

Article

Evaluation of Crack Coalescence and Stability of Jointed Rock Masses for Tunnel Engineering Using a Modified Coalescence Coefficient

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Abstract

Crack coalescence between discontinuities plays a critical role in the progressive failure and stability evolution of jointed rock masses. This study aims to develop a modified coalescence coefficient for quantifying crack coalescence behavior and evaluating its influence on rock mass stability. Uniaxial compression tests and heterogeneous numerical simulations were conducted to investigate the effects of crack dip angle, rock bridge dip angle, crack length, and rock bridge length on crack coalescence. The results revealed cubic polynomial relationships between the coalescence coefficient and crack dip angle, rock bridge dip angle, and crack length, whereas a linear relationship was identified with rock bridge length. Based on an $L_{25}(5^4)$ orthogonal design and nonlinear regression analysis, a modified coalescence coefficient applicable to jointed rock masses under static loading conditions was proposed. The coefficient was subsequently incorporated into the block theory implemented in GeoSMA-3D and applied to a tunnel excavation case. The results demonstrated that considering structural-plane coalescence significantly alters the geometry and volume of key blocks, leading to a more realistic stability assessment. The proposed coefficient provides an effective tool for improving block identification and stability evaluation in underground rock engineering.

Keywords: random weighting method; orthogonal experiment; uniaxial compression test; GeoSMA-3D; engineering stability evaluation



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1. Introduction

Crack propagation and coalescence constitute the primary mechanisms driving progressive failure in rock masses and have consequently received extensive attention in the field of rock mechanics. The initiation, propagation, interaction, and eventual coalescence of discontinuities strongly influence the strength, deformation characteristics, and stability of rock engineering structures. Therefore, understanding the mechanisms governing crack coalescence is of great significance for evaluating the mechanical behavior and failure processes of jointed rock masses.

The Griffith criterion is widely recognized as the most classical theoretical framework for describing crack propagation in brittle materials [1–3]. Nevertheless, numerous

experimental investigations have been conducted to further elucidate crack coalescence mechanisms and failure behaviors in rock materials.

Zhou et al. [4] systematically investigated the intrinsic mechanisms of crack propagation in red sandstone specimens containing a single flaw under uniaxial compression. Subsequently, Chen et al. [5,6] performed a series of compression–shear tests on sandstone specimens with double flaws. Their results demonstrated that flaw geometry significantly influences rock failure characteristics, and three typical crack coalescence patterns were identified. Furthermore, the effects of flaw eccentricity and overlap distance on the compressive–shear-bearing capacity were revealed.

Zhang et al. [7,8] examined the failure behavior and crack coalescence mechanisms of granite specimens containing prefabricated cracks. Their findings indicated that tensile stress dominates the formation of failure zones. In addition, they highlighted the significant role of inherent microcracks in governing crack propagation and fracture behavior. Pang and He, as well as Miao et al. [9–11], investigated crack initiation, propagation, and coalescence in specimens with different crack inclination angles. The results showed that, with increasing flaw inclination angle, the failure mode transitions from tensile failure to compressive–shear failure. Moreover, regardless of the initial crack orientation, wing tensile cracks consistently emerge as the first type of cracks in all specimens.

Zhang and She [12] analyzed the influence of fissure angle and rock bridge length on the mechanical properties and failure behavior of rocks through uniaxial compression tests combined with Digital Image Correlation. Tian et al. [13] quantitatively characterized the post-peak mechanical behavior of rocks containing Mode I, Mode II, and mixed-mode cracks, and established a peridynamics-based theoretical framework for rock fracture, incorporating multiple fracture criteria and virtual crack models.

In addition to experimental studies, numerous numerical investigations have been conducted to explore crack coalescence and failure mechanisms. Liang et al. [14] employed the CDEM method to analyze the influence of joint length and in situ stress conditions on blasting-induced stress evolution and crack propagation. Zheng and Liang [15] combined numerical simulation with damage mechanics theory to derive a constitutive model for discontinuous fracture in cracked sandstone specimens. Zhang et al. [16] integrated the finite element method (FEM) and finite volume method (FVM) to simulate the synchronous hydraulic fracturing process in rocks containing double cracks.

Yu et al. [17,18] provided in-depth insights into crack propagation behavior in various rock types under multiple influencing factors. Gong et al. [19] incorporated random field theory into conventional rock instability analysis to model material heterogeneity, effectively capturing the influence of intrinsic correlations on fracture behavior. Zhou et al. [20] proposed a particle-based model coupled with a discrete fracture network (DFN), elucidating the dynamic tensile response of rock masses and its associated uncertainties. Huang et al. [21] accurately simulated the dynamic evolution of hydraulic and natural fractures, thereby characterizing their interaction and propagation processes. Wang et al. [22] numerically investigated the effects of crack interaction, loading conditions, and heterogeneity on crack trajectories, stress fields, and energy release rates.

Overall, previous studies have extensively explored crack propagation and coalescence using fracture mechanics approaches [23,24] and advanced experimental and numerical techniques [25–27]. Although significant progress has been achieved in understanding crack coalescence mechanisms, most existing studies focus on failure behavior under specific crack geometries, and quantitative methods for characterizing coalescence and incorporating its effects into engineering stability assessment remain limited. In addition, the relationship between crack coalescence and practical engineering applications, such as block identification and stability evaluation, has received relatively little attention.

To address these issues, this study proposes a modified coalescence coefficient to quantitatively evaluate crack coalescence behavior and its engineering significance. Laboratory experiments, heterogeneous numerical simulations, orthogonal experimental design, and engineering case analysis were combined to investigate the effects of crack geometry on coalescence behavior and to establish a quantitative relationship between crack coalescence and rock mass stability. Specifically, the objectives are to: (1) investigate crack propagation and coalescence in red sandstone specimens containing two pre-existing cracks; (2) establish a modified coalescence coefficient considering crack dip angle, rock bridge dip angle, crack length, and rock bridge length; and (3) integrate the proposed coefficient into GeoSMA-3D [28,29] to evaluate the influence of structural-plane coalescence on block identification and rock mass stability.

2. Materials and Methods

2.1. Specimen Preparation and Experimental Program

In this paper, the electro-hydraulic servo rock shear rheological testing machine (TYJ-500 kN, Jilin Jinli Testing Technology Co., Ltd., Changchun, China) was used to perform uniaxial compression tests of rock specimens. All tests were conducted under displacement-controlled loading at a constant loading rate of 1 mm/min until specimen failure.

As shown in Figure 1, in order to reduce the boundary effect, the dimension of red sandstone samples are designed as 200 mm × 100 mm × 30 mm (length × width × thickness), the value of thickness refer to Yang's experimental results [30]. Other geometric parameters include the crack dip angle α (the two prefabricated cracks in each specimen have the same α), the rock bridge dip angle β , the crack length l , and the rock bridge length d , which were selected as the key variables affecting crack propagation and coalescence in this study.

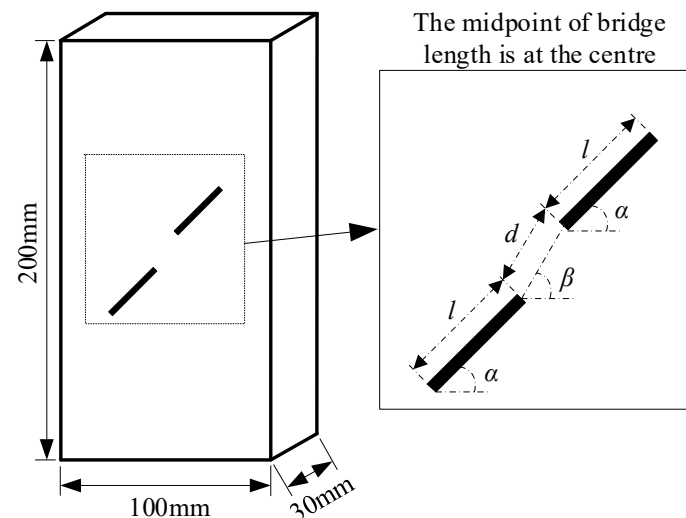


Figure 1. Dimension of samples and geometric parameters of cracks.

As shown in Table 1, four groups of tests were designed to investigate the effects of crack dip angle (α), rock bridge dip angle (β), crack length (l), and rock bridge length (d) on crack coalescence behavior. To ensure the comparability of the experimental results, all specimens were prepared from the same red sandstone block using identical preparation procedures. In each test group, only one geometric parameter was varied while the remaining parameters were kept constant. Specifically, set 1 investigated the effect of α , set 2 investigated the effect of β , set 3 investigated the effect of l , and set 4 investigated the effect of d . Considering the large number of parameter combinations involved, the

experimental program was intended primarily to identify the influence trends of geometric parameters on crack coalescence behavior.

Table 1. The test scheme.

Set 1			Set 2			Set 3			Set 4		
Sample No.	α (°)	Other Parameters	Sample No.	β (°)	Other Parameters	Sample No.	l (mm)	Other Parameters	Sample No.	d (mm)	Other Parameters
SJQ-1	15	$\beta = 60^\circ$ $l = 20$ mm $d = 15$ mm	SYQ-1	15	$\alpha = 45^\circ$ $l = 20$ mm $d = 15$ mm	SJC-1	10	$\alpha = 45^\circ$ $\beta = 60^\circ$ $d = 15$ mm	SJD-1	10	$\alpha = 45^\circ$ $\beta = 60^\circ$ $l = 20$ mm
SJQ-2	45		SJQ-2	45		SJC-2	15		SJD-2	20	
SJQ-3	60		SJQ-3	75		SJC-3	20		SJD-3	25	
SJQ-4	75		SJQ-4	105		SJC-4	25		SJD-4	30	
			SJQ-5	135							

2.2. Experimental Results

2.2.1. Failure Mode in Different α

The failure modes of samples with different α are shown in Figure 2.

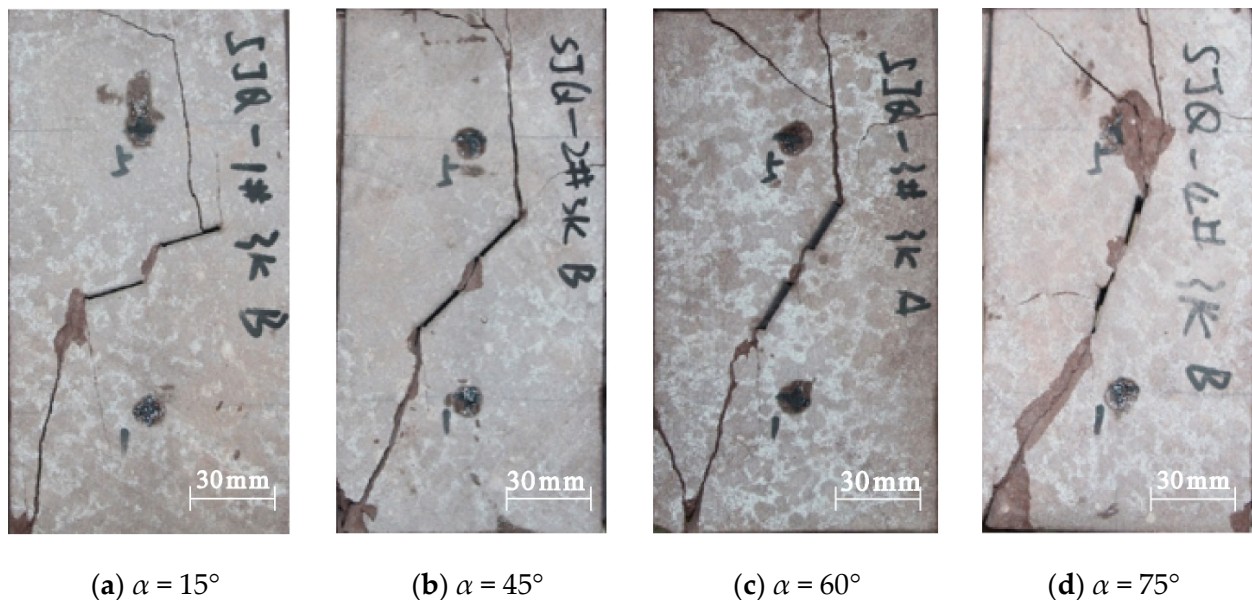


Figure 2. Failure mode of specimens with different α .

When α is 15° (Figure 2a), crack coalescence occurs. Two cracks initiate at the outer tip of the upper prefabricated crack. However, the main crack does not appear at the outer tip of the prefabricated crack, but near the middle of the prefabricated crack. Similarly, two cracks appear at the outer tip of the lower prefabricated crack, one of which extends to the edge of the specimen.

When α is 45° (Figure 2b), there is a coalescence-crack between the two prefabricated cracks. Two cracks occurred at the outer tip of the upper prefabricated crack. The small cracks did not extend upward to the specimen end, but extended to the lateral boundary, and the main crack extended upwards. Near the upper boundary, the main crack split into a few small cracks. Two cracks are also present at the outer tip of the lower prefabricated crack, but the small crack is much smaller than that observed when the crack inclination angle is 15° .

When α is 60° (Figure 2c), the crack propagation is similar to that measured in the test with a crack inclination of 45° . The difference is that the secondary crack at the outer tip of the upper crack is more obvious.

When α is 75° (Figure 2d), two obvious cracks appear at the end of the upper prefabricated crack, and the two cracks affect each other, eventually leading to the overall destruction of the specimen.

2.2.2. Failure Mode in Different β

The failure modes of samples with different β are shown in Figure 3.

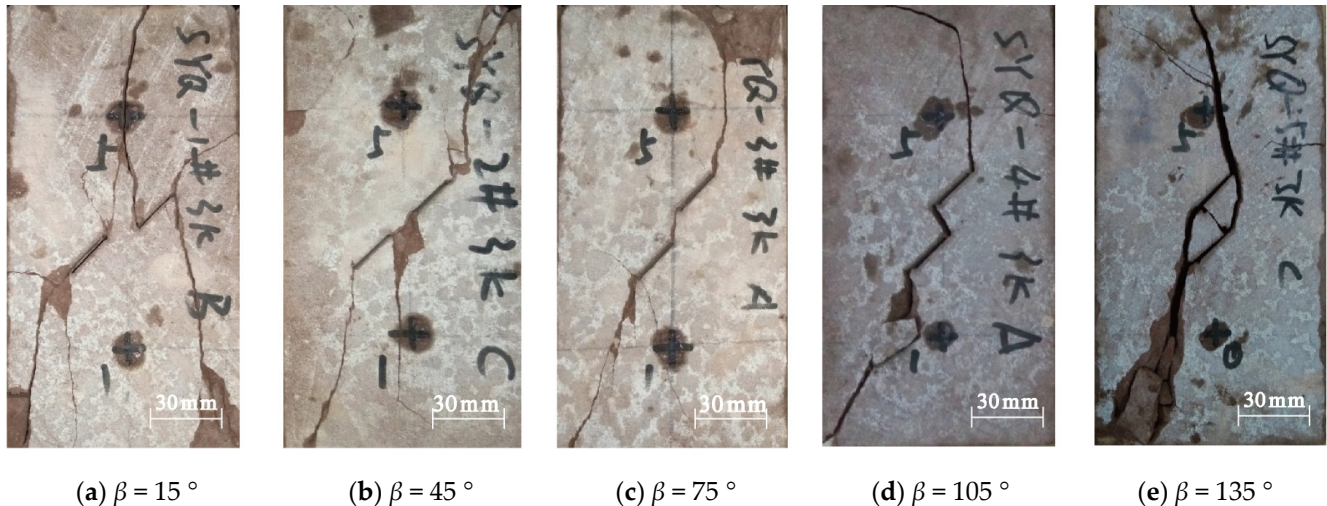


Figure 3. Failure mode of specimens in different β .

When β is 15° (Figure 3a), there is no direct coalescence-crack between the two prefabricated cracks. An anti-wing crack appears at the inner tip of the upper prefabricated crack, and a wing crack appears at the inner end of the lower prefabricated crack. The two cracks intersect and then extend to the upper boundary, eventually leading to a splitting failure of the specimen. In addition, another anti-wing crack appears at the outer tip of the upper prefabricated crack.

When β is 45° (Figure 3b), the anti-wing crack appears at the inner tip of the lower prefabricated crack, and the wing crack appears at the inner tip of the upper prefabricated crack. As the two cracks extend, they eventually intersect and extend to the lower boundary. However, this cannot directly result in the overall failure of the specimen. A wing crack appears at the outer tip of the upper prefabricated crack.

When β is 75° (Figure 3c), there is a coalescence-crack between the two prefabricated cracks. The wing cracks appear at the two tips of the prefabricated crack, and extend to the boundary, eventually leading to the destruction of the specimen.

When β is 105° (Figure 3d), the crack pattern at failure is the same as that observed for the specimen with an inclination angle of 75° .

When β is 135° (Figure 3e), a core failure zone is formed in the area between two prefabricated cracks. The wing cracks occur at the tip of the prefabricated cracks, and extend to the boundary, eventually leading to the destruction of the specimen.

2.2.3. Failure Mode in Different l

The test results of the specimens with different l are similar (Figure 4). The coalescence-crack appears between the two prefabricated cracks, and the wing cracks appear at the outer tips of the prefabricated cracks, and then extend to the outer boundary, eventually resulting in the failure of the specimen.

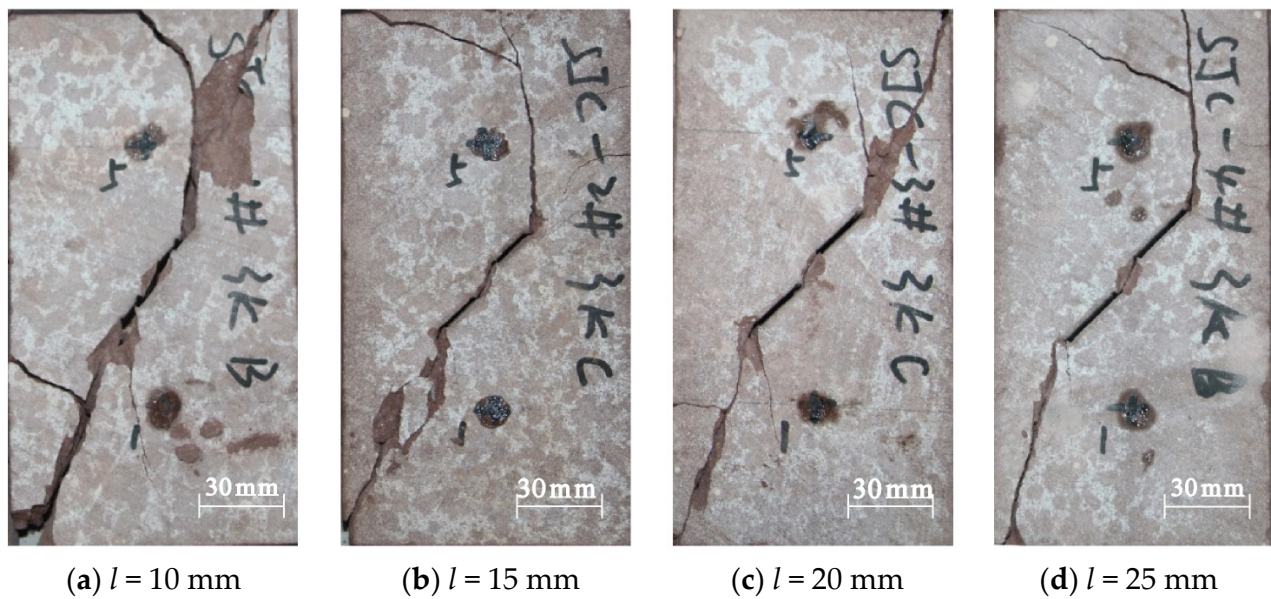


Figure 4. Failure mode of specimens with different l .

2.2.4. Failure Mode in Different d

When d is 10 mm and 20 mm, the crack propagation is similar, as shown in Figure 5. The coalescence-crack appears between the two prefabricated cracks; the wing crack appears at the outer tips of the prefabricated cracks and then extends to the outer boundary, eventually leading to the failure of the specimen. When d is 25 mm, in addition to the above-mentioned situation, an anti-wing crack appears at the inner tip of the lower prefabricated crack. When d is 30 mm, there is no direct coalescence-crack between the two prefabricated cracks. The anti-wing crack appears at the inner tip of the lower prefabricated crack, and the wing crack appears at the inner tip of the upper prefabricated crack. With the extension, the two cracks eventually intersect and extend to the lower boundary. Moreover, a few cracks are present at the outer tips of the two prefabricated cracks.

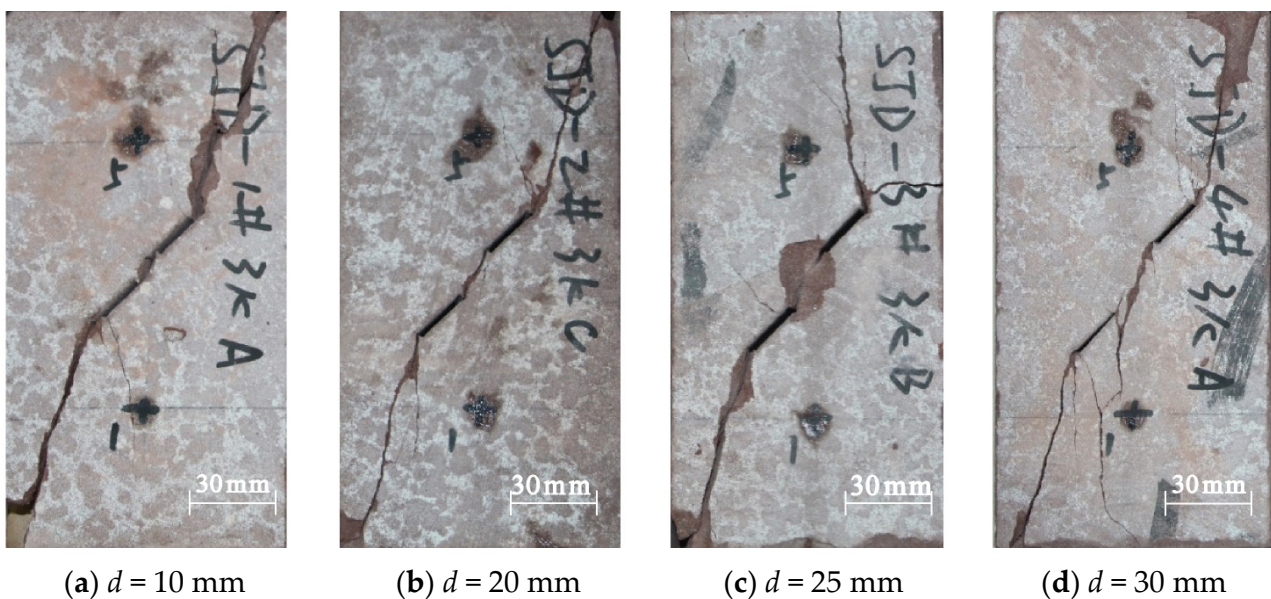


Figure 5. Failure mode of specimens in different d .

2.3. Analysis of Peak Stress

Figures 6–9 illustrate the influence of geometric parameters on the peak stress of the specimens. Overall, the peak stress exhibited distinct but systematic variations with crack geometry, indicating that crack interaction and coalescence behavior are strongly governed by the relative configuration of the two pre-existing cracks. Specifically, the peak stress increased with increasing crack dip angle α , which can be attributed to the reduced tendency for crack coalescence and the enhanced load-bearing capacity of the rock bridge. In contrast, the peak stress generally decreased with increasing crack length l because longer pre-existing cracks resulted in greater stress concentration and reduced effective load-carrying area. For the rock bridge dip angle β and rock bridge length d , the peak stress exhibited non-monotonic variations, reflecting the complex influence of crack interaction and stress redistribution on the coalescence process. Within the investigated parameter ranges, these trends could be reasonably described by quadratic polynomial functions for α , β , and d , and by a logarithmic function for l . It should be noted that the fitted relationships are intended to characterize the observed trends within the investigated parameter space rather than to serve as universal predictive models, and their applicability beyond the tested ranges requires further validation.

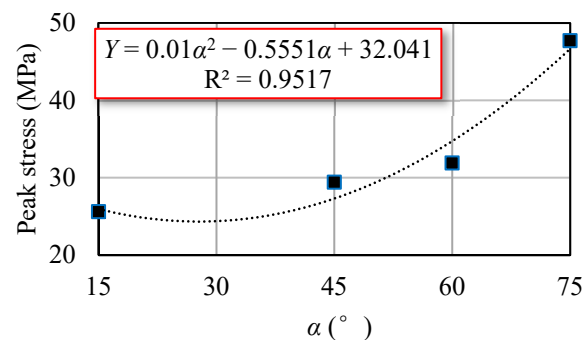


Figure 6. The peak stress with different α .

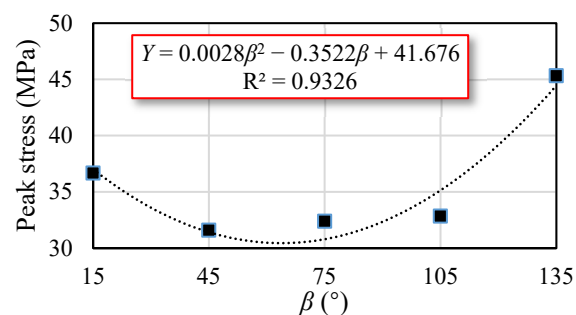


Figure 7. The peak stress with different β .

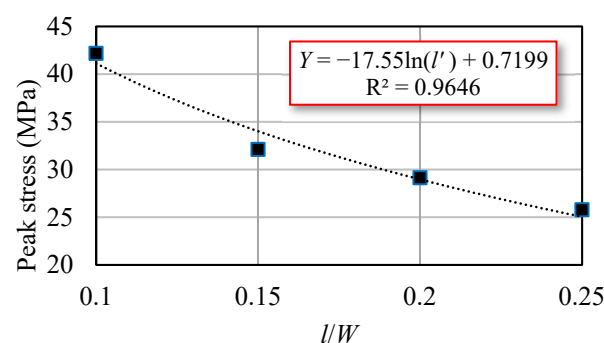


Figure 8. The peak stress with different l .

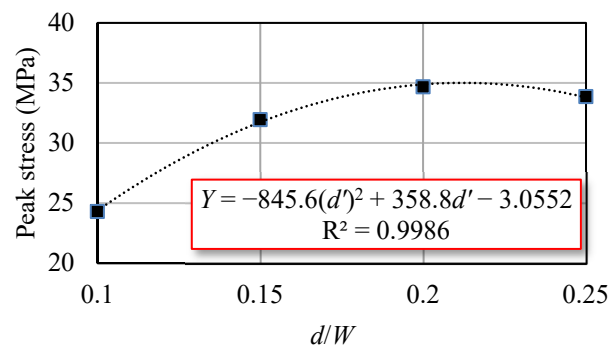


Figure 9. The peak stress with different d .

3. Numerical Simulation

3.1. Heterogeneity of Rock Mechanics Parameters

3.1.1. The Concept of Random Weighting

The random weighting method is a kind of statistical approach for small sample sizes based on the Bootstrap method, which is used in this investigation to analyze the mechanical parameters of rock mass statistically. The method reflects the inherent spatial variability in rock materials caused by mineral heterogeneity and micro-defects. Compared with conventional homogeneous modeling approaches, it can better capture the non-uniform mechanical response and progressive failure behavior of rock masses while maintaining relatively high computational efficiency.

The statistical sample of rock mechanical parameters is assumed to be $X(x_1, x_2, \dots, x_n)$, and its mean and variance are $\hat{x}$ and S_x^2 . The deviation from the true mean and variance of the sample population is denoted as

$$T_n^{(1)} = \hat{x} - \mu_x, \quad T_n^{(2)} = \frac{n}{n-1} S_x^2 - \sigma_x^2 \quad (1)$$

By using the Dirichlet variables, the random weighted statistics for $T_n^{(1)}$ and $T_n^{(2)}$ can be expressed as follows:

$$D_n^{(1)} = \sum v_i x_i - \hat{x} \quad D_n^{(2)} = \frac{n}{n-1} \sum v_i (x_i - \hat{x})^2 - \frac{n}{n-1} S_x^2 \quad (2)$$

$v_i = (v_1, v_2, \dots, v_n)$ obeys Dirichlet distribution $D(1, 1, \dots, 1)$. $D_n^{(i)}$ and $T_n^{(i)}$ have the same limit distribution, so the distribution of $D_n^{(i)}$ can theoretically be substituted for $T_n^{(i)}$.

For sample X , we can calculate j sets of random variables obeying $D(1, 1, \dots, 1)$, and then j sets of randomly weighted subsamples are obtained, so that the eigenvalues of rock mechanics parameters can be obtained using the estimation method.

3.1.2. Solution of Sample Mean and Variance

Uniaxial compression testing data were used to determine the elastic modulus of red sandstone. The elastic modulus sample space is $X(i) = (4.894; 3.815; 5.705; 4.863; 4.852; 4.558; 4.396; 4.906; 4.373; 4.639; 3.162; 4.851; 4.122; 5.647; 4.635; 3.442; 4.465; 5.003; 5.599; 3.745; 4.586; 3.633; 4.616; 5.146; 4.441; 4.848; 4.866; 4.42; 4.895; 5.653; 4.811; 4.546; 3.351; 4.882; 4.086; 4.292; 5.157; 3.5; 5.133; 5.599; 5.156)$. Unit is GPa.

From the statistical theory, we know that when j is infinite, $\hat{D}_n$ is an unbiased estimate of $E(D_n^{(i)})$. But in practice, j cannot be infinity. In order to determine the value of j , trial tests were conducted as follows: For each j value varying from 1000, 10,000, 100,000, to 1,000,000, the mean elastic modulus was calculated 10 times, as shown in Figures 10 and 11.

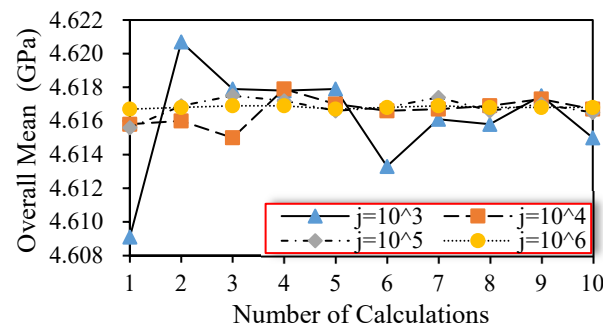


Figure 10. The overall mean value with different j .

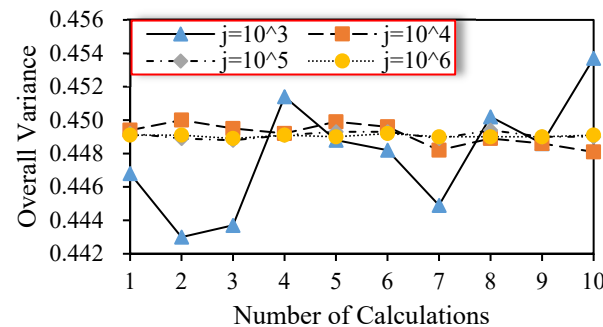


Figure 11. The overall variance with different j .

The results indicated that the mean and variance of the sample space for elastic modulus is almost constant when j is 1,000,000. Therefore, we take $j = 1,000,000$ for all subsequent calculations in this paper.

3.1.3. The Method of Generating Random Numbers

In this paper, the generalized beta (β) distribution was adopted. The processing method is as follows.

The probability density function of a standard beta distribution is as follows:

$$f(x) = \begin{cases} \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1} (1-x)^{q-1}, & 0 < x < 1 \\ 0, & x \leq 0 \text{ \& } x \geq 1 \end{cases} \quad (3)$$

where $p > 0$, $q > 0$; and Γ follows a gamma function.

The effective domain standard of beta distribution is (0,1), and the following calculation is conducted in this paper.

The weighted average of $X(i)$ is solved by Equation (4):

$$\hat{x}_{(j)} = \sum v_i x_i \quad (4)$$

Then, we can obtain a series of weighted solutions $\hat{X} = [\hat{x}_{(j)}]$; let $a = \min(X, \hat{X})$, $b = \max(X, \hat{X})$.

The normalization of X becomes:

$$Y_{(j)} = \frac{X_{(j)} - a}{b - a} \quad (5)$$

After transformation, $Y(j) \in [0, 1]$ satisfies the condition of beta distribution.

Then,

$$E(Y_{(j)}) = E\left(\frac{X_{(j)} - a}{b - a}\right) = \frac{E(X_{(j)}) - a}{b - a} \quad (6)$$

$$D(Y_{(j)}) = D\left(\frac{X_{(j)} - a}{b - a}\right) = \frac{D(X_{(j)})}{(b - a)^2} \quad (7)$$

The mean and variance of $Y(j)$ can be calculated using the mean and variance obtained by weighting $X(i)$. The parameters p, q can be obtained from the following equation:

$$\begin{cases} \mu(Y_{(j)}) = \frac{p}{p+q} \\ \sigma^2(Y_{(j)}) = \frac{pq}{(p+q)^2(p+q+1)} \end{cases} \quad (8)$$

A code is then used to generate the random number obeying $\beta(p, q)$ according to the number of simulation units. The generalized beta random number of the elastic modulus can be evaluated using Equation (5).

Finally, we use the fish statement to assign a generated random number to each discretized element of the model to achieve heterogeneity in the mechanical parameters. Figure 12 is a numerical model shear modulus cloud in *FLAC3D*. As can be seen from the figure, the method achieves full mesh coverage of the parameter's heterogeneity.

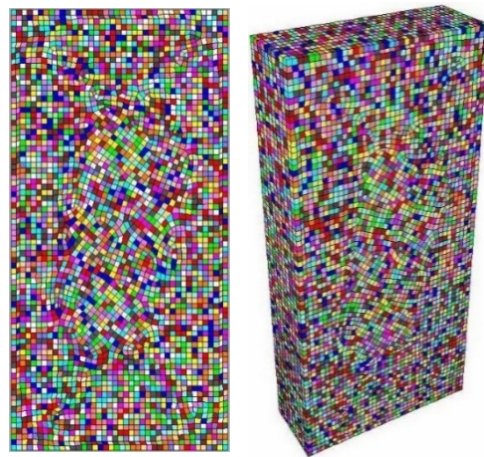


Figure 12. The shear modulus of the model.

3.2. Modeling and Validation

In this paper, the results obtained from the SYQ-4 test were used to evaluate the efficacy of the numerical model, as this configuration was considered representative of the crack propagation and coalescence behavior observed in the experimental program. After validation, the numerical model was applied to investigate a wider range of crack geometries. Although only one validation case is presented here, the comparison demonstrates that the model can reasonably reproduce the main failure characteristics of the tested specimens. The boundary conditions and physical parameters employed in the numerical simulation are summarized as follows.

3.2.1. The Boundary Conditions

A Z-direction displacement constraint is applied to the bottom surface of the specimen, and the top surface is loaded at a velocity of 2.5×10^{-8} m/step. In order to avoid the end effect, vaseline was daubed on the upper and lower surfaces of the specimens.

3.2.2. The Physical Parameters

The bulk modulus and shear modulus of solid elements of the specimen are explained as follows. The elastic modulus in this paper is obtained from the specimens. The sample space is: $X(i) = (4.894; 3.815; 4.853; 4.852; 4.558; 4.396; 4.906; 4.373; 4.639; 3.162; 4.851; 4.122;$

5.647; 4.635; 3.442; 4.465; 5.003; 5.599; 4.536; 4.361; 4.441; 4.848; 4.866; 4.42; 4.895; 5.653; 4.811; 4.546; 3.351; 4.882; 4.086; 4.292; 5.157; 3.5; 5.133; 5.599; 5.156) (unit: GPa). Using the generalized beta distribution, which was adopted to characterize the statistical variability in the elastic modulus within the observed sample range, the parameter eigenvalues were calculated as $p = 0.3200$ and $q = 0.5359$. Then, generalized beta random numbers are generated and the bulk and shear modulus of the unit are calculated. The other material properties of specimen are a Poisson's ratio of 0.22, cohesion of 19.2 MPa, internal friction angle of 41.2° and tensile strength of 0.4 MPa. The values of other material properties of specimen are obtained from References [30] and [31].

3.2.3. Result of Simulation

The measured experimental data are compared with the calculated simulation results, as shown in Figure 13.

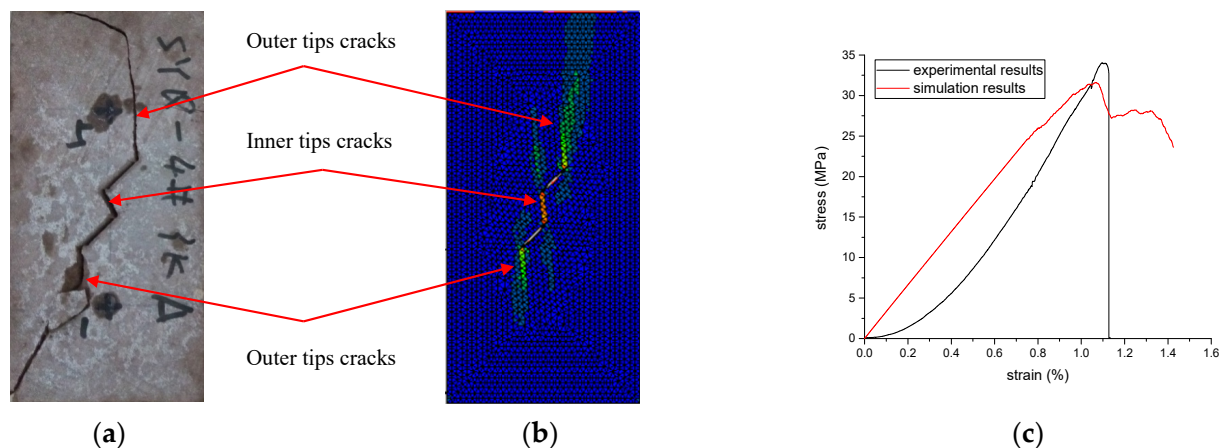


Figure 13. Comparison of experimental and simulated results: (a) the experimental result; (b) the simulated result; (c) the stress–strain curve.

It can be seen from the figure that the simulated crack pattern is almost the same as that observed in the test. Because the test is the uniaxial compression test, there is no strain-softening process. Therefore, the numerical calculation can only simulate the peak stress. The peak stress of the specimen measured in the experiment is 34.03 MPa, and the simulation peak value is 31.61 MPa. The two analyses could provide consistent results that differ by less than 7.1%. Therefore, it is believed that the numerical model can meet the calculation requirements.

3.3. Research on Scale Effect

Scale effect is one of the problems that cannot be neglected in engineering practice. To solve this problem, we take the size of the cracks' length as the benchmark, and take the dimension of the model as its corresponding multiple. The concrete scheme is as follows.

The geometric dimensions of models are length:width:thickness = 2:1:0.3, and the width W is 1/2/3/4/5/6/7/8/9 times the cracks size ($2l + d$) in the simulation. The simulation results are shown in Figure 14.

The results exhibited that the stress–strain curve of the specimen is almost uniform when $W/(2l + d) \geq 5$, so the specimen size when $W/(2l + d) = 5$ is taken as the standard size for follow-up study.

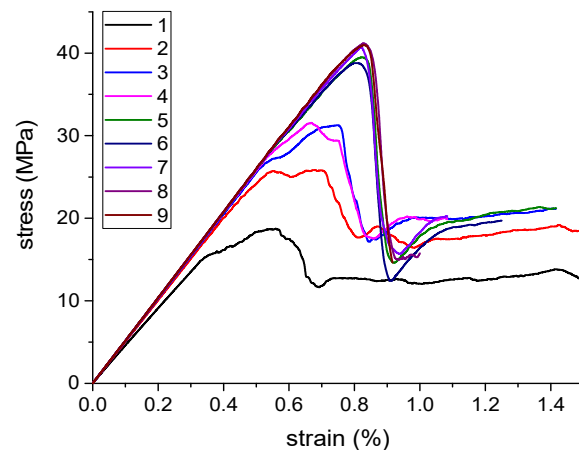


Figure 14. Stress–strain curve with different $W/(2l + d)$.

3.4. Correction of Crack Coalescence-Coefficient

In this paper, firstly, the coalescence-coefficient with the parameters of α , l , d and β is obtained by univariate analysis. Orthogonal tests were carried out to analyze the existing penetration coefficient, thereby correcting the existing coalescence-coefficients, which can be expressed by the following equation:

$$A = \frac{y(\alpha, \beta, l', d')}{Y(\alpha, \beta, l', d')} \quad (9)$$

where A is the coalescence-coefficient; y is the coalescence-stress (in this paper, when the strain in the center of the specimen is abruptly changed, the stress at the top of specimen is taken as the coalescence-stress); Y is the peak-stress; α is the crack inclination angle; β is the rock bridge inclination angle; l' is the ratio of the crack length to specimen width; and d' is the ratio of rock bridge length to specimen width.

3.4.1. Univariate Analysis

Univariate analysis was carried out for the above four factors (α , β , l and d). The crack length and rock bridge length were selected according to the percentage of specimen width, respectively. The simulation results are shown in Figures 15–18.

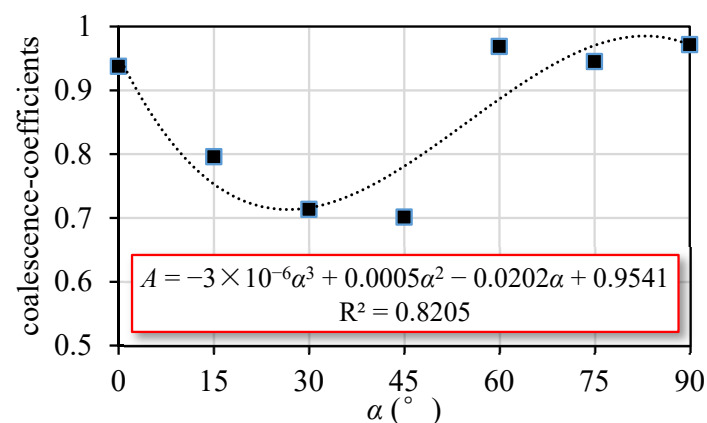


Figure 15. The coalescence-coefficient with different α .

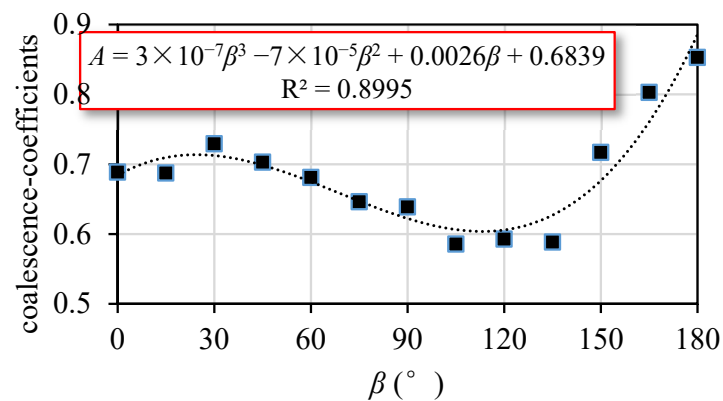


Figure 16. The coalescence-coefficient with different β .

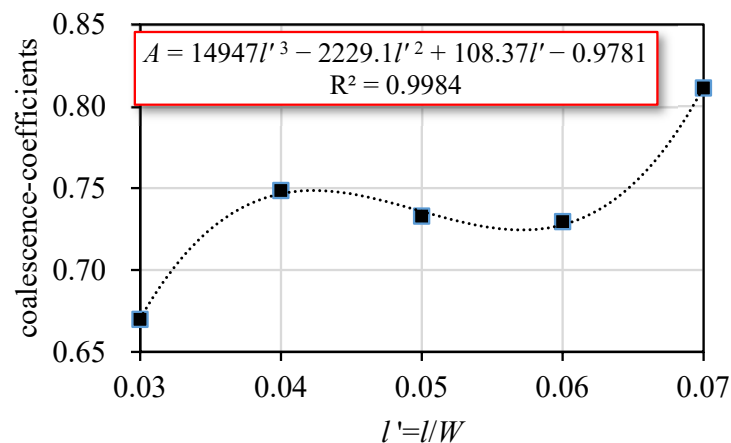


Figure 17. The coalescence-coefficient with different l .

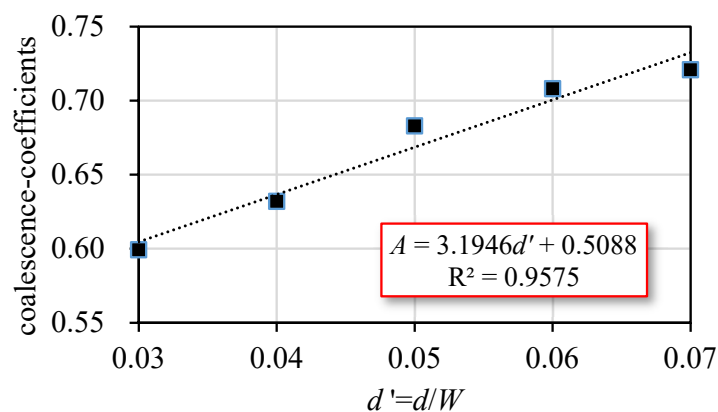


Figure 18. The coalescence-coefficient with different d .

The results show that there are cubic polynomial relationships, cubic polynomial relationships, cubic polynomial relationships and linear relationships between the coalescence-coefficient and crack inclination angle, rock bridge inclination angle, crack length, and rock bridge length.

To further evaluate the explanatory power of the investigated factors on the coalescence coefficient, the goodness-of-fit values (R^2) obtained from the regression analyses were compared. As shown in Figures 15–18, the crack length ratio l' exhibited the highest explanatory capability, with an R^2 value of 0.9984, indicating that crack length is the dominant factor governing the variation in the coalescence coefficient. The rock bridge length ratio d' also showed a strong correlation with the coalescence coefficient ($R^2 = 0.9575$). In

comparison, the rock bridge dip angle β and crack dip angle α yielded lower R^2 values of 0.8995 and 0.8205, respectively.

These results suggest that geometric size parameters (i.e., crack length and rock bridge length) exert a stronger influence on crack coalescence behavior than orientation parameters (i.e., crack dip angle and rock bridge dip angle). Physically, increasing the crack length enlarges the stress interaction zone between adjacent flaws and promotes crack linkage, whereas increasing rock bridge length directly alters the distance over which crack propagation must occur before coalescence is achieved. In contrast, the inclination angles primarily affect the propagation path and crack interaction mode, resulting in a comparatively weaker influence on the coalescence coefficient.

Therefore, among the four investigated variables, the relative explanatory power can be ranked as

$$l' > d' > \beta > \alpha$$

indicating that crack length is the most influential parameter controlling crack coalescence in the present study.

3.4.2. Orthogonal Test and Fitting Coalescence-Coefficient

To investigate the combined effects of crack dip angle (α), rock bridge dip angle (β), crack length (l), and rock bridge length (d) on the coalescence coefficient, an orthogonal experimental design based on the standard $L_{25}(5^4)$ orthogonal array was adopted. Four factors were considered, each at five levels, resulting in 625 possible combinations in a full-factorial design. To improve computational efficiency while maintaining representative coverage of the parameter space, the orthogonal design reduced the number of required simulations to 25.

The selection of the $L_{25}(5^4)$ orthogonal array ensured the orthogonality of the experimental design. In this array, each level of every factor appears the same number of times throughout the experimental matrix, and the factor levels are arranged according to the orthogonal table structure to minimize correlations among factors. Consequently, the main effect of each factor on the coalescence coefficient can be evaluated independently while substantially reducing the computational cost compared with a full-factorial design.

The factor levels were selected according to the parameter ranges employed in the laboratory tests and numerical simulations, ensuring that the orthogonal design adequately covered the practical variation range of the crack geometry and rock bridge characteristics. The design scheme of the orthogonal experiments is presented in Table 2.

Table 2. The scheme of the orthogonal test.

Level	Factor			
	α (°)	β (°)	l (mm)	d (mm)
1	15	30	9	9
2	30	60	12	12
3	45	90	15	15
4	60	120	18	18
5	75	150	21	21

A nonlinear regression analysis was performed on the results of numerical simulation, and the regression equation was calculated as follows:

$$A = f(\alpha) + g(\beta) + h(l') + k(d') + c \quad (10)$$

with the following definitions:

$$\begin{aligned}
 f(\alpha) &= 5.57 \times (\alpha + 1.35)^3; \\
 g(\beta) &= 71.03 \times (\beta + 2437.10)^3; \\
 h(l') &= -170.31 \times (l' - 0.01)^3; \\
 k(d') &= 0.39 \times d'; \\
 c &= -0.60.
 \end{aligned}$$

The trends reflected by these fitted functions are further illustrated in Figure 19, which shows the variation of the coalescence coefficient with each of the four factors. It should be noted that the proposed regression equation was derived from the investigated parameter ranges considered in the orthogonal experimental design. Therefore, the fitted relationships should be interpreted as quantitative descriptions of the observed trends within the investigated parameter space, and their applicability beyond these ranges requires further verification.

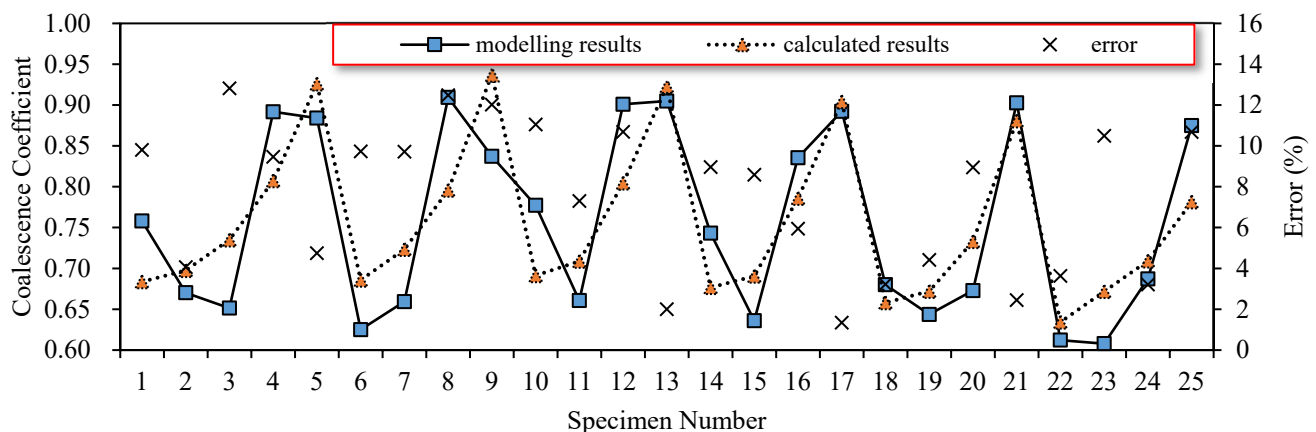


Figure 19. Curve of coalescence coefficient, changing with factors.

4. Engineering Applications

4.1. Engineering Model and Structural Plane Information

In the present study, a tunnel (Fantun Tunnel, located in Liaoning Province, and Horseshoe tunnel) is used as an illustrative example to demonstrate the effectiveness of the proposed method. The tunnel section's size is shown in Figure 20.

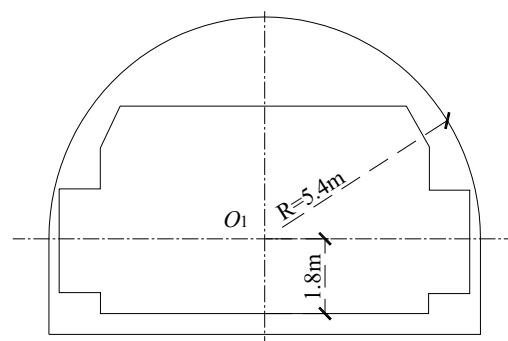


Figure 20. Section size of the tunnel.

A 2D plane strain cross-section of tunnel is modeled numerically using the software GeoSMA-3D. In order to reduce the boundary effect, the tunnel is modeled with width $W = 50$ m; height $H = 50$ m; and length $L = 100$ m.

There are 30 pieces of structural surface information obtained by field measurement. The information obtained by calculating the non-coalescence structural plane is shown in Table 3.

Table 3. Information of the non-coalescence structure surface.

Group No.	Structural Plane Number	X/(m)	Y/(m)	Z/(m)	Tendency/(°)	Inclination/(°)	Disk Radius/(m)
1	2	0.3	−133	−1.3	68	88	50
	9	11.1	167.6	−1.6	73	88	50
2	3	6.8	−2.8	0.7	77	86	70
	9	11.1	167.6	−1.6	73	88	50
3	21	13.2	69.2	−2.8	215	75	50
	25	−1.7	−36	−3.3	215	70	40
4	21	13.2	69.2	−2.8	215	75	50
	28	3.2	−6	−2.8	215	75	50
5	22	1.3	−65.4	−1.9	180	80	100
	29	1.3	6	−1.9	180	80	100
6	23	−12.9	123.4	3.8	225	70	80
	30	−2.9	13	3.8	225	70	80
7	25	−1.7	−36	−3.3	215	70	40
	28	3.2	−6	−2.8	215	75	50

4.2. Structural Plane Treatment and Coalescence Judgment

A local coordinate system (x' , y' , z') is obtained by rotating the three-dimensional Cartesian coordinate system along the z -axis and making the y -axis coincide with the structural plane (rotation angle $\theta = 90^\circ$ –inclination). The new coordinate system is then rotated along y' (rotation angle is the inclination angle of the structural plane) to get a quadratic transformation coordinate system (x'' , y'' , z''). After two coordinate transformations, the complex spatial problem can be transformed into a simple geometric problem.

After the coordinate transformation, we can use Formulas (11) and (12) to calculate the inclination angle of the rock bridge and the rock bridge length of the non-coalescence structure plane.

$$\beta = \frac{\arccos\left(\frac{|z''_B - z''_A|}{\sqrt{(x''_B - x''_A)^2 + (y''_B - y''_A)^2}}\right)}{\pi} \times 180 \quad (11)$$

$$d = \sqrt{(x''_A - x''_B)^2 + (y''_A - y''_B)^2 + (z''_A - z''_B)^2} \quad (12)$$

where the following hold:

β is the inclination angle of the rock bridge of the non-coalescence structural surface;

d is the rock bridge length of the non-coalescence structural planes (as presented in Figure 1);

(x''_A, y''_A, z''_A) and (x''_B, y''_B, z''_B) are the coordinates of A and B in Figure 21.

A and B are the nearest points on circle O''_1 and circle O''_2 .

The coalescence of non-coalescence structure surface is evaluated according to Equation (10). After calculation, we know that the seventh group of the non-coalescence structural plane will coalesce.

The newly generated facet information is shown in Table 4.

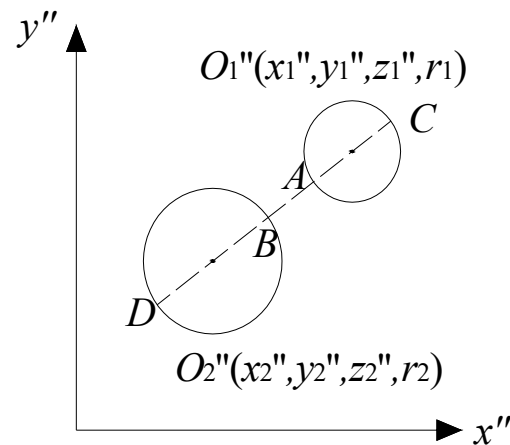


Figure 21. Schematic diagram of solution.

Table 4. The information of the new structure surface.

Structural Plane Number	X/(m)	Y/(m)	Z/(m)	Tendency/(°)	Inclination/(°)	Disk Radius/(m)
31	2.8	−23.5	−4.7	215	63.4	18.6

4.3. Engineering Stability Evaluation

In this paper, two cases were numerically simulated: one considers the structure plane coalescence, and another ignores the structural plane coalescence. The results of the spatial distribution of key blocks are as follows.

By comparing the two simulation results, it can be found that the two analyses provide a significant difference volume calculations of the key blocks (the volume of the key block highlighted in circles is 30.338 m³ for the first case, as shown in Figure 22a, and 0.703 m³ for the second case as depicted in Figure 22b).

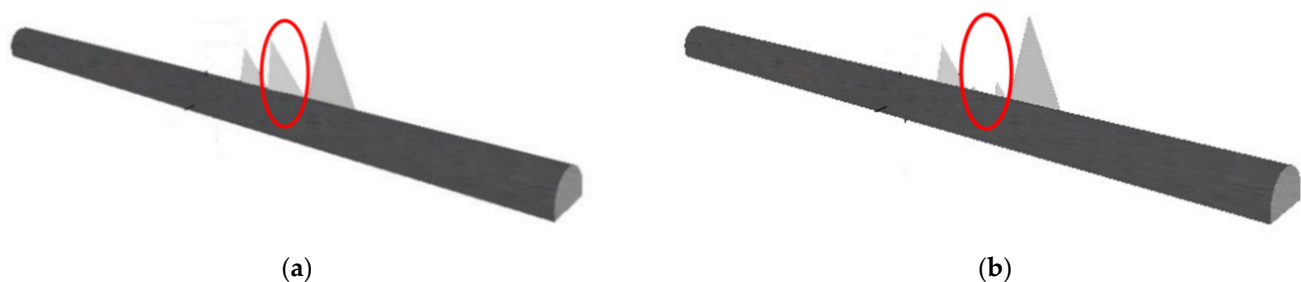


Figure 22. The distribution of key blocks in space: (a) considering coalescence; (b) without considering coalescence.

4.4. Applicability of the Proposed Method

The proposed coalescence-based stability assessment method is particularly suitable for jointed rock masses in which block stability is controlled by the geometric interaction and potential coalescence of discontinuities. This method is most effective for engineering scenarios such as tunnels, underground caverns, and rock slopes, where the spatial distribution of structural planes can be clearly identified and represented within the framework of block theory.

The present formulation was developed based on laboratory tests and numerical simulations conducted under static loading conditions. Therefore, it is primarily applicable to stability analyses involving pre-existing discontinuities subjected to quasi-static stress

environments. In addition, the method is most appropriate for rock masses with a limited number of dominant discontinuities, where potential coalescence between adjacent structural planes may significantly alter block geometry and stability.

It should be noted that the current method does not explicitly consider dynamic loading effects, hydraulic fracturing processes, thermal damage, freeze–thaw deterioration, or large-scale discrete fracture networks involving multiple simultaneous coalescence events. For such conditions, further modifications and validations of the coalescence coefficient are required before practical application.

5. Conclusions

(1) Within the investigated parameter ranges, with different α , the failure modes of specimens are very similar, and the peak stress exhibits an approximately quadratic polynomial relationship with α . With different β , the failure modes of specimens change a lot, and the peak stress exhibits an approximately quadratic polynomial relationship with β . With different l , the failure modes of specimens are quite similar, and the peak stress shows an approximately logarithmic relationship with l . With different d , when d reaches a certain size, there is no direct coalescence between the cracks, and the peak stress exhibits an approximately quadratic polynomial relationship with d .

(2) The random weighting method can achieve a heterogeneous distribution of the elastic modulus of rocks, enabling the heterogeneous simulation of mechanical parameters of rock materials. Through a study on scale effects, standard dimensions have been established for simulation research in this paper.

(3) Within the investigated parameter space, the coalescence coefficient exhibited cubic polynomial relationships with the crack dip angle, crack length, and rock bridge dip angle, and a linear relationship with the bridge length. An orthogonal test analysis of $L_{25} (5^4)$ was carried out. Through nonlinear regression analysis, the correlation between coalescence coefficient and four factors (including crack dip angle, bridge dip angle, crack length and bridge length) was proposed, providing a basis for correcting the coalescence coefficient for engineering design.

(4) A tunnel is used as an illustrative example to demonstrate the effectiveness of the proposed method. Based on the field data, the coordinate transformation method is used to determine whether the structural planes of the tunnel show non-coalescence. A spatial calibration of the new structure plane is carried out. The software GeoSMA-3D is used for stability analysis to compare with calculations using the proposed modified coalescence-coefficient. By comparing the simulation results, it has been observed that utilizing the modified coalescence-coefficient provides a more realistic assessment of crack coalescence and rock mass stability.

Future studies considering more complex geological conditions and additional engineering applications may further enhance the applicability and engineering value of the proposed coalescence coefficient.

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