

Article

Analysis of Slope Stability in Clayey Silt Under Steady-State Seepage and Research on Reinforcement with Anchor Plates

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Abstract

Clayey silt slopes are highly susceptible to instability under steady-state seepage, often exhibiting precursors to failure such as cracking at the slope crest. To address this cracking phenomenon under steady-state seepage conditions, a mechanical model characterizing the stability of clayey silt slopes was established. A MATLAB-based program was developed to implement the exhaustive method and derive the optimal solution. This study systematically analyzed the impacts of seepage-related parameters—including air entry value and pore size parameters—on slope stability and further optimized the design parameters of anchor plate support. Key research findings reveal that both the air entry value $1/\alpha$ and pore size parameter n exert significant influences on the stability of three-dimensional clayey silt slopes under seepage conditions. Specifically, when the air entry value $1/\alpha$ reaches 10 kPa, the slope safety factor reaches its minimum, rendering the slope most vulnerable to instability. Additionally, both the infiltration rate q and permeability coefficient k_s can alter the pore water pressure distribution within the slope, thereby affecting its stability, with the magnitude of this effect varying substantially across different scenarios. Considering both slope safety and project costs, the optimal embedment depth ratio for single-row anchor plates should be $K_T = 0.8$. Additionally, the stability of three-dimensional slopes can be enhanced by improving infiltration capacity and reducing slope angles. These findings provide theoretical guidance for anchor plate support in similar three-dimensional clayey silt slopes.

Keywords: unsaturated soil; steady-state seepage; tensile strength cut-off; anchor plate; three-dimensional slope



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1. Introduction

Rainfall, as a primary trigger for slope instability, is characterized by its sudden onset and severe consequences [1]. For example, in 2025, heavy rainfall in Yibin, Sichuan, caused a landslide that resulted in 2 injuries, 10 fatalities, and damage to over 100 hectares of crops. Slopes composed of clayey silt are particularly prone to instability under the influence of rainfall seepage [2,3]. Consequently, when slopes are subjected to complex rainfall conditions, seepage often triggers cracking at the crest, significantly increasing the risk of slope instability and posing a serious threat to human life [4–6]. Anchor plates, as flexible anchoring structures, form a flexible slope support system when combined with frame-beam support, effectively enhancing slope stability [7]. Therefore, conducting research

on the reinforcement of three-dimensional clayey silt slopes using anchor plates under steady-state seepage conditions holds significant theoretical and practical value.

Research on slope stability has been active both domestically and internationally. Methods for studying slope stability include model testing [8], theoretical analysis [9], and numerical simulation [10], among which limit analysis is a commonly used theoretical method. Zhang et al. [11] systematically examined the coupled effects of saturation-induced degradation and pore water pressure, incorporated the saturation-weakened rock mass strength into the Hoek–Brown failure criterion, and investigated the failure mechanism of rock slopes under pore water pressure using a three-dimensional horn-shaped failure model. Gao et al. [12] took the Erdaogou Landslide as a case study. When accounting for the spatial variability of geotechnical parameters, the coefficient of variation was found to be the primary factor governing the failure probability. In addition, the pile top displacement and maximum bending moment were significantly larger than the results from deterministic analysis. Li et al. [13] employed the upper-bound theorem of three-dimensional limit analysis to establish an innovative stability assessment framework for heterogeneous soil terraced slopes. In their research, the vertical spatial variability of soil properties and the topographic characteristics of terraces were explicitly integrated into the modeling framework. By implementing an optimization algorithm, the optimal upper-bound solution corresponding to the slope safety factor was successfully obtained. Drucker [14] initially introduced the concept of tensile cut-off, which restricts the tensile strength of soil to zero, leading to a nonlinear strength envelope in the tensile soil mass. Michalowski [15] established a calculation framework for the tensile strength cut-off mechanism of two-dimensional slopes under the plane strain condition. Park [16] extended this research to three-dimensional slopes, enabling more accurate calculation of the slope stability factor. Recently, Li et al. [17] incorporated tensile strength reduction into the stability analysis of reinforced slopes under seepage conditions. Based on the kinematic approach of limit analysis and integrated with FLAC^{3D} seepage modeling, they employed an optimization algorithm to identify the optimal upper-bound solution for the safety factor. The aforementioned scholars have conducted in-depth investigations into slope stability issues and related problems concerning tensile strength reduction. Nevertheless, effective and feasible solutions remain unavailable for the instability failure induced by tensile stress concentration in the soil mass at the slope crest.

To address issues such as slope instability caused by tensile failure in soil, it is essential to implement effective reinforcement and support methods. Currently, a relatively mature system of slope reinforcement methods has been established in the engineering field. Commonly used reinforcement methods primarily include anti-slip pile reinforcement [18], anchor bolt reinforcement [19], and reinforced soil reinforcement [20]. Zhang et al. [21] verified through laboratory tests and on-site monitoring that wheat straw and bamboo fibers, serving as environmentally benign reinforcing agents, are capable of effectively improving soil's mechanical characteristics and enhancing the anti-sliding stability of slopes, while additionally offering environmental benefits including cost-effectiveness and biodegradability; Zhou et al. [22] addressed slope safety risks induced by tensile failure occurring in the soil at the slope crest and carried out a comprehensive investigation into the working performance of anti-slip piles as well as the evolution of slope stability after the initiation of tensile cracks at the crest. They analyzed in detail the mechanisms by which factors such as crack location and crack length affect the lateral stress distribution within the piles, pile deformation, and the slope safety factor, thereby providing technical support for the design of anti-slip pile reinforcement for cracked slopes. Although the aforementioned scholars have verified the effectiveness of various traditional reinforcement materials, they have generally overlooked the tensile strength failure of soil mass at the slope crest under

steady infiltration conditions. The conventional Mohr–Coulomb criterion defines the tensile capacity of soil by directly extending the strength properties of the compressive region to the tensile region, which is inconsistent with real engineering scenarios. Accordingly, the traditional Mohr–Coulomb strength criterion needs to be modified to capture the actual rainfall-induced failure mechanism.

In summary, the research conducted by the aforementioned scholars has systematically addressed slope stability and support design, yielding fruitful results. However, under the influence of rainfall infiltration, slopes composed of clayey silt are prone to tensile stress concentration at the crest. The use of traditional Mohr–Coulomb strength criteria typically exaggerates the tensile stress in the soil, leading to discrepancies between calculated results and actual engineering conditions, and there is a lack of appropriate reinforcement design methods. Therefore, based on the upper bound theorem of limit analysis and considering tensile failure of the soil at the slope crest, this paper constructs a mechanical stability model for clayey silt slopes under steady-state seepage. Anchor plate support is adopted as the retaining structure, and its parameters are optimized to provide theoretical guidance for future anchor plate support applications in slope engineering.

2. Theoretical Basis

2.1. Tensile Strength Failure

In slope stability analysis, soil strength follows the Mohr–Coulomb (M-C) yield criterion only under compressive conditions; its tensile strength is derived by directly extending this criterion to the tensile zone. However, in engineering practice, soil mass does not possess the capacity to withstand such high tensile stresses. Its actual tensile behavior is far inferior to theoretical expectations, and its tensile strength is nearly zero. Consequently, to partially compensate for this shortcoming, Michalowski [15] established a tensile strength cut-off mechanism, which centers on revising the conventional Mohr–Coulomb criterion so as to control the tensile strength of soil.

As shown in Figure 1, in Mohr's stress space, the line segment representing the M-C criterion is denoted by OA . In Figure 1a, the Mohr stress circles C_1 and C_2 are associated with the uniaxial tensile strength and uniaxial compressive strength, respectively. The triaxial tensile strength is denoted by point O on circle C_1 , with the expression $f_{3t}^{M-C} = c \cdot \cot \varphi$, while the uniaxial tensile strength is characterized by point P on the same circle, with the expression $f_t^{M-C} = 2c \cos \varphi / (1 + \sin \varphi)$. In Figure 1b, the reformulated Mohr stress circle C_1 is utilized to depict the actual uniaxial tensile strength f_t , yielding an extra stress circle C_3 . This circle makes tangential contact with circle C_1 at point P and is tangent to the linear M-C failure envelope at point S . The line segment PS forms a segment of the strength envelope, replacing the original segment OS . Correspondingly, the modified yield criterion consists of two parts: the nonlinear segment PS and the linear segment SA , ensuring that the tensile stress at any point does not exceed the actual tensile strength. It is worth noting that the full tensile strength scenario in Figure 1a and the zero tensile strength scenario in Figure 1c can both be treated as boundary cases of the partial tensile strength scenario depicted in Figure 1b.

To account for the constraint of tensile strength, a dimensionless coefficient ξ is proposed to scale the actual uniaxial tensile capacity, expressed as

$$f_t = \xi f_t^{M-C} = \xi \cdot \frac{2c \cos \varphi}{1 + \sin \varphi} \quad (0 \leq \xi \leq 1) \quad (1)$$

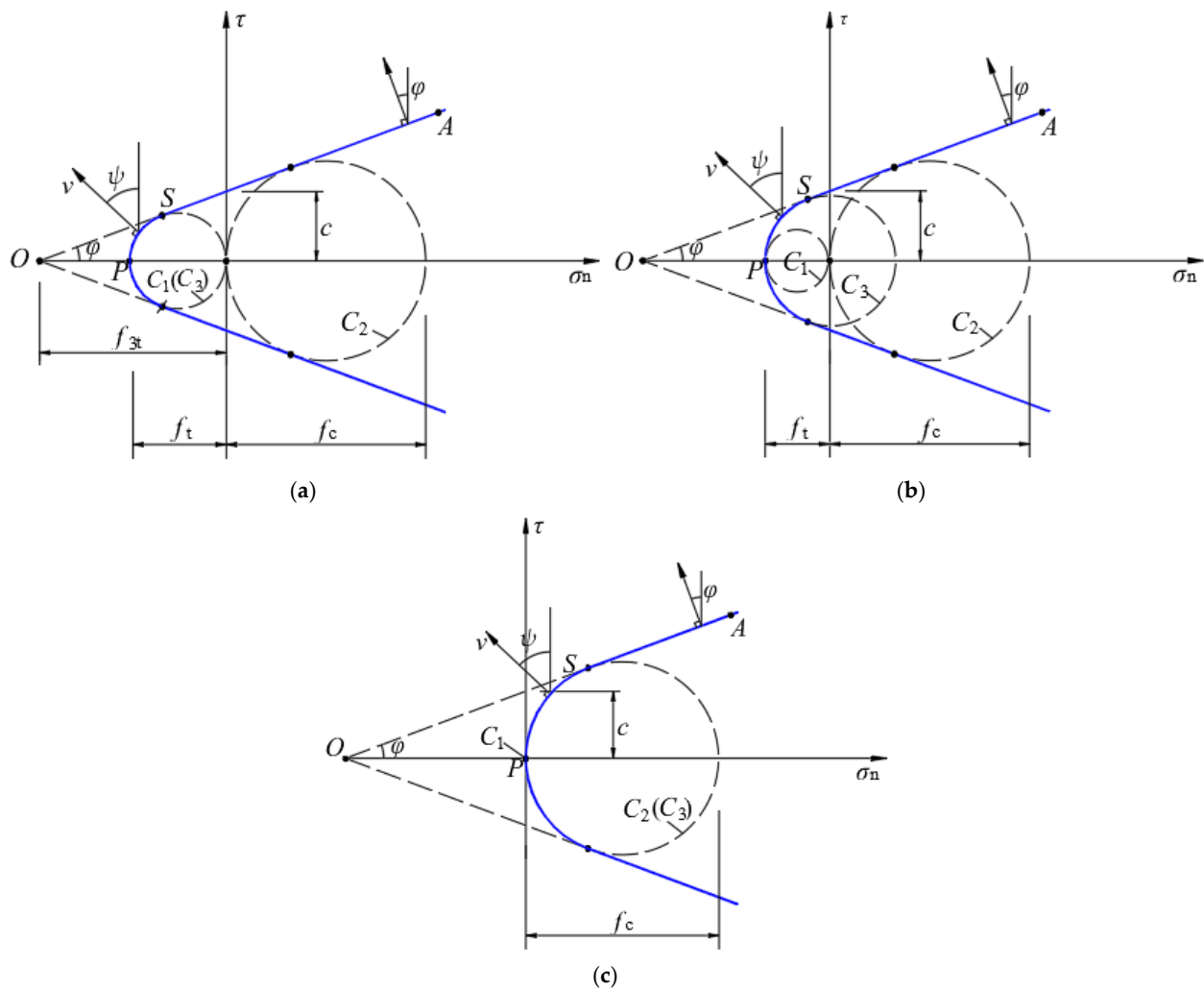


Figure 1. Revised Strength Envelope: (a) $\xi = 1$; (b) $0 < \xi < 1$; (c) $\xi = 0$.

The deformation behavior of the damaged region, which conforms to the tensile stress failure trajectory specified by the M-C criterion, can be characterized via the tensile truncation segment of this criterion. The per-unit-area internal energy dissipation rate, denoted as d_c , resulting from the cohesion c , is formulated as

$$d_c = v \cdot \left(f_c \frac{1 - \sin \psi}{2} + f_t \frac{\sin \psi - \sin \varphi}{1 - \sin \varphi} \right) \quad (2)$$

Here, v indicates the speed magnitude, f_c corresponds to the uniaxial tensile strength as defined in the M-C criterion function, and ψ is the dilatancy angle. Substituting Equation (2) into Equation (1) yields the expression

$$d_c = cv \cdot \left(\cos \varphi \frac{1 - \sin \psi}{1 - \sin \varphi} + 2\xi \frac{\sin \psi - \sin \varphi}{\cos \varphi} \right) \quad (3)$$

In situations where the linear segment of the strength envelope controls the failure mechanism, the stress state follows $\psi \equiv \varphi$. Accordingly, Equation (3) can be recast into a streamlined expression that is commonly adopted in engineering practice to evaluate the internal energy dissipation rate arising from the soil cohesion c , as presented below:

$$dc = cv \cos \varphi \quad (4)$$

2.2. Suction Stress and Apparent Cohesion Under Conditions of Steady Seepage

Drawing on prior research regarding the matric suction soil–water characteristic curve, Lu et al. [23] put forward a formulation for the effective stress in unsaturated soils under steady seepage conditions as follows:

$$\sigma' = \sigma - u_a - \sigma^s \quad (5)$$

where σ' denotes effective stress; σ represents total stress; u_a signifies pore air pressure; σ^s indicates suction stress.

Vahedifard et al. [24] derived the closed-form expression for the soil suction stress σ^s :

$$\sigma^s = -(u_a - u_w) \quad u_a - u_w \leq 0 \quad (6)$$

$$\sigma^s = -\frac{u_a - u_w}{\{1 + [\alpha(u_a - u_w)]^n\}^{(n-1)/n}} \quad u_a - u_w > 0 \quad (7)$$

In this context, u_w refers to pore water pressure, $(u_a - u_w)$ characterizes matric suction, and α and n are parameters describing the pore size distribution, where $1/\alpha$ stands for the air entry value and n is the pore size distribution index. This expression applies to saturated soils when matric suction $(u_a - u_w) \leq 0$ and to unsaturated soils when matric suction $(u_a - u_w) > 0$.

By incorporating the generalized Darcy's law and liquid-phase continuity conditions, Vahedifard et al. [25] derived an expression for the distribution of matric suction along the groundwater table in unsaturated soils:

$$u_a - u_w = -\frac{1}{\alpha} \ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha z} - \frac{q}{k_s} \right] \quad (8)$$

Here, q represents the infiltration volume per unit time, known as the infiltration rate. A positive value of $q > 0$ corresponds to the evaporation condition, a negative value of $q < 0$ corresponds to the infiltration condition, and $q = 0$ denotes the non-seepage condition; k_s represents the permeability coefficient of saturated soil; γ_w signifies the specific weight of water; and z indicates the vertical distance from any point within the slope above the groundwater table to the groundwater table itself.

Substituting Equation (8) into Equation (7) yields the analytical expression for the suction stress in unsaturated soil under steady-state seepage conditions:

$$\sigma^s = \frac{1}{\alpha} \frac{\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha z} - \frac{q}{k_s} \right]}{\left\{ 1 + \left\{ -\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha z} - \frac{q}{k_s} \right] \right\}^n \right\}^{(n-1)/n}} \quad (9)$$

By substituting Equation (9) into Equation (6), the shear strength and apparent cohesion of unsaturated soil under steady seepage can be derived in accordance with the effective stress-based M-C strength criterion. Their expressions are as follows [26]:

$$\tau_f = c + (\sigma - u_a) \tan \varphi + c_{app} \quad (10)$$

$$c_{app} = -\sigma^s \tan \varphi \quad (11)$$

Among these, τ_f denotes the shear strength of unsaturated soil under steady seepage conditions. c and φ represent the effective cohesion and effective internal friction angle, respectively. $\sigma - u_a$ is the effective stress perpendicular to the failure plane; c_{app} is the apparent cohesion induced by matric suction.

2.3. Ultimate Pull-Out Force of Anchor Plates

As an economical, efficient, and highly adaptable method of slope stabilization, anchor plate support involves embedding anchor plates deep into stable soil layers or rock within the slope by leveraging the frictional resistance at the interface between the anchor plates and the surrounding soil, the soil's pull-out resistance, and the structural strength of the anchor plates themselves. This effectively transfers tensile and shear stresses generated within the slope to the stable strata, thereby constraining slope deformation and suppressing adverse phenomena such as tensile failure and sliding instability. It is particularly suitable for slopes composed of materials such as clayey silty soils, which have low tensile strength and are easily affected by rainfall and seepage. As shown in Figure 2, when examining the effectiveness of anchor plate-reinforced slopes, Huang et al. [27] took into account the unsaturated nature of the soil. They conducted research on frame-anchor-plate-supported slopes, elucidated the force-bearing mechanism of the anchoring system, and derived and verified the expression for the anchor plate anchoring force T . This implies that the anchoring force is determined by the smaller value between the ultimate pull-out resistance of the anchor plate T_p and the pull-out resistance of the anchor rod T_s , given by

$$T = \min\{T_p, T_s\} \quad (12)$$

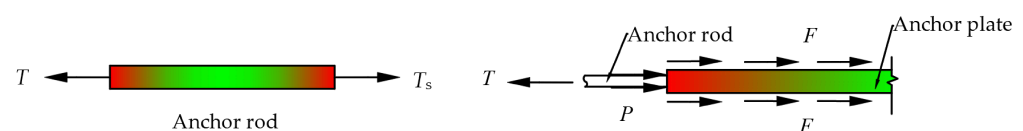


Figure 2. Anchoring Force Mechanism.

For an anchor rod, its maximum pull-out capacity T_s is governed by the material yield strength f_y and the cross-sectional area of the rod, and can be formulated as

$$T_s = A_s f_y = (\pi/4) D^2 f_y \quad (13)$$

The symbol D in the equation stands for the diameter of the reinforcing bar in the anchor rod.

The ultimate pull-out resistance T_p of an anchor plate is derived from two components: the frictional force F generated along its side interfaces with the surrounding soil, and the resistance P provided by the soil at its front end, expressed as

$$T_p = F + P \quad (14)$$

The expression for computing the friction force F is given as follows:

$$F = 2\tau_f L_a (L_b + L_h) \quad (15)$$

$$\tau_f = R_{\text{inter}} (c + \gamma Z \tan \varphi) \quad (16)$$

In this expression, τ_f denotes the ultimate shear strength at the interface of the anchor plate. The symbols L_a , L_b , and L_h correspond to the length, width, and thickness of the anchor plate, respectively. R_{inter} represents the interface strength reduction factor, while Z signifies the embedment depth of the anchor plate.

Since the anchor plate causes compressive deformation in the soil ahead of it as it is pulled out, thereby compacting the soil and increasing the anchor plate's resistance to pull-out, the soil resistance P can be expressed as [28]

$$P = \frac{(1-k)K_0K_p\gamma Z + 2c\sqrt{K_p}}{1-kK_p}L_bL_h \quad (17)$$

In the equation, k is the lateral pressure coefficient; K_0 denotes the coefficient of static earth pressure, $K_0 = 1 - \sin \varphi$; and K_p is the coefficient of passive earth pressure, $K_p = \tan^2(45^\circ + \varphi/2)$.

3. Three-Dimensional Rotational Failure Mechanism

In conventional slope stability analysis, the problem is typically simplified to a plane strain problem, i.e., the slope is assumed to have a unit length along its longitudinal axis. However, when the slope has a finite longitudinal length and exhibits significant three-dimensional mechanical effects, continuing to use the plane strain assumption will lead to computational errors, often resulting in an underestimation of the slope's stability. In light of this, this paper introduces a tensile strength failure criterion and establishes a three-dimensional stability analysis model for anchor-plate-reinforced slopes under steady-state seepage.

In view of this, this paper introduces the tensile strength failure criterion and establishes a three-dimensional stability analysis model for anchor-plate-reinforced slopes under steady-state seepage. As shown in Figure 3, the slope crest is AE , the slope surface is ED , the slope height is H , and the slope angle is β . Under the actions of rainfall and soil self-weight, the slope undergoes sliding failure. To describe the geometric shape of the failure mass, a polar coordinate system (θ, r) is established with the rotation center O as the pole. The rotating failure mass rotates around O at an angular velocity ω , intersecting the slope crest and the slope toe at points A and D , respectively; the actual failure shape of the slope is ADE . The angles between OA , OE , OD and the horizontal plane are denoted as θ_0 , θ_E , θ_h , respectively, where $OA = r_0$ and $OD = r_h$. The failure mass is composed of two logarithmic spirals intersecting at point P with an included angle of 2φ , and their expressions are $r = r_0e^{(\theta-\theta_0)\tan\varphi}$ and $r' = r_0'e^{-(\theta-\theta_0)\tan\varphi}$. For any point on the failure surface AD , the angle with the horizontal direction is θ , its linear velocity is v , and the angle between the velocity and the tangent is φ , where φ is the internal friction angle.

When considering the tensile strength failure of the soil at the slope crest, the logarithmic spiral angles change, and tensile stress is mainly concentrated near the slope crest. In this case, the logarithmic spiral is no longer applicable and must be modified to account for the effect of tensile stress. Specifically, the vertex P moves to P' , and points A and A' move to points B and B' , respectively, where the angle between OB and the horizontal plane is θ_m . After modification, the included angle between the two logarithmic spirals of the failure mass is 2ψ . Under the modified mechanism considering tensile strength failure at the slope crest, the dilatancy angle ψ is no longer equal to the internal friction angle but varies linearly along the failure surface, reaching its maximum value at the upper boundary point B , denoted as ψ_m . Point C is the common point connecting the upper and lower parts of the failure mass, and the angle between OC and the horizontal plane is θ_{tc} . At this time, the upper part of the failure mass (segments $P'C$ and $P'C'$) is governed by the nonlinear segment PS of the tensile cut-off envelope shown in Figure 1b, while the remaining parts $C'D'$ and CD are still controlled by the linear Mohr–Coulomb (M-C) segment of the envelope [29].

With the growth of the slope's lateral extent, the failure pattern progressively shifts from a three-dimensional failure mechanism toward a two-dimensional counterpart. To characterize this evolutionary trend, a block of width b is integrated into the symmetry plane of the failure mechanism, as depicted in Figure 4b. Under this scenario, the formula describing the total width B of the sliding block is expressed as

$$B = b_0 + b \quad (21)$$

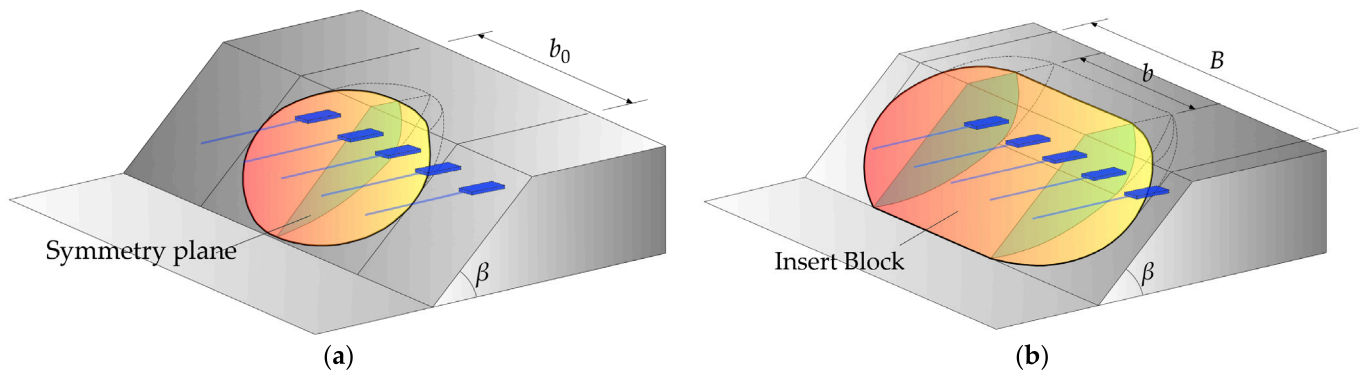


Figure 4. Schematic Diagram of Three-Dimensional Slope Model: (a) symmetry plane; (b) insert block b .

4. Analytical Solutions

4.1. Basic Assumption

In this paper, the following assumptions are adopted in the analysis using the upper bound method of limit analysis: (1) The soil of the clayey silt slope is an ideally elastoplastic material, which conforms to the Mohr–Coulomb strength criterion in the compression zone and is modified by the tensile strength cut-off mechanism in the tension zone; the soil follows the associated flow rule. (2) The failure mass of the clayey silt slope in Figure 3 is regarded as a rigid block, and its volume remains constant during the failure process. (3) The seepage inside the slope is steady-state seepage. The matric suction distribution in unsaturated soil conforms to the generalized Darcy's law and the liquid-phase continuity condition, and the pore water pressure is stably distributed along the slope. (4) The anchor plate is assumed as a rigid structure, and its own deformation is neglected.

4.2. Calculation of Gravitational Work in Sliding Earth Masses

As shown in Figure 4, the gravitational work of the sliding mass consists of two components: one is the work $\dot{W}_{\gamma\text{-horn}}$ exerted by the overlapping soil mass between the sliding body and the slope (Figure 4a), and the other is the work $\dot{W}_{\gamma\text{-insert}}$ exerted by the insert block (Figure 4b).

Work $\dot{W}_{\gamma\text{-horn}}$ exerted by the sliding body and the overlying soil mass on the slope is

$$\begin{aligned} \dot{W}_{\gamma\text{-horn}} &= 2\omega\gamma \left[\int_{\theta_m}^{\theta_E} \int_{d_1}^R \int_0^{x^*} (r_c + y)^2 \cos\theta dx dy d\theta \right. \\ &\quad \left. + \int_{\theta_E}^{\theta_h} \int_{d_2}^R \int_0^{x^*} (r_c + y)^2 \cos\theta dx dy d\theta \right] \\ &= \gamma\omega r_0^4 g_1 \end{aligned} \quad (22)$$

For the mathematical expressions of d_1 , d_2 , x^* and g_1 , see Appendix A.

The work $\dot{W}_{\gamma\text{-insert}}$ exerted by the weight of the insert block is

$$\dot{W}_{\gamma\text{-insert}} = \gamma \omega r_0^4 g_2 \quad (23)$$

In the equation, the expression for the g_2 function is provided in Appendix A.

The total work $\dot{W}_{\gamma\text{-3D}}$ generated by the gravitational force of the sliding soil mass in a three-dimensional slope is

$$\dot{W}_{\gamma\text{-3D}} = \dot{W}_{\gamma\text{-horn}} + \dot{W}_{\gamma\text{-insert}} \quad (24)$$

4.3. Internal Energy Dissipation Rate Calculation

(1) Sliding Soil Internal Energy Dissipation Rate

Since internal energy dissipation in slope soil occurs along the slip discontinuity BCD , the overall internal energy dissipation rate along this interface can be derived from the product of the differential area of the discontinuity, the soil's effective cohesion c , the apparent cohesion c_{app} , and the tangential velocity component $v \cos \varphi$ across the discontinuity. Consequently, the internal energy dissipation rate $D_{\text{s-horn}}$ associated with the sliding mass and overlying slope soil is formulated as

$$\begin{aligned} D_{\text{s-horn}} &= \int_S (c + c_{\text{app-horn}}) v \cos \varphi ds \\ &= 2\omega (c + c_{\text{app-horn}}) \int_{\theta_m}^{\theta_E} \int_0^{\alpha_1} f_{10} \cdot R(r_c + R \cos \alpha)^2 d\alpha d\theta \\ &\quad + 2\omega (c + c_{\text{app-horn}}) \int_{\theta_E}^{\theta_h} \int_0^{\alpha_2} f_{10} \cdot R(r_c + R \cos \alpha)^2 d\alpha d\theta \\ &= \omega (c + c_{\text{app-horn}}) r_0^3 g_3 \end{aligned} \quad (25)$$

For the mathematical expressions of α_1 , α_2 , f_{10} and g_3 , see Appendix A.

Rate of dissipation of internal energy generated by the insertion of a block $D_{\text{s-insert}}$:

$$D_{\text{s-insert}} = b \int_{\theta_m}^{\theta_h} (c + c_{\text{app-insert}}) v \cos \varphi \frac{r(\theta)}{\cos \varphi} d\theta = \omega (c + c_{\text{app-insert}}) r_0^3 g_4 \quad (26)$$

In the formula, z_0 denotes the vertical distance from the groundwater level to the toe of the slope. $c_{\text{app-horn}}$ and $c_{\text{app-insert}}$ represent the apparent cohesion induced by matric suction in the horn-shaped failure mass and the insert block, respectively. For details on the $c_{\text{app-horn}}$, $c_{\text{app-insert}}$, and g_4 expressions, see Appendix A, where $Z_{\text{3D-horn}}$ and $Z_{\text{3D-insert}}$ are the vertical distances from any point within the failure mass above the groundwater table to the groundwater table and from any point within the insert block above the groundwater table to the groundwater table, respectively.

The total internal energy dissipation of the sliding soil mass can be expressed as

$$D_{\text{T1}} = D_{\text{s-horn}} + D_{\text{s-insert}} \quad (27)$$

(2) Internal energy dissipation rate generated by anchor plates

As shown in Figure 3, for a three-dimensional slope reinforced with a single row of anchor plates, the radial distances from rotational center O to points F and K are defined as r_T and r'_T , respectively, with their corresponding polar angles designated as θ_T and θ'_T . X_T represents the horizontal distance from the anchor head to the slope toe, and L_X

corresponds to the horizontal projection length of the slope surface. The polar angle θ_T associated with point F can be derived as follows:

$$\theta_T = \arctan \frac{\sin \theta_h e^{(\theta_h - \theta_0) \tan \varphi} - \frac{X_T}{L_X} \frac{H}{r_0}}{\cos \theta_0 - \frac{L}{r_0} - \left(1 - \left(\frac{X_T}{L_X}\right)\right) \cdot \frac{H}{r_0} \cos \beta} \quad (28)$$

The internal energy dissipation associated with an individual anchor plate is given by [30]

$$dD_a = T v \tan \delta d\theta_T \quad (29)$$

In the equation, δ is the angle between the sliding surface velocity and the anchor plate, $\delta = \pi/2 - \theta_T$. Figure 3 indicates that, whether the force T is considered to act at surface point F or at a point on the sliding surface, the lever arms A and B corresponding to T are equal [31]; therefore, this paper assumes that the anchorage force T acts only on point F on the slope, thereby simplifying the calculation process. In this case, r_T can be expressed as

$$r_T = r_0 \frac{\sin(\theta_h + \beta)}{\sin(\theta_T + \beta)} e^{[(\theta_h - \theta_0) \tan \varphi]} \quad (30)$$

For the overlapping region between the sliding mass and the slope, the energy dissipation rate arising from the reactive force of the anchor plate can be formulated as D_{T-3D} :

$$D_{T-3D} = \omega N' Tr_T \sin \theta_T \quad (31)$$

In the equation, $N' = \left[2r_0 \sqrt{R^2 - d_2^2}\right] / S$ and S represent the lateral spacing of the anchor plates.

For the embedded section of the sliding block, the energy dissipation rate attributed to the resistance provided by the anchor plate is given by $D_{T-insert}$:

$$D_{T-insert} = \omega N Tr_T \sin \theta_T \quad (32)$$

In the equation, $N = b/S$.

The overall energy dissipation contributed by a single row of anchor plates can be expressed as

$$D_{T2} = D_{T-3D} + D_{T-insert} = \omega (N' + N) Tr_T \sin \theta_T \quad (33)$$

4.4. Safety Factor

On the basis of the work balance equation, during the entire evolution of slope instability and sliding, the external work applied to the slope soil mass corresponds to the internal energy dissipation rate generated by the deformation occurring within the soil mass throughout sliding, expressed as

$$\dot{W}_{\gamma-horn} + \dot{W}_{\gamma-insert} = D_{T1} + D_{T2} \quad (34)$$

Under steady-state seepage conditions, let F_{S1} denote the safety factor for a slope without anchor plate support. The expression for F_{S1} is

$$F_{S1} = \frac{D_{T1}}{\dot{W}_{\gamma-horn} + \dot{W}_{\gamma-insert}} \quad (35)$$

Using the Generalized Induction Method (GIM), the safety factor for an unsaturated soil slope supported by anchor plates is defined as F_S , where the expression for F_S is as follows:

$$F_S = \frac{D_{T1} + D_{T2}}{\dot{W}_{\gamma\text{-horn}} + \dot{W}_{\gamma\text{-insert}}} \quad (36)$$

Equation (36) can be expressed as $F_S = f(\theta_0, \theta_h, r_0)$. This is a multivariate implicit equation that admits infinitely many solutions, and the optimal solution must be sought subject to the constraints given by Equation (37). In this paper, an exhaustive search method is adopted, and a program is coded in MATLAB (2024) to obtain the optimal solution.

$$\begin{cases} 0 < \theta_0 < \theta_m < \theta_{tc} < \theta_h < \pi \\ 0 < r'_0/r_0 < 1 \\ 0 < (b_0 + b)/H < B/H \end{cases} \quad (37)$$

5. Comparison and Verification

5.1. Comparison 1

Michalowski and Drescher [32] calculated the stability number N_s of three-dimensional slopes using the upper bound method of limit analysis under non-seepage conditions. In this study, the tensile failure of soil at the slope crest and the reinforcing effect of anchor plates are neglected. Parameters identical to those in Michalowski and Drescher [32] ($c = 12$ kPa, $\varphi = 15^\circ$, $\gamma = 20$ kN/m³) are adopted, and seepage conditions are simplified to non-seepage conditions. On this basis, the stability numbers of three-dimensional slopes are computed for various slope angles and width-to-height ratios. As presented in Table 1, when compared with the results from Michalowski and Drescher [32], the maximum relative error is 2.7% and the minimum relative error is 0, with generally consistent outcomes. This demonstrates the feasibility of the calculation method employed in this paper.

Table 1. Comparison of research results under non-seepage conditions.

N	$\beta/(\circ)$	B/H	N_s		Relative Difference (Δ_1)/% ¹
			Michalowski and Drescher [32]	This Article	
1	30	2	28.911	28.424	1.69
2	30	3	25.693	25.692	0
3	30	5	23.843	23.931	0.37
4	30	10	22.691	22.805	0.5
5	45	2	14.875	14.771	0.7
6	45	3	13.731	13.717	0.1
7	45	5	12.984	12.991	0.05
8	45	10	12.494	12.509	0.12
9	60	2	10.527	10.396	1.24
10	60	3	9.748	9.742	0.06
11	60	5	9.266	9.278	0.13
12	60	10	8.927	8.956	0.33
13	75	2	8.167	7.947	2.7
14	75	3	7.502	7.438	0.86
15	75	5	7.077	7.076	0.02
16	75	10	6.809	6.819	0.15

¹ $\Delta_1 = \left| \frac{F_S(\text{This article}) - F_S(\text{Michalowski and Drescher})}{F_S(\text{Michalowski and Drescher})} \right| \times 100\%$.

5.2. Comparison 2

Concerning the stability assessment of three-dimensional slopes under seepage influences, Huang [33] incorporated the frame-beam anchor plate support system into the upper-bound theorem of limit analysis, proposing an innovative upper-bound solution methodology for slope stability evaluation. In the present work, by assuming no tensile failure at the slope crest and keeping all other parameters unchanged, the computed outcomes were compared with those of Huang [33]. As depicted in Figure 5, the evolution trend of the safety factor F_S is largely in line with that documented by Huang [33]. Further analysis based on Table 2 indicates that, relative to Huang [33], the maximum relative error in the calculation results of this study is 2.77%, while the minimum relative error is 0.03%. The two sets of results exhibit high consistency, thereby verifying the reliability of the conclusions drawn in this study.

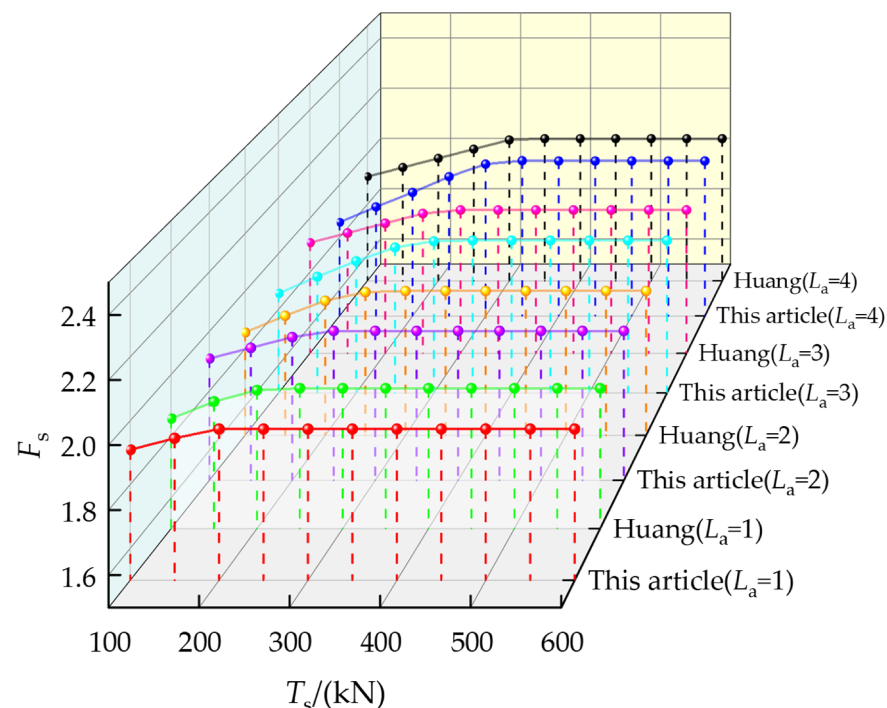


Figure 5. Safety factor comparison with Huang [33].

Table 2. Comparison of the results of this paper with existing findings.

N	L_a /(m)	T_s /(kN)	F_S		Relative Difference (Δ_2)/% ²
			Huang [33]	This Article	
1	1	100	1.857	1.909	2.77%
2	2	100	1.857	1.909	2.77%
3	3	100	1.857	1.909	2.77%
4	4	100	1.857	1.909	2.77%
5	1	200	1.949	1.974	1.27%
6	2	200	1.967	1.981	0.72%
7	3	200	1.973	1.981	0.43%
8	4	200	1.971	1.981	0.49%
9	1	300	1.955	1.974	0.94%
10	2	300	2.001	2.002	0.03%
11	3	300	2.044	2.030	0.71%
12	4	300	2.079	2.053	1.22%

Table 2. Cont.

N	$L_a/(m)$	$T_s/(kN)$	F_S		Relative Difference (Δ_2)/% ²
			Huang [33]	This Article	
13	1	400	1.955	1.974	0.94%
14	2	400	2.001	2.002	0.03%
15	3	400	2.047	2.030	0.83%
16	4	400	2.091	2.058	1.57%
17	1	500	1.955	1.974	0.94%
18	2	500	2.001	2.002	0.03%
19	3	500	2.047	2.030	0.83%
20	4	500	2.091	2.058	1.57%
21	1	600	1.955	1.974	0.94%
22	2	600	2.001	2.002	0.03%
23	3	600	2.047	2.030	0.83%
24	4	600	2.091	2.058	1.57%

$$^2 \Delta_2 = \left| \frac{F_S(\text{This article}) - F_S(\text{Huang})}{F_S(\text{Huang})} \right| \times 100\%.$$

6. Analysis of Results

6.1. Seepage Effect

In the presence of seepage flow, and accounting for the tensile-stress-induced cracking at the slope crest, this study analyzes the effects of different air entry value $1/\alpha$ and pore size parameter n on the stability of clayey silt slopes. The parameter values are as follows: $\gamma = 18 \text{ kN}\cdot\text{m}^{-3}$, $\gamma_w = 9.8 \text{ kN}\cdot\text{m}^{-3}$, $\varphi = 20^\circ$, $c = 20 \text{ kPa}$, $\beta = 60^\circ$, $B/H = 1$, $H = 10 \text{ m}$, $k_s = 1 \times 10^{-7} \text{ m}\cdot\text{s}^{-1}$, $q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$, and $z_0/H = 0$, $\zeta = 0$. As shown in Figure 6a, under the influence of different pore diameter parameters n , the three-dimensional slope safety factor increases with the air entry value. Furthermore, when the air entry value = 10 kPa, the slope safety factor F_{S1} is minimal, making the slope most prone to instability. As illustrated in Figure 6b, under varying air entry values $1/\alpha$, the safety factor of the three-dimensional slope displays a trend of initial decrease followed by gradual stabilization with respect to the pore size parameter n . Additionally, a larger air entry value $1/\alpha$ results in a larger safety factor F_{S1} and thus stronger slope stability.

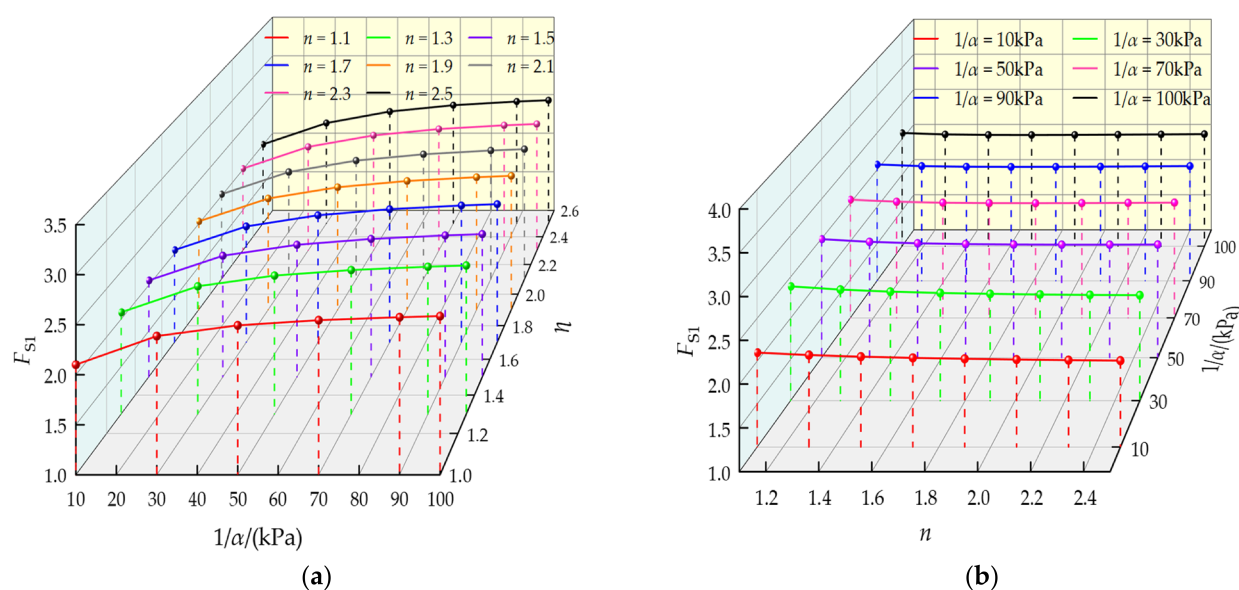


Figure 6. Effect of air entry value and aperture parameters on the safety factor under different conditions: (a) air entry value; (b) pore size parameter.

Further, with Table 3, it can be concluded that under the non-seepage condition ($q = 0$), taking the safety factor $F_{S1} = 2.020$ corresponding to the air entry value $1/\alpha = 10$ kPa as the reference, the safety factor F_{S1} increases to 2.176, 2.367, 2.468, 2.526 and 2.562, respectively, when the air entry value $1/\alpha$ rises to 20, 40, 60, 80 and 100 kPa. With the increase in the air entry value $1/\alpha$, the relative difference in the safety factor F_{S1} gradually decreases. When $1/\alpha$ exceeds 50 kPa, the relative differences Δ_3 between adjacent safety factors F_{S1} are 1.75%, 1.31%, 1.01%, 0.79% and 0.63% for each 10 kPa increment, all of which are less than 2%. Taking this as the judgment criterion, the safety factor F_{S1} can be regarded as tending to stabilize. Similarly, under the infiltration condition ($q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$), taking the safety factor $F_{S1} = 2.061$ at $1/\alpha = 10$ kPa as the reference, the safety factor F_{S1} increases to 2.278, 2.575, 2.749, 2.854 and 2.920, respectively, when $1/\alpha$ rises to 20, 40, 60, 80 and 100 kPa. With the increase of $1/\alpha$, the relative difference of F_{S1} gradually decreases. When $1/\alpha$ exceeds 70 kPa, the relative differences Δ_4 between adjacent safety factors F_{S1} are 1.62%, 1.26% and 1.00% for each 10 kPa increment, all of which are less than 2%. Taking this as the judgment criterion, the safety factor F_{S1} can be regarded as tending to stabilize. It can thus be seen that under steady-state infiltration conditions, a larger air entry value and a smaller pore size parameter led to greater matric suction in the slope soil, which strengthens the self-stability capacity of the slope.

Table 3. Effect of air entry value on Slope Safety Factor.

N	$1/\alpha/(\text{kPa})$	$q/(\text{m}\cdot\text{s}^{-1})$	F_{S1}	Relative Difference (Δ_3)/% ³	$1/\alpha/(\text{kPa})$	$q/(\text{m}\cdot\text{s}^{-1})$	F_{S1}	Relative Difference (Δ_4)/% ³
1	10	0	2.020		10	-3.14×10^{-8}	2.061	
2	20	0	2.176	7.18%	20	-3.14×10^{-8}	2.278	9.50%
3	30	0	2.288	4.86%	30	-3.14×10^{-8}	2.446	6.88%
4	40	0	2.367	3.35%	40	-3.14×10^{-8}	2.575	4.99%
5	50	0	2.425	2.38%	50	-3.14×10^{-8}	2.673	3.69%
6	60	0	2.468	1.75%	60	-3.14×10^{-8}	2.749	2.76%
7	70	0	2.501	1.31%	70	-3.14×10^{-8}	2.808	2.10%
8	80	0	2.526	1.01%	80	-3.14×10^{-8}	2.854	1.62%
9	90	0	2.546	0.79%	90	-3.14×10^{-8}	2.891	1.26%
10	100	0	2.562	0.63%	100	-3.14×10^{-8}	2.920	1.00%

$$^3 \Delta_3 = \Delta_4 = \left| \frac{F_{S1}(N=N_0) - F_{S1}(N=N_0+1)}{F_{S1}(N=N_0)} \right| \times 100\%.$$

Figure 7 examines the influences of infiltration rate q and permeability coefficient k_s on the stability of a clayey silt slope under diverse scenarios. The parameter values are as follows: $\gamma = 18 \text{ kN}\cdot\text{m}^{-3}$, $\gamma_w = 9.8 \text{ kN}\cdot\text{m}^{-3}$, $\varphi = 20^\circ$, $c = 20 \text{ kPa}$, $\beta = 60^\circ$, $B/H = 1$, $H = 10 \text{ m}$, $n = 2.5$, $1/\alpha = 100 \text{ kPa}$, $z_0/H = 0$, and $\xi = 0$. As shown in Figure 7a, the slope safety factor F_{S1} increases with the infiltration rate q . That is, the safety factor gradually rises as the seepage condition shifts from infiltration to evaporation, and the smaller the permeability coefficient k_s , the more significant its impact on slope stability. As illustrated in Figure 7b, under infiltration conditions ($q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$), F_{S1} increases with k_s ; the smaller k_s is, the more difficult it is for pore water pressure inside the slope to dissipate, making the slope more susceptible to instability. Under evaporation conditions ($q = 1.15 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$), F_{S1} decreases with k_s . This is because highly permeable soil has high hydraulic conductivity, which restricts surface matric suction under evaporation, resulting in a slower increase in effective stress in the shallow slope and thus reducing slope stability. Under non-seepage conditions ($q = 0$), the safety factor F_{S1} of the three-dimensional slope remains stable with varying k_s . It can be concluded that the infiltration rate affects the effective stress by changing the pore water pressure, thereby influencing slope stability. Meanwhile, the

permeability coefficient accelerates the rapid redistribution of pore water pressure, which exerts an adverse effect on the slope.

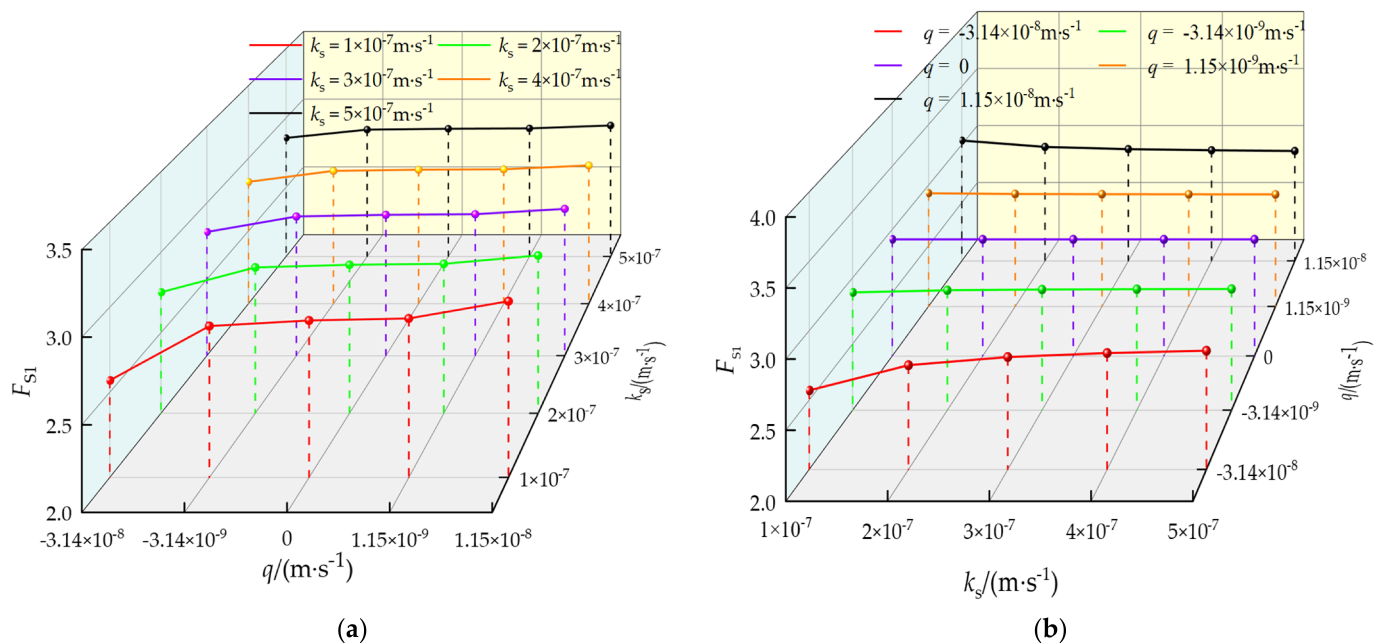


Figure 7. Effect of Infiltration Rate and Permeability Coefficient on Safety Factor under Different Conditions: (a) infiltration rate; (b) permeability coefficient.

In summary, (1) the air entry value $1/\alpha$ and pore size parameter n significantly influence slope stability, and (2) the infiltration rate q and permeability coefficient k_s exert differential effects on clayey silt slopes, both influencing the slope's self-stabilization capacity by affecting pore water pressure within the slope.

6.2. The Effect of Single-Row Anchor Plate Support on Slope Stability

Under rainfall conditions, the present work examines the influence of single-row anchor plate reinforcement on the safety factor F_S of a three-dimensional slope, with consideration given to the tensile strength failure criterion at the slope crest. The parameter values are as follows: $\gamma = 18 \text{ kN}\cdot\text{m}^{-3}$, $\gamma_w = 9.8 \text{ kN}\cdot\text{m}^{-3}$, $\varphi = 20^\circ$, $c = 20 \text{ kPa}$, $\beta = 60^\circ$, $B/H = 1$, $H = 10 \text{ m}$, $k_s = 1 \times 10^{-7} \text{ m}\cdot\text{s}^{-1}$, $q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$, $1/\alpha = 100 \text{ kPa}$, $L_a = 3 \text{ m}$, $L_b = 1 \text{ m}$, $L_h = 0.1 \text{ m}$, $R_{\text{inter}} = 0.3$, $T_s = 700 \text{ kN}$, $S = 3 \text{ m}$, $z_0 = 0$, and $\zeta = 0$. As shown in Figure 8, under the influence of different seepage rates q and slope angles β , the three-dimensional slope safety factor F_S exhibits a trend of an initial increase followed by a decrease with respect to the anchor plate burial depth ratio K_T (see Appendix A for the detailed expression). Notably, a smaller seepage discharge q within the slope soil mass correlates with a lower safety factor F_S , thereby amplifying the likelihood of slope failure. Conversely, the slope angle β exhibits an opposing trend: a smaller slope angle β corresponds to a greater safety factor F_S and enhanced stability of the slope.

Further, with Table 4, it can be seen that when $\varphi = 20^\circ$, taking the safety factor $F_S = 2.70$ at an embedment depth ratio $K_T = 0.1$ as the reference, the safety factor F_S increases to 2.76, 2.92, 3.12, 3.23 and 3.17, respectively, as the embedment depth ratio K_T rises to 0.2, 0.4, 0.6, 0.8 and 0.9. With the increase in K_T , the relative difference in the safety factor F_S gradually decreases. When K_T reaches 0.7, the relative differences Δ_5 between adjacent safety factors F_S are 1.5% and 1.89% for each 0.1 increment, both of which are less than 2%. Taking this as the judgment criterion, the safety factor F_S can be regarded as tending to stabilize. According to Equation (A24), when the anchor plate embedment depth ratio $K_T = 0.8$

(i.e., $Z_i \approx 8$ m), the safety factor F_S reaches its maximum value, and the reinforcement effect of single-row anchor plates is the best. Similarly, the same conclusion can be obtained when comparing other slope angles, i.e., when the slope angle β is 50° , 70° and 80° , respectively.

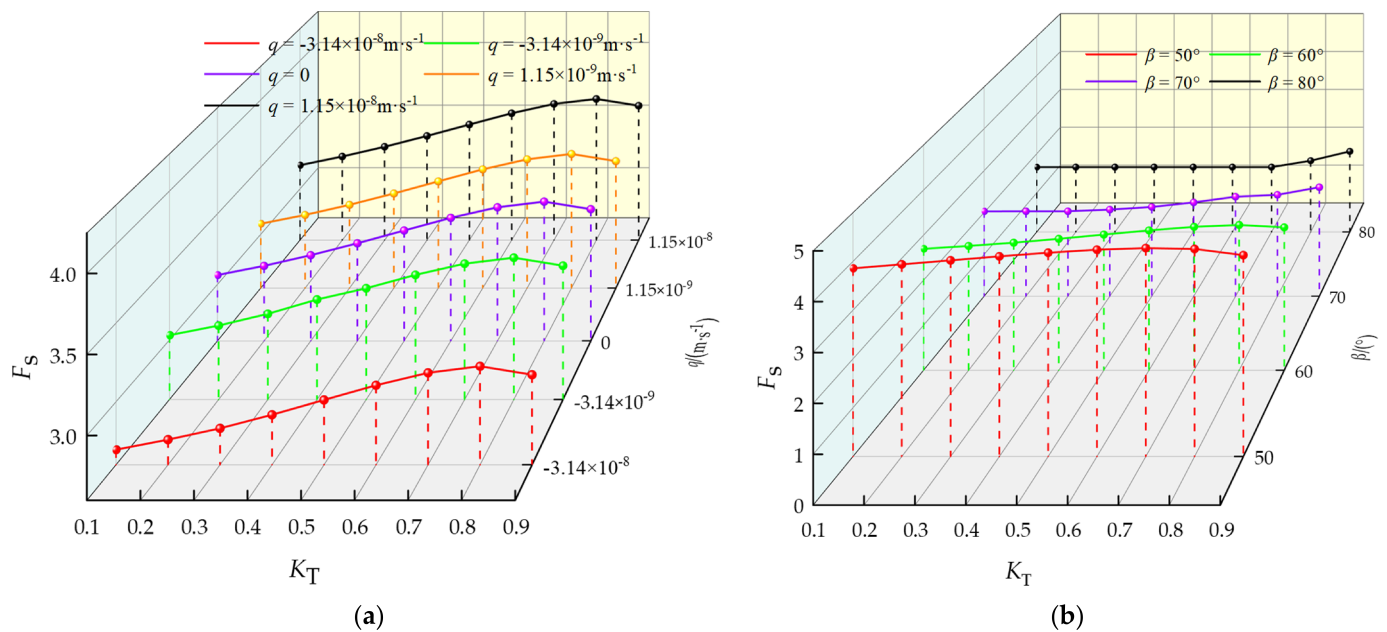


Figure 8. Effect of Anchor Plate Embedment Depth on Slope Safety Factor: (a) infiltration rate; (b) slope angle.

Table 4. Effect of Anchor Plate Embedment Depth Ratio on Slope Safety Factor.

N	K_T	$\varphi/(^\circ)$	F_S	Relative Difference (Δ_5)/% ⁴	K_T	$\varphi/(^\circ)$	F_S	Relative Difference (Δ_6)/% ⁴
1	0.1	20	2.70		0.1	30	4.74	
2	0.2	20	2.76	2.17%	0.2	30	4.93	3.85%
3	0.3	20	2.83	2.47%	0.3	30	5.14	4.09%
4	0.4	20	2.92	3.08%	0.4	30	5.38	4.46%
5	0.5	20	3.01	2.99%	0.5	30	5.63	4.44%
6	0.6	20	3.12	3.53%	0.6	30	5.88	4.25%
7	0.7	20	3.18	1.89%	0.7	30	6.05	2.81%
8	0.8	20	3.23	1.55%	0.8	30	6.12	1.14%
9	0.9	20	3.17	1.89%	0.9	30	5.95	2.86%

$$^4 \Delta_5 = \Delta_6 = \left| \frac{F_S(N=N_0) - F_S(N=N_0+1)}{F_S(N=N_0)} \right| \times 100\%.$$

As shown in Figure 9, the influence of different anchor plate parameters on the safety factor of a three-dimensional unsaturated soil slope is analyzed under tensile failure conditions. The parameter values are as follows: $\gamma = 18 \text{ kN}\cdot\text{m}^{-3}$, $\gamma_w = 9.8 \text{ kN}\cdot\text{m}^{-3}$, $\varphi = 20^\circ$, $c = 20 \text{ kPa}$, $\beta = 60^\circ$, $B/H = 1$, $H = 10 \text{ m}$, $k_s = 1 \times 10^{-7} \text{ m}\cdot\text{s}^{-1}$, $q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$, $1/\alpha = 100 \text{ kPa}$, $S = 3 \text{ m}$, $z_0 = 0$, $L_h = 0.1 \text{ m}$, $R_{\text{inter}} = 0.3$, $L_a = 3 \text{ m}$, $L_b = 1 \text{ m}$, $T_s = 700 \text{ kN}$, and $\zeta = 0$. As illustrated in Figure 9a, across varying seepage velocities, the safety factor F_S of the three-dimensional slope follows a pattern of “initial growth followed by plateauing” with respect to the ultimate tensile capacity T_s of the anchor bolts. When T_s rises to 300 kN , F_S attains its peak, corresponding to the maximum slope stability. At this critical point, the failure mode of the anchor plate transitions from tensile rupture to pull-out failure, where the upper limit of F_S is dictated by the ultimate pull-out capacity T_p of the anchor plate. In Figure 9b,c, across various seepage scenarios, the three-dimensional slope’s safety factor F_S exhibits a linear growth with both the anchor length L_a and width L_b . When $L_a = 0.5$

or $L_b = 0.2$, F_s attains its minimum value, signaling the weakest slope stability. At this stage, the single-row anchor plate's ultimate pull-out capacity T_p stays below the ultimate tensile strength T_s of the anchor rod, indicating that the load-carrying capacity of this reinforcement system meets the basic design requirements. Hence, in actual engineering practice, the ultimate tensile strength T_s derived from anchor parameters A_s and f_y shall be ensured to be at least the ultimate pull-out capacity T_p of the anchor plate. Furthermore, emphasis should be placed on expanding the physical dimensions of the anchor plate to elevate its pull-out resistance, alongside selecting cost-efficient anchors to curtail overall project expenses.

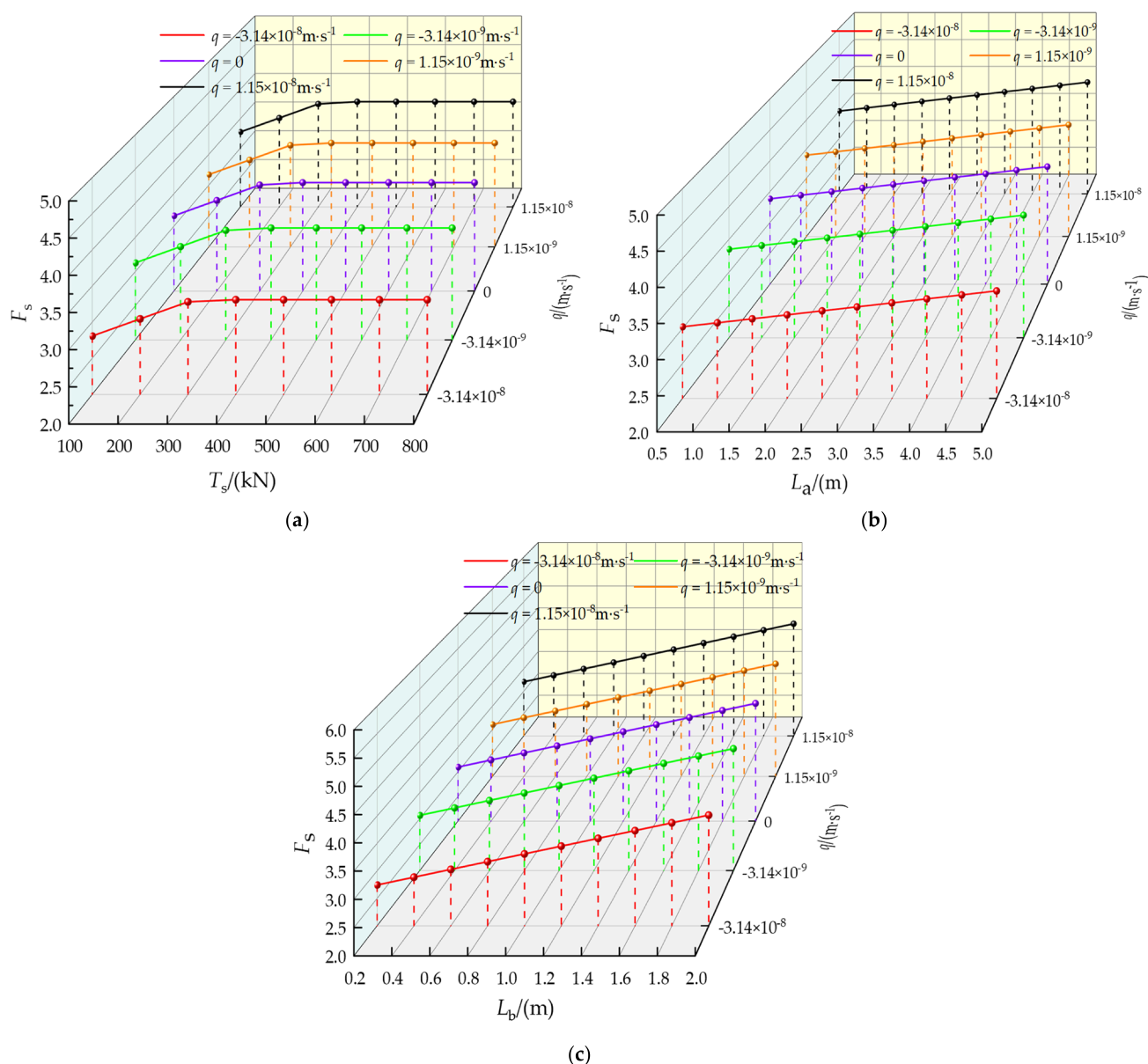


Figure 9. Safety Factor of Slopes Under Different Anchor Plate Parameters: (a) ultimate tensile strength of anchor bolts; (b) anchor plate length; (c) anchor plate width.

In summary, (1) across diverse seepage scenarios, the burial depth ratio associated with single-row anchor reinforcement exerts a notable influence on slope stability, with peak reinforcement efficacy occurring at a burial depth ratio $K_T = 0.8$, and (2) when selecting anchor plate reinforcement for slopes, priority should be given to optimizing anchor plate parameters to increase the pull-out resistance, thereby enhancing slope stability.

7. Engineering Applications

7.1. Engineering Case Study

A clayey silt slope suffers from concentrated rainfall, mostly heavy downpours, during the rainy season. Affected by rainfall seepage, cracks easily occur at the slope crest, which further induces slope instability. Therefore, anchor plate support is adopted to reinforce this slope [34]. The slope has a height of 10 m, a slope angle of 60° , and a width-to-height ratio $B/H = 2$. The soil parameters are as follows: cohesion $c = 20$ kPa, internal friction angle $\varphi = 20^\circ$, infiltration rate $q = -3.14 \times 10^{-8} \text{ m}\cdot\text{s}^{-1}$, permeability coefficient $k_s = 1 \times 10^{-7} \text{ m}\cdot\text{s}^{-1}$, air entry value $1/\alpha = 50$ kPa, pore size parameter $n = 2.5$, tensile strength reduction factor $\xi = 0$, and groundwater-level-to-slope height ratio $z_0/H = 0$. According to the relevant specification [28], the dimensions of the anchor plate are $3 \text{ m} \times 1 \text{ m} \times 0.08 \text{ m}$, the spacing S of anchor bolts is 5 m, the diameter D is 10 mm, the yield strength $f_y = 100$ MPa, and the anchor plate is embedded at 8 m below the slope crest.

Based on the method established in this paper, the safety factors of the slope under different working conditions are calculated and listed in Table 5. The results show that under the basic load condition, the safety factor is 1.46 without support, and it increases to 2.01 after adopting anchor plate support, with a relative difference increase of nearly 37.6%. Under the rainfall condition, the safety factor is 0.97 without support, and it rises to 1.48 after reinforcement, with a relative difference increase of approximately 52.5%. It can be concluded that single-row anchor plate support can significantly enhance slope stability under both basic load and rainfall conditions, and the reinforcement effect is more prominent under rainfall conditions, indicating that this support design has good applicability.

Table 5. Slope Safety Factor under Different Operating Conditions.

Serial Number N	Operating Conditions	Safety Factor F_S		Relative Difference (Δ_7)/% ⁵	Stable State (After Support)
		Unanchored Support	Anchor Plate Support		
1	Basic load	1.46	2.01	37.6%	Stable
2	Rainfall	0.97	1.48	52.5%	Stable

$$^5 \Delta_7 = \left| \frac{F_S(\text{Anchor plate support}) - F_S(\text{Unanchored plate Support})}{F_S(\text{Unanchored plate Support})} \right| \times 100\%.$$

7.2. Slope Failure Plane

As shown in Figure 10, based on the above engineering case, the morphologies of the slope slip surfaces under different working conditions are plotted. It can be seen from Figure 10c that under rainfall conditions without anchor plate support, the geometric morphology of the slip surface is stable, and the slope stability is at the baseline level. By comparing Figures 10a and 10b, it can be found that the slope failure range does not change significantly when only the basic load is altered or support is added, whereas the overall stability is improved. Further comparison with Figure 10d reveals that the slope failure range expands after applying anchor plate support under rainfall conditions, which is mainly reflected in the increase in both the width and depth of the slope crest. This phenomenon indicates that for the slope crest cracking induced by rainfall, anchor plate support can restrain the tensile cracking deformation at the slope crest and transfer part of the load to the deep part of the slope body, thus causing the slip surface to develop toward the deep and rear parts of the slope crest. Although the sliding range increases, the overall slope stability is still improved due to the anchoring force provided by the anchor plates.

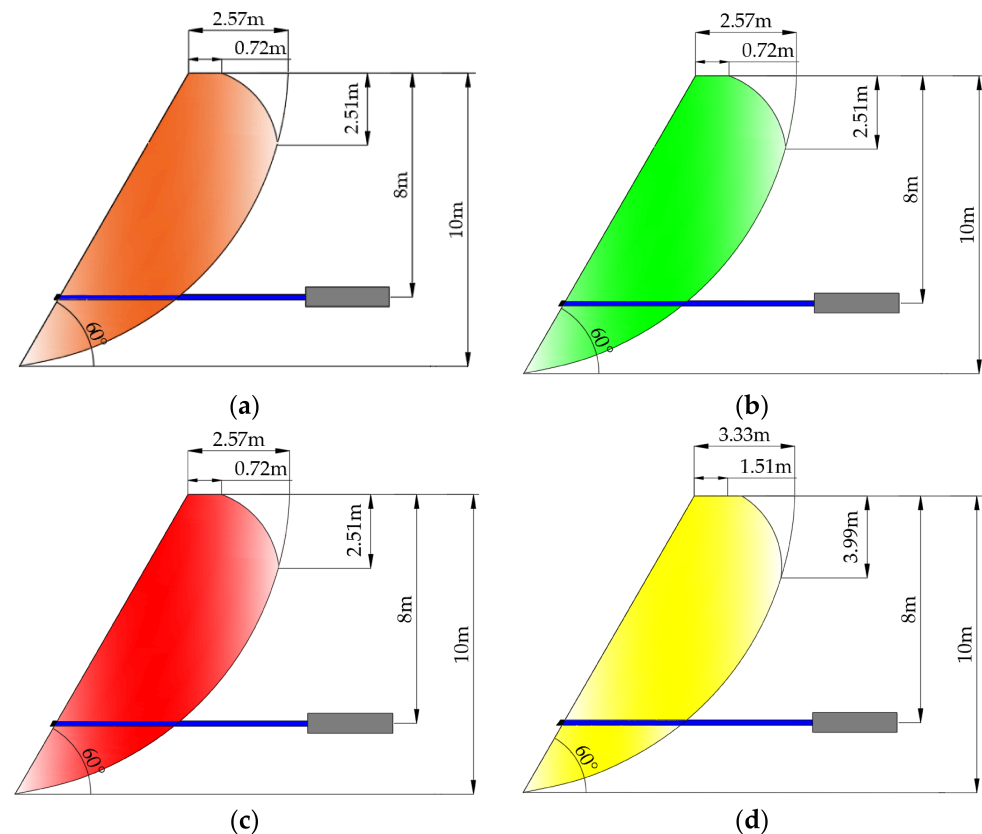


Figure 10. Slope Failure Surface: (a) unloaded and unsupported; (b) unloaded and supported; (c) under rainfall and unsupported; (d) under rainfall and supported.

8. Conclusions

- (1) Without considering tensile strength failure at the slope crest, the results are consistent with existing findings, validating the correctness of the proposed method.
- (2) Under steady-state infiltration, the air entry value and pore size parameter significantly affect the stability of three-dimensional clayey silt slopes. The larger the air entry value $1/\alpha$ and the smaller the pore size parameter n , the better the slope stability. Compared with the air entry value $1/\alpha$, the pore size parameter n exerts a weaker influence on slope stability.
- (3) The infiltration rate q and permeability coefficient k_s exert significant effects on clayey silt slopes. As the infiltration intensity q shifts from infiltration to evaporation, the air content in soil pores increases, which reduces the pore water pressure and consequently improves slope stability. The influence of the permeability coefficient k_s on slope stability varies under different infiltration conditions. Both parameters affect slope stability by altering the pore water pressure within the slope soil.
- (4) Balancing safety and cost-effectiveness, the optimal reinforcement effect for three-dimensional clayey silt slopes reinforced with anchor plates is achieved at a single-row anchor plate embedment depth ratio $K_T = 0.8$. Additionally, slope stability can be enhanced by improving infiltration capacity and reducing slope angles.
- (5) This paper aims to establish a mechanical model for three-dimensional clayey silt slopes reinforced with anchor plates under steady-state infiltration, considering tensile strength failure. At present, the analysis is only carried out for single-row anchor plates, and complex factors such as the synergistic mechanism between multi-row anchor plates and the spatial variability of soil mechanical parameters have not been involved. Future research will focus on the above issues to further improve the engineering applicability of the model.

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Appendix A

$$a = \frac{\sin(\theta_h - \theta_0)}{\sin \theta_h} - \frac{e^{(\theta_h - \theta_0) \tan \varphi'} \sin \theta_h - \sin \theta_0}{\sin \theta_h \sin \beta} \sin(\theta_h + \beta) \quad (\text{A1})$$

$$f_1 = \frac{1}{2} \begin{cases} \frac{\sin \theta_0}{\sin \theta_m} e^{\int_{\theta_m}^{\theta} \tan \psi d\theta} + \frac{r_0'}{r_0} \frac{\sin \theta_m}{\sin \theta_0} e^{-\int_{\theta_m}^{\theta} \tan \psi d\theta} & (\theta_m < \theta < \theta_{tc}) \\ e^{(\theta - \theta_0) \tan \varphi} + \frac{r_0'}{r_0} e^{-(\theta - \theta_0) \tan \varphi} & (\theta_{tc} < \theta < \theta_h) \end{cases} \quad (\text{A2})$$

$$f_2 = \frac{1}{2} \begin{cases} \frac{\sin \theta_0}{\sin \theta_m} e^{\int_{\theta_m}^{\theta} \tan \psi d\theta} - \frac{r_0'}{r_0} \frac{\sin \theta_m}{\sin \theta_0} e^{-\int_{\theta_m}^{\theta} \tan \psi d\theta} & (\theta_m < \theta < \theta_{tc}) \\ e^{(\theta - \theta_0) \tan \varphi} - \frac{r_0'}{r_0} e^{-(\theta - \theta_0) \tan \varphi} & (\theta_{tc} < \theta < \theta_h) \end{cases} \quad (\text{A3})$$

$$d_1 = \frac{r_0 \sin \theta_0}{\sin \theta} - r_c, d_2 = \frac{r_0 \sin \theta_h}{\sin \theta} e^{(\theta_h - \theta_0) \tan \varphi'} - r_c, x^* = \sqrt{R^2 - y^2} \quad (\text{A4})$$

$$g_1 = 2 \int_{\theta_m}^{\theta_h} \left[\left(\frac{2}{3} f_1 f_2^2 + \frac{1}{8} f_2^2 f_3 - \frac{1}{2} f_1^2 f_3 - \frac{2}{3} f_1 f_3^2 - \frac{1}{4} f_3^3 \right) \sqrt{f_2^2 - f_3^2} + \left(\frac{1}{2} f_1^2 f_2^2 + \frac{1}{8} f_2^4 \right) \cdot \arccos \left(\frac{f_3}{f_2} \right) \right] \cos \theta d\theta \quad (\text{A5})$$

$$f_3 = \begin{cases} \frac{\sin \theta_0}{\sin \theta} - f_1 & \theta_m < \theta < \theta_E \\ e^{(\theta_h - \theta_0) \tan \varphi} \frac{\sin(\theta_h + \beta)}{\sin(\theta + \beta)} - f_1 & \theta_E < \theta < \theta_h \end{cases} \quad (\text{A6})$$

$$g_2 = \frac{b}{H} (f_4 - f_5 - f_6 - f_7 + f_8 + f_9) \left[e^{(\theta_h - \theta_0) \tan \varphi} \sin \theta_h - \sin \theta_0 \right] \quad (\text{A7})$$

$$f_4 = \frac{1}{3(1 + 9 \tan^2 \varphi)} \left[(3 \tan \varphi \cos \theta_h + \sin \theta_h) e^{3(\theta_h - \theta_0) \tan \varphi} - (3 \tan \varphi \cos \theta_0 + \sin \theta_0) \right] \quad (\text{A8})$$

$$f_5 = \frac{1}{6} \left(2 \cos \theta_0 - \frac{L}{r_0} \right) \frac{L \sin \theta_0}{r_0} \quad (\text{A9})$$

$$f_6 = \frac{1}{6} \frac{H}{r_0} \frac{\sin(\theta_h + \beta)}{\sin \beta} e^{(\theta_h - \theta_0) \tan \varphi} \left[\cos \theta_0 - \frac{L}{r_0} + \cos \theta_h e^{(\theta_h - \theta_0) \tan \varphi} \right] \quad (\text{A10})$$

$$f_7 = \frac{1}{3(1 + 9 \tan^2 \varphi)} \left[(3 \tan \varphi \cos \theta_{tc} + \sin \theta_{tc}) e^{(\theta_h - \theta_0) \tan \varphi} - (3 \tan \varphi \cos \theta_0 + \sin \theta_0) \right] \quad (\text{A11})$$

$$f_8 = \frac{1}{6} \left(\frac{1}{\tan^2 \theta_0} - \frac{1}{\tan^2 \theta_m} \right) \sin^3 \theta_0 \quad (\text{A12})$$

$$f_9 = \frac{1}{3} \frac{\sin^3 \theta_0}{\sin^3 \theta_m} \int_{\theta_m}^{\theta_{tc}} e^{3 \frac{\theta_{tc} - \theta_m}{\psi_m - \varphi} \ln \frac{\cos \psi}{\cos \psi_m}} \cos \theta d\theta \quad (\text{A13})$$

$$\frac{H}{r_0} = \sin \theta_h e^{(\theta_h - \theta_0) \tan \varphi} - \sin \theta_0 \quad (\text{A14})$$

$$\frac{L}{r_0} = \frac{\sin(\theta_0 + \beta) - \sin(\theta_h + \beta) e^{(\theta_h - \theta_0) \tan \varphi}}{\sin \beta} \quad (\text{A15})$$

$$\alpha_1 = \arccos(d_1/R), \alpha_2 = \arccos(d_2/R) \quad (\text{A16})$$

$$f_{10} = \begin{cases} \frac{1}{\cos \psi} \left(\cos \varphi \frac{1 - \sin \psi}{1 - \sin \varphi} + 2\zeta \frac{\sin \psi - \sin \varphi}{\cos \varphi} \right) & \theta_m < \theta < \theta_{tc} \\ 1 & \theta_{tc} < \theta < \theta_h \end{cases} \quad (\text{A17})$$

$$g_3 = 2 \int_{\theta_m}^{\theta_h} f_{10} \left[\left(2f_1 f_2 + \frac{1}{2} f_2 f_3 \right) \sqrt{f_2^2 - f_3^2} + \left(f_1^2 f_2 + \frac{1}{2} f_3^3 \right) \arccos \frac{f_3}{f_2} \right] d\theta \quad (\text{A18})$$

$$c_{\text{app-horn}} = -\frac{\tan \varphi}{\alpha} \frac{\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha Z_{3D-\text{horn}}} - \frac{q}{k_s} \right]}{\left\{ 1 + \left\{ -\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha Z_{3D-\text{horn}}} - \frac{q}{k_s} \right] \right\}^n \right\}^{(n-1)/n}} \quad (\text{A19})$$

$$c_{\text{app-insert}} = -\frac{\tan \varphi}{\alpha} \frac{\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha Z_{3D-\text{insert}}} - \frac{q}{k_s} \right]}{\left\{ 1 + \left\{ -\ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha Z_{3D-\text{insert}}} - \frac{q}{k_s} \right] \right\}^n \right\}^{(n-1)/n}} \quad (\text{A20})$$

$$z_{3D-\text{horn}} = r_0 e^{(\theta_h - \theta_0) \tan \varphi} \sin \theta_h - (r_c + R \cos \alpha) \sin \theta + z_0 \quad (\text{A21})$$

$$z_{3D-\text{insert}} = r_0 e^{(\theta_h - \theta_0) \tan \varphi} \sin \theta_h - r_0 e^{(\theta - \theta_0) \tan \varphi} \sin \theta + z_0 \quad (\text{A22})$$

$$g_4 = \frac{b}{H} \cdot (\sin \theta_h e^{(\theta_h - \theta_0) \tan \varphi} - \sin \theta_0) \cdot \left[\frac{\sin^2 \theta_0}{\sin^2 \theta_m} \int_{\theta_m}^{\theta_{tc}} e^{2 \frac{\theta_{tc} - \theta_m}{\psi_m - \varphi} \ln \frac{\cos \psi}{\cos \psi_m}} \frac{1}{\cos \eta} \cdot \left(\cos \varphi \frac{1 - \sin \psi}{1 - \sin \varphi} + 2\zeta \frac{\sin \psi - \sin \varphi}{\cos \varphi} \right) d\theta + \frac{e^{2(\theta_h - \theta_0) \tan \varphi} - e^{2(\theta_{tc} - \theta_0) \tan \varphi}}{2 \tan \varphi} \right] \quad (\text{A23})$$

$$X_T = \frac{Z_i}{H} = \frac{H - H \cdot K_T}{\tan \beta} \quad (\text{A24})$$

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