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Enhanced Feature Selection via Hierarchical Concept Modeling

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Abstract: The objectives of feature selection include simplifying modeling and making the results more understandable, improving data mining efficiency, and providing clean and understandable data preparation. With big data, it also allows us to reduce computational time, improve prediction performance, and better understand the data in machine learning or pattern recognition applications. In this study, we present a new feature selection approach based on hierarchical concept models using formal concept analysis (FCA) and a decision tree (DT) for selecting a subset of attributes. The presented methods are evaluated based on all learned attributes with 10 datasets from the UCI Machine Learning Repository by using three classification algorithms, namely decision trees, support vector machines (SVM), and artificial neural networks (ANN). The hierarchical concept model is built from a dataset, and it is selected by top-down considering features (attributes) node for each level of structure. Moreover, this study is considered to provide a mathematical feature selection approach with optimization based on a paired-samples *t*-test. To compare the identified models in order to evaluate feature selection effects, the indicators used were information gain (IG) and chi-squared (CS), while both forward selection (FS) and backward elimination (BS) were tested with the datasets to assess whether the presented model was effective in reducing the number of features used. The results show clearly that the proposed models when using DT or using FCA, needed fewer features than the other methods for similar classification performance.

Keywords: formal concept analysis; feature selection methods; hierarchical concept model; the paired-samples *t*-test; classification

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1. Introduction

At present, big data are often confronted, and dataset sizes have increased dramatically in machine learning classification applications. Feature selection is typically applied in these classification tasks. The aim of feature selection is to improve the predictive performance of classifiers by removing redundant or irrelevant features. A single feature subset to generate a binary classifier tends to have features that are useful for distinguishing these specific classes but useless for distinguishing others [1–3]. Feature selection is a common task in applications of pattern recognition, data mining, and machine learning since it can help improve prediction quality, reduce computation time, and allow building more human-understandable models. Thus, it is used in many applications before training a classifier [3]. However, while there are a lot of state-of-the-art approaches for feature selection in standard feature space [4], only a few approaches for feature selection in hierarchical feature space have been proposed in the literature [1,5].

Hierarchical modeling is one approach to feature selection for enhanced predictive performance of classifiers. It is not only advantageous for identifying the hierarchical model itself but is also helpful for selecting a feature subset for each node [1,5–8]. Classes in a hierarchical structure have both parent–children relationships and sibling relationships [2,9,10]. Moreover, classes with a parent–children relationship are similar to each other and may share common features for classification, while distinguishing between classes with a sibling relationship may require different features [1,8,11]. The authors developed a method for joint feature selection and hierarchical classifier design using genetic algorithms. Thus, this work focuses on using hierarchical structure in feature selection to improve the predictive performance in classification. In this study, we designed a feature selection algorithm based on the hierarchical information structure.

The hierarchical models were built by using two strategies: bottom-up and top-down. The bottom-up strategy starts by placing each object in its own cluster and then merges these atomic clusters into larger and larger clusters until all the objects are in a single cluster or until some other termination conditions are satisfied. Later, the top-down strategy performs the reverse by starting with all objects in one cluster. It subdivides the cluster into smaller and smaller pieces until each object forms a cluster on its own or until some other termination conditions are met, such as a desired number of clusters or that the distance between the two closest clusters is above a certain threshold [12–14]. Thus, this work applied both bottom-up and top-down approaches, which were performed with formal concept analysis (FCA) and a decision tree, respectively. Both FCA and the decision tree approach were considered for feature selection in order to reduce the attribute count while still retaining the efficiency of data classification.

In the context of feature selection, the hierarchical approach provides distinct advantages over non-hierarchical or flat methods, particularly when applied to complex datasets where relationships among features are not uniform. Traditional feature selection techniques, while effective for certain tasks, often operate in a flat structure and may overlook nuanced, multi-level relationships between features. Hierarchical models, such as those using FCA and DT, address these limitations by organizing features in a structured, multi-level manner that reflects inherent data patterns more accurately. Thus, FCA is advantageous for building hierarchical feature selection models because it organizes features based on generalization and specialization relationships. This lattice-based hierarchy helps identify clusters of related features and isolate distinctive attributes within classes. FCA's structured approach reduces noise and redundancy more effectively than flat methods, as it respects the hierarchical dependencies among features. Meanwhile, the DT model is well suited for hierarchical feature selection due to its top-down approach, which prioritizes attributes based on their classification power at each level. By constructing a hierarchy where critical features emerge at the root levels, DT minimizes the feature space efficiently. This targeted selection enables better classification performance with fewer attributes, particularly in cases where some features are only relevant at certain levels within the dataset hierarchy. For these reasons, hierarchical models can better manage complex data with interrelated features. FCA's logical structure and DT's decision nodes allow these models to adapt to varying feature relevance across levels, providing more accurate classification outcomes than flat selection methods. This adaptability is particularly beneficial in tasks where features exhibit varying importance depending on context or subclass.

FCA [15–18], invented by Rudolf Wille in 1982, is a method for data analysis based on concept lattice. It is widely used in information science to describe attributes and objects of information that can be represented in a hierarchical structure. FCA provides the relationships for generalization and specialization among concepts in the concept lattice. This method is rarely applied to feature selection in classification or other tasks. This study applied the advantage of general knowledge through calculation and selection of a root node in the hierarchical structure. The alternative approach of decision trees uses these powerful and popular tools for classification and prediction. Such classifier has a tree structure, where each node is either a leaf node or a branching decision node [11,19]. This

approach is widely used to perform feature selection. The advantages of decision trees are intuitive appeal for knowledge expression, simple implementation, and high classification accuracy. Thus, this work applied both FCA and decision trees to reduce the number of features that need to be collected and also to provide better classification accuracy. Due to the different patterns of selecting general knowledge in both FCA and decision trees in hierarchical concept structure, FCA will be constructed a concept lattice structure based on logic and set theory while decision trees will be built a tree structure based on information gain. Moreover, the decision tree in this study is considered to provide a mathematical selection of the features with the optimization level based on paired-samples *t*-test. Next, the proposed models were tested and evaluated by using three popular algorithms for classification—namely, decision tree, support vector machine (SVM), and artificial neural network (ANN)—to assess the predictive performances with the presented approach using data from the UCI repository.

This article is organized as follows. Section 2 briefly describes the machine learning techniques applied in this work. Section 3 presents the research methodology, followed by the experimental results and discussion in Section 4. Finally, Section 5 concludes the article.

2. Related Works

Feature selection methods have been used for a wide variety of applications [1,2]. Such methods proceed to eliminate unfavorable features that might be noisy, redundant, or irrelevant and could penalize the performance of a classifier [2]. Feature selection thus contributes to a reduction in the dimensionality of data and a restriction of the inputs, which can contain missing values in one or several features [1,4]. The various feature selection methods fall into three main categories: filters, wrappers, and embedded methods [2–4,20]. The first category of filter methods selects the features by using weights that indicate correlations between each feature and a class. The largest weights are selected in rank order. The calculated weights may be based on information gain [1,3,20], chi-square [21,22], or other saliency measures. The second category is wrapper methods that evaluate feature subsets by directly using a learning algorithm and selecting features based on their impact on the model's performance [3,20]. While this appears reasonable as a direct approach, it tends to be too computationally expensive. Finally, embedded methods incorporate feature selection within the learning algorithm during the training process [1–3]. Currently, several studies have explored innovative methods to improve learning frameworks, particularly in meta-learning, robustness, and advanced classification strategies. For instance, Liu et al. investigated the adaptability of model-agnostic meta-learning (MAML) in NLP applications, providing insights into optimizing task generalization [23]. Similarly, the robustness of learning frameworks was examined in adversarial scenarios by Yang et al., emphasizing the importance of reliability in sensitive domains like healthcare [24]. Additionally, Ju et al. introduced hypergraph-enhanced semi-supervised classification, highlighting the utility of complex relational modeling [25]. These works provide valuable insights that contextualize and contrast our hierarchical feature selection approach. In prior studies, many feature selection methods were developed to identify the most relevant and informative features from a dataset. Selecting the right set of features can improve model performance, reduce overfitting, enhance interpretability, and speed up the learning process. This study focuses on applying a hierarchical concept model to select the features from available data.

In recent years, feature selection based on a hierarchical information structure has been proposed because of its rational learning inside the structure. Zhao et al. [26] proposed a hierarchical feature selection framework by considering the parent–child relationship, sibling relationship, and family relationship. These relationships were modeled and implemented using a data matrix concept [5]. Tuo et al. [13] presented the hierarchical feature selection with subtree-based graph regularization by exploring two-way dependence among different classes. Zhao et al. [5] designed a feature selection strategy for hierarchical classification based on fuzzy rough sets to compute the lower and upper approximations of classes organized in a class hierarchy. The authors developed an efficient hierarchical

feature selection algorithm based on sibling nodes [5]. Huang et al. [27] proposed a feature selection framework based on semantic and structural information of labels for hierarchical feature selection by transforming to semantic regularization and adapted the proposed model to a directed acyclic graph case. Liu et al. [28] proposed a robust hierarchical feature to reduce the adverse effects of data outliers and learn relatively robust and discriminative feature subsets for hierarchical classification.

Hierarchical feature selection has gained attention due to its effectiveness in managing structured data relationships and optimizing feature sets within multi-level classification tasks. Conventional feature selection methods, like filter, wrapper, and embedded approaches, often struggle to capture the complex, layered interactions between features in datasets where hierarchical relationships exist. This has led to the development of hierarchical feature selection techniques, among which FCA and DT have been widely applied. While FCA and DT form a foundational basis for hierarchical selection, other methods have emerged, such as fuzzy rough sets and multi-granularity clustering structures, which provide additional insights. For instance, the fuzzy rough sets method combines the robustness of rough set theory with the flexibility of fuzzy logic, allowing for the handling of uncertain or imprecise data in a hierarchical format. Studies show that fuzzy rough sets are particularly effective for data with varying levels of uncertainty across different hierarchical layers, making them well suited for classification tasks where feature relevance may be ambiguous or context-dependent [5]. The multi-granularity clustering structures approach organizes features at different granularity levels, allowing for a flexible clustering of features based on varying levels of detail. Multi-granularity structures can adapt to both broad and specific feature relationships, providing a more nuanced hierarchy than flat clustering methods. They have shown promise in applications requiring both high-level summaries and detailed feature differentiation. These alternative methodologies highlight the flexibility of hierarchical models in capturing feature dependencies across multiple layers, further emphasizing the limitations of traditional, non-hierarchical methods in structured classification tasks [12]. Recent advancements have explored hybrid methodologies that combine hierarchical approaches with other selection techniques to enhance the adaptability and performance of feature selection [16]. The hybrid hierarchical-filter methods such as information gain or chi-square allow for the quick preliminary reduction of features before building a hierarchical structure. This hybrid approach leverages the strengths of each method, achieving both computational efficiency and structured feature prioritization. Hybrid methods incorporating fuzzy logic into hierarchical models, such as fuzzy FCA, enhance flexibility in handling complex and imprecise feature relationships. This advancement allows for more nuanced feature selection in datasets where feature relevance is context-dependent or fluctuates across hierarchical levels.

All the above-mentioned studies assumed specific relationships between categories for hierarchical regularization. However, most of the existing hierarchical feature selection methods are not robust when dealing with the inevitable data outliers, resulting in a serious inter-level error propagation problem in the classification process that follows. Thus, FCA is applied in this current study because the previous experimental studies have shown that FCA is not sensitive to outliers [7]. FCA provides a well-defined mathematical framework to discover implicit and explicit knowledge in an easily understood format by using formal context and a Hasse diagram that clearly represents the concepts' generalization/specialization relationships. In addition, FCA can construct an informative concept hierarchy providing valuable information on various specific domains. To support this work, Trabelsi et al. [15] also applied the FCA method to present a new filter for feature selection, called H-Ratio, which can identify pertinent features from data based on the Shannon entropy, also known as the diversity Shannon index, which reflects how many different types there are in a dataset to select features without considering classification accuracy. However, the classification accuracy is an important aspect of selection. Thus, this work applied the classification accuracy to formulate for feature selection from concept lattice.

3. The Proposed Models for Feature Selection

FCA uses a concept lattice structure to systematically organize features based on their generalization and specialization relationships. By identifying clusters of features, FCA can pinpoint attributes that contribute significantly to classification accuracy. In this hierarchical structure, each level of the concept lattice corresponds to a specific set of feature combinations that add distinct value to the classification task. Features that do not contribute to these essential relationships are pruned, allowing FCA to retain a streamlined set of features that still preserves the necessary classification information. This lattice-based approach reduces the dimensionality of the feature set without compromising accuracy, as FCA naturally filters out features that may add noise or redundancy to the model. The retained features are those that contribute most to accurate classification outcomes, ensuring that the reduced feature set is both efficient and effective.

The proposed approaches based on hierarchical models focused on two methods, namely FCA and DT. FCA and DT were applied to build the hierarchical structures for feature selection from data so that each node in the structure represented a feature (attribute). The choice of FCA and DT for hierarchical feature selection was motivated by their ability to structure features in a multi-level hierarchy. Unlike traditional flat feature selection methods, which may overlook complex relationships between features, FCA and DT enable the preservation of feature dependencies across levels. This hierarchical organization allows for more informed feature selection, enhancing the model's classification accuracy and interpretability, especially in datasets with intricate class structures. An overview of the proposed models is depicted in Figure 1.

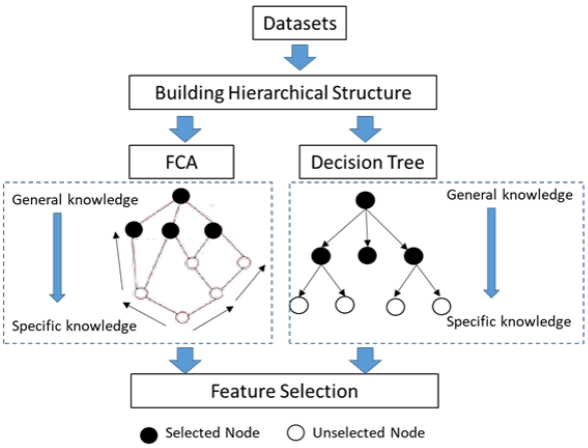


Figure 1. A conceptual overview of the proposed models.

In developing the hierarchical models for feature selection using FCA and DT, we make the following assumptions: (1) We assume that the dataset contains hierarchical relationships among features that can be effectively captured through a concept lattice or decision tree structure. This structure is assumed to reflect generalization and specialization among features, allowing us to reduce feature dimensionality meaningfully. (2) Each level in the hierarchical model is assumed to contain features that independently contribute to classification accuracy. Redundant features within a level are minimized under the assumption that the FCA and DT structures accurately filter irrelevant or less useful attributes. (3) It is assumed that the feature contributions to classification accuracy remain consistent across the hierarchy. This means that a feature's impact at one level should either be retained or improved at higher levels. This allows us to optimize feature selection sequentially through the hierarchy.

Initially, the hierarchical structure form was built, where the top node represents general knowledge, whereas the bottom represents specific knowledge. Next, the top node of structure was chosen by considering each level of structure. In this study, we selected a

feature for each level of the structure, and then the selected feature was used to represent data for classification in the next step of evaluating the performance.

Afterward, the relationship between the number count of features (attributes) and classification accuracy was considered. We found that the number of attributes for each level affects classification accuracy at some level. We attempted to optimize the feature selection from the hierarchical structure. Thus, the paired-samples *t*-test was applied to choose appropriate features for each level. This approach has advantages, including the estimation of mean differences between paired observations and the identification of statistically significant changes in two related measurements. The paired design helps manage individual differences, minimizing the impact of confounding variables and optimizing data economy by leveraging the paired nature of observations. Furthermore, it serves as an efficient method for testing differences between two related conditions or measurements, offering a straightforward and practical means for comparing means within the same group. We performed feature selection based on a hierarchical structure as follows.

On level l of hierarchical structure, for any $l \in \{1, 2, \ldots, k\}$, we defined the total accuracy as follows.

$$Total\,Acc_l = \sum_{i=1}^{n} \frac{Acc_{l,i}}{n} \tag{1}$$

where $Acc_{l,i} \in \{0, 1\}$ is the ith accuracy on level l of the decision tree, and the values 1 and 0 mean that prediction of $Acc_{l,i}$ is true or false, respectively. To find the set of chosen levels, we denoted by $SoCL$ where level 0 is definitely chosen. Next, we consider sequentially the next level of decision tree by using the two-samples z test between total accuracy on level l and $l-1$ defined as

$$\begin{aligned} Z_{l,l-1} &= \frac{Total\,Acc_l - Total\,Acc_{l-1}}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n} + \frac{1}{n}\right)}} \\ &= \sqrt{\frac{n}{2\hat{p}\hat{q}}}(Total\,Acc_l - Total\,Acc_{l-1}) \end{aligned} \tag{2}$$

where $\hat{p} = \frac{\sum_{i=1}^{n} Acc_{l,i} + \sum_{i=1}^{n} Acc_{l-1,i}}{n+n}$ and $\hat{q} = 1 - \hat{p}$ for any $l \in \{1, 2, \ldots, k\}$. The level l will be added to set $SoCL$ depending on the level of significance α. This means we use α to consider the significant difference between total accuracy of level l and $l-1$. If the total accuracy of level l is significantly different from the total accuracy of level $l-1$, we will add l into the set $SoCL$. Otherwise, we do not add l into the set $SoCL$ and stop the level finding process. Table 1 shows some values of the significance level α and their corresponding confidence intervals for z.

Table 1. The mapping from α to confidence interval for z.

α	0.01	0.05	0.1
z	$-2.576 \leq z \leq 2.576$	$-1.96 \leq Z \leq 1.96$	$-1.645 \leq z \leq 1.645$

For example, when we consider the level of significance α=0.01, if $|Z_{(l,l-1)}| \leq 2.576$, then we add l to the set $SoCL$. Otherwise, l is not added to set $SoCL$, and we stop the process. We denote the set of chosen attributes by $SoCA$. If m is the last level added to the set $SoCL$, we have set $SoCL = \{0, 1, 2, \ldots, m\}$ and set $SoCA = \bigcup_{i=0}^{m} Attr_i$, where $Attr_i$ is the set of all attributes on level i. We present the proposed feature selection algorithm using FCA and decision tree, presented as Algorithm 1.

Algorithm 1: Feature selection for FCA and decision tree.

Input : The training dataset comprises all attributes and n instances.
: Tree in decision tree or concept lattice including k level.
: α and z // for example: $\alpha = 0.01, z = [-2.576, 2.576]$
Output : Set of selected attributes ($SoCA$).
Method :

1. $SoCL = \{0\}, l = 0$
2. $SoCA = \{Attr_l\}$
3. $TotalAcc[l] = \sum_{i=1}^{n} \frac{Acc_{l,i}}{n}$ // Accuracy of using selected attributes in level l
4. For $++l$ to k // l=1 to k
5. $\hat{p} = (\sum_{i=1}^{n} Acc_{l,i} + \sum_{i=1}^{n} Acc_{l-1,i})/(n+n)$
6. $\hat{q} = 1 - \hat{p}$
7. $Z_{l,l-1} = \sqrt{\frac{n}{2\hat{p}\hat{q}}}(TotalAcc[l] - TotalAcc[l-1])$
8. If ($|Z_{(l,l-1)}| \leq 2.576$)
9. $SoCL = \{0\, l\}$
10. $SoCA = \bigcup_{i=0}^{l} Attr_i$
11. Else Break();
12. End For
13. Return ($SoCA$)

4. Research Methodology

4.1. Dataset Description and Tools

This study obtained all required datasets from the UCI machine learning repository [29]. These datasets belong to many different fields, such as engineering, social science, and business. Among them, we selected 11 datasets covering many areas for our evaluation. Table 2 shows the different samples of data used. We also provide the number of attributes, instances, and classes. Moreover, in the final column, we describe the attribute data type in each dataset. These data are used to demonstrate the applicability and performance of feature selection. Afterwards, classification is used to evaluate the accuracy when each dataset is randomly split into training and test sets.

Table 2. Datasets used in this study.

Dataset	No. Attributes	No. Instances	No. Classes	Data Type
Dermatology	33	366	6	Integer
Glass	10	214	7	real
Iris	4	150	3	real
Lung-cancer	56	32	3	Integer
Movement	91	360	15	Integer
Pageblocks	10	5473	5	Integer, real
Segmentation	19	2310	7	Integer, real
Soybean	35	307	19	Integer
Tunadromd	242	4465	2	Integer
Wine	13	178	3	Integer, real
Zoo	17	101	7	Nominal, Integer

This study used R for data management and analysis, specifically R-3.5.1 for Windows and RStudio Version 1.1.456. They were used to classify using the DT, SVM, and ANN classifiers. We also used Weka to provide the previous feature selection algorithms compared to our models. The feature selection methods tested were filter methods (information gain

and chi-square), wrapper methods, and forward selection. In addition, in the proposed model using FCA for feature selection, we applied Concept Explorer (ConExp) [30] to generate the hierarchical structure for FCA. ConExp has the functionality to create and edit contexts, generating lattices from contexts and finding the relative attribute that actually occurs in the context and the basic relationship that is true in the context.

4.2. The Experimental Design

We designed experiments for evaluating the performance in classification tasks. The overall scheme is illustrated in Figure 2.

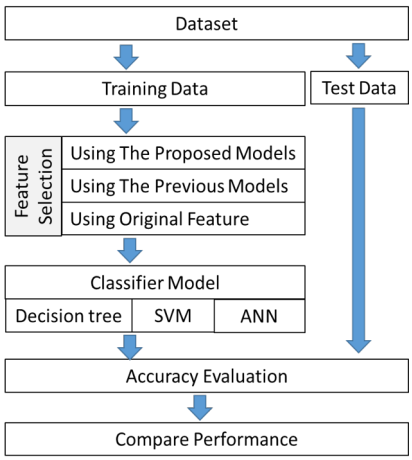


Figure 2. The experimental design.

In these numerical experiments, datasets from the UCI Machine Learning Repository (Asuncion, 2007) were employed, each divided into training and testing groups. The training set was processed with feature selection using three methods: the proposed models, prior models, and the original features. Initially, the proposed model involves the utilization of FCA and decision tree within a hierarchical concept model. These structures were created, and significant features were selected, as explained in Section 3. Subsequently, traditional feature selection methods such as IG, CS, FS, and BS were employed to compare the proposed feature selection model and the original features. These conventional methods are widely utilized for feature selection and operate based on filter (e.g., IG and CS) and wrapper (e.g., FS and BS) methods. Finally, the original features were used as a baseline for comparison. Next, the selected feature(s) for each dataset were used to train a classifier, namely DT, SVM, and ANN. These models are popular classifier models. Afterward, the test data were used to evaluate the classification accuracies of these classifiers. The accuracies served as the performance measures for each model.

Subsequently, the chosen feature(s) for each dataset were employed to train classifiers, specifically, DT, SVM, and ANN. These models are well known in the field of classification. Following the training phase, the test data were utilized to assess the classification accuracies of these classifiers. The accuracies then served as performance metrics for each model.

5. Results and Discussion

5.1. The Original Performance Results

Practically, the datasets in Table 2 were run using 10-fold cross-validation, which is the most used method. Cross-validation metrics further validated the stability and reliability of our selected features, providing key metrics such as mean accuracy, F1-score, and standard deviation. In this work, we selected accuracy as the primary metric to gauge the stability of model performance across different data splits. Accuracy was chosen for its straightforward interpretation and its widespread use as an effective measure of a model's ability to correctly classify instances across classes. For this, each dataset was divided into

complementary subsets, performing the analysis on one subset, called the training set, and validating the analysis on the other subset, called the testing set. The results from our experiments for classification accuracy are shown in Table 3. To improve the interpretability of the classification accuracy results, we provide a color-coded heatmap visualization for Table 3 shown in Figure 3. This heatmap illustrates classification performance across different classifiers—DT, SVM, and ANN—for each dataset. Darker shades indicate higher classification accuracy, making it easier to identify which classifiers performed best for each dataset at a glance.

Table 3. Summary of datasets and classifier performances.

Dataset	No. Attributes	The Classification Accuracy (%)		
		Org-DT	Org-SVM	Org-ANN
Dermatology	33	84.99	84.50	85.09
Glass	10	100.0	95.26	99.05
Iris	4	78.67	76.98	77.00
Lung-cancer	56	80.83	80.56	81.00
Movement	91	73.06	72.69	72.88
Pageblocks	10	97.94	96.05	97.88
Segmention	19	89.09	88.63	88.70
Soybean	35	96.00	95.98	96.00
Tunadromd	242	96.64	99.10	99.22
Wine	13	83.17	80.82	82.98
Zoo	17	96.09	95.53	95.72

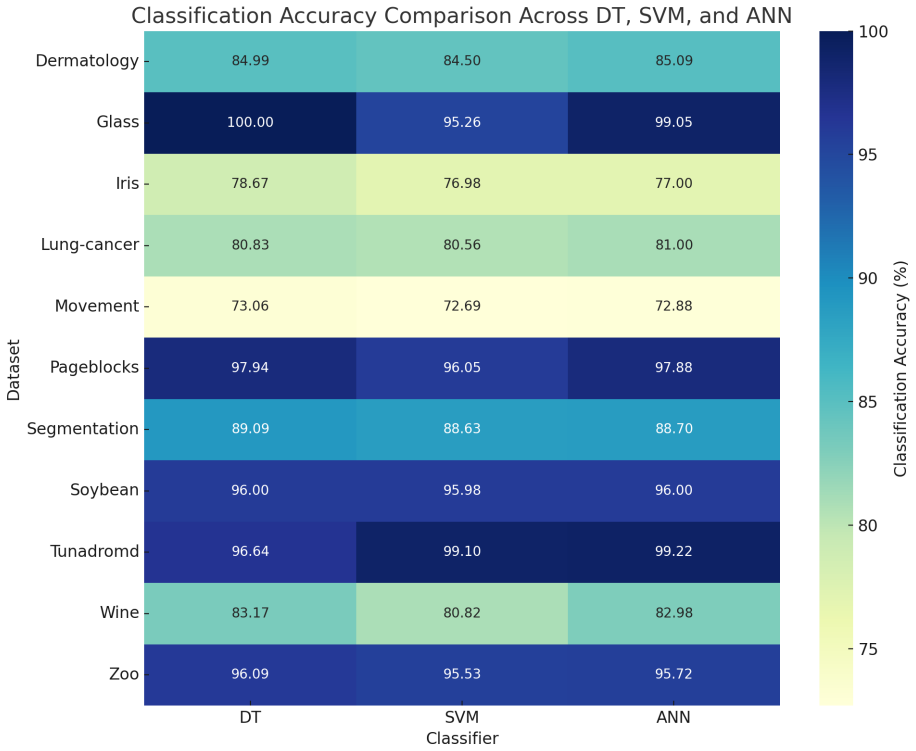


Figure 3. The classification accuracy comparison across DT, SVM, and ANN.

In this experiment, the input data had not been reduced; instead, all original attributes were used for training. In Table 3, the results show the standard classification accuracy for

each dataset when using DT, SVM, and ANN. Mostly, DT was the superior one. The accuracy results are used as benchmarks for evaluation the feature selection in the next section.

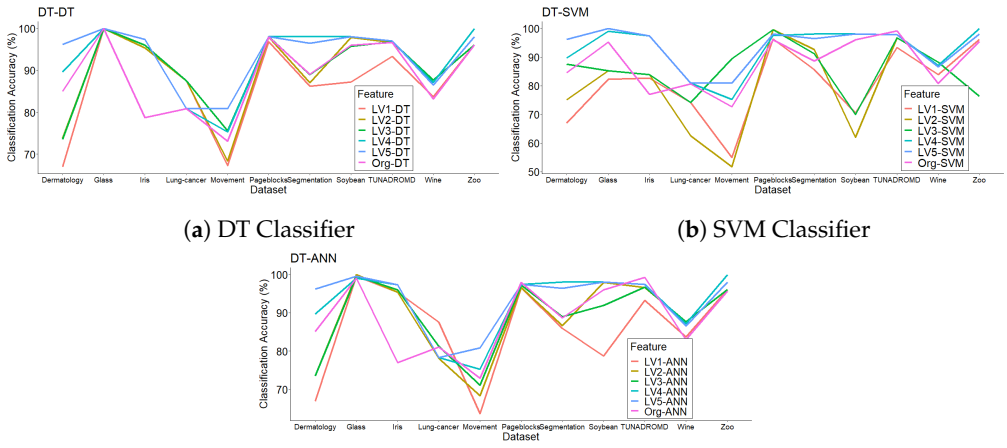
5.2. The Hierarchical Concept Model Performance

This section describes the proposed model for feature selection based on hierarchical concept model using DT or FCA. Thus, the results will be divided into three parts as follows.

5.2.1. Feature Selection Using Decision Tree

In order to evaluate our models, we used the well-known software RapidMiner Studio to generate a tree structure using the DT algorithm. The datasets in Table 2 were used to create tree structures. Next, we selected the attributes from each level of the tree structure. In this experiment, we selected attributes by filtering from the first to the fifth level of the tree. Table 4 shows the classification accuracy (%) in each level of the decision tree. We compared classification accuracy with our feature selection using DT to the benchmark with original features (without using feature selection at all) in Section 5, as shown in Figure 4.

From Figure 4, we see that the classification accuracy did not differ between the original and the proposed models for each level. Indeed, if we add the number of attributes from each level, it leads to a trend of improving accuracy.



(**a**) DT Classifier (**b**) SVM Classifier

(**c**) ANN Classifier

Figure 4. The comparison DT-SVM-ANN classifiers with features selected using DT from the first to the fifth level and with all original features.

5.2.2. Feature Selection Using FCA

FCA's ability to create a structured concept lattice enables it to systematically reduce the number of features while retaining classification robustness. Within the concept lattice, FCA organizes features according to their generalization and specialization relationships, which helps distinguish between essential and redundant attributes. Features that consistently contribute to accurate classification outcomes are preserved within the hierarchical structure, while those that do not support the classification process are pruned. This lattice-based hierarchy allows FCA to efficiently identify a minimal yet comprehensive feature subset that captures the necessary information for classification. As a result, the feature set derived using FCA maintains a level of classification accuracy comparable to the original dataset while achieving significant dimensionality reduction. This is particularly beneficial in large datasets, where FCA's structure can simplify models, reducing computational load without compromising accuracy.

Table 4. The classification accuracy on using each level of decision tree for feature selection.

Dataset	The Classification Accuracy (%) Using Decision Tree-Based Feature Selection																		
	LV.1				LV.2				LV.3				LV.4				LV.5		
	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	Attribute	DT	SVM	ANN
Dermatology	1	66.94	66.95	66.94	3	73.77	75.15	73.50	5	73.50	87.45	73.50	6	89.65	89.65	89.65	8	96.16	96.16	96.16
Glass	1	100.00	82.24	99.53	3	100.00	85.19	100.00	4	100.00	85.19	99.53	5	100.00	99.05	99.05	7	100.00	100.00	99.52
Iris	1	95.33	82.67	95.33	2	95.33	83.83	95.33	3	96.00	83.83	96.00	4	97.33	97.33	97.33	4	97.33	97.33	97.33
Lung-cancer	1	87.50	74.17	87.50	3	87.50	62.50	78.12	5	87.50	74.17	81.25	9	80.83	80.83	78.33	11	80.83	80.83	78.33
Movement	1	67.22	55.00	63.61	2	68.33	51.67	68.33	5	75.56	89.44	71.11	8	75.28	75.28	75.28	17	80.83	80.83	80.83
Page-blocks	1	96.86	96.29	96.53	2	97.94	99.60	96.55	5	98.06	99.60	97.20	7	98.06	97.44	97.44	9	98.05	98.05	97.44
Segmentation	1	86.15	85.67	85.89	2	87.06	92.64	86.67	5	89.00	91.21	89.00	9	98.00	98.00	98.00	10	96.41	96.41	96.41
Soybean	1	87.23	70.50	78.72	3	97.87	62.00	97.87	5	95.74	70.00	91.87	11	98.00	98.00	98.00	15	98.00	98.00	98.00
Tunadromd	1	93.28	93.28	93.28	2	96.64	96.64	96.64	4	96.86	96.75	96.75	6	96.98	97.76	97.42	6	96.98	97.76	97.42
Wine	1	83.71	83.82	83.71	3	87.64	88.24	87.64	5	87.64	88.24	87.64	7	87.16	87.16	87.16	10	86.50	86.50	86.50
Zoo	1	96.04	96.09	96.04	3	96.04	76.27	96.04	5	96.04	76.27	96.04	7	100.00	100.00	100.00	8	98.00	98.00	98.00

The other hierarchical method, FCA, was then applied to generate the hierarchical construct. We applied the ConExp program to build the concept lattice structure. Similarly, we selected the attributes from each level of concept lattice from the first to the fifth level. The classification accuracy results are shown in Table 5. From this table, we generate the color-coded heatmap showed as Figure 5 showing classification accuracy across various levels (LV.1 to LV.5) for different classifiers (DT, SVM, ANN) using FCA-based feature selection. This visualization highlights the classification performance across datasets, with darker shades representing higher accuracy, facilitating a quick comparison across levels and classifiers.

The results show that as the levels increase from LV.1 to LV.5, certain datasets, such as Pageblocks, Movement, and Tunadromd, show improvements in classification accuracy. This pattern suggests that higher levels in the FCA hierarchy incorporate additional relevant features, enhancing the classifiers' ability to make accurate predictions. This improvement across levels aligns with FCA's role in selecting more specific feature combinations as the hierarchy expands, allowing for more refined data representation. For datasets like Dermatology and Zoo, accuracy remains relatively stable across all levels, with minor fluctuations. This stability indicates that FCA-based selection effectively identifies a core set of important features early on, and additional levels do not significantly alter the classification performance. This result suggests that, for these datasets, FCA's early levels capture the most relevant information, making deeper feature selection levels less impactful. The heatmap highlights variability in performance across different classifiers. For instance, ANN achieves high accuracy on datasets such as Segmentation and Soybean at all levels, while SVM and DT sometimes lag in accuracy. This variability underscores the adaptability of FCA-based selection: certain classifiers, especially those more sensitive to specific features or complex interactions (like ANN), benefit more from the hierarchical structure than others. In some cases, specific levels offer optimal feature sets. For example, Glass and Iris datasets achieve high accuracy with minimal levels (LV.1 or LV.2), indicating that these datasets require only a few select features for effective classification. This observation supports the advantage of FCA-based selection in balancing dimensionality reduction with performance, especially for simpler datasets where adding more features would not yield substantial accuracy gains. A few datasets, such as Lung-cancer and Zoo, demonstrate lower accuracy across most levels, particularly for certain classifiers. This pattern may reflect inherent complexities in these datasets that are not fully captured by FCA-selected features or limitations in the classifiers used. It suggests that alternative selection strategies or more complex models may be required to handle such cases effectively.

Likewise, we compared classification accuracy of our feature selection using FCA to the benchmark original features (without using feature selection at all) in Section 5.1, as shown in Figure 6.

From Figure 6, we see that the classification accuracy is not different between the original and the proposed model at each level. Indeed, if we add the number of attributes from each level, the trend in accuracy is increasing.

From the testing above, we find that the classification accuracy is not different between the original and the proposed model for each level. Thus, we can compare using paired-samples *t*-tests as mentioned in Section 3.

On considering the relationship between the number of selected attributes and the average classification accuracy for datasets, as shown in Figure 7, we found that the proposed model using a decision tree gave a better classification result than the model using FCA.

Table 5. The classification accuracy when using each level of FCA concept lattice for feature selection.

Dataset	The Classification Accuracy (%) Using FCA-Based Feature Selection																			
	LV.1				LV.2				LV.3				LV.4				LV.5			
	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN	No. Attribute	DT	SVM	ANN
Dermatology	13	77.58	77.58	77.58	21	77.58	77.58	77.58	24	77.58	77.58	77.58	28	77.58	77.58	77.58	33	77.58	77.58	77.58
Glass	1	100.00	92.60	99.55	3	94.87	93.96	93.96	5	93.01	91.17	88.38	10	93.01	91.17	88.38	10	93.01	91.17	88.38
Iris	2	94.67	94.00	94.00	3	73.33	49.33	58.67	4	80.00	84.00	84.00	4	80.00	84.00	84.00	4	80.00	84.00	84.00
Lung-cancer	5	97.50	97.50	97.50	8	49.17	55.83	55.25	10	49.17	55.83	55.83	16	49.17	55.83	55.83	32	49.17	55.83	55.83
Movement	7	63.33	60.28	60.28	12	63.33	60.28	60.28	19	99.44	99.17	99.17	28	99.44	99.17	99.17	32	99.44	99.17	99.17
Page-blocks	4	90.39	85.31	85.31	4	90.39	85.31	85.31	7	99.80	99.85	99.85	10	99.80	99.85	99.85	10	99.80	99.85	99.85
Segmentation	8	99.70	92.81	99.70	10	99.70	91.02	99.70	19	57.36	91.02	97.49	19	57.36	91.02	97.49	19	57.36	91.02	97.49
Soybean	2	100.00	100.00	100	12	83.50	87.50	87.50	16	61.50	68.00	64.00	35	61.50	68.00	64.00	35	61.50	68.00	64.00
Tunadromd	5	93.28	93.84	93.28	13	94.96	96.64	97.42	19	96.19	97.87	97.98	34	96.75	98.88	98.88	44	96.75	99.10	99.33
Wine	3	81.50	83.76	83.76	6	69.71	68.56	68.56	9	78.14	81.50	81.50	13	78.14	81.50	81.50	13	78.14	81.50	81.50
Zoo	7	88.18	88.18	88.18	10	88.18	88.18	88.18	12	61.55	61.55	59.55	17	61.55	61.55	59.55	17	61.55	61.55	59.55

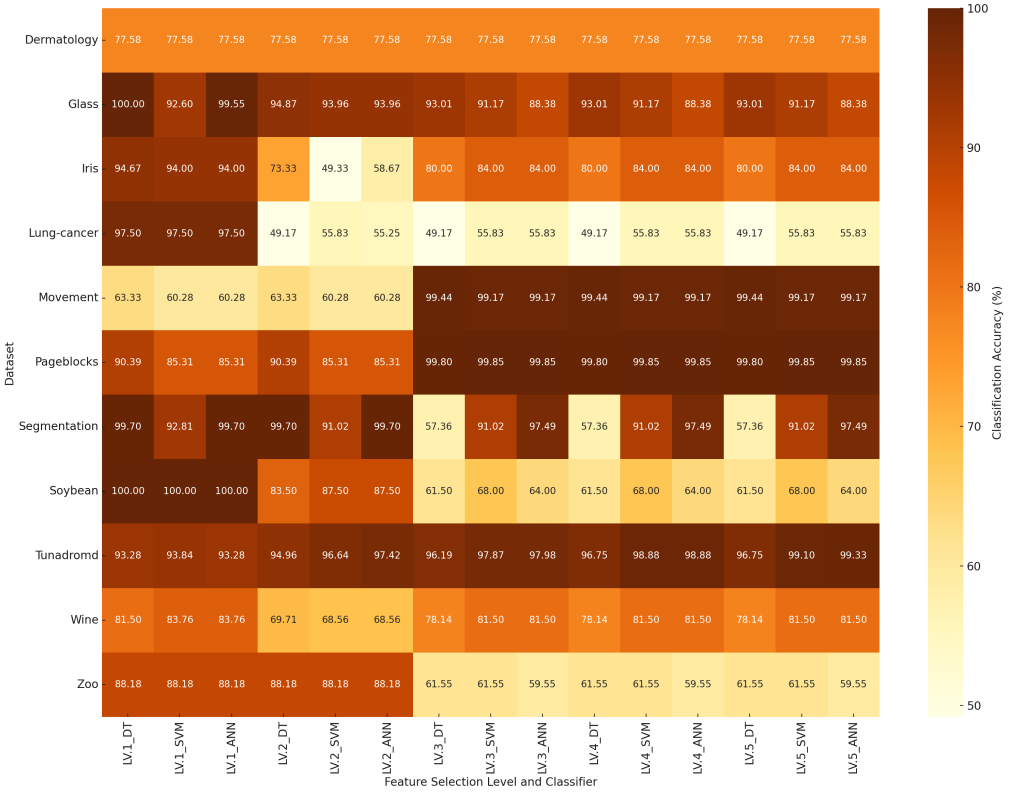
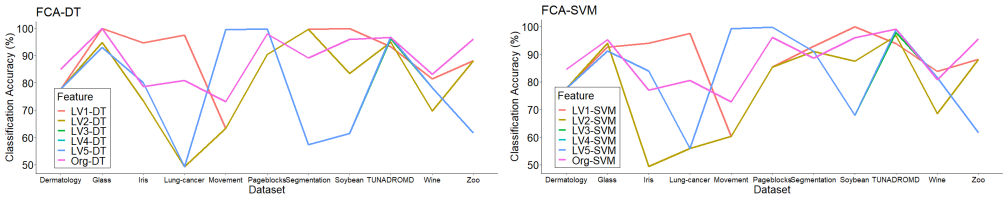
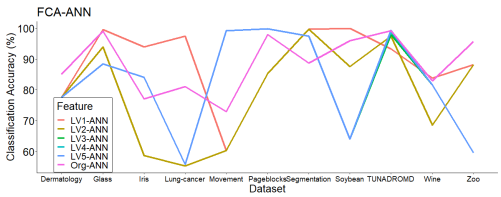


Figure 5. The classification accuracy using FCA-based feature selection across levels and classifiers (Table 5).

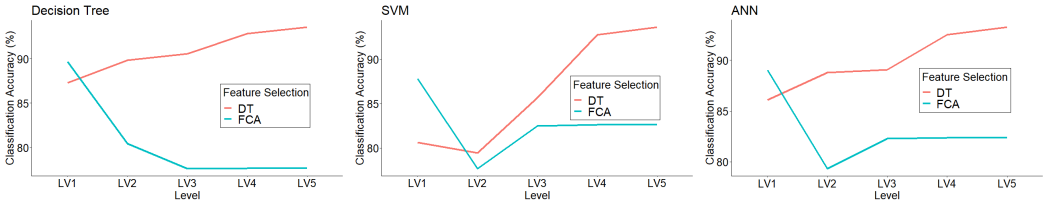


(**a**) DT Classifier (**b**) SVM Classifier



(**c**) ANN Classifier

Figure 6. A comparison DT-SVM-ANN classifiers with feature selection using FCA from the first to the fifth level and with all original features.

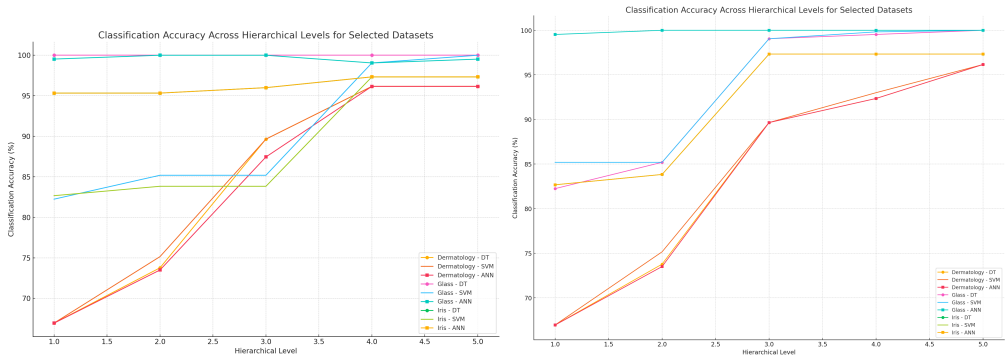


(**a**) DT Classifier (**b**) SVM Classifier (**c**) ANN Classifier

Figure 7. A comparison of average classification accuracies with each classifier model between features selected using DT or FCA.

5.2.3. An Example of Feature Selection Optimization

In our feature selection, we select the top node of structure to be representative of dataset. However, the proper selection of relevant attribute leads to an efficient classification. For example, in Tables 4 and 5, we chose datasets (Dermatology, Glass, and Iris) and applied three classifiers (DT, SVM, and ANN) to illustrate a line plot of classification accuracy across hierarchical levels (Levels 1 to 5), as shown in Figure 8. This visualization demonstrates how classification performance improves with higher hierarchical levels, providing insight into the incremental impact of feature selection at each level for each classifier and dataset. Both graphs show that as the hierarchical level increases, accuracy gradually improves. However, after reaching a certain level, the accuracy plateaus and remains constant. By systematically adjusting the number of hierarchical levels (from LV.1 to LV.5) and observing corresponding accuracy changes, we gained insights into the optimal level configurations for each dataset. Specifically, we used paired-samples statistical tests to compare accuracy across levels, ensuring that each selected level added meaningful improvement without redundant features. Thus, we experimented with using the paired-samples t-test to optimize the selected level. We use the experimental results from Table 4 to demonstrate the proper selection of the node attribute in each level. The Dermatology dataset was investigated to show the calculation for selecting attribute in each level of the tree. We assume the hierarchical structure of the Dermatology dataset, and the classification accuracies are shown in Figure 9.



(**a**) (**b**)

Figure 8. The classification accuracy across hierarchical levels for selected datasets. (**a**) Feature selection using decision tree. (**b**) Feature selection using FCA.

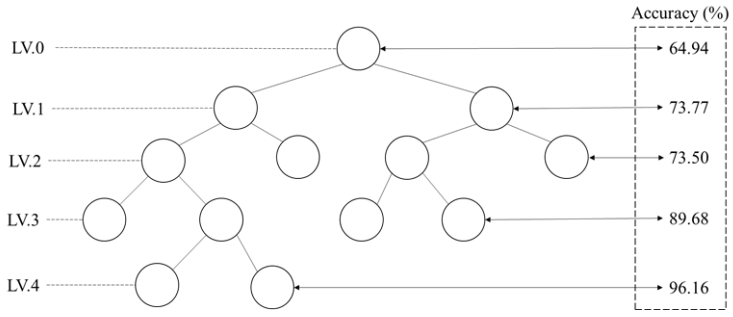


Figure 9. An example hierarchical structure using decision tree and the accuracies with selected features.

From Figure 9, we can apply the proposed feature selection in an example calculation. The initial attribute node (or root) is selected with an accuracy of 64.94%. Next, we consider the next step for selection or not by using the Equation (2) where n= 366, $\hat{p} = \frac{\sum_{l=i}^{n} Acc_{l,i} + \sum_{l=i}^{n} Acc_{l-1,i}}{n+n} = \frac{245+270}{366+366} = 0.704$, $\hat{q} = 1 - \hat{p} = 0.296$, thus

$$Z_{l,l-1} = \sqrt{\frac{366}{22 * 0.704 * 0.296}}(73.77 - 64.94) = 42.18.$$

From Table 1, if we consider the level of significance 0.01, if $|Z_{l,l-1}| \leq 2.576$, then we select level 1 (LV.1 in Figure 9). Next, we still consider the next level (LV.2). We calculate $z_{2,1} = -1.61$ and find that this value is less than z in Table 1. Thus, LV.2 is not selected, and we stop the process. Thus, in this structure, we choose attributes from the structure in LV.0 to LV.2.

In addition, we estimate for all datasets which levels should be chosen. We applied the statistical paired-samples *t*-test to estimate this across all the datasets shown in Table 6, where x is the mean of the accuracy values of the data set for each level and SD is the standard deviation. We find that the feature selection from each level depends on the classifier model. Namely, we can use two levels in a decision tree classifier, whereas we should select four levels for using SVM or ANN classifiers.

Table 6. The paired-samples *t*-test results.

Methods	Level	x ± SD	Paired *t*-Test	Sig.
DT	LV.1	86.69 ± 8.39	2.97	**0.016**
	LV.2	89.14 ± 6.74	2.55	**0.031**
	LV.3	91.00 ± 7.13	1.59	0.146
	LV.4	92.43 ± 3.07	2.22	0.053
	LV.5	92.51 ± 2.17	3.03	0.014
SVM	LV.1	79.34 ± 3.12	4.88	**0.001**
	LV.2	77.70 ± 4.07	4.15	**0.002**
	LV.3	84.54 ± 3.52	2.85	**0.019**
	LV.4	92.27 ± 0.94	2.46	**0.036**
	LV.5	93.21 ± 0.88	1.56	0.153
ANN	LV.1	86.28 ± 12.67	2.55	**0.031**
	LV.2	88.00 ± 6.50	3.20	**0.011**
	LV.3	89.51 ± 6.20	2.59	**0.029**
	LV.4	92.02 ± 3.45	2.35	**0.043**
	LV.5	92.85 ± 3.33	1.65	0.133

5.3. The Performances of Prior Feature Selection Approaches

This section evaluates the classification performance of feature selection using IG, CS, FS, and BS. IG and CS algorithms function as filter methods, while FS and BS algorithms operate as wrappers. These algorithms were implemented in the RapidMiner Studio program, representing traditional feature selection methods for comparison with our proposed model. The 10 datasets listed in Table 2 were employed for training and testing through a 10-fold cross-validation approach. The experimental results, presented in Table 7, present the use of DT, SVM, and ANN as classifiers, with the "*No.*" column indicating the number of selected attributes. The classification accuracies reported in this table will be further compared in the subsequent section.

To improve clarity and facilitate comparisons, we provide a color-coded heatmap visualization for Table 7, shown in Figure 10. This heatmap represents the classification accuracy results across various feature selection methods (IG, CS, FS, BS) and classifiers (DT, SVM, ANN) for each dataset. The color intensity in each cell corresponds to the accuracy level, with darker shades indicating higher accuracy. Figure 10 illustrates the classification performance for each method and classifier on different datasets. This visual format allows for quick identification of the most effective feature selection method and classifier combination for each dataset. The heatmap makes it easier to compare performance across different configurations, addressing a reviewer's suggestion to include enhanced data interpretation.

As shown in the results, the IG and CS methods prioritize features with strong discriminatory power, which directly boosts classifier performance, particularly in datasets with well-defined feature relevance, such as Dermatology and Glass. IG, in particular, enhances the performance of DT and SVM by focusing on features that most impact classification decisions. Similarly, FS and BS methods refine the feature space by iteratively selecting or removing features based on their importance. This targeted approach leads to high accuracy in datasets like Pageblocks and Segmentation, where complex feature relationships are effectively reduced to only the most critical attributes for classification. Feature importance also varies across classifiers. For instance, DT benefits from feature selection methods that retain features with hierarchical significance, aligning well with its split-based structure. In contrast, SVM performs best with methods that maintain a broad set of influential features, such as those identified by CS. ANN, on the other hand, benefits from a more nuanced feature selection approach, as seen in the results for Soybean and Wine. The layered structure of ANN models can leverage diverse feature sets, making it especially effective with IG and FS methods that capture non-linear relationships within the data. The highest classification accuracy occurs when feature importance aligns with each classifier's inherent strengths. For example, datasets with high inter-feature relevance perform well with BS and FS, which selectively retain essential features while discarding irrelevant ones. This alignment between feature importance and classifier capabilities ensures that the chosen features directly support classification goals, resulting in improved model performance.

Table 7. The classification performances of prior feature selection approaches.

Dataset	IG				CS				FS				BS			
	DT	SVM	ANN	No.	DT	SVM	ANN	No.	DT	SVM	ANN	No.	DT	SVM	ANN	No.
Dermatology	92.35	93.00	92.35	12	86.07	87.29	86.07	7	85.79	87.85	84.64	9	85.79	87.85	85.79	33
Glass	99.53	100.00	100.00	4	99.53	99.82	100.00	3	100.00	100.00	100.00	9	100.00	100.00	100.00	3
Iris	95.33	95.33	95.00	2	95.33	94.22	95.33	2	96.00	96.04	96.00	1	96.00	96.04	96.67	3
Lung-cancer	68.50	68.50	69.00	2	87.50	86.50	86.50	16	87.50	87.50	83.50	55	87.50	87.50	87.50	2
Movement	66.64	66.65	65.45	16	75.00	74.51	76.02	34	76.94	82.50	77.85	88	79.44	82.50	83.33	6
Pageblocks	97.92	97.92	97.80	4	98.19	97.63	98.02	5	98.14	98.06	97.94	8	98.32	98.12	98.06	4
Segmentation	90.91	89.33	90.87	10	88.61	88.61	88.52	7	89.22	89.48	80.78	6	89.48	89.52	97.36	10
Soybean	97.87	100.00	98.87	14	100.00	100.00	98.87	10	100.00	100.00	97.87	2	100.00	100.00	100.00	34
Wine	82.58	82.50	82.00	5	83.15	83.15	84.22	6	85.96	88.22	90.45	9	87.08	88.48	89.89	10
Zoo	99.00	99.50	100.00	8	95.09	96.56	96.00	4	96.04	96.08	100.00	6	96.04	96.08	96.04	16

5.4. Comparison of Performances

We compared the classification accuracies of our feature selection using decision trees in Section 5.2 with others in Section 5.3, as illustrated in Figure 11. The figure depicts the classification performance after utilizing different numbers of attributes at each level (from Level 1 to 3, from top to bottom respectively) for each classification method denoted as Figure 11a–c, respectively. Detailed data can be found in Tables 4 and 7. From the graphs in each dataset, considering the accuracy of attribute selection at each level, it appears that the performance is relatively consistent with other feature selection methods. For some datasets, the presented method may exhibit superior accuracy from the initial level, such as the Soybean dataset and Segmentation dataset. Conversely, the presented method may have lower accuracy for certain datasets, as observed in the Dermatology dataset.

Similarly, we conducted a comparison of the classification accuracy of our models using FCA in Section 5.2 with others in Section 5.3, illustrated in Figure 12. The figure demonstrates the classification performance after employing varying numbers of attributes at each level, with detailed data available in Tables 5 and 7. Upon examining the accuracy trends from this graph, we observe a similar pattern to use the decision tree approach,

where some datasets may exhibit comparable performance to other methods, such as the Movement dataset, while others may show comparatively lower classification accuracy.

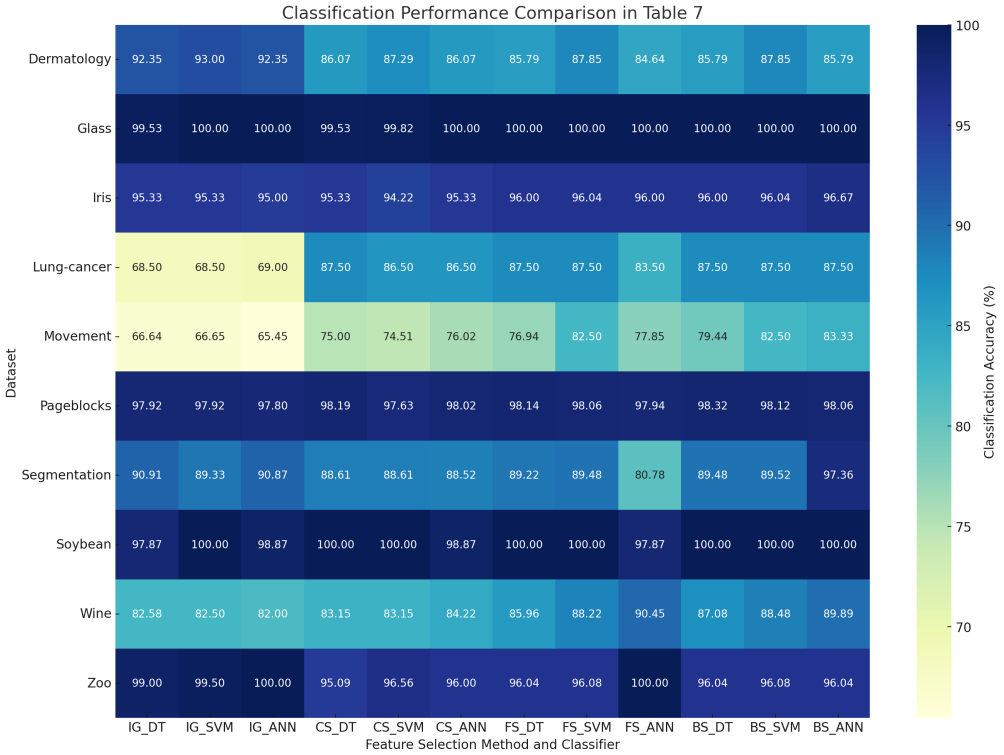
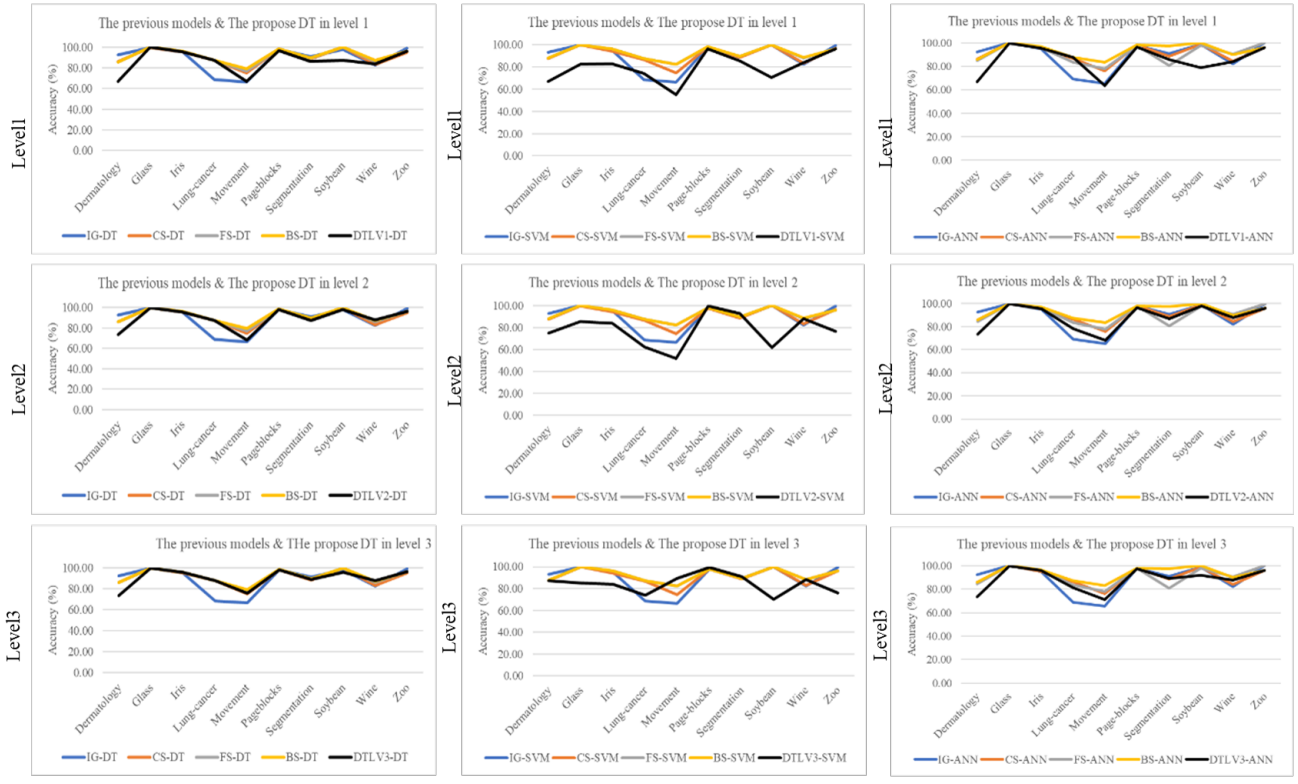


Figure 10. The classification performance comparison in Table 7.

From the above results, the outcomes obtained by employing a hierarchical structure for feature selection before the classification task indicate that it could lead to either favorable or less favorable results. This corresponds to the observed patterns seen in other feature selection methods used for comparison. Importantly, the classification performance of the proposed method does not consistently appear inferior. However, the presented key point in this study is its ability to significantly reduce the number of attributes compared to previous methods. To highlight the comparative advantages of the hierarchical concept model (using FCA and DT), we compared its performance with traditional feature selection methods, such as flat filters and wrappers, across multiple datasets. Our results reveal specific cases where the hierarchical model demonstrates notable benefits over traditional approaches. The hierarchical concept model's structured approach is particularly advantageous in datasets with layered feature relationships and complex class structures. Hierarchical methods like FCA and DT can capture multi-level dependencies between features. For instance, in datasets where features have a natural hierarchy or are interdependent, these methods outperform traditional flat methods by preserving relational structures within the data. This leads to a more meaningful reduction in feature space and improved classification accuracy. In datasets with many classes or intricate class relationships, the hierarchical approach organizes features according to relevance at each level, leading to better feature prioritization and reducing the computational load. For example, in the Soybean dataset, FCA's concept lattice helped isolate critical features while minimizing noise, thereby enhancing performance compared to non-hierarchical methods.

Considering the count of selected features, the results show clearly that the DT retained fewer features than the other methods, as shown in Figure 13. This figure will illustrate the number of selected attributes at each level from 1 to 3 in comparison with other methods that exhibit classification performance as shown in Figures 6 and 7 mentioned earlier. Figure 13a displays the count of selected attributes from the decision tree structure, while Figure 13b shows the count of selected attributes from the FCA structure. From as the results, the

hierarchical structure using decision tree method can be applied to select the feature for data dimension reduction yields favorable outcomes. This is due to the decision tree's method, which involves a top-down data entry process using specific filtering and selecting root node. However, FCA produces less satisfactory results due to the creation of a layer-wise structure designed for bidirectional ordering—both from top to bottom and from bottom to top. This may lead to effect of feature selection. The experimental results confirm that FCA's concept lattice can achieve dimensionality reduction without compromising classification accuracy. By systematically identifying and retaining only the most informative features, FCA produces a reduced feature set that achieves comparable accuracy to the original, full feature set. This performance validates FCA's effectiveness in filtering out features that are irrelevant or redundant while preserving those that contribute to robust classification outcomes. Compared to traditional feature selection methods, FCA's lattice structure organizes attributes in a way that maintains high classification reliability while minimizing the feature set. This approach optimally balances computational efficiency and model interpretability with performance, making FCA an ideal choice for applications that require both robust classification and efficient feature reduction.

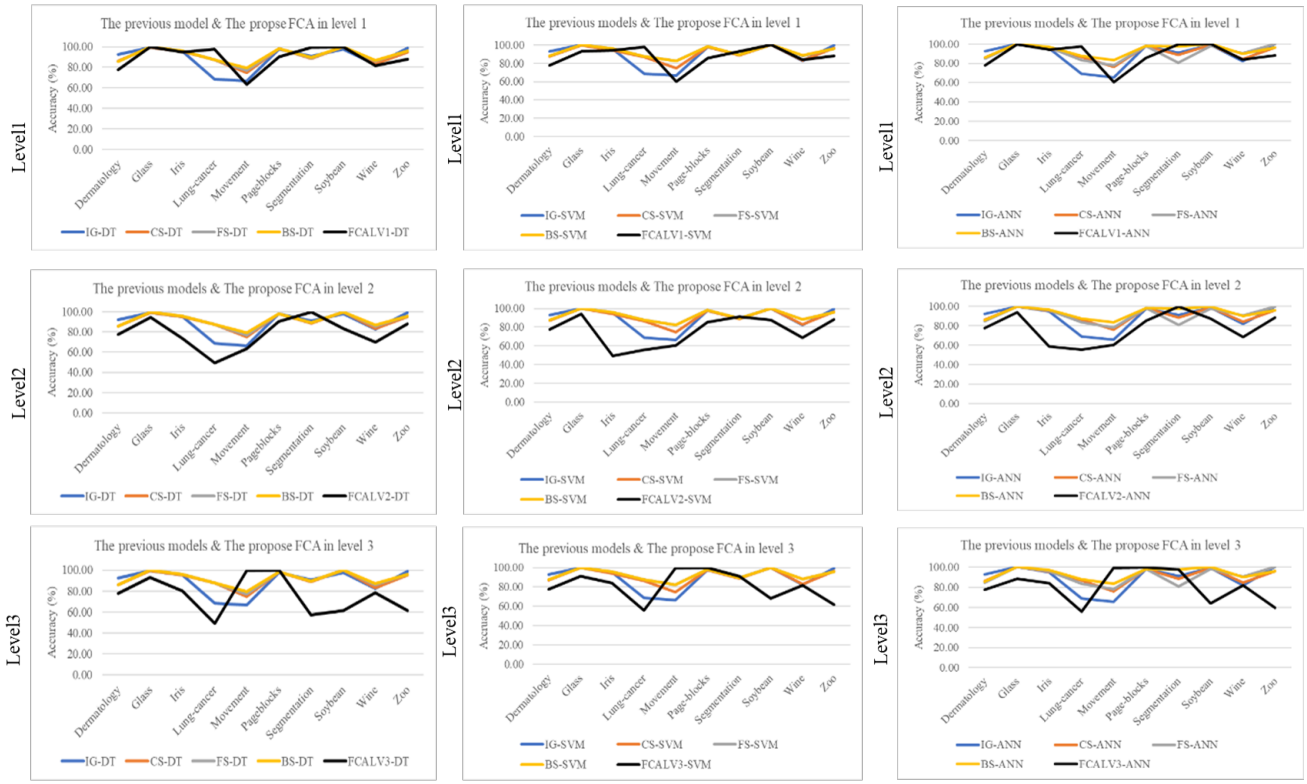


(a) DT classifier (b) SVM classifier (C) ANN classifier

Figure 11. Comparison of proposed feature selection using DT on each level and others with DT-SVM-ANN classifiers.

While the hierarchical feature selection model demonstrated robust performance with DT, SVM, and ANN classifiers, certain conditions could further enhance its results. For instance, tuning the parameters for each classifier, such as adjusting hyperparameters in SVM or layer configurations in ANN, could improve classification accuracy. Moreover, increasing the hierarchical levels of feature selection allows finer distinctions among features, which can be beneficial when working with high-dimensional datasets. This model can be adapted to other classification algorithms, such as k-nearest neighbors (k-NN) and random forest, by modifying the selection criteria within the hierarchical structure. The flexibility in adapting this model to other classifiers makes it a versatile approach across different machine learning tasks.

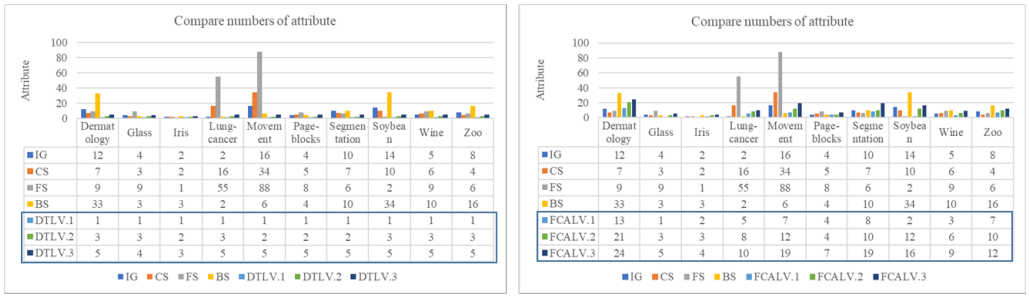
This hierarchical feature selection model is particularly beneficial in applications requiring efficient and accurate feature reduction. For example, in medical diagnosis systems, where high-dimensional data (e.g., from genetic or imaging data) can impact computational efficiency, this model enables the reduction of features without sacrificing diagnostic accuracy. Another practical use case is in financial analysis for credit scoring, where a subset of relevant features can streamline decision-making processes. Additionally, environmental monitoring systems, where IoT data often contains redundant attributes, can benefit from this model's ability to filter and retain only the most informative features.



(a) DT classifier (b) SVM classifier (C) ANN classifier

Figure 12. Comparison of proposed feature selection using FCA in each level to others on using DT-SVM-ANN classifiers.

However, one interesting limitation is class imbalance. Class imbalance can impact the performance of feature selection methods, as features correlated with the minority class may be underrepresented or overlooked during selection. To mitigate this, we employed re-sampling based on using cross-validation metrics to ensure balanced representation across classes. However, this is still a limitation of this study. Our analysis indicates that while the hierarchical feature selection method performs robustly in balanced datasets, its effectiveness in imbalanced datasets improves with re-sampling techniques. Future work could explore integrating class-sensitive feature scoring directly into the hierarchical selection framework, ensuring equitable consideration of features across all classes. Moreover, we plan to address key challenges, such as multicollinearity and class imbalance, through established techniques like variance inflation factor (VIF) analysis, SMOTE for balancing classes, and comparison with advanced feature selection methods. Future studies will involve comprehensive testing on both benchmark and synthetic datasets to validate the efficacy of the hierarchical framework.



(a) Using DT method (b) Using FCA method

Figure 13. The number of selected features by (**a**) DT and (**b**) FCA methods in each level, compared to the other selection methods.

6. Conclusions

Feature selection methods are used in a wide variety of applications. Such methods proceed to eliminate unfavorable features that may be noisy, redundant or irrelevant, as these can penalize the performance of a classifier. Feature selection reduces the dimensionality of data and restricts the inputs to a subset of the original, but the features can have missing values. In this paper, we propose a new feature selection method based on a hierarchical concept model to improve the performance of supervised learning. We applied a decision tree and FCA to build the hierarchical structures from data to generate knowledge-based systems. In these hierarchical structures that embed knowledge, the top and bottom nodes represent general and specific knowledge, respectively. For this reason, we selected the top node of each structure to be representative of the dataset. However, the proper selection of relevant attributes leads to an efficient classification. Thus, we also used paired-samples *t*-tests to optimize the selected levels in the hierarchical structures. In this study, numerical experiments used available software, RapidMiner Studio v. 9.2.0 and ConExp v. 1.3 programs, to generate the hierarchical structures for decision tree and FCA, respectively. Afterward, the attributes in top node of the obtained structure were selected to use in learning classifier models. Three popular classifier algorithms were used in this study: decision trees, SVM, and ANN. To compare the performances of our feature selection methods with available alternatives: all original features (without using feature selection) and the prior feature selection methods. We used 10 datasets from the UCI Machine Learning Repository for classification. The prior feature selection methods tested were IG, CS, FS, and BS. The results show clearly that the proposed models with DT and FCA gave smaller set of retained attributes than the other methods, while still classification performance remained comparable. In other words, the new approach provided superior data reduction to the least dimensionality without performance sacrifice. In summary, this study demonstrates the strengths of hierarchical feature selection methods, specifically FCA and DT, over traditional flat methods in scenarios involving complex feature interdependencies or layered class structures. The hierarchical model's ability to retain structured relationships and prioritize relevant features enables it to outperform conventional approaches in both feature reduction and classification accuracy for multi-level datasets. This makes hierarchical methods a valuable approach for feature selection in data-intensive applications requiring nuanced feature analysis.

Future work could focus on refining the hierarchical feature selection model by exploring more dynamic methods of defining hierarchical relationships among features. Integrating adaptive methods, such as reinforcement learning, to determine optimal hierarchical levels could further improve model efficiency and adaptability to diverse datasets. Additionally, investigating other classifiers, including ensemble models like gradient boosting machines (GBMs) and random forest, could provide insights into the model's adaptability across various machine learning paradigms.

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