

Article

Calculation Method for Torsional Angle of Main Cables of Suspension Bridges with Spatial Cables

Shengbo Chai ^{1,*}, Qingyu Zhao ¹, Cailin Zheng ² and Shang Chen ²¹ School of Architecture and Civil Engineering, Xi'an University of Science and Technology, No. 58 Yanta Rd., Xi'an 710054, China; 24204053001@stu.xust.edu.cn² Ankang Housing and Urban-Rural Housing Construction Bureau, Ankang 725000, China; czcl2025zcl@163.com (C.Z.); cs2025cs@163.com (S.C.)

* Correspondence: shengbo@xust.edu.cn

Abstract

In suspension bridges employing spatial cables, the main cables undergo torsion during the construction process. Clarifying the distribution pattern of the main cable torsional angle can guide the positioning and installation of cable clamps and mitigate issues such as wire waving. This paper provides a theoretical calculation method for the distribution law of the torsion angle of the main cable. Firstly, it is assumed that the load form that causes the torsion of the main cable is the uniform load along the cable length and the span length. The theoretical calculation formulas of the torsion angle are derived, respectively, and the two formulas are compared. Analysis shows that under a uniformly distributed load, when the main cable is deflected at a certain angle, the torque is non-uniformly distributed along the cable, and the torsional angle follows a fourth-order parabolic curve along the span. A model test on spatial cable deflection was conducted, and the measured torsional angles were compared with theoretical results. The findings indicate that the measured torsional angle variation along the span is generally consistent with theoretical calculations. When the deflection angle of the spatial cable is large, the theoretical value exceeds the measured value. Furthermore, a finite element model was employed to calculate the torsional angle, torque, and bending moment during cable deflection. The results reveal that the distribution of the torsional angle depends on the ratio of the moment of inertia and the polar moment of inertia of the main cable section. The smaller the polar moment of inertia relative to the moment of inertia, the closer the torsional angle is to the spatial deflection angle. When the polar moment of inertia is relatively small, except in the end regions, the distribution of torque obtained from simulations is largely consistent with the theoretical values.

Keywords: suspension bridge; torsion angle calculation; model test; flexural stiffness; torsional stiffness



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1. Introduction

Compared to traditional parallel cable systems, suspension bridges with spatial cables employ inclined main cable planes to form a three-dimensional cable system, significantly enhancing the overall lateral stiffness, torsional stability, and wind resistance of the bridge [1]. During construction, the main cables transition from a vertical plane in the unloaded state to a plane with a specific horizontal inclination, undergoing torsional deformation [2]. Accurately determining the torsional angle of the main cables is crucial for guiding the installation and positioning of cable clamps and preventing issues such as

wire waving. However, there is still a lack of research on the variation law of the torsional angle induced by the lateral displacement of spatial main cables during construction.

Current research on the torsional behavior of spatial main cables primarily relies on model tests and numerical simulations. Qi et al. [3] simulated the system transformation process of the Hangzhou Jiangdong Bridge. Their experimental results indicated that the torsional stiffness of the main cable is not constant but increases with the spatial inclination angle. Zhang et al. [4], through a scaled model test of the Liede Bridge, found that the maximum lateral torsional angle of the cable clamps occurs at the midspan of the main and side spans, and the torsional angle of the clamps is approximately proportional to the lateral displacement of the main cable. Li et al. [5–7], via a scaled model test of the Hangzhou Jiangdong Bridge, discovered that the stiffness of the main cable is relatively low when the hanger load is small; setting an accurate pre-deflection angle for the cable clamps can effectively reduce the torsional angle during system transformation; and when the direction of the hanger load passes through the centroid of the main cable cross-section, no additional torque is generated on the main cable. Qi et al. [2] established an analytical model considering tension-torsion coupling, derived a formula for calculating the anti-torsional moment, and revealed that the torsional stiffness of the main cable increases nonlinearly with both cable tension and torsional angle. Li et al. [8] theoretically analyzed the enhancement of the torsional stiffness of the main girder due to tension-torsion coupling effects and inter-wire friction. Combined with ANSYS (2021R1) simulation, it is shown that the torsional change in the main cable in the numerical simulation needs to be realized by nonlinear iteration.

In the traditional analysis of the main cable of the suspension bridge, the main cable is usually regarded as a flexible structure, ignoring the bending stiffness of the main cable. Relevant studies have proved that the influence of the bending stiffness of the main cable on the mechanical behavior of the main cable cannot be ignored. Tian et al. [9,10] proposed an analytical method for determining the target configuration of spatial main cable considering the bending and torsion deformation of the main cable. The numerical calculation shows that the target configuration of the spatial main cable is highly sensitive to its bending stiffness and torsion stiffness. Zhang et al. [11] derived the form-finding expression of the main cable considering the bending stiffness of the main cable, and found that the influence of the bending stiffness of the main cable on the vibration performance of the bridge is mainly related to the bending stiffness ratio of the main cable to the stiffening beam. Sun et al. [12] proposed an analytical method for the chord substitution method considering the influence of axial geometric stiffness and bending geometric stiffness on the flexible-rigid coupling multi-stiffness (FSCM) characteristics of multi-cables in the construction stage, and verified the effectiveness of the method in combination with actual engineering cases. Chen et al. [13] studied the bending performance of structural cables through bending tests. The results show that pre-tension can improve the effective bending stiffness of cables, and the enhancement effect increases with the increase in pre-tension, but decreases with the increase in cable size. Arena et al. [14] proposed a fully nonlinear model of the cable, and explored the influence of the bending stiffness and torsional stiffness of the cable under different boundary conditions. The research shows that the initial pretension state of the cable has an effect on the formation of the boundary layer, the peak stress in the boundary layer and its spatial distribution range. Grigorjeva et al. [15,16] explored the influence of the bending stiffness of the cable on the static performance. The research shows that appropriately increasing the bending stiffness of the main cable can effectively reduce the displacement, internal force and stress of the suspension system. Ni et al. [17,18] constructed a finite element model of large diameter sag cable considering bending stiffness. The analysis results show that ignoring

bending stiffness will lead to unacceptable prediction error of high-order natural frequency of cable. Sun et al. [19] established a dimensionless continuum model of a suspension bridge to explore the influence of the bending stiffness of the 3D cable system on the free bending vibration of the structure. The results show that the lateral inclination of the sling will suppress the influence of the bending stiffness on the structural frequency. Bendalla et al. [20] proposed a numerical modeling framework for cables, and verified it by analytical solutions. The results show that the self-weight distribution and the bending stiffness of the cable have a significant influence on the distribution and shape of the internal force of the cable. When the bending and torsional stiffness of the main cable are considered, the entire cable can be idealized as a beam-like structure. The large deformation behavior of such beam structures can be analyzed using nonlinear finite element theory [9]. Sheng F et al. [21] developed theoretical derivations and corresponding numerical methods for the nonlinear analysis of linear structures undergoing large tensile strains and finite deformations, with simulation results validating the accuracy of the proposed model. Wang D et al. [22] combined theoretical analysis with finite element modeling to investigate the mechanical behavior of main cables. The study revealed that wires in the lower layers are more susceptible to global slip at the saddle zones, and that pre-tensioning can effectively reduce the extent of micro-slip regions. Wang L et al. [23] established a non-uniform slip analysis model for main cables using the steel strand element method, with experimental verification demonstrating the non-uniform characteristics of strand slippage within saddle sections. Meier C et al. [24] proposed a novel discrete model based on Kirchhoff-Love theory for geometrically exact finite element analysis of slender beams. Theoretical and numerical verification confirmed the method's particular suitability for highly slender beam configurations. Liu Y et al. [25] introduced a discrete geometrically exact elastic rod model incorporating improved discrete curvature formulation, enabling effective analysis of deformation patterns including bending and torsion in slender structural components. These theoretical and numerical models provide valuable references for torsional analysis of main cables.

Existing studies have analyzed the mechanical mechanisms of spatial main cable torsion through scaled model tests or numerical simulations, but a mathematical relationship between the lateral deflection angle and the torsional angle of the spatial main cable has not been established. This paper derives theoretical formulas to calculate the torsional angle of the main cable induced by lateral deflection, validates these formulas through model tests, and uses finite element model (FEM) to analyze the influence of the bending and torsional stiffness of the main cable on its torsion, thereby elucidating the mechanism behind the variation in the torsional angle. This study aims to derive a mathematical relationship between the lateral deflection and the torsional angle of the main cable, a relationship that has not been clearly established in previous research.

2. Distribution Law of Torsion Angle of Spatial Main Cables

In the construction process of a suspension bridge with spatial cables, the main cable is converted from a natural drooping surface to an inclined surface with a certain lateral deflection angle. In this process, the main cable is twisted. Based on the following basic assumptions, the distribution law of torsion angle of spatial cable is analyzed:

- (1) The main cable is constrained by the saddle at the saddle, and its torsion angle is 0° ;
- (2) The loads from the hangers act through the central axis of the main cable, thus no local torsion is induced due to eccentric loading;
- (3) The torsion of the main cable is caused by the overall deflection of the main cable, and the spatial deflection angle of the main cable is equal to the maximum torsion angle of the main cable.

The overall geometric configuration of the main cable follows a parabolic shape, described by the equation:

$$y = ax^2 \quad (1)$$

where $a = 4f/L^2$, L is the distance between two saddles.

2.1. Transverse Uniformly Distributed Load Along the Cable Length

As shown in Figure 1, assuming that the main cable is subjected to a transverse uniform load q_1 , which is the uniform load along the cable length, the torque of the main cable at x can be expressed as:

$$T_x = \int_0^x q_1 y dx = \frac{q_1 a}{3} x^3 \quad (2)$$

where T_x represents the main cable torque at x .

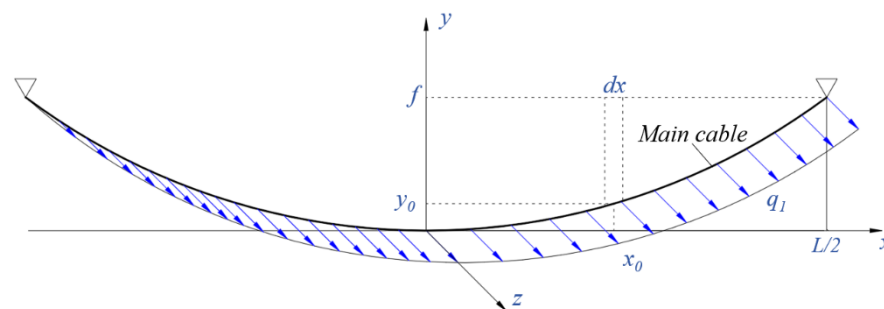


Figure 1. Distribution of uniform load q_1 along the cable length.

Correspondingly the relative torsion angles of the two cross-sections of the main cable with a distance of dx are expressed as:

$$d\varphi = \frac{T_x dl}{GJ} = \frac{q_1 a}{3GJ} x^3 \sqrt{1 + y'^2} dx \quad (3)$$

where G , J and l are the shear modulus of the main cable material, the polar moment of inertia of the main cable section and the main cable length.

Therefore, at the coordinate x , the torsion angle φ_x of the main cable relative to the saddle position is expressed as:

$$\varphi_x = \int_x^{L/2} d\varphi = \frac{q_1 a}{3GJ} \int_x^{L/2} x^3 \sqrt{1 + y'^2} dx \quad (4)$$

Through the calculation Equation (4), we can obtain:

$$\varphi_x = \frac{q_1}{720GJa^3} \left[(1 + a^2 L^2)^{3/2} (3a^2 L^2 - 2) - 2(1 + 4a^2 x^2)^{3/2} (6a^2 x^2 - 1) \right] \quad (5)$$

Since the torsional stiffness of the main cable is unknown, let $m = \frac{q_1}{720GJa^3}$, then Equation (5) can be expressed as:

$$\varphi_x = m \left[(1 + a^2 L^2)^{3/2} (3a^2 L^2 - 2) - 2(1 + 4a^2 x^2)^{3/2} (6a^2 x^2 - 1) \right] \quad (6)$$

where m is an unknown quantity. However, Equation (6) can be solved by introducing boundary conditions. When $x = 0$, $\varphi = \theta$, θ is the angle between the main cable plane and the vertical plane. When $x = L/2$, $\varphi = 0$.

2.2. Transverse Uniformly Distributed Load Along Span Length

Assuming that the uniform load q_2 that causes the torsion of the main cable is uniformly distributed along the span length, Equation (4) can be written as:

$$\varphi_x = \int_x^{L/2} d\varphi = \frac{q_2 a}{3GJ} \int_x^{L/2} x^3 dx \quad (7)$$

Through the calculation Equation (7), we can obtain:

$$\varphi_x = \int_x^{L/2} d\varphi = \frac{q_2 a}{12GJ} \left(\frac{L^4}{16} - x^4 \right) \quad (8)$$

Since the torsional stiffness of the main cable is unknown, let $n = \frac{q_2 a}{12GJ}$, then Equation (8) can be expressed as:

$$\varphi_x = n \left(\frac{L^4}{16} - x^4 \right) \quad (9)$$

where n is an unknown quantity, and the solution of Equation (9) is the same as that of Equation (6).

It can be seen from the theoretical derivation that the two calculation methods can calculate the torsion angle of the main cable according to the deflection angle of the main cable, and the torsion angle exhibits in a four-time parabola along the main cable.

2.3. Comparison of Two Calculation Methods

In order to explore the difference between the two theoretical calculation methods, the calculation results of Equations (6) and (9) are compared when the main cable deflection angle θ is 20° , 30° and 40° , respectively, as listed in Table 1, and the variation trends of the torsional angle along the span direction were compared, as shown in Figure 2.

Table 1. Comparison of results between Equations (6) and (9).

Deflection Angles Horizontal Distance from the Midspan (m)	20°			30°			40°		
	Equation (6)	Equation (9)	Relative Error (%)	Equation (6)	Equation (9)	Relative Error (%)	Equation (6)	Equation (9)	Relative Error (%)
0	20.00	20.00	0.00	30.00	30.00	0.00	40.00	40.00	0.00
0.473	20.00	20.00	−0.02	30.00	29.99	0.02	39.99	39.99	0.01
0.783	19.98	19.97	0.03	29.96	29.96	0.02	39.95	39.95	0.01
1.032	19.93	19.92	0.04	29.89	29.88	0.04	39.86	39.84	0.04
1.329	19.80	19.78	0.11	29.70	29.68	0.08	39.60	39.57	0.09
1.637	19.54	19.50	0.21	29.31	29.25	0.21	39.08	39.00	0.21
1.958	19.06	18.98	0.40	28.58	28.47	0.40	38.11	37.96	0.40
2.267	18.29	18.17	0.65	27.43	27.25	0.67	36.58	36.33	0.68
2.566	17.17	16.99	1.02	25.75	25.49	1.00	34.33	33.98	1.02
3.147	13.46	13.19	1.99	20.19	19.79	1.97	26.92	26.38	1.99
3.691	7.35	7.12	3.17	11.03	10.68	3.17	14.71	14.23	3.24
3.951	3.21	3.09	3.76	4.82	4.63	3.86	6.42	6.17	3.91
4.12	0.00	0.00	-	0.00	0.00	-	0.00	0.00	-

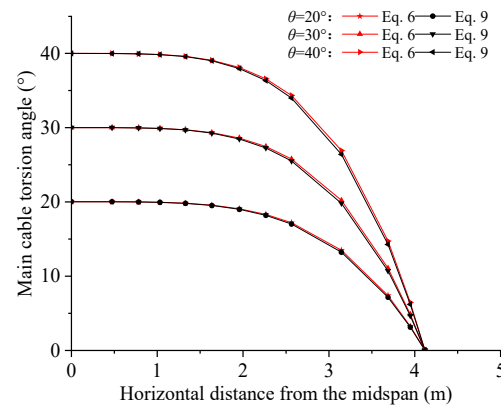


Figure 2. Comparison of variation trends between the results of Formulas (6) and (9). As observed from Figure 2, it can be seen that the results calculated by the two formulas are basically the same under different deflection angles. The discrepancy between the results obtained from the two formulas increases with the distance from the mid-span. And the values calculated by Equation (6) are slightly greater than those from Equation (9), yet the maximum relative difference in Equation (9) compared to Equation (6) does not exceed 4%. This indicates that the distribution pattern of the uniformly distributed load has a limited influence on the torsional angle of the main cable.

3. Experiment and Results

3.1. Design of Test Model

In order to verify the accuracy of the theoretical formula, two models of the main span and side span of the spatial main cable are designed. In the test, the linear shape of the main cable is approximately parabolic. The model size of the middle span is shown in Figure 3. The horizontal height of the two saddles is consistent. The horizontal distance between the two ends of the main cable is $L = 8.24$ m, and the mid-span vector height of the main cable is $f_1 = 1.205$ m. According to the coordinate system shown in Figure 3, the linear equation of the main cable is $y_1 = 0.071 x^2$.

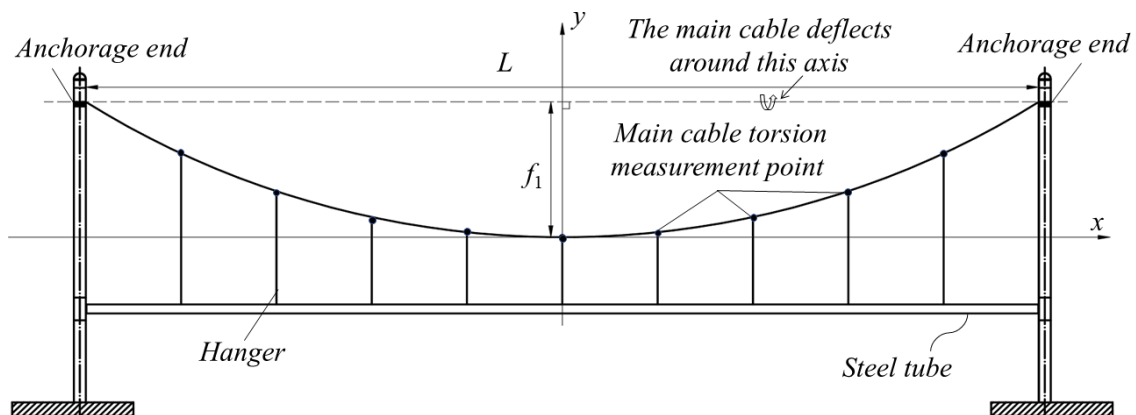


Figure 3. The main span model facade of the spatial main cable.

The model size of the side span is shown in Figure 4. The horizontal height of the two saddles is different. The vertical height difference between the two ends of the main cable is $H = 1.89$ m, the horizontal distance between the two ends of the main cable is $L = 8.24$ m, and the vector height of the mid-span position of the main cable is $f_2 = 0.445$ m. According to the coordinate system shown in Figure 4, the main cable shape equation is $y_2 = 0.0243 x^2$.

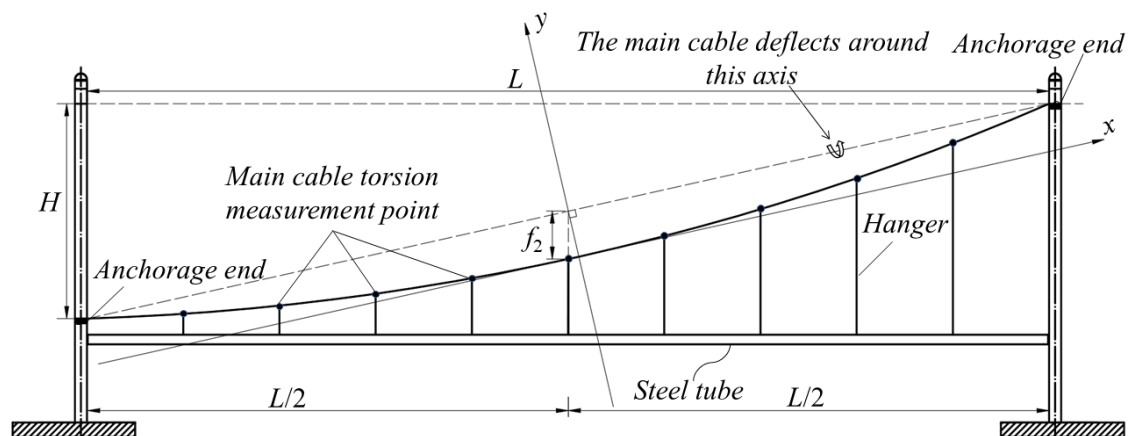


Figure 4. The side span model facade of the spatial main cable.

The main cable is deflected by laterally moving the steel tube at the lower end of the sling, as shown in Figure 5.

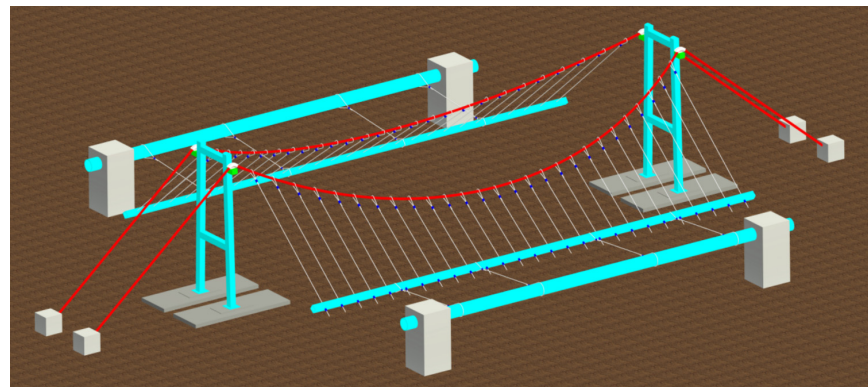


Figure 5. Schematic diagram of transverse deflection of main cable model.

As shown in Figures 6 and 7, the main cable is bundled by 19 wire ropes with a diameter of 5mm and is tightly bundled with a steel hoop. 24 positions along the length are selected as the measuring points, and the transverse aluminum square tube is set at the measuring point, which is adjusted to the level with the level ruler, and the two are adhered. The aluminum square tube rotates with the torsion of the main cable. The torsion angle φ of the main cable at each measuring point was calculated by the height difference between the two ends of the aluminum square tube.

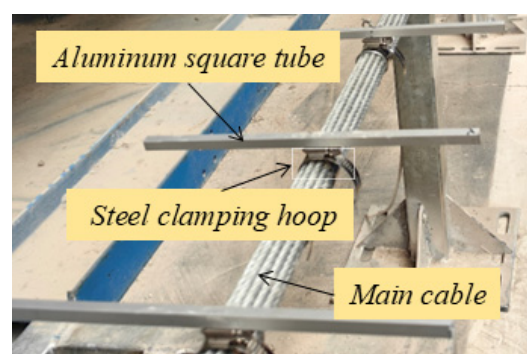


Figure 6. Steel hoop and aluminum square tube.

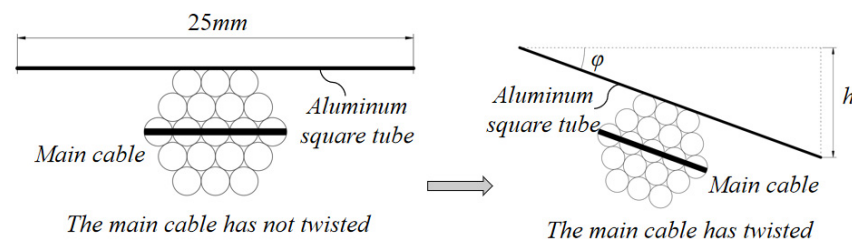


Figure 7. Test torsion angle calculation diagram.

3.2. Test Process

In order to ensure that the deflection angle of each position of the main cable is consistent, the main cable rotates around the connecting line of the two saddles. The test took the lateral displacement of the main cable as the control variable, and the deflection angle of the main cable can be calculated by the lateral displacement. The steel wire rope of the lifting rod bypassed the main cable and rides over the main cable to form a straddle-type structure, so as to ensure that the lifting rod does not generate additional torque on the main cable when it deflects. As shown in Figure 8.

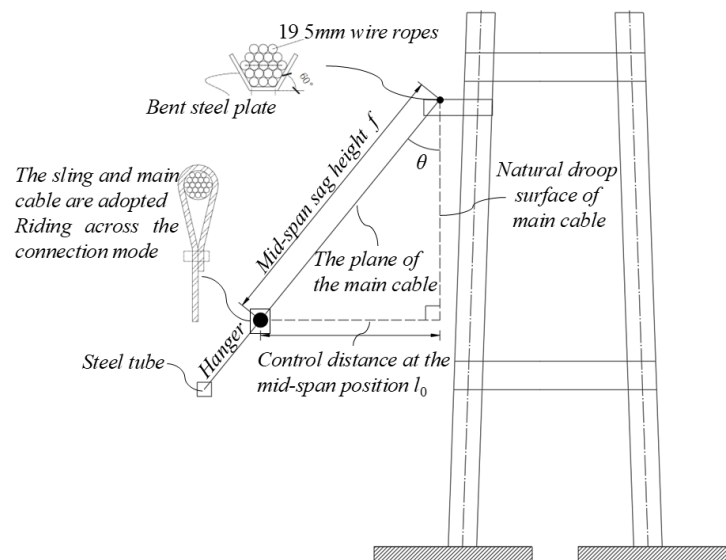


Figure 8. View along the bridge.

The test conditions are as follows:

- (1) Select 24 measurement control points along the length direction of the main cable, and level the aluminum square tube on the cable clamp;
- (2) Determine the main cable deflection angle θ_i and calculate the deflection angle θ_i ($i = 1, 2, 3, \dots$) corresponding to the lateral offset distance l_k of the main cable;
- (3) Tensioning the sling to make the main cable control point move the corresponding control distance;
- (4) Fix the sling, measure the height difference between the two ends of the aluminum square tube and calculate the torsion angle φ ;
- (5) The main cable deflection angle θ_{i+1} ($\theta_{i+1} > \theta_i$) is determined again, and the steps (2)~(5) are repeated.

3.3. Comparison of Theoretical Calculation and Experimental Results

Because the calculation results of the two formulas derived from the theory are basically the same, this paper only compares the measured values obtained from the experiment with the calculation results of Equation (6).

In the main span deflection test of the spatial main cable, the comparison between the measured value and the theoretical value of the main cable torsion angle is shown in Figure 9.

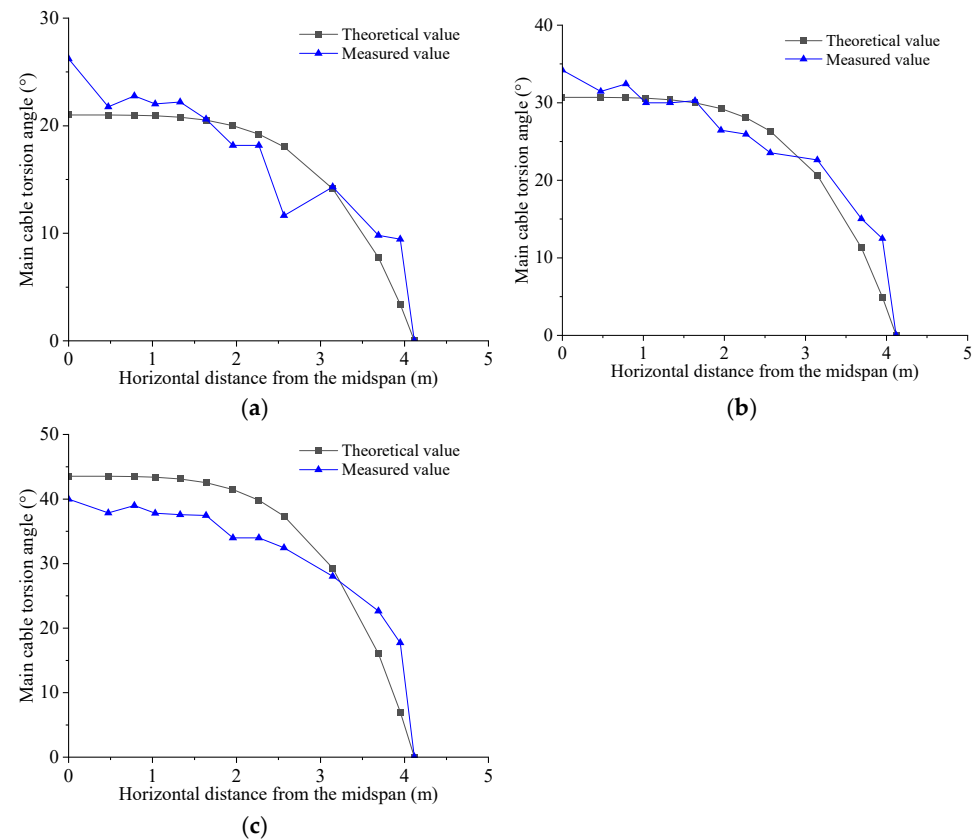


Figure 9. Comparison between measured and theoretical values in the main span test. (a) Deflection angle $\theta = 21^\circ$; (b) Deflection angle $\theta = 30.69^\circ$; (c) Deflection angle $\theta = 43.53^\circ$.

As shown in Figure 9, as the deflection angle increases, the measured torsional angle of the main cable also increases, but it is slightly lower than the theoretical value. Although the measured and theoretical values follow the same trend, a numerical discrepancy exists. With an increase in the deflection angle of the main cable, the maximum torsional angle is smaller than the deflection angle.

Analysis indicates that the main cable used in the test was composed of multiple wire strands bundled together, resulting in non-uniform torsional stiffness across sections. This non-uniformity led to an uneven distribution of the torsional angle along the cable, thereby causing the deviation from the theoretically calculated values.

In the side span spatial main cable deflection test, the comparison between the measured value and the theoretical value of the main cable torsion angle is shown in Figure 10.

As observed from Figure 10, when the main cable is deflected, the measured torsion angle of the main cable at the middle of the span is close to the deflection angle of the main cable, and the variation trend of the measured torsion angle is basically consistent with the theoretical value. With the increase in the deflection angle of the main cable, the measured torsion angle of the main cable is smaller than the theoretical value. This is because the main cable is not a completely flexible member. After the main cable is twisted, the strands are closer, and the bending and torsional stiffness are increased, resulting in the measured torsion angle being smaller than the theoretical value.

Upon returning the main cable to its initial natural sagging state after the test, the torsional angle was not completely eliminated. This indicates that the main cable exhibits

certain plastic characteristics during the torsion process, and relative slip occurs among the wire strands.

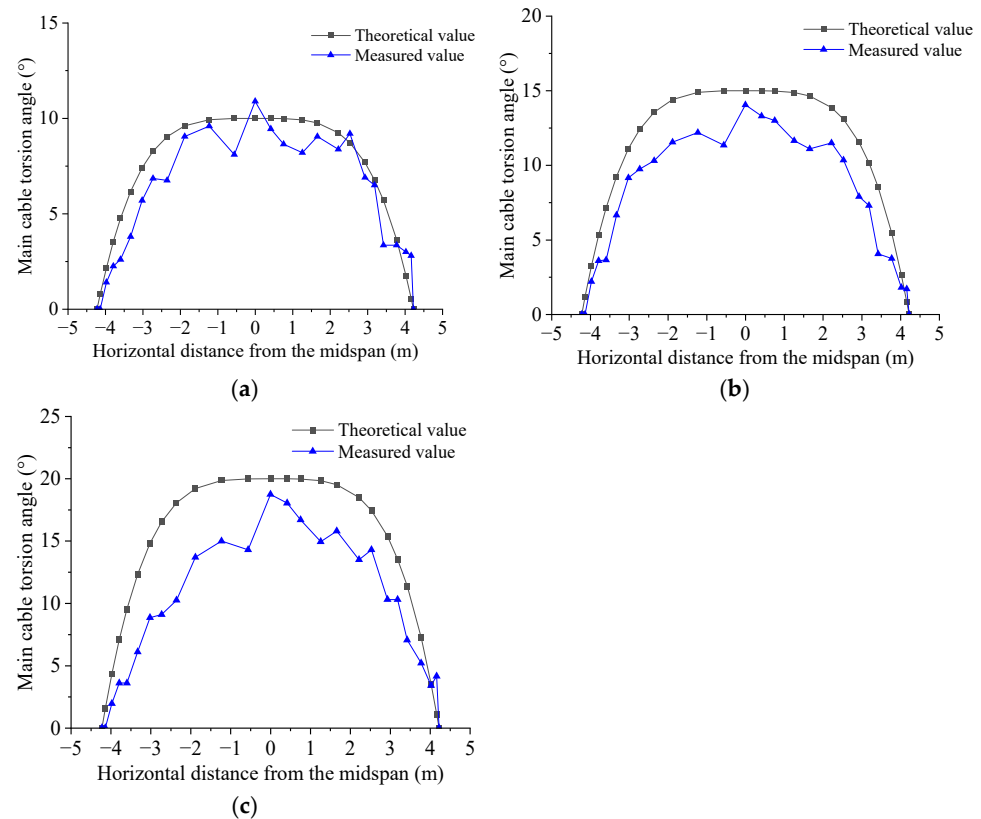


Figure 10. Comparison between measured and theoretical values in the side span test. (a) Deflection angle $\theta = 10^\circ$; (b) Deflection angle $\theta = 15^\circ$; (c) Deflection angle $\theta = 20^\circ$.

4. Finite Element Analysis

4.1. Basic Parameters

In order to verify the accuracy of the theoretical formula, the FEM of the main cable is established in Ansys (2021R1). The model size is the same as the test model of the main cable in the middle span spatial. The FEM has 121 nodes and 120 elements. The main cable is simulated by the elements' type of BEAM 4, and the ends are consolidated. Firstly, the initial state of the main cable is found by iteration, and then the load is applied to calculate the distribution of the torsion angle and torque of the main cable along the span length of the main cable when the main cable is deflected.

The definition of material properties and cross-sectional properties of the element is shown in Table 2. The data are to ignore the gap between the wire strands in the main cable and regard the main cable as a whole.

The main cable is composed of multiple wire strands. Under torsional and bending deformations, relative movement occurs among the individual strands within the cable. During bending deformation, the strands are subjected to normal stress, resulting in relative deformation along the cable length. In torsional deformation, shear stress induces relative deformation perpendicular to the cable length. Consequently, the actual torsional and bending stiffness of the main cable are significantly lower than those of an equivalent solid cross-section. To account for this effect in calculations, the moment of inertia and polar moment of inertia are reduced accordingly. The cross-sectional properties of the FEM under various simulation conditions are summarized in Table 3.

Table 2. Model material and section parameters.

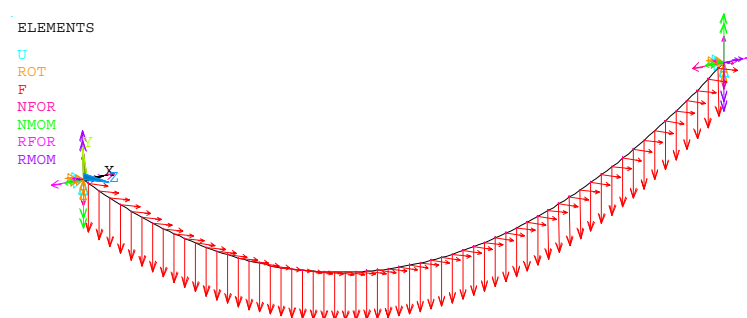
Basic Parameter	Symbol	Unit	Value
Span length	L	m	8.24
Designed vertical sag	f	m	1.205
Cross-sectional area	A	m^2	0.00049
Young modulus	E	Gpa	210
Poisson's ratio	μ	/	0.3
Shear modulus	G	Gpa	80.77
Density	D	N/m^3	76,930
Flexural moment of inertia	I	m^4	1.917×10^{-8}
Polar moment of inertia	J	m^4	3.83×10^{-8}

Table 3. Cross-sectional properties of the FEM for simulation cases.

Combination	Working Condition	Flexural Moment of Inertia	Polar Moment of Inertia	Polar Moment of Inertia/Moment of Inertia
1	1	I	J	I/J
	2		$0.5 J$	$2 I/J$
	3		$0.1 J$	$10 I/J$
	4		$0.01 J$	$100 I/J$
2	5	$0.1 I$	$0.1 J$	I/J
	6		$0.05 J$	$2 I/J$
	7		$0.01 J$	$10 I/J$
	8		$0.001 J$	$100 I/J$
3	9	$0.01 I$	$0.01 J$	I/J
	10		$0.005 J$	$2 I/J$
	11		$0.001 J$	$10 I/J$
	12		$0.0001 J$	$100 I/J$

This paper only simulates the situation when the main cable is deflected by 15° . According to the actual situation, the concentrated load is applied to the model nodes. In order to ensure the convergence of the calculation, the magnitude of the concentrated load applied is adjusted according to the moment of inertia: when the moment of inertia is I , $0.1 I$, $0.01 I$, 5000 N, 1000 N, 135 N are taken, respectively. The direction of the applied load is consistent with the deflection direction of the main cable, and it is decomposed into horizontal and vertical components acting on each node.

This paper specifically simulates the case where the main cable is deflected by 15° . Concentrated loads were applied to the model nodes based on practical conditions. To ensure computational convergence, the magnitude of the applied concentrated loads was adjusted according to the moment of inertia: for moments of inertia of I , $0.1 I$, and $0.01 I$, the applied loads were set to 5000 N, 1000 N, and 135 N, respectively. The direction of the applied load was consistent with the deflection direction of the main cable and was decomposed into horizontal and vertical components acting on each node. The single main cable model in Ansys is shown in Figure 11.

**Figure 11.** The single main cable model in Ansys.

4.2. The Results of the FEM Are Compared with the Measured Values

According to the section characteristics and the application of load, the comparison between the torsion angle calculated by FEM and the measured value is shown in Figure 12.

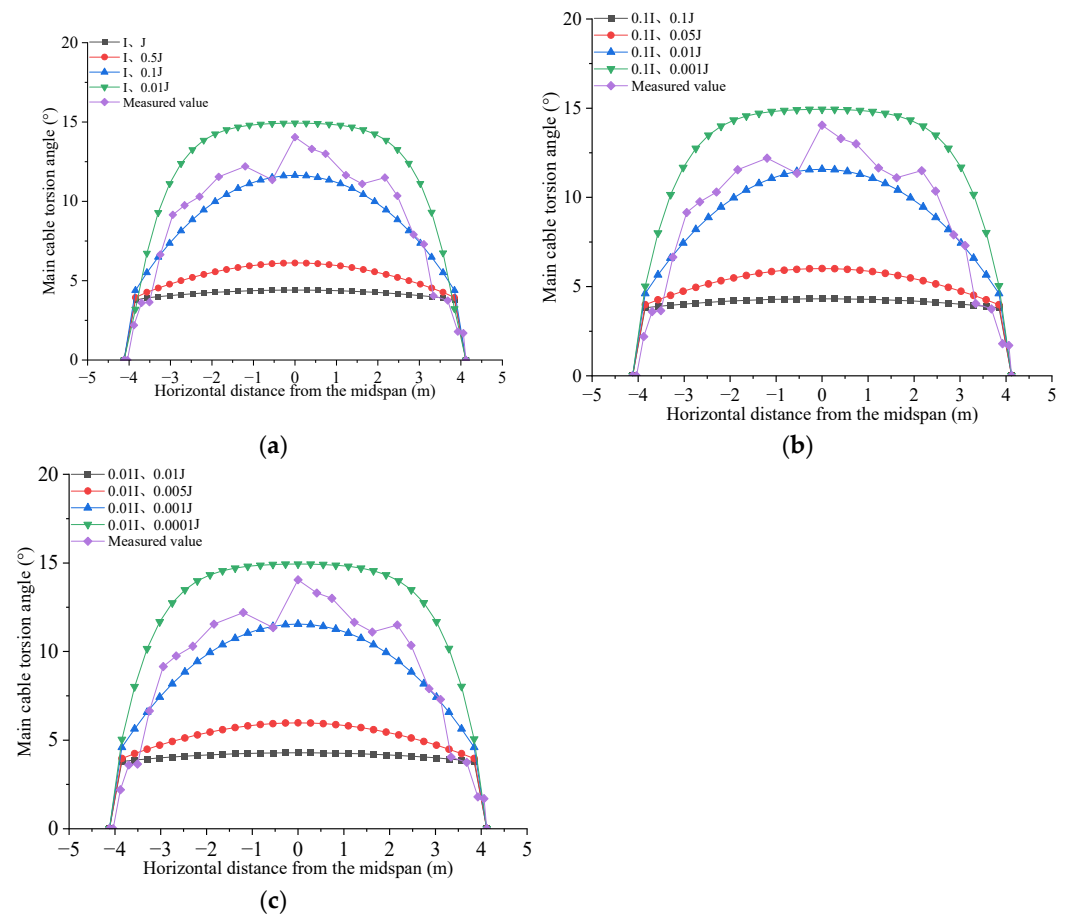


Figure 12. Comparison of the torsional angle under various simulation cases with measured values. (a) The torsion angle when the moment of inertia is I ; (b) The torsion angle when the moment of inertia is $0.1 I$; (c) The torsion angle when the moment of inertia is $0.01 I$.

It can be seen from Figure 12a that the torsion angle of the main cable varies greatly under different polar moments of inertia, and the distribution law is different. The main cable is regarded as a solid section. When the torsional stiffness is not reduced, the maximum torsional angle is about 4° , which is much smaller than the deflection angle of the main cable in spatial. With the decrease in the polar moment of inertia, the torsion angle increases gradually. When the polar moment of inertia is reduced to $0.1 J$, the torsion angle of the main cable increases to about 11° . When it is reduced to $0.01 J$, the torsion angle of the main cable is basically consistent with the overall deflection angle of the main cable, reaching 15° . At this time, the finite element simulation value is basically consistent with the theoretical value.

Comparing Figure 12a–c, it can be seen that when the overall deflection angle of the main cable is the same, the ratio of the moment of inertia to the polar moment of inertia is the same, and the corresponding torsion angle is basically the same. For example, J in Figure 12a, $0.1 J$ in Figure 12b and $0.01 J$ in Figure 12c, the ratio of moment of inertia to polar moment of inertia is I/J ; in Figure 12a, $0.1 J$; (b), $0.01 J$; (c), $0.001 J$, the ratio of moment of inertia to polar moment of inertia is $10 I/J$; as well as $0.01 J$ in Figure 12a, $0.001 J$ in (b) and $0.0001 J$ in (c), the ratio of moment of inertia to polar moment of inertia is $100 I/J$. The three kinds of comparisons show that the corresponding torsion angle distribution is the

same, and the values are basically the same. It can be seen that when the deflection angle of the main cable is constant, the distribution of the torsion angle depends not only on its torsional stiffness, but also on the ratio of the moment of inertia and the polar moment of inertia. Keeping the ratio of the moment of inertia and the polar moment of inertia unchanged, the size and distribution of the torsion angle are basically unchanged.

4.3. Analysis of Torque and Bending Moment Results

The main cable torque distribution of the FEM is calculated when the inertia moment is I , $0.1 I$ and $0.01 I$, respectively, as shown in Figure 13.

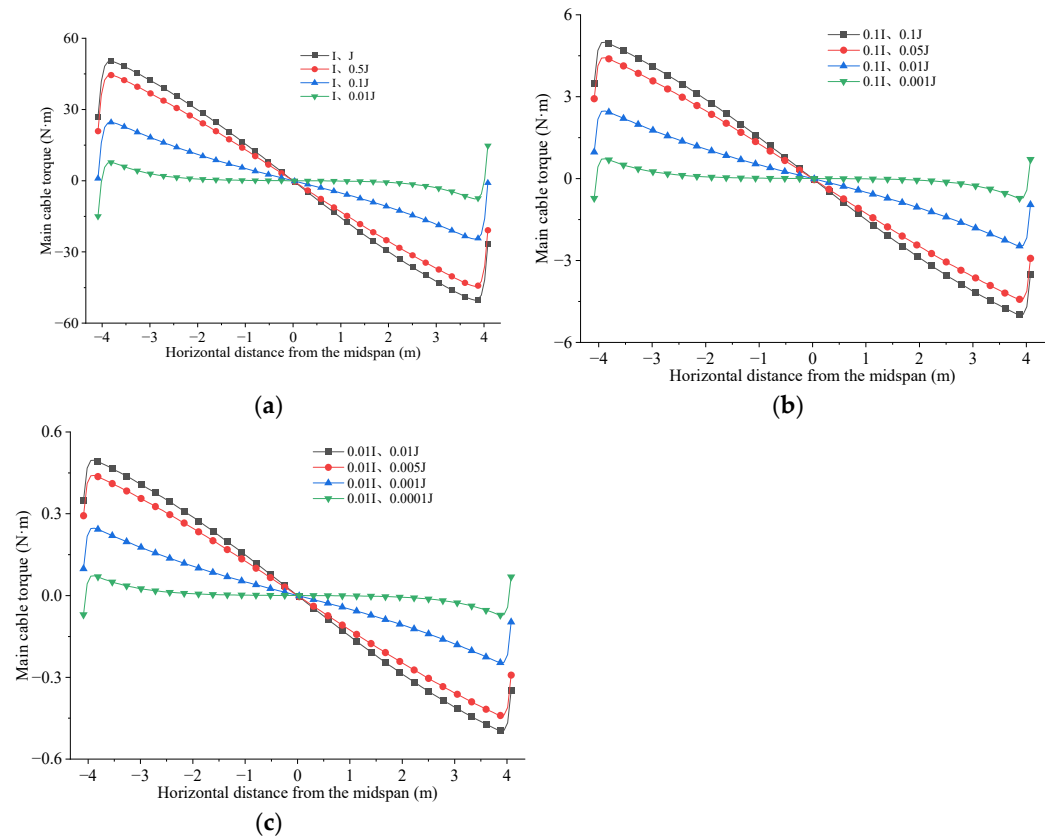


Figure 13. Torque in the main cable under various simulation cases. (a) The torque when the moment of inertia is I . (b) The torque when the moment of inertia is $0.1 I$. (c) The torque when the moment of inertia is $0.01 I$.

From Figure 13, it can be seen that the main cable torque is larger at the end area, but the maximum torque position is not at the end, but a certain distance from the end. The torque decreases from the maximum position to the mid-span, and the mid-span torque is 0. The torque of the main cable is related to the polar moment of inertia. The greater the polar moment of inertia, the greater the torque, but the two are not nonlinear. In Figure 13a, when the polar moment of inertia is $0.1 J$, the maximum torque is $25 \text{ N}\cdot\text{m}$, and when the polar moment of inertia is $0.01 J$, the maximum torque is $7.4 \text{ N}\cdot\text{m}$.

The torque variation curve of $0.01 J$ in Figure 13a is the same as that of $0.001 J$ in Figure 13b and $0.0001 J$ in Figure 13c. It can be seen that the variation in torque from the maximum value of the end to the mid-span is not only related to the polar moment of inertia, but also to the ratio of the polar moment of inertia to the moment of inertia.

The torque of the main cable can be solved theoretically according to Equation (2) in this paper. Compared with the torque when the moment of inertia is I and the polar moment of inertia is $0.01 J$ and $0.1 J$, because the actual polar moment of inertia is unknown, only the variation law of torque is compared here. First, the torque is non-dimensionalized,

and then the torque obtained by the finite element and the theoretically calculated torque are normalized (the maximum value is set to 1). The theoretical and finite element values of the normalized torque are shown in Table 4 and Figure 14.

Table 4. Comparison of Normalized Torque between Finite Element Analysis and Theoretical Model.

Horizontal Distance from the Midspan (m)	0.01 J	0.0 J	Theoretical Value
−4.09	−1.95	0.04	1.23
−3.54	0.77	0.92	0.80
−2.99	0.38	0.74	0.48
−2.44	0.18	0.57	0.26
−1.89	0.08	0.42	0.12
−1.34	0.03	0.29	0.04
−0.79	0.01	0.16	0.01
−0.24	0.00	0.05	0.00
0.31	0.00	−0.06	0.00
0.86	−0.01	−0.18	−0.01
1.41	−0.04	−0.30	−0.05
1.96	−0.09	−0.44	−0.14
2.51	−0.20	−0.59	−0.28
3.06	−0.42	−0.76	−0.52
3.61	−0.84	−0.95	−0.85
4.09	1.91	−0.04	−1.23

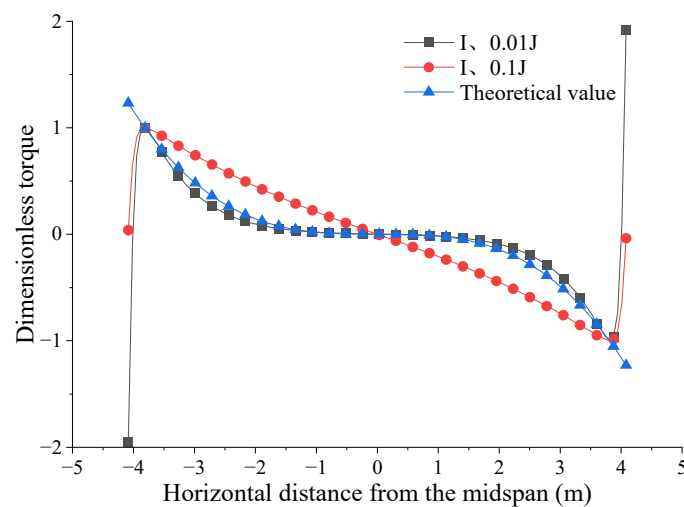


Figure 14. Normalized torque.

It can be seen from Figure 14 that except for the end of the main cable, the theoretical value of the torque is very close to the change rule when the moment of inertia is I and the polar moment of inertia is $0.01 J$. It can be seen from Equation (2) that the theoretical value of torque increases nonlinearly from the mid-span to the end of the main cable, while in the finite element model, the torque of the main cable changes abruptly at the end.

Through the FEM, the distribution of the bending moment of the main cable is calculated when the inertia moment I , $0.1 I$ and $0.01 I$ are calculated, respectively. As shown in Figure 15.

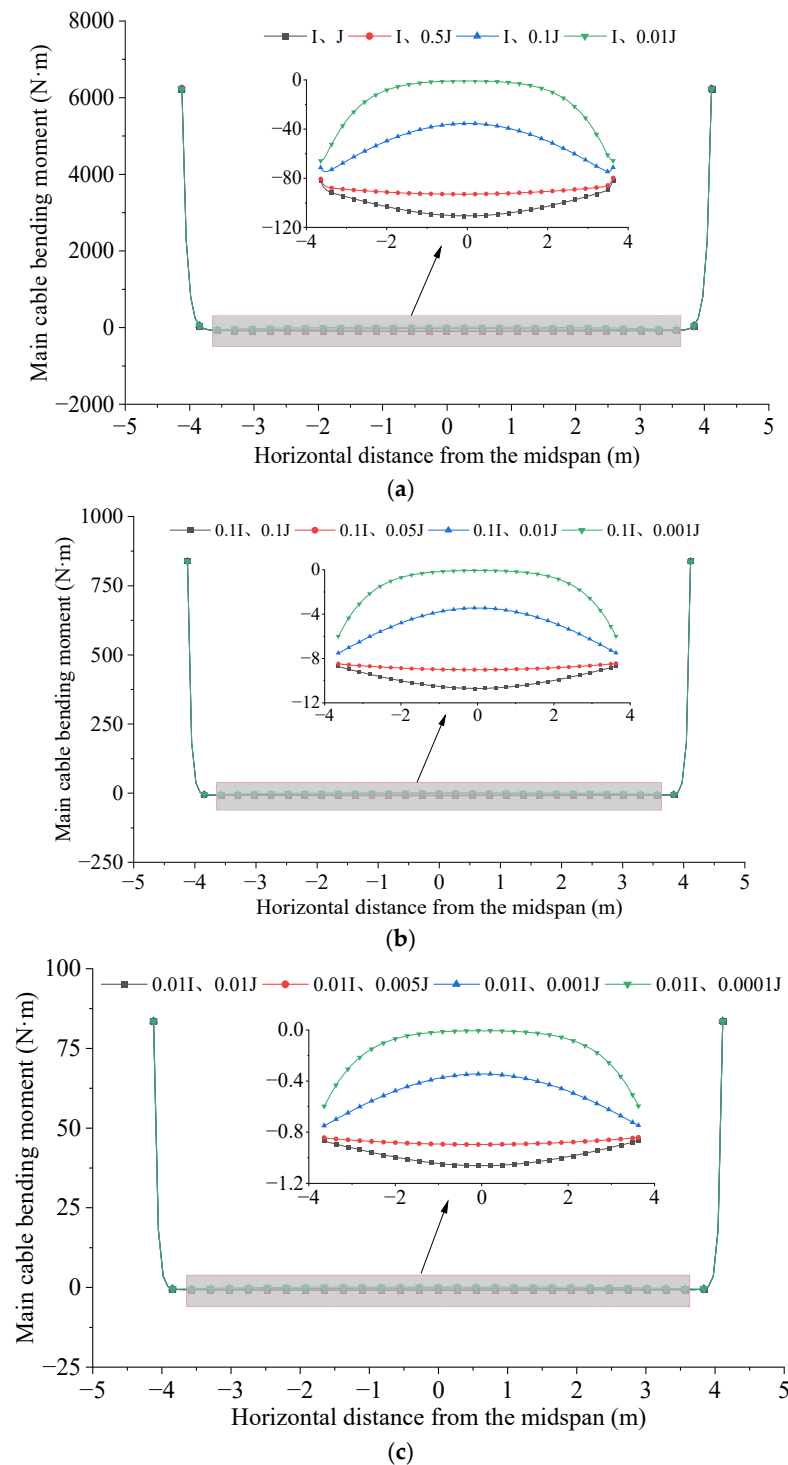


Figure 15. Bending moment of the main cable under various simulation cases. (a) The bending moment of the main cable when the moment of inertia is I . (b) The bending moment of the main cable when the moment of inertia is $0.1 I$. (c) The bending moment of the main cable when the moment of inertia is $0.01 I$.

It can be seen from Figure 15 that the bending moment of the main cable in the end area is larger and much larger than that in other positions. Near the end, the bending moment of the main cable with different polar moments of inertia is relatively close. When the moment of inertia is I in Figure 15a, the bending moment of the end is 6222.4~6226.2 N·m. In Figure 15b, when the moment of inertia is $0.1 I$, the end bending moment is 837.9~838.5 N·m; in Figure 15c, when the moment of inertia is $0.01 I$, the end

bending moment is 83.4~83.5 N·m. It can be seen that the end bending moment of the spatial cable mainly depends on its bending stiffness, and the influence of the change in the polar moment of inertia on the end bending moment of the spatial cable can be neglected.

Except for the end area (between $-3.7\sim 3.7\text{m}$), when the moment of inertia is constant, the bending moment of the main cable is related to the polar moment of inertia. The greater the polar moment of inertia, the greater the absolute value of the bending moment at the mid-span position.

It can be seen from the bending moment diagram that the bending deformation of the spatial main cable mainly occurs in the end area, which is consistent with the actual situation. Because the degrees of freedom in all directions of the main cable end are constrained, the end deformation is the most severe in the conversion process of the main cable from the vertical plane to the inclined plane. After the main cable is deflected to the inclined plane, its own linear shape is basically unchanged, only the transformation of spatial position, so the bending moment of the main cable is very small except the end.

5. Conclusions

This study develops a theoretical method for calculating the torsional angle of the main cable and verifies its validity through model tests and finite element analysis. The proposed method enables the determination of the torsional angle at any position along the cable based on its deflection angle, providing specific numerical guidance for the positioning and installation of cable clamps during the construction of spatial cable systems. The following conclusions are drawn:

(1) The distribution pattern of the uniformly distributed load—whether applied along the cable length or along the span length—has a negligible influence on the torsional angle of the main cable. The results calculated by the derived formulas under both assumptions are consistent, demonstrating that the torsional angle follows a fourth-order parabolic variation along the span.

(2) Deflection tests were conducted on both the main span and side span spatial cable models. The measured torsional angles along the span showed a variation trend consistent with theoretical predictions, confirming that the derived formulas can effectively estimate the torsional angle caused by cable deflection.

(3) Finite element analysis, modeling the main cable as a solid cross-section, revealed that the maximum torsional angle is smaller than the spatial deflection angle. When the polar moment of inertia was reduced to 0.01 times its original value, the torsional angle at midspan became equal to the spatial deflection angle, matching the theoretical value and slightly exceeding the measured torsional angle.

(4) Finite element results indicated that when the polar moment of inertia is relatively small, the torque distribution obtained from the model is consistent with theoretical predictions except in the end regions. Near the saddle supports, the main cable primarily resists bending moments, with the bending moment at the ends significantly larger than that at other locations.

It should be noted that the analysis of how bending and torsional stiffness affect the torsional angle of the main cable was conducted using a parametric finite element approach based on adjusted cross-sectional properties. This methodology primarily reveals qualitative influence patterns, whereas establishing a precise quantitative relationship between these parameters and the torsional angle may be considered for future investigation.

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