jadeite [29,30], ruby [31,32], manganese aluminum garnet [33], emerald [34], tourmaline [35], and citrine [36] in the CIE 1976 $L^*a^*b^*$ uniform color space by color cards for color comparison [37,38] or by measuring color parameters with instruments. GIA and Howard Rubin evaluated the color of gemstones using GemSet and GemDialogue, respectively [39]. The colorimetric method is subjective, and it can only be used for qualitative evaluation. Gem Research SwissLAB (GRS) used colorimetric methods to classify the color of rubies and sapphires into five levels: Intensity, Intensity to Vivid, Vivid, Vivid to Deep, and Deep. Zhang Hui et al. [40] and Wang Rong et al. [41] discussed the application of color measurement methods in the color evaluation of jadeite using colorimeters and fiber optic spectrometers, respectively. Liujun Lin graded the color of sapphires using the GEM-3000 UV-visible spectrophotometer and the K-means algorithm [42]. Therefore, the color of gemstone obtained by this method, ignoring the influence of gemstone optical effects on its color, was closer to the body color of the gemstone.

The above research progress has confirmed the complexity and inaccuracy of evaluating gemstone colors through the human eye. It is necessary to comprehensively consider the impact of potential factors on the optimal evaluation method of faceted blue sapphires. Mao Xiumin et al. selected an optimal lighting source for various gemstones (red, yellow, green, blue, and purple) by taking photos [43]. This demonstrates that taking images of gemstones, digitizing image information, and combining the optical effects of faceted gemstones in a standard evaluation environment to simulate their color could objectively and accurately evaluate the color of gemstones.

The aim of this study was to explore the feasibility of quantitatively evaluating a sapphire’s color by analyzing gemstone images. The quantitative method established in this study can effectively overcome the subjectivity and inaccuracy of the color evaluation by human eye. And this also helps to establish a method and system that is suitable for industrial batch color evaluations of gemstones, as well as provides a theoretical basis for the development of intelligent instruments for the color evaluation of gemstones.

2. Materials and Methods

A total of 50 faceted blue sapphires were prepared, including 25 oval-shaped, faceted blue sapphires (LBR) and 25 pear-shaped, faceted blue sapphires (LBP). These natural blue sapphires were produced in Sri Lanka and heated to make the color more attractive. The size of the sapphire samples LBR01 to LBR14 were 4 mm × 6 mm (oval), while that of LBR31 to LBR40, LBP01 to LBP15, and LBP71 to LBP80 were 3.5 mm × 4.5 mm (oval-shaped), 4 mm × 6 mm (pear-shaped), and 4.5 mm × 5.5 mm (pear-shaped), respectively.

2.1. Manual Color Extraction

The sapphires were placed in a light source box with D65 as the light source. The sapphire images were taken by a Canon EOS 200D which manufacturer was Canon (China), and with a neutral gray color card as the background. And the photograph condition was set as follows: a resolution of 720 dpi, an aperture value of f/8, and an exposure time of 1/200 s. The captured images were removed from the background by Adobe Photoshop CC 2018, and the width was adjusted to 300 pixels with the length changing proportionally. The sapphires' colors were divided into four regions based on the human perception of gemstone colors: brilliance, "fire", body color, and darkness. The color was manually selected in each region by Adobe Photoshop CC 2018 software based on the size of the region and the degree of color change (as shown in Figure 1). For Sample LBR01, Regions 1, 2, and 3 represent the brilliance, with one pixel color extracted from each region as a representative. The light shining on these three facets was mainly reflected to the human eye. But the amount of light reflected to the human eye was different, and the color seen was also different. As shown in Table 1, Numbers 1, 2, and 3 represent these three facets, and their color parameters and simulated color blocks are also listed. Regions 4, 5, 6, 7, 8, and 9 represent "fire", with one pixel color extracted from each region as a representative. Region 10 represents darkness, with one pixel color extracted as a representative. Region 11 represents body color, with one pixel color extracted as a representative.

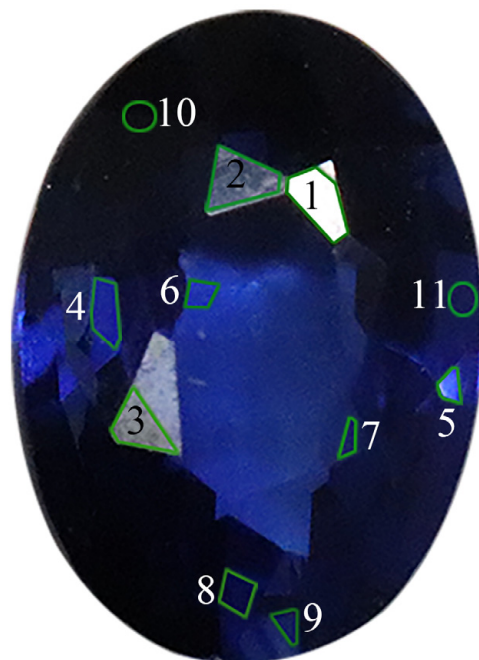


Figure 1. Schematic diagram of the manual color extraction on the Sample LBR01 image.

Table 1. The color parameters of Sample LBR01.

Number	Region	L	a	b	Simulated Color Block
1	Brilliance	100	0	0	
2	Brilliance	34	0	−16	
3	Brilliance	57	−1	−3	
4	Fire	22	30	−62	
5	Fire	53	19	−72	
6	Fire	27	30	−67	
7	Fire	17	12	−35	
8	Fire	7	14	−32	
9	Fire	14	12	−34	
10	Darkness	4	0	−2	
11	Body Color	12	12	−31	

Samples with noticeable optical effects were selected to extract the color parameters and to obtain representative color data. Six oval-shaped blue sapphires were selected (LBR01, LBR02, LBR03, LBR32, LBR33, and LBR34), and 73 sets of color parameters were manually extracted (including L, a, b, C, and h). Eleven pear-shaped blue sapphires were selected (LBP01, LBP02, LBP03, LBP04, LBP05, LBP07, LBP08, LBP14, LBP71, LBP72, and LBP80), and 120 sets of color parameters were manually extracted.

The optical properties of each facet on the gemstones were nearly identical. Theoretically, there was a certain correlation among the color parameters of each pixel in the photographed image of the gemstone. Therefore, 193 sets of manually selected color parameters were subjected to correlation analysis. And correlation analysis was conducted on the color parameters of each region separately due to the significant differences in color among the four regions.

2.2. Color Simulation Model

2.2.1. Principle

According to the program written in MATLAB R2020b, which can read the color parameters of all pixels in a sapphire image, each color region of the faceted blue sapphires was defined by the range of color parameter obtained through manual color extraction. The color parameters obtained were divided into four groups: brilliance, fire, body color, and darkness.

The color chroma of the samples was evaluated by a professional gemstone inspector. And the evaluation result showed that the sapphires in this study exhibited intense blue color, with chroma values (C) greater than or equal to 85. The inspector primarily focused on the most vivid part (the “fire” region) of the gemstone during the evaluation process. Therefore, the “fire” region was chosen as the primary area for simulating gemstone colors. And two methods were employed for color simulation.

Method One (Statistical Method): The average, median, and mode of the **L**, **a**, and **b** values in the “fire” region were selected, respectively. These three sets of data were subjected to color simulation, and the simulated color blocks were compared with the gemstone colors. Since the color parameter reads by the program retains fourteen decimal places, all of the obtained color parameters did not appear to be duplicate values. Therefore, the color parameters undertook the integer process to obtain the mode used in this study.

Method Two (Function Simulation Method): There was a negative linear correlation between the gemstones’ chroma and Parameter **b**. The chroma (C) and **b** in the “fire” region was linearly simulated to obtain the corresponding linear formulas. The mean, median, and mode of **b** were selected separately and substituted into the above formula to calculate the corresponding C value. The average, median, and mode of the selected **b** corresponded to multiple lightness (L), and the average of these L values was calculated as the L value used to simulate the color. The following used LBR01 as an example.

2.2.2. Color Simulation Model (LBR01) Establishment

Step One: The points with a chroma greater than or equal to 85 in the “fire” region of Sample LBR01 were selected, as shown in **Zone A** in Figure 2b and is represented by purple dots. The average, median, and mode of **L**, **a**, and **b** were calculated, as shown in Table 2. The color was simulated using the data in Table 2. This step was both the complete step of the statistical method and the first step of the function simulation method.

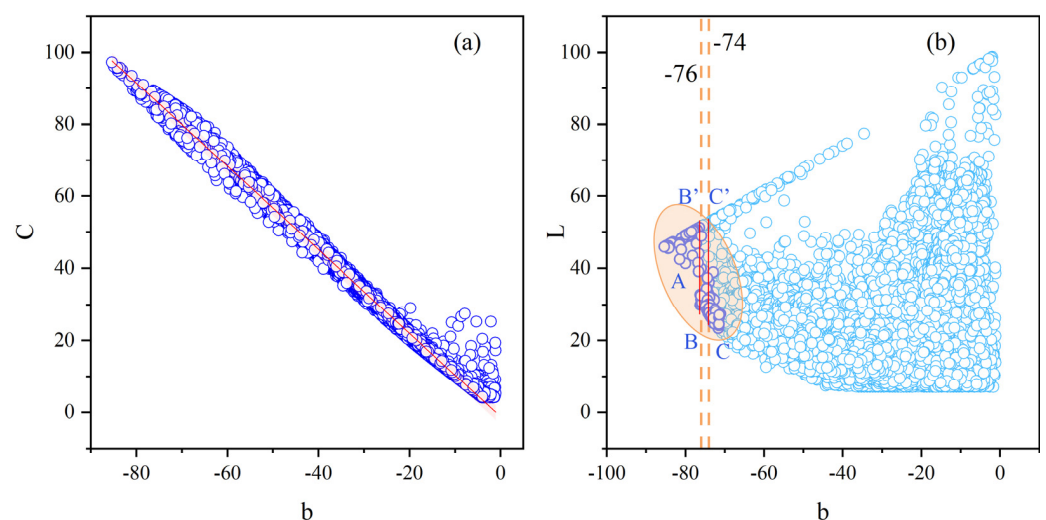


Figure 2. Relationship between the color Parameter **b** and chroma (C) (a), and the distribution of lightness (L) and **b** values for LBR01 (b).

Table 2. Simulation results of the color for Sample LBR01.

	L	a	b	Simulated Color Block	LBR01
Method 1 (Statistical Method)	Average	Average	Average		
	35.11	44.46	−75.55		
	Median	Median	Median		
	29.97	45.56	−74.08		
	Mode	Mode	Mode		
	28.00	46.00	−74.00		
Method 2 (Function Simulation Method)	Average				
	30.71	42.00	−75.55		
	Median				
	29.80	41.13	−74.08		
	Mode				
	29.80	41.08	−74.00		

Step Two: **C** and **b** were linearly fitted (Figure 2a), and the relationship between **C** (chroma) and **b** was as follows:

$$C_{\text{fire}} = -1.20 - 1.16 \times b_{\text{fire}}, R^2 = 0.997.$$

Step Three: The **b** values (average, median, and mode) taken in the first step were substituted into the formula obtained in the second step to obtain the corresponding **C** values. And **C** could be calculated from **a** and **b** by the following formula:

$$C = \sqrt{a^2 + b^2}.$$

Therefore, the corresponding **a** value could be obtained by $a = \sqrt{C^2 - b^2}$.

Step Four: As shown in Figure 2, each **b** value corresponded to multiple **L** values. The line segment **BB'** represented the corresponding **L** when the value of **b** was the average of all **b** values in **Zone A**, and the line segment **CC'** represented the corresponding **L** when the value of **b** were the median or mode of all **b** values in **Zone A**. The average of the **L** values corresponding to each **b** value was calculated and used as the final **L** value. And the color was simulated in Table 2.

3. Results and Discussion

3.1. Color Parameter Distribution

The color parameters were manually extracted, as shown in Figure 3. The regions of the brilliance, fire, body color, and darkness are represented by “L, H, B, D”, respectively. The horizontal axis was represented by “Sample Category-Color Region-Color Parameter”. For example, “LBR-L-L” represents the lightness (L) in the brilliance region of the oval-shaped, faceted sapphires.

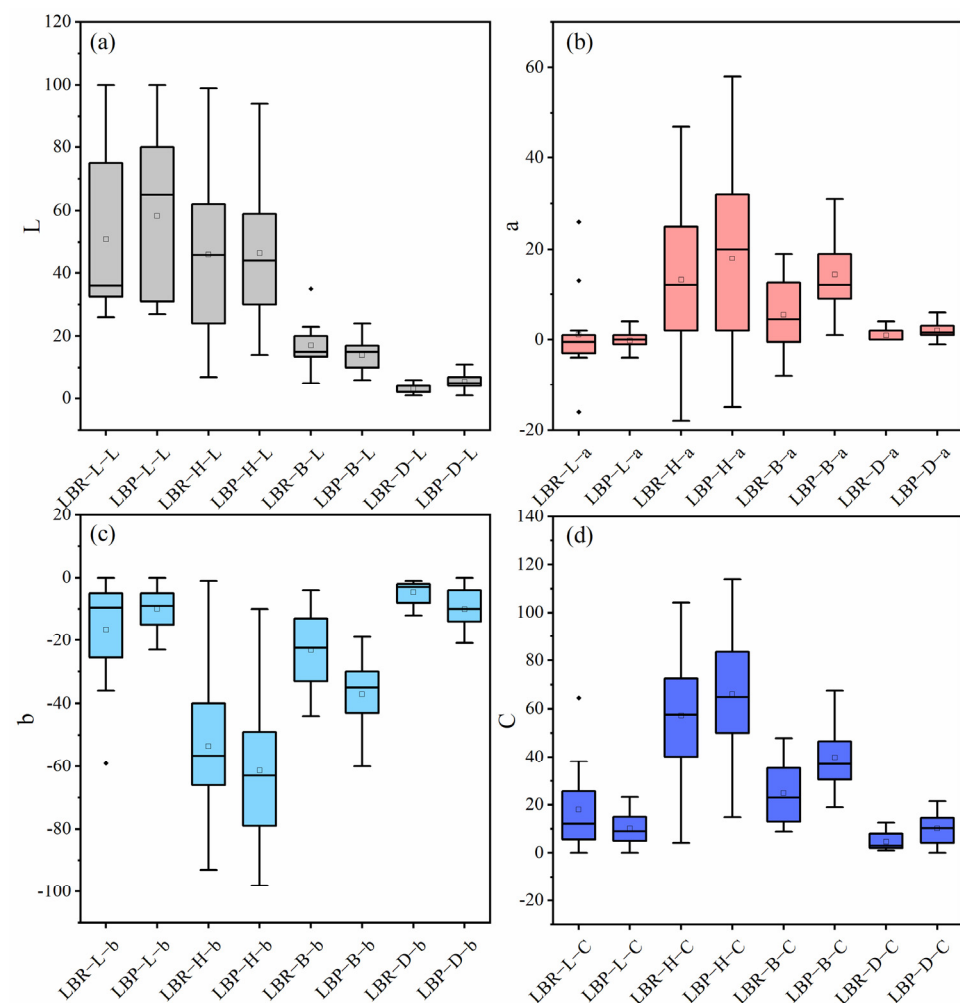


Figure 3. Distribution of the color parameters for the manually extracted colors of sapphires. (a) A box plot of the lightness for each region of the oval- and pear-shaped sapphires; (b) a box plot of the color parameter *a* for each region of the oval- and pear-shaped sapphires; (c) a box plot of the color parameter *b* for each region of the oval- and pear-shaped sapphires; and (d) a box plot of the chroma for each region of the oval- and pear-shaped sapphires.

Among the various color parameters, the lightness value of the brilliance region was the highest, followed by “fire”, then body color, and darkness. The “fire” region had the highest *a* value, followed by the body color, the darkness, and the brilliance. The chroma (*C*) and absolute values of *b* in the “fire” region were the highest, followed by body color, with the darkness and brilliance having similar absolute values of *b* and *C*. In the CIE 1976 $L^*a^*b^*$ uniform color space, there were three coordinate axes, including the L^* axis, a^* axis, and b^* axis. The L^* axis represents lightness, with the black at the bottom end corresponding to a lightness of $L^* = 0$ and the white at the top end corresponding to a lightness of $L^* = 100$. The a^* axis and b^* axis together represent color characteristics, with the positive direction of the a^* axis representing variations toward red (magenta), the

negative direction representing variations toward green, the positive direction of the b^* axis representing variations toward yellow, and the negative direction representing variations toward blue.

3.2. Function Simulation Results

Theoretically, the chroma (C) of the blue sapphires had the greatest correlation with b due to their blue tone. The correlations of C and b , L and b , and L and C of the sapphires were fitted with functions (Figure 4).

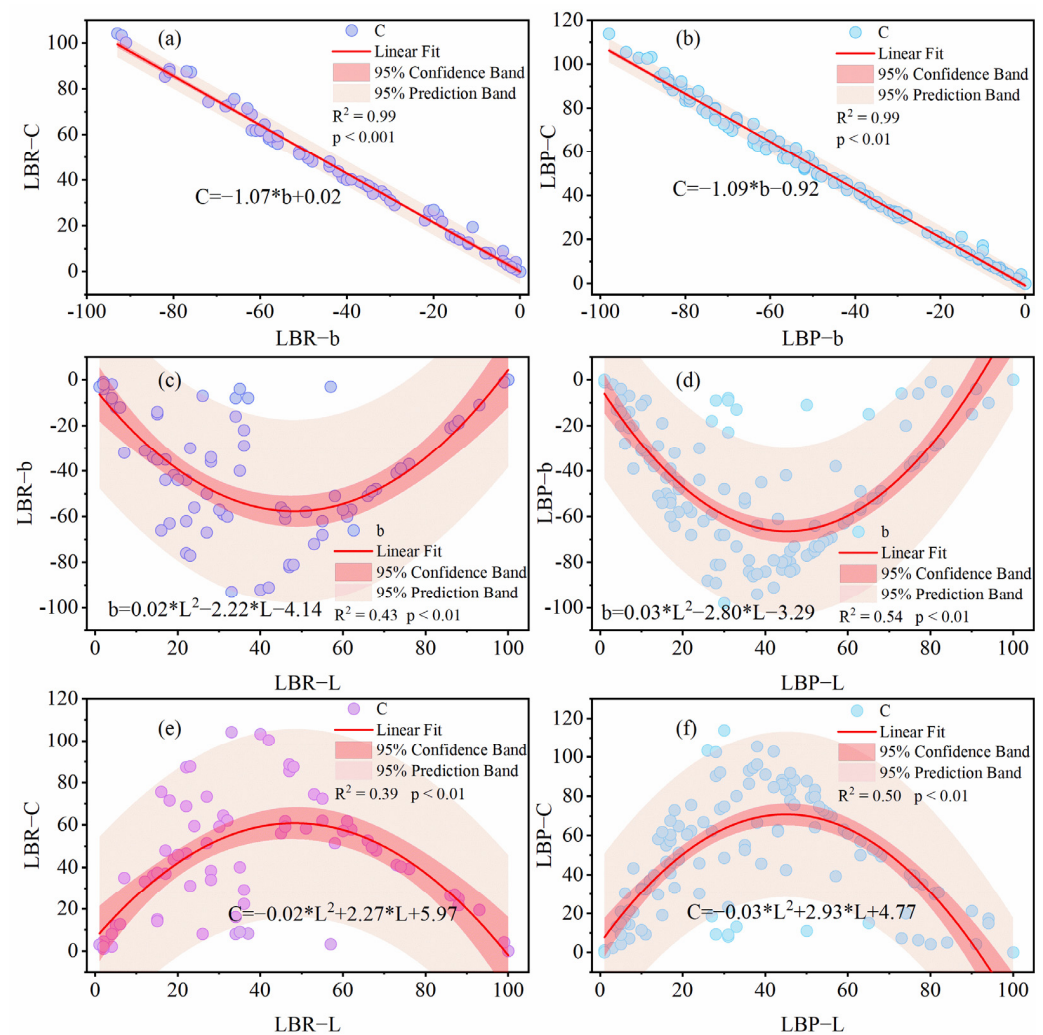


Figure 4. Function fitting of the sapphire color parameters. (a) Linear fitting between b and C for oval-shaped sapphires; (b) linear fitting between b and C for pear-shaped sapphires; (c) function fitting between L and b for oval-shaped sapphires; (d) function fitting between L and b for pear-shaped sapphires; (e) function fitting between L and C for oval-shaped sapphires; and (f) function fitting between L and C for pear-shaped sapphires.

As shown in Figure 4, the chroma (C) and b values of the two sapphire cut shapes exhibited a linear negative correlation, meaning that the chroma (C) was linearly positively correlated with the absolute value of b . The relationship between the chroma (C) and b values of the oval-shaped sapphires conformed to the formula $C_{LBR} = -1.07 \times b_{LBR} + 0.02$, $R^2 = 0.99$ (Figure 4a), while that of the pear-shaped sapphires conformed to the formula $C_{LBP} = -1.09 \times b_{LBP} - 0.92$, $R^2 = 0.99$ (Figure 4b). In the CIE 1976, $L^* a^* b^*$ is the uniform color space, and the negative direction of the b^* axis represents variations toward blue. The

smaller the b value, the larger the absolute value of b in the negative direction of the b^* axis, and the greater the chroma (C).

For both the cut shapes of the sapphires, the lightness (L) showed a quadratic correlation with b and C (Figure 4c–f). As the L increased, the b value first decreased and then increased, meaning the absolute value of b first increased and then decreased (Figure 4c,d). Similarly, as L increased, the C value first increased and then decreased (Figure 4e,f). This rule can be best illustrated by the following extreme example. The color was black, with its chroma of 0, when the lightness was at its lowest. And the color became visible and the chroma increased, until it reached its extremum, as the lightness increased. The lightness continued to increase until 100, the color became white, and the chroma (C) returned to 0. The function fitting results were as follows:

The relationship between the b values and the lightness of the oval-shaped sapphires conformed to the formula $b_{LBR} = 0.02 \times L_{LBR}^2 - 2.22 \times L_{LBR} - 4.14$, $R^2 = 0.43$ (Figure 4c). When $L_{LBR} = 55.5$, the value of b_{LBR} was at its minimum (-65.745), with an L_{LBR} of 55.5. This meant that the blue perception of the sapphires increased with an increase in lightness when the lightness of the oval-shaped sapphires was less than 55.5. When the lightness of the oval-shaped sapphires exceeded 55.5, the blue perception of the sapphires decreased as the lightness increased.

The relationship between the C values and lightness of the oval-shaped sapphires conformed to the formula $C_{LBR} = -0.02 \times L_{LBR}^2 + 2.27 \times L_{LBR} + 5.97$, $R^2 = 0.39$ (Figure 4e). The C_{LBR} was at its maximum (70.38) with an L_{LBR} of 56.75. This meant that the chroma of the sapphires increased with an increase in the lightness when the lightness of the oval-shaped sapphires was less than 56.75. When the lightness of the oval-shaped sapphires exceeded 56.75, the chroma of the sapphires decreased as the lightness increased.

The relationship between the b values and lightness of the pear-shaped sapphires conformed to the formula $b_{LBP} = 0.03 \times L_{LBP}^2 - 2.80 \times L_{LBP} - 3.29$ ($R^2 = 0.54$) (Figure 4d). The value of b_{LBP} was at its minimum (-68.62) with an L_{LBP} of 46.67. This meant that the blue perception of the sapphires increased with an increase in the lightness when the lightness of the pear-shaped sapphires was less than 46.67. When the lightness of the pear-shaped sapphires exceeded 46.67, the blue perception of the sapphires decreased as the lightness increased.

The relationship between the C values and lightness of the pear-shaped sapphires conformed to the formula $C_{LBP} = -0.03 \times L_{LBP}^2 + 2.93 \times L_{LBP} + 4.77$ ($R^2 = 0.50$) (Figure 4f). The C_{LBP} was at its maximum (76.31) with an L_{LBP} of 48.83. This meant that the chroma of the sapphires increased with the increase in lightness when the lightness of the pear-shaped sapphires was less than 48.83. When the lightness of the pear-shaped sapphires exceeded 48.83, the chroma of the sapphires decreased as the lightness increased.

In summary, when b or C reached its extremum, the L value of the oval-shaped sapphires was greater than that of the pear-shaped sapphires. The extremum of C for the oval-shaped sapphires was smaller than that for the pear-shaped sapphires, while the extremum of b for the oval-shaped sapphires was larger than that for the pear-shaped sapphires. The absolute value of the oval-shaped sapphires' quadratic and first-order coefficient were smaller than that of the pear-shaped sapphires' coefficient in the formula for the quadratic correlation between the lightness (L) and the b and C values. This indicates that the chroma and blue perception of the oval-shaped sapphires varied less with lightness. This was because the oval-shaped sapphires had a fourth-order symmetry, while the pear-shaped sapphires had a second-order symmetry.

3.3. Comparison of Methods

The statistical method and function simulation method were used to simulate the color parameters in this study, with both of them dividing into three sub-methods (Figure 5). The colors of the two types of sapphire with different cut shapes were classified as the same color grade in the qualitative evaluation. But the color of each sapphire was not completely identical; instead, they had subtle differences. The purpose of the quantitative

evaluation was to accurately distinguish the sapphire colors and reduce the limitations of the human eye evaluation. The standard deviation of each parameter was calculated (as shown in Figure 5) to observe the degree of dispersion of the color parameter. The method that gained a higher degree of dispersion (larger standard deviation) for color parameters made it easier to distinguish the color of each sapphire, which can be considered the optimal method.

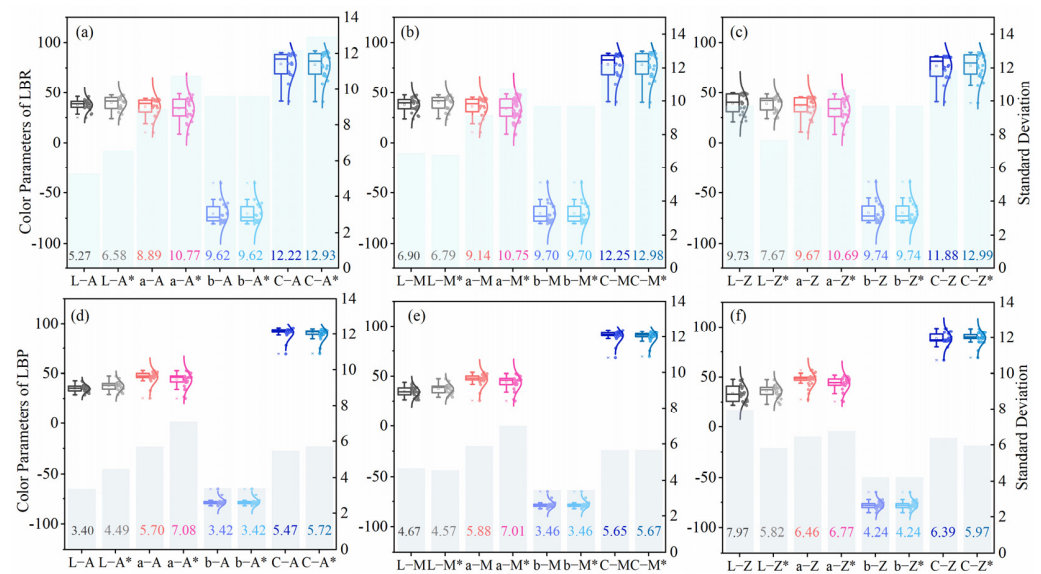


Figure 5. Box plots of the parameter distribution and standard deviation bar charts for the simulated sapphires' color. (a) Box plots of the parameter distribution and standard deviation bar chart for the oval-shaped sapphires' color that were simulated using the average; (b) box plots of the parameter distribution and standard deviation bar chart for the oval-shaped sapphires' color that were simulated using the median; (c) box plots of the parameter distribution and standard deviation bar chart for the oval-shaped sapphires' color that were simulated using the mode; (d) box plots of the parameter distribution and standard deviation bar chart for the pear-shaped sapphires' color that were simulated using the average; (e) box plots of the parameter distribution and standard deviation bar chart for the pear-shaped sapphires' color that were simulated using the median; and (f) box plots of the parameter distribution and standard deviation bar chart for the pear-shaped sapphires' color that were simulated using the mode.

The parameters for simulating the two types of cut sapphire colors are shown in Figure 5, where A, M, and Z represent the average, median, and mode, respectively. For example, L-A, a-A, b-A, and C-A represent the lightness, *a* value, *b* value, and the chroma of the sapphire color obtained using the average of the “fire” region in Method 1 (statistical method), while L-A*, a-A*, b-A*, and C-A* represent these same four parameters in Method 2 (function simulation method). The average of each set of the simulated color parameters was calculated to compare the advantages and disadvantages of the two simulation methods in evaluating color.

The average and standard deviation of each set of *b* values obtained through the function simulation method and statistical method were similar (as shown in Table 3). The negative direction of the *b* value in the color space represents the blue perception. Therefore, the two methods used to simulate sapphire color had similar effects on its primary blue hue. To select the optimal method, it was necessary to observe the average and standard deviation of the other color parameters, namely L, *a*, and C.

Table 3. Comparison of the simulated color parameters for the sapphires.

		L	a	b	C
Average	LBR	L-A (37.94) < L-A* (40.02)	a-A (36.01) > a-A* (34.03)	b-A (−70.23) = b-A* (−70.23)	C-A (79.16) > C-A* (78.31)
		L-M (38.12) < L-M* (40.08)	a-M (36.23) > a-M* (33.92)	b-M (−70.01) = b-M* (−70.01)	C-M (78.45) > C-M* (78.06)
		L-Z (39.45) > L-Z* (38.96)	a-Z (36.45) > a-Z* (33.64)	b-Z (−69.45) = b-Z* (−69.45)	C-Z (77.00) < C-Z* (77.43)
	LBP	L-A (35.40) < L-A* (37.61)	a-A (46.72) > a-A* (43.93)	b-A (−78.12) = b-A* (−78.12)	C-A (91.21) > C-A* (89.79)
		L-M (34.77) < L-M* (37.59)	a-M (47.04) > a-M* (43.90)	b-M (−78.10) = b-M* (−78.10)	C-M (90.71) > C-M* (89.75)
		L-Z (34.00) < L-Z* (36.32)	a-Z (47.52) > a-Z* (43.50)	b-Z (−77.56) = b-Z* (−77.56)	C-Z (88.44) < C-Z* (89.09)
Standard Deviation	LBR	L-A (5.27) < L-A* (6.58)	a-A (8.89) < a-A* (10.77)	b-A (9.62) = b-A* (9.62)	C-A (12.22) < C-A* (12.93)
		L-M (6.90) > L-M* (6.79)	a-M (9.14) < a-M* (10.75)	b-M (9.70) = b-M* (9.70)	C-M (12.25) < C-M* (12.98)
		L-Z (9.73) > L-Z* (7.67)	a-Z (9.67) < a-Z* (10.69)	b-Z (9.74) = b-Z* (9.74)	C-Z (11.88) < C-Z* (12.99)
	LBP	L-A (3.40) < L-A* (4.49)	a-A (5.70) < a-A* (7.08)	b-A (3.42) = b-A* (3.42)	C-A (5.47) < C-A* (5.72)
		L-M (4.67) > L-M* (4.57)	a-M (5.88) < a-M* (7.01)	b-M (3.46) = b-M* (3.46)	C-M (5.65) < C-M* (5.67)
		L-Z (7.97) > L-Z* (5.82)	a-Z (6.46) < a-Z* (6.77)	b-Z (4.24) = b-Z* (4.24)	C-Z (6.39) > C-Z* (5.97)

Except for the lightness (**L**) of the oval-shaped sapphires simulated using the mode, the lightness average of the sapphire simulated by the function simulation method was higher than that simulated by the statistical method. Therefore, the simulated color by the function simulation method had higher lightness. The average of the **a** values of the sapphire color (both oval- and pear-shaped) simulated by the function simulation method was higher than those simulated by the statistical method. Except for the chroma (**C**) of sapphire color (both oval- and pear-shaped) simulated by the mode, the chroma's average of the sapphire simulated by the function simulation method was lower than that simulated by the statistical method. This indicates that the simulated color by the statistical method was more saturated.

The standard deviations of the various parameters for the oval-shaped sapphires were generally greater than those for the pear-shaped sapphires, indicating that the color differences in the oval-shaped sapphires were larger than those in the pear-shaped sapphires. The standard deviation of the **b** value obtained from the two methods was similar when using the average, median, or mode as the representative value. This indicated that the influence of the two methods on the sapphire's primary hue (blue) was similar. The standard deviation of the **a** value simulated by the function simulation method was greater than that of the **a** value calculated by the statistical method. This showed that the functional simulation method was better than the statistical method in distinguishing red and green hues. The standard deviation of the chroma was larger for the function simulation method than for the statistical method except for the chroma (**C**) of the pear-shaped sapphires that were simulated using the mode. The standard deviation of the lightness for the function simulation method was smaller than that for the statistical method for the other simulated sapphire color, except for the lightness (**L**) of both the oval- and pear-shaped sapphires that were simulated using the average.

Among the six sub-methods, the standard deviation of the function simulation method based on the **b** average values was all greater than that of the statistical method. Therefore, the function simulation method using the average of **b** as the representative was able to maximize the dispersion of the obtained sapphire's color parameters (**L**, **a**, **b**, and **C**), which was considered the optimal method. In the process of evaluating the gemstone color with the human eye, it could only be divided into 3–5 levels due to the insensitivity of the human eye to subtle color changes, which is conducive to ensuring the accuracy of the evaluation results. The function simulation method has the ability to grade and evaluate the color of gemstones with minimal color differences, which is beyond the capabilities of the human eye. Moreover, the evaluation results of the function simulation method have better stability and reliability compared to human eyes.

4. Conclusions

(1) The lightness in the brilliance region was the highest, followed by the “fire”, the body color, and the darkness regions. The “fire” region had the highest **a** value, followed by the body color, then the darkness, and finally the brilliance regions. The absolute value of **b** and the chroma (**C**) in the “fire” region were the highest, followed by the body color, with the darkness and brilliance regions having similar absolute values of **b** and chroma (**C**).

(2) The chroma (**C**) of the sapphires was linearly negatively correlated with the **b** value. The lightness (**L**) showed a quadratic correlation with both **b** and **C**. The **L** values of the oval-shaped sapphires when **b** or **C** reached their extremums were greater than those of the pear-shaped sapphires. The color of the oval-shaped, faceted sapphires was able to achieve the highest saturation, higher than even that of the pear-shaped, faceted sapphires.

(3) The average and standard deviation of each set of the **b** values obtained through the function simulation method and statistical method were similar, which indicated that the two methods had similar effects on the primary hue of the sapphires. Only the function simulation method using the average as the representative value to simulate the sapphire color was able to maximize the dispersion of the obtained sapphire parameters (**L**, **a**, **b**, and **C**), which was considered the optimal method.

The function simulation method (using the average of **b** as the representative) provided a reference for the quantitative evaluation of other faceted gemstones' colors. In the future, a software and hardware facility system may be designed that can evaluate the various aspects of gemstones combined with technologies such as AI and VR, including color, clarity, cut, and weight. And this study provides intuitive and accurate evaluation tools for gemstone evaluators through a highly simulated virtual environment.

Author Contributions: Conceptualization, G.Y. and F.L.; methodology, G.Y. and F.L.; validation, F.L., B.Z. and M.L.; formal analysis, G.Y. and B.Z.; writing—original draft preparation, G.Y., F.L., B.Z. and M.L.; writing—review and editing, G.Y. and F.L.; visualization, B.Z. and F.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Acknowledgments: We would like to thank the laboratory of the School of Gemmology of the China University of Geosciences (Beijing) for its technical support for this paper, as well as the laboratory teacher Yuan Ye and the laboratory assistants for their help.

Conflicts of Interest: The authors declare no conflicts of interest.

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