

Article

A Two-Dimensional Cloud Model for Early Warning of Tailings Dam Failure Risk Considering Probability and Consequence Coupling

Zhengjun Ji ¹, Guocai Yan ², Yaoyao Meng ³, Menglong Wu ^{2,3,*} and Lizhen Zhao ^{2,3}

¹ Wuhan Yuanqi Technology Co., Ltd., Wuhan 430075, China

² School of Emergency Management, Henan Polytechnic University, Jiaozuo 454000, China

³ Safety and Emergency Management Research Center, Henan Polytechnic University, Jiaozuo 454000, China; menggyaoyao@hpu.edu.cn

* Correspondence: wumenglong@hpu.edu.cn

Abstract

The accurate assessment of tailings dam operational status and timely risk warnings are critical for ensuring their safe operation. To address the limitations of existing models in managing complex environments and multidimensional risk factors, this study proposes an early warning model for tailings dam operational status based on a two-dimensional cloud model. First, a comprehensive early warning system is developed to assess the probability and consequences of dam failure, using risk probability and consequences as two-dimensional coordinates, incorporating the randomness and fuzziness of uncertainty described by cloud theory, and transforming qualitative data into quantitative conclusions. Next, a genetic algorithm optimizes the projection pursuit model to determine weights, and weighted numerical features are utilized to enhance the classification of early warning levels. Furthermore, the two-dimensional cloud model is enhanced by introducing a proximity coefficient to replace the membership function, with the resulting cloud map visualized using a forward cloud generator. Finally, the early warning level of the tailings dam's operational status is determined based on the clustering of cloud droplets and the proximity coefficient. Empirical application to five tailings dams in Hubei Province confirms the model's effectiveness and practicality. The results demonstrate that the model effectively addresses the complexity and uncertainty of tailings dam operational status, delivers accurate warnings, and provides robust decision support for emergency response.

Keywords: tailings dam failure; risk early warning; cloud model; uncertainty quantification; risk assessment

Academic Editor: Nikolaos Koukoulas

Received: 31 January 2026

Revised: 20 February 2026

Accepted: 24 February 2026

Published: 27 February 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

Tailings reservoirs are critical infrastructure in the mining industry, engineered to safely store waste materials generated during ore processing. They play an essential role in balancing mineral resource extraction with environmental protection. However, tailings dam failures are among the most severe hazards in global mining, often causing significant ecological damage and posing substantial risks to downstream communities' lives and property [1]. Rising global demand for metals and minerals has driven rapid

expansion in the scale and capacity of mining operations, leading to a significant increase in the number and volume of tailings storage facilities and, consequently, heightened safety risks. Developing an effective early warning and risk prediction system to prevent tailings dam instability is a pressing priority in engineering geology and environmental safety.

The safety of tailings reservoirs is fundamentally governed by the construction methodologies implemented during the embankment raising process. In contemporary engineering practice, upstream, centerline, and downstream raises remain the three primary techniques, each characterized by idiosyncratic material placement protocols, phreatic line management, and load-transfer mechanisms. Although the upstream method is highly cost-effective, its reliance on previously deposited tailings as a foundation for subsequent stages renders the structure inherently more susceptible to tailings heterogeneity, suboptimal drainage, and saturation-induced strength loss. In contrast, the downstream method facilitates the construction of each successive raise via engineered fill on the downstream side; despite necessitating considerably more material and land, this typically yields superior structural redundancy and seismic resilience. The centerline method maintains a stationary crest axis by utilizing a hybrid of deposited tailings and engineered fill, where stability is critically predicated upon the rigorous management of internal drainage and pore water pressure. Recent stability assessments and engineering syntheses consistently indicate that in scenarios of inadequate drainage, upstream and modified/centerline methods may manifest an amplified sensitivity to seepage softening and cumulative deformation. While the downstream methodology is relatively robust, it remains vulnerable to extreme hydrological triggers—such as overtopping—and internal erosion under adverse boundary conditions [2,3]. Concurrently, global tailings governance and design philosophies are undergoing a paradigm shift toward minimized water impoundment and comprehensive life-cycle risk mitigation, advocating for an integrated framework of design–construction–operation–monitoring and risk-informed decision-making [4]. From a technical standpoint, filtered tailings and dry stacking have gained prominence as safer alternatives by significantly mitigating the risk of liquefaction-induced instability through reduced saturation. Additionally, reinforcement strategies employing geosynthetics, such as geogrids and geotextiles, have been extensively investigated to enhance both deformation control and seepage safety within tailings embankments [5,6].

Against this engineering background, tailings dam failure risk factors arise from the complex interplay of geological, hydrological, operational, and climatic factors [6–8]. The nonlinear coupling of these multidimensional variables makes risk assessment an inherently complex systems problem. Traditional studies have primarily focused on failure mechanism analysis and monitoring technology improvements. In recent years, research has increasingly shifted from a purely engineering-focused perspective to an interdisciplinary approach, integrating insights from geotechnical engineering, environmental science, and disaster risk management [9]. In particular, the evolution of tailings dam safety management has transitioned towards an integrated ‘life-cycle’ risk control philosophy. Recent critical reviews have emphasized that the structural integrity of tailings facilities is not only a function of construction techniques but is increasingly dependent on the synergistic management of design–operation–monitoring sequences. This paradigm shift highlights the necessity of viewing tailings reservoirs as dynamic systems where geotechnical parameters must be constantly re-evaluated against evolving environmental stressors [10]. For example, Wu et al. [11] comprehensively reviewed the evolution of tailings dam risk assessment methods, from empirical judgment to probabilistic modeling, highlighting the role of multi-factor coupling models in enhancing prediction accuracy. Recent findings suggest that integrating multi-source real-time monitoring data and quantifying

uncertainty propagation significantly improves the responsiveness and reliability of early warning systems.

Despite significant advancements in tailings dam risk identification, current studies still face limitations in addressing multidimensional coupling and inherent uncertainties. Many existing models rely on linear assumptions or static frameworks, which fail to adequately capture the complex interplay between risk probability and consequence severity in dynamic environmental conditions. Ma et al. [12] noted that most models overly focus on individual variables, neglecting nonlinear interdependencies among multiple factors. Traditional deterministic or single-dimensional probabilistic models often fail to account for the coupled nature of probability and consequence in real-world scenarios, leading to biased risk classifications and delayed early warning responses [13]. Moreover, many approaches inadequately address the fuzziness and stochasticity of monitoring data, such as those arising from geological disturbances or rainfall-induced seepage, resulting in oversimplified risk characterizations. Although some studies have developed multi-criteria frameworks, their ability to integrate qualitative and quantitative data and model uncertainty under dynamic environmental conditions, such as climate variability, remains limited [14]. This methodological gap limits the translation of research findings into high-precision, real-time early warning tools, particularly in complex tailings management scenarios. As noted in the systematic analysis of global tailings failure mechanisms, the interplay between stochastic environmental triggers and fuzzy geotechnical responses creates a 'prediction hurdle' that traditional linear models cannot surmount [15]. While the existing literature has established robust frameworks for macroscopic risk identification, there remains a critical need for quantitative tools capable of synthesizing multi-source qualitative perceptions and quantitative monitoring data while explicitly modeling the propagation of uncertainty.

In recent years, researchers have increasingly adopted probabilistic modeling and cloud theory to systematically address the uncertainties and fuzzy boundaries of tailings dam risks [16,17]. Hu et al. [18] developed an early warning model for tailings dam instability using a one-dimensional associative cloud model, elucidating the combined effects of multiple parameters. Wang et al. [19] proposed a multi-criteria decision-making framework using an asymmetric trapezoidal cloud model, effectively capturing the coexistence of fuzziness and randomness in risk-level distributions. Hao et al. [20] integrated probability weighting with multi-layer fuzzy inference to develop a multi-indicator coupled risk assessment model for quantitatively evaluating tailings dam instability risks. Collectively, these studies mark a methodological shift from traditional static assessments to probabilistic-fuzzy hybrid modeling, offering a robust theoretical foundation for uncertainty quantification in risk evaluation.

Meanwhile, data-driven approaches have emerged as a promising direction for tailings dam safety analysis. Das et al. [21] used Differential Interferometric Synthetic Aperture Radar (DInSAR) data to dynamically predict dam deformation patterns and identify key factors contributing to instability. Pacheco et al. [22] applied artificial intelligence algorithms combined with multi-sensor data fusion to enable automated classification of tailings dam risk levels. Compared to conventional analytical methods, these models exhibit greater responsiveness to high-dimensional data inputs, though their black-box nature often limits interpretability. Chen et al. [23] and Zhang et al. [24] emphasized that combining the interpretability of cloud models with the nonlinear learning capabilities of machine learning algorithms represents a critical future direction for risk modeling and intelligent early warning systems.

Despite these advancements, two primary challenges persist in practical applications. First, the stability and generalizability of many models under small-sample conditions remain inadequately validated. Second, a unified theoretical framework that bridges

qualitative risk perception and quantitative modeling has yet to be established. To address these gaps, this study proposes a two-dimensional cloud model-based early warning system for tailings dam failure risk. By comprehensively evaluating failure probability and its consequences, the proposed framework overcomes the limitations of existing models in risk prediction and early warning. The two-dimensional cloud model explicitly accounts for uncertainty and fuzziness, enabling an accurate representation of ambiguous risk boundaries. This approach enhances early warning precision, supports adaptive evaluation, and provides proactive, targeted risk information for effective decision-making. The specific objectives of this study are: (1) to develop a targeted indicator system and classification standard for tailings dam failure risk assessment; (2) to construct and optimize a two-dimensional cloud model for integrated risk evaluation; and (3) to validate the model through empirical application, thereby providing operational insights for enhancing safety management and early warning practices.

The primary contribution of this study lies in developing an integrated modeling framework that simultaneously addresses the multidimensional, fuzzy, and uncertain characteristics of risk systems. This framework advances traditional risk assessment methods by enabling synergistic analysis of failure probability and consequence severity, offering novel theoretical and methodological insights for tailings dam safety management and disaster early warning.

The remainder of this paper is structured as follows: Section 2 presents the risk assessment indicator system and grading standards for tailings dam failures; Section 3 addresses the determination of weights for risk warning indicators; Section 4 details the construction of the two-dimensional cloud model-based early warning framework; Section 5 presents and validates the model's empirical results; and Section 6 discusses the findings and outlines future research directions.

2. Tailings Dam Failure Disaster Risk Warning Indicator System and Grading Standards

2.1. Construction of the Tailings Dam Failure Disaster Risk Warning Indicator System

The scientific validity and accuracy of the risk warning system directly determine the effectiveness of its alerts. Therefore, establishing an accurate, comprehensive, and objective risk warning indicator system for tailings dam failures is of paramount importance. Guided by the principles of systematics, representativeness, scientific rigor, and feasibility, this study employs inherent and controllable risk attributes to comprehensively identify tailings dam failure risks, thereby establishing a universal risk warning indicator system, as illustrated in Figure 1.

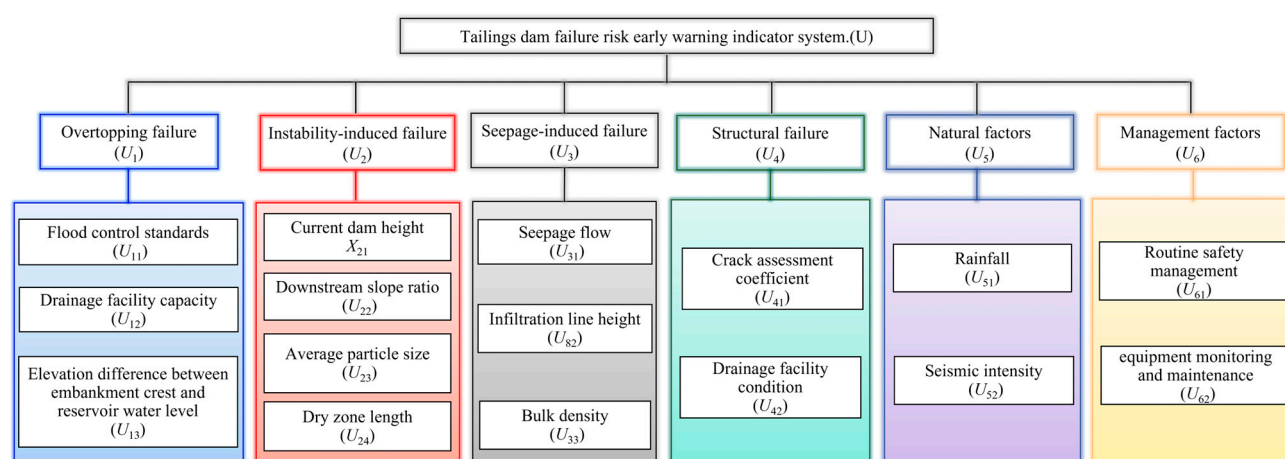


Figure 1. Tailings dam failure risk warning indicator system.

Detailed explanations of the tailings dam failure risk warning indicators are provided in Table 1. The direction of an indicator indicates the correlation between its values and the dam's safety (inversely related to failure risk). A positive indicator implies that higher values correspond to enhanced dam safety (lower risk), whereas a negative indicator suggests that higher values denote increased risk (reduced safety).

Table 1. Analysis of tailings dam failure risk warning indicators.

Indicators	Attributes	Definition of Indicators	Unit	Direction
U_{11}	Quantitative	Flood defense standard, capable of withstanding a flood with a return period of n years	Year	Positive
U_{12}	Qualitative	Capacity coefficient of the tailings dam drainage system	Constant	Positive
U_{13}	Quantitative	Vertical distance between the water surface in the dam and the crest of the beach	m	Negative
U_{21}	Quantitative	Current accumulation height of the tailings dam	m	Positive
U_{22}	Quantitative	Steepness of the downstream area of the tailings dam	Percentage	Negative
U_{23}	Quantitative	Average particle size of the tailings accumulated in the tailings dam	mm	Negative
U_{24}	Quantitative	Horizontal distance from the crest of the beach to the waterline in the dam	m	Negative
U_{31}	Quantitative	Water volume in a cross-section perpendicular to the direction of seepage flow per unit time	L/s	Negative
U_{32}	Quantitative	Location of the free surface of the seepage water in the dam body	m	Negative
U_{33}	Quantitative	Weight of the accumulated tailings in the tailings dam	t/m ³	Positive
U_{41}	Qualitative	Cracking conditions in the lateral, longitudinal, and horizontal directions of the dam body	Constant	Negative
U_{42}	Quantitative	Completeness of the drainage facilities	Constant	Positive
U_{51}	Quantitative	Impact of rainfall on the tailings dam	Constant	Negative
U_{52}	Quantitative	Seismic level of the region where the tailings dam is located	Degree	Negative
U_{61}	Qualitative	Daily safety management status of the tailings dam	Constant	Positive
U_{62}	Qualitative	Condition rate of monitoring equipment in the tailings dam	Constant	Positive

It should be noted that the thickness of the dam body was not explicitly listed as a separate indicator in this system. Instead, this factor is implicitly represented by related geometric parameters—specifically, the current dam height (U_{21}) and the steepness of the downstream slope (U_{22}). These two indicators together capture the influence of the dam's thickness on stability, ensuring that the effects of dam thickness are reflected in the risk model despite not being directly measured.

2.2. Tailings Dam Failure Disaster Risk Warning Grading

2.2.1. Risk Warning Classification Standards

The scientific establishment of safety classification standards for tailings reservoirs is a pivotal element in the safety evaluation framework for tailings dams. The rationality and accuracy of these standards directly affect the reliability of risk assessment outcomes and the efficacy of safety-related engineering decisions. To fully capture the coupled interactions between failure probability and consequence severity, this study develops a two-dimensional evaluation model comprising five levels for both probability and consequence dimensions. In this model, both the probability dimension (P) and the

consequence dimension (C) are categorized into five levels, extremely low, low, moderate, high, and extremely high, thereby establishing a systematic framework for risk classification.

Drawing on relevant technical specifications for tailings reservoirs, expert opinions, and prior research [25,26], the overall safety status of tailings reservoirs is classified into five levels: extremely dangerous (I), severely dangerous (II), moderately dangerous (III), slightly dangerous (IV), and safe (V).

Through the Cartesian product of the two dimensions ($P \times C$), 25 risk units are generated. The comprehensive risk level is subsequently determined via the risk function:

$$R = f(P, C) \quad (1)$$

The specific definitions are as follows:

Extremely dangerous (I), $R_I \in [20, 25]$: This indicates an extremely hazardous state with a critical risk of dam failure, necessitating immediate emergency intervention to ensure safety.

Severely dangerous (II), $R_{II} \in [15, 20]$: The dam is in a very high-risk state, necessitating strengthened safety management and immediate risk mitigation measures.

Moderately dangerous (III), $R_{III} \in [10, 15]$: Some safety hazards exist; however, the dam can continue operating under enhanced management, maintenance, and monitoring.

Slightly dangerous (IV), $R_{IV} \in [5, 10]$: The dam's condition does not affect normal operations but requires regular inspections and potential hazard assessments.

Safe (V), $R_V \in [0, 5]$: The dam meets all relevant safety standards, with no significant risks, and can operate normally.

The classification thresholds are determined based on the Chinese National Standard GB 39496-2020: Safety Regulations for Tailings Reservoirs [27], which systematically outlines safety management requirements and classification criteria for tailings dams. This standard also incorporates practical experience from tailings reservoir management, as well as input from experts and industry stakeholders. By referencing this standard, the scoring intervals in Table 2 are established on the dual foundation of authoritative regulations and engineering practice, ensuring the scientific, systematic, and operational integrity of the evaluation system.

Table 2. Risk probability grading standards.

Indicators	Classification Standards	Scoring Range	Indicators	Classification Standards	Scoring Range
U_{11}	(500, 1000]	[0.0, 0.5]	U_{32}	(70, 100]	[3.5, 5.0]
	(200, 500]	[0.5, 1.5]		(30, 70]	[1.5, 3.5]
	(100, 200]	[1.5, 2.5]		(15, 30]	[0.8, 1.5]
	(50, 100]	[2.5, 3.5]		(5, 15]	[0.4, 0.8]
	(0, 50]	[3.5, 5.0]		(0, 5]	[0.0, 0.4]
U_{12}	[0.0, 0.2]	[4.0, 5.0]	U_{33}	(2.2, 2.6]	[4.0, 5.0]
	(0.2, 0.4]	[3.0, 4.0]		(1.9, 2.2]	[3.0, 4.0]
	(0.4, 0.6]	[2.0, 3.0]		(1.5, 1.9]	[2.0, 3.0]
	(0.6, 0.8]	[1.0, 2.0]		(1.0, 1.5]	[1.0, 2.0]
	(0.8, 1]	[0.0, 1.0]		(0.0, 1.0]	[0.0, 1.0]
U_{13}	(10, 15]	[4.0, 5.0]	U_{41}	[0.0, 0.2]	[4.0, 5.0]
	(5, 10]	[3.0, 4.0]		(0.2, 0.4]	[3.0, 4.0]
	(2, 5]	[2.0, 3.0]		(0.4, 0.6]	[2.0, 3.0]
	(1, 5]	[1.0, 2.0]		(0.6, 0.8]	[1.0, 2.0]
	(0, 1]	[0.0, 1.0]		(0.8, 1]	[0.0, 1.0]
U_{21}	(80, 110]	[4.0, 5.0]	U_{42}	(0.8, 1]	[4.0, 5.0]
	(50, 80]	[3.0, 4.0]		(0.6, 0.8]	[3.0, 4.0]

U_{22}	(30, 50]	[1.6, 3.0)	U_{51}	(0.4, 0.6]	[2.0, 3.0)
	(15, 30]	[0.8, 1.6)		(0.2, 0.4]	[1.0, 2.0)
	(0, 15]	[0.0, 0.8]		[0.0, 0.2]	[0.0, 1.0]
	[0, 1]	[4.0, 5.0]		(100, 200]	[4.0, 5.0]
	(1, 3]	[3.0, 4.0)		(50, 100]	[3.0, 4.0)
	(3, 5]	[2.0, 3.0)		(25, 50]	[2.0, 3.0)
	(5, 8]	[1.0, 2.0)		(10, 25]	[1.0, 2.0)
	(8, 10]	[0.0, 1.0]		[0, 10]	[0.0, 1.0]
U_{23}	[0.8, 1.0]	[3.0, 5.0]	U_{52}	[12, 15]	[4.0, 5.0]
	(0.6, 0.8]	[1.5, 3.0)		(9, 12]	[3.0, 4.0)
	(0.4, 0.6]	[0.5, 1.5)		(6, 9]	[2.0, 3.0)
	(0.2, 0.4]	[0.25, 0.5)		(3, 6]	[1.0, 2.0)
	(0.0, 0.2]	[0.0, 0.25)		[0, 3]	[0.0, 1.0]
U_{24}	(100, 150]	[3.3, 5.0]	U_{61}	[0.0, 0.2]	[4.0, 5.0]
	(70, 100]	[2.3, 3.3)		(0.2, 0.4]	[3.0, 4.0)
	(50, 70]	[1.6, 2.3)		(0.4, 0.6]	[2.0, 3.0)
	(20, 50]	[0.70, 1.6)		(0.6, 0.8]	[1.0, 2.0)
	(0, 20]	[0.0, 0.70)		(0.8, 1]	[0.0, 1.0]
U_{31}	(15, 20]	[3.5, 5.0]	U_{62}	[0.0, 0.2]	[4.0, 5.0]
	(8, 15]	[2.4, 3.5)		(0.2, 0.4]	[3.0, 4.0)
	(1, 8]	[1.4, 2.4)		(0.4, 0.6]	[2.0, 3.0)
	(0.01, 1]	[0.60, 1.4)		(0.6, 0.8]	[1.0, 2.0)
	(0, 0.01]	[0.0, 0.60)		(0.8, 1]	[0.0, 1.0]

Furthermore, in defining the five-level threshold intervals (R_I – R_V), this study adheres to internationally accepted principles of risk matrix methodology while considering the specific characteristics of tailings reservoir safety management in China. This approach ensures that the evaluation model strikes a balance between international consistency and regional adaptability. Specifically, low-probability–low-consequence combinations correspond to low-risk zones, high-probability–high-consequence combinations to high-risk zones, and intermediate zones are formed through weighted aggregation to ensure a smooth transition. This methodology ensures the accuracy and rationality of risk classification.

2.2.2. Risk Probability Warning Grading

The threshold values for grading each indicator were established based on national safety guidelines, particularly China’s national standard GB 39496-2020, “Safety Regulations for Tailings Ponds,” which delineates comprehensive safety measures and classification criteria for tailings dams. This approach ensures that the scoring intervals presented in Table 2 are firmly rooted in established regulatory frameworks and practical experience.

Table 2 outlines the risk probability grading standards, which employ discrete numerical intervals to delineate varying levels of risk probability. For each indicator, the probability is partitioned into multiple intervals, each associated with a specific range of risk scores. These interval values correspond to different risk levels for the tailings dam, where higher scores signify an elevated probability of risk. This standardized approach enables the systematic quantification of risks for individual tailings dams, thereby facilitating timely warnings and informed decision-making.

2.2.3. Risk Consequence Warning Grading

A combined qualitative and quantitative approach is employed to provide a comprehensive evaluation of the consequences associated with tailings dam failure events,

encompassing five key aspects: environmental impact, social impact, personnel safety, property safety, and emergency capabilities. The grading standards for risk consequences are systematically established, as illustrated in Table 3.

Table 3. Tailings dam failure impact assessment standards.

Assessment Dimension	Consequence				
	Negligible [0, 0.2)	Acceptable [0.2, 0.4)	Severe [0.4, 0.6)	Very Severe [0.6, 0.8)	Catastrophic [0.8, 1]
Environmental impact A_E	No significant impact	Minor regional environmental pollution or damage, recovery is relatively fast	Mild regional environmental pollution or damage, may recover	Severe regional environmental pollution or damage, recovery is difficult	Comprehensive destruction of the ecosystem, recovery is difficult
Social impact A_S	Minimal social impact, rapid recovery	The majority of the public understands conflicts are easily resolved	A small portion of the public reacts strongly, which may lead to conflict	Strong social reaction may trigger small-scale group events	Strong social reaction may trigger large-scale group events
Personnel safety A_R	$E = 0$	$E \leq 1$	$1 < E \leq 3$	$3 < E \leq 10$	$E > 10$
Property loss A_P	Within 50,000 RMB	50,000 RMB to 1 million RMB	1 million RMB to 5 million RMB	5 million RMB to 10 million RMB	Above 10 million RMB
Emergency capability A_M	Rapid emergency response, no impact	Good emergency response, limited impact	Strong emergency response, localized impact	Slow emergency response, delayed rescue	Emergency failure, slow rescue

In Table 3, the personnel safety measurement coefficient is determined based on the population within 1 km downstream of the tailings dam, as presented in Equation (2).

$$E = A_{R1} + \frac{A_{R2}}{3} + \frac{A_{R3}}{60} \quad (2)$$

In Equation (2), the terms $A_{R2}/3$, and $A_{R3}/60$ are used to convert injuries of different severities into an equivalent-fatality scale, so that personnel casualties can be expressed as a single comparable consequence indicator. Specifically, following a commonly adopted convention in safety risk assessment, one fatality is treated as approximately equivalent to three serious injuries or sixty minor injuries; therefore, A_{R2} and A_{R3} are weighted by 1/3 and 1/60, respectively.

Here, A_{R1} , A_{R2} , and A_{R3} denote the numbers of fatalities, serious injuries, and minor injuries resulting from the tailings dam failure, respectively.

$$S = A_E + A_S + A_R + A_P + A_M \in [0, 5] \quad (3)$$

Among them, A_E stands for environmental impact; A_S stands for social impact; A_R stands for personnel safety; A_P stands for property loss; and A_M stands for emergency capability.

Here, A_E denotes environmental impact; A_S denotes social impact; A_R denotes personnel safety; A_P denotes property loss; and A_M denotes emergency capability.

3. Determination of the Tailings Dam Failure Risk Warning Indicator System Weights

The Real-coded Accelerating Genetic Algorithm Projection Pursuit Model (RAGA-PPM) is employed to determine the weights of the tailings dam failure risk warning indicator system. Tailings dam risk-warning data are typically characterized by high

dimensionality, small sample sizes, pronounced heterogeneity, and weak linear correlations, rendering traditional linear analysis methods insufficient for elucidating underlying risk patterns. Consequently, an approach that can identify nonlinear structures under limited data conditions, while preserving robustness and interpretability, is critical for reliable risk assessment.

The Projection Pursuit Model (PPM) offers an effective solution for such conditions. It excels in handling small-sample, non-Gaussian, and multi-source heterogeneous datasets, achieving a balance among discriminative capability, stability, and transparency. By optimizing a task-oriented projection index function, the PPM identifies the low-dimensional projection direction that optimally reveals the intrinsic structure and hierarchical features of the data.

In this study, the PPM was utilized to extract a composite feature variable most sensitive to tailings dam risk classification. This projection variable was subsequently integrated with the TDCM to establish a unified framework that integrates high-dimensional feature fusion with uncertainty quantification and classification. This integration enhances the accuracy, stability, and interpretability of the risk assessment process, delivering a scientifically robust and practically viable tool for intelligent early warning in complex tailings dam systems.

The RAGA-PPM is adopted to determine the weights of the tailings dam failure risk warning indicator system. RAGA-PPM integrates the optimization capabilities of genetic algorithms with the dimensionality reduction strengths of the projection pursuit model, effectively combining subjective and objective information to derive representative weights across multiple dimensions, thereby offering a flexible and scientifically rigorous weighting mechanism [28]. It not only optimizes weight allocation but also mitigates issues such as subjectivity, complexity, and computational errors inherent in conventional methods. It reduces biases arising from data endogeneity to some extent, making it well-suited to the practical requirements of tailings dam failure risk warning.

The core principle of PPM involves projecting high-dimensional data onto a low-dimensional subspace to identify projections that effectively capture the structure and features of the high-dimensional space for data analysis. The modeling steps of the RAGA-PPM are outlined as follows [29].

Step1: Normalization of evaluation indicators

Let the sample set of indicator values in the evaluation ranking table be denoted as $\{x^*(i, j) | i = 1, 2, \dots, n; j = 1, 2, \dots, p\}$ represents the value of the i -th sample for the j -th indicator, and n and p denote the sample size and the number of indicators, respectively. To eliminate dimensional differences among the indicators and standardize the range of variation for each indicator value, Equations (4) and (5) are used to normalize both positive and negative indicators, respectively. Specifically, positive indicators are standardized using Equation (4), whereas negative indicators are standardized using Equation (5).

$$x(i, j) = \frac{x^*(i, j) - x_{\min}(j)}{x_{\max}(j) - x_{\min}(j)} \quad (4)$$

$$x(i, j) = \frac{x_{\max}(j) - x^*(i, j)}{x_{\max}(j) - x_{\min}(j)} \quad (5)$$

In the equations, $x_{\max}(j)$ and $x_{\min}(j)$ denote the maximum and minimum values of the j -th indicator, respectively, while $x(i, j)$ represents the normalized indicator value.

Furthermore, to ensure comparability of evaluation results across diverse datasets, this study adheres strictly to principles of standardization and consistency in the normalization process, thereby subjecting all datasets to uniform scoring thresholds and normalization criteria. This standardized approach maintains result comparability, ensures

consistent scaling of outcomes, and effectively mitigates biases arising from localized standard variations.

Step2: Constructing the projection indicator function

The p -dimensional data is integrated into a one-dimensional projection value $z(i)$ along the projection direction $a = \{a(1), a(2), a(3), \dots, a(p)\}$, that is, the p -dimensional data is reduced to a one-dimensional projection value $z(i)$ in the direction of a .

$$z(i) = \sum_{j=1}^p a(j)x(i, j), i = 1, 2, \dots, n \quad (6)$$

In Equation (6), $a(j)$ denotes the projection direction vector (i.e., the weight vector) in the projection pursuit model, consisting of the weights for the p risk indicators.

The projection indicator function can be constructed as follows:

$$Q(a) = S_z D_z \quad (7)$$

In the equation, S_z represents the standard deviation of the projection value $z(i)$, and D_z denotes the local density of the projection value $z(i)$, as follows:

$$S_z = \sqrt{\frac{\sum_{i=1}^n (z(i) - E(z))^2}{n-1}} \quad (8)$$

$$D_z = \sum_{i=1}^n \sum_{j=1}^n (R - r(i, j)) \cdot u(R - r(i, j)) \quad (9)$$

The kernel radius R is a key parameter in the local density estimation D_z . Based on the optimization principles of projection pursuit, R must balance the resolution of local structures with the stability of the density estimate. In this study, R was determined as $0.1 S_z$, which has been empirically proven effective for preventing over-smoothing while filtering stochastic noise in small-sample datasets.

Step3: Optimizing the projection indicator function

The optimal projection direction is the one that maximizes the value, potentially exposing the high-dimensional data's feature structure. The best projection direction can be estimated by solving the maximization problem of the projection indicator function.

Maximization objective function:

$$\max Q(a) = S_z \cdot D_z \quad (10)$$

Constraint function:

$$\text{s. t. } \sum_{j=1}^p a^2(j) = 1 \quad (11)$$

Equation (11) represents a complex linear optimization problem, with the set of coefficients $\{a(j)|j = 1, 2, \dots, p\}$ serving as the optimization variables. To address this high-dimensional global optimization challenge, the Real-coded Accelerating Genetic Algorithm (RAGA)—which simulates biological mechanisms of natural selection and chromosomal information exchange within populations—was employed. To eliminate the subjectivity inherent in manual weight assignment, this study integrates RAGA with the Projection Pursuit Model (PPM) to construct the RAGA-PPM for objective, data-driven determination of indicator weights.

In this framework, a projection index function is first established to maximize the dispersion of samples along the projection direction, thereby yielding an information-optimal projection vector. The genetic algorithm parameters were configured as follows: initial population size $n = 500$ to enhance search diversity; crossover probability $P_c = 0.8$ to ensure global search capability; mutation probability $P_m = 0.2$ to maintain population vitality; and number of optimization variables $m = 16$, corresponding to the 16 risk-indicator weights. Each mutation direction was determined by six random numbers to balance perturbation and convergence, and the acceleration cycle was set to 100 generations to ensure stable convergence of the projection function.

The algorithm incorporated adaptive mutation and elitism retention strategies to prevent premature convergence and preserve high-quality solutions throughout the evolutionary process. Consequently, an optimal weight vector was obtained and applied to the comprehensive risk calculation. Through this data-driven optimization procedure, the weight allocation achieves objectivity and reproducibility, effectively avoiding biases associated with subjective weighting schemes.

This complex linear optimization problem, with $\{a(j)|j=1, 2, \dots, p\}$ as the optimization variables, can be solved using the RAGA. The RAGA—inspired by biological evolutionary mechanisms such as natural selection and chromosomal information exchange within populations—is capable of addressing high-dimensional global optimization problems. By combining real-coded RAGA with PPM, the optimal projection direction in high-dimensional space can be obtained. This optimal projection direction reflects the ranking of the overall evaluation for each early warning indicator and the importance of each indicator in influencing the tailings dam's overall safety status. Therefore, normalizing this optimal projection direction yields the weights for each indicator.

Accordingly, the weight vector generated by RAGA-PPM will be used as an input parameter in the TDCM to infer the final risk level.

4. Tailings Dam Breach Risk Early Warning Model Construction

In tailings dam failure risk early warning, multidimensionality, fuzziness, and uncertainty are critical factors that influence prediction outcomes. Tailings dam risk assessment involves not only analyzing multiple independent factors but also their complex interactions. Traditional methods typically focus on a single dimension, such as geological or hydrological factors, often overlooking interactions in multidimensional space. To address this, this study proposes a dual-axis framework based on probability and consequence dimensions, which fully captures multidimensionality in risk assessment, thereby offering a novel approach for comprehensive evaluation.

In tailings dam failure risk early warning, key parameters such as dam stability and seepage coefficients lack clear boundaries, exhibiting fuzzy ranges. Such fuzzy information is often challenging to quantify using traditional precise methods. However, by incorporating entropy and hyper-entropy parameters from the cloud model, these influences can be effectively quantified and expressed. Entropy measures information uncertainty, while hyper-entropy reflects the system's uncertainty degree. This approach ensures the stability and reliability of risk warnings under highly fuzzy conditions.

Tailings dam failure risk encompasses not only external uncertainties, such as geological disturbances and extreme climate changes, but also variations in the dam's design and operation, all contributing to assessment complexity. In this study, uncertainty is quantified via the hyper-entropy parameter in the cloud model. This parameter measures uncertainty propagation in the tailings dam system and, by depicting dynamic changes, helps identify key factors for risk early warning. By incorporating probability and consequence dimensions into the two-dimensional cloud model, this study comprehensively models the multidimensionality, fuzziness, and uncertainty of tailings dam risks. Specifically, the probability dimension reflects failure likelihood, while the consequence dimension describes potential impacts of instability, such as casualties and property damage. This dual-dimensional framework not only enhances risk identification accuracy but also improves model adaptability, allowing flexible adjustments under varying environmental conditions and providing reliable early warning information.

4.1. Two-Dimensional Early Warning Cloud Model

The two-dimensional cloud model, an extension of the one-dimensional cloud model, simultaneously considers the probability and severity of risks. It effectively

addresses the uncertainty and fuzziness inherent in risk indicator levels [30]. By employing cloud drops to represent probability distributions, the model quantitatively evaluates early warning outcomes for tailings dam breach risks, enhancing its suitability for dynamic and complex decision-making environments.

The two-dimensional cloud model is characterized by digital attributes, including expectation, entropy, and hyper-entropy. Assuming the two-dimensional random function f follows a normal distribution, E_x and E_y denote the expectations of the influencing factors, while E_{nx} and E_{ny} represent their standard deviations. The cloud drop (x_i, y_i, μ_i) , as defined in Equation (12), constitutes the two-dimensional cloud model.

It should be noted that while the normal distribution is adopted here for general risk characterization, it may not be universal for all risk factors. For rare and catastrophic events with ‘heavy-tailed’ characteristics, such as high-intensity seismic activities (U_{52}), extreme value statistics—specifically the Gumbel distribution—may provide a more accurate mathematical representation. Future versions of the model will explore the integration of such extreme value clouds to enhance warning precision under black-swan scenarios.

$$\begin{cases} (x_i, y_i) = f(E_x, E_y, E_{nx}, E_{ny}) \\ (P_{xi}, P_{yi}) = f(E_{nx}, E_{ny}, H_{ex}, H_{ey}) \\ \mu_i = \exp\left\{-\frac{1}{2}\left[\frac{(x_i - E_x)^2}{E_{nx}^2} + \frac{(y_i - E_y)^2}{E_{ny}^2}\right]\right\} \end{cases} \quad (12)$$

In this equation, x_i and y_i represent the coordinates of the cloud drop; P_{xi} and P_{yi} are the conditional coordinates of the cloud drop; H_{ex} and H_{ey} denote the hyper-entropy of the influencing factors; and μ_i indicates the membership degree.

The operational principle of the two-dimensional cloud model generator is illustrated in Figure 2. Upon inputting the mathematical expectations, entropies, hyper-entropies, and the number of cloud droplets N , a set of random numbers is generated. Subsequently, the membership degree of each cloud droplet is computed according to Equation (12), with this procedure iterated until N cloud droplets are produced.

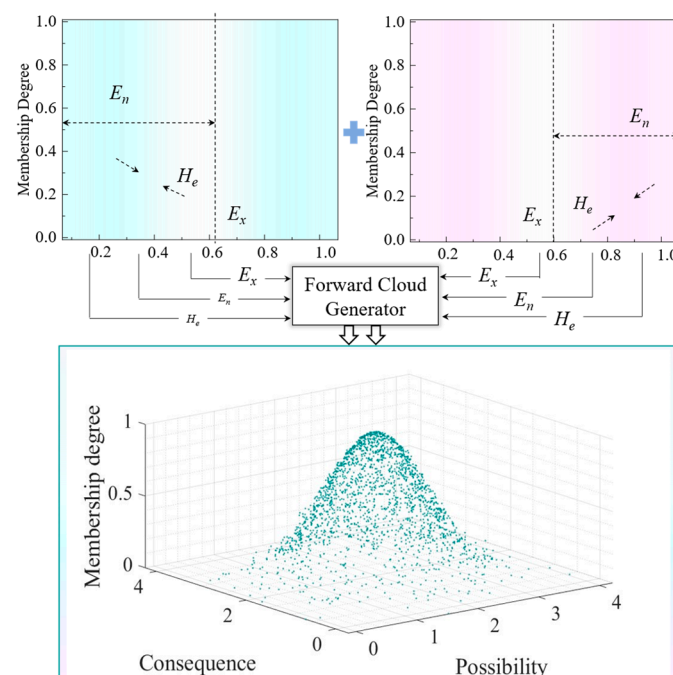


Figure 2. Two-dimensional cloud model generator (visualized in 3D).

4.2. Two-Dimensional Standard Cloud Model

A two-dimensional early warning cloud model is developed to assess the risk levels associated with tailings dam failures. This model provides a comprehensive framework for evaluating both the probability and consequences of individual indicator events within the tailings dam risk early warning system. The computational approach for the two-dimensional early warning cloud model, which integrates both probability and consequence, is presented in Equation (13).

$$\begin{cases} E_x = \frac{1}{m} \sum_{i=1}^n x_i \\ E_n = \sqrt{\frac{\pi}{2}} \times \frac{1}{n} \sum_{i=1}^n |x_i - E_x| \\ H_e = \sqrt{|S^2 - E_n^2|} \\ S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - E_x)^2 \end{cases} \quad (13)$$

In the equation, E_x denotes the expected value of the sample, E_n represents the entropy, and H_e refers to the hyper-entropy, corresponding to the central value, fuzziness, and uncertainty of each risk grade, respectively. Here, n is the number of samples, x_i represents the individual sample values, and S^2 is the sample variance.

The digital characteristics of the cloud model- E_x , E_n , and H_e -represent the central value, fuzziness, and second-order uncertainty of the risk concept, respectively. The calculation of E_n is formally derived from the 3σ rule in statistics. For a normal cloud model, the probability of cloud drops falling within the range of $[E_x - 3E_n, E_x + 3E_n]$ is 99.73%. By defining $E_n = (R_{jmax} - R_{jmin})/6$, the entire risk scoring interval is statistically covered by the corresponding risk level. Physically, E_n captures the inherent measurement fuzziness and boundary ambiguity of the dam's operational status.

In the cloud model theory proposed by Li et al., [31] according to the 'atomization threshold' theory, $H_e \approx E_n/3$ serves as the critical upper limit before atomization (severe dispersion) occurs. This condition allows for a certain degree of entropy fluctuation while preventing conceptual confusion, making it an empirically justified threshold in practical cloud model applications. Under these parameter settings, each risk grade corresponds to a reproducible normal cloud model, which enables the quantitative representation of fuzzy boundaries between risk levels and provides a robust foundation for characterizing uncertainty within the risk assessment framework.

Based on the practical requirements for tailings dam breach risk early warning and the established classification system for tailings dam breach disaster risks, the safety risk probability and consequences associated with the early warning indicators are divided into five subintervals. The j -th evaluation interval is defined as $[R_j^{min}, R_j^{max}]$, and the calculation method for the five early warning cloud evaluation intervals is presented in Equation (14).

$$\begin{cases} E_x^{(k)} = \frac{R_j^{min} + R_j^{max}}{2} \\ E_n^{(k)} = \frac{R_j^{max} - R_j^{min}}{6} \\ H_e^{(k)} \in [\frac{E_x}{100}, \frac{E_n}{10}] \end{cases} \quad (14)$$

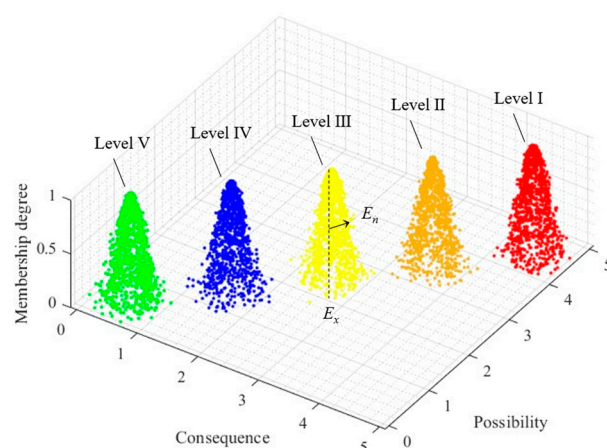
In the equation, $E_x^{(k)}$, $E_n^{(k)}$, and $H_e^{(k)}$ represent the expectation, entropy, and hyper-entropy of the evaluation cloud, respectively.

Based on the predefined risk early warning levels for tailings dam breaches, five two-dimensional standard clouds are constructed. These standard clouds serve as reference benchmarks for the early warning clouds. The numerical characteristics of these standard clouds are determined using Equation (14), with the results presented in Table 4.

Table 4. Results of early warning level quantification.

Early Warning Classification	Classification Description	Early Warning Level Quantification Range	Standardized Cloud Digital Features
Extremely dangerous (I)	The risk is unacceptable and must be mitigated.	[20, 25]	(22.5, 0.83, 0.05)
Severely dangerous (II)	The risk is acceptable once mitigated.	[15, 20)	(17.5, 0.83, 0.05)
Moderately dangerous (III)	The risk is acceptable, but certain precautions must be implemented.	[10, 15)	(12.5, 0.83, 0.05)
Slightly dangerous (IV)	The risk is deemed acceptable.	[5, 10)	(7.5, 0.83, 0.05)
Safe (V)	The risk is considered negligible.	[0, 5)	(2.5, 0.83, 0.05)

Based on the numerical characteristics of the standard clouds derived from Equation (14) and the principles of the two-dimensional cloud generator illustrated in Figure 2, the visualized early warning levels for tailings dam breach risks are presented in Figure 3.

**Figure 3.** Two-dimensional standard clouds for different early warning levels.

4.3. Integrated Early Warning Cloud Model

The integrated early warning cloud represents the comprehensive early warning level for tailings dam breach risk. The first-level early warning cloud is derived from second-level early warning clouds, and the integrated early warning cloud is calculated based on the first-level cloud values.

Consider n early warning clouds, denoted as $N_1(E_{x1}, E_{n1}, H_{e1})$, $N_2(E_{x2}, E_{n2}, H_{e2})$, ..., $N_n(E_{xn}, E_{nn}, H_{en})$, each with corresponding weights $\omega_j (j = 1, 2, \dots, n)$. The weighted integrated early warning cloud is determined using Equation (15).

$$\begin{cases} E_x = \sum_{j=1}^n \omega_j E_{xj} \\ E_n = \sqrt{\frac{1}{n} \sum_{j=1}^n \omega_j E_{nj}^2} \\ H_e = \sqrt{\frac{1}{n} \sum_{j=1}^n \omega_j H_{ej}^2} \end{cases} \quad (15)$$

In the equation, E_x , E_n , and H_e denote the expectation, entropy, and hyper-entropy of the weighted integrated early warning cloud, respectively.

4.4. Cloud Model Similarity Determination

The visualization of the two-dimensional cloud-based early warning results may introduce perceptual biases. To address this, a similarity coefficient is introduced to quantitatively determine the final risk warning level for each indicator. The calculation method is presented in Equation (16):

$$Z = \frac{1}{\sqrt{(E_{x,p}^{(k)} - E_{x,p}^*)^2 + (E_{x,c}^{(k)} - E_{x,c}^*)^2}} \quad (16)$$

where Z represents the similarity coefficient; $E_{x,p}^{(k)}$ and $E_{x,c}^{(k)}$ are the expectation values of the standard clouds in the probability and consequence dimensions for the k -th risk level, respectively; $E_{x,p}^*$ and $E_{x,c}^*$ are the corresponding expectation values of the actual tailings dam cloud.

The higher the value of Z , the closer the actual sample cloud is to the standard cloud, indicating greater similarity and stronger correspondence to the given risk level.

5. Empirical Study on Tailings Dam Breach Risk Early Warning

5.1. Study Area Description

The proposed two-dimensional cloud model was validated using nine representative tailings dams in Hubei Province, China. The study area is situated in the hilly terrain of southeastern Hubei, featuring pronounced topographic relief. Regional mountain ranges predominantly trend northwest–southeast (NW–SE), with the terrain sloping gradually from north to south and west to east. The highest elevation is Sifeng Mountain (487.84 m a.s.l.), while the lowest is near Shenghongqing Village (33 m a.s.l.), resulting in substantial topographic variation that facilitates the formation of tailings impoundments through natural high-potential geomorphological conditions.

Climatically, the region falls within the subtropical continental monsoon zone, exhibiting four distinct seasons. The annual mean temperature is approximately 17 °C, with mean annual precipitation around 1400 mm, primarily concentrated from April to September. This abundant and seasonal rainfall significantly impacts dam performance by promoting seepage, infiltration, and erosion, thereby elevating risks of piping, slope instability, and dam failure.

From a tectono-geological perspective, the mining area is situated on the western flank of the fore-arc structure within the Huaiyang arcuate fold system. Structurally, it occupies the northern limb of the Tieshan compound anticline and the southern margin of the Tiemen granite intrusion. The area features intense faulting and folding, dominated by NW-trending multi-order fault systems that serve as preferential pathways for groundwater flow and seepage. The strata primarily comprise Mesoproterozoic and Sinian sedimentary rocks, interspersed with localized granite intrusions and volcanoclastic layers. Rock weathering varies considerably, and fault-fracture and shear zones locally undermine dam foundation stability.

Based on rock mass discontinuities and structural characteristics, the regional rock mass is classified into three engineering geological types: blocky, stratified, and loose structural assemblages. From an engineering geology perspective, the area constitutes a high-potential energy zone with dense tailings dam distribution, where rugged topography and stratigraphic conditions foster an environment prone to anthropogenic debris flows. Upstream catchment areas for the dams are extensive, while downstream regions include residential zones and ecologically sensitive areas. Thus, dam safety is paramount for regional environmental protection and socioeconomic stability. Dam failures could initiate debris flows, heavy metal release, and ecological pollution, posing grave threats to downstream communities and ecosystems.

The mining area has a long history of mineral extraction and features a well-established safety management system. Enterprise records and field surveys indicate that the mines have instituted a Safety Production Committee and a Department of Safety and Environmental Protection, which implement a structured framework for risk grading and hazard identification. All tailings dams undergo tiered monitoring and periodic safety

evaluations, with long-term data on dam surface deformation and seepage accumulated over years, furnishing reliable baseline information for this study.

In summary, the study area displays a complex geological and structural setting, marked by varied geomorphology, dynamic hydrological processes, and potential tectonic risks. This unique “high-energy hilly terrain–faulted rock mass–elevated reservoir” configuration renders the region an ideal natural laboratory for validating the proposed tailings dam risk assessment model. Multi-site empirical analyses in this area enable comprehensive evaluation of the two-dimensional cloud model’s applicability and robustness under intricate geological conditions.

Field survey data for the five tailings dams in the mining area are presented in Table 5.

Table 5. Field survey data of tailings dams.

Indicators	Tailings Pond Identifier				
	#1	#2	#3	#4	#5
U_{11}	100.00	100.00	200.00	500.00	100.00
U_{12}	0.84	0.91	0.61	0.64	0.77
U_{13}	4.50	4.85	3.85	4.04	3.96
U_{21}	16.16	56.72	19.57	26.78	17.82
U_{22}	7.98	7.70	8.74	8.64	8.37
U_{23}	0.11	0.08	0.16	0.24	0.16
U_{24}	92.15	83.46	72.43	84.38	93.06
U_{31}	1.21	3.41	0.40	0.80	1.40
U_{32}	15.73	9.87	11.21	9.23	12.51
U_{33}	2.17	1.89	2.60	1.88	2.36
U_{41}	0.38	0.46	0.36	0.23	0.13
U_{42}	0.18	0.28	0.17	0.22	0.28
U_{51}	0.31	0.39	0.21	0.12	0.13
U_{52}	0.73	0.79	0.39	0.18	0.26
U_{61}	12.00	20.00	14.00	19.00	14.00
U_{62}	2.30	1.60	1.60	1.80	1.60

5.2. Model Validation

Based on the data presented in Tables 2 and 3, further quantitative analysis was performed on each risk early warning indicator to delineate the ranges of probability and severity. Quantitative indicators were determined by experts using actual operational data from the tailings dam system and historical operating conditions of the tailings dams. Qualitative indicators were quantified using a five-expert scoring panel based on the descriptive criteria in Table 2. The final input value is the arithmetic mean of the expert scores to mitigate individual bias. By integrating the expert evaluation results, the digital characteristics of probability and severity for the early warning cloud model were calculated using Equation (12). The results are presented in Table 6.

Table 6. Digital characteristics of probability and consequences for early warning indicators.

Indictors	Tailings Pond Identifier				
	#1	#2	#3	#4	#5
U_{11}	(2.45, 0.06, 0.04)	(2.45, 0.05, 0.03)	(1.50, 0.03, 0.01)	(0.45, 0.05, 0.02)	(2.45, 0.05, 0.03)
	(1.25, 0.08, 0.05)	(1.75, 0.08, 0.04)	(2.25, 0.08, 0.03)	(2.25, 0.08, 0.03)	(1.50, 0.08, 0.02)
U_{12}	(0.50, 0.07, 0.03)	(0.45, 0.05, 0.02)	(2.00, 0.07, 0.02)	(1.25, 0.05, 0.04)	(1.50, 0.03, 0.02)
	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)
U_{13}	(1.70, 0.07, 0.02)	(2.85, 0.05, 0.03)	(2.65, 0.05, 0.02)	(2.80, 0.03, 0.02)	(2.70, 0.03, 0.02)

	(2.25, 0.08, 0.02)	(3.25, 0.08, 0.03)	(2.75, 0.08, 0.04)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)
U_{21}	(0.95, 0.05, 0.01)	(3.20, 0.07, 0.03)	(1.05, 0.05, 0.01)	(1.45, 0.05, 0.01)	(0.90, 0.03, 0.01)
	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)
U_{22}	(0.30, 0.07, 0.01)	(1.70, 0.07, 0.01)	(0.15, 0.05, 0.03)	(0.15, 0.05, 0.03)	(0.15, 0.05, 0.03)
	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)
U_{23}	(0.13, 0.04, 0.02)	(0.05, 0.02, 0.01)	(0.15, 0.01, 0.01)	(0.25, 0.03, 0.02)	(0.15, 0.01, 0.01)
	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)
U_{24}	(2.90, 0.13, 0.03)	(2.70, 0.07, 0.02)	(2.30, 0.03, 0.02)	(2.50, 0.03, 0.02)	(3.25, 0.03, 0.03)
	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)
U_{31}	(1.70, 0.10, 0.02)	(1.75, 0.08, 0.03)	(1.00, 0.03, 0.02)	(1.35, 0.05, 0.01)	(1.40, 0.03, 0.01)
	(1.75, 0.08, 0.03)	(3.25, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)
U_{32}	(0.90, 0.03, 0.01)	(0.60, 0.03, 0.01)	(0.60, 0.03, 0.02)	(0.55, 0.01, 0.01)	(0.70, 0.03, 0.02)
	(4.75, 0.08, 0.05)	(2.75, 0.08, 0.04)	(2.75, 0.08, 0.04)	(2.75, 0.08, 0.05)	(2.75, 0.08, 0.04)
U_{33}	(3.75, 0.08, 0.04)	(2.85, 0.05, 0.03)	(4.80, 0.07, 0.05)	(2.85, 0.05, 0.03)	(4.25, 0.05, 0.04)
	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.02)	(2.25, 0.08, 0.02)
U_{41}	(1.75, 0.08, 0.03)	(2.20, 0.07, 0.04)	(3.55, 0.05, 0.03)	(3.20, 0.03, 0.02)	(4.50, 0.03, 0.01)
	(2.25, 0.08, 0.02)	(3.25, 0.08, 0.03)	(2.75, 0.08, 0.04)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)
U_{42}	(3.50, 0.10, 0.03)	(3.95, 0.05, 0.01)	(1.95, 0.05, 0.01)	(0.85, 0.05, 0.02)	(1.15, 0.05, 0.03)
	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)
U_{51}	(1.25, 0.08, 0.01)	(1.65, 0.05, 0.02)	(1.25, 0.05, 0.02)	(1.55, 0.05, 0.01)	(1.25, 0.05, 0.01)
	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)
U_{52}	(0.60, 0.07, 0.02)	(0.50, 0.06, 0.02)	(0.50, 0.03, 0.02)	(0.55, 0.02, 0.01)	(0.50, 0.03, 0.01)
	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.02)
U_{61}	(1.80, 0.07, 0.02)	(2.25, 0.05, 0.02)	(1.25, 0.05, 0.01)	(1.50, 0.03, 0.01)	(1.50, 0.03, 0.01)
	(2.25, 0.08, 0.02)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)	(1.75, 0.08, 0.03)
U_{62}	(3.00, 0.07, 0.02)	(3.00, 0.07, 0.02)	(2.00, 0.07, 0.02)	(1.50, 0.03, 0.01)	(1.50, 0.03, 0.01)
	(1.75, 0.08, 0.03)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)	(2.25, 0.08, 0.02)

Using Equations (5) to (9), the initial population size for the parent generation is set to 500, with a crossover probability (P_c) of 0.8, a mutation probability (P_m) of 0.2, 16 optimization variables, six random numbers for determining the mutation direction, and 100 acceleration iterations. The projection index function is optimized using the RAGA to determine the optimal projection vector.

Based on the optimal projection direction, the influence levels of indicators for tailings dam failure risk are ranked as follows: U_{13} , U_{32} , U_{12} , U_{11} , U_{33} , U_{31} , U_{61} , U_{42} , U_{22} , U_{51} , U_{23} , U_{52} , U_{41} , U_{62} , U_{21} , U_{24} . After normalizing the optimal projection vector, the weights of the tailings dam failure risk early warning indicators are derived, as presented in Figure 4.

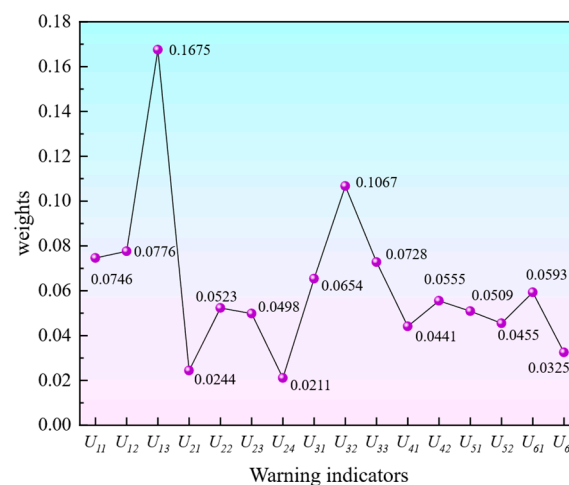


Figure 4. Distribution of early warning indicator weights.

To verify the reliability and repeatability of the RAGA-PPM optimization process, 20 independent runs were conducted using the same initial dataset. As summarized in Table 7, the statistical analysis reveals that the weight vectors for all 16 indicators are highly stable, with the coefficient of variation (CV) for the primary weights (e.g., U_{13} and U_{32}) remaining below 1.5%. Furthermore, the final risk classification results for the five tailings dams were consistently 100% identical across all independent trials. This high level of convergence consistency proves that the RAGA effectively overcomes the randomness inherent in genetic operators and provides a robust foundation for subsequent cloud-based early warning.

Table 7. Statistical analysis of 20 independent RAGA-PPM runs.

Indicator	Mean Weight	Std. Dev.	Indicator	Mean Weight	Std. Dev.
U_{11}	0.0746	± 0.0009	U_{32}	0.1067	± 0.0014
U_{12}	0.0776	± 0.0008	U_{33}	0.0728	± 0.0011
U_{13}	0.1675	± 0.0018	U_{41}	0.0441	± 0.0006
U_{21}	0.0244	± 0.0004	U_{42}	0.0555	± 0.0007
U_{22}	0.0523	± 0.0007	U_{51}	0.0509	± 0.0008
U_{23}	0.0498	± 0.0006	U_{52}	0.0455	± 0.0005
U_{24}	0.0211	± 0.0003	U_{61}	0.0593	± 0.0009
U_{31}	0.0654	± 0.0009	U_{62}	0.0325	± 0.0004

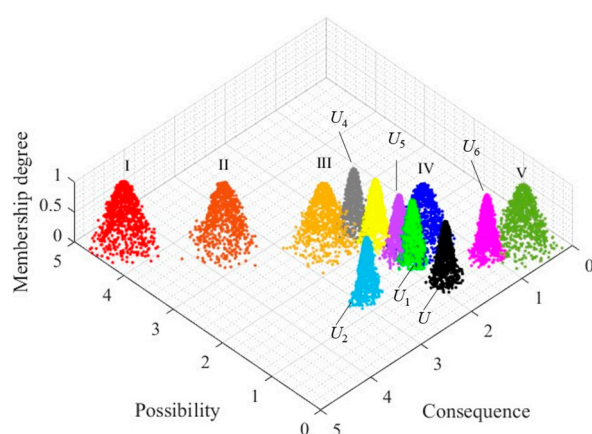
Using the initial digital characteristics of the early warning clouds, the weighted cloud digital characteristics for each tailings dam were computed by incorporating the weights of the respective early warning indicators. The results are presented in Table 8.

Table 8. Weighted digital characteristics of probability and consequences for early warning indicators.

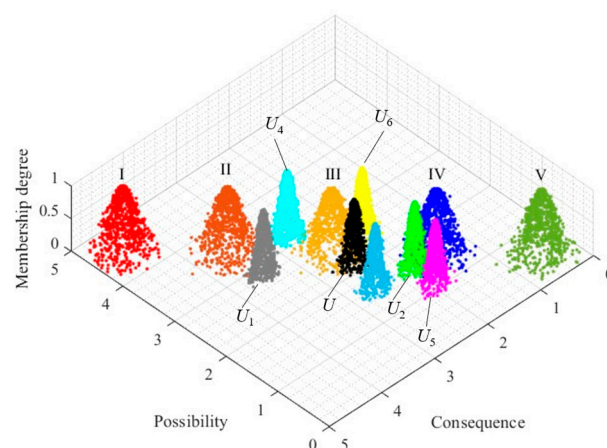
Tailings	Composite Indicator	Digital Characteristics	Primary Indicators	Digital Characteristics
#1	U	(1.57, 0.07, 0.03) (2.04, 0.08, 0.03)	U_1	(1.58, 0.07, 0.03) (1.90, 0.08, 0.03)
			U_2	(0.72, 0.07, 0.02) (1.82, 0.08, 0.03)
			U_3	(1.96, 0.06, 0.02) (3.06, 0.08, 0.04)
			U_4	(2.73, 0.09, 0.03) (1.97, 0.08, 0.03)
			U_5	(0.91, 0.07, 0.01) (1.15, 0.08, 0.03)
			U_6	(2.05, 0.07, 0.02) (1.91, 0.08, 0.02)
#2	U	(2.13, 0.05, 0.02) (2.55, 0.08, 0.03)	U_1	(2.84, 0.05, 0.04) (3.47, 0.08, 0.04)
			U_2	(1.54, 0.05, 0.01) (1.90, 0.08, 0.03)
			U_3	(1.58, 0.05, 0.02) (2.59, 0.08, 0.03)
			U_4	(3.18, 0.06, 0.02) (2.69, 0.08, 0.02)
			U_5	(1.07, 0.05, 0.02) (1.95, 0.08, 0.02)
			U_6	(2.52, 0.06, 0.02) (1.92, 0.08, 0.03)
#3	U	(1.65, 0.04, 0.02) (2.19, 0.08, 0.03)	U_1	(2.20, 0.05, 0.02) (2.51, 0.08, 0.03)
			U_2	(0.61, 0.03, 0.02) (2.00, 0.08, 0.03)
			U_3	(1.96, 0.04, 0.03) (2.33, 0.08, 0.03)
			U_4	(2.66, 0.05, 0.02) (2.19, 0.08, 0.03)
			U_5	(0.90, 0.04, 0.02) (1.75, 0.08, 0.03)
			U_6	(1.52, 0.06, 0.01) (1.93, 0.08, 0.03)
#4	U	(1.04, 0.04, 0.02) (1.06, 0.08, 0.03)	U_1	(1.38, 0.04, 0.02) (1.08, 0.08, 0.02)
			U_2	(0.74, 0.04, 0.02) (0.50, 0.08, 0.02)
			U_3	(1.25, 0.03, 0.01) (1.09, 0.08, 0.04)
			U_4	(0.89, 0.04, 0.02) (1.27, 0.08, 0.03)

#5	U	(1.71, 0.03, 0.01) (1.95, 0.08, 0.03)	U_5	(1.21, 0.04, 0.01) (1.43, 0.08, 0.02)
			U_6	(0.50, 0.03, 0.01) (1.65, 0.08, 0.02)
			U_1	(2.35, 0.03, 0.01) (1.95, 0.08, 0.02)
			U_2	(0.72, 0.03, 0.01) (1.82, 0.08, 0.03)
			U_3	(1.94, 0.04, 0.02) (2.33, 0.08, 0.03)
			U_4	(2.63, 0.04, 0.02) (1.75, 0.08, 0.03)
			U_5	(0.90, 0.04, 0.01) (1.75, 0.08, 0.03)
			U_6	(1.50, 0.03, 0.01) (1.93, 0.08, 0.02)

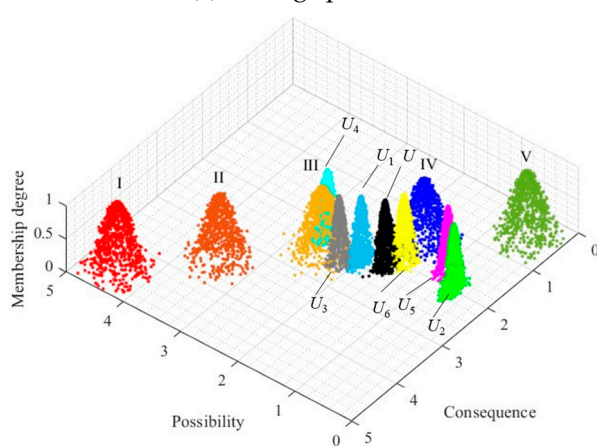
Using Equation (12) and the cloud digital characteristics presented in Table 7, along with the two-dimensional cloud generator illustrated in Figure 2, cloud diagrams are generated for tailings dam failure risk early warning levels I through V. These diagrams, depicting secondary indicators including cap failure, instability failure, seepage damage, structural damage, natural environmental factors, and management factors, as well as the integrated early warning indicator for tailings dam failure risk, are presented in Figure 5.



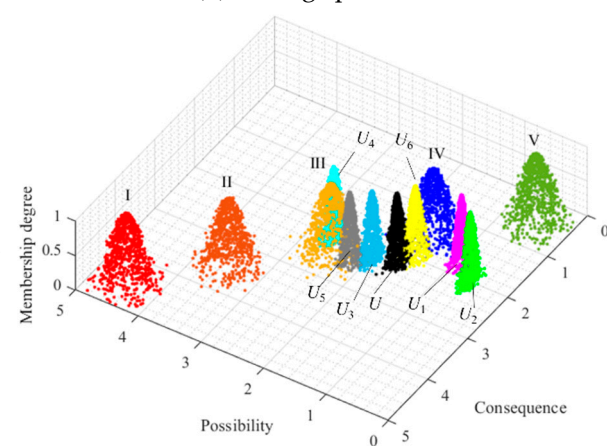
(a) Tailings pond #1



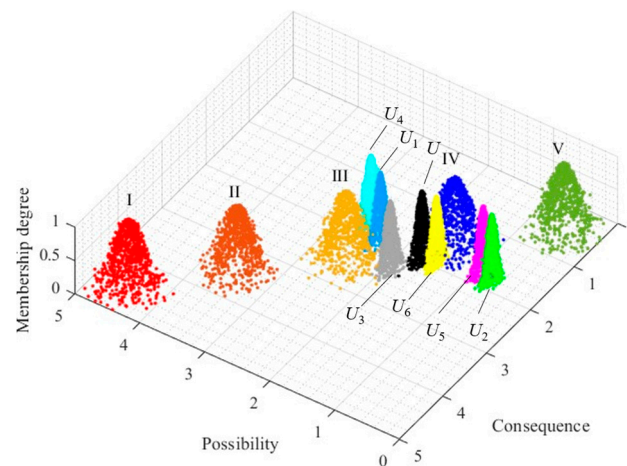
(b) Tailings pond #2



(c) Tailings pond #3



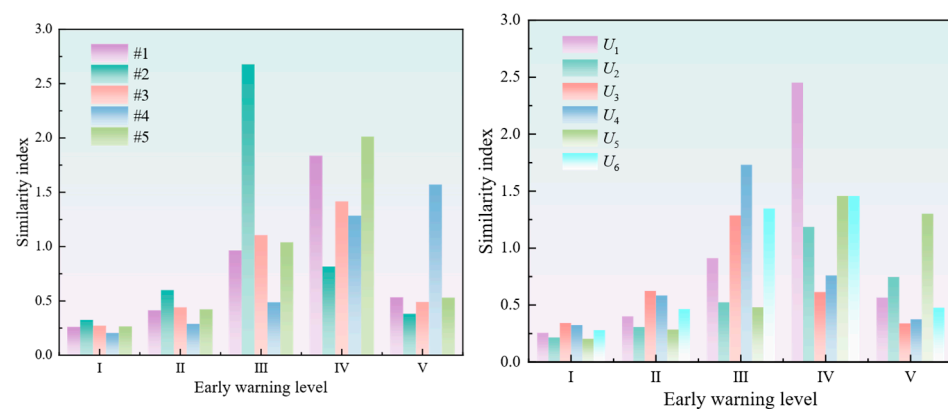
(d) Tailings pond #4



(e) Tailings pond #5

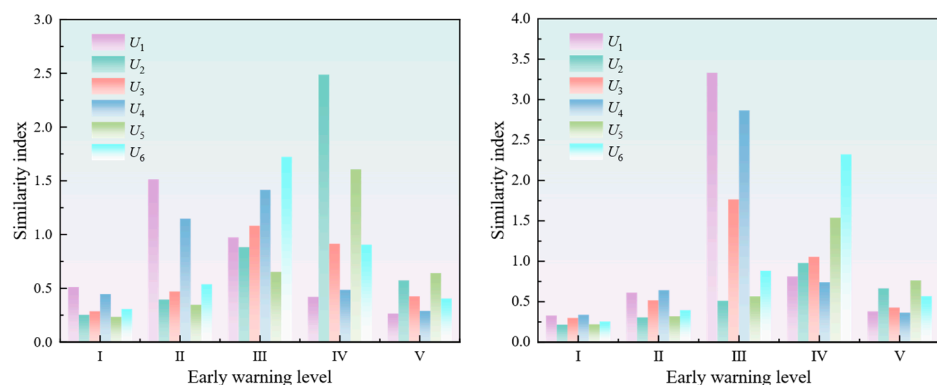
Figure 5. Integrated risk early warning cloud diagram.

As illustrated in Figure 5, Tailings Dams #1 and #4 are positioned between Levels IV and V, whereas Tailings Dams #2, #3, and #5 lie between Levels III and IV. The risk warning levels for the five tailings dams were evaluated using the developed tailings dam breach risk early warning model. Similarity coefficients between each tailings dam and the five standard warning clouds were computed using Equation (14). The similarity coefficients for the secondary risk indicators and the integrated risk early warning for each tailings dam are presented in Figure 6.



(a) Different tailings ponds

(b) Tailings pond #1



(c) Tailings pond #2

(d) Tailings pond #3

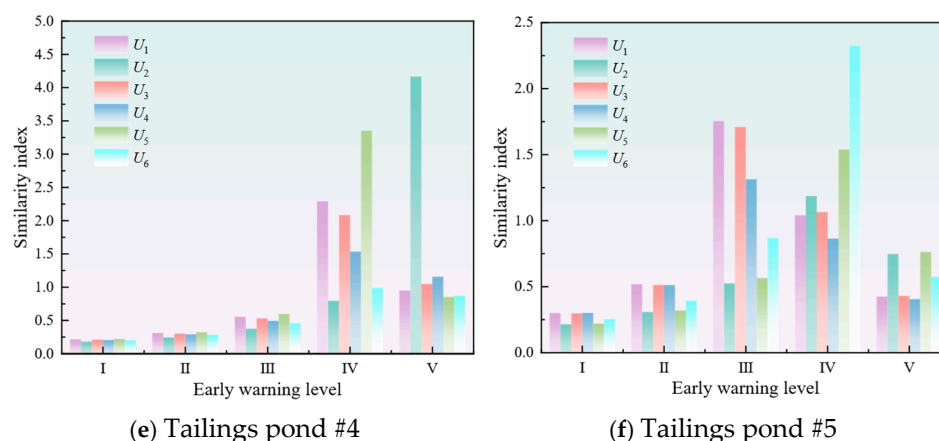


Figure 6. Similarity degree of different warning levels.

Based on the similarity coefficient analysis in Figure 6, the similarity coefficient of tailings dam #1 with level IV is 1.8365, exceeding that with level III (0.9638) and level V (0.5333). This indicates that the breach risk for this tailings dam is relatively low. Among the secondary risk indicators, seepage damage (U_3) and structural damage (U_4) exhibit high similarity values of 1.2854 and 1.7308, respectively, identifying them as the primary risk sources. Consequently, enhanced stability monitoring and effective seepage prevention measures are essential for tailings dam #1.

The similarity coefficient of tailings dam #2 with level III is 2.6784, markedly exceeding those with level II (0.5998) and level IV (0.8167). This suggests that the breach risk for this tailings dam is relatively high. Among the secondary risk indicators, seepage damage (U_3), structural damage (U_4), and management factors (U_6) show values of 1.0818, 1.4163, and 1.7231, respectively, indicating that risks are primarily concentrated in these areas. Notably, seepage damage and structural damage may induce dam instability, necessitating intensified monitoring of water infiltration and structural integrity. Furthermore, the elevated similarity of management factors reveals deficiencies in risk control and management protocols, underscoring the need for immediate improvements to the management system.

The similarity coefficient of tailings dam #3 with level IV is 1.8365, surpassing that with level III (0.9638) and level V (0.5333). This indicates that the breach risk for this tailings dam is relatively low. However, certain similarity coefficients suggest a potential shift toward level III risk, warranting heightened attention to emerging hazards. Among the secondary risk indicators, cap failure (U_1), seepage damage (U_3), and structural damage (U_4) display values of 0.9104, 1.2854, and 1.7308, respectively, signifying their substantial influence on overall risk. Specifically, seepage damage and structural damage may precipitate dam instability, emphasizing the imperative to bolster monitoring and risk management in these domains.

The similarity coefficient of tailings dam #4 with level V is 1.571, exceeding that with level IV (1.2854), indicating a relatively safe state. Overall, the risk level remains manageable and controlled. Among the secondary indicators, natural factors (U_5) and management factors (U_6) exhibit elevated similarity coefficients within level IV; however, the aggregate risk is low. Regular monitoring for natural disaster prevention and enhancements to emergency measure efficacy are recommended.

The similarity coefficient of tailings dam #5 with level IV is 2.0137, substantially surpassing those with level III (1.0389) and level V (0.5295). This indicates a low breach risk, with minimal likelihood of escalation to level III. Among the secondary risk indicators, cap failure (U_1), seepage damage (U_3), and structural damage (U_4) emerge as primary

factors exerting significant influence. Thus, intensified monitoring and management of these elements are imperative.

Overall, all tailings dams display elevated risks associated with seepage damage (U_3) and structural damage (U_4), with seepage damage being especially pronounced in tailings dams #3 and #4. Furthermore, tailings dams #3 and #5 require attention to cap failure (U_1), necessitating enhanced protective facilities and environmental monitoring.

In summary, all tailings dams should bolster seepage and structural monitoring systems, refine management mechanisms, and proactively address potential environmental variations across early warning levels. This analysis furnishes explicit guidance for safety management and risk mitigation, delivering targeted recommendations for prevention and control to relevant authorities. Such measures facilitate optimization of the safety assurance framework and the implementation of timely, effective risk controls.

5.3. Comparative Model Analysis

To rigorously validate the effectiveness and superiority of the proposed two-dimensional cloud model (TDCM) for tailings dam failure risk early warning, a systematic comparative analysis was performed against three established models: the traditional one-dimensional cloud model (ODCM), the fuzzy comprehensive evaluation (FCE) model, and the Bayesian network (BN) model. The comparison encompassed several key dimensions, including risk classification consistency, prediction accuracy, robustness, computational complexity, parameter sensitivity, and model interpretability.

(1) Comparison with the ODCM

Structurally, the proposed TDCM introduces a dual-axis risk representation by integrating both the probability dimension (P) and the consequence dimension (C). This design addresses the inherent limitation of the one-dimensional cloud model, which is confined to characterizing risk fuzziness along a single dimension. Table 9 presents the comparative risk level outputs from both models for five representative tailings dam samples.

Table 9. Comparison of early warning results from different models.

Tailings Pond Identifier	Similarity Degree					TWDC	ODCM	Actual Classification
	I	II	III	IV	V			
#1	0.2614	0.4132	0.9638	1.8365	0.5333	IV	IV	IV
#2	0.3258	0.5998	2.6784	0.8167	0.3818	III	III	III
#3	0.2726	0.4411	1.1053	1.4162	0.4892	IV	IV	IV
#4	0.2050	0.2886	0.4876	1.2854	1.571	V	V	V
#5	0.2646	0.4223	1.0389	2.0137	0.5295	IV	IV	IV

The results indicate that the TDCM and ODCM yield generally consistent risk level classifications, thereby confirming the fundamental effectiveness of the TDCM. However, the TDCM exhibits superior stability and smoother risk score distributions when identifying boundary-level samples (IV–V). This advantage stems from its cloud droplet reconstruction mechanism, which dynamically balances risk information in the probability–consequence space, facilitating natural and continuous transitions between risk categories. Consequently, this mechanism effectively alleviates the abrupt changes and ambiguous boundaries typically observed in the ODCM.

(2) Comparison with the FCE Model

The FCE model is widely applied in engineering risk analysis; however, its dependence on membership functions and manually assigned weights introduces substantial subjectivity and constrains its robustness. In this study, the same indicator system and weighting scheme were used to compare the evaluation outcomes of the FCE model and the proposed TDCM. The results indicate that the FCE model achieved an overall

prediction accuracy of 60.0%, underestimating the risk levels for Tailings Reservoirs #2 and #3 relative to actual observations. In contrast, the TDCM attained 100% prediction accuracy, aligning fully with the measured classifications. Further analysis showed that under $\pm 10\%$ input perturbations, the FCE model's output risk score fluctuated by $\pm 6.5\%$, compared to only $\pm 3.9\%$ for the TDCM, demonstrating the latter's superior stability and resilience to disturbances.

This marked difference arises from the TDCM's adaptive adjustment mechanism involving entropy (E_n) and hyper-entropy (H_e), which automatically modulates the distribution of fuzzy information and facilitates the simultaneous handling of fuzziness and randomness. As a result, the TDCM exhibits enhanced robustness and reliability in managing incomplete information and fuzzy boundary conditions.

(3) Comparison with the BN Model

The BN model effectively captures conditional dependencies in complex systems; however, its predictive performance is highly sensitive to sample size and prior probability distributions. In cases of limited datasets or missing information, prediction biases are prone to arise. Under identical sample conditions, a BN model was constructed in this study using Maximum A Posteriori (MAP) estimation. The results show that the BN attained an overall classification accuracy of 80%, with an overestimation of the risk level for Tailings Reservoir #1. This deviation stems from the BN's reliance on variable independence assumptions and inadequate specification of prior distributions.

In contrast, the proposed TDCM does not require large sample sizes or prior assumptions. Its cloud droplet generation mechanism enables self-adaptive uncertainty absorption by modeling the joint distribution of probability and consequence in a two-dimensional risk space, thereby substantially mitigating input biases. Furthermore, the TDCM's low computational complexity renders it well-suited for real-time early warning and dynamic management of tailings dam safety.

To thoroughly assess the performance of the models, a quantitative comparison was conducted across four key dimensions: accuracy, robustness, computational efficiency, and interpretability. The comparative results for the four models are presented in Table 10.

Table 10. Comparison of model performance.

Model	Classification Accuracy (%)	Variance of Risk Scores (σ^2)	Computational Complexity (Relative)	Interpretability	Applicable Scenarios
TDCM	100	0.039	Medium	High	Multi-factor, and high-uncertainty environments
ODCM	100	0.061	Low	Medium	Single-dimensional risk analysis
FCE	60	0.072	High	Medium	High fuzziness in information
BN	80	0.058	Medium	Low	Sufficient sample conditions

The results demonstrate that the TDCM exhibits significant advantages in risk classification accuracy, adaptability to uncertain information, and multidimensional data fusion. Its dual-dimensional cloud droplet generation mechanism not only enhances the model's precision and stability but also signifies a theoretical advancement from fuzzy qualitative evaluation to quantitative and interpretable risk assessment.

Based on the systematic comparative analysis, the following conclusions can be drawn:

(1) At the structural level, the TDCM achieves a dual-dimensional representation of risk in terms of probability and consequence, offering a more comprehensive understanding of the underlying risk mechanisms compared to single-dimensional models;

(2) At the performance level, the TDCM outperforms both the fuzzy comprehensive evaluation (FCE) model and the Bayesian network (BN) model in classification accuracy and robustness, rendering it particularly suitable for small-sample and high-uncertainty environments;

(3) At the application level, the TDCM demonstrates high interpretability and real-time applicability, rendering it well-suited for dynamic monitoring and early warning systems in tailings dam safety management.

Therefore, the TDCM provides a comprehensive early warning framework that integrates theoretical innovation with engineering practicality. It establishes a solid foundation for intelligent and dynamic risk prediction, offering a forward-looking approach to tailings dam safety management.

5.4. Sensitivity Analysis

To assess the stability and robustness of the TDCM in the presence of input parameter uncertainty, a sensitivity analysis was performed on its key parameters. Tailings Reservoir No. 1 was chosen as the representative case study. Five key indicators—flood control standards (U_{11}), current dam height (U_{21}), seepage flow (U_{31}), rainfall (U_{51}), and routine safety management (U_{61})—were individually perturbed by $\pm 10\%$, with all other parameters held constant. The risk scores and associated risk levels were subsequently recalculated and compared against the baseline results to quantify the model's sensitivity to these perturbations. The results of this sensitivity analysis are summarized in Table 11.

Table 11. Summary of risk-level and score variation before and after parameter perturbation ($\pm 10\%$).

Identifier	Perturbation Parameter	Original Risk Level	Risk Level After Perturbation (+10%)	Risk Level After Perturbation (−10%)	Variation in Risk Level
#1	Flood control standards (U_{11})	IV	IV	IV	Unchanged
#2	Current dam height (U_{21})	III	III	III	Unchanged
#3	Seepage flow (U_{31})	IV	IV	IV	Unchanged
#4	Rainfall (U_{51})	V	V	V	Unchanged
#5	Routine safety management (U_{61})	IV	IV	IV	Unchanged

The analysis results indicate that within the $\pm 10\%$ perturbation range, the risk levels of all samples remained consistent, with no cross-grade variations observed. Only minor fluctuations in the calculated risk scores occurred under a few boundary conditions, with a maximum variation rate of less than 3.9%. These findings demonstrate that the TDCM exhibits strong robustness and stability, even in the presence of input uncertainty, subjective differences in expert weighting, and noise in monitoring data.

This robustness can be attributed to three underlying mechanisms:

(1) Structural self-balancing property. By incorporating both probability and consequence dimensions, the TDCM decomposes the uncertainty and fuzziness of risk into two interrelated spaces, thereby significantly mitigating the influence of single-parameter fluctuations on the overall risk output. Even when factors such as flood control standards or seepage flow vary, the averaging effect inherent in cloud droplet generation and numerical feature computation ensures stable risk scores.

(2) Weight coupling and compensatory effects. Within the multidimensional indicator system, the model captures interdependencies among indicator weights. When a single factor (e.g., rainfall or dam height) is perturbed, other relatively stable indicators offset its influence through compensatory weighting effects. For instance, an increase in rainfall

may slightly elevate the risk value, but if flood control standards are sufficient, the overall risk level remains unchanged.

(3) Fuzzy absorption capacity of entropy and hyper-entropy. Entropy quantifies the uncertainty of indicators, while hyper-entropy reflects the fluctuation range of fuzzy information within the system. Since the model constrains the distribution characteristics of these quantities during parameterization, perturbations do not cause abrupt changes in risk outcomes but rather propagate smoothly through the risk space, thereby enhancing system stability.

Based on the systematic comparative analysis, the following conclusions can be drawn:

(1) At the structural level, the TDCM achieves a dual-dimensional representation of risk in terms of probability and consequence, providing a more comprehensive reflection of the underlying risk mechanism compared with single-dimensional models.

(2) At the performance level, the TDCM outperforms both the fuzzy comprehensive evaluation (FCE) model and the Bayesian network (BN) model in terms of classification accuracy and robustness, making it particularly suitable for applications involving small samples and high uncertainty.

(3) At the application level, the TDCM demonstrates strong interpretability and real-time capability, rendering it well-suited for dynamic monitoring and early warning systems in tailings dam safety management.

Overall, the two-dimensional cloud model exhibits a stable response to both natural factors (flood control standards, seepage flow, rainfall) and engineering management factors (dam height, routine safety management coefficient), indicating high adaptability under complex, multi-source, and incomplete data conditions. From a mechanistic perspective, the TDCM achieves continuity and stability in risk level determination through the self-adjusting properties of cloud droplet distributions and the dynamic balance of multi-dimensional information, effectively avoiding boundary mutation phenomena.

Therefore, the sensitivity analysis results provide compelling evidence for the robustness and reliability of the two-dimensional cloud model. Even under moderate input uncertainty or perturbation, the model consistently preserves the coherence and interpretability of risk classification. This feature not only enhances the scientific validity and practical applicability of the model in multi-factor tailings dam risk assessment but also establishes a solid theoretical foundation for its broader application under diverse geological, climatic, and management conditions.

5.5. Model Stability and Generalization Validation

To address the limitations of the finite sample size and further verify the model's robustness against measurement uncertainties, a stochastic simulation-based validation (similar to the bootstrapping approach) was conducted. For each of the five tailings dams, 100 synthetic datasets were generated by injecting Gaussian random noise ($\sigma = 5\%$) into the original 16 risk indicators. The risk labels were recalculated for these 500 synthetic cases using the proposed TDCM framework.

As summarized in Table 12, the model exhibits exceptional classification stability, with an average consistency rate of 98.6%. Even under multi-factor simultaneous perturbation, the similarity coefficients Z fluctuated within a narrow range, and cross-level misclassifications occurred only in 1.4% of the trials—primarily in samples near the risk boundaries (e.g., Dam #2 and #5). This high degree of stability confirms that the TDCM's dual-axis cloud representation effectively absorbs environmental noise and measurement errors, providing a reliable early warning tool for real-world tailings management.

Table 12. Classification stability results under stochastic data perturbation (100 runs per dam).

Identifier	Original Risk Level	Mean Similarity Score ($\bar{Z}$)	Std. Deviation of Score (σ_z)	Classification Stability (%)	Deviation Distribution
#1	IV	1.8365	± 0.084	99%	IV→V (1%)
#2	III	2.6784	± 0.125	97%	III→II (3%)
#3	IV	1.4162	± 0.062	99%	IV→III (1%)
#4	V	1.5710	± 0.051	100%	None
#5	IV	2.0137	± 0.093	98%	IV→III (2%)
Average	--	--	--	98.6%	Robust

6. Discussion and Future Work

This study addresses the critical challenge of early warning for tailings dam failure risks by proposing a novel framework based on a two-dimensional cloud model for quantitative risk assessment and early warning. By integrating the probabilistic likelihood of dam failure with its associated consequences, the proposed approach establishes an operable, interpretable, and environmentally adaptive early warning system, validated through multiple real-world case studies.

The core innovation of this study is the development of a dual-dimensional risk representation framework that simultaneously addresses risk randomness (probability) and fuzziness (uncertainty). Traditional deterministic or single-dimensional fuzzy models often fail to capture the stochastic characteristics of risk occurrence and the uncertain range of failure consequences inherent in tailings dam systems. By introducing a three-parameter system comprising expectation, entropy, and hyper-entropy, the proposed TDCM establishes a mechanism for uncertainty propagation and dynamic quantification of fuzzy boundaries, thereby extending theoretical understanding from single-axis fuzzy expression to dual-axis probability–consequence cognition.

Within this framework, the probability dimension characterizes the occurrence patterns of dam failure events, while the consequence dimension quantifies the impact range of potential disasters, forming a risk-space structure that more accurately reflects real-world operating conditions. The TDCM exhibits high robustness and interpretability, even in small-sample or incomplete data scenarios, thereby providing a novel theoretical foundation for modeling complex uncertainties in engineering systems.

Model results indicate that structural and seepage failures are the dominant factors influencing overall risk levels in tailings dams, consistent with prior findings [32,33]. This consistency underscores the necessity of monitoring these key indicators and aligns with observations by other researchers. For instance, Chen et al., [23] found that maintaining structural stability (e.g., sufficient shear strength under extreme rainfall) is critical for preventing tailings dam failures, while Zhang et al., [24] demonstrated that seepage-related pathways (such as internal water lenses) can directly precipitate dam breaches. Such evidence from the literature reinforces the importance of focusing on structural integrity and seepage control in risk assessments. Accordingly, our risk distribution analysis reveals that dam stability and seepage conditions carry the highest weights in determining risk levels, aligning closely with the primary causes of tailings dam failures. Thus, the proposed model exhibits strong explanatory power in capturing high-impact factors and identifying emerging risk trends.

Compared to traditional FCE and BN models, the TDCM exhibits superior performance in classification accuracy, robustness, and interpretability. Notably, FCE methods have been widely applied in tailings dam risk assessments, and BN techniques are known for incorporating uncertainty in dam safety analysis. However, these approaches generally rely on expert-driven weight assignments and static frameworks [20]. Meanwhile,

data-driven machine learning models have emerged as alternatives—Pacheco et al., [22] applied artificial intelligence to generate synthetic data for tailings dam stability analysis—but such models often sacrifice interpretability. Dempster–Shafer evidence theory has also been employed to fuse multi-source information for dam safety evaluation, providing an uncertainty-handling framework that still requires sufficient evidence from diverse inputs. In contrast, the TDCM introduces a multi-parameter uncertainty propagation mechanism and enables real-time dynamic risk representation. By coupling entropy and hyper-entropy in its modeling process, the TDCM effectively captures the diffusion characteristics of risk information in multidimensional spaces, overcoming the excessive reliance on expert experience typical of traditional fuzzy models. Overall, the TDCM integrates flexibility and robustness, providing a novel quantitative pathway for tailings dam risk management.

Given the limited sample size, model robustness was validated through parameter perturbation experiments. When key input variables (including indicator weights and individual values) were perturbed by $\pm 10\%$, risk levels remained unchanged, indicating stable responses under moderate perturbations. Further multi-indicator joint perturbation analysis showed that risk level fluctuations did not exceed 0.4 grade intervals, demonstrating the TDCM's strong robustness even under multi-factor interference. These results confirm that the TDCM exhibits greater response stability than traditional fuzzy models under structural parameter uncertainty and input variability.

Despite these positive findings, several limitations must be acknowledged. First, the study sample was drawn primarily from five tailings reservoirs in a mining area in Hubei Province, resulting in limited sample size and regional concentration; thus, the generalizability of the conclusions requires further verification. Second, although the model effectively processes probabilistic and consequential risk dimensions, its performance may be constrained under extremely complex or highly coupled conditions. Additionally, some input data were obtained from static field surveys and expert assessments, lacking validation under dynamic conditions.

To enhance the model's generalizability and scientific rigor, future research will focus on several directions: (1) quantitative benchmarking against multiple approaches, involving comparative experiments on identical datasets to evaluate differences in accuracy, stability, and computational efficiency; (2) cross-regional and multi-scenario validation, extending the application to diverse geological, climatic, and operational conditions to test robustness across contexts; (3) uncertainty propagation analysis, integrating Monte Carlo and bootstrap techniques to construct confidence intervals for risk levels based on parameter distributions; (4) integration with real-time monitoring data, embedding the model into online systems to explore its adaptability for dynamic risk evolution tracking and early warning decision-making.

These efforts will further enhance the model's external validity and applicability, providing a solid foundation for the intelligent, quantitative, and dynamic management of tailings dam risks.

In summary, the proposed two-dimensional cloud model offers a scientifically grounded and practically feasible solution for assessing tailings dam failure risks. By jointly considering the probability and consequence dimensions of risk, the model more accurately captures the true risk state of tailings reservoirs and serves as a forward-looking, actionable tool for safety management. With continued advancements in real-time monitoring and artificial intelligence algorithms, the model's predictive accuracy and applicability are expected to improve further, thereby laying a solid foundation for data-driven tailings dam risk early warning systems.

7. Conclusions

This study addresses the challenges of complexity, fuzziness, and uncertainty in tailings dam failure risk early warning by proposing an integrated TDCM-based framework for comprehensive risk assessment and early warning. The proposed approach demonstrates high levels of scientific rigor and practical applicability, contributing to both theoretical innovation and engineering relevance.

From a theoretical perspective, this study establishes a probability–consequence dual-dimensional risk representation framework, overcoming the inherent limitations of traditional one-dimensional fuzzy models. By introducing the parameters of entropy and hyper-entropy, the model quantitatively characterizes both uncertainty and fuzziness within the risk system. Furthermore, the proposed dual-layer cloud droplet generation mechanism enables a self-balancing distribution of risk information in the probability–consequence space, resulting in smoother and more continuous risk classifications while significantly mitigating the abrupt boundary transitions observed in conventional models. These methodological advances provide a novel theoretical foundation for uncertainty quantification in complex engineering systems.

From a practical perspective, the TDCM was applied to five representative tailings reservoirs in Hubei Province, accurately reflecting the spatial distribution of risk levels and exhibiting outstanding reliability and computational stability. The model outperformed comparative approaches in terms of classification accuracy, robustness, and interpretability, offering a quantitative and real-time decision-support tool for safety supervision and management authorities. In addition, the modular and transferable structure of the TDCM enables its extension to other complex risk scenarios, such as landslides, mine geological hazards, and reservoir seepage risks.

Nevertheless, several limitations should be acknowledged. The current dataset is relatively small and regionally concentrated within Hubei Province, and certain parameters still rely on expert judgment. Moreover, the model validation was primarily based on static data. Future research should aim to (1) perform cross-regional validation under diverse geological and climatic conditions; (2) integrate machine learning and deep learning techniques to develop a hybrid “cloud model–data-driven” intelligent framework; (3) establish a dynamic risk evolution analysis system for real-time early warning; and (4) apply Monte Carlo and Bootstrap techniques to quantitatively assess the uncertainty of model outputs.

In conclusion, the proposed TDCM extends the probabilistic–fuzzy integrated framework for risk assessment at the theoretical level and provides an effective tool for intelligent, quantitative, and interpretable management of tailings dam failure risks in practice. This work lays a solid scientific foundation for the development of dynamic safety early warning systems for complex engineering infrastructures.

Author Contributions: Investigation, Validation, Data curation, Supervision, Software, Z.J. and G.Y.; Visualization, Methodology, Y.M.; Writing—Original draft, Funding acquisition, Conceptualization, M.W.; Project administration, Resources, L.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Fundamental Research Funds for the Universities of Henan Province, grant number NSFRF2502113” and “Doctoral Fund Project of Henan Polytechnic University, grant number B2025-91”.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The datasets used and analysed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest: Author Zhengjun Ji is an employee of Wuhan Yuanqi Technology Co., Ltd. The company had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results. The remaining authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DInSAR	Differential Interferometric Synthetic Aperture Radar
FCE	Fuzzy Comprehensive Evaluation
MAP	Maximum A Posteriori
ODCM	One-dimensional Cloud Model
PPM	Projection Pursuit Model
RAGA	Real-coded Accelerating Genetic Algorithm
TDCM	Two-dimensional Cloud Model

References

- Islam, K.; Murakami, S. Global-scale impact analysis of mine tailings dam failures: 1915–2020. *Glob. Environ. Change* **2021**, *70*, 102361. <https://doi.org/10.1016/j.gloenvcha.2021.102361>.
- Pooraskarparast, B.; Bento, A.M.; Baron, E.; Matos, J.C.; Dang, S.N.; Fernandes, S. Fluid–soil–structure interactions in semi-buried tanks: Quantitative and qualitative analysis of seismic behaviors. *Appl. Sci.* **2023**, *13*, 8891. <https://doi.org/10.3390/app13158891>.
- Singh, S.; Kumar, A.; Sitharam, T.G. Stability assessment of a tailings pond considering the effect of staged construction and embankment raising rate: A numerical study. *Min. Metall. Explor.* **2023**, *40*, 851–869. <https://doi.org/10.1007/s42461-023-00772-8>.
- Maduka, R.I.; Igwe, O. Suitability of mine waste rocks as earth tailing storage embankment materials: Environmental risk assessments. *Model. Earth Syst. Environ.* **2022**, *8*, 1825–1849. <https://doi.org/10.1007/s40808-021-01182-9>.
- Cacciuttolo, C.; Pastor, A.; Valderrama, P.; Atencio, E. Process water management and seepage control in tailings storage facilities: Engineered environmental solutions applied in Chile and Peru. *Water* **2023**, *15*, 196. <https://doi.org/10.3390/w15010196>.
- Wang, X.K.; Zhang, H.Y.; Wang, Y.N.; Hou, W.H.; Wang, J.Q.; Peng, H.G. A decision support system for comments-adjusted ranking of hotels. *J. Oper. Res. Soc.* **2025**, *76*, 1880–1897. <https://doi.org/10.1080/01605682.2024.2446683>.
- Aswathi, T.S.; Jakka, R.S. Seismic analysis of hybrid tailings dams: Insights into stability and responses. *Bull. Eng. Geol. Environ.* **2024**, *83*, 56. <https://doi.org/10.1007/s10064-024-03541-z>.
- Ge, Y.F.; Li, Z.H.; Tang, H.M.; Chen, Q.; Wen, Z.X. Efficient rock joint detection from large-scale 3D point clouds using vectorization and parallel computing approaches. *Geosci. Front.* **2025**, *16*, 102085. <https://doi.org/10.1016/j.gsf.2025.102085>.
- Tian, S.; Dai, X.; Wang, G.; Lu, Y.; Chen, J. Formation and evolution characteristics of dam breach and tailings flow from dam failure: An experimental study. *Nat. Hazards* **2021**, *107*, 1621–1638. <https://doi.org/10.1007/s11069-021-04649-1>.
- Afrazi, M.; Jahed Armaghani, D.; Afrazi, H.; Fattahi, H.; Samui, P. Real-time monitoring of tunnel structures using digital twin and artificial intelligence: A short overview. *Deep. Undergr. Sci. Eng.* **2025**, *early view*. <https://doi.org/10.1002/dug2.70029>.
- Wu, M.; Ye, Y.; Hu, N.; Wang, Q.; Tan, W. Scientometric analysis on the review research evolution of tailings dam failure disasters. *Environ. Sci. Pollut. Res.* **2023**, *30*, 13945–13959. <https://doi.org/10.1007/s11356-022-24937-y>.
- Ma, L.; Zhang, C.; Ma, C.; Li, X. Spatial evolution analysis of tailings flow from tailings dam failure based on MacCormack-TVD. *Appl. Sci.* **2025**, *15*, 1757. <https://doi.org/10.3390/app15041757>.
- Lyu, Z.; Chai, J.; Xu, Z.; Qin, Y.; Cao, J. A comprehensive review on reasons for tailings dam failures based on case history. *Adv. Civ. Eng.* **2019**, *1*, 4159306. <https://doi.org/10.1155/2019/4159306>.
- Du, C.; Niu, B.; Yi, F.; Jiang, X.; Liang, L. Impact of inundation range of overtopping dam break of tailings pond under actual terrain conditions. *PLoS ONE* **2023**, *18*, e0295056. <https://doi.org/10.1371/journal.pone.0295056>.
- Afrazi, M.; Huat, C.Y.; Onifade, M.; Khandelwal, M.; Shonuga, D.O.; Fattahi, H.; Jahed Armaghani, D. Modeling the Abrasive Index from Mineralogical and Calorific Properties Using Tree-Based Machine Learning: A Case Study on the KwaZulu-Natal Coalfield. *Mining* **2025**, *5*, 48. <https://doi.org/10.3390/mining5030048>.

16. Wang, X.K.; Deng, M.H.; Hou, W.H.; He, L.; Qu, F.; Wang, J.Q. Probabilistic linguistic group decision-making based on evidential reasoning considering correlations between linguistic terms. *Int. J. Fuzzy Syst.* **2023**, *25*, 3001–3015. <https://doi.org/10.1007/s40815-023-01550-w>.
17. Ge, Y.F.; Liu, G.; Tang, H.M.; Zhao, B.B.; Xiong, C.R. Comparative analysis of five convolutional neural networks for landslide susceptibility assessment. *Bull. Eng. Geol. Environ.* **2023**, *82*, 377. <https://doi.org/10.1007/s10064-023-03408-9>.
18. Hu, S.; Qu, M.; Yuan, Y.; Pan, Z. Coupling cloud theory and concept hierarchy construction early warning thresholds for deformation safety of tailings dam. *Nat. Hazards* **2024**, *120*, 8827–8849. <https://doi.org/10.1007/s11069-024-06553-w>.
19. Wang, X.K.; Wang, S.H.; Zhang, H.Y.; Wang, J.Q.; Li, L. The recommendation method for hotel selection under traveller preference characteristics: A cloud-based multi-criteria group decision support model. *Group Decis. Negot.* **2021**, *30*, 1433–1469. <https://doi.org/10.1007/s10726-021-09735-0>.
20. Hao, T.; Zheng, X.; Wang, H.; Xu, K.; Yu, Y. Development of a method for weight determination of disaster-causing factors and quantitative risk assessment for tailings dams based on causal coupling relationships. *Stoch. Environ. Res. Risk Assess.* **2023**, *37*, 749–775. <https://doi.org/10.1007/s00477-022-02316-w>.
21. Das, S.; Priyadarshana, A.; Grebby, S. Monitoring the risk of a tailings dam collapse through spectral analysis of satellite InSAR time-series data. *Stoch. Environ. Res. Risk Assess.* **2024**, *38*, 2911–2926. <https://doi.org/10.1007/s00477-024-02713-3>.
22. Pacheco, F.; Hermosilla, G.; Piña, O.; Villavicencio, G.; Allende-Cid, H.; Palma, J.; Valenzuela, P.; García, J.; Carpanetti, A.; Minatogawa, V.; et al. Generation of synthetic data for the analysis of the physical stability of tailing dams through artificial intelligence. *Mathematics* **2022**, *10*, 4396. <https://doi.org/10.3390/math10234396>.
23. Chen, Z.; Xie, C.; Xiong, G.; Shen, J.; Yang, B. Using the morgenstern–price method and cloud theory to invert the shear strength index of tailings dams and reveal the coupling deformation and failure law under extreme rainfall. *Sustainability* **2023**, *15*, 6106. <https://doi.org/10.3390/su15076106>.
24. Zhang, H.; Li, Q.; Wang, J.; Fu, B. The impact of lenses on the seepage failure of tailings dam. *PLoS ONE* **2024**, *19*, e0305425. <https://doi.org/10.1371/journal.pone.0305425>.
25. Chang, M.; Qin, W.; Wang, H.; Wang, H.; Wang, C.; Zhang, X. Study on the evolution of a flooded tailings pond and its post-failure effects. *Water* **2022**, *15*, 173. <https://doi.org/10.3390/w15010173>.
26. Chu, C.; Xu, G. Application of grey deformation prediction model optimized by double coefficient for tailings dam. *Math. Probl. Eng.* **2022**, *1*, 6103860. <https://doi.org/10.1155/2022/6103860>.
27. GB 39496-2020; Safety Regulations for Tailings Reservoirs. Standardization Administration of China: Beijing, China, 2020.
28. Liang, Y.; Guo, D.; Huang, Z.; Jiang, X. Prediction model for coal-gas outburst using the genetic projection pursuit method. *Int. J. Oil Gas Coal Technol.* **2017**, *16*, 271–282. <https://doi.org/10.1504/IJOGCT.2017.087042>.
29. Song, C.; Xu, Z.; Li, B. Water eutrophication evaluation based on the improved projection pursuit regression model under the hesitant fuzzy environment. *Int. J. Inf. Technol. Decis. Mak.* **2024**, *23*, 361–380. <https://doi.org/10.1142/S0219622023500074>.
30. Duan, Z.; Chen, J.; Xie, J.; Li, Q.; Zhang, H.; Chen, C. Study on the evolution characteristics of dam failure due to flood overtopping of tailings ponds. *Water* **2024**, *16*, 2406. <https://doi.org/10.3390/w16172406>.
31. Li, D.; Cheung, D.; Shi, X.; Ng, V. Uncertainty reasoning based on cloud models in controllers. *Comput. Math. Appl.* **1998**, *35*, 99–123. [https://doi.org/10.1016/S0898-1221\(97\)00282-4](https://doi.org/10.1016/S0898-1221(97)00282-4).
32. Melo, M.; Eleutério, J. Probabilistic analysis of floods from tailings dam failures: A method to analyze the impact of rheological parameters on the HEC-RAS Bingham and Herschel-Bulkley models. *Water* **2023**, *15*, 2866. <https://doi.org/10.3390/w15162866>.
33. Edraki, M.; Baumgartl, T.; Manlapig, E.; Bradshaw, D.; Franks, D.M.; Moran, C.J. Designing mine tailings for better environmental, social and economic outcomes: A review of alternative approaches. *J. Clean. Prod.* **2014**, *84*, 411–420. <https://doi.org/10.1016/j.jclepro.2014.04.079>.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.