

Article

Dynamic Dependence and Tail Risk in Technology, Cryptocurrency and Commodity Markets

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Abstract

This study examines the evolution of dependence structures and tail risk transmission among technology equities, Bitcoin, Gold, and Crude Oil during 1 January 2016–1 January 2026. The analysis focuses on NVIDIA (NVDA), AMD, Tesla (TSLA), Bitcoin (BTC), Gold and Oil, covering major disruptions including the COVID-19 pandemic and the Russia–Ukraine conflict. An integrated methodological framework combines DCC-GARCH modeling, R-vine copulas, tail dependence analysis, complexity measures and machine learning-based forecasting techniques. The findings reveal volatility persistence and time-varying correlations, especially between technology equities and BTC during crisis periods. Regime analysis reveals that dependence structures are not stable in time. Lower-tail dependence intensifies during periods of market stress, indicating increased downside risk transmission. Gold remains weakly connected to the other assets, while Bitcoin has the strongest exposure to extreme downside co-movements. Complexity analysis based on the Scale-Dependent Lyapunov Exponent (SDLE) indicates heterogeneous dynamics across scales, characterized by local divergence and stabilization at broader scales. Forecast results based on Random Forest and XGBoost models provide limited predictive gains over benchmark specifications, suggesting that dependence and tail risk modeling offer better insight than short-horizon return predictions. These results are important for monitoring tail risk transmission for financial stability policies.

Keywords: DCC-GARCH; vine copula; tail dependence; financial risk analytics; digital financial ecosystems

1. Introduction

The rapid expansion of digital financial ecosystems and artificial intelligence (AI)-driven analytics has transformed modern financial markets. Technological innovation, cryptocurrencies and AI-based forecasting have intensified the interconnectedness between technology equities, digital assets and commodity markets. As financial systems become more and more integrated and data-intensive, understanding dynamic dependence, volatility transmission and tail risk propagation has become essential for intelligent financial systems and digital risk monitoring.

The growing importance of technology-oriented firms, such as NVIDIA, Advanced Micro Devices and Tesla, together with the emergence of cryptocurrencies such as Bitcoin, has created new forms of financial interconnectedness. At the same time, traditional assets such as Gold and Crude Oil continue to contribute to global financial stability and portfolio diversification. Extreme global events, such as the COVID-19 pandemic and the Russia–



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Ukraine conflict, amplified market uncertainty and volatility spillovers across digital and financial assets [1,2].

Linear models are often unable to capture the complex relationships during periods of market stress. Advanced econometric techniques based on multivariate volatility and copula dependence modeling have become relevant in financial research. The Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model by Engle (2002) [3] allows dynamic estimation of time-varying volatility and conditional correlations among financial assets. Vine copula models provide a flexible framework for modeling nonlinear, asymmetric, and tail-dependent relationships in high-dimensional systems. References [1,4,5] prove that vine copula approaches are very effective at capturing systemic risk transmission and extreme co-movement in financial markets.

Tail dependence has become important in financial risk management because extreme market events are rarely symmetric. Rotated copulas, survival copulas, and asymmetric dependence structures can capture joint downside or upside crashes that conventional correlation measures overlook. References [6,7] emphasize that vine copula–GARCH frameworks improve the analysis of contagion, portfolio vulnerability and systemic risk during turbulent market conditions.

AI and machine learning techniques have become important in financial forecasting and digital business analytics. Ensemble learning models such as Random Forests and Extreme Gradient Boosting (XGBoost) can identify nonlinear patterns and hidden interactions in financial datasets. Chung (2024) [8] shows that machine learning models often outperform econometric approaches in predictive tasks involving financial volatility and return forecasting. As shown by [9,10], the integration of AI with copula-based dependence structure represents an emerging research direction in intelligent financial systems.

Recent advances in AI have moved beyond purely data-driven prediction toward hybrid data–physics fusion frameworks. Song et al. [11] developed a Generative Data–Physics (GenDP) architecture that integrates physics-informed generative models, including Diffusion Models, Variational Autoencoders (VAEs), and Transformers, with copula-based dependency reconstruction for hierarchical reliability assessment under uncertainty. Their framework combines domain-specific physical constraints with generative AI to improve distributional learning, tail characterization, and dependency modeling in complex engineering systems. Data–physics fusion methods aim to preserve structural consistency and probabilistic coherence while modeling uncertainty propagation.

This study proposes an integrated framework combining the DCC-GARCH model, vine copula dependence, tail dependence analysis, complexity measures and machine learning techniques to explore the interactions among technology stocks, cryptocurrencies, and commodity markets. The analysis focuses on six representative assets extracted from Yahoo Finance during 1 January 2016–1 January 2026: NVIDIA (NVDA), AMD, Tesla (TSLA), Bitcoin (BTC), Gold, and Crude Oil.

This study addresses the following testable question: Does the tail dependence structure between AI-exposed technology equities, Bitcoin and traditional assets shift across crisis regimes, and is that shift statistically distinguishable from the unconditional full-sample structure? We formally test regime-specific shifts in tail dependence within the AI-equity–crypto–commodity nexus, combining R-vine copula estimation with block-bootstrap inference on DCC-GARCH conditional correlations across sub-periods. We characterize multiscale complexity via Scale-Dependent Lyapunov Exponents with surrogate data validation, and benchmark tree ensemble return forecasters against naive baselines using Diebold–Mariano tests. The asset universe—NVDA, AMD, TSLA (technology equities), Bitcoin (the leading cryptocurrency), Gold, and WTI Crude Oil (cyclical commodity)—was

chosen to span the technology–crypto–commodity nexus that characterizes the post-2016 AI investment cycle.

The findings have relevant implications for intelligent financial information systems, digital risk analytics, and systemic risk monitoring under conditions of geopolitical and financial uncertainty.

The paper is structured as follows. Section 2 reviews the relevant literature on dynamic dependence, tail risk transmission, multiscale complexity, and machine learning applications in financial markets. Section 3 presents the data, variable selection, and methodological framework. Section 4 reports the empirical findings. Finally, Section 5 summarizes the main findings, discusses their implications for systemic risk monitoring and financial stability, and outlines directions for future research.

2. Literature Review

2.1. Digitalization, Artificial Intelligence and Financial Market Integration

The accelerated digitalization of financial systems has transformed the modern investment environment, increasing the interconnectedness between technology equities, cryptocurrencies and commodity markets. AI, cloud computing and big data analytics have intensified the speed and complexity of financial decision-making processes. Technology-oriented firms such as NVDA, AMD, and Tesla influence global markets because of their central role in AI infrastructure, semiconductors and digital information systems [12].

Studies emphasize that digital financial ecosystems exhibit stronger cross-market integration and faster contagion transmission than traditional financial systems. Bouri et al. (2021) [13] show that cryptocurrencies and technology assets become synchronized during periods of uncertainty and speculative expansion. Yousaf et al. (2024) [14] demonstrate that AI-related assets and fossil fuel markets exhibit volatility interconnectedness that varies across time horizons, frequencies, and market conditions, during periods of heightened uncertainty and market stress. The expansion of intelligent financial systems has stimulated the use of AI-driven predictive analytics in business intelligence and financial decision support systems. According to [15], machine learning and digital analytics improve risk monitoring and portfolio management in volatile financial environments.

2.2. Dynamic Volatility and DCC-GARCH Models

Financial markets are characterized by heteroskedasticity, volatility clustering and dynamic dependence structure [16]. Linear models sometimes overlook these features, especially during periods of market stress and crises. Consequently, multivariate volatility frameworks such as the DCC-GARCH model have become widely used. The DCC-GARCH model by [3] enables simultaneous estimation of conditional volatilities and time-varying correlations across assets. Recent studies confirm the effectiveness of the DCC-GARCH model in analyzing cryptocurrencies, technology stocks and commodity markets. Malhotra and Gupta (2019) [17] reveal significant volatility spillovers and time-varying correlations between cryptocurrencies and Asian equity markets. Bitcoin exerts the strongest influence on future stock market volatility among the cryptocurrencies studied. Shahzad et al. (2020) [18] find that Gold serves as a stronger and more stable safe haven and hedging asset than Bitcoin for G7 stock markets, although Bitcoin has safe-haven properties in the Canadian market. Mugrabi (2025) [19] introduces a three-step segmentation procedure in the DCC-GARCH model to detect non-simultaneous structural breaks and shift-contagion episodes. The study demonstrates the model's effectiveness in identifying volatility and correlation shifts associated with the Global Financial Crisis, the European Sovereign Debt Crisis, and the post-COVID-19 geopolitical and monetary shocks.

2.3. Vine Copulas and Tail Dependence Structures

Copula approaches are flexible instruments to model symmetric and nonlinear dependence observed during extreme market conditions. They separate marginal distributions from dependence structures and allow the modeling of tail risk and asymmetric co-movements. According to [20], vine copulas model complex nonlinear interactions and tail dependence in financial systems. It has been proven that lower-tail dependence tends to dominate during financial crises, indicating stronger co-movements during market downturns than during expansions. Reboredo and Ugolini (2018) [21] show that cryptocurrencies exhibit higher downside contagion with traditional financial markets under stress conditions. During market distress, they amplify systemic risk through liquidity-driven selloffs and portfolio rebalancing [22]. Naeem et al. (2023) [23] report asymmetric tail dependence between energy markets, digital assets, and technology equities during geopolitical crises.

2.4. Multiscale Complexity in Financial Markets

Modern financial markets are often interpreted as complex adaptive systems characterized by nonlinear interactions, persistence and multiscale stochastic dynamics. Complexity measures such as SDLE have become tools for understanding financial instability and predictability.

Partida et al. (2022) [24] show that Bitcoin and Ethereum price volatility series exhibit chaotic, fractal, and hierarchical structure. Visibility graph analysis confirms long-term dependence, multifractality, and the absence of purely random behavior in cryptocurrency markets.

The Lyapunov exponent measures sensitivity to initial conditions and chaotic instability. Lahmiri and Bekiros (2018) [25] reveal that Bitcoin markets exhibit nonlinear dynamics characterized by chaos and multifractality, with stronger uncertainty and persistent fluctuations emerging during high-price and turbulent market regimes. However, the applicability of classical Lyapunov exponents in financial systems remained debated since financial time series are stochastic, nonstationary and multiscale. Gao et al. (2007) [26] argue that traditional chaos indicators may generate misleading interpretations when applied directly to noisy economic and financial data. To deal with this limitation, Gao et al. (2013) [27] introduce the Scale-Dependent Lyapunov Exponent (SDLE), a multiscale framework designed to characterize divergence dynamics across different scales. Their results show that financial systems may simultaneously exhibit local divergence and global stabilization. This suggests that market complexity is scale-dependent rather than purely chaotic. Gao et al. (2020) [28] analyze complexity changes in the US and Chinese stock markets and show that financial market complexity evolves in time in response to economic transformations and macroeconomic uncertainty. Financial markets are interpreted as heterogeneous multiscale systems characterized by nonlinear stochastic interactions rather than low-dimensional deterministic chaos.

2.5. Artificial Intelligence and Machine Learning in Financial Forecasting

Artificial intelligence and machine learning techniques have transformed financial forecasting and business intelligence systems. Fischer and Krauss (2018) [29] prove that long short-term memory (LSTM) networks outperform traditional machine learning models in predicting S&P stock movements by exploiting high-volatility and short-term reversal patterns with low exposure to systematic market risk. Gu et al. (2020) [30] demonstrate that tree-based models and neural networks improve the prediction of asset risk premiums by dealing with complex nonlinear interactions among financial signals such as momentum, liquidity, and volatility.

3. Materials and Methods

The methodological framework combines econometric volatility modeling, nonlinear dependence analysis, chaos diagnostics, and AI techniques to investigate the behavior of digital financial ecosystems composed of technology equities, cryptocurrencies, and commodity assets.

Daily adjusted closing prices were extracted from Yahoo Finance for the following assets: NVIDIA (NVDA), Advanced Micro Devices (AMD), Tesla (TSLA), Bitcoin (BTC-USD), Gold Futures (GC = F), and Crude Oil Futures (CL = F). The prices covered the period from 1 January 2016 to 1 January 2026.

The asset basket was constructed to span three distinct but interconnected segments of the post-2016 investment landscape: AI-focused technology equities (NVDA, AMD, TSLA), the leading cryptocurrency by market capitalization (Bitcoin), and two traditional asset classes (Oil, Gold). This composition omits two categories: AI-focused cryptocurrencies (e.g., FET, RNDR, AGIX) and aggregate market indices (e.g., the Nasdaq-100 or a diversified AI-equity basket). This is a deliberate limitation and not an oversight. AI-focused cryptocurrencies such as FET, RNDR, and AGIX have shorter and less liquid trading histories, which would reduce the sample period. Similarly, including broad market indices could mask the asset-specific tail dependence and network analysis.

We partition the sample into five economically motivated sub-periods defined by the identifiable structural events in global financial markets. The pre-COVID-19 period (January 2016–19 February 2020) represents the baseline expansion phase, ending on the date NVDA, AMD, and TSLA traded near their pre-pandemic highs. The COVID-19 shock period (20 February–31 December 2020) begins on the first day of the broad equity selloff triggered by pandemic fears, when major equity indices declined approximately 34% over the following month, and extends through year-end. The post-COVID-19 recovery period (January 2021–23 February 2022) spans the liquidity-driven bull market fueled by monetary and fiscal stimulus, ending the day before the Russian invasion of Ukraine. The Russia–Ukraine conflict period (24 February–31 December 2022) captures the combined shock of the invasion, the commodity price surge, and the Federal Reserve’s most aggressive tightening cycle since 1980. All these events compressed equity and crypto valuations simultaneously. The post-crisis period (January 2023–January 2026) represents the subsequent normalization phase. Break dates are consistent with the event-study approach used in the crisis contagion literature [31,32].

Since Bitcoin trades on a 365-day calendar while equities and futures operate on an approximately 252-day exchange calendar, return series were aligned to the intersection of trading days common to all six assets. This discarded approximately 28% of Bitcoin’s raw daily observations (weekend and holiday prices), consistent with standard practice in multi-asset DCC-GARCH studies requiring a common time index.

The daily logarithmic returns are computed as in Equation (1):

$$r_t = \ln(P_t) - \ln(P_{t-1}) \quad (1)$$

where r_t denotes the logarithmic return and P_t represents the adjusted closing price at time t . According to [33], logarithmic returns are preferred because they stabilize variance and allow additive aggregation over time.

3.1. Dynamic Conditional Correlation GARCH

To estimate time-varying volatility and dynamic correlations among the assets, the DCC-GARCH model proposed in [3] is employed. The return-generating process is defined by Equation (2):

$$r_t = \mu_t + \varepsilon_t \quad (2)$$

where the residual term is $\varepsilon_t = \sigma_t z_t$. μ_t is the conditional mean (expected value) of the series at time t . It represents the predictable component of returns. z_t is the standardized white-noise process. The conditional variance follows a GARCH(1,1) specification as in Equation (3):

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3)$$

where $\omega > 0$ is a constant, α measures short-run volatility effects, and β captures long-run persistence. The conditional covariance matrix is decomposed as in Equation (4):

$$H_t = D_t R_t D_t \quad (4)$$

where D_t is the diagonal matrix of conditional standard deviations and R_t represents the dynamic conditional correlation matrix. Following [3], the correlation dynamics evolve according to Equation (5):

$$Q_t = (1 - a - b)\bar{Q} + a z_{t-1} z'_{t-1} + b Q_{t-1} \quad (5)$$

where $\bar{Q}$ denotes the unconditional covariance matrix. a and b are scalar parameters satisfying $a + b < 1$. This framework captures volatility spillovers and contagion effects within interconnected financial systems [34].

The DCC-GARCH framework was selected in preference to alternative multivariate volatility models such as BEKK-GARCH, GO-GARCH, asymmetric DCC, and regime-switching correlation models. BEKK-GARCH becomes overly parameterized in systems with multiple assets. GO-GARCH relies on latent factor decompositions that may reduce the direct interpretability of pairwise dependence structures. Regime-switching models help identify latent market states but increase model complexity. Since the primary objective of this study is to generate standardized residuals and dynamic conditional correlations for subsequent R-vine copula estimation, the DCC-GARCH specification is preferred.

3.2. Vine Copula Methodology

Traditional multivariate copulas become restrictive in high-dimensional systems. Vine copulas overcome this limitation by decomposing a multivariate density into a cascade of bivariate copulas. The vine copula density can be written as in Equation (6):

$$f(x_1, \dots, x_n) = \prod_{k=1}^n f_k(x_k) \prod c_{ij|D} \quad (6)$$

where $c_{ij|D}$ represents pair-copula densities and D is the conditioning set. The study employs an R-vine structure selected using the Akaike Information Criterion (AIC). The empirical analysis employs several advanced copula families capable of capturing nonlinear, asymmetric, and tail-dependent structures frequently observed in financial and energy markets. In contrast to traditional linear correlation measures, these copulas allow the modeling of extreme co-movements, contagion effects, and asymmetric dependence during market stress periods.

The Student- t copula extends the Gaussian copula by incorporating symmetric tail dependence. It is particularly appropriate for modeling financial crises and contagion phenomena where extreme events occur simultaneously across markets. The Student- t copula is defined as [35]:

$$C_{\Sigma, \nu}(u, v) = t_{\Sigma, \nu} \left(t_{\nu}^{-1}(u), t_{\nu}^{-1}(v) \right) \quad (7)$$

where $t_{\Sigma, \nu}$ denotes the multivariate Student- t distribution, Σ is the dependence matrix, and ν represents the degrees of freedom. The Student- t copula exhibits both upper- and lower-tail dependence: $\lambda_L = \lambda_U > 0$. This property makes it suitable for modeling simultaneous extreme losses and gains.

The Gumbel copula belongs to the Archimedean copula family and captures upper-tail dependence. Its functional form is [36]:

$$C_{\theta}(u, v) = \exp(-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{1/\theta}) \quad (8)$$

where $\theta \geq 1$ measures dependence intensity. The upper-tail dependence coefficient is $\lambda_U = 2 - 2^{1/\theta}$, while the lower-tail dependence coefficient is $\lambda_L = 0$. Gumbel copulas are useful for modeling speculative booms and synchronized upward movements in financial assets.

The Joe copula is another Archimedean copula designed to model strong upper-tail dependence. Its cumulative distribution function is [37]:

$$C_{\theta}(u, v) = 1 - [(1 - u)^{\theta} + (1 - v)^{\theta} - (1 - u)^{\theta}(1 - v)^{\theta}]^{1/\theta} \quad (9)$$

where $\theta \geq 1$. The upper-tail dependence coefficient is $\lambda_U = 2 - 2^{1/\theta}$, while the lower-tail dependence coefficient is $\lambda_L = 0$. This copula is appropriate for capturing extreme positive co-movements and exuberant market phases. The VineCopula package in R automatically selects the optimal copula family and corresponding rotation according to the Akaike Information Criterion (AIC). Consequently, rotated copulas, including 90°, 180° (survival), and 270° transformations, may emerge endogenously in the estimation procedure.

The Survival Gumbel copula is the rotated version of the standard Gumbel copula and captures lower-tail dependence instead of upper-tail dependence. The survival transformation is [37]:

$$C^{180}(u, v) = u + v - 1 + C(1 - u, 1 - v) \quad (10)$$

The Survival Gumbel (180-degree rotated) copula is especially relevant for modeling joint crashes and downside contagion in financial systems. The Survival Gumbel copula reverses the Gumbel tail dependence structure, yielding $\lambda_L = 2 - 2^{1/\theta}$, $\lambda_U = 0$. The 90-degree rotated Gumbel copula is:

$$C^{90}(u, v) = v - C(1 - u, v) \quad (11)$$

while the 270-degree rotated Gumbel copula is:

$$C^{270}(u, v) = u - C(u, 1 - v) \quad (12)$$

The 90° and 270° rotated Gumbel copulas exhibit asymmetric mixed-tail dependence, where dependence is concentrated in opposite tails rather than purely in the upper or lower tail, making them suitable for modeling nonlinear negative spillovers and asymmetric contagion effects.

The Survival Joe (180-degree rotated) copula is the rotated version of the standard Joe copula and captures lower-tail dependence instead of upper-tail dependence. The survival transformation is defined as [37]:

$$C^{180}(u, v) = u + v - 1 + C(1 - u, 1 - v) \quad (13)$$

The Survival Joe copula reverses the Joe tail-dependence structure, yielding $\lambda_L = 2 - 2^{1/\theta}$, $\lambda_U = 0$. The 90-degree rotated Joe copula is:

$$C^{90}(u, v) = v - C(1 - u, v) \quad (14)$$

while the 270-degree rotated Joe copula is:

$$C^{270}(u, v) = u - C(u, 1 - v) \quad (15)$$

The 90° and 270° rotated Joe copulas also generate asymmetric mixed-tail dependence, capturing directional and nonlinear dependence structures characterized by opposite tail co-movements frequently observed during periods of financial stress and market dislocations.

The BB1 copula (also called the Clayton–Gumbel copula) combines lower- and upper-tail dependence simultaneously. It belongs to the flexible two-parameter Archimedean family. Its functional form is [37]:

$$C(u, v) = [1 + ((u^{-\theta} - 1)^\delta + (v^{-\theta} - 1)^\delta)^{1/\delta} - 1]^{-1/\theta} \quad (16)$$

where $\theta > 0$, $\delta \geq 1$. The BB1 copula combines both upper- and lower-tail dependence, allowing flexible modeling of asymmetric extreme co-movements: $\lambda_L = 2^{-1/(\theta\delta)}$, $\lambda_U = 2 - 2^{1/\delta}$. Its rotated versions capture nonlinear directional spillovers and mixed-tail asymmetric dependence structures frequently observed during financial crises and periods of heightened market contagion.

The BB8 copula (also called the Joe–Frank copula) is a two-parameter Archimedean copula that combines the Joe copula’s tail-asymmetry structure with a Frank-type mixing parameter. Its cumulative distribution function is [37,38]:

$$C(u, v, \theta, \delta) = \delta^{-1} [1 - \{1 - [1 - (1 - \delta)^\theta]^{-1} [1 - (1 - \delta u)^\theta] [1 - (1 - \delta v)^\theta]\}^{1/\theta}] \quad (17)$$

where $\theta \geq 1$, $0 \leq \delta \leq 1$. The independence copula is obtained as $\theta \rightarrow 1$ or $\delta \rightarrow 0$. The BB8 copula reduces to the Joe copula when $\delta = 1$, while the Frank copula is obtained as $\theta \rightarrow \infty$. The BB8 copula does not exhibit tail dependence except when $\delta = 1$, in which case $\lambda_U = 2 - 2^{1/\theta}$, $\lambda_L = 0$. This property makes BB8 well suited to modeling asset pairs with strong overall dependence (high Kendall’s τ) but negligible co-movement during extreme events. Its rotated (180-degree) version, the Survival BB8 copula, reverses this structure to capture lower-tail dependence instead of upper-tail dependence.

The Tawn copula [39], as an extension of the Gumbel copula, incorporates three parameters rather than one, allowing for asymmetric dependence structures that the single-parameter Gumbel copula cannot capture. It is defined as:

$$C(u, v, \theta, \delta_1, \delta_2) = (1 - \delta_1)t - (1 - \delta_2)(1 - t) + [(\delta_1(1 - t))^\theta + (\delta_2 t)^\theta]^{1/\theta} \quad (18)$$

where $t = \frac{\log(v)}{\log(uv)}$, $0 \leq \delta_1, \delta_2 \leq 1$, $\theta \in [1, \infty)$. When $\delta_1 = \delta_2$, the Tawn copula reduces to the Gumbel copula, with δ_1, δ_2 acting as skewness parameters that determine the asymmetry of the dependence structure. The Tawn copula can be divided into two sub-types, each fixing one asymmetry parameter to 1 so that the copula density is either left- or right-skewed (i.e., $\delta_1 = 1$ or $\delta_2 = 1$). The variant fitted in this study (family code 114, the 180-degree rotated Tawn type 1) corresponds to one such skewed configuration. The upper-tail dependence coefficient is given by $\lambda_U = \delta_1 + 1 - (\delta_1^\theta + 1)^{1/\theta}$ or $\lambda_U = \delta_2 + 1 - (\delta_2^\theta + 1)^{1/\theta}$. The Tawn copula’s flexibility relative to Gumbel makes it well suited to asset pairs where upper-tail co-movement is present but markedly asymmetric.

The independence copula represents the simplest copula structure and corresponds to the case of stochastic independence between variables. The independence copula is defined as [40]:

$$C(u, v) = uv \quad (19)$$

In this setting, no dependence structure, tail dependence, or nonlinear interaction exists between the variables. Consequently, the associated variables evolve approximately independently in the conditional dependence framework.

3.3. Tail Dependence—Theoretical Aspects

Tail dependence measures the probability of simultaneous extreme movements among assets. The upper-tail dependence coefficient is defined as:

$$\lambda_U = \lim_{u \rightarrow 1^-} P(Y > F_Y^{-1}(u) | X > F_X^{-1}(u)) \quad (20)$$

The lower-tail dependence coefficient is:

$$\lambda_L = \lim_{u \rightarrow 0^+} P(Y \leq F_Y^{-1}(u) | X \leq F_X^{-1}(u)) \quad (21)$$

λ_U measures co-boom behavior and λ_L measures co-crash behavior. Tail dependence analysis is essential for understanding systemic risk transmission and financial contagion [41].

3.4. Scale-Dependent Lyapunov Exponent—Theoretical Aspects

Lyapunov exponents are frequently used to evaluate the average rate of divergence between nearby trajectories in a dynamic system. In financial applications, the direct interpretation of a single global Lyapunov exponent remains controversial since financial time series are characterized by fluctuations, nonstationarity, volatility clustering and multiscale structures [26,42]. Under such conditions, conventional chaos indicators may generate misleading conclusions with respect to deterministic chaos. To address these limitations, Gao et al. (2013) [27] propose SDLE, a multiscale framework designed to analyze divergence across different scales in noisy and nonlinear systems. SDLE evaluates how the local divergence rate evolves in time as a function of the distance scale between neighboring trajectories in the reconstructed phase space. SDLE is defined as:

$$\lambda(\varepsilon) = \frac{\langle \ln d_{t+\Delta t} - \ln d_t \rangle_\varepsilon}{\Delta t} \quad (22)$$

where d_t denotes the Euclidean distance between neighboring trajectories at time t ; $d_{t+\Delta t}$ represents the corresponding distance after the evolution interval Δt . The operator $\langle \cdot \rangle_\varepsilon$ denotes the average over all trajectory pairs whose initial separations belong to a distance interval centered around the scale ε . The phase-space reconstruction is obtained through the delay coordinate embedding method introduced in [43]. Unless otherwise stated, the baseline reconstruction employs an embedding dimension $m = 2$ and embedding delay $\tau = 1$. The embedding dimension specifies the number of delayed coordinates used to reconstruct the phase space, whereas the embedding delay specifies the time lag between successive coordinates. Robustness is evaluated using alternative embedding dimensions $m \in \{2, 3, 4\}$ and embedding delays $m \in \{1, 2\}$ in the empirical part.

Let x_t denote the financial return series. The embedded vectors are constructed as:

$$V_t = (x_t, x_{t-\tau}, x_{t-2\tau}, \dots, x_{t-(m-1)\tau}) \quad (23)$$

where m is the embedding dimension and τ is the embedding delay parameter. Positive SDLE values indicate local divergence dynamics and sensitivity to small perturbations, while negative values signify convergence and stabilization effects.

3.5. Artificial Intelligence Models

Extreme Gradient Boosting (XGBoost) and Random Forest (RF) models are employed in this study to capture nonlinear predictive relationships among technology stocks, cryptocurrencies, and commodity markets. These machine learning techniques are particularly suitable for financial time series because they can model nonlinear interactions, volatility clustering, asymmetric spillovers, and complex hidden structures that are often inadequately represented by traditional linear econometric models. XGBoost is based on gradient boosting and sequential tree optimization [44]. RF relies on bootstrap aggregation and ensemble learning through multiple decision trees [45]. To enhance predictive capability, the feature space includes lagged returns and rolling volatility measures. One-period lagged returns and five-day rolling standard deviations are generated for each asset and used as explanatory variables. The prediction framework can be represented as:

$$Y_{t+1} = f(X_t) + \varepsilon_t \quad (24)$$

where Y_{t+1} denotes the future return of the target asset. X_t is the vector of lagged financial indicators and volatility measures. $f(\cdot)$ represents the nonlinear mapping learned by the machine learning algorithm, and ε_t is the prediction error term. Overall, 80% of the observations are allocated to the training sample, while the remaining 20% are reserved for out-of-sample testing.

XGBoost minimizes the following objective function [44]:

$$\mathcal{L} = \sum_{i=1}^n l(y_i, \hat{y}_i) + \sum_{k=1}^K \Omega(f_k) \quad (25)$$

where $l(\cdot)$ denotes the loss function. $\Omega(f_k)$ is the regularization term, n is the sample size and K is the number of boosting iterations. f_k is an individual regression tree. In gradient boosting, each successive tree is trained on the residuals generated by previous trees, allowing the model to iteratively improve predictive accuracy.

RF aggregates predictions from multiple decision trees:

$$\hat{f}(x) = \frac{1}{B} \sum_{k=1}^B f_k(x) \quad (26)$$

where B is the number of trees and $f_k(x)$ denotes the prediction from tree b . Breiman (2001) [45] proved that RF captures nonlinear interactions while reducing overfitting risk. Forecasting performance is evaluated using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE).

The Appendix A contains preliminary diagnostic tests.

4. Empirical Results

Figure 1 presents the cumulative returns, daily returns, and drawdown dynamics of NVIDIA (NVDA), AMD, Tesla (TSLA), Bitcoin (BTC), Gold, and Oil from 1 January 2016 to 1 January 2026.

The upper panel illustrates cumulative returns. NVDA exhibits the strongest long-term performance among all assets, especially after 2023. This reflects the rapid expansion of AI technologies and increased investor demand for semiconductor companies associated

with AI infrastructure. BTC also demonstrates cumulative returns with larger fluctuations, indicating speculative dynamics and high market uncertainty. AMD and TSLA show positive long-run growth trajectories, even though their cumulative returns remain below those of NVDA. Gold and Oil exhibit comparatively lower cumulative growth; this points to their roles as defensive and commodity-based assets.

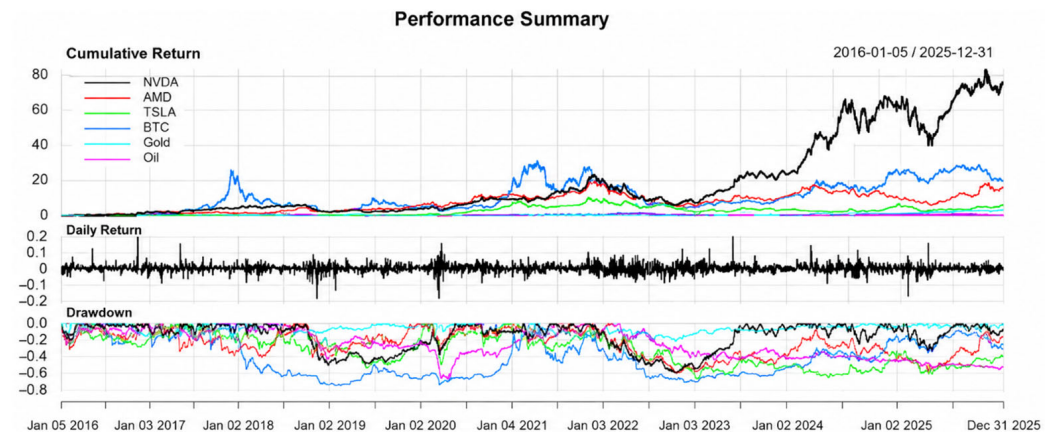


Figure 1. Performance summary of technology stocks, Bitcoin, Gold, and Oil.

The middle panel displays daily returns. All financial assets exhibit pronounced volatility clustering. Bitcoin and technology stocks have the largest return fluctuations, confirming the presence of heteroskedasticity observed in high-growth and speculative markets. Several extreme return spikes are visible during periods associated with major global shocks, including the COVID-19 crisis and subsequent inflationary and geopolitical disturbances.

The lower panel presents drawdown dynamics, which measure cumulative losses relative to previous peaks. Bitcoin experiences the deepest drawdowns, reaching losses exceeding 70%. This points to its highly speculative nature and vulnerability to abrupt market corrections. Technology stocks, particularly TSLA and AMD, also experience downside episodes during periods of market stress. NVIDIA, despite its strong cumulative performance, undergoes temporary but significant corrections associated with volatility in the technology sector. Gold exhibits comparatively smaller drawdowns. Oil displays cyclical fluctuations, being sensitive to global economic conditions, geopolitical events, and supply–demand shocks.

Table 1 reports the descriptive statistics of daily log returns for NVIDIA (NVDA), AMD, Tesla (TSLA), Bitcoin (BTC), Gold, and Oil. The arithmetic mean returns are positive for all assets, indicating positive average performance in the last decade. NVDA and BTC exhibit the highest average returns, reflecting the expansion of AI and cryptocurrency markets. Gold and Oil display smaller average returns, consistent with their traditional roles as commodity and defensive assets.

The standard deviation values reveal noticeable differences in market risks across assets. BTC has the highest volatility (0.0423), followed by Tesla, AMD, and NVIDIA. It confirms the speculative and unstable nature of cryptocurrency and technology markets. Gold displays the lowest volatility (0.0097), supporting its safe-haven characteristics and lower sensitivity to speculative shocks. Oil also exhibits volatility due to geopolitical uncertainty, global supply–demand imbalances, and energy markets.

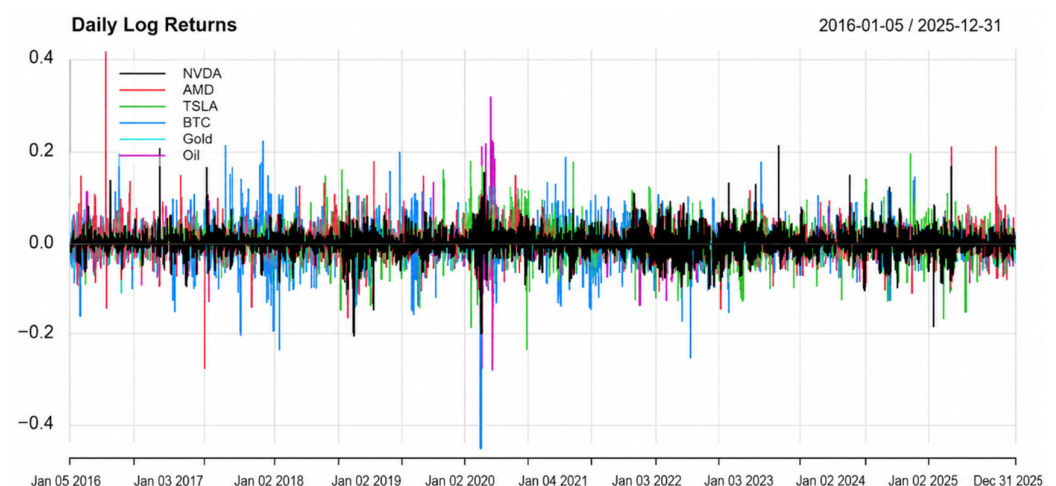
The skewness coefficients indicate asymmetric return distributions. Bitcoin and Gold exhibit negative skewness. AMD displays strong positive skewness (0.6239), suggesting the presence of positive return episodes associated with speculative rallies and technological innovation cycles. NVIDIA and Tesla show moderate skewness.

Table 1. Performance summary of technology stocks, Bitcoin, Gold, and Oil.

Statistic	NVDA	AMD	TSLA	BTC	Gold	Oil
Minimum	−0.2077	−0.2775	−0.2365	−0.4647	−0.0591	−0.2822
Quartile 1	−0.0130	−0.0171	−0.0174	−0.0153	−0.0041	−0.0126
Median	0.0028	0.0007	0.0013	0.0019	0.0007	0.0020
Arithmetic Mean	0.0022	0.0018	0.0014	0.0021	0.0006	0.0004
Geometric Mean	0.0017	0.0011	0.0007	0.0012	0.0005	0.0000
Quartile 3	0.0185	0.0202	0.0198	0.0204	0.0058	0.0137
Maximum	0.2181	0.4206	0.2045	0.2251	0.0578	0.3196
SE Mean	0.0006	0.0007	0.0007	0.0008	0.0002	0.0006
LCL Mean (0.95)	0.0010	0.0003	−0.0001	0.0005	0.0002	−0.0007
UCL Mean (0.95)	0.0034	0.0032	0.0029	0.0038	0.0009	0.0015
Variance	0.0010	0.0013	0.0014	0.0018	0.0001	0.0008
Stdev	0.0311	0.0366	0.0372	0.0423	0.0097	0.0288
Skewness	−0.0120	0.6239	−0.0585	−0.5699	−0.2624	0.0694
Kurtosis	5.7433	10.5480	4.2086	9.8484	3.9786	24.7331

The kurtosis statistics are larger than the Gaussian benchmark value of 3 for all assets, especially for Oil (24.7331), AMD (10.5480), and Bitcoin (9.8484). These elevated kurtosis values indicate leptokurtic behavior characterized by fat tails and extreme observations.

The series in Figure 2 exhibits pronounced volatility clustering. The single most extreme observation in the dataset belongs to Bitcoin, which fell by approximately 47% in one day during the March 2020 COVID-19-induced crash—far exceeding any single-day move in the equity or commodity series. Bitcoin’s return band is also visibly wider than the other five series across the full sample, not only during crises. Gold and Oil display the narrowest and calmest return bands throughout, reflecting their comparatively low unconditional volatility and supporting their characterization as the lower-risk members of the asset basket.

**Figure 2.** Daily log returns of technology stocks, Bitcoin, Gold, and Oil.

The correlation matrix in Figure 3 reveals heterogeneity in the dependence structure among technology stocks, cryptocurrency, precious metals, and energy markets. Strong positive correlations are observed between NVDA and AMD (0.62). It reflects their common exposure to semiconductors and AI sectors. NVDA also exhibits moderate correlation (0.43) with TESLA, indicating interconnected dynamics within technology markets.

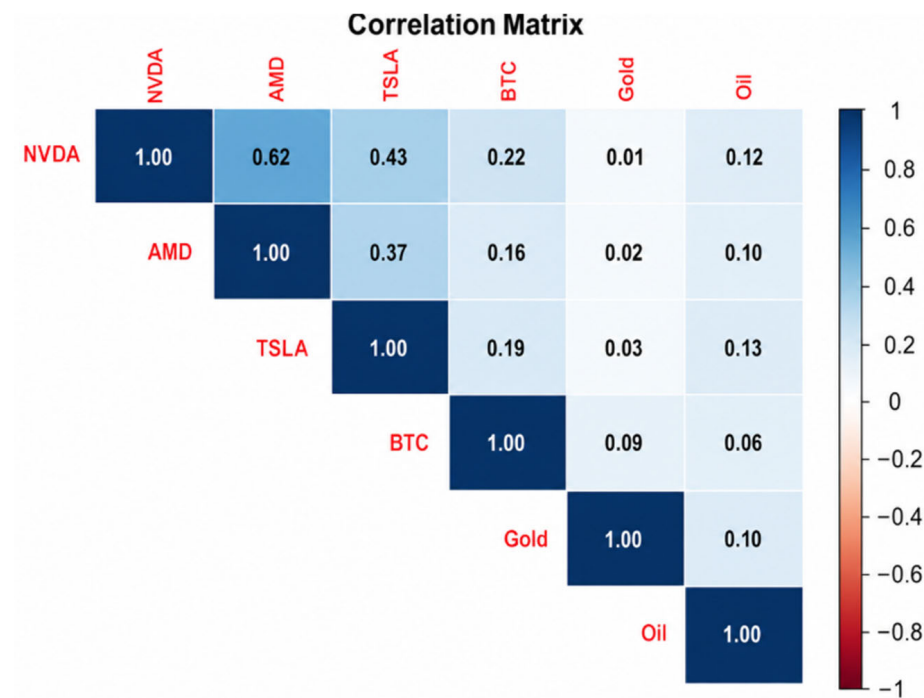


Figure 3. Correlation matrix of daily returns.

AMD and TESLA display a moderate correlation (0.37), suggesting partial co-movement driven by common investor sentiment, technological innovation cycles and macroeconomic conditions affecting growth-oriented equities.

BTC exhibits weak positive correlations with technology assets, ranging from 0.16 to 0.22. The highest relationship is between BTC and NVDA (0.22), which may reflect the integration between cryptocurrency markets and technology-driven speculative investment behavior.

Gold has very weak correlations with all assets. These results support the traditional role of Gold as a defensive or safe-haven asset characterized by independent price dynamics during periods of financial turbulence [46,47]. Similarly, Oil has positive weak correlations with the remaining assets. Stronger correlations are observed for Tesla (0.13) and NVDA (0.12).

4.1. DCC-GARCH(1,1) Model

The DCC-GARCH(1,1) model is estimated under a multivariate Student-*t* distribution, which is appropriate for financial data characterized by heavy tails, excess kurtosis, and extreme observations (see Table 2). The DCC-GARCH(1,1) framework combines univariate GARCH volatility specifications with a dynamic conditional correlation structure, allowing both volatilities and correlations to evolve. The model includes six financial series and 2508 observations, providing a sufficiently large sample for reliable estimation of time-varying volatility and dependence structures. A total of 60 parameters is estimated, reflecting the complexity of the multivariate volatility and correlation system. The use of the multivariate Student-*t* distribution indicates that the model accounts for fat tails and non-normal return distributions frequently observed in financial and cryptocurrency markets. This aligns with the high kurtosis values identified in the descriptive statistics. The reported log-likelihood value (35,598.51) and average log-likelihood (14.19) suggest a strong statistical fit of the model to the observed data.

Table 2. DCC-GARCH(1,1) model estimation results.

Item	Value
Distribution	Multivariate Student- <i>t</i> (mvt)
Model	DCC(1,1)
Number of Parameters	60
[VAR GARCH DCC UncQ]	[0 + 42 + 3 + 15]
Number of Series	6
Number of Observations	2508
Log-Likelihood	35,598.51
Average Log-Likelihood	14.19

The estimated GARCH(1,1) results for NVIDIA (NVDA) in Table 3 indicate significant volatility clustering and strong persistence in returns. The ARCH coefficient ($\alpha_1 = 0.0903$) confirms that recent shocks increase current volatility, while the large GARCH coefficient ($\beta_1 = 0.8866$) suggests persistent volatility. The persistence measure $\alpha_1 + \beta_1 = 0.9769$ is close to unity, indicating long memory effects and slow dissipation of volatility shocks. The significant AR(1) and MA(1) terms reveal short-run dependence in returns, while the Student-*t* shape parameter confirms fat tails and non-normality in NVIDIA.

Table 3. Optimal parameter estimates—NVDA.

Parameter NVDA	Estimate	Std. Error	t-Value	p-Value
μ	0.002841	0.000449	6.32694	0.000000
AR(1)	−0.470427	0.149661	−3.14328	0.001671
MA(1)	0.430102	0.151716	2.83491	0.004584
ω	0.000027	0.000012	2.36137	0.018207
α_1	0.090257	0.022593	3.99497	0.000065
β_1	0.886552	0.029443	30.11125	0.000000
Shape	4.582858	0.448075	10.22787	0.000000

Tables 4–8 collectively confirm the existence of strong volatility persistence, nonlinear dependence, and heavy-tailed behavior across technology stocks, Bitcoin, Gold, and Oil. The estimated GARCH coefficients (β_1) are large and statistically significant for almost all assets, indicating that volatility shocks dissipate slowly and that financial turbulence tends to persist. TSLA and Gold report β_1 values above 0.95. For Gold, however, ω , α_1 and the shape parameter are not statistically significant ($p = 0.906$, 0.565 , and 0.264 , respectively), indicating the univariate model is poorly identified for this asset (see Table 7). This pattern indicates a poorly identified univariate GARCH model for Gold within this sample. The large β_1 estimate reflects a diffuse, slowly mean-reverting volatility process, but the model cannot reliably distinguish short-run shock amplification (α_1) from background noise. We note that its conditional volatility is best characterized as slowly evolving and insensitive to day-to-day return shocks, with the precise dynamics uncertain given the insignificance of the shock response.

Bitcoin and Oil display substantial persistence, while AMD exhibits lower persistence effects.

The ARCH coefficients (α_1) reveal that short-run volatility reactions are strongest for Bitcoin, AMD, and Oil, indicating that these assets react more intensely to recent market shocks. Gold shows an insignificant ARCH effect, implying weaker short-run volatility transmission and reinforcing its role as a relatively stable defensive asset.

Differences also emerge in the ARMA dynamics. Significant AR(1) and MA(1) coefficients for NVDA, AMD, TSLA, and Oil suggest short-run return dependence and temporary

predictability in these markets. Oil is particularly distinct because it displays strong positive AR(1) and negative MA(1) effects. It reflects cyclical adjustments and rapid corrections associated with geopolitical and energy market shocks. Bitcoin and Gold exhibit statistically insignificant ARMA terms, indicating that their returns behave closer to stochastic or weakly predictable processes.

Table 4. Optimal parameter estimates—AMD.

Parameter AMD	Estimate	Std. Error	t-Value	p-Value
μ	0.001438	0.000567	2.53748	0.011165
AR(1)	−0.764097	0.203314	−3.75822	0.000171
MA(1)	0.733920	0.214857	3.41585	0.000636
ω	0.000182	0.000111	1.63181	0.102720
α_1	0.107173	0.040228	2.66416	0.007718
β_1	0.762624	0.113655	6.71002	0.000000
Shape	4.034099	0.342695	11.77170	0.000000

Table 5. Optimal parameter estimates—TSLA.

Parameter TSLA	Estimate	Std. Error	t-Value	p-Value
μ	0.001407	0.000559	2.51621	0.011863
AR(1)	−0.801443	0.214639	−3.73391	0.000189
MA(1)	0.785807	0.222450	3.53251	0.000412
ω	0.000009	0.000008	1.10195	0.270482
α_1	0.040354	0.008931	4.51867	0.000006
β_1	0.955576	0.001884	507.23958	0.000000
Shape	4.271318	0.382602	11.16386	0.000000

Table 6. Optimal parameter estimates—BTC.

Parameter BTC	Estimate	Std. Error	t-Value	p-Value
μ	0.001536	0.000482	3.18897	0.001428
AR(1)	−0.211671	0.331299	−0.63891	0.522881
MA(1)	0.174407	0.333025	0.52370	0.600483
ω	0.000036	0.000016	2.30657	0.021079
α_1	0.107497	0.016671	6.44801	0.000000
β_1	0.891503	0.019470	45.78809	0.000000
Shape	3.098419	0.148214	20.90500	0.000000

Table 7. Optimal parameter estimates—Gold.

Parameter Gold	Estimate	Std. Error	t-Value	p-Value
μ	0.000616	0.000168	3.66492	0.000247
AR(1)	−0.053409	0.179440	−0.29764	0.765977
MA(1)	0.019112	0.180045	0.10615	0.915463
ω	0.000001	0.000009	0.11810	0.905990
α_1	0.035677	0.061963	0.57579	0.564758
β_1	0.955111	0.052586	18.16276	0.000000
Shape	4.280354	3.835360	1.11602	0.264412

Table 8. Optimal parameter estimates—Oil.

Parameter Oil	Estimate	Std. Error	t-Value	p-Value
μ	0.001165	0.000364	3.20225	0.001364
AR(1)	0.725274	0.117044	6.19660	0.000000
MA(1)	−0.744774	0.113343	−6.57099	0.000000
ω	0.000021	0.000005	4.21420	0.000025
α_1	0.101273	0.015322	6.60983	0.000000
β_1	0.862559	0.018349	47.00751	0.000000
Shape	6.522709	0.839862	7.76640	0.000000

The estimated Student-*t* shape parameters further confirm heavy tails and excess kurtosis across the analyzed assets. Bitcoin has the lowest shape parameter, indicating the fattest tails and highest probability of extreme events, which is consistent with its speculative nature and elevated risk profile. Oil also exhibits substantial tail risk, while technology stocks display moderate but still significant non-normality. Gold differs because its shape parameter is statistically insignificant, having weaker tail dependence and lower sensitivity to extreme speculative episodes. Although the GARCH coefficient is large and significant for Gold, the insignificance of ω , α_1 , and the shape parameter suggests the model has limited power to characterize Gold's short-run volatility dynamics.

The DCC parameter estimates describe the overall dynamic correlation structure among all analyzed assets (see Table 9). The significant DCC coefficient ($\alpha_1 = 0.0112$) indicates that conditional correlations respond to new market shocks. The large persistence parameter ($\beta_1 = 0.9794$) reveals highly persistent correlation across the multivariate system. The persistence measure $\alpha_1 + \beta_1 = 0.9907$ suggests that correlation shocks dissipate slowly in time. The significant multivariate Student-*t* shape parameter confirms the existence of fat tails and extreme joint movements across markets.

Table 9. DCC parameters.

Parameter	Estimate	Std. Error	t-Value	p-Value
DCC α_1	0.011243	0.002244	5.00948	0.000001
DCC β_1	0.979423	0.005775	169.59363	0.000000
Multivariate Shape	5.521875	0.153894	35.88103	0.000000

The information criteria (Table 10) evaluate the goodness-of-fit and parsimony of the estimated DCC-GARCH model. Lower values of the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Shibata criterion, and Hannan–Quinn criterion indicate a better balance between model fit and complexity. The negative values obtained for all criteria suggest that the estimated DCC-GARCH specification provides a suitable representation of the volatility and dynamic dependence structure among the analyzed assets.

Table 10. Information criteria for the DCC-GARCH model.

Criterion	Value
Akaike (AIC)	−28.340
Bayesian (BIC)	−28.201
Shibata	−28.341
Hannan–Quinn	−28.290

The time-varying conditional volatilities estimated from the DCC-GARCH model reveal substantial heterogeneity in volatility behavior (see Figure 4). Bitcoin exhibits the highest and most persistent volatility, with pronounced volatility spikes during major market turbulence episodes, particularly around 2017–2018 and the COVID-19 crisis in 2020. The speculative nature and elevated uncertainty characterizing cryptocurrency markets are also confirmed in [48]. AMD and TSLA also display volatility fluctuations, indicating strong sensitivity to technological innovation cycles, investor sentiment, and macroeconomic shocks. NVIDIA exhibits persistent smoother volatility dynamics, although several sharp increases are visible during periods associated with rapid growth in artificial intelligence markets and broader financial instability. Oil volatility also experiences significant spikes, especially during periods of global economic disruption and energy market uncertainty, reacting to geopolitical and supply–demand shocks. Gold exhibits the lowest and most stable conditional volatility. The coexistence of volatility clustering and persistent volatility among most assets confirms the presence of nonlinear heteroskedastic dynamics.

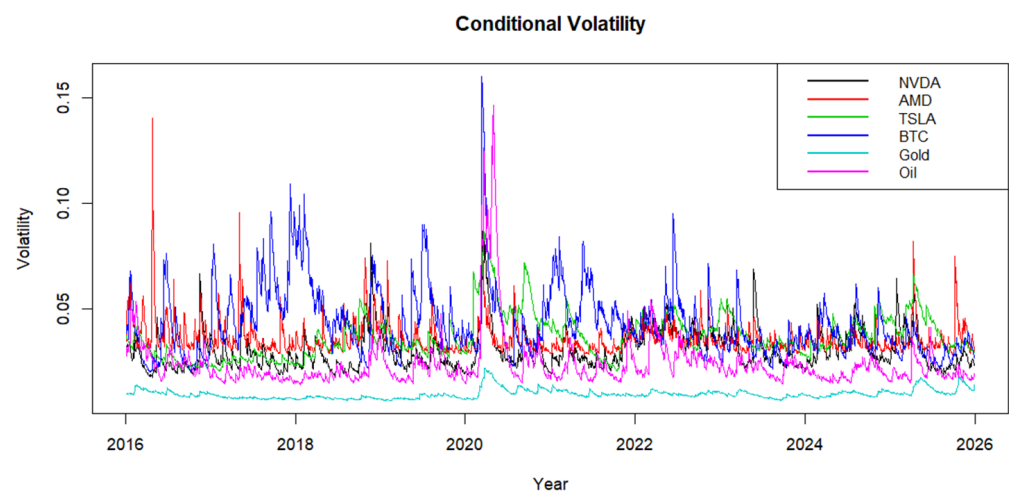


Figure 4. Conditional volatility dynamics under the DCC-GARCH model.

To illustrate the dynamic correlation between assets, we chose three pairs because they capture different dimensions of financial interconnectedness and diversification. NVDA–BTC reflects the interaction between artificial intelligence equities and cryptocurrency markets, NVDA–Gold examines the relationship between speculative technology assets and a traditional safe-haven asset, and BTC–Gold evaluates whether Bitcoin behaves similarly to Gold during periods of global uncertainty. These pairs are particularly relevant for analyzing contagion, hedging, and time-varying dependence during major geopolitical and financial crises such as the COVID-19 pandemic and the Russia–Ukraine conflict [49].

The dynamic correlation between NVDA and BTC in Figure 5 remains positive, indicating increasing interconnectedness between AI-related technology equities and cryptocurrency markets. Correlations rise sharply during the COVID-19 pandemic in 2020–2021 and intensify further during the post-pandemic technology boom. They reach values above 0.40 in 2022. During periods of heightened uncertainty and monetary expansion, stronger contagion effects and synchronized speculative behavior occur. Following the Russia–Ukraine war beginning in early 2022, the correlation remains high, reflecting global risk transmission across digital and technology-oriented assets. Similar evidence of increased crypto–equity integration during crisis periods is documented in [49,50].



Figure 5. Dynamic conditional correlation between NVDA and BTC.

The NVDA–Gold correlation (Figure 6) fluctuates around zero and occasionally becomes negative, suggesting weak and unstable dependence between technology equities and the traditional safe-haven asset. During the COVID-19 crisis and the Russia–Ukraine conflict, several negative correlation episodes emerge, indicating that Gold maintains diversification and hedging properties during periods of heightened stock market volatility. Compared with Bitcoin, Gold remains less connected to speculative technology market dynamics, reinforcing its defensive role in portfolio management. These findings are consistent with the literature identifying Gold as a safe-haven asset during financial and geopolitical crises [51,52].

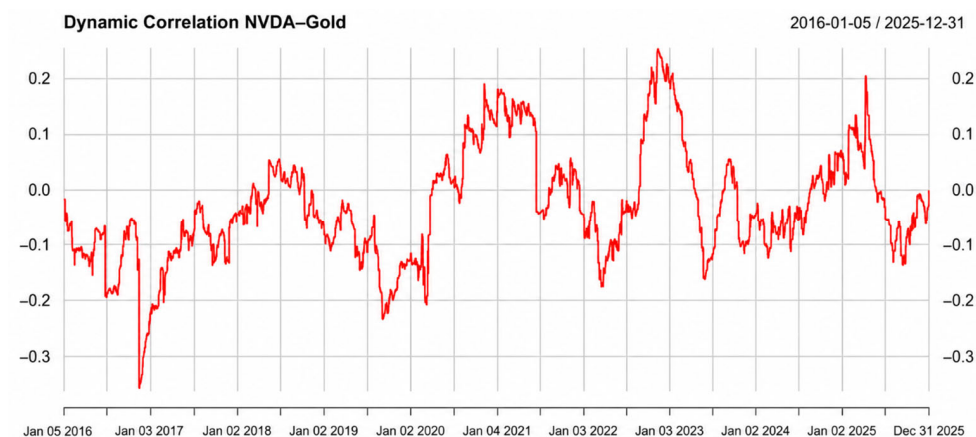


Figure 6. Dynamic conditional correlation between NVDA and Gold.

The BTC–Gold correlation exhibits moderate variability and remains weak, indicating that Bitcoin does not consistently behave as a traditional safe-haven asset (Figure 7). During the COVID-19 pandemic, temporary increases in correlation are observed, suggesting short-lived periods of common reactions to global uncertainty. The correlation becomes unstable during the Russia–Ukraine conflict, alternating between positive and negative values. This behavior suggests that Bitcoin and Gold respond differently to geopolitical shocks and systemic risk. The results support similar findings that Bitcoin behaves more as a speculative digital asset than as a stable safe haven during crisis periods [49,53].



Figure 7. Dynamic conditional correlation between BTC and Gold.

4.2. Tail Dependence

The optimal bivariate copula families selected for each asset pair, together with Kendall's τ , lower-tail dependence (LTD) and upper-tail dependence (UTD), are shown in Table 11. The vine structure reveals that NVIDIA (NVDA) occupies a central position in the dependence network, acting as the main linkage between technology equities, cryptocurrency markets, and traditional assets. This result is consistent with NVIDIA's dominant role in the post-2016 AI investment cycle and confirms its importance in transmitting information and market shocks across asset classes.

Table 11. R-vine copula structure: pairwise and conditional dependence estimates.

Tree	Pair	Copula	Par	Par2	τ (Kendall)	UTD	LTD
1	Gold—BTC	Gumbel (4)	1.0644	—	0.0605	0.0821	0.0000
1	BTC—NVDA	Surv. Gumbel (14)	1.1302	—	0.1152	0.0000	0.1534
1	NVDA—TSLA	Surv. BB8 (20)	2.1612	0.8790	0.2771	0.0000	0.0000
1	NVDA—Oil	Surv. Gumbel (14)	1.0733	—	0.0683	0.0000	0.0925
1	NVDA—AMD	Surv. BB8 (20)	4.3528	0.7927	0.4820	0.0000	0.0000
2	Gold—NVDA BTC	t (2)	−0.0423	9.7411	−0.0269	0.0059	0.0059
2	BTC—TSLA NVDA	t (2)	0.1086	27.1672	0.0693	0.0001	0.0001
2	TSLA—Oil NVDA	Surv. Gumbel (14)	1.0394	—	0.0379	0.0000	0.0519
2	TSLA—AMD NVDA	Gaussian (1)	0.1540	—	0.0984	0.0000	0.0000
3	Gold—TSLA BTC, NVDA	Indep. (0)	—	—	0.0000	0.0000	0.0000
3	BTC—Oil TSLA, NVDA	Indep. (0)	—	—	0.0000	0.0000	0.0000
3	BTC—AMD TSLA, NVDA	Surv. BB8 (20)	1.1252	0.8849	0.0369	0.0000	0.0000
4	Gold—Oil BTC, TSLA, NVDA	t (2)	0.0900	10.6875	0.0574	0.0091	0.0091
4	Gold—AMD BTC, TSLA, NVDA	Indep. (0)	—	—	0.0000	0.0000	0.0000
5	Oil—AMD Gold, BTC, TSLA, NVDA	Indep. (0)	—	—	0.0000	0.0000	0.0000

Note: “|” denotes the conditioning set for higher-tree (partial) copulas. Surv. = survival (180-rotated) copula. τ = Kendall's tau; UTD = upper-tail dependence coefficient; LTD = lower-tail dependence coefficient. Indep. = independence copula (no significant residual dependence after conditioning). Par2 is reported only for two-parameter families (Student-*t*, Survival BB8); single-parameter families show “—”. All estimates obtained from a single AIC-selected family search applied to pseudo-observations derived from DCC-GARCH standardized residuals.

The first tree reflects the strongest unconditional dependence. The NVDA—AMD pair exhibits the highest Kendall's τ (0.4820), indicating higher dependence between the two semiconductor companies. However, both upper- and lower-tail dependence coefficients are equal to zero, suggesting that although the assets move together during normal market conditions, they do not exhibit significant joint extreme events. Similarly, the NVDA—TSLA relationship shows moderate dependence ($\tau = 0.2771$) without meaningful tail dependence.

Bitcoin displays asymmetric dependence patterns. The BTC—NVDA pair is characterized by a Survival Gumbel copula with Kendall's τ equal to 0.1152 and a lower-tail

dependence coefficient of 0.1534. This indicates that Bitcoin and NVIDIA become more strongly connected during adverse market conditions than during positive market phases. The presence of lower-tail dependence suggests a non-negligible probability of simultaneous extreme losses, providing evidence of downside contagion between cryptocurrency and AI-related technology markets.

Gold exhibits weak dependence with Bitcoin ($\tau = 0.0605$) through a standard Gumbel copula, implying limited co-movement and weak upper-tail dependence (UTD = 0.0821). Likewise, the dependence between NVDA and Gold remains indirect and weak throughout the vine structure.

The relationship between NVDA and Oil is also described by a Survival Gumbel copula with weak dependence ($\tau = 0.0683$) and lower-tail dependence (LTD = 0.0925). This asymmetric dependence structure indicates that extreme negative movements in Oil and technology markets are more likely to occur simultaneously than extreme positive movements.

The conditional dependence relationships appearing in Trees 2–5 are weaker than those in the first tree. Most conditional Kendall's τ values remain below 0.10, indicating that a large share of the dependence structure is already explained by the primary connections identified in Tree 1. Several higher-order relationships are modeled by the independence copula, particularly Gold–TSLA, BTC–Oil, Gold–AMD, and Oil–AMD. These results imply that after accounting for the major dependence channels, the remaining conditional associations become statistically negligible.

The Student- t copulas identified in Tree 2 and Tree 4 (Gold–NVDA | BTC, BTC–TSLA | NVDA, and Gold–Oil | BTC, TSLA, NVDA) exhibit very small but non-zero symmetric tail dependence. The corresponding Kendall's τ coefficients remain low, indicating that extreme co-movements between these assets are not frequent.

The copula parameters (Par and Par2) characterize the shape and dependence structure of the selected pair-copula families. For Student- t copulas, Par corresponds to the correlation parameter and Par2 represents the degrees of freedom. For Gumbel-type copulas, Par measures the dependence intensity, with values closer to one indicating weaker dependence. For BB8 copulas, the two parameters jointly determine the nonlinear dependence structure and are therefore interpreted primarily through Kendall's τ and the corresponding tail-dependence coefficients.

To assess the statistical reliability of the dependence relationships reported in Table 11, we evaluated parameter uncertainty, tail-dependence significance, and goodness of fit for each fitted pair. Parameter uncertainty was obtained via a parametric bootstrap: for each pair, we simulated 100 synthetic samples of 500 observations from the fitted copula using its estimated parameters, re-estimated the copula parameters on each simulated sample by maximum likelihood, and constructed 95% confidence intervals from the resulting bootstrap distribution. The same bootstrap draws were used to derive 95% confidence intervals for the implied upper- and lower-tail dependence coefficients. Goodness of fit was assessed using White's (1982) [54] information matrix equality test for elliptical and single-parameter Archimedean families, and Kendall's process-based test [55,56] for the BB1/BB6/BB7/BB8 family and its rotations, since these two tests cover disjoint subsets of copula families within the VineCopula implementation in R Studio 2023.12.1 Build 402.

For the first tree, the confidence intervals indicate that the dependence parameters of the selected copulas are different from the independence benchmark. The Gold–BTC pair exhibits a weak but statistically significant upper-tail dependence, with the estimated UTD confidence interval ranging from 0.0002 to 0.1549, suggesting a limited probability of joint extreme positive returns. The Survival Gumbel copulas selected for BTC–NVDA and NVDA–Oil reveal significant lower-tail dependence, with LTD confidence intervals

of [0.0877, 0.2105] and [0.0258, 0.1620], respectively. Downside contagion is more relevant than upside contagion for relationships involving Bitcoin and Oil.

The strongest dependence relationship is observed between NVDA and AMD. The Survival BB8 copula parameter is significant, with narrow confidence intervals for both parameters. Although the tail-dependence coefficients remain close to zero, the large Kendall's τ reported in Table 11 confirms strong dependence between the two semiconductor companies. A similar pattern is observed for the NVDA–TSLA pair, where dependence is high but not concentrated in the extreme tails.

In Tree 2, conditional dependence relationships become weaker after controlling for the influence of central nodes. Gold–NVDA conditional on BTC exhibits a confidence interval for the correlation parameter that includes zero ($[-0.1256, 0.0225]$), indicating that the residual dependence is statistically weak. Likewise, BTC–TSLA conditional on NVDA displays only weak positive dependence. The Survival Gumbel copula selected for TSLA–Oil conditional on NVDA suggests weak downside dependence, although the corresponding lower-tail confidence interval is small.

The higher-order trees reveal a decline in conditional dependence strength. Several pairs are modeled by the independence copula, including Gold–TSLA | BTC, NVDA, BTC–Oil | TSLA, NVDA, Gold–AMD | BTC, TSLA, NVDA, and Oil–AMD | Gold, BTC, TSLA, NVDA. This result indicates that most dependence is already captured by the lower vine trees and that additional conditional relationships contribute little explanatory power.

The tail-dependence confidence intervals in Table 12 provide further evidence that extreme co-movements are limited within the system. Except for BTC–NVDA and NVDA–Oil, most confidence intervals for UTD and LTD remain close to zero. Therefore, systemic contagion appears concentrated in a few specific channels.

Table 12. Parameter uncertainty, tail-dependence confidence intervals, and goodness-of-fit results for the R-vine copula structure.

Tree	Pair	Par 95% CI	Par2 95% CI	UTD 95% CI	LTD 95% CI	GOF p (Method)
1	Gold–BTC	[1.0002, 1.1317]	—	[0.0002, 0.1549]	[0.0000, 0.0000]	0.36 (white)
1	BTC–NVDA	[1.0692, 1.1912]	—	[0.0000, 0.0000]	[0.0877, 0.2105]	0.64 (white)
1	NVDA–TSLA	[1.7116, 4.2783]	[0.1067, 0.5309]	[0.0000, 0.0000]	[0.0000, 0.0000]	0.00 (kendall)
1	NVDA–Oil	[1.0192, 1.1388]	—	[0.0000, 0.0000]	[0.0258, 0.1620]	0.46 (white)
1	NVDA–AMD	[3.4276, 6.0000]	[0.0635, 0.6624]	[0.0000, 0.0000]	[0.0000, 0.0000]	0.00 (kendall)
2	Gold–NVDA BTC	$[-0.1256, 0.0225]$	[5.1848, 30.0000]	[0.0000, 0.0000]	[0.0000, 0.0391]	0.64 (white)
2	BTC–TSLA NVDA	[0.0007, 0.1994]	[7.5737, 30.0000]	[0.0000, 0.0000]	[0.0000, 0.0361]	0.32 (white)
2	TSLA–Oil NVDA	[1.0002, 1.0915]	—	[0.0000, 0.0000]	[0.0002, 0.1129]	0.08 (white)
2	TSLA–AMD NVDA	[0.0820, 0.2280]	—	[0.0000, 0.0000]	[0.0000, 0.0000]	0.00 (white)
3	Gold–TSLA BTC, NVDA	—	—	[0.0000, 0.0000]	[0.0000, 0.0000]	n/a (independence)
3	BTC–Oil TSLA, NVDA	—	—	[0.0000, 0.0000]	[0.0000, 0.0000]	n/a (independence)
3	BTC–AMD TSLA, NVDA	[1.0010, 6.0000]	[0.0010, 1.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]	0.80 (kendall)
4	Gold–Oil BTC, TSLA, NVDA	[0.0030, 0.1796]	[5.6147, 30.0000]	[0.0000, 0.0000]	[0.0000, 0.0570]	0.64 (white)
4	Gold–AMD BTC, TSLA, NVDA	—	—	[0.0000, 0.0000]	[0.0000, 0.0000]	n/a (independence)
5	Oil–AMD Gold, BTC, TSLA, NVDA	—	—	[0.0000, 0.0000]	[0.0000, 0.0000]	n/a (independence)

Note: “|” denotes the conditioning set for higher-tree (partial) copulas. 95% confidence intervals obtained via a parametric bootstrap (100 replications per pair; simulated sample size capped at 500 observations per replication). “—” indicates a single-parameter family with no second parameter, or an independence copula with no parameters to bootstrap. GOF p = goodness-of-fit p -value; method follows BiCopGofTest's family support (“white” = White's (1982) [54] information matrix equality test, used for elliptical/Archimedean families; “kendall” = Kendall's process-based test of Genest et al. (2006) [55], used for the BB1/BB6/BB7/BB8 family and its rotations). Independence-copula pairs (Tree 3–5) have no associated GOF test, as there are no parameters to test. For Tree 1 pairs, pseudo-observations are exact; for Tree 2–5 pairs, pseudo-observations are exact; for Tree 2–5 pairs, pseudo-observations are approximated from the marginal pseudo-observations of u_{all} , and these higher-tree results should be interpreted as approximate. A GOF p -value below 0.05 indicates the fitted family is formally rejected despite being AIC-optimal among the candidate set; this is the case for the NVDA–TSLA, NVDA–AMD, and TSLA–AMD | NVDA pairs.

Although the AIC criterion selected the Survival BB8 copula for the strongest technology–equity pairs, the goodness-of-fit tests reject these specifications at conventional

significance levels. This indicates that the selected copulas represent the best-performing families within the candidate set but do not completely reflect the empirical dependence structure. Consequently, the reported dependence network should be interpreted as describing the dominant dependence patterns rather than an exact characterization of the underlying multivariate distribution.

To address the research question of the study, we extended the empirical analysis beyond the full-sample R-vine copula estimation by introducing a regime-specific dependence analysis. The sample was partitioned into five economically meaningful sub-periods (pre-COVID-19, COVID-19 shock, recovery, Russia–Ukraine conflict, and post-crisis). Copula models were re-estimated within each regime using the same DCC-GARCH filtering and AIC copula selection.

Table 13 reports the 95% confidence interval for the difference between each regime’s mean DCC-GARCH conditional correlation and the full-sample mean, obtained via block-bootstrap. For each pair and regime, we tested whether the regime-specific mean correlation differs significantly from the full-sample mean by computing a 95% block-bootstrap confidence interval for their difference. A block length has 10 trading days and 1000 replications. This procedure accounts for the serial dependence inherent in conditional correlation series. If this confidence interval excludes zero, the regime-specific correlation is considered statistically distinguishable from the full-sample baseline. The observed difference is unlikely to be attributable to sampling noise alone, and the regime can be said to exhibit a different dependence level. Conversely, if the interval includes zero, the data cannot rule out that the difference is zero, and the regime’s correlation cannot be distinguished from the unconditional, full-sample estimate.

Table 13. Block-bootstrap 95% CI for (regime mean correlation—full-sample mean correlation).

Pair	Regime	Diff 95% CI Lower	Diff 95% CI Upper	Median Diff	Distinguishable?
NVDA–BTC	Pre-COVID-19	−0.1638	−0.1390	−0.1510	Yes
NVDA–BTC	COVID-19 shock	0.0919	0.1278	0.1103	Yes
NVDA–BTC	Recovery	0.0976	0.1309	0.1139	Yes
NVDA–BTC	Ukraine	0.3511	0.3763	0.3640	Yes
NVDA–BTC	Post-crisis	0.0024	0.0292	0.0162	Yes
NVDA–Gold	Pre-COVID-19	−0.1422	−0.1153	−0.1290	Yes
NVDA–Gold	COVID-19 shock	0.2225	0.2624	0.2421	Yes
NVDA–Gold	Recovery	0.0835	0.1183	0.1010	Yes
NVDA–Gold	Ukraine	0.1563	0.1839	0.1708	Yes
NVDA–Gold	Post-crisis	−0.0057	0.0274	0.0114	No
BTC–Gold	Pre-COVID-19	−0.0367	−0.0169	−0.0265	Yes
BTC–Gold	COVID-19 shock	0.2170	0.2404	0.2293	Yes
BTC–Gold	Recovery	−0.1526	−0.1292	−0.1415	Yes
BTC–Gold	Ukraine	0.0430	0.0638	0.0537	Yes
BTC–Gold	Post-crisis	−0.0415	−0.0117	−0.0261	Yes

The results show that the regime-specific correlation is statistically distinguishable from the full-sample mean in 14 of 15 pair–regime combinations (93%). Only NVDA–Gold during the post-crisis period fails to differ significantly from the full-sample baseline, as its confidence interval ([−0.0057, 0.0274]) includes zero.

The most striking result concerns the NVDA–BTC pair during the Russia–Ukraine conflict. In this case, the correlation shift is +0.364 (95% CI [0.351, 0.376])—about three times larger than any other deviation in Table 13, including the COVID-19 shock itself (+0.110). The linkage between this AI-exposed equity and Bitcoin became much tighter

during the 2022 geopolitical and monetary tightening shock than at any other point across the ten-year sample.

The COVID-19 shock window shows a coordinated increase across all three pairs simultaneously—NVDA–BTC (+0.110), NVDA–Gold (+0.242), and BTC–Gold (+0.229). The pre-COVID-19 regime shows negative deviations for all three pairs (−0.151, −0.129, and −0.027 respectively). The higher average correlation observed over the full sample is mainly driven by the stronger dependence recorded during the COVID-19, recovery, and Russia–Ukraine crisis periods.

For BTC–Gold, the dependence structure is also regime-dependent. Dependence strengthens during the COVID-19 shock ($\tau = 0.2106$) compared with the pre-COVID-19 period ($\tau = 0.0546$) but disappears during the recovery phase when an independence copula is selected. The relationship re-emerges during the Ukraine conflict before returning to relatively weak levels in the post-crisis period. This relationship is event-driven.

Regime-specific copulas were re-estimated on DCC-GARCH standardized residuals within each sub-period using the same AIC-based family selection procedure employed for the full-sample analysis. The full-sample copula family and Kendall's τ are reported for comparison in Table 14. For the NVDA–BTC relationship, dependence is weak during the pre-COVID-19 period ($\tau = 0.0235$) but increases during subsequent crisis episodes. Kendall's τ rises to 0.1608 during the COVID-19 shock, rises to 0.2441 during the recovery phase, and reaches 0.3618 during the Russia–Ukraine conflict. The selected copula family changes repeatedly across sub-periods. The full-sample Survival Gumbel estimate ($\tau = 0.1152$) therefore masks regime heterogeneity.

Table 14. Regime-specific copula family and Kendall's τ versus full-sample estimates.

Pair	Regime	Regime-Specific Family	Regime τ	Full-Sample Family	Full-Sample τ
NVDA–BTC	Pre-COVID-19	Rotated Tawn 1	0.0235	Survival Gumbel	0.1152
NVDA–BTC	COVID-19 shock	Survival Joe	0.1608	Survival Gumbel	0.1152
NVDA–BTC	Recovery	Frank	0.2441	Survival Gumbel	0.1152
NVDA–BTC	Ukraine	Rotated Tawn 1	0.3618	Survival Gumbel	0.1152
NVDA–BTC	Post-crisis	Gaussian	0.1379	Survival Gumbel	0.1152
NVDA–Gold	Pre-COVID-19	Rotated BB1 (90°)	−0.0928	Student- <i>t</i>	−0.0160
NVDA–Gold	COVID-19 shock	Student- <i>t</i>	0.0797	Student- <i>t</i>	−0.0160
NVDA–Gold	Recovery	Independence	0.0000	Student- <i>t</i>	−0.0160
NVDA–Gold	Ukraine	Student- <i>t</i>	0.0910	Student- <i>t</i>	−0.0160
NVDA–Gold	Post-crisis	Student- <i>t</i>	−0.0210	Student- <i>t</i>	−0.0160
BTC–Gold	Pre-COVID-19	Joe	0.0546	Gumbel	0.0605
BTC–Gold	COVID-19 shock	Clayton	0.2106	Gumbel	0.0605
BTC–Gold	Recovery	Independence	0.0000	Gumbel	0.0605
BTC–Gold	Ukraine	Joe	0.0968	Gumbel	0.0605
BTC–Gold	Post-crisis	Gumbel	0.0549	Gumbel	0.0605

The NVDA–Gold pair exhibits a different pattern. Dependence remains weak throughout the sample and alternates between positive, negative, and insignificant values. The recovery period is characterized by an independence copula, while the remaining sub-periods are associated with weak Student-*t* or rotated BB1 structures.

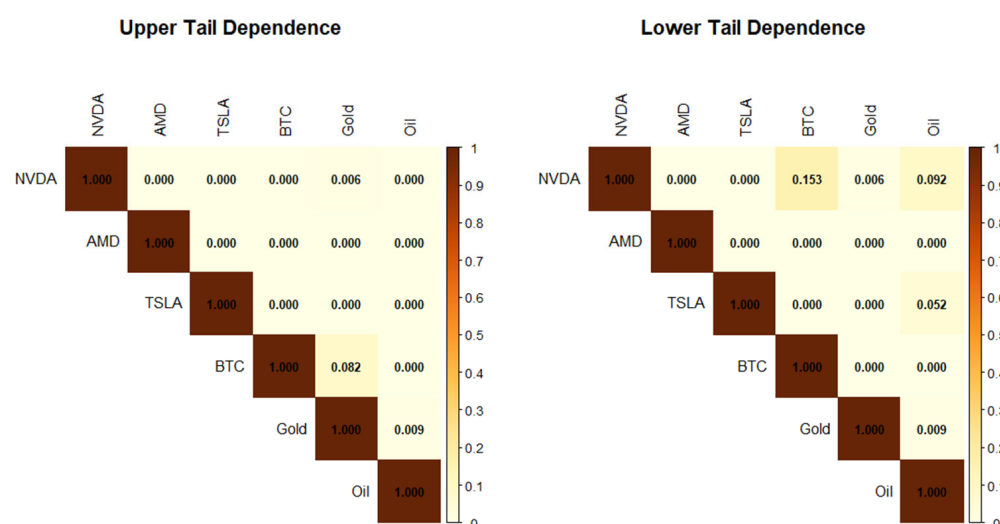
Table 15 reports the bivariate copula family legend used to interpret the family names referenced in Table 11, following the family numbering convention defined in the VineCopula R package's BiCop() function documentation.

Table 15. Copula family legend.

Family Code	Copula Type	Letter
0	Independence	I
1	Gaussian (Normal)	N
2	Student- <i>t</i>	t
4	Gumbel	G
14	Rotated Gumbel (180°; “Survival Gumbel”)	SG
20	Rotated BB8 (180°; “Survival BB8”)	SBB8

Note: Copula family codes follow the numbering convention implemented in the VineCopula package.

The tail dependence matrices in Figure 8 reveal asymmetry in extreme co-movements. Lower-tail dependence is stronger than upper-tail dependence, indicating that joint extreme losses are more likely than joint extreme gains. The strongest lower-tail dependence is noticed for the NVDA–BTC pair (0.153), followed by NVDA–Oil (0.092) and TSLA–Oil (0.052). Dependence intensifies during adverse market conditions, particularly for asset pairs involving Bitcoin and, to a lesser extent, Oil. Upper-tail dependence is weak throughout the network. The largest upper-tail dependence coefficient is observed for the BTC–Gold pair (0.082), while the other pairs exhibit values close to zero. Extreme positive market movements are rarely synchronized

**Figure 8.** Upper- and lower-tail dependence structures.

The hierarchical structure of the estimated R-vine copula model is shown in Figure 9. Tree 1 reflects the strongest unconditional dependence relationships. NVDA (node 1) occupies the central position in the network, connecting directly with AMD, TSLA, Bitcoin, and Oil, while Gold is linked through Bitcoin. This configuration identifies NVIDIA as the principal dependence hub within the technology–cryptocurrency–commodity system.

The strongest first-order relationships are concentrated among technology-related assets, NVDA–AMD and NVDA–TSLA. Bitcoin has direct connections with both NVDA and Gold. Technology equities are the core of the dependence network. Bitcoin acts as an intermediary between digital and traditional asset classes.

Trees 2 and 3 represent conditional dependence relationships after accounting for the strongest links identified in Tree 1. Several conditional connections involving Bitcoin, Tesla, and Gold are present, indicating that some dependence persists even after controlling for the primary technology sector relationships. The magnitudes of these conditional dependencies are weaker than those observed in the first tree, as confirmed by the small Kendall’s τ values reported in Table 11.

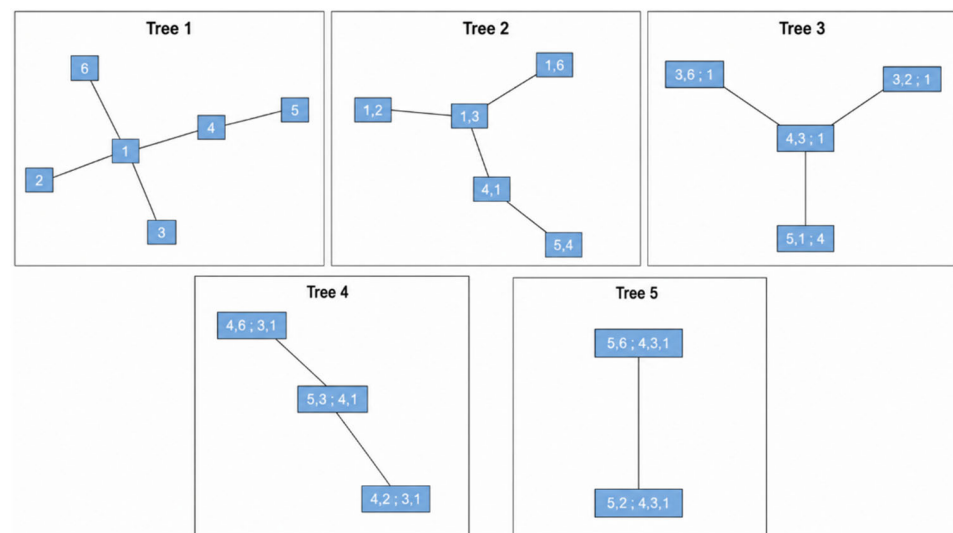


Figure 9. Hierarchical R-vine copula tree structure for the dependence network. Note: Asset mapping: 1 = NVDA, 2 = AMD, 3 = TSLA, 4 = BTC, 5 = Gold, 6 = Oil.

The higher-order trees (Trees 4 and 5) contain only a small number of conditional relationships. Several pairs are modeled by independence copulas, indicating that most of the dependence structure has already been captured by the lower trees. This progressive reduction in network complexity confirms that the dominant interactions are concentrated in the first levels of the vine hierarchy. Higher-order conditional dependencies contribute only marginally to the overall dependence structure.

4.3. Scale-Dependent Lyapunov Exponent

To further investigate nonlinear dynamics across multiple scales, we apply the Scale-Dependent Lyapunov Exponent (SDLE) framework proposed by [27]. Unlike the Lyapunov exponent, which provides a single measure of divergence, SDLE captures how local trajectory divergence evolves across different scales. Table 16 reports summary statistics for SDLE. The Mean SDLE values are positive for all assets, indicating local divergence dynamics. Oil and AMD exhibit slightly higher mean values. However, the observed differences across assets remain moderate and should not be interpreted as statistically significant distinctions in nonlinear complexity. The Positive Scale Share is introduced in this study as the proportion of analyzed scales for which the estimated SDLE values remain positive. The Positive Scale Share values are close to 0.5 for all assets, implying that divergence and convergence mechanisms coexist across different temporal scales. This result supports the multiscale nature of financial markets emphasized by Gao [26,27]. The minimum SDLE values are negative, while the maximum values are positive for all assets. It follows that the analyzed markets are characterized by scale-dependent stochastic complexity.

Table 16. Summary statistics of SDLE.

Asset	Mean SDLE	Positive Scale Share	Minimum SDLE	Maximum SDLE
NVDA	0.1262	0.5000	−0.5756	1.0699
AMD	0.1383	0.5385	−0.4805	1.1062
TSLA	0.0815	0.5185	−0.4861	0.9639
BTC	0.0759	0.4815	−0.4679	0.9941
Gold	0.0992	0.5185	−0.6519	1.0373
Oil	0.1628	0.5217	−0.4079	1.1522

To assess whether the SDLE results reported above are robust to methodological choices, and to test directly whether stochastic noise might be misinterpreted as deterministic complexity, we conducted two additional checks: a sensitivity analysis across embedding parameters and a surrogate data comparison. Table 17 reports the Mean SDLE and Positive Scale Share statistics across a grid of embedding dimensions ($m \in \{2, 3, 4\}$) and time delays ($\tau \in \{1, 2\}$) for each asset. The Positive Scale Share remains within a narrow range (0.41–0.57) across the entire parameter grid for all six assets. NVDA shows the most stable estimate, and Oil shows the most variable estimate.

Table 17. SDLE sensitivity to embedding dimension (m) and delay (τ).

Asset	m	τ (Delay)	Mean SDLE	Positive Scale Share
NVDA	2	1	0.1262	0.5000
NVDA	2	2	0.0723	0.5000
NVDA	3	1	0.0178	0.4231
NVDA	3	2	0.0674	0.4615
NVDA	4	1	0.0296	0.4615
NVDA	4	2	−0.0032	0.4643
AMD	2	1	0.1383	0.5385
AMD	2	2	0.1121	0.5600
AMD	3	1	0.0307	0.4615
AMD	3	2	0.1608	0.5556
AMD	4	1	0.0075	0.4400
AMD	4	2	0.0817	0.5185
TSLA	2	1	0.0815	0.5185
TSLA	2	2	0.1481	0.5556
TSLA	3	1	0.0183	0.4615
TSLA	3	2	0.0536	0.5185
TSLA	4	1	0.0016	0.4074
TSLA	4	2	0.0701	0.5172
BTC	2	1	0.0759	0.4815
BTC	2	2	0.1365	0.5185
BTC	3	1	0.0477	0.5000
BTC	3	2	0.1053	0.5556
BTC	4	1	0.0136	0.4231
BTC	4	2	0.0916	0.5172
Gold	2	1	0.0992	0.5185
Gold	2	2	0.0906	0.5200
Gold	3	1	0.0356	0.4800
Gold	3	2	0.0280	0.5000
Gold	4	1	0.0190	0.4444
Gold	4	2	−0.0314	0.4615
Oil	2	1	0.1628	0.5217
Oil	2	2	0.2231	0.5652
Oil	3	1	0.0939	0.4783
Oil	3	2	0.0143	0.4348
Oil	4	1	0.0370	0.4545
Oil	4	2	−0.0072	0.4091

To test whether the SDLE signal reflects deterministic temporal structure or is instead consistent with stochastic noise sharing the same marginal distribution, we constructed surrogate series for each asset by randomly shuffling its return series. This procedure preserves the marginal distribution of returns (and therefore any heavy-tailedness or skewness already present) while completely destroying temporal dependence. Table 18 compares the Mean SDLE of the original series against its shuffled surrogate. For AMD, BTC, and Gold, the difference is small (0.0115, −0.0294, and −0.0395, respectively), indicating that a

great portion of their apparent multiscale complexity is statistically indistinguishable from i.i.d. noise with the same marginal distribution. In contrast, NVDA, TSLA, and particularly Oil exhibit larger differences (0.0572, 0.0345, and 0.0852, respectively), indicating that their SDLE signals reflect temporal structure beyond what marginal non-Gaussianity alone would produce.

Table 18. SDLE surrogate data check ($m = 2, \tau = 1$).

Asset	Original Mean SDLE	Surrogate Mean SDLE	Difference
NVDA	0.1262	0.0690	0.0572
AMD	0.1383	0.1268	0.0115
TSLA	0.0815	0.0470	0.0345
BTC	0.0759	0.1053	−0.0294
Gold	0.0992	0.1387	−0.0395
Oil	0.1628	0.0775	0.0852

In Figure 10, the SDLE functions are represented for the analyzed assets. The SDLE curves are positive at small scales and gradually decline toward zero or negative values as the scale increases. This pattern indicates that local divergence dynamics are more pronounced at smaller scales. Larger scales are characterized by increasing stabilization and convergence effects.

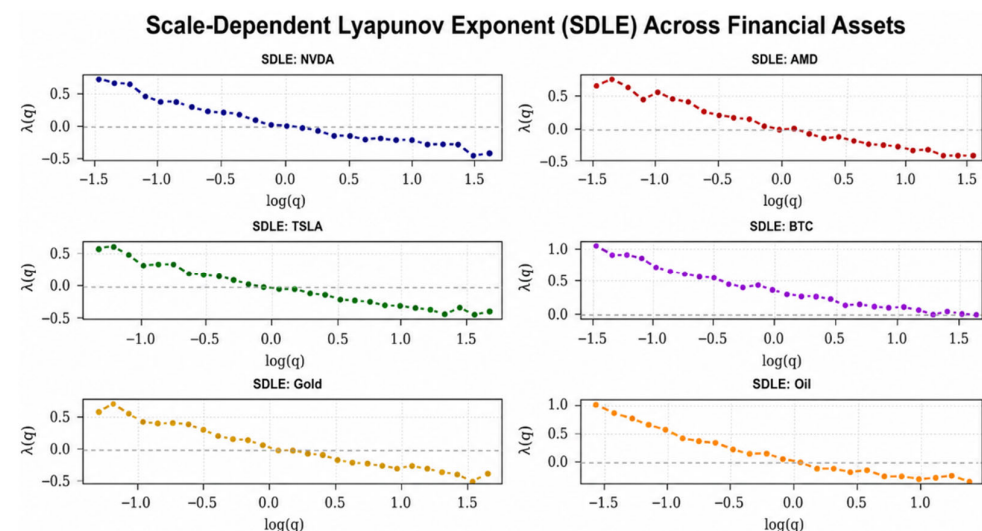


Figure 10. SDLE across financial assets.

The positive scales observed at small scales suggest that trajectories in the phase space tend to diverge locally. Such behavior is visible for Oil and AMD, which display relatively higher SDLE values in the small-scale regions. BTC and TSLA have lower SDLE amplitudes, even if the multiscale patterns present similarities for all assets.

Short-term market fluctuations may exhibit local instability effects, while larger-scale market dynamics remain stabilized through long adjustment mechanisms.

4.4. Predictive Modeling of Financial Returns Using XGBoost and Random Forest

The machine learning models were estimated using a chronological time-series split with an 80–20% training–testing partition. The training sample spans from 11 January 2016 to 29 December 2023, while the out-of-sample test period extends from 2 January 2024 to 30 December 2025. Although the raw financial series were downloaded starting from 1 January 2016, the effective estimation sample begins on 11 January 2016 due to lag

construction, rolling-window volatility estimation, and removal of missing observations generated by the preprocessing procedure.

The variable importance analysis in Table 19 and Figure 11 reveals that Oil market variables are the dominant predictive drivers. Oil_vol5 emerges as the most influential predictor under both XGBoost and RF estimation. Short-term energy market volatility contributes to forecasting cross-asset return dynamics.

Table 19. Variable importance ranking from XGBoost and Random Forest models.

Rank	Variable	XGBoost Gain	Random Forest Importance
1	Oil_vol5	0.2263	0.2515
2	Oil_lag1	0.1404	0.1807
3	TSLA_lag1	0.1081	0.1310
4	NVDA_lag1	0.1056	0.1386
5	Gold_vol5	0.0915	0.1294
6	NVDA_vol5	0.0564	0.1400
7	BTC_lag1	0.0538	0.1207
8	Gold_lag1	0.0491	0.1301
9	BTC_vol5	0.0491	0.1376
10	TSLA_vol5	0.0485	0.1061
11	AMD_lag1	0.0362	0.1052
12	AMD_vol5	0.0350	0.1061

Notes: lag1 denotes one-period lagged returns. vol5 represents 5-day rolling volatility.

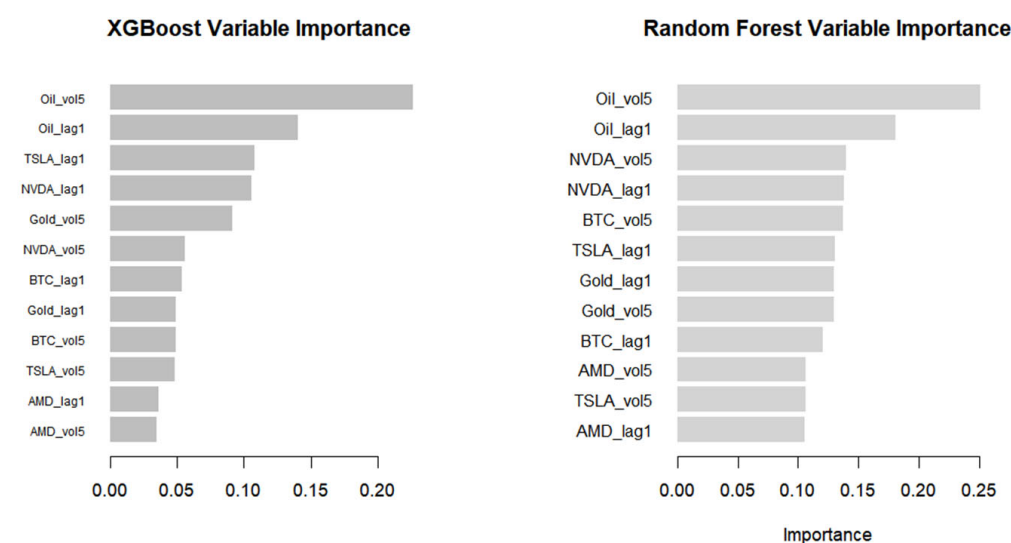


Figure 11. Variable importance comparison between XGBoost and Random Forest.

The strong contribution of Oil_lag1 suggests that Oil price persistence and volatility transmission mechanisms exert influence on broader financial market behavior. This result is economically intuitive, as Oil markets are sensitive to macroeconomic conditions, geopolitical shocks, and inflationary expectations. All of them propagate across global financial systems.

Technology sector variables also display predictive importance. TSLA_lag1 and NVDA_lag1 rank among the most influential predictors, reflecting the increasing systemic relevance of large-cap technology firms in financial markets. The importance of NVDA_vol5 suggests that volatility clustering within AI-related technology stocks contains meaningful information about future market movements.

Bitcoin-related variables (BTC_lag1 and BTC_vol5) have moderate importance. Cryptocurrency dynamics contribute to nonlinear information transmission, although their

influence remains below that of Oil and major technology assets. Gold variables (Gold_lag1 and Gold_vol5) also demonstrate non-negligible importance.

Methodologically, the two machine learning models exhibit distinct importance structures. XGBoost assigns a larger share of explanatory power to a relatively concentrated subset of dominant variables, particularly Oil-related predictors, indicating stronger nonlinear prioritization within the boosting framework. RF distributes importance more evenly across variables, suggesting a more diversified ensemble learning structure. XGBoost importance is measured using normalized gain, while Random Forest importance is based on mean decrease in predictive impurity.

The multi-panel Figure 12 compares actual returns with forecasts generated by the XGBoost and RF models during the out-of-sample test period. Both AI models capture the general direction and short-run fluctuations in returns. Extreme spikes and abrupt market shocks are more difficult to predict accurately. The predicted series remain smoother than the actual returns. This is consistent with machine learning forecasting models in highly volatile financial markets.

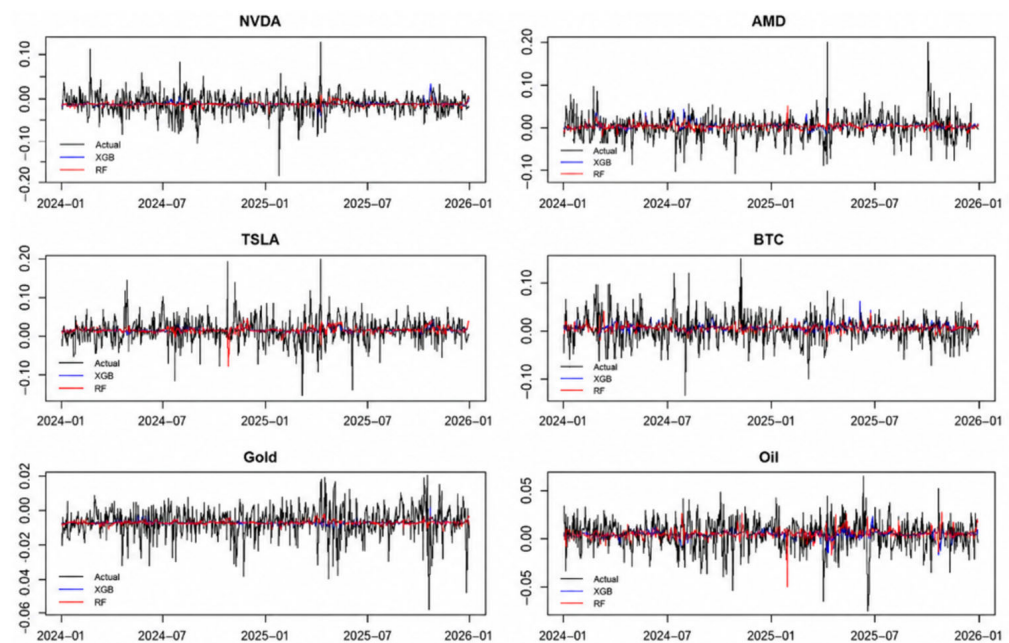


Figure 12. Actual versus predicted returns using XGBoost and Random Forest models.

The largest deviations between actual and predicted returns occur during periods associated with heightened geopolitical and macroeconomic uncertainty. Volatility clustering and sudden jumps can be observed around the aftermath of the Russia–Ukraine war beginning in 2022, persistent inflationary pressures, tightening monetary policy by the Federal Reserve, and instability in global energy markets.

Technology-related assets such as NVDA, AMD, and TSLA fluctuate more due to sensitivity to AI sector expectations, semiconductor supply chains, and speculative investor sentiment. Bitcoin is also unstable, reflecting the cryptocurrency market turbulence and changing investor sentiment during crisis periods.

Gold displays lower volatility and more stable predictions. Oil returns exhibit intermittent sharp movements related to energy supply disruptions and geopolitical tensions linked to the Russia–Ukraine conflict.

To assess whether the tree ensemble forecasts provide genuine predictive value beyond trivial baselines, we benchmark XGBoost and Random Forest against three naive models estimated on the identical chronological train/test split: a random walk (predicting zero return), a historical mean (predicting the training sample average return), and a first-order

autoregressive model fitted on each asset's own return series. We additionally test the statistical significance of pairwise forecast accuracy differences using the Diebold–Mariano test [57] under squared error loss [58].

Table 20 shows that the machine learning models (XGBoost and Random Forest) do not outperform the naïve benchmark models (random walk, historical mean, and AR(1)), as the benchmark models consistently achieve lower RMSE values across all six assets. The AR(1) model provides the best forecasting performance for NVDA, BTC, and Gold, while the historical mean benchmark performs best for TSLA and Oil.

Table 20. Forecasting performance: XGBoost and Random Forest versus naive benchmarks (RMSE).

Asset	σ (Uncond.)	XGB RMSE	RF RMSE	RW RMSE	HM RMSE	AR(1) RMSE
NVDA	0.03106	0.03270	0.03243	0.03218	0.03207	0.03194
AMD	0.03643	0.03497	0.03479	0.03373	0.03374	0.03374
TSLA	0.03725	0.04044	0.04041	0.03958	0.03957	0.03957
BTC	0.04228	0.03121	0.03101	0.03056	0.03054	0.03052
Gold	0.00969	0.01140	0.01153	0.01146	0.01142	0.01070
Oil	0.02837	0.01958	0.01996	0.01908	0.01910	0.01911

Notes: σ = unconditional standard deviation of daily log returns in the test period (the “noise floor”). RW = random walk (predicts zero return); HM = historical mean (predicts the train sample mean return); AR(1) = first-order autoregressive benchmark fitted on the target asset's own return series. All models use an identical chronological 80/20 train/test split (no random shuffling). Lower RMSE indicates better performance.

The Diebold–Mariano test results (Table 21) indicate that XGBoost and Random Forest have statistically equivalent predictive accuracy for most assets, while both machine learning models significantly outperform the random walk benchmark for AMD, TSLA, and Oil. Random Forest significantly outperforms XGBoost for Gold and Oil.

Table 21. Diebold–Mariano tests for equal predictive accuracy.

Asset	DM: XGB vs. RF	<i>p</i> -Value	DM: XGB vs. RW	<i>p</i> -Value	DM: RF vs. RW	<i>p</i> -Value
NVDA	1.201	0.230	1.693	0.091	0.905	0.366
AMD	0.712	0.477	3.103	0.002 ***	2.816	0.005 ***
TSLA	0.122	0.903	2.411	0.016 **	2.317	0.021 **
BTC	0.872	0.383	1.857	0.064	1.528	0.127
Gold	−2.240	0.026 **	−0.722	0.471	0.895	0.371
Oil	−2.507	0.013 **	2.162	0.031 **	3.440	0.001 ***

Notes: DM statistic and two-tailed *p*-value under squared error loss ($h = 1$). A negative DM (XGB vs. RF) statistic indicates RF achieved lower forecast error than XGBoost for that asset. H_0 : equal predictive accuracy. ** $p < 0.05$; *** $p < 0.01$.

5. Conclusions

This study has examined the dynamic interactions among technology stocks, Bitcoin, Gold and Oil using an integrated framework combining the DCC-GARCH model, vine copulas, tail dependence, complexity measures and AI forecasting models. The results reveal strong volatility persistence and time-varying dependence structure among digital and financial assets. Technology stocks and BTC exhibit stronger interconnectedness during the COVID-19 pandemic and the Russia–Ukraine conflict, confirming the increasing integration of AI-related financial ecosystems.

The DCC-GARCH results indicate persistent conditional correlations and volatility clustering. The vine copula framework identifies asymmetric dependence structures, especially in the lower tails. BTC-related pairs have stronger downside contagion, meaning that extreme negative shocks propagate more intensely during crisis periods. Gold is weakly connected to speculative assets, preserving its diversification role. The complexity

analysis reveals heterogeneous multiscale dynamics characterized by weak persistence and local divergence effects at lower scales and increasing stabilizations at larger scales. The machine learning models provide a complementary perspective on return dynamics. Their out-of-sample forecasting performance does not consistently exceed that of the benchmark models. Oil-related variables remain among the most influential predictors, highlighting the importance of energy markets within the technology–cryptocurrency–commodity network.

The estimated dependence network can be useful for portfolio allocation, risk management, and systemic risk surveillance. The identification of NVIDIA as a central node within the technology–cryptocurrency–commodity network suggests that shocks originating in AI-related technology stocks may propagate rapidly across multiple asset classes. Portfolio managers can use the dynamic dependence structure as an early-warning indicator when adjusting portfolio exposures during periods of heightened uncertainty. From a regulatory perspective, the estimated R-vine dependence network and tail-dependence measures can support systemic risk monitoring by identifying assets that act as contagion transmitters and by detecting periods when downside co-movements intensify. Supervisory authorities may apply these network-based indicators to monitor cross-market vulnerability, assess financial stability risks, and design scenarios involving simultaneous shocks to technology, cryptocurrency, and commodity markets.

Several limitations can be acknowledged. First, the analysis focuses only on six assets, which may not fully describe the complexity of digital financial ecosystems. Second, the machine learning is limited only to XGBoost and Random Forest models, excluding deep learning architectures such as LSTM or Transformer networks. Structural breaks and regime-switching behavior [59] may affect dependence dynamics, particularly during extreme geopolitical and macroeconomic events. Finally, macroeconomic variables not included in this study, such as interest rates, inflation, or monetary policy indicators, may influence volatility transmission and systemic risk.

Future research may extend the analysis by including other cryptocurrencies, AI-related firms, ESG assets, and green-energy indices. Contagion mechanisms may be understood better by the integration of high-frequency data and real-time risk monitoring systems. The combination of vine copulas and deep learning architectures may improve predictive accuracy. The exploration of regime-switching vine copulas or quantum machine learning approaches may capture better nonlinear financial complexity and evolving dependence structures in digitalized financial systems.

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Conflicts of Interest: The author declares no conflicts of interest.

Appendix A. Preliminary Diagnostic Tests

To ensure the suitability of the proposed econometric and machine learning framework, a series of preliminary diagnostic tests was performed on the return series. Specifically, unit root and stationarity tests (ADF, PP, and KPSS), nonlinearity tests (BDS), volatility clustering diagnostics (ARCH-LM), serial correlation tests (Ljung–Box), and normality tests (Jarque–Bera) were conducted. These tests provide statistical evidence regarding the stochastic properties of the analyzed financial assets and support the application of DCC-GARCH, vine copula, and scale-dependent complexity methodologies.

Table A1 reports the results of the stationarity tests. The Augmented Dickey Fuller (ADF) and Phillips–Perron (PP) unit root tests reject the null hypothesis of a unit root for all assets, while the KPSS test fails to reject the null hypothesis of stationarity. Therefore, all return series can be considered stationary and appropriate for subsequent empirical analysis.

Table A1. Unit root tests.

Asset	ADF <i>p</i> -Value	PP <i>p</i> -Value	KPSS <i>p</i> -Value
NVDA	0.0100	0.0100	0.1000
AMD	0.0100	0.0100	0.1000
TSLA	0.0100	0.0100	0.1000
BTC	0.0100	0.0100	0.1000
Gold	0.0100	0.0100	0.0958
Oil	0.0100	0.0100	0.1000

Table A2 presents additional diagnostic evidence regarding the statistical properties of the analyzed return series. The BDS test rejects the null hypothesis of independent and identically distributed (i.i.d.) observations for NVDA, AMD, TSLA, Bitcoin, and Oil, indicating the presence of nonlinear dependence in these markets. The BDS statistic for Gold is not statistically significant, suggesting weaker evidence of nonlinear dependence and a behavior closer to an i.i.d. process. This finding is consistent with the traditional role of Gold as a stable and safe-haven asset.

The ARCH-LM test reveals significant conditional heteroskedasticity across all assets, confirming the existence of volatility clustering and supporting the application of GARCH-type models. The Jarque–Bera test strongly rejects normality for all return series, indicating heavy-tailed and non-Gaussian distributions. The Ljung–Box statistics provide evidence of serial dependence for NVDA, AMD, TSLA, and Oil, whereas Bitcoin and Gold exhibit weaker autocorrelation patterns.

Table A2. Nonlinearity, volatility clustering and distributional diagnostics.

Asset	BDS Statistic	BDS <i>p</i> -Value	ARCH-LM <i>p</i> -Value	Ljung–Box <i>p</i> -Value	Jarque–Bera <i>p</i> -Value
NVDA	8.0663	0.0000	3.20×10^{-15}	1.03×10^{-4}	0.0000
AMD	3.7362	0.0002	9.77×10^{-4}	7.20×10^{-3}	0.0000
TSLA	5.1548	0.0000	1.40×10^{-23}	2.54×10^{-2}	0.0000
BTC	7.6661	0.0000	4.93×10^{-10}	6.88×10^{-2}	0.0000
Gold	1.1270	0.2597	1.30×10^{-30}	2.02×10^{-1}	0.0000
Oil	9.3294	0.0000	8.13×10^{-138}	3.98×10^{-7}	0.0000

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