

Article

Modeling and Experimental Evaluation of Fixed-Bed Convective Drying of Composite Peat-Coal Fuel Pellets

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Abstract

Drying of fragile granules of bulk materials has to be carried out in a fixed bed to prevent particle damage. This way of drying protects the granules, but the lack of mixing of the particles leads to an uneven distribution of heat and mass transfer conditions in the bed. It is necessary to be able to predict the distribution of drying conditions inside the bed to select a rational drying mode. In this paper, a mathematical model for drying bulk media in a fixed bed is proposed, its parametric identification is carried out, and the model is verified by comparison with experimental results. The numerical model is based on explicit difference schemes using the mathematical apparatus of Markov chains. The experimental study was carried out on a laboratory dryer with the detection of local drying parameters according to the height of the fixed bed. The numerical and experimental results were obtained at the fixed temperature (90 °C) and gas velocity (0.5 m·s⁻¹). It has been shown that at a certain bed height, areas with relative gas humidity above 100% are formed within it, which negatively affects the efficiency of the drying. The proposed model allows identifying the indicated ineffective drying modes, and has proven to be a reliable tool for predicting the moisture ratio of the material with an MAPE of less than 7%.

Keywords: drying; pellets; biofuel; coal waste; fixed bed; numerical modeling

1. Introduction

The continued growth of the Earth's population is associated with an increase in energy consumption, the annual growth rate of which is projected to be 1.8% per year (according to forecasts until 2030) [1]. Although energy production and consumption vary from region to region, the global energy balance is approximately 80% dependent on fossil fuels [1,2]. Coal usage provides approximately 40% of all electricity generated worldwide [3]. Researchers are fairly unanimous in the fact that the usage of solid fossil fuels continues to be a key component in providing global energy security in the following decades [1,2]. At the same time, the use of coal in the energy sector faces internal and external challenges. A bottleneck that should be considered as an internal industry challenge is the high level of waste that is difficult to use for energy purposes (in the extraction of precious coal products, 1 ton of hard coal produces 0.4 tons of extractive waste materials) [3]. It is widely known that the instability of the weight and size characteristics of fuel particles makes their use difficult [4]. Furnaces and boiler units are capable of working effectively with fuel particles of fairly



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narrow ranges of characteristics, so the fuel mass is processed into granules (pellets), for which these units can be configured to operate. Therefore, it is convenient to pre-process the fuel mass into granules (pellets), for the combustion of which new installations are developed, or existing ones are improved [4,5]. Better combustion parameters have been observed both in pellet boilers and when combined with coal [6].

External challenges associated with the predominance of fossil fuels include a wide range of environmental problems. Air pollution from coal combustion and the frequent and serious environmental consequences worldwide caused by accidents at coal dumps and waste storage facilities, wastewater contamination, and spontaneous combustion are driving the demand for more responsible ways of disposing of coal industry waste [3,7].

Converting (pelletization) of heterogeneous coal waste into compact, energy-intensive particles with a narrow range of characteristics can be a very effective way to mitigate these external and internal challenges [8]. The efficiency of burning granulated fuel depends on many factors: the composition of the granules, the bed height, the air supply method, etc. The efficiency of the combustion process of fuel pellets from agricultural and industrial waste under various combustion conditions ranged from 27% to 70% [9].

The pellets can be manufactured as a composite fuel to increase carbon neutrality. The composition for this purpose can include various biomaterials (in addition to coal). The range of biomass that can be used as a fuel component is very wide (woody materials, agricultural residues (straw, hay, sunflower), food waste, etc.) [10–12]. A peat or the agricultural residues in the composition is considered as a fuel additive (having a heat of combustion comparable to coal), but also acts as a binding component [8,13].

The density, heat release during combustion, and moisture content of peat and agricultural waste differ slightly. The agricultural crop residues should be considered a renewable resource, so interest in their consumption and processing for inclusion in energy production is consistently high [14,15]. The peat resources are also considered to be practically unlimited and are included in the concept of “slowly renewable biomass” [13,16].

The pellets production consists of four main stages: raw material preparation, granule formation, drying of formed granules, and cooling of the finished product [5,17]. The energy consumption for the drying process of formed pellets is approximately 25% of the total energy required for their production [18]. According to some estimates, almost 90% of thermal energy is spent on evaporation of moisture [19].

Reducing drying costs represents a significant reserve for reducing the cost of pellets and increasing their competitiveness. Various drying methods are widely used in the production of pellets, and the choice between them has no systematic basis [19–21]. Convective dryers are the most commonly used, but the choice of the interaction mode of particles and the flow of drying agent is also difficult and is based more on some engineering art than on an accurate calculation prediction [21–23]. In particular, significant uncertainty in this choice is introduced by the fact that the formed particles of the granules do not have high strength and can be abraded or milled during the drying process against the background of mixing (for example, in the fluidization process) [21,24].

Thus, drying of fuel pellets, as a process, is carried out as a technological operation in the preparation of pellets, but can also be considered as the initial stage of combustion or thermal conversion of fuel. Thus, it is common to distinguish three main stages in the thermal treatment of biomass in the form of pellets: (1) water evaporation, (2) combustion of volatile components, and (3) oxidation of fixed carbon [25]. Considering that drying is an endothermic process, an important issue is minimizing energy consumption for this process.

The chief aim of this study is to substantiate a rational regime of convective drying of composite pellets (coal with biomass) in a batch-type dryer with a fixed bed.

2. Materials and Methods

2.1. Theoretical Methods

Nowadays, the most applied approach (and continuing to gain popularity [26]) for modeling of particulate processing is built on the basis of a combination of computational fluid dynamics (CFD) and a discrete element method (DEM). This approach involves decomposing the object under consideration to the level of representative volumes of differentially small sizes [27–29]. On the one hand, this ensures high information content during modeling (although the computational cost is also high), but on the other hand, the models involve many parameters, the identification of which is an expert-level task, and therefore difficult to implement in an engineering laboratory [30–32]. If there is serious hope of overcoming the computational cost [30], then the parameter estimation forms an additional and stable layer of complexity [33].

The opposite approach involves considering the entire drying material as an object with lumped parameters, which increases parametric and computational accessibility but reduces the information content of the modeling [34,35].

Some mathematical approaches allow one to flexibly adjust the step of the mathematical grid in search of a compromise between information content and computational and parametric complexity. Attempts to use such approaches (for example, models based on cellular automata [36,37] or Markov chains [38,39]) are currently being intensified. The mathematical apparatus of the Markov chain theory was used as the basis for modeling. The Markov chain model (MCM) provides high information content in describing an object and an acceptable computational cost, accessible to modern home computers [30]. When the object of study is a particulate medium, this approach is especially convenient because it calculates particle motion through the transition probability matrix and the Monte Carlo method without solving the complicated dynamic formulae [31].

In practical terms, the general idea of the MCM is to represent the object as a number of representative volume elements with a small but finite size. The marked elements form a chain of cells, the evolution of which over time allows us to describe the object [30]. The main object of the study is the fixed-bed drying batch column. This column is separated into n domains of length $\Delta x = H/n$ (where H is the height of the apparatus). Any properties are considered to be ideally averaged within each such cell [30].

The chain of cells is characterized by a system state vector. The state vector characterizes the distribution of any observed additive property over the chain of domains [30].

The basic principle of representing the apparatus with a Markov chain is illustrated schematically in Figure 1. Shown here is a gas cell chain and a bulk solids cell chain (dryer space). The corresponding cells of the chains can exchange some properties (moisture and heat transfers). Each chain is described by state vectors containing a set of selected physical properties.

It is required to track several basic properties to describe drying. For each property, it is necessary to introduce the state vector; therefore, in the proposed model, the following vectors are defined (index “g” denotes the relation to the gas phase, index “s” denotes the relation to the solid phase): S —vectors of the mass content of the solids and gas, [kg]; M —vectors of the mass of water in the gas and in the solids, [kg]; Q —vectors of the heat content, [J] [39]. Within the framework of this work, the vectors are given in bold. For example, the total solid mass distribution is presented as the column state vector $S_s = \{S_{si}\}$, the gas content distribution is presented as the state vector $S_g = \{S_{gi}\}$, where $i = 1, 2, \dots, n$ (the unit corresponds to the lower cell of the device).

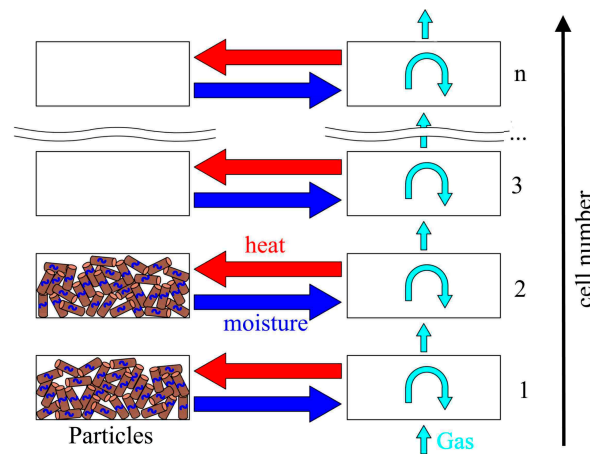


Figure 1. The graphic sketch of the model representation of the phase movement.

The developing model is based on the Markov chains approach with a discrete state space and discrete time. The latter means that the change in any state vector is calculated only at discrete moments in time, which are separated by a finite interval Δt . The current time can be calculated as $t_k = (k - 1) \cdot \Delta t$, where k is the number of the time step ($k = 1, 2, \dots, N$; where N is the total number of calculations) [30,40].

The evolution of the state vectors is described, as a general rule, using recurrent matrix procedures, in which each next state vector is obtained by multiplying the transition matrix P by the current state vector. Thus, the gas distribution along the chain is described as follows [30]:

$$S_g^{k+1} = P_g^k \cdot S_g^k + G, \quad (1)$$

where G is the vector of sources and discharge for the gas phase, providing the “loading” of the gas in the lower cell and the “unloading” of the gas from the upper [30]. Provided that the “loading” and removal of air from the column are coordinated, vector G has the form:

$$G = \begin{bmatrix} g \cdot V \cdot \rho_g \\ 0 \\ \dots \\ 0 \\ -g \cdot V \cdot \rho_g \end{bmatrix}, \quad (2)$$

where g is a dimensionless fraction of incoming gas; V is the cell volume, $[m^3]$; ρ_g is the air density, $[kg \cdot m^{-3}]$.

In the common case, particles can move (for example, as a result of fluidization or particles’ shrinkage during dehydration), then the state vector S_s changes, which is described by a similar recurrent procedure [30]. In the present study, the process is carried out at a gas velocity lower than the minimum fluidization velocity, and there is no shrinkage of the material; the state vector S_s does not change. However, we retain this state vector (in fact, stationary), since it determines the nature of the filtration of the drying agent along the bed height. The study [39] presented the relationship between the local gas velocity U in the cell and the mass content of the particulate phase in it as follows:

$$U = \frac{U_0}{1 - \pi(S_s / 8 \cdot S_{max})^{\frac{2}{3}}}, \quad (3)$$

where S_s and S_{max} are the mass of randomly packed particles and the maximum value of the mass of a given material in cells (corresponding to the true density).

It is assumed in the proposed model that the gas is filtered through the bed of material in the ideal displacement mode. In this way, the proportion of gas removed from a cell in one time step can be interpreted as a transition probability for the transition matrix and calculated as follows [30]:

$$g = \frac{U \cdot \Delta t}{\Delta x}, \quad (4)$$

Then the matrix of transition probabilities for the gas phase will have the following form [30]:

$$P_g = \begin{bmatrix} 1-g & 0 & 0 & 0 & 0 \\ g & 1-g & 0 & 0 & 0 \\ 0 & g & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1-g & 0 \\ 0 & 0 & 0 & g & 1 \end{bmatrix}. \quad (5)$$

Thus, the vectors M for the masses of moisture inside the particles (index “s”) and in the gas phase (index “g”) will evolve according to the following recurrent procedures:

$$M_s^{k+1} = M_s^k - \beta^k * F^k * \Delta X^k * \rho^k \cdot \Delta t, \quad (6)$$

$$M_g^{k+1} = P_g \cdot (M_g^k + \beta^k * F^k * \Delta X^k * \rho^k \cdot \Delta t) + M_f^k \cdot \Delta t, \quad (7)$$

where M_s is the vector of moisture masses in the cells for solid, [kg]; β is the vector of local moisture transfer coefficients, [m·s⁻¹]; F is the vector of moisture transfer surface in the cells, [m²]; $\Delta X^k = X^k - X_f$ is the vector of local values for the driving force of the mass transfer process (here X is the vector of moisture content in the cells for solid, [kg·kg⁻¹]; X_f is the value of minimum moisture content for solid, [kg·kg⁻¹]); M_g is the vector of moisture masses in the cells for the gas phase, [kg]; M_f is the vector of moisture source for the gas chain, [kg·s⁻¹]; the operator “*” means element by element multiplication of vectors.

The evolution of the thermal state of the dryer is described by the followingsimilar recurrent procedures [39]:

$$Q_s^{k+1} = Q_s^k - \beta^k * F^k * \Delta X^k * \rho^k * q \cdot \Delta t + \alpha^k * F^k * \Delta T^k \cdot \Delta t, \quad (8)$$

$$Q_g^{k+1} = P_g \cdot (Q_g^k - \alpha^k * F^k * \Delta T^k \cdot \Delta t) + Q_f^k \cdot \Delta t, \quad (9)$$

where Q_s is the vector of heat content for solid, [J] ($Q_s = T_s * c * \rho \cdot V$, where c is the vector of specific heat capacity of wet particles, [J·K⁻¹]; ρ is the vector of wet solid density, [kg·m⁻³]; $\rho^k = (M_s^k + S_s^k) / V$; where V is a cell volume, [m³]); q is the vector of values of the enthalpy of evaporation, [J·kg⁻¹]; Q_g is the vector of heat content in the cells for the gas phase, [J]; α is the vector of local heat transfer coefficients, [J·s⁻¹·m⁻²·K⁻¹]; Q_f is the vector of heat source for the gas chain, [J·s⁻¹]; $\Delta T^k = T_g^k - T_s^k$ is the vector of local values of the driving force of the heat transfer process (T_s is the vector of particle temperature, [K]; T_g is the vector of gas temperature, [K]).

Equations (1)–(9) are fairly obvious balance equations expressing the processes of heat and mass exchange between the phases (Equations (6)–(9)) and the movement of phases along the height of the apparatus (Formulas (1)–(5)). The most interesting parameters during the drying process are also easy to interpret in vector format. The current values (for the k -th calculation step) of the elements of the moisture content vector $X = \{X_i\}$ can be determined as follows [39]:

$$X_i^k = \frac{M_{s(i)}^k}{S_{s(i)}^1}, \quad (10)$$

The heat and mass transfer coefficients can be estimated based on the Chilton–Colburn analogy for heat and mass transfer processes. In this way, the mass transfer coefficient is estimated by replacing the Nusselt (Nu) and Prandtl (Pr) numbers in Equation (9) with Sherwood (Sh) and Schmidt (Sc) numbers respectively [41]:

$$Nu_i^k = \frac{\alpha_i^k d}{\lambda_{pi}^k} = 0.033 \cdot (Re_i^k)^{1.3} Pr^{0.3}, \quad (11)$$

$$Sh_i^k = \frac{\beta_i^k d}{D_i^k} = 0.033 \cdot (Re_i^k)^{1.3} Sc^{0.3}, \quad (12)$$

where λ is the gas conductivity, [$\text{W m}^{-1}\text{K}^{-1}$]; D is the mass diffusivity coefficient, [$\text{m}^2 \cdot \text{s}^{-1}$]; Re is the Reynolds number [41]:

$$Re_i^k = \frac{U \cdot d}{v_i^k}, \quad (13)$$

where d is the characteristic particle size (equivalent particle diameter), [m]; v is the gas kinematic viscosity, [$\text{m}^2 \cdot \text{s}^{-1}$].

Despite the fact that Equations (11) and (12) are not often used to calculate heat and mass transfer in a fixed bed (it is considered more valid for a fluidized state), it has shown good applicability with the cellular model, since in it the calculation is carried out based on local state parameters.

For the practical use of the proposed model for the purpose of predicting drying, it is necessary to identify the model parameters related to the physics of the processes taking place.

Equations (1)–(13) express the basic model concepts and are essentially fairly obvious balance relationships. To provide the proposed model with predictive capabilities, it is necessary to identify the model parameters, which is carried out mainly using formulas known from the literature (partially, the physical parameters of the materials were determined independently by the research group using standard laboratory tests with the materials).

The model parameters calculated based on the results presented in the professional scientific literature or determined experimentally are presented in Table 1. This table also presents dependencies for drying process parameters, which will be further determined experimentally. Many calculation formulas have been adjusted to match model representations, meaning they are calculated separately for the i -th cell and the k -th moment in time.

Table 1. Characteristics of model identification parameters.

Model Parameter	Equation	Source
Particle effective heat capacity, [$\text{J} \cdot \text{kg}^{-1} \text{K}^{-1}$]	$c_i^k = \sum y_{si}^k c_{si}^k + y_{wi}^k c_{wi}^k$ where y denotes the mass fraction of each phase: solid (subscripted "s"), water (subscripted "w").	[42]
Particle effective heat conductivity, [W/m K]	$\lambda_{pi}^k = \sum f_{si}^k \lambda_{si}^k + f_{wi}^k \lambda_{wi}^k$ where λ_s and λ_w are the components of the heat conductivity: solid (subscripted "s"), water (subscripted "w").	[42]
Particle effective density, [$\text{kg} \cdot \text{m}^{-3}$]	$\rho_i^k = \sum f_{si}^k \rho_{si}^k + f_{wi}^k \rho_{wi}^k$ where ρ_s and ρ_w are the components of the density: solid (subscripted "s"), water (subscripted "w").	[42]

Table 1. Cont.

Model Parameter	Equation	Source
Moisture ratio for solid	$MR_i^k = \frac{X_i^k - X_e}{X_i^1 - X_e}$ where X_e is the equilibrium moisture content	(17) [39]
Enthalpy of evaporation, [J·kg ^{−1}]	$q_i^k = 2889.4 \cdot 10^3 \left[1 - \frac{273 + T_{s(i)}^k}{647.13} \right]^m$ where m is an empirically established parameter $m = 0.3199 - 3.276 \cdot 10^{-4} (273 + T_{s(i)}^k) + 6.1596 \cdot 10^{-7} (273 + T_{s(i)}^k)^2$	(18) [43]
Mass diffusivity, [m ² ·s ^{−1}]	$D_i^k = 1.87 \cdot 10^{-10} \frac{(273 + T_{s(i)}^k)^{2.072}}{p}$, where p is total pressure, [atm]	(19) [44]
Diameter of a sphere equivalent to a particle of volume, [m]	$d = \sqrt[3]{6 \frac{V}{\pi}}$	(20) [45]
Water thermal conductivity, [W/m K]	$\lambda_{wi}^k = \lambda_0 0.16^m$, where $m = 1 - \left(\frac{1 - T_{si}^k}{1 - T_b} \right)^{0.2}$, T_b is the normal boiling point temperature, [K]	(21) [46]
Thermal conductivity of dry coal, [W/m K]	$\lambda_{s1} = 1.11$ (determined experimentally)	
Thermal conductivity of dry peat, [W/m K]	$\lambda_{s2} = 0.049$	[12]
Water specific heat capacity, [J·kg ^{−1} K ^{−1}]	$c_{w} = \frac{(4245 - 1.841 T_s)}{\rho_w}$	(22) [47]
Peat specific heat capacity, [J·kg ^{−1} K ^{−1}]	1720	[12]
Coal specific heat capacity, [J·kg ^{−1} K ^{−1}]	1382 (determined experimentally)	
Saturated vapor pressure of water, [kPa]	$p_{wi}^k = 0.61078 \exp \left(\frac{17.27 \cdot (T_g^k - 273.15)}{T_g^k - 35.85} \right)$	(23) [48]
Partial vapor pressure of water, [kPa]	$p_{gi}^k = 4.61 \rho_{wi}^k (T_g^k - 35.85)$	(24) [49]
Relative humidity	$RH_i^k = \frac{p_{gi}^k}{p_{wi}^k} 100\%$	(25) [50]
Root mean square error	$RMSE = \sqrt{\frac{\sum_{i=1}^K (Predicted_i - Actual_i)^2}{K}}$ where K is the number of observations	(26) [51]
Mean absolute percentage error	$MAPE = 100 \frac{1}{K} \sum_{i=1}^K \left \frac{Predicted_i - Actual_i}{Actual_i} \right $ where K is the number of observations (when processing moisture ratio data (less than 1), all values were pre-multiplied by 100)	(27) [51]

Computer simulations were performed using spatio-temporal decomposition with the column axial step of $\Delta x = 0.01$ m and the time step $\Delta t = 0.005$ s (the calculations were performed in the computing package MATLAB ver. R2016). The dryer geometry and process parameters applied in the simulations were the same as those observed during the experiments.

2.2. Experimental Procedures

The aim of the experimental part of the study is to validate the capabilities of the proposed model for predicting the parameters of the convective drying process of freshly formed pellets in a batch-type column with a packed bed.

A schematic representation of the experimental setup is shown in Figure 2. The working space of the drying chamber 1 (DIY device) is a column of rectangular cross-section with sides of 0.12 m by 0.06 m. The height of the column was 1 m. A metal mesh 2 with a cell size of 1 mm was located at the bottom of the column and served as a gas distributor. Air for maintaining the drying process was pumped by a centrifugal blower 3 (model VR 140-15 No. 4) through an electric air heater 4 (DIY device), a surge tank 5 (DIY device), a control valve 6 and a digital variable area vertically mounted flow meter 7 (model RIZYR-RPS-250). The total flow rate and gas temperature were also controlled by a thermal anemometer 8 (model testo 400) equipped with a separate thermal probe. A multichannel thermo-hygrometer 9 (model ITVMod 2605/8) was used to measure local values of the gas temperature and the gas relative humidity. Its sensors were inserted inside the drying column so that their measuring part was located in the center of its cross-section, but at different heights. The lower sensor was mounted at a height of 4 cm from the level of the gas distribution plate. Each subsequent sensor was located 4 cm higher than the previous one. Thus, the fourth sensor (the last one) was located at a height of 16 cm from the gas distributor.

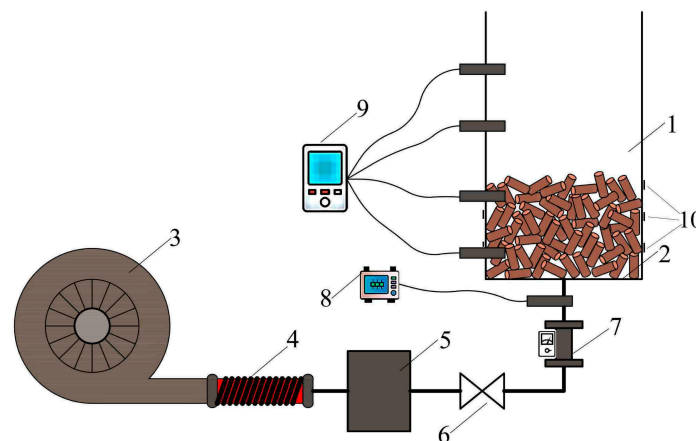


Figure 2. The graphic sketch of the experimental setup: 1—drying chamber; 2—gas distributor; 3—centrifugal blower; 4—electrical heater; 5—surge tank; 6—control valve; 7—flow meter; 8—thermal anemometer; 9—multichannel thermo-hygrometer; 10—control windows.

The drying material used in this study was cylindrical granules obtained by extrusion in a screw granulator, with the granules being squeezed out through a die. The obtained bulk media was a monofraction consisting of cylindrical particles with a diameter of 5 mm and a height of 7 mm. The diameter of a sphere equivalent to a particle of volume was used in the calculation part of the study (Equation (20)). The raw material for granulation was a moistened peat mixture (60% peat and 40% coal dust calculated on a dry weight basis). The granules loaded into the column had the initial moisture content of $0.27 \text{ kg} \cdot \text{kg}^{-1}$ and a bulk density of $1257 \text{ kg} \cdot \text{m}^{-3}$. Along the height of the column, on both sides, there were also small round windows 10 with lids, through which particles were periodically removed for weighing (the diameter of the window was 2 cm). The lower windows were at a height of 2 cm. Each subsequent pair of windows was located 4 cm higher than the previous two, respectively. At regular time intervals, the windows were opened, and several particles were collected (the total sample size, collected through the different windows, was 10). These particles were then weighed. The particle mass was used to calculate the average

moisture content of the bed. After weighing, the particles were returned to the apparatus. The mass of absolutely dry material was determined by drying 100 g of particles in a drying oven at a temperature of 105 °C.

The sample set of 10 particles is not statistically representative; it was chosen to ensure the stability of the particle bed geometry (to ensure that the particles within the bed remain as positionally stable as possible). The bed's stability was dictated by the need to capture local characteristics of the drying process. After each experiment, the material was unloaded and weighed as a whole. The values obtained in this manner differed from the sample results by less than 5%. However, the description of the transient process based on the experimental results presented in Figure 3 should be considered as indicative.

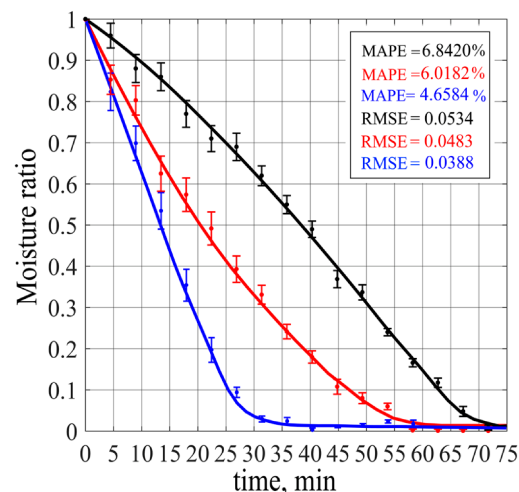


Figure 3. Calculated (lines) and experimental (markers) values of the solids moisture ratio: blue—for the bed height of 4 cm, red—for the bed height of 8 cm, black—for the bed height of 12 cm.

The empty column was heated for 5 min with air at a given temperature (the superficial gas velocity was $0.2 \text{ m}\cdot\text{s}^{-1}$). At the next stage, the solid sample was loaded into the column from the top, and the superficial gas velocity was set to $U_0 = 0.5 \text{ m}\cdot\text{s}^{-1}$. The gas temperature at the column inlet was 90 °C, and the relative humidity was 4%.

After this, the particles were dried in a fixed-bed state while local parameters and integral parameters of the process were recorded at equal time intervals. The experiment was conducted at different solid bed heights: 4 cm, 8 cm, and 12 cm. A separate experiment (for each bed height) was repeated three times, and the results were averaged.

3. Results and Discussion

The experimental (markers) and calculated (lines) changes in the MR in the fixed-bed material during drying are shown in Figure 3. To be more precise, this figure shows the curves of changes in the moisture content of the material in the apparatus over time for three loading options (three different heights of the fixed bed). It is evident that at the smallest of the bed heights (masses) considered, its dehydration by its nature is largely similar to the process observed in an individual particle: at the beginning there is an almost linear section with a constant drying rate, then nonlinearity is observed, associated with a decrease in the drying rate due to limitations associated with the migration of moisture from the depth of the solid.

The qualitative difference between the individual curves in Figure 3 is obvious: starting at a certain sample mass, the concavity of the curves changes to convexity. From a bed height of 8 cm, the drying curve becomes convex, and the drying process slows rapidly. The explanation for this phenomenon will be given further in the context of this study.

With a further increase in the mass and height of the material bed, the situation becomes even more noticeable, and the drying conditions worsen.

Figure 4a shows the calculated graph of the distribution of relative air humidity in the apparatus during the entire drying period at the bed height of 8 cm. It can be seen that during the entire drying period, the relative humidity of the air inside the bed at any height does not exceed 100%. This situation is also confirmed experimentally: Figure 4b shows the comparison of the calculated values of the gas relative humidity with the experimental ones (obtained from the thermo-hygrometer sensors located at the height of 4 cm and 8 cm from the gas distribution plate). It should be emphasized here that the maximum value of the relative humidity of the gas at the top of the bed at some point in time is about 90%, which obviously slows down the removal of moisture, but does not stop it completely.

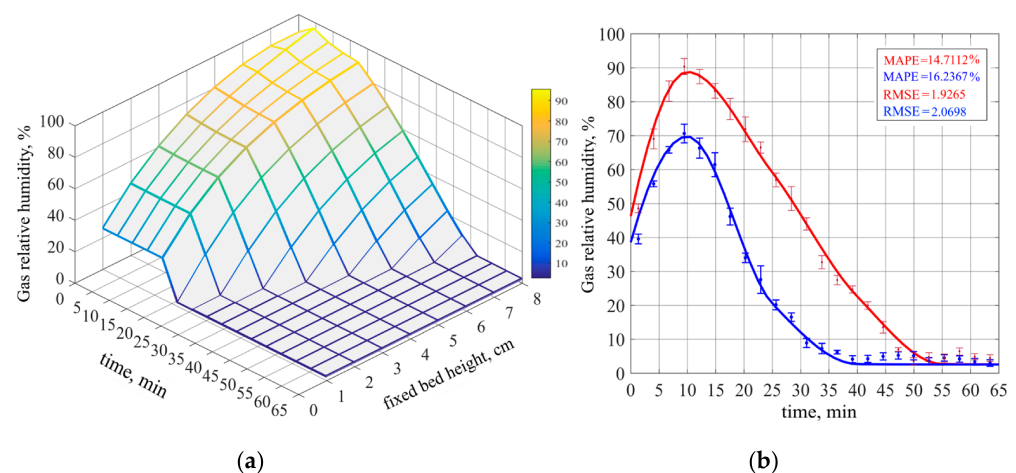


Figure 4. (a) Calculated distribution of the relative gas humidity inside the particle bed with its height of 8 cm; (b) calculated (lines) and experimental (markers) values of the relative gas humidity inside the particle bed with its height of 8 cm: blue—at a height of 4 cm, red—at a height of 8 cm.

Figure 5 characterizes the change in the temperature state of the material bed over time. Figure 5a shows that the temperature distribution evens out as the entire material dries, reaching thermal equilibrium with the gas. This demonstrates the qualitative consistency of the model.

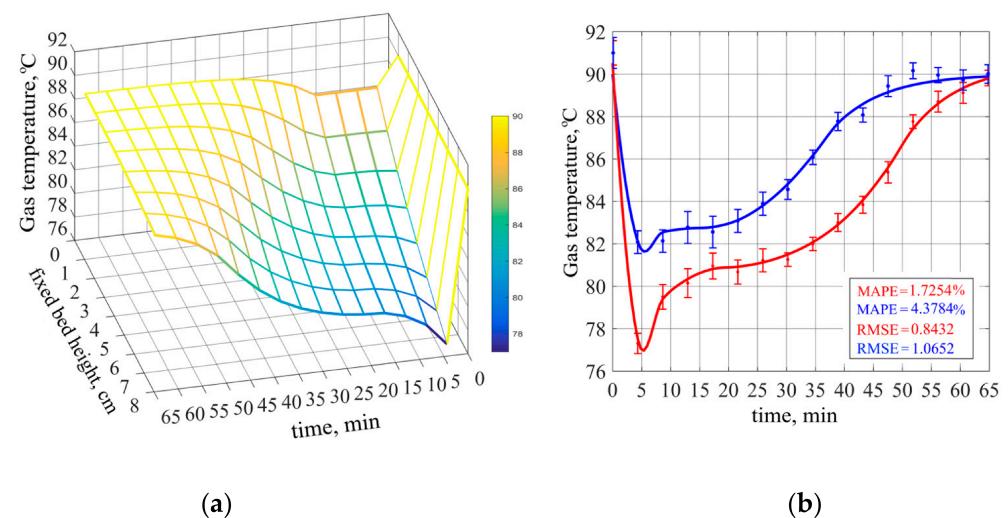


Figure 5. (a) Calculated distribution of the gas temperature inside the particle bed with its height of 8 cm; (b) calculated (lines) and experimental (markers) values of the gas temperature inside the particle bed with its height of 8 cm: blue—at a height of 4 cm, red—at a height of 8 cm.

Figure 5b shows a comparison of the calculated and experimental temperature values at a height of 4 cm and at a height of 8 cm (the total bed height is 8 cm). The calculated and experimental results are in good agreement. Furthermore, it is clear that the maximum temperature decrease at each of these heights (which indirectly corresponds to the maximum moisture removal rate) slightly precedes the corresponding maxima on the relative air humidity curves (Figure 4b). Also, when comparing Figures 4b and 5b, it is clear that the dehydration process ends before temperature equilibrium is reached (which is also qualitatively expected).

Overall, it should be noted that the nature of moisture movement within the study turned out to be more significant. Figure 6 shows a scenario for the distribution of relative air humidity for a fixed bed of 12 cm high. It can be seen that at the 4 cm mark, the graph of relative humidity over time is similar in shape to those shown in Figure 4. However, the shape of these graphs for higher-lying cells in the beds is different, more extended over time. It is clear that for cells at heights of 8 cm and 12 cm, there are fairly long periods of time when relative humidity values are above 100%. The temperature curves obtained for moist air against this background (shown in Figure 7) are qualitatively similar to those observed for the bed of lower height (Figure 5). However, due to the greater height (and mass) of the bed, the temperature drops to lower values, and its recovery is characterized by periods of practically constant values.

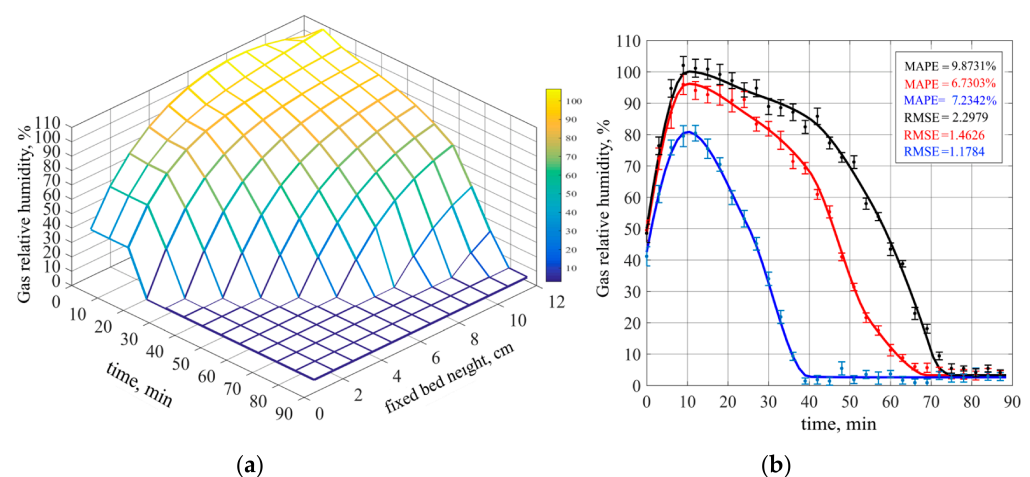


Figure 6. (a) Calculated distribution of the relative gas humidity inside the particle bed with its height of 12 cm; (b) calculated (lines) and experimental (markers) values of the relative gas humidity inside the particle bed with its height of 12 cm: blue—at a height of 4 cm, red—at a height of 8 cm, black—at a height of 12 cm.

Relative humidity values exceeding 100% in some areas of the dense bed indicate that the material is being moistened rather than dried at certain stages of processing. Naturally, from a practical standpoint, such conditions should be avoided, as they will delay the time it takes for the bulk material to reach the required final moisture content.

It is important to note that this excess relative humidity of 100% has a very clear numerical justification in the proposed model. During the recurrent transition time Δt , the same amount of air enters any cell as is removed from it. However, the amount of moisture entering a cell with the same amount of air is not the same. If air with higher humidity enters and is removed with lower humidity, then moisture will accumulate in this volume. This is precisely what leads to the air becoming oversaturated with moisture. Thus, it was the mechanical movement of the drying agent that led to moisture accumulation in certain volumes of the layer, in quantities that caused localized increases in relative humidity even above 100%. The state in which the degree of saturation exceeds the equilibrium value

is quite achievable and is described in a number of works as supersaturation [52–54]. It should also be emphasized that in this case, the onset of this condition is caused not only by intensive interphase heat and mass transfer, but also by the mechanical movement of moist air.

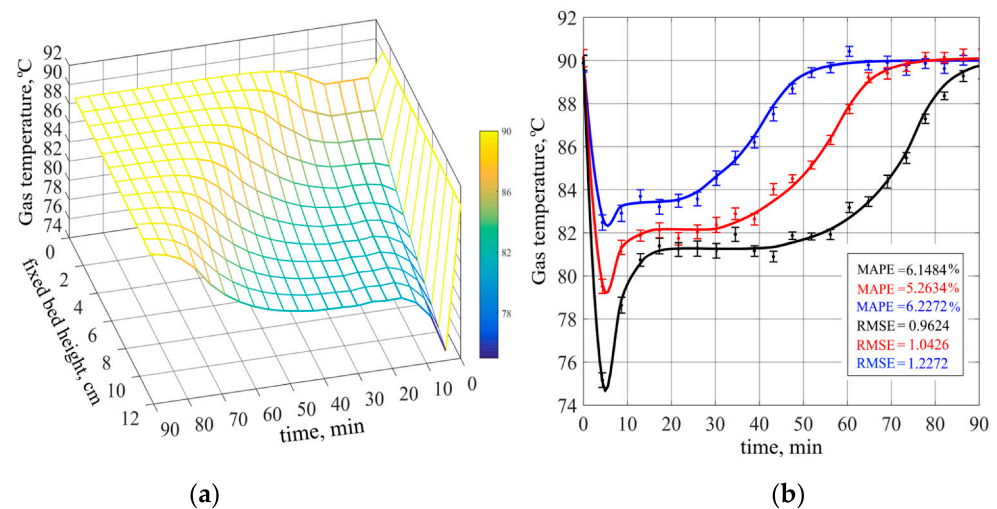


Figure 7. (a) Calculated distribution of the gas temperature inside the particle bed with its height of 12 cm; (b) calculated (lines) and experimental (markers) values of the gas temperature inside the particle bed with its height of 12 cm: blue—at a height of 4 cm, red—at a height of 8 cm, black—at a height of 12 cm.

4. Conclusions

In this paper, we investigated the drying of freshly formed pellets in the fixed-bed column. Drying of this material using active particle mixing is impossible due to its brittleness. Drying in a fixed bed is possible, but it results in significant unevenness in drying conditions across the bed height. To account for this unevenness, we used the MCM, which turned out to be computationally simple enough to use and identify its parameters from literary sources. At the same time, its use made it possible to predict the unevenness of drying conditions along the height of the fixed bed. We did not strive for great spatial detail, so a 1 cm high cell was used as a representative bed volume. As a result, any forecast in the calculation plan was completed within a few minutes using office-level computers.

The obtained predictions and the conducted laboratory experiments are in preliminary agreement, at least for the chosen design and operating mode of the drying column. During the studies on drying fixed beds of material of different heights, the existence of regimes was observed when the fixed-bed height is sufficient to achieve relative air humidity above 100 percent (supersaturation). The supersaturation of moist air was demonstrated both at the experimental and numerical levels. These regimes are obviously non-rational and undesirable from a practical point of view (since they clearly increase the drying time). It was also shown that the appearance of supersaturation zones is related not only to the intensity of heat and mass transfer but also to the nature of air movement. Therefore, such conditions can be prevented by varying the bed height and the drying agent velocity. Identifying and explaining the reasons for the emergence of these regimes is a significant result. Further studies of fixed-bed drying and the formulation and solution of optimal process control problems are planned.

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Abbreviations

The following abbreviations are used in this manuscript:

MCM	Markov chain model
CFD	Computational fluid dynamics
DEM	Discrete element method

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