

Article

Experimental and CFD Investigation of Bubble Dynamics in Geldart Group B Fluidized Beds: A Comparative 2D and 3D Analysis

Zhu Yang ^{1,2} , Germán Mazza ³ , Maarten Vanierschot ^{4,5} , Renaud Ansart ⁶ and Yimin Deng ^{7,*} 

¹ Dongfang Boiler Co., Ltd., Zigong 643001, China

² Clean Energy Low-Carbon Thermal Utilization Technology and Equipment Key Laboratory of Sichuan Province, Chengdu 611731, China

³ Institute for Research and Development in Process Engineering, Biotechnology, and Alternative Energies, PROBIEN, CONICET-National University of Comahue, Neuquén 8300, Argentina; german.mazza@probien.gob.ar

⁴ Department of Mechanical Engineering, Applied Mechanics and Energy Conversion, Group T, KU Leuven, Celestijnenlaan 300, 3001 Leuven, Belgium; maarten.vanierschot@kuleuven.be

⁵ Material Science, Innovation and Modelling (MaSIM), North-West University, Private Bag X2046, Mmabatho 2745, South Africa

⁶ Laboratoire de Génie Chimique (LGC), Centre National de la Recherche Scientifique (CNRS), National Polytechnic Institute of Toulouse (Toulouse INP), University of Toulouse, 31000 Toulouse, France; renaud.ansart@toulouse-inp.fr

⁷ School of Chemistry and Chemical Engineering, Beijing Institute of Technology, Beijing 102248, China

* Correspondence: dengyimin0718@163.com

Abstract

Gas–solid bubbling fluidized beds involving Geldart Group B particles are fundamental to numerous industrial thermochemical processes, where bubble dynamics dictate the efficiency of heat and mass transfer. However, accurately predicting these complex hydrodynamic behaviors remains a challenge due to the non-linear coupling of phase interactions. This study presents a comprehensive validation of 2D and 3D Eulerian–Eulerian Two-Fluid Models (TFM) against an extensive experimental dataset. A ‘core-flow’ consistency principle is adopted, demonstrating that the 3D cylindrical simulation provides a physically equivalent representation of the central bubbling dynamics in the rectangular experimental bed. A key innovation of this work is a novel post-processing framework that bridges raw CFD datasets and quantitative bubbling metrics. Unlike traditional threshold-based segmentation or localized probe measurements, which are often limited by spatial resolution and noise sensitivity, the integrated use of Autodesk 3DS Max for volumetric reconstruction and customized MATLAB (R2024a) algorithms allows for the seamless processing of heterogeneous 2D and 3D data. This methodology significantly enhances the capability to track complex bubble coalescence and breakup events while improving batch-processing efficiency, providing a high-fidelity alternative for analyzing gas–solid flow patterns in complex geometries. The results show that both experimental data and 2D simulations align with Werther’s correlation, yielding Mean Relative Errors (MRE) of 8.2% and 10.5%, respectively. In contrast, the 3D simulation tracks Darton’s prediction closely with a lower MRE of 7.4%, demonstrating superior concordance in volumetric bubble growth. The core innovation lies in the definition of a clear dimensional choice framework: 2D simulations are computationally sufficient and accurate for predicting macro-scale bubble heights and frequencies under pseudo-2D or narrow-bed constraints. However, 3D simulations are strictly necessary when evaluating unconstrained radial expansion, core-flow dynamics, and precise volumetric bubble diameters (d_v) where full multi-directional degrees of freedom dictate hydrodynamics.



Academic Editor: Dmitry Portnikov

Received: 10 April 2026

Revised: 28 May 2026

Accepted: 28 May 2026

Published: 25 June 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and

conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

Keywords: group B particles; bubbles; 3D gas-fluidized beds; experiments; CFD simulation

1. Introduction

Gas–solid bubbling fluidized beds are widely used in different industrial processes and operations. Mixing, drying, heat exchange, and chemical reactors are typical examples where fluidization enhances heat and mass transfer rates and the easy handling of solids [1]. The hydrodynamic characteristics of fluidized beds provide a fundamental basis for operating and optimizing such units, such as the characterization of the bubbling phenomenon, with mostly the bubble sizes and their frequencies, which is crucially important. The bubble diameter itself is linked with mostly bed geometry, distributor characteristics, excess superficial gas velocity, and the level in the bed. The literature provides a range of experimental and simulated values for these properties [2]. Earlier works by [3–6] were further refined by Renaud et al. [7,8]. In gas–solid fluidized beds, one of the main modeling difficulties is the motion of two phases, where the interface is unknown and transient, as already demonstrated by [9,10].

When a model of fluidized bed is conceived, it should include detailed approaches to consider local phenomena and to connect them with the overall hydrodynamics of the bed.

Even though CFD tools are widely used to study bubbles' behavior, it is essential to validate CFD models with experimental data and correlations.

Bubble characteristics can be studied experimentally by means of intrusive techniques, such as optical fiber probes [11] or non-intrusive techniques, such as X-rays and capacitance tomography [12,13], or those based on pressure fluctuation measurements [14]. The literature correlations are mostly of an empirical nature. For Geldart Group A (fine) particles [15], a stable bubble size was noticed as a result of continuous bubble splitting and re-coalescence [16,17]. For Group B particles, various experimental data were correlated, but with a difference of about 50%. It is, hence, hardly possible to extrapolate predictions with confidence to systems that are different from the original one. Since these correlations are frequently used to validate CFD models, the critical evaluation presented by Karimipour and Pugsley [18,19] is of great value. Furthermore, the choice of closure models and physical boundary parameters significantly impacts the simulation of bubble patterns. As demonstrated by Parvathaneni et al. [20], Eulerian simulations are highly sensitive to interphase drag and solid–wall frictional contacts during the transition between bubbling and jetting regimes, making rigorous bubble statistics extraction essential. Additionally, Parvathaneni and Buwa [21] highlighted that wall specular coefficients and frictional parameters dictate localized mixing and segregation behaviors, which directly supports the necessity of evaluating 3D geometric boundaries over simplified 2D representations.

Experimental and three-dimensional (3D) CFD analyses of a cylindrical gas–solid bubbling fluidized bed, filled with Geldart group B particles, were conducted [22]. Experimental data regarding bubble characteristics were obtained from optical and pressure probe measurements based on the maximum entropy method (MEM) proposed by Sobrino et al. [23]. Rüdisüli et al. [24] developed a bubble-linking algorithm based on regression techniques to evaluate optical probe signals and measure the main bubble characteristics, such as rise velocity and size distribution in a bed. A digital image analysis technique (DIAT) was developed based on the use of a non-intrusive video camera coupled with image analysis in order to study fluidization dynamics in a 2D bubbling bed. The developed technique allows the simultaneous measurement of several bubble properties, such as size, velocity, bed height, and bubble-phase hold-up. The results

were compared with correlations, showing very good agreement. Later, simulations of a pseudo-2D fluidized bed using the Discrete Element Method (DEM) and a code based on the flood fill method for data analysis using ANSYS-CFX were performed. The obtained values of bubble diameter, aspect ratio, and velocity showed very good consistency with those that were derived from empirical correlations and experimental data reported in the literature.

Experimental upgrades were proposed by Li et al. [25] by using electrical capacitance tomography (ECT), while Maurer et al. [26] used X-ray tomography or acquired data by means of magnetic resonance imaging (MRI) combined with an ultrashort echo time (UTE) to obtain a higher resolution. Sobrino et al. [27] considered the results from 3D CFD simulations to validate MEM. Both probability density functions of chord lengths obtained from virtual optical probe signals were converted into diameter distributions by means of MEM, and the bubble volume was directly extracted based on the tomography reconstruction of an instantaneous 3D field—good agreement was obtained.

Although there are many reports in the literature regarding bubble characterization, most are restricted to experimental or 2D CFD data. In this work, CFD analysis was carried out using an Eulerian–Eulerian two-fluid model in 2D and 3D schemes, which provided information regarding the macroscopic hydrodynamics of phases, including bubble formation and movement. Autodesk USMax 2017-student version software was adopted here as a tool for post-processing CFD data. This novel approach allows for the prediction of bubble properties (size, shape, rise velocity, and frequency) directly from voidage data obtained via 3D simulations.

2. Experimental Setup, Materials, and Image Processing Methods

Experiments were carried out in a transparent two-dimensional rectangular fluidized bed (0.70 m height and a cross-sectional area of 0.60×0.025 m), thus allowing video imaging. A perforated plate distributor of 30 mbar pressure dropped at a superficial bed velocity of 0.3 m/s. A blower supplied the fluidizing air, with the flow rate measured by means of a mass flow meter with a resolution of 0.02 m/s. SiO₂ sand was used as bed material, with the average particle size, sphericity, roughness, and absolute density measured by common techniques [28]. The results of Table 1 define the sand as a Group B particle. Particle shapes were imaged using a microscope, as shown in Figure 1. The mean sphericity was 0.8, while the mean roundness was 0.6, corresponding to moderate sphericity and subrounded particles [16].

Table 1. Particle properties.

Property	Value
Average particle size, d_p [μm]	472.4
Absolute particle density, ρ_s [kg/m^3]	2781
Particle sphericity	0.8
Particle roundness	0.6

The minimum fluidization velocity was experimentally measured by the common pressure drop versus gas flow rate measurement [29]. The resulting value of U_{mf} was 0.2 m/s, which is in very good agreement with Nazmul Hossain et al. [30].

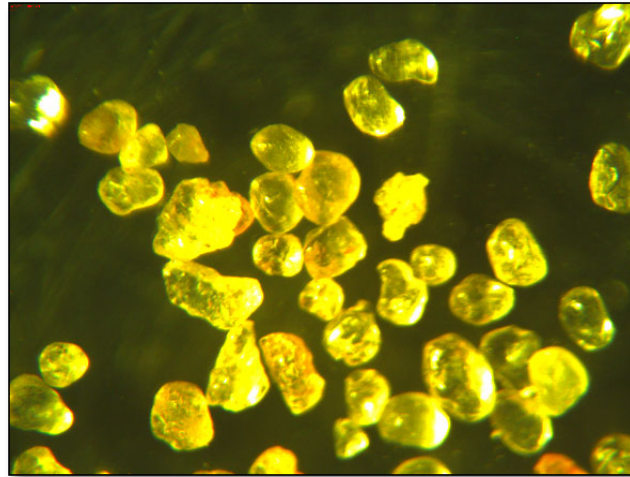


Figure 1. Microscopic image of sand showing roundness and sphericity.

The fluidization behavior was recorded by a high-speed digital camera at 60 fps with 1920×1080 resolution. Intense backlight illumination was used to improve image quality. Bubbles were clearly detected as transparent voids. A reference frame was added in the upper right side of the bed to relate the picture dimensions to the real size of the fluidized bed. The experimental setup is shown in Figure 2. The settled bed height was 0.32 m. To avoid image uncertainty by overlapping and coalescing bubbles, the superficial air velocity was set at 0.34 m.

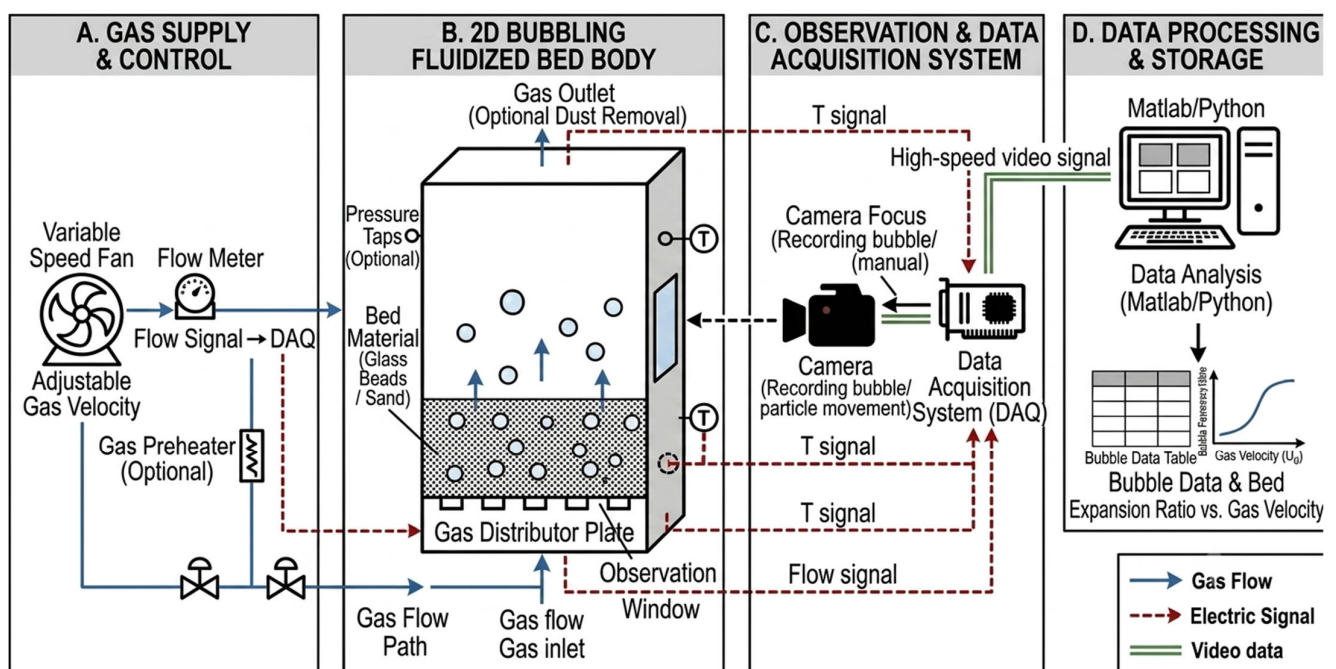


Figure 2. Experimental rig and methods.

Grayscale images were converted into binary images, using a threshold as shown in Figure 3. This threshold was determined by a comparative analysis between RGB images and binary images until the value that best represented the bubble phase was reached. In order to perform image treatment, a code was developed using MATLAB's Image Processing Toolbox. The reference frame is shown in the top right side of the experimental bed. The code reads the original image in TIFF format and uses a conversion factor based

on the reference frame to transform pixels into millimeters, creating a file with raw data of the area, perimeter, and center of mass of each bubble.

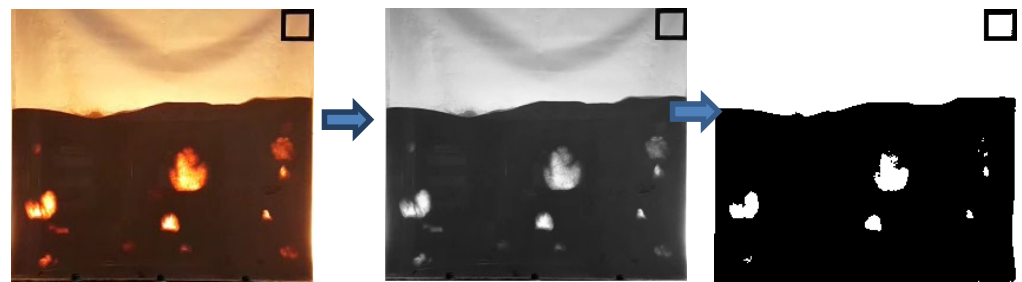


Figure 3. Experimental image analysis process: RGB image transformed into greyscale and then converted into binary by means of a threshold.

For CFD analysis, bubbles were identified using the simulated air volume fraction (α_g) within the imaged bubble boundary. There is no exact definition of bubble boundary, yet different researchers have used different standards to define it, such as 0.75, 0.80, or 0.85. The selection of the gas-phase volume fraction threshold (α_g) is a critical parameter for defining bubble boundaries in Eulerian–Eulerian simulations. In this work, $\alpha_g = 0.8$ was adopted as the criterion for bubble identification. To justify this choice, a sensitivity analysis was performed by varying the threshold between 0.75, 0.80, and 0.85. The results indicated that while a lower threshold ($\alpha_g = 0.75$) slightly increased the detected mean bubble diameter by approximately 4.5% due to the inclusion of the diffuse emulsion–bubble interface, the overall trend of bubble growth with height remained remarkably consistent across all three cases. The choice of 0.8 aligns with the established literature for Geldart Group B particles [4,22], where it has been identified as a robust value that balances the capture of the bubble core while minimizing the interference of bed fluctuations. This threshold ensures a precise representation of the bubble phase without overestimating its spatial extent in sand–air systems.

It is critical to define the explicit validity domain of the proposed framework. The quantitative findings and predictive criteria established in this study are strictly applicable to Geldart Group B particles operating within early bubbling regimes ($U_0 = 0.34$ m/s, $U_{mf} = 0.2$ m/s) in small-to-medium-scale fluidization geometries. Due to the inherent assumptions within the Eulerian–Eulerian Two-Fluid Model (TFM) and the specific bubble-tracking thresholds ($\alpha_g = 0.80$), these results do not directly generalize to (i) highly cohesive Geldart Group C or very coarse Group D solids; (ii) high-velocity fluidization regimes such as slugging, turbulent, or circulating fluidization, where regime transitions alter bubble topology, and (iii) large industrial-scale reactors, where severe wall-slugging effects are absent.

3. CFD Modeling

Fluidized bed hydrodynamics can be simulated by means of CFD by using the two-fluid model (TFM), which is based on an Euler–Euler approach [31,32]. This method defines both phases (gas and solid) as interpenetrating continua; therefore, a set of conservation equations is derived, with a similar structure for both phases.

ANSYS-Fluent 17.2 software was used in the present study. This code has different Euler–Euler multiphase models available, but for fluidized beds, Eulerian modeling is the most appropriate (ANSYS Theory Guide 17.1, 2016). It solves momentum and continuity equations for each phase, and coupling is achieved through the pressure and interphase exchange coefficient. Governing equations for this model are shown in Table 2. For

granular flows, the properties were obtained from the application of kinetic theory. Table 3 summarizes the closure models selected for the present study.

Table 2. Governing equations for TFM.

Conservation of mass: $\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{v}_g) = 0$ $\frac{\partial}{\partial t}(\alpha_s \rho_s) + \nabla \cdot (\alpha_s \rho_s \vec{v}_s) = 0$
Conservation of Momentum: $\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{v}_g) + \nabla \cdot (\alpha_g \rho_g \vec{v}_g \vec{v}_g) = -\alpha_g \nabla p + \nabla \cdot \vec{\tau}_g + \alpha_g \rho_g \vec{g} - K_{gs}(\vec{v}_g - \vec{v}_s)$ $\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{v}_s) + \nabla \cdot (\alpha_s \rho_s \vec{v}_s \vec{v}_s) = -\alpha_s \nabla p - \nabla p_s + \nabla \cdot \vec{\tau}_s + \alpha_s \rho_s \vec{g} + K_{gs}(\vec{v}_g - \vec{v}_s)$

Table 3. Summary of closure models.

Interphase Exchange Coefficient $K_{gs} = \frac{3}{4} C_D \frac{\alpha_s \alpha_g \rho_g \vec{v}_s - \vec{v}_g }{d_s} \alpha_g^{-2.65} \text{ for } \alpha_g > 0.8$ $K_{gs} = 150 \frac{\alpha_s (1 - \alpha_g) \mu_g}{\alpha_g d_s^2} + 1.75 \frac{\rho_g \alpha_s \vec{v}_s - \vec{v}_g }{d_s} \text{ for } \alpha_g \leq 0.8$ $C_D = \frac{24}{\alpha_g Re_s} \left[1 + 0.15 (\alpha_g Re_s)^{0.687} \right] \text{ Drag Coefficient}$ $\text{With } Re_s = \frac{\rho_g d_s \vec{v}_s - \vec{v}_g }{\mu_g}$
Solids Pressure $p_s = \alpha_s \rho_s \Theta_s + 2 \rho_s (1 + e_{ss}) \alpha_s^2 g_{0,ss} \Theta_s$ $g_{0,ss} = \left[1 - \left(\frac{\alpha_s}{\alpha_{s,max}} \right)^{\frac{1}{3}} \right]^{-1} \text{ Radial Distribution Function}$
Solids Shear Stresses $\mu_s = \mu_{s,col} + \mu_{s,kin} + \mu_{s,fr} \text{ Shear viscosity}$ $\mu_{s,col} = \frac{4}{5} \alpha_s \rho_s d_s g_{0,ss} (1 + e_{ss}) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}} \alpha_s \text{ Collisional Viscosity}$ $\mu_{s,kin} = \frac{10 d_s \rho_s \sqrt{\Theta_s} \pi}{96 \alpha_s (1 + e_{ss}) g_{0,ss}} \left[1 + \frac{4}{5} \alpha_s g_{0,ss} (1 + e_{ss}) \right]^2 \alpha_s \text{ Kinetic Viscosity}$

CFD simulations of the bed were performed in 2D and 3D environments, keeping the same bed cross-section and experimental conditions. Three-dimensional simulations were carried out in a cylindrical bed due to the fact that this geometry is most commonly used in industrial applications. The advantage of 3D simulations is that tri-dimensional images of the phase may be obtained, allowing preservation of all the degrees of freedom in the bubble and solid phases' motion [33]. The disadvantage is its high computational cost and the more complex methods needed for image post-processing and the study of bubble parameters.

For mesh validation, grids with different degrees of refinement were tested. The coarsest mesh that reproduced the bed pressure drop at a superficial gas velocity corresponding to a fluidized bed with less than 5% error. compared with the experimental results. was identified and adopted for further simulations. In this regard, a 444,312-cell grid was employed in a 3D simulation, while only 48,923 cells were needed in the 2D simulations.

For fluidized bed simulations, transient treatment was used. Both 3D and 2D simulations were carried out under a double-precision mode, and the pressure-based solver was adopted. Second-order upwind discretization schemes were used, and pressure–velocity coupling was obtained using the phase-coupled SIMPLE algorithm. The boundary conditions used were:

- Velocity Inlet at the bottom of the bed, where the superficial gas velocity was set;
- Pressure Outlet at the top of the bed, set to atmospheric conditions;
- Wall, with a no-slip condition set for the air-phase and a Secularity Coefficient of 0.5 set for the solid-phase.

The main model parameters and conditions applied in this study are shown in Table 4.

Table 4. Parameter description.

Parameter	Value
ρ_g [kg/m ³]	1.2
μ_g [Pa·s]	1.789×10^{-5}
ρ_s [kg/m ³]	2781
d_p [μm]	472.4
Specularity Coefficient	0.5
Restitution coefficient	0.9
Superficial air velocity [m/s]	0.35
Initial solid volume fraction α_s	0.53
Initial bed height [m]	0.32
Time step	0.001
Max. Iterations/Time step	40
Convergence criteria	0.001

It should be noted that while the experimental measurements were conducted in a pseudo-2D rectangular bed, the 3D simulations utilized a cylindrical geometry. This choice was motivated by two primary considerations. First, the cylindrical domain represents a more generalized configuration for industrial-scale reactors, allowing the validation of the Euler–Euler TFM in a context that is applicable to standard circular geometries. Second, from a computational perspective, the cylindrical symmetry facilitates an optimized mesh structure that significantly reduces the occurrence of skewed cells at corner regions, thereby enhancing numerical stability and convergence efficiency compared to a 3D rectangular mesh with similar cell counts.

To ensure the accuracy and numerical stability of the CFD model, grid and time-step independence studies were systematically performed using the time-averaged pressure drop (ΔP) and the mean bubble frequency (f_b) at a bed height of $h = 150$ mm as evaluation metrics. Three mesh densities—coarse (approx. 45,000 cells), medium (approx. 120,000 cells), and fine (approx. 280,000 cells)—were tested. The relative difference in both ΔP and f_b between the medium and fine meshes was less than 1.8%, justifying the selection of the medium mesh to balance accuracy and computational economy. Furthermore, three time steps (1.0×10^{-4} s, 5.0×10^{-4} s, and 1.0×10^{-3} s) were compared. A time step of 5.0×10^{-4} s was selected since it guaranteed a maximum Courant number below 0.25 and ensured strict convergence within 20 iterations per time step.

4. Characterization of Bubble Parameters

To convert raw numerical data into quantifiable hydrodynamics, a structured post-processing framework was implemented. The sequential steps for isolating, thresholding,

and reconstructing the bubble structures from both 2D and 3D CFD datasets are visually illustrated in Figure 4, Figures 5 and 6, respectively.

Two-dimensional simulations generate a greyscale image of bubbles, as shown in Figure 4. The equivalent diameter of the cylindrical 3D bed was carefully selected to represent the hydraulic conditions of the rectangular setup. These images were post-processed using a code similar to that developed for experimental images in order to obtain binary images and data concerning the area, perimeter, and bubble center of mass from them.

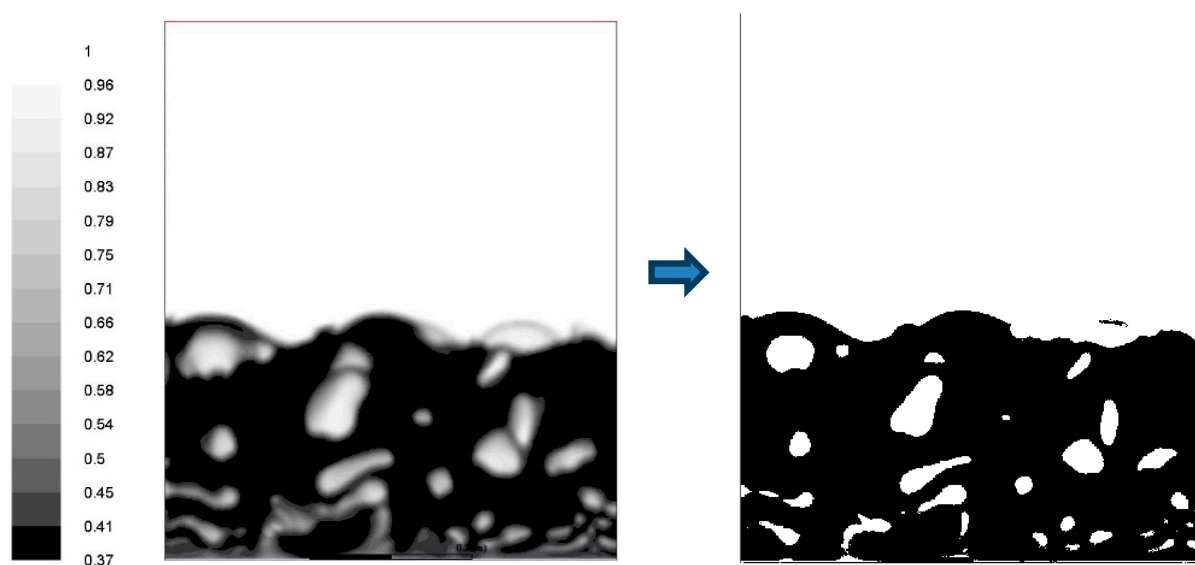


Figure 4. Two-dimensional CFD image analysis process: grayscale image obtained from Fluent turned into a binary image.

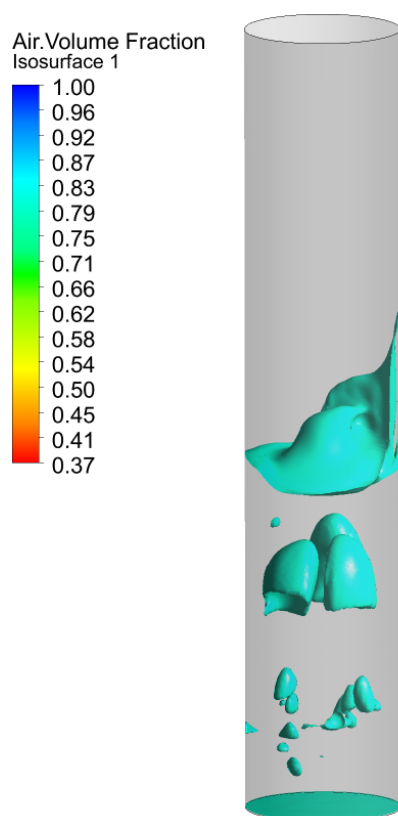


Figure 5. Iso-surface of air volume fraction obtained for 3D CFD simulations.

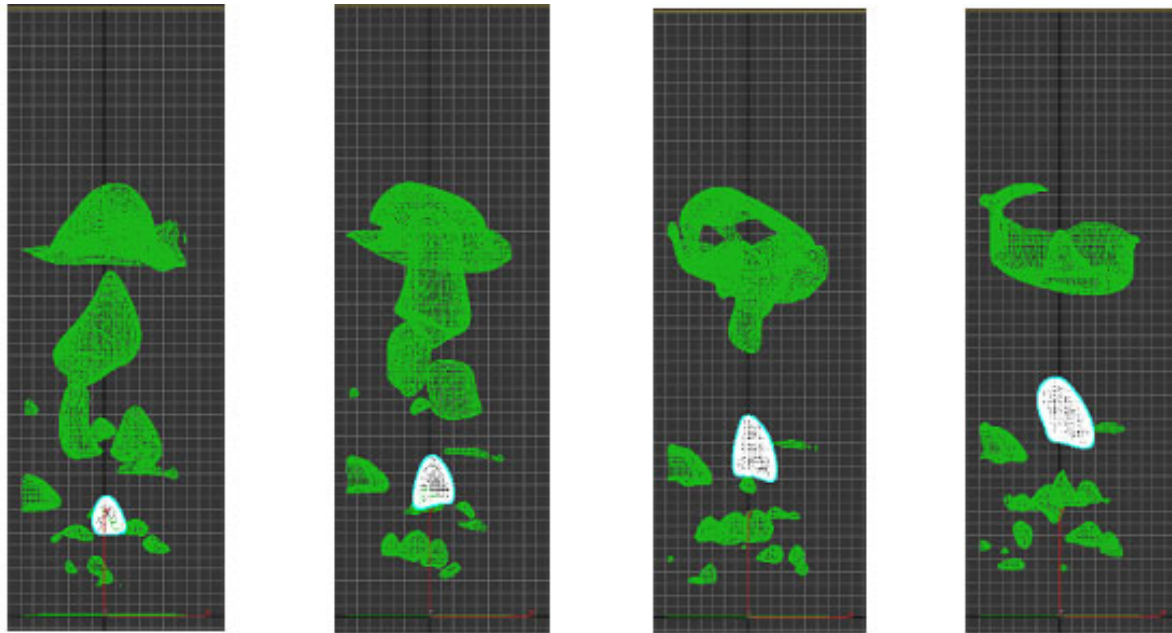


Figure 6. Consecutive images in the 3DS Max environment following rising bubbles.

To perform 3D bubble analysis, it is necessary to generate tri-dimensional images that exclusively represent the bubble phase, called iso-surfaces. These images were generated every 10 time steps, using α_g as the reference parameter to create them, as shown in Figure 5. These iso-surfaces were exported from ANSYS-Fluent to Autodesk 3DS Max 2017-student version software in order to isolate each bubble and obtain its area, volume, and mass center.

A second self-developed MATLAB code was used to calculate the bubble mean diameter, frequency, shape factor (for 2D images), and sphericity (for 3D images). The results of the mean values averaged in time are presented graphically as a function of vertical position. The code thus pairs off bubbles in consecutive images, taking into account the maximum possible value of displacement along the horizontal and vertical axes. The core of the tracking algorithm is a proximity-based matching criterion, which is further refined to handle complex topological changes. To address bubble coalescence (merging), the algorithm identifies instances where multiple centroids in frame t correspond to a single enlarged area in frame $t + \Delta t$ within a localized search radius. Conversely, bubble breakup (splitting) is detected when a single parent bubble fragmentizes into multiple child centroids that collectively conserve the original projected area. For boundary conditions, bubbles whose centroids approach the computational or experimental domain edges within a defined tolerance are flagged as ‘truncated’ and excluded from velocity averaging to prevent systematic underestimation of the rise velocity.

Then, time-averaged rising bubbles’ velocities are calculated and presented graphically as a function of vertical position. This tracking procedure is exemplified in Figure 6, which shows consecutive images processed by Autodesk 3DS Max, allowing one to follow the rise of a particular bubble tagged in white.

To ensure a rigorous comparison between the 2D/3D simulations and experimental data, four key bubble parameters are defined. The extraction of these parameters relies on the integrated post-processing framework combining volumetric reconstruction and image analysis.

4.1. Bubble Diameter (d_b)

The bubble diameter is defined based on the equivalent area (2D) or volume (3D). For experimental and 2D CFD images, d_b is the diameter of a circle with an area equal to the detected bubble. In 3D simulations, the volume-equivalent diameter (d_v) is calculated by:

$$d_v = \left(\frac{6V_b}{\pi} \right)^{1/3} \quad (1)$$

where V_b is the bubble volume reconstructed via the 3DS Max framework.

4.2. Bubble Frequency (f_b)

Bubble frequency is determined as the number of bubbles passing through a specific horizontal cross-section per unit time. In this study, frequency is monitored at varying heights (h) above the distributor to track the evolution of bubble population.

4.3. Bubble Shape Factor (Φ)

To quantify the deviation from a perfect sphere/circle, the shape factor is defined as the ratio of the minimum to maximum dimensions of the bubble. For 3D bubbles, this reflects the aspect ratio of the reconstructed ellipsoid, providing insight into the degree of flattening during rise.

4.4. Bubble Rise Velocity (U_b)

The rise velocity is calculated by tracking the displacement of the bubble centroid (Δy) over a discrete time interval (Δt). For the 3D cylindrical domain, only bubbles rising within the core region are considered to ensure consistency with core-flow dynamics and minimize wall-induced interference. To minimize misidentification, a secondary criterion based on area consistency ($\pm 20\%$ tolerance) was implemented to ensure the paired bubbles in consecutive frames represent the same physical entity.

5. Results and Discussion

To address the potential systematic bias introduced by the geometric inconsistency between the rectangular experimental bed and the cylindrical 3D simulation, the comparison focuses on the core-flow dynamics where wall effects are localized. Previous sensitivity studies on gas–solid systems [22] have indicated that the central bubble properties (diameter and rise velocity) remain largely independent of the bed's cross-sectional shape, provided the equivalent diameter is maintained and the aspect ratio is sufficiently large. In this study, the hydraulic diameter of the cylindrical domain was matched to the effective width of the experimental bed to minimize the 'wall-induced drag' discrepancies. This approach ensures that the observed dimensionality effects (2D vs. 3D) are a result of volumetric bubble development rather than purely geometric artifacts.

The fluidization behavior can be illustrated by means of instantaneous air volume fraction contour graphics. Figure 7 shows the fluidization process in the studied cylindrical bed. In Figure 7a, the time evolution of air volume fraction contours, corresponding to an axial plane passing through the middle of the bed, is presented, while in Figure 7b, the 3D iso-surfaces corresponding to the same instants are shown.

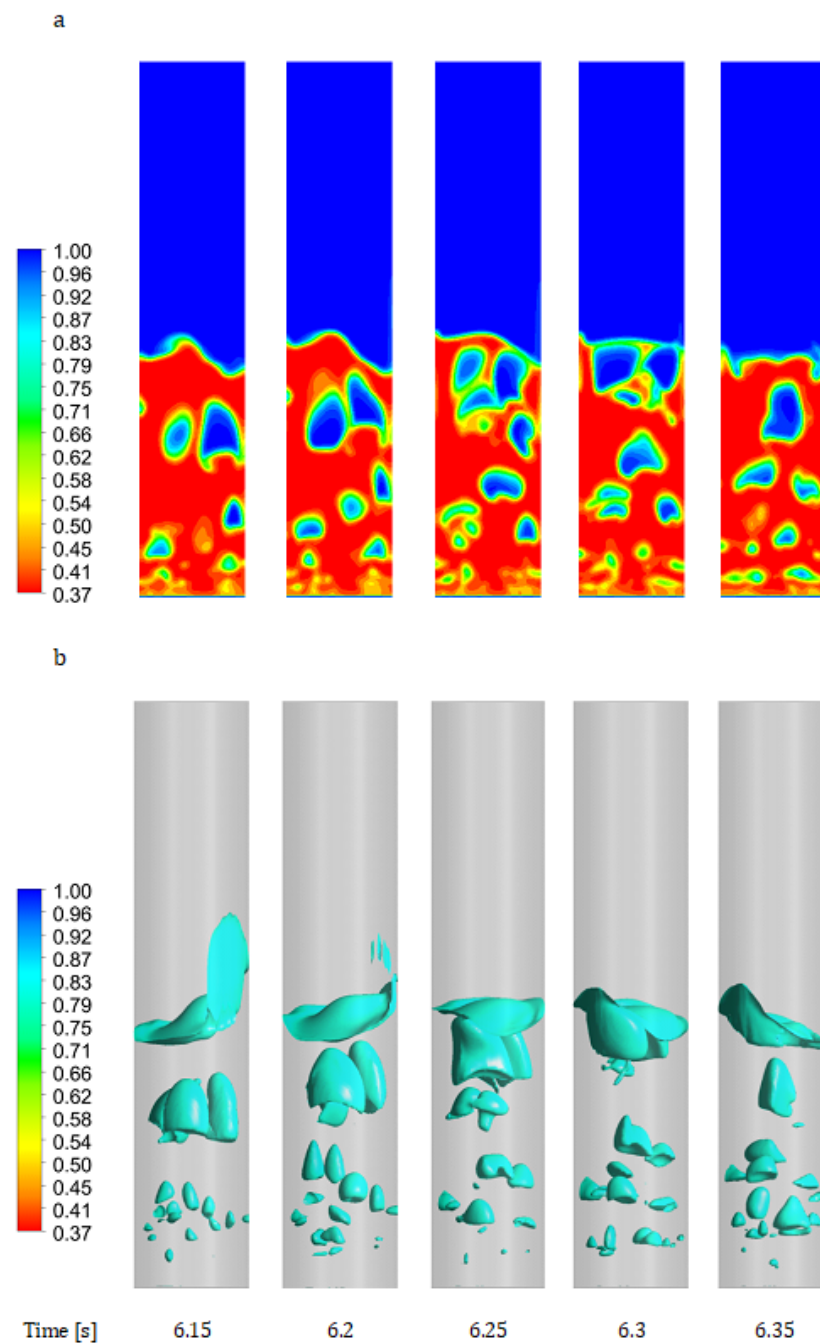


Figure 7. Time evolution of bubbles in the fluidized bed. (a) Contours of air volume fraction. (b) Iso-surfaces of air volume fraction in a 3D environment for the contours of Figure 7a contours.

5.1. Evaluation of Bubble Diameter and Growth

Bubble diameter values resulting from experimental and simulated data as a function of vertical coordinates above the distributor are shown in Figure 8a–c, where a wide scatter can be seen. Bubble diameters d_b for experimental and 2D CFD analyses were estimated while considering the equivalent diameter of a circle that has the same area (A_b) as the bubble:

$$d_b = \sqrt{\frac{4 A_b}{\pi}} \quad (2)$$

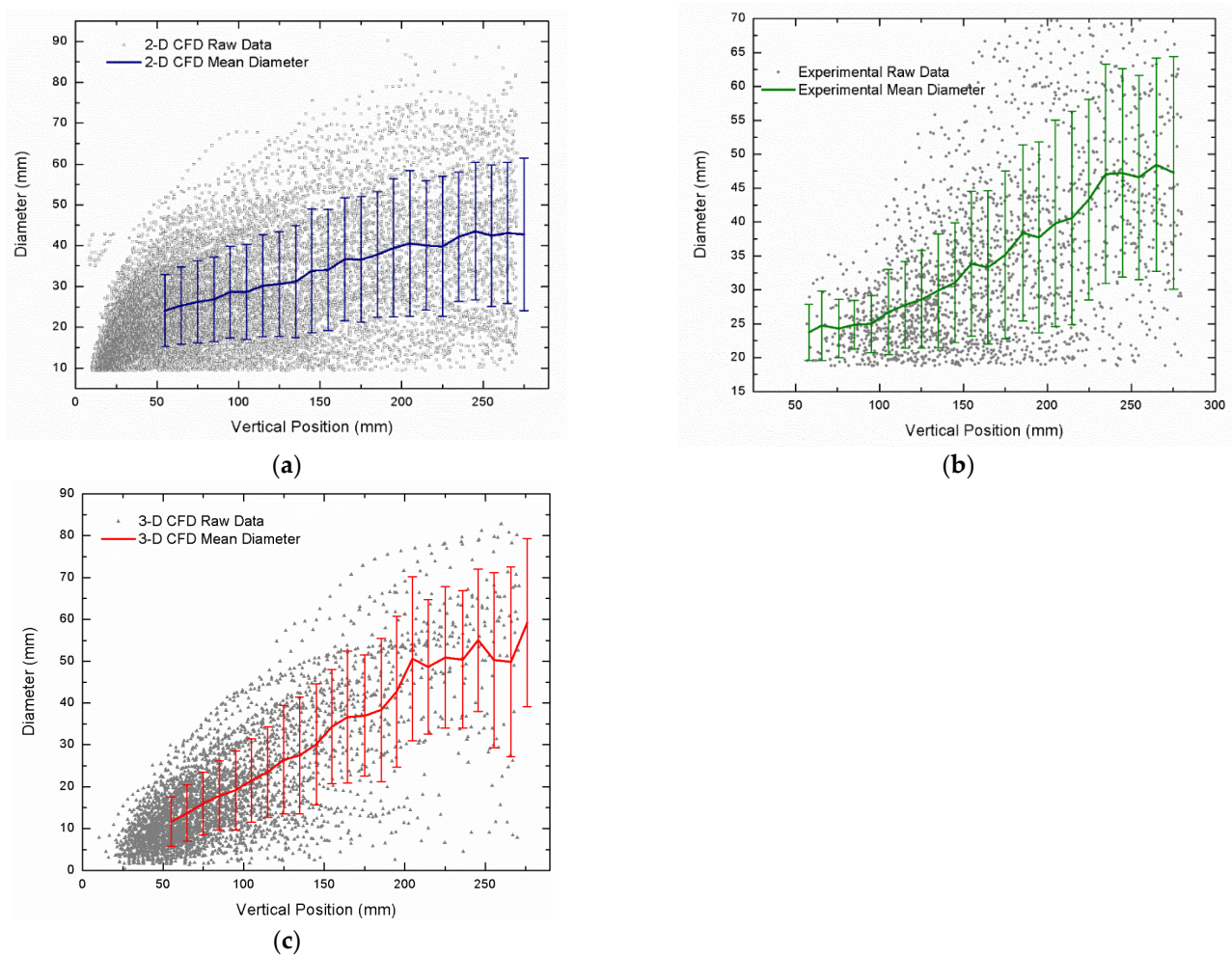


Figure 8. Data for bubble diameters and the corresponding mean diameters as a function of vertical position for experimental (a), 2D CFD (b), and 3D CFD (c) results.

For 3D bubbles, their diameters can be estimated from their volume or from their area. In the present work, the volume basis was applied:

$$d_v = \sqrt[3]{\frac{6 V_b}{\pi}} \quad (3)$$

As observed, bubble diameters always increase with vertical position; however, with a significant fraction of small bubbles at all vertical positions, these correspond to a wide distribution of bubble sizes due to coalescence and splitting phenomena.

The bubble mean diameter as a function of vertical position above the distributor was compared with the bubble diameter predicted by correlations proposed by Darton [34] and Werther [35]. Figure 9 shows the corresponding results, confirming that bubbles grow steadily as they rise in the bed, approximately following a power law, which is in good agreement with the correlation mentioned.

$$d_v = 0.54 g^{-0.2} (U_0 - U_{mf})^{0.4} \left(h + \frac{4}{N_{OR}^{0.5}} \right)^{0.8} \quad \text{Darton} \quad (4)$$

$$d_v = 0.853 \left(1 + 0.272 (U_0 - U_{mf}) \right)^{1/3} (1 + 0.0684h)^{1.21} \quad \text{Werther, for Geldart B-type particles} \quad (5)$$

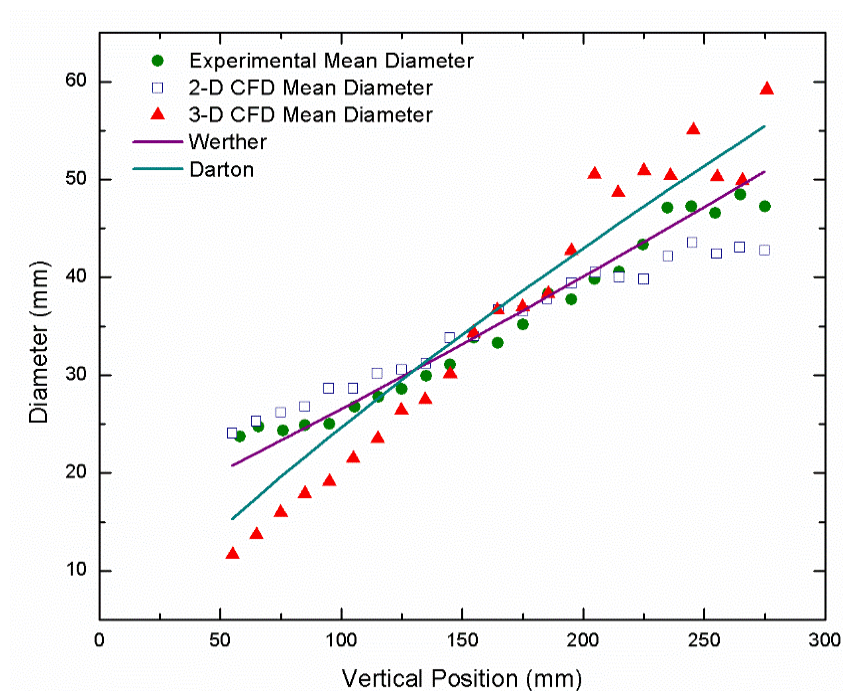


Figure 9. Bubble mean diameter versus vertical position.

The observed divergence in correlation applicability—where experimental and 2D CFD results align with Werther’s prediction while 3D results favor Darton’s model—offers fundamental insights into the role of geometric constraints. It is hypothesized that 2D domains inherently restrict the lateral expansion of bubbles, effectively mimicking the ‘narrow bed’ assumptions prevalent in Werther’s development. Conversely, 3D simulations permit unconstrained radial development and volumetric growth, which more closely adhere to the ‘volume-expansion’ principles underlying Darton’s correlation. This finding suggests that the selection of empirical benchmarks must be strictly harmonized with the dimensionality of the modeling approach to ensure predictive accuracy.

To quantitatively evaluate the goodness-of-fit between the observed bubble diameters and the empirical correlations, the Mean Relative Error (MRE) was calculated in Table 5. The results indicate that the experimental data and 2D CFD simulations exhibit a high degree of consistency with Werther’s correlation, yielding MRE values of 8.2% and 10.5%, respectively, compared to a significantly higher MRE (exceeding 18%) when matched with Darton’s model. Conversely, the 3D CFD dataset demonstrates a superior alignment with Darton’s prediction, with an MRE of only 7.4%, whereas its deviation from Werther’s correlation increases to 14.2%. These quantitative metrics provide robust evidence for the dimensionality-dependent applicability of the studied correlations.

Table 5. Quantitative error metrics (MRE and RMSE) comparing experimental data, CFD-2D, and CFD-3D models against empirical correlations at various bed heights.

Comparison Baseline	Evaluation Case	Mean Relative Error (MRE, %)	Root Mean Square Error (RMSE, mm)
Werther’s Correlation	Experimental Data	8.2%	3.41
Werther’s Correlation	CFD-2D Simulation	10.5%	4.85
Darton’s Correlation	CFD-3D Simulation	7.4%	2.92

5.2. Analysis of Bubble Frequency and Coalescence

Figure 10 represents the bubble frequency at vertical positions from the distributor. For its evaluation, the bed was divided into 23 equal horizontal slices, and the number of

bubbles at each section was calculated. Then, the frequency was estimated as the number of bubbles at each horizontal bed section divided by the total number of bubbles throughout the bed during the experiment and by the total lapse of time.

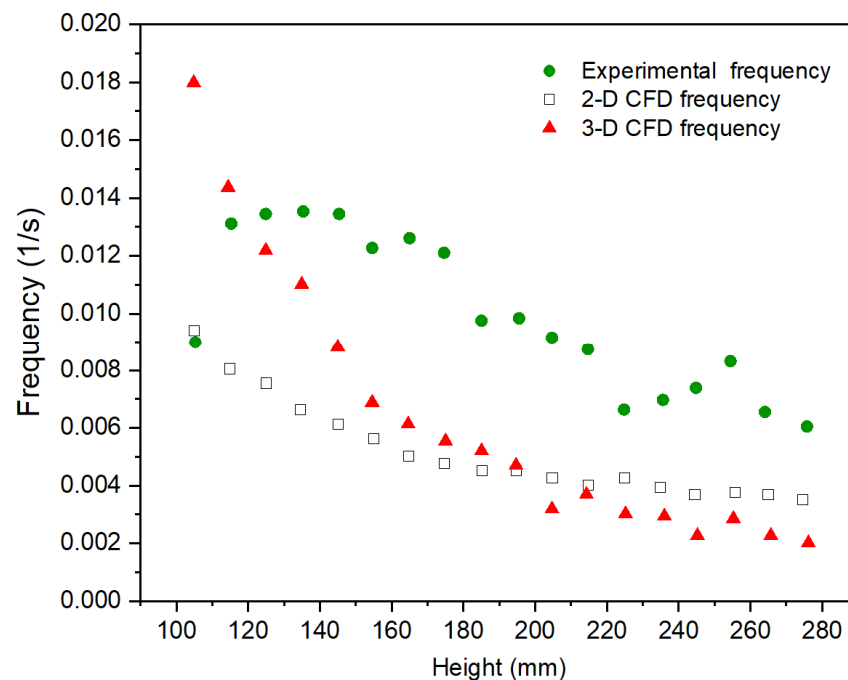


Figure 10. Bubble frequency results.

Both the 2D and 3D CFD simulation results show a decrease in bubble frequency at all vertical positions, following an almost logarithmic law. Meanwhile, the experimental results present an increase of up to 110 mm axial position, followed by the same decreases that were shown in the simulation results. The behavior observed in the experimental results is due to the fact that at lower vertical positions, bubbles are small and hard to visualize because they do not go through the bed and, therefore, are not transparent, which allows for their observation in images. At a certain vertical position, most bubbles are big enough to enter through the bed and are able to be taken into account for this study.

5.3. Evolution of Bubble Shape and Sphericity

Shape factor and sphericity are equivalent parameters for 2D and 3D bubble shape analysis, respectively. They are evaluated as:

$$\eta = \frac{\pi d_B}{P} \quad (6)$$

$$\psi = \frac{\pi(d_v)^2}{(A_b)} \quad (7)$$

Figure 11a shows the experimental and 2D CFD shape factor results, and Figure 11b presents the 3D CFD sphericity values as a function of vertical position. In all cases, the bubbles have an almost round or spherical shape within the vertical positions. Despite the fluctuations observed, both parameter values are within 0.8.

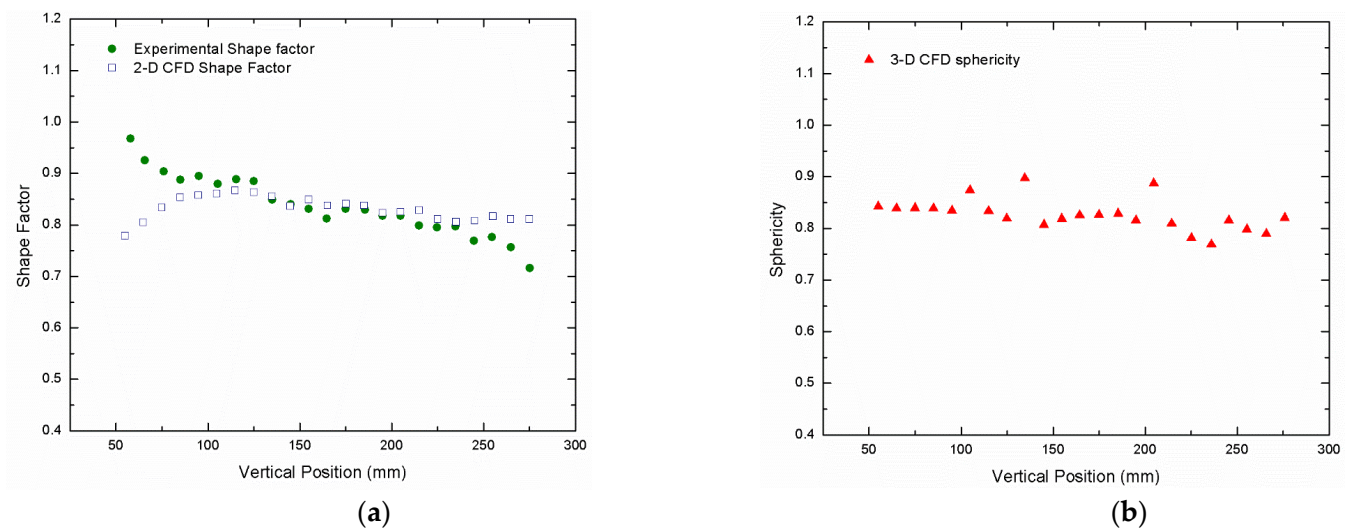


Figure 11. (a) Bubble shape factor for 2D images and (b) bubble sphericity for 3D images.

5.4. Comparison of Bubble Rise Velocity

In evaluating the bubble rise velocity, it is imperative to address the geometric discrepancy between the rectangular experimental bed and the cylindrical 3D computational domain. As established in Section 3, the comparison is predicated on the ‘core-flow’ consistency principle. By focusing on bubble dynamics within the central region of the bed—where wall-induced drag and friction effects are minimized—the 3D cylindrical model serves as a physically equivalent representation of the experimental setup.

Bubble rise velocity is calculated as the vertical displacement of its mass center divided by the time step between both recording instants, as follows:

$$v_b = \frac{y_{i+1} - y_i}{\Delta t} \quad (8)$$

The results were compared with the predictions proposed by Davidson and Harrison [32], and Kunii and Levenspiel [36]:

$$v_{br} = 0.71\sqrt{gd_B} \quad \text{single bubble}$$

$$v_b = v_{br} + (U_o - U_{mf}) \quad \text{bubbling bed}$$

$$v_b = 0.0032 \left((U_o - U_{mf}) + 11.3\sqrt{d_v} \right) D^{1.35} + v_{br} \quad \text{Kunii and Levenspiel}$$

Figure 12 shows an increment in the bed vertical coordinate, confirming that larger bubbles have higher velocities. The results from the 2D CFD analysis show good agreement with both experimental and Davidson and Harrison velocity values. The 3D CFD velocity results are, however, the most elevated due to the fact that 2D bubbles are affected by wall effects [37]. The slightly higher velocities observed in 3D TFM results, compared to the experimental data, can be further explained by this geometric equivalence. While the core-flow definition ensures scientific comparability, the 3D cylindrical domain provides an unconstrained radial environment that more accurately reflects the rise of the fastest, non-truncated bubbles in the bed center, which are often difficult to isolate purely in a narrow rectangular experimental setup. Moreover, in 2D fluidized beds, the bubble velocity seems to reach a maximum value when becoming close to the eruption height [38].

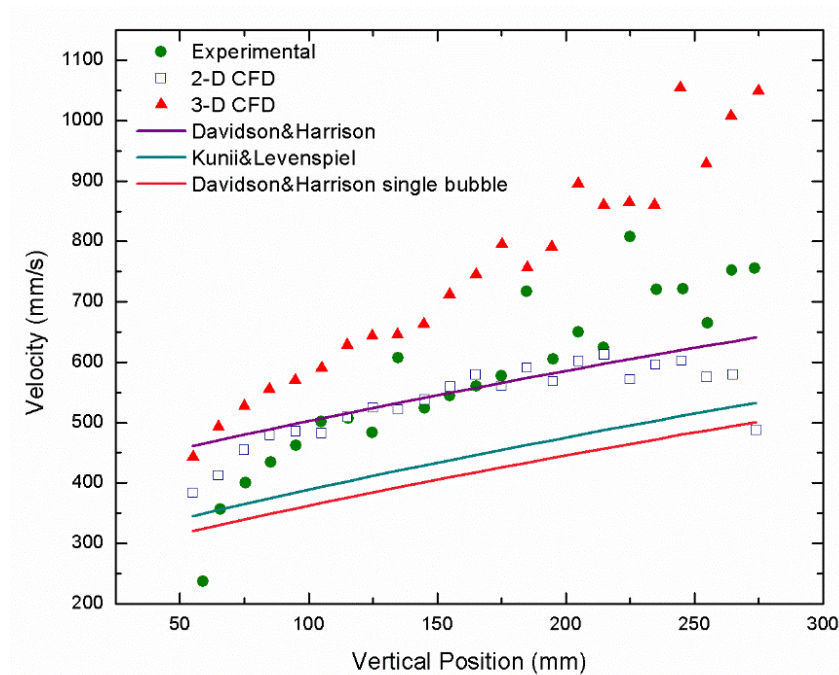


Figure 12. Bubble rise velocity versus vertical position. A good agreement is obtained between Davidson and Harrison and 2D data (experimental and CFD).

When studying the velocity coefficient, described as $\varphi = v_b / \sqrt{g d_b}$, it can be seen that φ remains almost constant at every vertical position [38], as shown in Figure 13. Therefore, bubble rise velocity can be described as $v_b = \varphi * \sqrt{g d_b}$, which has the same form as the correlation of single bubble velocity in the fluidized bed proposed by Davidson and Harrison in 1963 ($v_b = 0.71 * \sqrt{g d_b}$), with constants of almost 0.95 for the experimental and 2D CFD simulations results and 1.15 for the 3D CFD simulations.

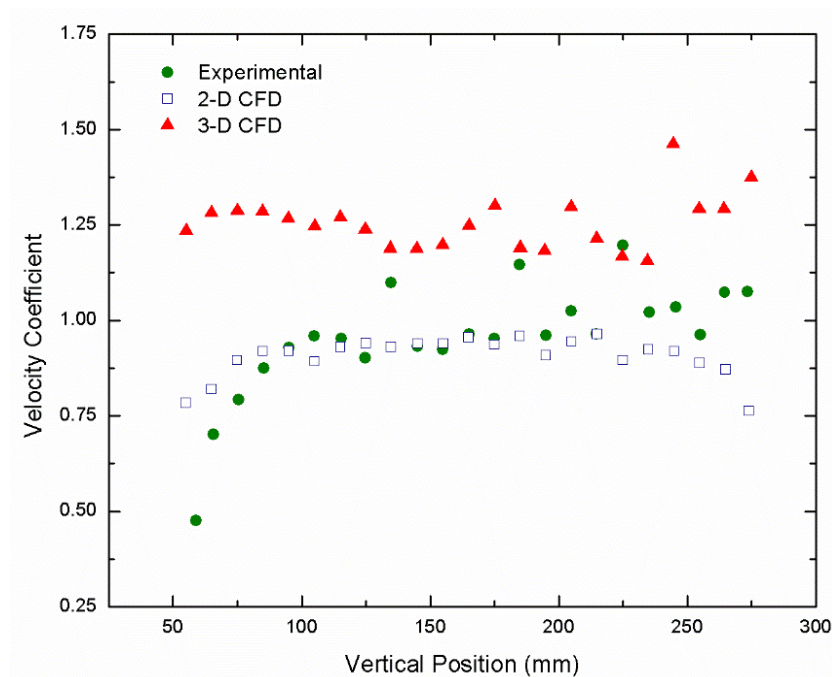


Figure 13. Velocity coefficients for different vertical positions: experimental and 2D/3D CFD simulation results.

6. Conclusions

This study successfully validated the Eulerian–Eulerian Two-Fluid Model (TFM) for predicting bubble dynamics in Geldart Group B gas-fluidized beds by integrating comprehensive experimental data with multi-dimensional CFD simulations. The results demonstrate that the ANSYS-Fluent 17.2 framework, complemented by a novel post-processing methodology utilizing Autodesk 3DS Max and MATLAB image analyses, accurately captures essential bubbling characteristics, including diameter, frequency, and shape factors. A significant finding is the dimensionality-dependent performance of empirical correlations: while experimental measurements and 2D simulations align closely with Werther's predictions, the 3D CFD results exhibit superior concordance with Darton's correlation. This divergence underscores the necessity of accounting for volumetric expansion and spatial bubble distribution when modeling sand-based fluidized systems at higher dimensions.

Furthermore, the analysis of bubble rise velocity reveals that 3D simulations yield slightly higher values compared to both 2D models and experimental observations, exceeding the predictions established by the Davidson and Harrison correlation. This phenomenon is primarily attributed to the reduced influence of wall friction in the 3D computational domain, which allows for a more realistic representation of high-velocity bubbles rising through the bed core without wall-induced retardation. The increasing dispersion of bubble diameters in the upper regions of the bed further confirms the dominant role of coalescence phenomena in governing phase hydrodynamics. While 3D TFM provides higher fidelity for volumetric bubble behavior, 2D simulations remain a computationally efficient proxy for preliminary screening of Group B fluidization, provided that Werther-type wall-constrained correlations are applied. Overall, the integrated approach developed in this work provides a robust and powerful tool for the high-fidelity analysis of air–sand bubbling fluidized beds, offering valuable insights into the design and scale-up of industrial thermochemical reactors.

While this study establishes a robust framework for Geldart Group B particles under ambient conditions, certain limitations warrant further investigation. The current post-processing algorithms assume a high contrast between phases, which may require recalibration for Geldart Group A (fine) or C (cohesive) particles due to more diffuse inter-phase boundaries. Furthermore, while the model is validated for spherical sand particles, the impact of particle non-sphericity and extreme operational conditions (e.g., high temperature or high pressure) remains to be explored. Future work will extend this 3D validation methodology to reactive systems and non-isothermal flows to broaden its applicability in industrial-scale chemical reactors.

Author Contributions: Methodology, Y.D.; Software, R.A.; Validation, Y.D.; Investigation, Germán Mazza; Data curation, Z.Y.; Writing—original draft, Z.Y., G.M., M.V., R.A., and Y.D.; Writing—review and editing, Z.Y., G.M., M.V., R.A., and Y.D.; Supervision, M.V. and R.A.; Funding acquisition, Y.D. All authors have read and agreed to the published version of the manuscript.

Funding: The authors gratefully acknowledge the support received from the National Natural Science Foundation of China (Grant No. 22408021), the Science and Technology Program of Xinjiang Uyghur Autonomous Region (2024A01006), and the China Postdoctoral Science Foundation (2025T181111).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data is available on the request to the corresponding author.

Conflicts of Interest: Author Zhu Yang was employed by the Dongfang Boiler Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Nomenclature

Greek Symbols

α_g	Volume fraction of the gas phase	-
α_s	Volume fraction of the solid phase	-
ρ_s	Solid density	kg/m ³
ρ_g	Gas density	kg/m ³
ϕ_w	Specularity coefficient at the wall	-
τ_s	Solid phase stress tensor	Pa

Variables and Parameters

d_b	Equivalent bubble diameter	mm
d_v	Volumetric bubble diameter	mm
d_p	Mean particle diameter	μm
e_{ss}	Particle-particle restitution coefficient	-
f_b	Bubble frequency	Hz
g	Acceleration due to gravity	m/s ²
h	Height above the gas distributor	mm
N_{OR}	Number of active orifices on the distributor	-
U_0	Superficial gas velocity	m/s
U_{mf}	Minimum fluidization velocity	m/s

Abbreviations

CFD	Computational Fluid Dynamics
MRE	Mean Relative Error
RMSE	Root Mean Square Error
TFM	Two-Fluid Model

References

1. Baeyens, J.; Li, S.; Dewil, R.; Zhang, H.; Deng, Y. Fluidized Bed Technology: Challenges and Perspectives. *IOP Conf. Ser. Earth Env. Sci.* **2022**, *952*, 012010. [[CrossRef](#)]
2. Wu, X.; Li, Y.; Zhu, X.; Huang, L.; Zhu, X. Experimental study on fluidization behaviors of walnut shell in a fluidized bed assisted by sand particles. *RSC Adv.* **2018**, *8*, 40279–40287. [[CrossRef](#)] [[PubMed](#)]
3. Ding, J.; Gidaspow, D. A bubbling fluidization model using kinetic theory of granular flow. *AIChE J.* **1990**, *36*, 523–538. [[CrossRef](#)]
4. Reuge, N.; Cadoret, L.; Coufort-Saudejaud, C.; Pannala, S.; Syamlal, M.; Caussat, B. Multifluid Eulerian modeling of dense gas–solids fluidized bed hydrodynamics: Influence of the dissipation parameters. *Chem. Eng. Sci.* **2008**, *63*, 5540–5551. [[CrossRef](#)]
5. Lundberg, L.; Soria-Verdugo, A.; Pallarès, D.; Johansson, R.; Thunman, H. The role of fuel mixing on char conversion in a fluidized bed. *Powder Technol.* **2017**, *316*, 677–686. [[CrossRef](#)]
6. Deng, Y.; Dewil, R.; Appels, L.; Van Tulden, F.; Li, S.; Yang, M.; Baeyens, J. Hydrogen-enriched natural gas in a decarbonization perspective. *Fuel* **2022**, *318*, 123680. [[CrossRef](#)]
7. Neau, H.; Pigou, M.; Fede, P.; Ansart, R.; Baudry, C.; Méricoux, N.; Laviéville, J.; Fournier, Y.; Renon, N.; Simonin, O. Massively parallel numerical simulation using up to 36,000 CPU cores of an industrial-scale polydispersed reactive pressurized fluidized bed with a mesh of one billion cells. *Powder Technol.* **2020**, *366*, 906–924. [[CrossRef](#)]
8. Neau, H.; Ansart, R.; Baudry, C.; Fournier, Y.; Méricoux, N.; Koren, C.; Laviéville, J.; Renon, N.; Simonin, O. HPC challenges and opportunities of industrial-scale reactive fluidized bed simulation using meshes of several billion cells on the route of Exascale. *Powder Technol.* **2024**, *444*, 120018. [[CrossRef](#)]
9. Jiang, H.; Liao, D.; Li, D.; Chen, Y. Industrial Regenerator Model for SMT Technology. *ACS Omega* **2023**, *8*, 9334–9345. [[CrossRef](#)] [[PubMed](#)]
10. Zhang, H.; Kong, W.; Tan, T.; Gilles, F.; Baeyens, J. Experiments support an improved model for particle transport in fluidized beds. *Sci. Rep.* **2017**, *7*, 10178. [[CrossRef](#)] [[PubMed](#)]
11. Sobrino, C.; Acosta-Iborra, A.; Santana, D.; de Vega, M. Bubble characteristics in a bubbling fluidized bed with a rotating distributor. *Int. J. Multiph. Flow* **2009**, *35*, 970–976. [[CrossRef](#)]

12. Kong, W.; Wang, B.; Baeyens, J.; Li, S.; Ke, H.; Tan, T.; Zhang, H. Solids mixing in a shallow cross-flow bubbling fluidized bed. *Chem. Eng. Sci.* **2018**, *187*, 213–222. [\[CrossRef\]](#)
13. Kong, W.; Li, S.; Ke, H.; Zhang, H.; Baeyens, J. The use of ultrasound probes to monitor multi-phase behavior in opaque systems. *Particuology* **2019**, *45*, 91–97. [\[CrossRef\]](#)
14. Bi, C.-X.; Jing, W.-Q.; Zhang, Y.-B. Broadband acoustic holography from intensity measurements with a three-dimensional pressure-velocity probe. *J. Acoust. Soc. Am.* **2015**, *138*, 2929–2936. [\[CrossRef\]](#) [\[PubMed\]](#)
15. Geldart, D. Types of gas fluidization. *Powder Technol.* **1973**, *7*, 285–292. [\[CrossRef\]](#)
16. Kong, W.; Baeyens, J.; Flamant, G.; Tan, T.; Zhang, H. Solids Flow in a “Particle-in-Tube” Concentrated Solar Heat Absorber. *Ind. Eng. Chem. Res.* **2019**, *58*, 4598–4608. [\[CrossRef\]](#)
17. Deng, Y.; Sabatier, F.; Dewil, R.; Flamant, G.; Le Gal, A.; Gueguen, R.; Baeyens, J.; Li, S.; Ansart, R. Dense upflow fluidized bed (DUFb) solar receivers of high aspect ratio: Different fluidization modes through inserting bubble rupture promoters. *Chem. Eng. J.* **2021**, *418*, 129376. [\[CrossRef\]](#)
18. Karimipour, S.; Pugsley, T. Application of the particle in cell approach for the simulation of bubbling fluidized beds of Geldart A particles. *Powder Technol.* **2012**, *220*, 63–69. [\[CrossRef\]](#)
19. Karimipour, S.; Pugsley, T. A critical evaluation of literature correlations for predicting bubble size and velocity in gas–solid fluidized beds. *Powder Technol.* **2011**, *205*, 1–14. [\[CrossRef\]](#)
20. Parvathaneni, S.; Karmakar, S.; Buwa, V.V. Eulerian simulations of bubbling and jetting regimes in a fluidized bed. *Particuology* **2023**, *75*, 50–68. [\[CrossRef\]](#)
21. Parvathaneni, S.; Buwa, V.V. Eulerian multifluid simulations of segregation and mixing of binary gas-solids flow of particles with different densities. *Chem. Eng. Sci.* **2021**, *245*, 116901. [\[CrossRef\]](#)
22. Acosta-Iborra, A.; Sobrino, C.; Hernández-Jiménez, F.; de Vega, M. Experimental and computational study on the bubble behavior in a 3-D fluidized bed. *Chem. Eng. Sci.* **2011**, *66*, 3499–3512. [\[CrossRef\]](#)
23. Sobrino, C.; Almendros-Ibáñez, J.A.; Santana, D.; Vázquez, C.; de Vega, M. Maximum entropy estimation of the bubble size distribution in fluidized beds. *Chem. Eng. Sci.* **2009**, *64*, 2307–2319. [\[CrossRef\]](#)
24. Rüdisüli, M.; Schildhauer, T.J.; Biollaz, S.M.A.; Ruud van Ommen, J. Bubble characterization in a fluidized bed by means of optical probes. *Int. J. Multiph. Flow* **2012**, *41*, 56–67. [\[CrossRef\]](#)
25. Cao, Z.; Ji, L.; Xu, L. Iterative Reconstruction Algorithm for Electrical Capacitance Tomography Based on Calderon’s Method. *IEEE Sens. J.* **2018**, *18*, 8450–8462. [\[CrossRef\]](#)
26. Maurer, S.; Durán, S.R.; Küntle, M.; Biollaz, S.M.A. Influence of interparticle forces on attrition and elutriation in bubbling fluidized beds. *Powder Technol.* **2016**, *291*, 473–486. [\[CrossRef\]](#)
27. Sobrino, C.; Acosta-Iborra, A.; Izquierdo-Barrientos, M.A.; de Vega, M. Three-dimensional two-fluid modeling of a cylindrical fluidized bed and validation of the Maximum Entropy method to determine bubble properties. *Chem. Eng. J.* **2015**, *262*, 628–639. [\[CrossRef\]](#)
28. Deng, Y.; Dewil, R.; Appels, L.; Zhang, H.; Li, S.; Baeyens, J. The Need to Accurately Define and Measure the Properties of Particles. *Standards* **2021**, *1*, 19–38. [\[CrossRef\]](#)
29. Geldart, D. *Gas Fluidization Technology*; John Willy: Chichester, UK, 1986.
30. Hossain, N.; Metcalfe, R. Performance analysis of a 2D numerical model in estimating minimum fluidization velocity for fluidized beds. *Particuology* **2023**, *77*, 116–127. [\[CrossRef\]](#)
31. van Wachem, B.G.M.; Almstedt, A.E. Methods for multiphase computational fluid dynamics. *Chem. Eng. J.* **2003**, *96*, 81–98. [\[CrossRef\]](#)
32. Zhang, D.; Anjum, T.; Chu, Z.; Cross, J.S.; Ji, G. Simulation of multiphase flow with thermochemical reactions: A review of computational fluid dynamics (CFD) theory to AI integration. *Renew. Sustain. Energy Rev.* **2025**, *221*, 115895. [\[CrossRef\]](#)
33. López-Quesada, G.; Acosta-Iborra, A.; Sánchez-Rodríguez, M.R.; Salas-Colera, E.; Hernández-Jiménez, F. Preliminary CFD simulations of a lab-scale novel design of a particle receiver for CSP applications. *Results Eng.* **2024**, *24*, 103360. [\[CrossRef\]](#)
34. Darton, R.C.; Lanauze, R.; Davidson, J.; Harrison, D. Bubble Growth Due To Coalescence in Fluidized Beds. *Trans. Inst. Chem. Eng.* **1977**, *55*, 274–280.
35. Werther, J.; Wein, J. Expansion Behavior of Gas Fluidized Beds in the Turbulent Regime. *AIChE Symp. Ser.* **1994**, *301*, 31–44.
36. Kunii, D.; Levenspiel, O. *Fluidization Engineering*, 2nd ed.; Butterworth-Heinemann: Newton, MA, USA, 1991.
37. Baeyens, J.; Geldart, D. An investigation into slugging fluidized beds. *Chem. Eng. Sci.* **1974**, *29*, 255–265. [\[CrossRef\]](#)
38. Chen, S.; Shi, J.; Yuan, J.; He, M.; Li, Y.; Zhu, L.; Liu, J.; Wang, J.; Xie, G. Multiscale CFD Simulation of Multiphase Erosion Process in a Connecting Pipe of Industrial Polycrystalline Silicon Unit. *Processes* **2023**, *11*, 2510. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.