

Article

Study on Fracture Behavior of GH4169 Superalloy Considering Crack Closure Effect: Combining Numerical Modeling and BSL 3D DIC

Zechang Li ^{1,†}, Bin Kuang ^{2,†}, Bin Wang ³, Xing Sun ^{1,2}, Xinlong Yang ³, Bo Liu ¹, Qihong Fang ¹, Huimin Xie ⁴, Wei He ^{1,*} and Yanhuai Ding ^{2,*}

¹ State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan Provincial Key Laboratory of Intelligent Laser Manufacturing, Hunan University, Changsha 410082, China

² School of Mechanical Engineering and Mechanics, Xiangtan University, Xiangtan 411105, China

³ Aviation Powerplant Research Institute, AECC Hunan, Zhuzhou 412002, China

⁴ Applied Mechanics Laboratory, School of Aerospace Engineering, Tsinghua University, Beijing 100084, China

* Correspondence: hewei2008@hnu.edu.cn (W.H.); yhding@xtu.edu.cn (Y.D.)

† These authors contributed equally to this work.

Abstract

As a critical aerospace structural material, the fatigue crack propagation behavior and fatigue life of the nickel-based GH4169 superalloy are directly related to the service safety of engineering components. The crack closure effect is one of the key factors influencing the fatigue life of metallic materials. At present, the finite element method (FEM) is widely used to investigate fatigue crack propagation in metals. However, the commercial software ABAQUS 2021b employs the conventional Paris law for crack growth simulation, which neglects the influence of crack closure. In addition, ABAQUS cannot simultaneously perform fatigue life prediction and crack path prediction within a single numerical model. To overcome these limitations, the bi-prism-based single-lens (BSL) three-dimensional digital image correlation (3D DIC) technique was employed to experimentally investigate the crack closure behavior during fatigue crack propagation in GH4169 compact tension (CT) specimens. A new parameter, termed the crack opening ratio (COR), was introduced to quantitatively characterize the crack closure effect. Furthermore, a self-developed plugin was implemented on the ABAQUS platform through secondary development, enabling the numerical model to incorporate the influence of crack closure during fatigue crack propagation. The plugin automatically records the crack tip coordinates at each propagation step, calculates the stress intensity factors near the crack tip, and predicts the corresponding fatigue life, thereby integrating crack path prediction and fatigue life prediction within a unified framework. The results demonstrate that the COR effectively characterizes the crack closure effect in the numerical model, and the predicted fatigue life agrees with experimental results within an 11% deviation once the crack reaches a certain length.

Keywords: crack closure effect; BSL 3D DIC; numerical simulation methods; fatigue crack propagation; GH4169

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1. Introduction

Nickel-based GH4169 superalloy, as a high-performance critical material, is widely used in the manufacturing of turbine blades, high-temperature components for gas turbines, and core parts for nuclear reactors due to its excellent service performance. It holds an important position in the aerospace and power engineering fields [1]. However, during complex service conditions, fatigue cracks inevitably initiate and propagate in these components, eventually leading to failure and posing significant safety risks. Therefore, in-depth investigation of the fatigue behavior and crack propagation mechanisms of GH4169 is of considerable scientific and engineering importance [2,3].

In recent years, the combination of in situ experimental techniques with numerical simulation methods has emerged as an important approach for analyzing fatigue behavior [4]. Among numerical methods, the extended finite element method (XFEM) has shown unique advantages in fatigue crack propagation research due to its ability to handle discontinuities without the need for remeshing. XFEM is commonly used to simulate the stress field near the crack tip, calculate stress intensity factors (SIFs) [5], and predict crack propagation rates [6], crack propagation paths [7], or structural life [8]. For example, Ghandriz et al. [9] used XFEM to study the fracture behavior of additively manufactured polymer materials with different layering orientations, developing an anisotropic cohesive zone model capable of predicting the crack propagation paths of additively manufactured materials. Chen et al. [10] proposed a three-dimensional crack surface direct update strategy that does not require level set updating. This strategy integrates interaction integral methods with cyclic overload hysteresis models, allowing accurate prediction of crack paths and efficient calculation of fatigue life for structures subjected to complex loading spectra.

The discovery of the crack closure effect in the 1970s triggered widespread research [11]. The use of XFEM to analyze crack closure effects has become a common approach in modern studies [12,13]. Crack opening displacement (COD) is a widely adopted indicator for characterizing the crack closure effect [14–16]. In experimental studies, digital image correlation (DIC) technology has become a core method for acquiring crack tip mechanical parameters, including COD [17], SIFs [18], and displacement fields [19] due to its advantages of non-contact, full-field measurement, and high precision. Based on these advances, previous studies introduced a new parameter, the crack opening ratio (COR), to more intuitively characterize the crack closure effect [20,21]. Combining 3D DIC with fatigue fracture mechanics has provided new insights into crack growth behavior in high-temperature alloys. Xiao B. [22] revealed the mechanism of overload-induced crack closure effect in GH4169 under single-peak overload at elevated temperatures, while He W. [23] further utilized the COR technique based on bi-prism-based single-lens (BSL) 3D DIC to quantify the crack closure effect of GH4169.

In this study, a custom ABAQUS plugin was developed through secondary programming to simulate the fatigue crack propagation behavior of GH4169 compact tension (CT) specimens at room temperature. The overall framework of this study is illustrated in Figure 1. First, the methods implemented in the plugin are introduced, including the calculation of SIFs, the characterization of crack closure effect based on COR, the determination of the crack opening load ratio (COLR) and crack opening load (COL), and the crack tip localization technique used in the numerical analyses. Subsequently, a systematic experimental program was conducted to obtain the constitutive data of the GH4169 superalloy, modify Paris law to account for the effect of crack closure, and acquire the COLR and COL of the actual specimens to validate the accuracy of the numerical model. In the numerical simulations, crack growth was calculated using the modified the Paris law. During this process, the plugin automatically updates and records the crack tip coordinates, crack growth increment Δa , and the corresponding number of cycles ΔN , while simultaneously

calculating the SIFs around the crack tip and extracting the displacement fields after each propagation step. Based on the obtained crack tip coordinates and displacement field data, the COD and COR are further calculated to characterize the crack closure effect in the numerical model. The COL and effective stress intensity factor are then determined, enabling prediction of the remaining fatigue life of the specimens.

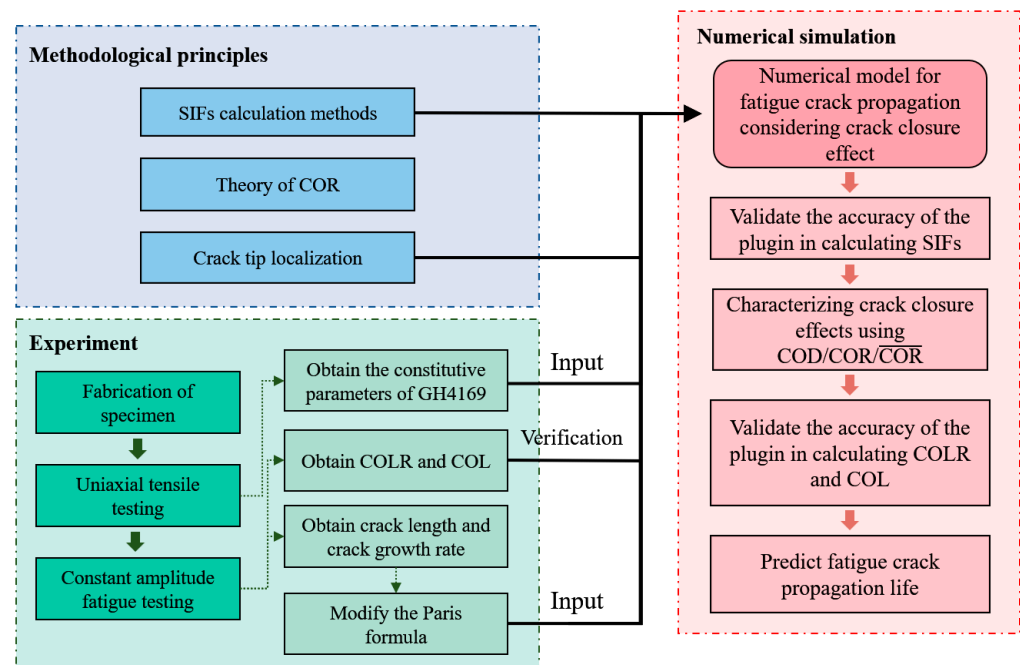


Figure 1. The framework and experimental procedure route of the present study.

2. Methodological Principles

2.1. SIF Calculation Method

SIFs are critical parameters that characterize the strength of the stress singularity in the vicinity of a crack tip. To accurately extract SIFs from experimentally obtained full-field displacement data, it is essential to employ a mathematical model that is consistent with the asymptotic displacement field near the crack tip.

The Williams series expansion provides a complete analytical description of the displacement field in the vicinity of a crack tip [24]. In this formulation, the displacement field is expressed as an infinite series of functions of the polar coordinates r and θ , multiplied by a set of undetermined coefficients. Among these coefficients, the lowest-order terms ($n = 1$) are linearly related to the mode I and mode II SIFs. Based on this theoretical framework, Yoneyama et al. [25] developed a practical methodology for extracting mixed-mode SIFs from experimental data. In this approach, the discrete displacement components (u_i, v_i, w_i) obtained from DIC measurements, defined with the crack tip as the origin, are substituted into a truncated form of the Williams series expansion (as given in Equations (1)–(3)). In addition to the coefficients associated with the SIFs, rigid-body displacement terms (u_0, v_0, w_0) are included to account for any overall rigid-body motion present during the experiment.

$$v_i = \sum_{n=1}^m \frac{r_i^{\frac{n}{2}}}{2G} \left\{ \begin{array}{l} a_n \left[\left(k - \frac{n}{2} + (-1)^n \right) \sin \frac{n}{2} \theta_i - \frac{n}{2} \sin \left(\frac{n}{2} - 2 \right) \theta_i \right] \\ + b_n \left[\left(k - \frac{n}{2} + (-1)^n \right) \right] \cos \frac{n}{2} \theta_i + \frac{n}{2} \sin \left(\frac{n}{2} - 2 \right) \theta_i \end{array} \right\} \quad (1)$$

$$u_i = \sum_{n=1}^m \frac{r_i^{\frac{n}{2}}}{2G} \left\{ \begin{aligned} &a_n \left[\left(k + \frac{n}{2} + (-1)^n \right) \cos \frac{n}{2} \theta_i - \frac{n}{2} \cos \left(\frac{n}{2} - 2 \right) \theta_i \right] \\ &- b_n \left[\left(k + \frac{n}{2} - (-1)^n \right) \right] \sin \frac{n}{2} \theta_i - \frac{n}{2} \sin \left(\frac{n}{2} - 2 \right) \theta_i \end{aligned} \right\} \quad (2)$$

$$w_i = \sum_{n=1}^m \frac{2r_i^{\frac{n-1}{2}}}{G} [c_n \sin(n - \frac{1}{2})\theta_i] \quad (3)$$

Figure 2 illustrates the local crack tip coordinate system adopted in this work. For the plane stress condition of the CT specimen under investigation, the primary displacements of interest are u_i and v_i . Therefore, Equations (1) and (2) are utilized in our analysis to determine the mode I SIF K_I and mode II SIF K_{II} . The general form including the out-of-plane term w_i (Equation (3)) is provided for completeness, acknowledging its relevance for three-dimensional or anti-plane shear problems. G is the shear modulus, and $k = (3 - \mu)/(1 + \mu)$ is the Kolosov constant under the plane stress assumption. The CT specimen studied in this paper belongs to the plane stress condition. The parameter m specifies the order of the Williams series expansion, which is set to 4th order in this study [26]. The coefficients, determined via least-squares fitting, are directly correlated with the SIFs [25]. Ultimately, the K_I and K_{II} can be determined from experimentally acquired displacement field data through this computational framework:

$$K_I = \sqrt{2\pi} a_n \quad (4)$$

$$K_{II} = -\sqrt{2\pi} b_n \quad (5)$$

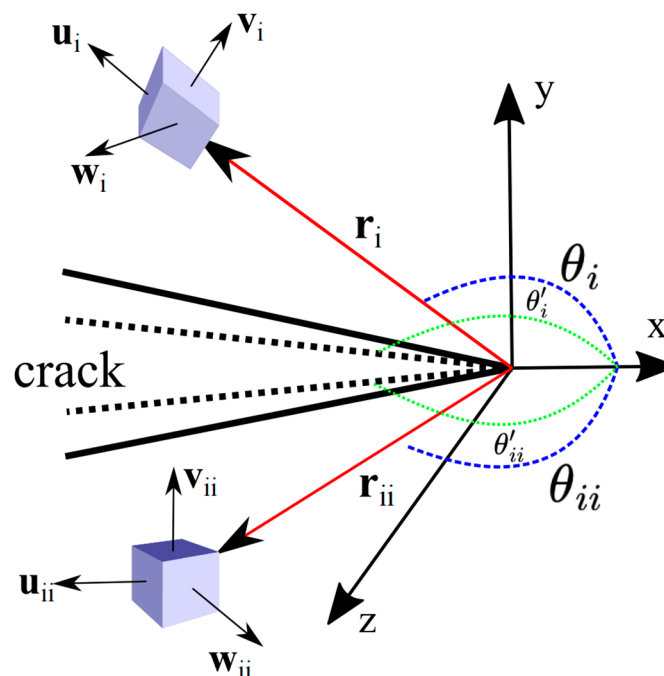


Figure 2. Schematic of the local crack tip coordinate system and displacement field components.

The theoretical mode I SIF $K_{I_{theo}}$ (denoted as K_{theo}) for the CT configuration employed in this study, which conforms to the ASTM E647 Standard Test Method for Measurement of Fatigue Crack Growth Rates, is given by [27]:

$$K_{\text{theo}} = \frac{P}{B\sqrt{W}} \frac{2+\alpha}{(1-\alpha)^{\frac{3}{2}}} (0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4) \quad (6)$$

In this equation, P represents the instantaneous applied load; in the present study, the maximum load (P_{max}) and minimum load (P_{min}) were used in Equation (6) to calculate the maximum and minimum stress intensity factors, K_{max} and K_{min} ; and B denotes the specimen thickness, $\alpha = a/W$, where a corresponds to the crack length and W signifies the specimen width.

2.2. Crack Opening Ratio

Duan Q Y et al. [21] proposed two novel parameters, the COR and $\overline{\text{COR}}$, to evaluate fatigue crack closure states. As illustrated in Figure 3, δ_{st}^n represents the crack opening displacement measured at a specific location n behind the crack tip at a certain moment t during the fatigue cycle. This value is directly obtained using DIC with a virtual extensometer and reflects the instantaneous crack opening at location n at time t . Meanwhile, δ_{max}^n denotes the maximum crack opening displacement at the same location n within the same fatigue cycle, corresponding to the moment when the crack is fully opened. It represents the peak opening at that position during the cycle. Accordingly, the COR is defined as:

$$\text{COR}_n(st) = \frac{\delta_{st}^n}{\delta_{\text{max}}^n} \times 100\% \quad (7)$$

$$\overline{\text{COR}}(st) = \frac{\text{COR}_1(st) + \text{COR}_2(st) + \dots + \text{COR}_n(st)}{n} \quad (8)$$

COR indicates the percentage of the instantaneous crack opening at location n and time t relative to the maximum opening in the same cycle, ranging from 0% to 100%. When $\text{COR} = 0\%$, the crack is fully closed at that location; when $\text{COR} = 100\%$, the crack has reached its maximum opening state. This parameter is independent of the measurement location and provides a robust characterization of the crack closure level.

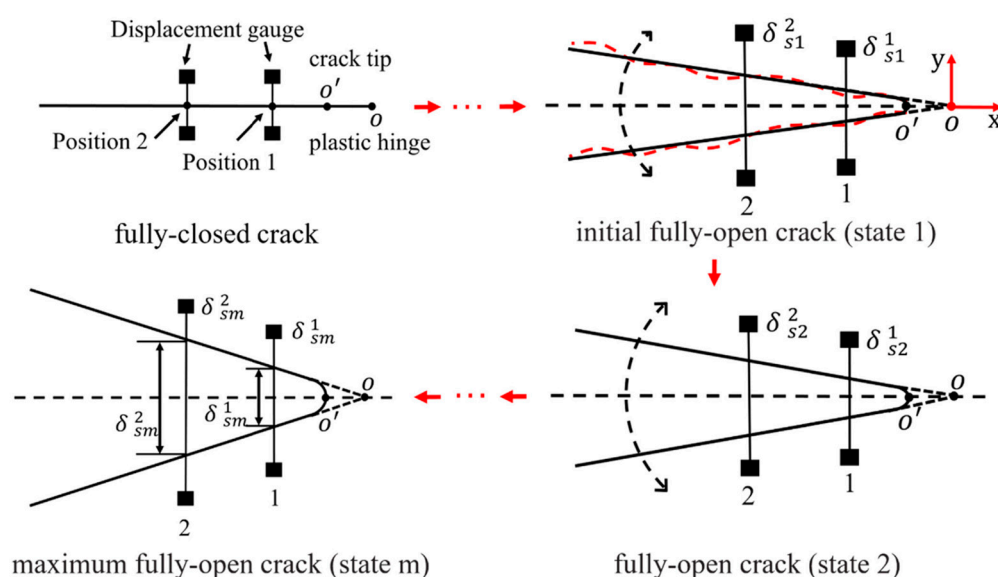


Figure 3. Crack opening process [21].

Based on the COR theory, they further proposed that using COR to characterize the crack closure effect exhibits strong robustness. When the crack closure effect exists, the

slopes of the curves describing the relationships between load and COD, COR, or $\overline{\text{COR}}$ continue to change before the crack is fully opened. However, once the crack is fully open, the applied load $P(t)$ and COD, COR, or $\overline{\text{COR}}$ exhibit a linear relationship with a constant slope. At this stage, the intersection of the extrapolated linear fit with the horizontal axis corresponds to the COLR, which is related to the COL P_{open} by the following expression:

$$P_{\text{open}} = \text{COLR} \times P_{\text{max}} \quad (9)$$

2.3. Crack Tip Location Method

Crack tip localization plays a crucial role in fatigue fracture mechanics, as it provides the basis for accurately characterizing crack closure effects, calculating SIFs, and predicting structural fatigue life. In the experimental program of this study, a novel crack tip localization method based on an immune algorithm was introduced [28]. This algorithm is established on the theoretical framework of the Williams displacement field model, which effectively correlates the crack tip displacement field, crack tip position, and fracture mechanics parameters, thereby enabling automatic crack tip localization with a maximum error of only 0.14%, as shown in Figure 4. The immune algorithm formulates the crack tip localization task as an optimization problem. In this framework, the antigen corresponds to the objective function, defined as the discrepancy between the experimentally measured DIC displacement field U_{exp} and the theoretical displacement field $F(X_c, Y_c)A$, derived from the Williams displacement field model. The antibodies represent candidate crack tip coordinates (X_c, Y_c) . The algorithm first randomly generates a set of initial antibodies within the region of interest (ROI), corresponding to potential crack tip locations, thereby forming the initial population. The objective function value (i.e., affinity) of each antibody is then evaluated, with smaller values indicating a higher degree of matching. Antibodies with high affinity are selected and subjected to cloning, mutation, and selection operations, enabling iterative evolution of the population.

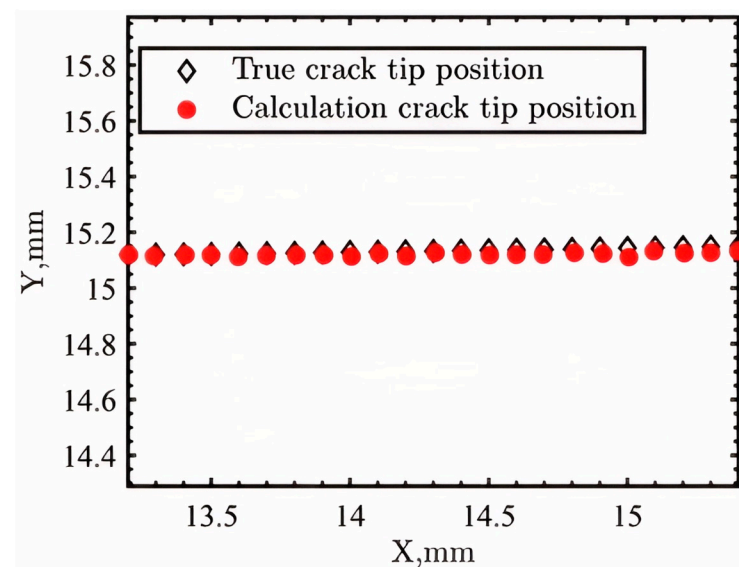


Figure 4. Accuracy of crack tip localization based on an immune algorithm [28].

This process mimics the adaptive mechanism of the biological immune system, allowing the algorithm to effectively avoid local optima without requiring an accurate initial guess. Ultimately, the population converges globally to the optimal solution that minimizes the objective function, yielding an accurate crack tip location. During experimental data processing, this method enables rapid extraction of crack tip positions from DIC images, significantly improving the efficiency of obtaining key parameters such as crack

opening load and crack growth rate, thereby providing reliable support for subsequent analyses of fatigue crack behavior.

The following section presents the crack tip coordinate updating method employed in the plugin developed in this study. The design of the plugin is based on finite element modeling using the maximum circumferential stress criterion. The fundamental assumption of this criterion is that crack propagation initiates at the crack tip and proceeds in the direction of the maximum principal tensile stress in the tangential direction, as illustrated in Figure 5, where the X-axis is aligned parallel to the specimen edge. The formula for calculating the crack deflection angle φ_0 is expressed as follows [29]:

$$\varphi_0 = \arccos\left(\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right) \quad (10)$$

The crack deflection angle is calculated using the SIFs obtained through the methodology described in Section 2.1. Subsequently, the finite element model iteratively updates the crack tip coordinates (X_1, Y_1) for the next propagation step according to the user-defined crack growth increment Δa and Equations (11) and (12). In this study, the value of Δa is set to 0.05 mm, which is approximately three times the mesh size.

$$X_1 = X_0 + \Delta a \quad (11)$$

$$Y_1 = Y_0 + \Delta a \times \varphi_0 \quad (12)$$

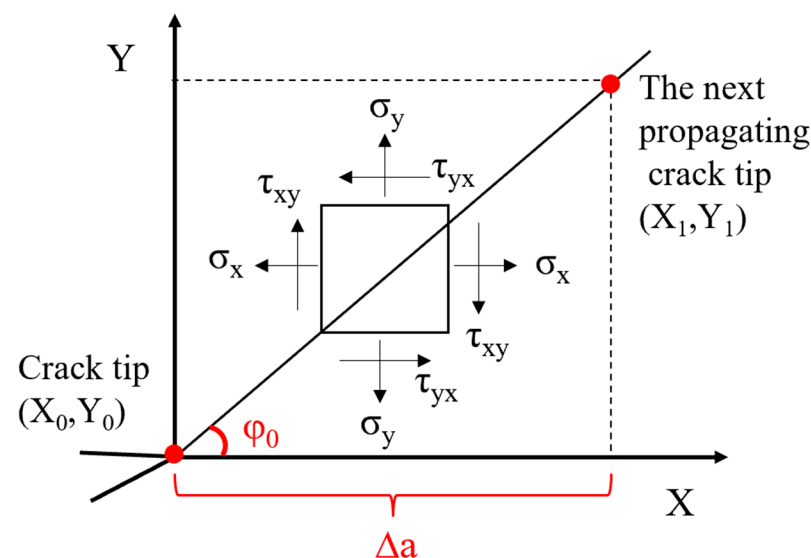


Figure 5. Crack propagation angle.

3. Experimental Steps

3.1. Sample Preparation

The GH4169 superalloy employed in this study was in accordance with the GB/T 14992-2005 standard [30], and its chemical composition is listed in Table 1. To eliminate residual stresses in the base material, the raw material underwent the solution heat treatment process illustrated in Figure 6. As shown in Figure 6a, the base material was placed in a high-temperature furnace for primary solution heat treatment at $1160\text{ }^{\circ}\text{C} \times 12\text{ h}$, followed by air cooling. The treated base material was subsequently processed into standard CT specimens (designed according to GB/T 6398-2017 “Metallic materials—Standard test method for fatigue crack growth rates” [31]) through a wire electrical discharge machining (WEDM) process, as shown in Figure 7. Subsequently, the CT specimens underwent a

secondary solution heat treatment as depicted in Figure 6b, employing a solution treatment at $1160\text{ }^{\circ}\text{C} \times 2\text{ h}$, followed by air cooling.

Table 1. Chemical composition and content (%) of GH4169.

Ni	Mn	Si	Cu	Cr	Mo	Al	Ti
52.15	0.054	0.13	0.028	19.4	3.1	0.78	1.11
Nb	Co	B	C	P	S	Fe	/
5.12	0.06	0.005	0.037	0.001	0.004	Rest	/

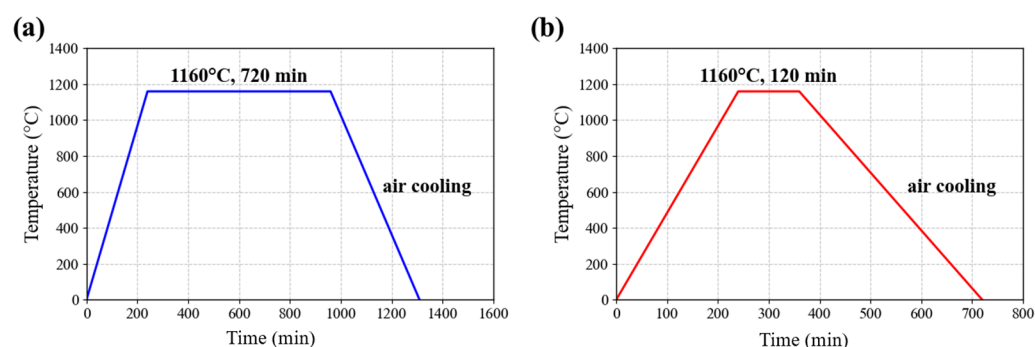


Figure 6. Heat treatment process temperature profile: (a) heating curve of solid solution heat treatment before wire cutting; (b) heating curve for solid solution heat treatment after wire cutting.

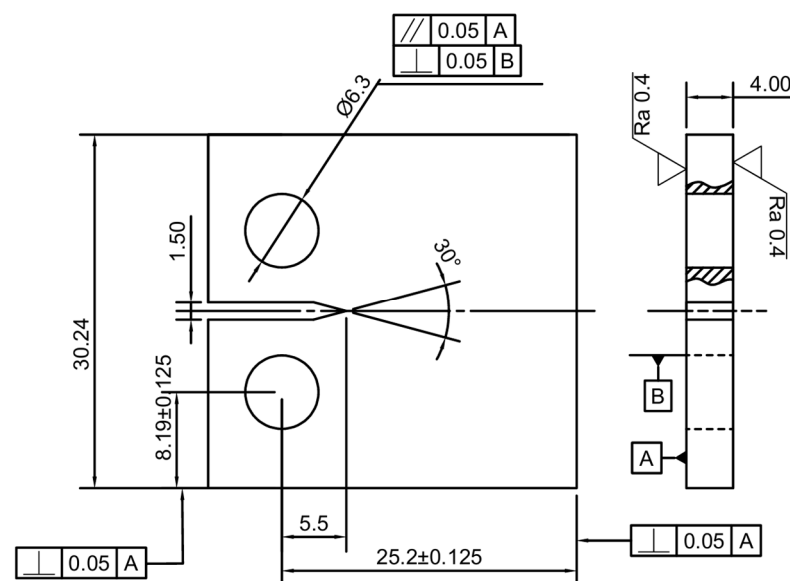


Figure 7. CT specimen size of nickel-based superalloy GH4169 (Unit: mm).

3.2. Sample Mechanical Parameters

Uniaxial tensile tests were conducted on GH4169 superalloy using an MTS high- and low-temperature electromechanical universal testing machine in accordance with the GB/T 228.1 standard [32]. The fundamental mechanical properties obtained from the tensile tests are summarized in Table 2. The corresponding true stress–strain curves are presented in Figure 8.

Table 2. Fracture mechanics parameters of heat-treated GH4169 material.

Treatment	Young's Modulus	Poisson's Ratio	Yield Stress	Yield Strain
Before	189 GPa	0.3	730 MPa	0.4573%
After	187 GPa	0.3	804 MPa	0.6282%

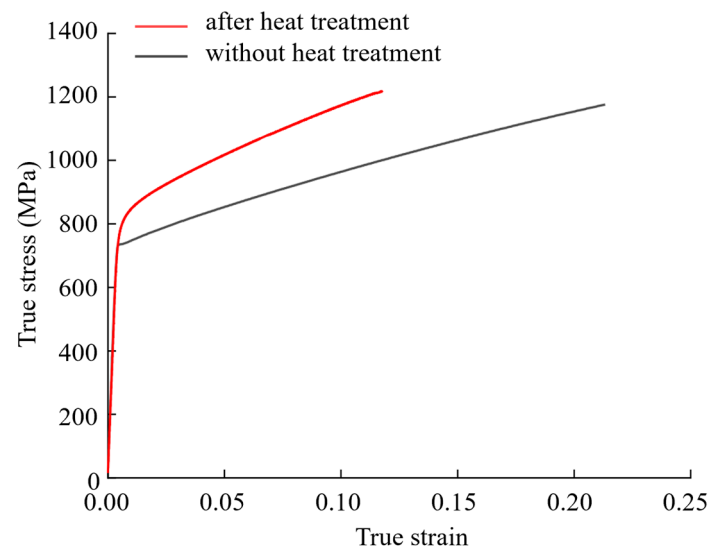


Figure 8. True stress–strain curve.

3.3. Constant Amplitude Fatigue Test

3.3.1. BSL 3D DIC System

The experimental setup employed a BSL 3D DIC system for image acquisition. As specifically configured in Figure 9, the system comprised a CMOS camera (Daheng MER2-502-79U3M, Beijing, China), a double telecentric lens (Xenoplan 1:5), and a biprism.

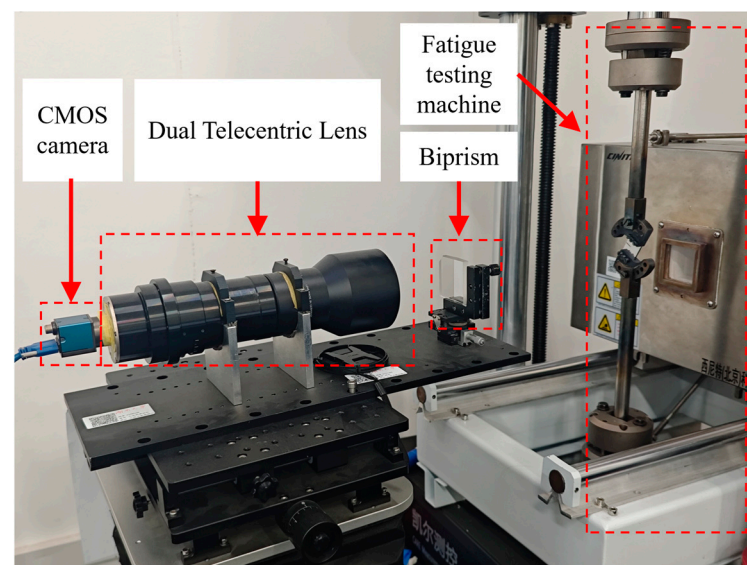


Figure 9. BSL 3D DIC system.

3.3.2. Constant Amplitude Fatigue Test Parameters

Drawing upon our previous work [23], a variable-frequency image acquisition strategy was implemented to achieve load-image synchronization. As illustrated in Figure 10, the loading process comprises two phases: During conventional fatigue loading, sinusoidal waveform loading ($f = 10$ Hz, $R = 0.1$) is applied with loads ranging from 300 N to 3000 N. The image acquisition phase employs frequency reduction to 0.05 Hz combined with stepwise load control where each 100 N load increment triggers synchronized image capture by the BSL 3D DIC system. Speckle patterns were sprayed on the specimen to facilitate the extraction of displacement fields during the loading process [33]. Based on the acquired images, the displacement field data matrix of the ROI is obtained through DIC

processing. By combining this matrix with the immune algorithm, accurate crack tip localization can be achieved. Virtual displacement extensometers (VDEs) are arranged within the ROI, and the extracted data are further used to calculate COD and COR. Figure 11 illustrates the arrangement of VDEs within the ROI: each square represents one VDE. Two rows of VDEs are placed along the upper and lower sides of the crack surface, with 10 VDEs in each row. In the X-direction, the nearest distance between the VDEs and the crack tip is set to 2 pixels, and subsequent VDEs are placed at intervals of 2 pixels. In the Y-direction, the distance between the VDEs and the crack surface is set to 6 pixels. Moreover, the ROI is defined as 180×150 pixels, with a subset radius of 26 pixels, a subset step size of 0 pixels, and a spatial resolution of $17.25 \mu\text{m}/\text{pixel}$.

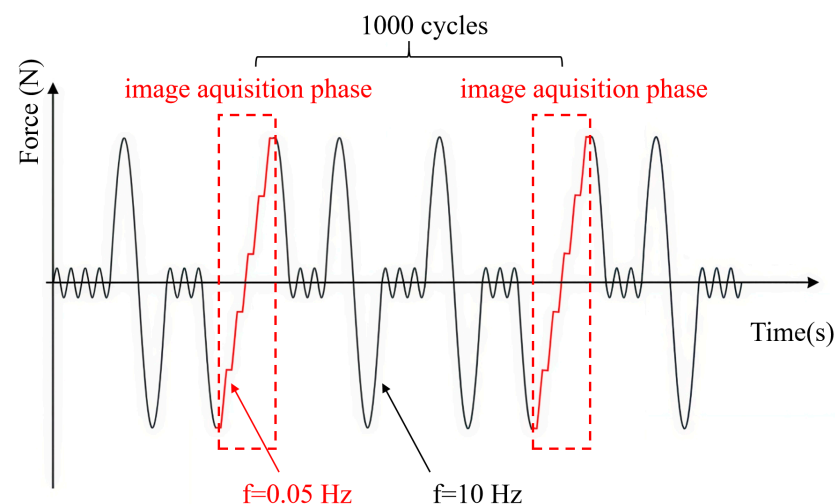


Figure 10. Load-DIC measurement strategy during fatigue experiment.

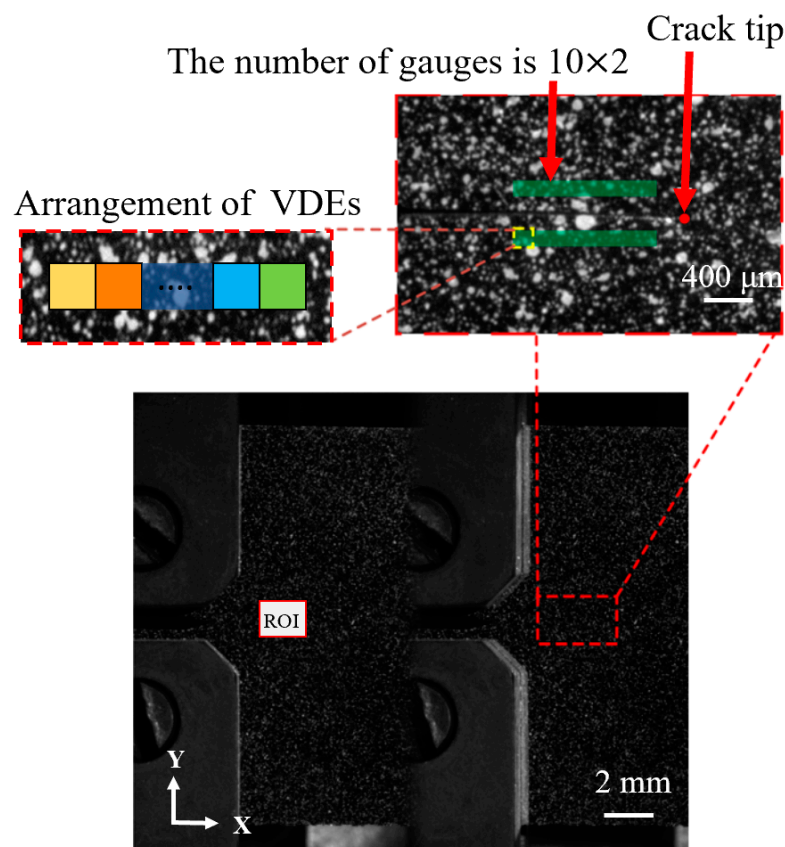


Figure 11. Arrangement of VDEs on both sides of cracks.

4. Results and Discussion

4.1. Fatigue Experiment Results

Through experiments, we obtained the variation pattern of the $\overline{\text{COR}}$ with normalized load at different crack lengths during the crack propagation process of the GH4169 CT specimen, as shown in Figure 12. The $\overline{\text{COR}}$ is defined as the mean value of COR at multiple positions behind the crack tip. Based on the fundamental theory of characterizing the crack closure effect using the COR, it can be observed from Figure 12a that with increasing crack length, the slope of the $\overline{\text{COR}}$ -load curve becomes steeper, indicating a more pronounced crack closure effect. This suggests that during crack propagation, the influence of the crack closure effect on the deformation behavior near the crack tip progressively increases. Figure 12b illustrates the variation of COLR under different loading cycles. Before the transition to the rapid crack growth stage, the COLR of the specimen remains relatively stable. Once the rapid crack growth stage is reached, however, the COLR decreases rapidly until it eventually disappears.

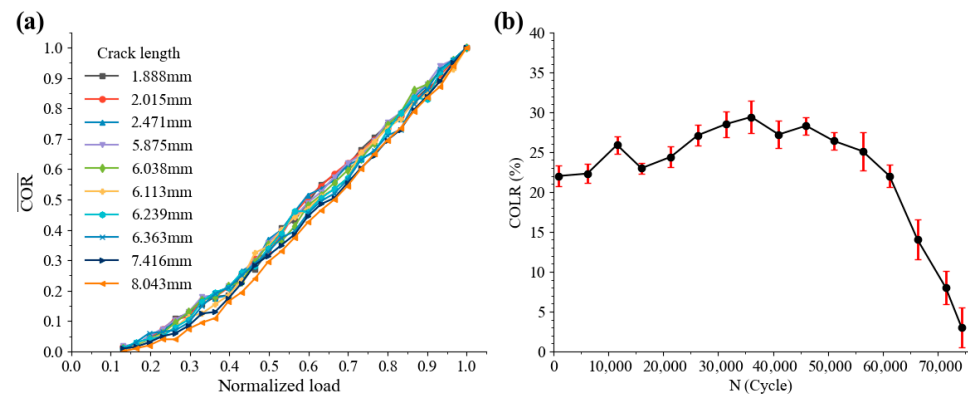


Figure 12. (a) $\overline{\text{COR}}$ -load curves at different crack lengths; (b) the COLR- N curves.

Meanwhile, the crack length (a) and crack growth rate (da/dN) were systematically measured through experiments, and their variations with the number of loading cycles are shown in Figure 13. The classical Paris law can be expressed as $\lg\left(\frac{da}{dN}\right) = m \cdot \lg(\Delta K) + \lg(c)$, where the parameters m and c are typically obtained by fitting the experimentally measured relationship between da/dN and ΔK . To modify the Paris law by incorporating the crack closure effect, the effective SIF range ΔK_{eff} was employed in place of ΔK in the classical formulation [27,34], with $\Delta K_{eff} = K_{max} - K_{open}$, where K_{open} denotes the crack-opening SIF. Accordingly, by applying nonlinear regression analysis using the least-squares method on the experimental data, as shown in Figure 14, the modified Paris law was obtained: $\frac{da}{dN} = 23.2491 \times \Delta K_{eff}^{2.5882}$, where da/dN is in mm/cycle and ΔK_{eff} is in $\text{MPa} \cdot \text{m}^{1/2}$.

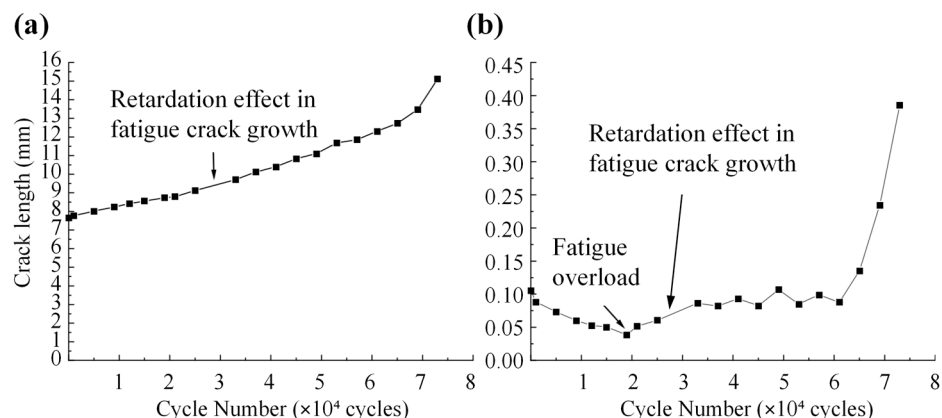


Figure 13. Crack growth manifestations: (a) relationship between crack length and number of cycles; (b) relationship between crack growth rate and number of cycles.

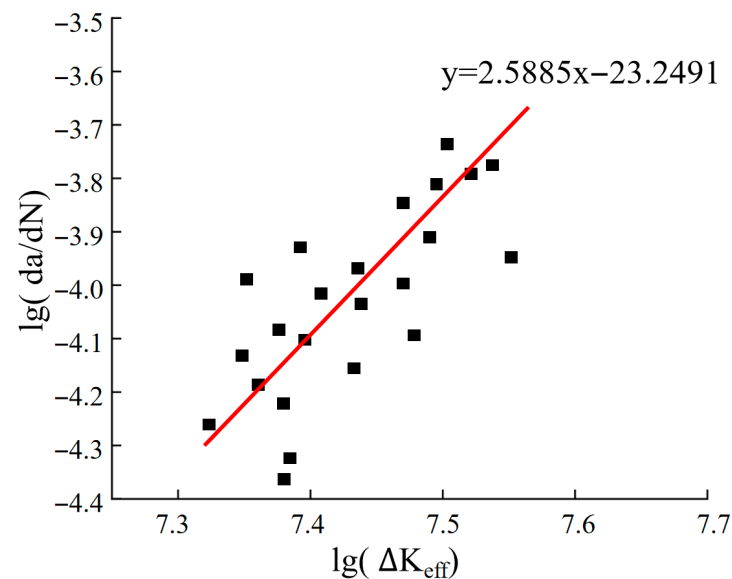


Figure 14. Modified Paris law.

4.2. Characterization of Crack Closure Effect

To simulate the crack closure effect observed during the experimental process, a new numerical simulation plugin was developed in ABAQUS in this work. This plugin enables the extraction of multiple mechanical parameters during the fatigue crack propagation process of GH4169 CT specimens and automatically updates the crack tip position. The overall framework of the numerical simulation is illustrated in Figure 15.

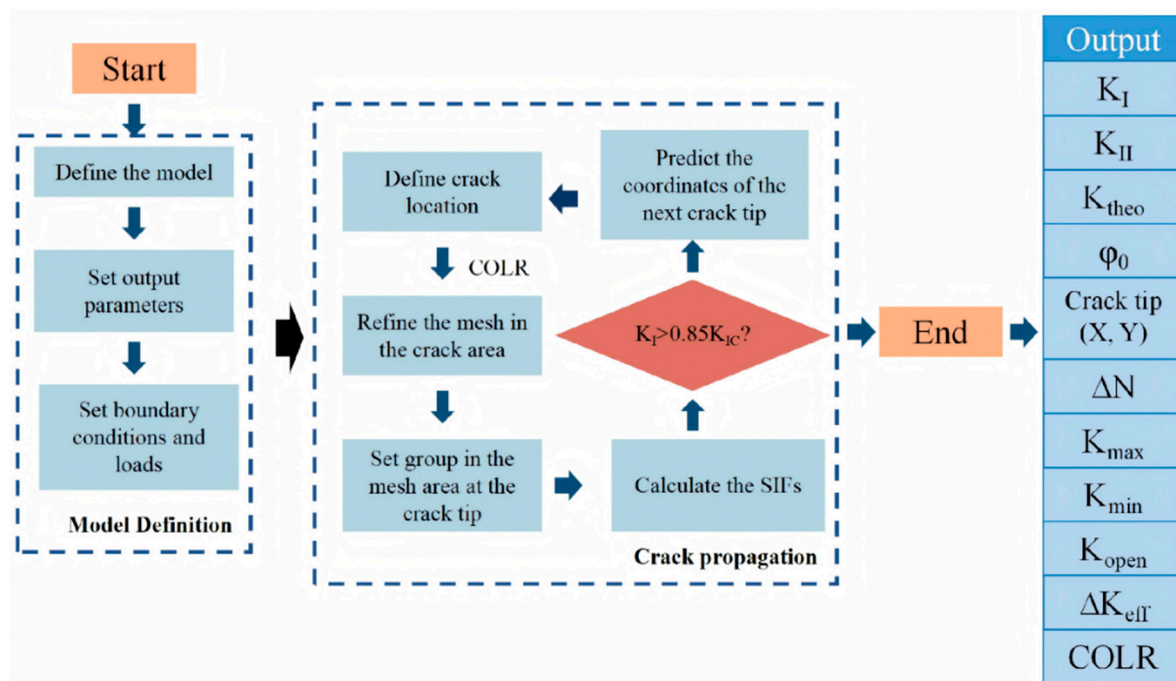


Figure 15. Flowchart of the numerical simulation method considering the crack closure effect.

The functionality of the plugin is mainly divided into two parts: model definition and crack propagation. In the model definition stage, the specimen geometry, material parameters, boundary conditions, initial crack position, and mesh generation are first defined.

The plugin then imports the material's true stress–strain data file and the COLR reference values obtained in Section 3.3.2. In this stage, the mesh type in the crack propagation region is set to CPE4R: a four-node bilinear plane strain quadrilateral element with reduced integration and hourglass control—with an element size of 15 μm . During the crack propagation stage, a nonlinear analysis considering the crack closure effect is performed. The plugin extracts the displacement field near the crack tip and calculates the SIFs based on the method described in Section 2.1. The new crack tip coordinates are then determined according to the procedure presented in Section 2.3. The number of load cycles ΔN required for a given crack extension Δa is computed using the modified Paris law, and the process iterates as the crack gradually propagates. When the calculated K_I exceeds 0.85 times the fracture toughness K_{IC} , the crack propagation becomes unstable, indicating that the structure has entered a high-risk state. At this point, the simulation automatically terminates, and the corresponding result data are exported.

We validated the accuracy of the plugin in calculating SIFs. As shown in Figure 16a, the red region represents a circular region of interest (ROI) with a radius of 1.2 mm centered at the crack tip. The program automatically exports nodal displacements and coordinates within this region to compute SIF K_I using the Williams formula. Figure 16b presents the comparison between K_I and K_{theo} , each as a function of crack length. The results demonstrate good agreement between experimental and theoretical values, confirming the high computational accuracy of this method.

In this study, the SIFs were extracted using a least-squares fitting approach based on the Williams series expansion and full-field displacement data (Yoneyama method), and its agreement with theoretical solutions is validated in Figure 16. This method can directly utilize full-field displacement data obtained from DIC, thereby avoiding the positional sensitivity errors that may arise from single-point measurements such as strain gauges. Moreover, by simultaneously fitting rigid-body displacement terms, it effectively eliminates the interference caused by unavoidable global rigid-body motion in experiments. However, the application of this method entails certain prerequisites and limitations. Specifically, it relies on the assumption of linear elasticity; therefore, the ROI selected for fitting must lie within the K-dominant zone near the crack tip, where plastic deformation is negligible. In this study, the ROI radius was set to 1.2 mm (approximately 3–5 times the plastic zone size) to satisfy this requirement. Other SIF evaluation methods, such as J-integral-based equivalence or crack face displacement extrapolation, have their respective applicable scenarios. The full-field displacement fitting method employed here, owing to its natural compatibility with digital image correlation techniques and strong resistance to noise, represents a powerful tool for characterizing the crack tip fields under both static and fatigue conditions in experimental and numerical analyses.

The COD, COR, and $\overline{\text{COR}}$ versus normalized load (P/P_{max}) curves obtained from this model are presented in Figure 17. By performing linear fitting and extrapolation of data from different nodes on the crack opening plastic hinge, the intersection point with the x-axis corresponds to the COLR. As shown in Figure 17a, numerical simulation results derived from COD-normalized load curves exhibit COLR variations within the range of 21.64% to 32.44%. In contrast, the COLR values obtained through COR-normalized load and $\overline{\text{COR}}$ -normalized load curves are 27.29% and 27.17%, respectively. These results clearly demonstrate that the COR and $\overline{\text{COR}}$ parameters provide more accurate characterization of crack closure effects.

This superior stability of COR can be attributed to its intrinsic physical definition. While COD measures the absolute separation at a discrete point, which is influenced by both the true crack face opening and the local elastic deformation of the surrounding material, COR is defined as the ratio of COD to its maximum value at the same point. This normalization inherently minimizes the contribution of bulk elastic compliance, making

COR primarily sensitive to the relative separation of the crack faces—the direct manifestation of the closure effect. Consequently, the transition from a nonlinear to a linear load–COR relationship is more abrupt, enabling a more precise and consistent identification of the crack opening load.

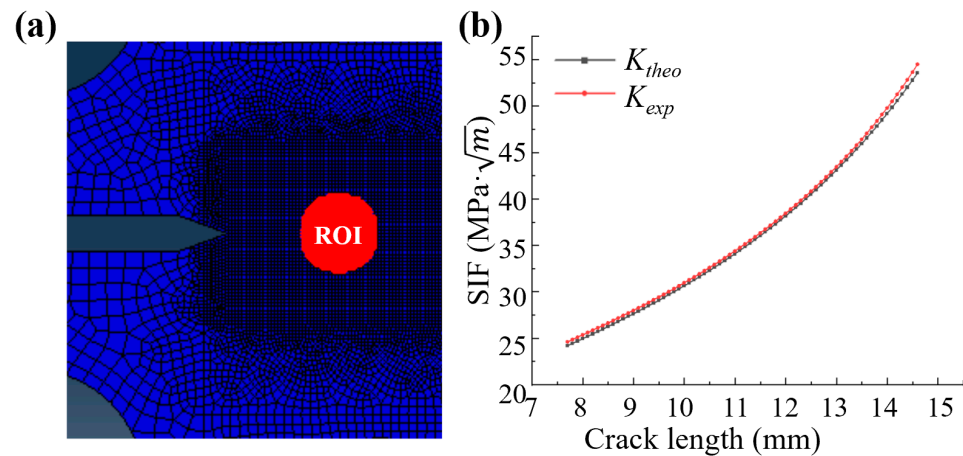


Figure 16. (a) ROI near the crack tip; (b) comparison of accuracy of SIF.

In summary, the results confirm that both COR and $\overline{\text{COR}}$ provide a more accurate, robust, and physically grounded characterization of crack closure effects compared to the conventional COD-based method. Their use leads to a more reliable determination of the COL, which is critical for accurately calculating the ΔK_{eff} in fatigue life predictions.

We validated the accuracy of crack opening load data calculated by the plugin. Figure 18 demonstrates good agreement between experimental fatigue test data and numerical simulation results. Minor fluctuations observed in the 11–14 mm crack length range are attributed to experimental errors. This comparison confirms the reliability and effectiveness of the proposed numerical simulation methodology for subsequent analysis.

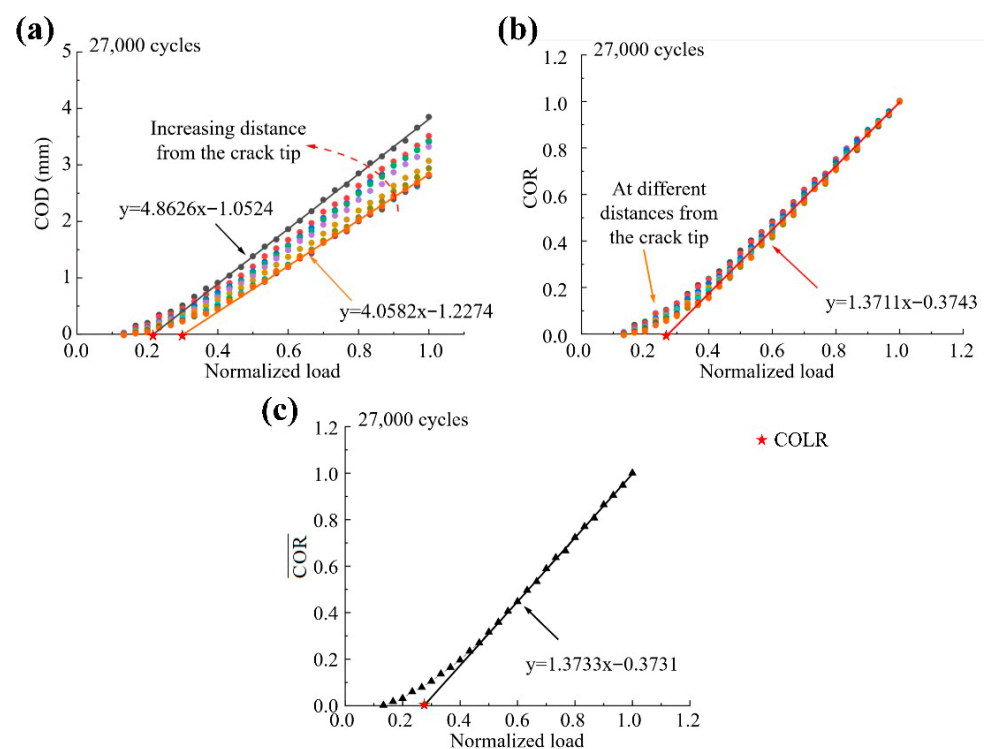


Figure 17. Characterization of crack closure effect: (a) COD–load variation curve, (b) COR–load variation curve, and (c) $\overline{\text{COR}}$ –load variation curve.

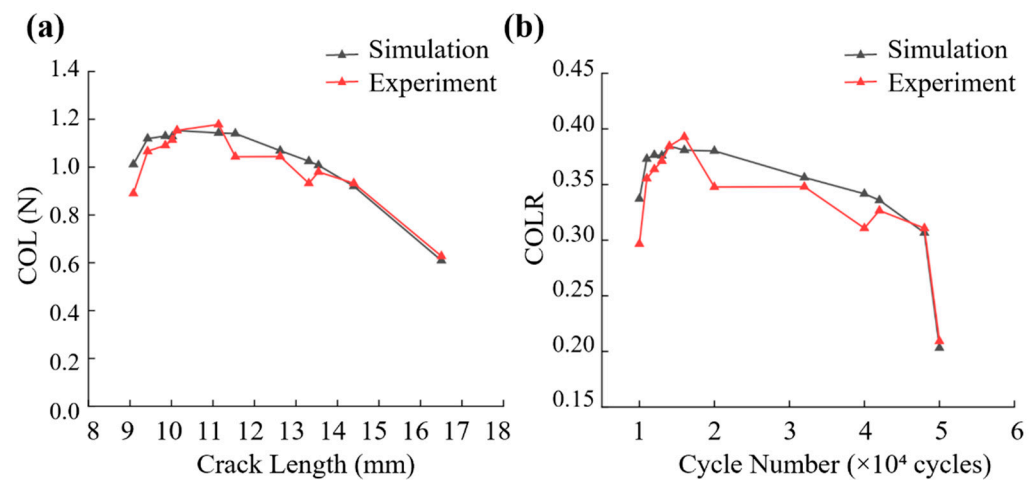


Figure 18. Comparison of the accuracy of crack opening loads: (a) variation curve of crack opening load with crack length; (b) variation curve of COLR with crack length.

4.3. Crack Extension Life Analysis

Finally, fatigue tests were conducted on specimens of the same material and structure, and the experimental results were compared with the predictions from the numerical simulations, as shown in Figure 19a. The program-predicted fatigue life was 85,000 cycles, corresponding to a final crack length of 15 mm, whereas the experimentally measured fatigue life was 77,000 cycles, with a final crack length of 14.5 mm at specimen fracture. Figure 19b presents the error analysis between the numerical and experimental fatigue lives. It can be observed that when the crack length reaches 10 mm, the deviation in fatigue life prediction between the simulation and experimental results is approximately 11%. This overestimation primarily stems from the inherent simplifications of the numerical model compared with physical reality. Several factors may contribute to this discrepancy: (i) crack closure effects may be overestimated in the simulations, resulting in a reduced ΔK_{eff} and, consequently, an underestimation of the actual driving force for crack growth; (ii) the monotonic constitutive model employed may not adequately represent the cyclic softening behavior of the material under fatigue loading, leading to non-conservative predictions of crack tip stress fields; and (iii) the current model treats the material as a homogeneous isotropic body, while the microstructural heterogeneity, initial defects, and complex fracture morphology of the actual material may be important factors leading to the discrepancy between simulated and experimental fatigue life. Despite this non-conservative tendency, the deviation ($\approx 11\%$) falls within the acceptable scatter band typically associated with fatigue life predictions, particularly given the inherent complexity of simulating crack closure phenomena. This agreement demonstrates the overall robustness of the developed framework. Future efforts will focus on refining the crack closure estimation methodology and incorporating cyclic plasticity constitutive models to improve predictive accuracy and ensure conservative assessments for engineering applications.

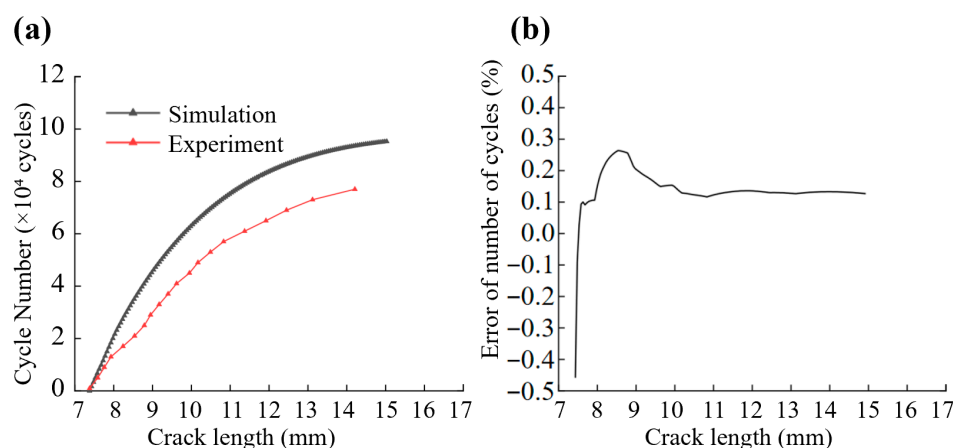


Figure 19. Comparative analysis of numerical simulated life and experimental life: (a) crack length versus cycle time; (b) error curve of simulated and experimental values.

5. Conclusions

In this work, the crack closure effect during the fatigue crack propagation process of GH4169 CT specimens was analyzed experimentally. Based on the ABAQUS platform, a user-defined plugin was developed through secondary programming to establish a fatigue crack propagation model that accounts for the crack closure effect. This model is capable of predicting the crack propagation path, recording the crack tip coordinates, calculating the stress intensity factors near the crack tip, and predicting the fatigue life of the specimen. Using the displacement field data obtained from the numerical model, the COD, COR, and $\overline{\text{COR}}$ were calculated to quantitatively characterize the crack closure effect during crack propagation. When the crack propagated to a certain length, the model successfully predicted the fatigue life of the GH4169 CT specimen within an average deviation of 11% from the experimental results, demonstrating its reliability. Overall, this study provides a more comprehensive and effective tool for investigating fatigue crack propagation behavior in nickel-based superalloys.

6. Future Work

Future work will focus on extending the present framework in several directions. First, theoretical models such as the Westergaard and Tanaka formulations will be explored under complex loading paths and three-dimensional crack configurations, in order to enhance the predictive capability for mixed-mode conditions and non-planar crack propagation. Second, multiscale experimental techniques, including SEM, EBSD, and DIC, will be combined to quantitatively reveal the influence of microstructural features on crack closure effects and crack path evolution and to investigate viable approaches for incorporating these effects into constitutive models. Finally, the performance of the proposed model under variable-amplitude loading spectra will be further assessed, with the aim of improving the robustness and generalization capability of fatigue crack growth predictions.

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