

Article

Differentiable and Self-Auditing Transient Dynamics Solver for Ball Bearings: OpenBEARD Cross-Verified Against ADORE

Xinlu Yu ^{1,*}, Kai Wang ¹, Yuchen Han ¹ and Yingqian Fu ²

¹ School of Mechanical Engineering and Automation, College of Science and Technology, Ningbo University, Ningbo 315300, China; 2511170024@nbu.edu.cn (K.W.); 2411170020@nbu.edu.cn (Y.H.)

² Key Laboratory of Impact and Safety Engineering, Ningbo University, Ministry of Education, Ningbo 315211, China; 1111081012@nbu.edu.cn

* Correspondence: yuxinlu@nbu.edu.cn

Abstract

A transient multibody dynamics simulation of rolling-element bearings is the basis for the design of high-speed rotating machinery; however, the established solvers are proprietary, cannot be used with automatic differentiation, and offer no built-in measure of their own physical consistency. We present OpenBEARD, an open-source, fully differentiable transient dynamics solver for angular-contact ball bearings. The solver steps a $40 + 13Z$ -component state (inner ring, cage, and Z balls with quaternion attitude, plus guide-patch, lumped-thermal, and energy-audit states) forward in time under coupled Hertzian contact, Hamrock–Dowson and full-multigrid elastohydrodynamic lubrication, thermal–elastohydrodynamic traction, and centrifugal/press-fit clearance models, using nondimensionalized implicit stiff time integration. A built-in metriplectic conservation audit checks energy closure, the second law per dissipation channel, and the gyroscopic-power identity at every output step. OpenBEARD is cross-verified against two published ADORE references of Gupta. For a high-speed NASA angular-contact ball bearing, the quasi-static contact loads, angles, stresses, and centrifugal force match the published values to within 0.3%, and the ball spin and orbital velocities and the spin-axis orientation to $\leq 0.1\%$. The inner-race spin-to-roll ratio—a slip-derived secondary quantity that is the most model-sensitive metric in this class of solvers—differs from the NASA quasi-static reference by 8.8%. In the separate caged BallBearingTestCase benchmark, the corresponding quasi-static difference is 3.2%, and the transient settled value is $\approx 16\%$ above the ADORE step-100 snapshot; these bounded offsets reflect different spin-moment constitutive models. The BallBearingTestCase comparison—a caged bearing under combined thrust and radial load—matches the per-ball contact angles and loads to within 0.23% RMS, and a single published dynamic snapshot (step 100) agrees with the transient contact mechanics to within a few percent. The built-in energy-closure residual stays of order 10^{-5} with no second-law violations. In the fully transient regime, race control emerges as a dynamical attractor of the coupled traction balance—ball-spin states perturbed by $\pm 12\%$ converge to a single outer-race-control solution—rather than the kinematic hypothesis assumed by quasi-static theory. OpenBEARD is released under the MIT license.

Keywords: rolling-element bearing; multibody dynamics; elastohydrodynamic lubrication; transient simulation; verification; code-to-code cross-verification; open-source software; automatic differentiation; energy conservation; race control



Academic Editor: Alexandre Carvalho

Received: 24 June 2026

Revised: 9 July 2026

Accepted: 10 July 2026

Published: 13 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article

distributed under the terms and

conditions of the [Creative Commons](#)

[Attribution \(CC BY\)](#) license.

1. Introduction

Rolling-element bearings are critical components of nearly all rotating machinery, and their internal dynamics—contact loads, contact angles, cage motion, ball slip, friction, and heat generation—set the fatigue life, efficiency, and stability, particularly in high-speed applications such as aero-engine mainshaft bearings and machine-tool spindles [1]. At high speed the centrifugal loading of the rolling elements shifts the contact loads between the inner and outer raceways and tilts the ball spin axis, producing the well-known race-control behavior that a quasi-static analysis can only approximate [2]. An accurate prediction of these effects requires a transient, multibody treatment in which every rolling element, the cage, and the rings are stepped forward in time under fully coupled contact, lubrication, traction, and thermal models.

A large body of work models these internal dynamics at several levels of detail. The simplest models reduce the bearing to a few lumped degrees of freedom, extended to coupled thermal and lubrication conditions [3,4] and to explicit finite-element and planar-multibody treatments that resolve the bearing clearances [5,6]. At the system level these bearing models are built into rotor-bearing and aero-engine studies of nonlinear vibration, skidding, and instability [7–10]. A parallel line of research focuses on localized raceway or rolling-element defects—lumped-parameter [11–15], finite-element [16], and high-speed [17–21] defect-dynamics models—which motivates the development of general-purpose, differentiable solvers whose gradient-based inverse capabilities can ultimately be applied to defect identification (Section 4.5). The most detailed approach to the defect-free internal dynamics steps every rolling element, the cage, and the rings forward as rigid bodies under fully coupled contact, traction, and thermal models—the transient multibody-dynamics method first developed in ADORE [22,23] and used to predict cage and frictional instabilities [24,25].

Despite this range of work, the general-purpose solvers in the most detailed class remain proprietary. The reference solver is ADORE (Advanced Dynamics of Rolling Elements), the code of P. K. Gupta, which has been an industry and research benchmark for more than three decades; Gupta’s monograph also documents ADORE’s experimental-validation history [23]. ADORE and the commercial bearing-dynamics packages that followed [26,27] are mature, but they share three traits that limit their use as research platforms. First, they are closed-source executables, so their constitutive models cannot be inspected, replaced, or extended by the user. Second, their kernels are not differentiable, which rules out the gradient-based sensitivity analysis, design optimization, and inverse parameter identification that are increasingly important in data-driven engineering. Third, they give no built-in, quantitative measure of their own physical consistency: correctness is shown by external validation rather than by a self-check that the discrete solution conserves the invariants of the continuous model.

This paper presents OpenBEARD, an open-source transient multibody dynamics solver for angular-contact ball bearings (deep-groove bearings being the zero-contact-angle special case), implemented in the Julia language [28] and built on the SciML differential-equation ecosystem [29]. OpenBEARD is designed to be quantitatively consistent with the established benchmark while adding the features it lacks. Its entire right-hand side is zero-allocation and automatic-differentiation-compatible [30]; it integrates the fully general six-degree-of-freedom element motion with modern adaptive implicit stiff integrators; it resolves the elastohydrodynamic conjunction with a full-multigrid Reynolds solver [31] in addition to the classical Hamrock–Dowson correlation [32]; and it includes a metriplectic conservation audit that measures energy closure, the second law, and the gyroscopic-power identity at every step.

The contribution of this paper is twofold. We describe the solver and its governing models (Section 2), and we report a systematic verification and code-to-code cross-verification of the solver against two published ADORE references of Gupta (Section 3). The novelty is not that the individual physical closures—Hertz contact, Hamrock–Dowson film thickness, traction, Newton–Euler rigid-body dynamics, and Jones–Harris quasi-static initialization—are new. Rather, the contribution is the open, fully differentiable, fully coupled transient implementation, together with a built-in conservation audit that makes the numerical solution itself physically checkable. The verification uses the conservation audit; the cross-verification compares, per quantity, against the published reference values for a high-speed thrust-loaded bearing and a caged combined-load bearing. We show sub-percent agreement on the quasi-static contact mechanics and an energy-closure residual of order 10^{-5} . Direct experimental validation is outside the present paper and is identified as future work. Section 4 discusses the capabilities that the open, differentiable formulation adds beyond the benchmark, and the transient ball spin-axis dynamics.

2. Materials and Methods

2.1. Solver Architecture and State Vector

OpenBEARD is organized into seven data-oriented layers: an immutable configuration layer; a stateless physics layer (Hertz contact, lubrication, traction, drag, thermal conductance, kinematics, cage guidance); coordinate transforms; a nondimensionalization layer; a quasi-static equilibrium solver; a dynamics layer; and an input/output layer. The conservation audit is a separate diagnostic module. All physics functions in the integration hot loop are written as generic, zero-allocation pure functions that accept both double-precision floats and dual numbers, so the kernel is differentiable end-to-end (Figure 1).

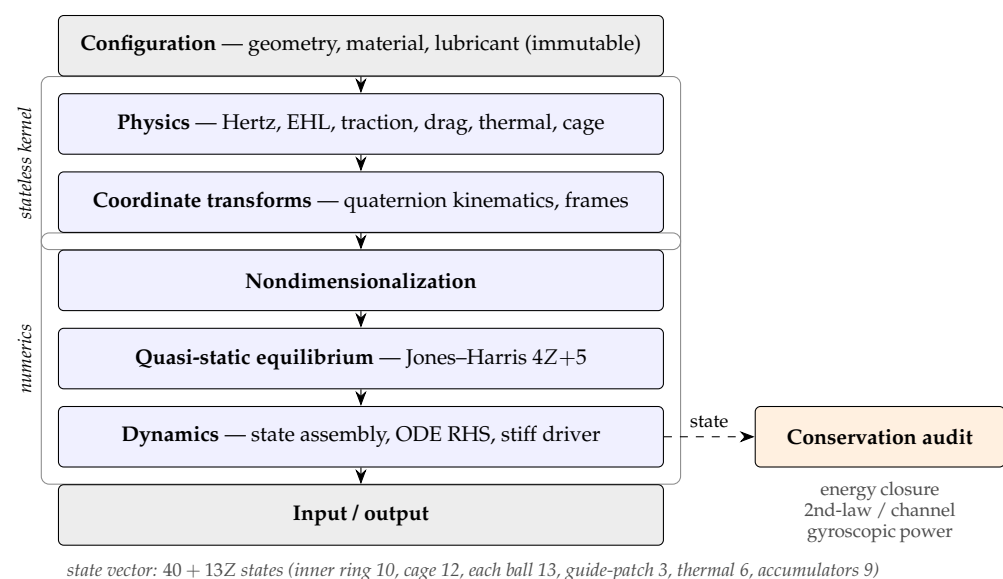


Figure 1. Seven-layer data-oriented architecture of OpenBEARD. Solid arrows show the build and data-flow direction, from the immutable configuration to the input/output layer; the physics-through-dynamics hot loop is zero-allocation and automatic-differentiation-compatible. The conservation audit (dashed) is a separate diagnostic that reads the dynamics state at every output step.

For a bearing of Z balls the dynamic state vector has $40 + 13Z$ components (hereafter states). In this count, a state component is a first-order unknown of the ODE system, not an independent mechanical degree of freedom. The inner ring is described by five kinematic degrees of freedom—three translations and two tilts, with spin prescribed by the shaft speed—and by the five corresponding velocities, giving ten first-order compo-

nents. The cage has six kinematic degrees of freedom and six corresponding velocities, giving twelve components. Each ball has six mechanical degrees of freedom; in the state vector these are represented by three cylindrical-coordinate position components, a four-component unit quaternion subject to the unit-norm constraint for attitude, and six velocity components, giving thirteen first-order components per ball. The remaining eighteen states carry no companion velocities—a three-component guide-patch contact history, a six-node lumped-parameter thermal network, and nine energy/work accumulators used by the audit—so the non-ball part totals $10 + 12 + 3 + 6 + 9 = 40$. The governing equations that advance these states are given in Section 2.3.

2.2. Model Scope and Simplifying Assumptions

The present model is intended as a transient rigid-body bearing-dynamics solver, not as a full structural, wear, or computational-fluid-dynamics model. The rings, cage, and balls are treated as rigid bodies except for local Hertzian contact deformation and the closed-form elastic growth terms used in the clearance model. Plasticity, wear, surface damage, material fatigue, and permanent raceway geometry changes are not included. The outer ring is fixed to ground, the inner ring spin is prescribed by the shaft speed, and shaft/housing flexibility enters only through the imposed boundary conditions and the Lamé/centrifugal clearance correction. The lubricant is assumed to be fully supplied and Newtonian within the traction and film-thickness closures used here; starvation, aeration, debris, non-Newtonian rheology, and detailed lubricant transport through narrow rotating gaps are outside the present scope. Heat transfer is represented by a six-node lumped thermal network, so local fluid-flow structures, turbulent recirculation, and oil-jet impingement patterns are not spatially resolved. These assumptions define the applicability envelope of the current paper; the validity bounds of the individual closures are given in Sections 2.3, 2.5 and 2.6.

2.3. Governing Equations of Motion

The solver steps the fully coupled rigid-body motion of every bearing element forward in time. The inner ring is driven about its axis at the prescribed shaft speed (ramped smoothly from rest, with the drive reaction torque recovered from its rigid-body balance) and the outer ring is fixed to ground; the remaining free coordinates—the three translations and two tilts of the inner ring, the six cage coordinates, and the position and attitude of each of the Z balls—are advanced by the Newton–Euler equations written in their working frames,

$$m_b \dot{\mathbf{v}}_b = \sum_k \mathbf{F}_k^{(b)}, \quad (1)$$

$$\mathbf{I}_b \dot{\boldsymbol{\omega}}_b + \boldsymbol{\omega}_b \times \mathbf{I}_b \boldsymbol{\omega}_b = \sum_k \mathbf{M}_k^{(b)}, \quad (2)$$

where m_b and $\mathbf{I}_b$ are the body mass and inertia tensor, and $\mathbf{v}_b, \boldsymbol{\omega}_b$ its translational and angular velocity. For a ball $\mathbf{I}_b = J_b \mathbf{1}$ is isotropic, so the gyroscopic term $\boldsymbol{\omega}_b \times \mathbf{I}_b \boldsymbol{\omega}_b$ vanishes identically and is retained only for the inner ring and the cage. The right-hand sides collect the Hertzian normal load (with its squeeze-film contact damping), elastohydrodynamic traction, cage pocket and guide-land contact, aerodynamic drag and churning, gravity, the rolling-resistance and spin moments, and—in the rotating ball frame—the centrifugal and Coriolis contributions defined in the constitutive models below. The contact angles α_i, α_o entering these forces are those in Figure 2. Each ball attitude is carried as a unit quaternion $\mathbf{q}_b$ and advanced by

$$\dot{\mathbf{q}}_b = \frac{1}{2} \mathbf{q}_b \otimes [0, \boldsymbol{\omega}_b] + \lambda (1 - \|\mathbf{q}_b\|^2) \mathbf{q}_b, \quad (3)$$

in which the second term is a Baumgarte stabilization [33] that keeps the quaternion at unit norm continuously, avoiding the discrete re-normalization callbacks that would otherwise spoil the step history of multistep integrators. Frictional heat generated in the conjunctions is carried by the six-node lumped-parameter thermal network

$$C_i \dot{T}_i = \sum_j G_{ij} (T_j - T_i) + \dot{H}_i, \quad i = 1, \dots, 6, \quad (4)$$

where C_i is the nodal heat capacity, G_{ij} the inter-node conductance (from film, forced-convection $Nu = 2 + 0.6 Re^{1/2} Pr^{1/3}$ (Ranz–Marshall [34,35], applied to the ball–oil and race–oil nodes as an effective forced-convection closure), and solid-conduction correlations), and $\dot{H}_i$ the local frictional heat; the nodal temperatures feed back into the lubricant viscosity and the thermal expansion. In this lumped network the thermal submodel does not solve the local oil-flow field; it only requires an equivalent convective conductance $G_{conv} = hA$ between a solid node and the oil node. The heat-transfer coefficient is therefore closed as $h = Nu k_{oil} / L_c$, with Re formed from the local relative solid–oil speed and the chosen characteristic length L_c , and Pr from the lubricant properties. The Ranz–Marshall form is used for this conductance because it preserves the isolated-body conduction limit ($Nu \rightarrow 2$ as $Re \rightarrow 0$) while adding the standard forced-convection scaling with Reynolds and Prandtl numbers. The rationale is therefore not that the bearing lubricant flow is identical to the original droplet or sphere experiments, but that the unresolved oil-side transport is represented by a single correlation-based conductance with the correct stagnant-film limit and a monotonic dependence on Re and Pr . Oil-jet, starved, highly turbulent, or strongly thermally stratified regimes—where body rotation, recirculation, narrow gaps, and centrifugal transport modify the local heat transfer—require replacing or calibrating this closure against experiment or CFD.

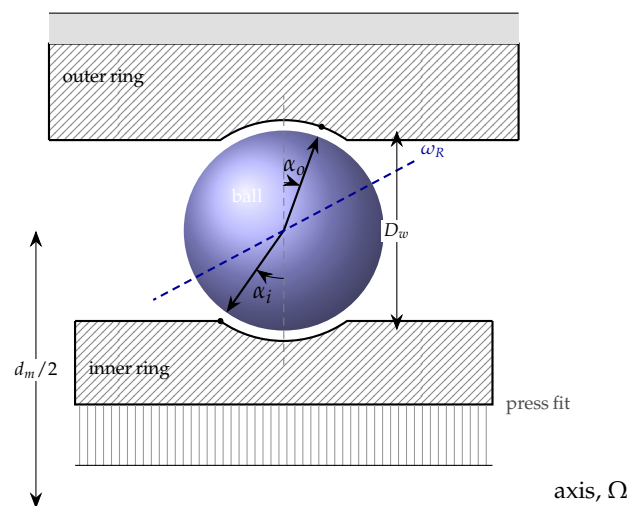


Figure 2. Radial cross-section of the angular-contact ball bearing. Each ball is cradled between the inner and outer raceway grooves and contacts them along load lines inclined to the radial plane by the contact angles α_i and α_o ; at high speed the ball centrifugal force flattens the outer contact and steepens the inner one—the race-control split—while the ball spin axis ω_R tilts away from the bearing axis. d_m is the pitch diameter, D_w the ball diameter, and the inner ring is mounted on the shaft with an interference (press) fit.

2.4. Contact Mechanics and Structural Damping

Each ball/raceway conjunction is modeled as a Hertzian elliptical point contact within the angular-contact kinematics in Figure 2. The load–deflection relation is $Q = Y \delta^{3/2}$ with

$$Y = \frac{4\sqrt{2} E'}{3 \sqrt{\Sigma\rho} \delta^{*3/2}}, \quad (5)$$

where E' is the composite contact modulus, $1/E' = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$, $\Sigma\rho$ the curvature sum, and δ^* the dimensionless contact approach; δ^* and the ellipse semi-axes follow from the curvature-difference parameter $F_\rho = |\rho_{\text{cross}} - \rho_{\text{roll}}|/\Sigma\rho$ through complete elliptic integrals. Because the model is fully coupled, the contact angle is not fixed: at every evaluation α_i, α_o are recovered from the instantaneous ball/groove geometry, and the curvature sum $\Sigma\rho(\alpha)$, the ellipse semi-axes, and the stiffness Y are re-evaluated at that angle, so each conjunction follows its load- and position-dependent osculation rather than a frozen nominal value. The contact is regularized with a C^1 cubic Hermite toe $\delta \mapsto \delta_{\text{sm}}$ so that no force is produced for $\delta \leq 0$ while the exact Hertz law is recovered for $\delta \geq \varepsilon_c$.

Structural damping is added as a squeeze-film term $F_d = c_{\text{sq}} \dot{\delta} g(\delta)$ that is linear in the contact approach velocity $\dot{\delta}$, with a constant per-contact coefficient $c_{\text{sq}} = 2\zeta \sqrt{K_H m_b}$ (critical-damping scaling on a representative linearized contact stiffness K_H , ball mass m_b , and assigned modal damping ratio ζ) and a C^1 gate $g(\delta) = \delta_{\text{sm}}/\delta \in [0, 1]$ that removes the force at $\delta \leq 0$. The approach velocity is the projection of the ball-raceway relative velocity onto the instantaneous contact normal, $\dot{\delta} = \sin \alpha \Delta v_x + \cos \alpha \Delta v_r$, and the inner and outer conjunctions of each ball are evaluated independently. A unilateral anti-adhesion truncation, applied only during separation, keeps the total normal force compressive and renders the dissipated power $P = (Q - F_{\text{Hertz}}) \dot{\delta}$ non-negative by construction.

2.5. Lubrication, Traction, and Drag

The central lubricant film thickness follows the Hamrock–Dowson correlation [32],

$$\frac{h_c}{R} = 2.69 U^{0.67} G^{0.53} \frac{1 - 0.61 e^{-0.73 \kappa_e}}{W^{0.067}}, \quad (6)$$

with the dimensionless speed $U = \eta_0 \bar{u}/(2E'R)$, materials $G = 2\alpha_{pv} E'$, and load $W = Q/(2E'R^2)$ groups, the contact ellipticity κ_e , and the mean entrainment velocity $\bar{u}$; here E' is the composite modulus of Equation (5), so the combination $2E'$ entering the groups is the plane-strain (reduced) modulus of the Hamrock–Dowson convention. The film is reduced for inlet shear heating by a thermal factor $\phi_T = 1/(1 + 0.1 Q_m^{0.64})$ with thermal-loading group $Q_m = 4\beta\mu_0 \bar{u}^2/k$ (optionally replaced by the stronger Harris–Kotzalas reduction parameterized by $L = \mu_0 \beta \bar{u}^2/k$; both benchmark cases in Table 1 use the Q_m form), is gated to zero on unloaded conjunctions, and is optionally multiplied by a correction factor $\phi(U, W, \kappa_e)$ obtained from a full-multigrid (FMG–FAS) solution of the two-dimensional point-contact Reynolds equation [31]. The minimum film,

$$\frac{h_{\min}}{R} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68 \kappa_e}), \quad (7)$$

sets the film parameter $\Lambda = h_{\min}/\sigma$ (σ the composite surface roughness) that gauges the mixed-lubrication regime.

The traction is integrated over the contact ellipse; at each point the isothermal shear follows a four-parameter law [36],

$$\kappa(u) = (A + Bu) e^{-Cu} + D, \quad (8)$$

capped by the limiting shear stress and softened for thermal thinning by a Nahme factor $\phi_T^{\text{tr}} = 1/\sqrt{1 + \beta \Delta T}$, where ΔT is the contact flash-temperature rise from an Archard moving-source estimate [37]. A mixed-lubrication blend weights asperity-borne boundary

shear against the film shear by $w_b = \operatorname{erfc}(\Lambda/\sqrt{2})$, and an ultra-low-speed regularization removes the slip-reversal singularity from the Jacobian. The integration gives the rolling and lateral traction forces, the spin friction moment, the Heathcote rolling resistance, and the sliding and spin heat that drives the thermal network of Equation (4). Aerodynamic drag, Palmgren viscous drag, and three-dimensional churning complete the non-conservative force set.

The speed range of these EHL and traction closures is governed by the same film and heat-transfer indicators used inside the model. At the low-speed end, the film parameter $\Lambda = h_{\min}/\sigma$ marks departure from full-film lubrication; when Λ is small, the prediction should be interpreted as the mixed-lubrication blend above rather than as a pure EHL solution. At the high-speed end, the traction law includes thermal softening through ϕ_T^{tr} , but the present closure does not resolve starvation or oil-supply limits; the same supply-regime caveats as the thermal conductance closure above apply.

2.6. Operating Clearance: Centrifugal Growth and Press Fit

The inner-ring raceway grows radially at speed; the rotating-annulus plane-stress solution gives a centrifugal growth

$$u_{\text{cf}} = c \omega^2, \quad (9)$$

quadratic in the inner-ring angular speed ω , with c a geometry- and material-dependent coefficient. At low and intermediate speed the operating clearance is instead set by the inner-ring interference (press) fit, modeled with the Lamé thick-ring solution for a solid same-material shaft,

$$u_b = \frac{1}{2} I \frac{a}{b}, \quad (10)$$

with I the diametral interference and a, b the bore and raceway radii. Both terms follow the sign convention that a positive value is an outward shift of the raceway that consumes operating clearance (i.e., adds preload). Because the fit is gradually relieved as the spinning ring grows off the shaft, the two contributions are combined not by simple addition but through a C^1 smooth maximum,

$$u^* = \frac{1}{2} \left(u_b + u_{\text{cf}} + \sqrt{(u_b - u_{\text{cf}})^2 + \varepsilon^2} \right), \quad (11)$$

so that the press fit dominates the operating clearance at low speed and the centrifugal growth at high speed, with a smooth transition between them. This is a C^1 approximation to the true behavior, in which the interference-fit bore contact pressure is continuously relieved by the centrifugal field of the spinning ring until it drops to zero and the ring lifts off the shaft; a more rigorous treatment would solve the combined Lamé–centrifugal contact pressure as a single boundary-value problem rather than taking the smooth maximum of the two limiting solutions. For the present geometries the approximation is used over the reported elastic, fully lubricated speed envelope: the caged-bearing sweep of 1909–20,000 rpm and the benchmark operating points at 20,000 and 25,000 rpm (Table 1). As an auxiliary range check, OpenBEARD and ADORE Evaluation (version 9.20)/AdoreQS (version 8.50) quasi-static program were run on the same distributed caged BallBearingTest-Case input from 50 to 40,000 rpm. This scan used ADORE quasi-static evaluation output generated from the distributed example; it is distinct from the published ADORE transient trajectory, for which only the step-0 and step-100 snapshots are available. OpenBEARD converged over the whole scanned range. Using a 5% maximum mean-error criterion over the contact angles, contact loads, centrifugal force, and cage speed ratio, agreement with the ADORE quasi-static evaluation remained below the threshold through 36,250 rpm and crossed it between 36,250 and 36,500 rpm. Within the reported manuscript range,

the crossover between press-fit-dominated and centrifugally dominated clearance is confined to the narrow 14–15,000 rpm band. At 40,000 rpm the maximum mean discrepancy increased to 9.86%, dominated by the inner contact-angle/load split. Outside this range—for example during very low-speed startup, beyond the high-speed agreement envelope, or under large thermal gradients—the smooth maximum should be replaced by a coupled elastic ring/shaft contact calculation. The raceway offset that actually enters the contact kinematics adds the static thermal expansion $\delta r_{\text{thermal}}$ (a preset operating input) to this fit/centrifugal part,

$$\delta r_{\text{op}} = \delta r_{\text{thermal}} + u^*, \quad (12)$$

and the same δr_{op} both seeds the quasi-static initial state and drives the transient kernel, keeping the warm start consistent with the dynamics.

2.7. Quasi-Static Initialization

Initial conditions come from a classical Jones–Harris $4Z + 5$ quasi-static equilibrium [1]: four unknowns per ball plus five global ring displacements, solved by the Levenberg–Marquardt method of NonlinearSolve.jl with residuals normalized to order unity. The equilibrium includes the ball centrifugal force, the gyroscopic moment (optionally neglected to reproduce the benchmark’s quasi-static convention), and the race-control kinematics, and it calibrates the contact stiffness, damping, and thermal conductances of the transient kernel.

2.8. Nondimensionalization, Time Integration, and Implementation

The state is nondimensionalized with the characteristic length, force, and time scales

$$L_0 = r_{\text{ball}}, \quad Q_0 = Y_i L_0^{3/2}, \quad T_0 = \sqrt{m_{\text{ball}}/Y_i} L_0^{-1/4}, \quad (13)$$

where Y_i is the inner-race Hertz stiffness of Equation (5) and m_{ball} the ball mass; this maps nanometer displacements, gigapascal stresses, and kilohertz frequencies onto order-unity quantities and conditions the Jacobian. The nondimensional equations are stepped forward with the QNDF quasi-constant-step backward-differentiation-formula integrator of OrdinaryDiffEq.jl, which reuses its Jacobian across many steps, using a relative tolerance of 10^{-6} , block-wise absolute tolerances (10^{-7} mechanical, 10^{-3} thermal), and an optional arrowhead sparse-Jacobian coloring; the transient is integrated to between 13 and 100 ms, depending on the diagnostic. Integrating a 13 ms transient of the $Z = 18$ caged bearing (274 states) takes on the order of ten seconds on a single desktop CPU core, excluding one-time just-in-time compilation. OpenBEARD is implemented in Julia [28] (version ≥ 1.10): the quasi-static equilibrium uses NonlinearSolve.jl and the transient uses OrdinaryDiffEq.jl, while the $40 + 13Z$ right-hand side is written generically so that it accepts dual numbers, allowing its exact Jacobian to be assembled by forward-mode automatic differentiation (ForwardDiff.jl) for sensitivity and optimization studies. Version v0.9.0 is released under the MIT license, together with the scripts that regenerate every figure and table.

2.9. Conservation Audit

Three independent audits run on the computed solution at every output step: an energy-closure monitor, an angular-momentum balance, and a metriplectic decomposition [38] that splits the generalized forces into conservative, gyroscopic, and dissipative channels (Figure 3). The audit is a diagnostic only; the integrator is a general-purpose BDF, not a structure-preserving metriplectic scheme. The energy closure verifies

$$\Delta E_{\text{mech}} - \Delta W_{\text{ext}} + \Delta H_{\text{gen}} \approx 0, \quad (14)$$

and the angular-momentum balance verifies that the change in total angular momentum equals the impulse of the external torques, $\Delta \mathbf{L}_{\text{total}} \approx \int \mathbf{M}_{\text{ext}} dt$. From the metriplectic split, a per-channel second-law audit checks that each sign-definite damper—squeeze film, drag, churning, structural, pocket, pilot, and cage-oil shear—removes energy on its own, while the coupled traction/spin/rolling family, whose roll, lateral, spin, and Heathcote sub-projections carry cross-terms and may individually take either sign, is required to be dissipative only in aggregate. A gyroscopic-power check verifies the rigid-body identity $\boldsymbol{\omega} \cdot (\boldsymbol{\omega} \times \mathbf{I} \boldsymbol{\omega}) \equiv 0$. A further power-residual check compares the kinetic-energy rate from the integrator's own state derivative against the force-power ledger, revealing curvilinear-coordinate and Coriolis bookkeeping errors that the integrated closure cannot see.

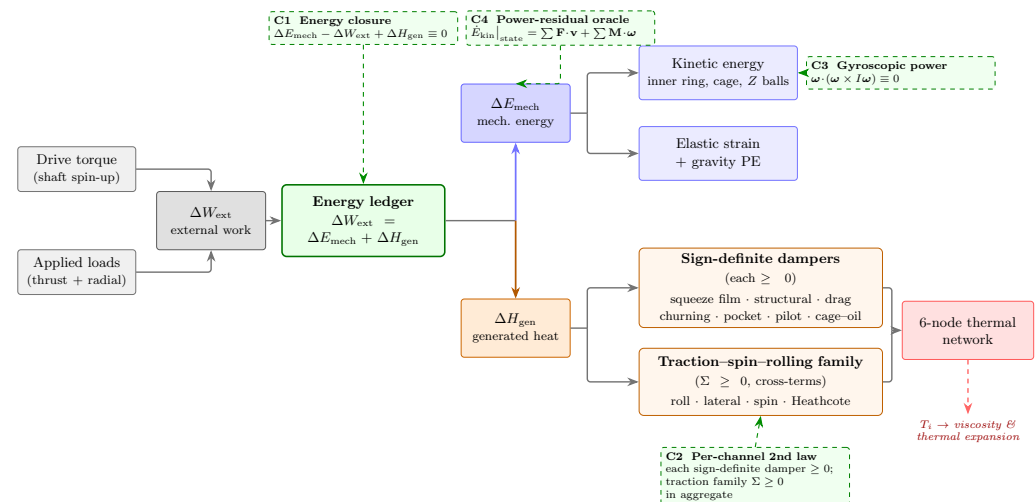


Figure 3. Structure of the built-in metriplectic conservation audit. The external work ΔW_{ext} (drive + applied loads) enters the energy ledger, which partitions it into the mechanical-energy change ΔE_{mech} (kinetic plus elastic and gravitational) and the generated heat ΔH_{gen} ; the heat is attributed to the sign-definite damper channels (each second-law non-negative on its own) and to the coupled traction–spin–rolling family (non-negative only in aggregate), and feeds the six-node thermal network, which in turn updates the lubricant viscosity and thermal expansion. Green dashed callouts mark the four invariants the audit verifies at every output step: energy closure (C1), the power-residual oracle (C4), the gyroscopic-power identity (C3), and the per-channel second law (C2).

2.10. Reference Solutions and Benchmark Cases

The solver is cross-verified against two independent, published ADORE references. The primary benchmark is the high-speed angular-contact ball bearing from the NASA ultra-high-speed test program [39], subsequently used by Gupta [40]: bore 120 mm, pitch diameter 155 mm, outer diameter 190 mm, $Z = 15$ balls of diameter 20.6375 mm, free contact angle 24° , outer/inner groove conformity 0.52/0.54, M-50 VIMVAR bearing steel (elastic modulus 166.8 GPa and Poisson ratio 0.28 at the operating temperature, as tabulated by the reference), lubricated by MIL-L-23699 jet oil, carrying a pure thrust load of 22,240 N at an inner-ring speed of 25,000 rpm and an operating temperature of 492 K. Its reference values are taken from the published quasi-static race-control solution of Gupta [40] (Table 2 in that reference). The second case is the caged BallBearingTestCase distributed as a worked example with ADORE [23]: a $d_m = 140$ mm, $Z = 18$ bearing of 19.05 mm balls in AISI 52100 steel, under a combined 4500 N thrust and 2000 N radial load at 20,000 rpm; its reference values are the published step-0 quasi-static solution of that example. The two cases complement each other—a cageless pure-thrust bearing and a caged combined-load bearing—and their input parameters are listed in Table 1. The NASA bearing therefore has direct experimental pedigree: the NASA tests measured bearing temperature, power loss, operating contact angle, and skidding trends at speeds up to 25,000 rpm and thrust

loads up to 22,240 N [39]. The same experimentally based NASA case was used by Gupta as an ADORE validation/benchmark case [40], so the reference solution adopted here is not detached from experimental evidence. In the present paper, however, the quantities compared in the two main benchmark tables are the published ADORE quasi-static and transient outputs of Gupta [23,40], not the raw NASA measurements. As a limited direct experimental consistency check, the NASA report gives an inner-race operating contact angle of approximately 25° at the maximum-speed/load condition [39]; OpenBEARD predicts 24.89° for the corresponding thrust-bearing condition, as reported in the Results section. Power loss, temperature, and skidding are not used here as direct validation quantities because they depend on test-rig thermal boundary conditions, lubricant churning/drag, and cage-speed measurements that are outside the present cageless contact-dynamics benchmark. The main comparison is therefore reported, in strict verification-and-validation terminology, as a code-to-code cross-verification with experimental heritage and external support, rather than a new direct experimental validation of OpenBEARD.

Table 1. Input parameters of the two benchmark cases.

Parameter	NASA Benchmark	BallBrg Example
Pitch diameter d_m (mm)	155	140
Ball diameter D_w (mm)	20.6375	19.05
Number of balls Z	15	18
Free contact angle ($^\circ$)	24	25
Groove conformity f_o/f_i	0.52 / 0.54	0.52 / 0.54
Rolling-element material	M-50 VIMVAR	AISI 52100
Elastic modulus E (GPa)	166.8	197.75
Poisson ratio ν	0.28	0.276
Lubricant	MIL-L-23699	MIL-L-7808
Inner-ring speed (rpm)	25,000	20,000
Axial load (N)	22,240	4500
Radial load (N)	0	2000
Operating temperature (K)	492	323
Cage	cageless	caged, Z pockets

Both cases use the four-parameter traction law (Tevaarwerk–Johnson from [36], SKF parameter set) $\kappa(u) = (A + Bu)e^{-Cu} + D$ with $A = -0.016$, $B = 0.0383$, $C = 1.718$, $D = 0.016$.

3. Results

We first verify that the discrete solver conserves the invariants of the continuous model, and then cross-verify it quantitatively against two independent published ADORE references—the cageless NASA thrust bearing and the caged BallBearingTestCase under combined load.

3.1. Verification: Conservation Audit

The built-in audit is run on a 100 ms transient of each benchmark geometry (Table 1)—the cageless high-speed NASA thrust bearing and the caged BallBearingTestCase under combined load, two deliberately dissimilar regimes. Because the NASA bearing has no cage, its dynamic run carries only a non-interacting cage placeholder in the state vector (contributing $<0.2\%$ of the dissipation), leaving the dynamics effectively cageless.

Table 2 reports the global first-law balance, which Figure 4 resolves in time. For both bearings the drive work ΔW_{ext} splits, at every instant, into the mechanical-energy rise ΔE_{mech} (the ball complement spinning up from the warm start) and frictional heat ΔH_{gen} , and the closure residual $\Delta E_{\text{mech}} - \Delta W_{\text{ext}} + \Delta H_{\text{gen}}$ remains at the 10^{-6} – 10^{-5} level relative to the mechanical energy throughout.

Table 2. Energy-conservation audit over a 100 ms transient of each benchmark geometry (ball-windage drag disabled). ΔE_{mech} , ΔW_{ext} , and ΔH_{gen} are the changes in mechanical energy, external (drive+load) work, and generated heat over the run.

Quantity	NASA (Cageless, Thrust)	BallBrg (Caged, Combined)
ΔE_{mech} (J)	+188	+59.4
ΔW_{ext} (J)	+795	+241
ΔH_{gen} (J)	+607	+181
Closure / E_{mech}	3.0×10^{-6}	1.3×10^{-5}
Second-law violations	0/2001	0/2001
$\max \boldsymbol{\omega} \cdot (\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega}) $ (W)	3×10^{-18}	5×10^{-20}
Mean external power $\bar{P}_{\text{ext}}$ (W)	7950	2410
max power residual (W)	82 (1.0%)	9 (0.4%)

Figure 5a resolves the BallBrg heat generation over a 100 ms transient into its three leading dissipation channels, each with a distinct temporal signature. Sliding traction (red) rises steeply during the first ~ 3 ms warm-start as the balls undergo large macroscopic slip, then flattens sharply once they lock into steady outer-race-control kinematics. Spin dissipation (blue) remains near zero during the same warm-start interval, then grows at a nearly constant rate as differential spin at the ball/raceway contacts settles into a steady regime. Pilot-land dissipation (green) remains zero until the cage first contacts the inner-ring guide surface (0–3 ms); it then exhibits a characteristic staircase pattern—each step corresponding to an intermittent cage/guide-land contact event during the cage precession cycle, with plateaux between contacts. Once the transient has settled ($t \gtrsim 20$ ms), all three channels converge to constant dissipation rates, with the pilot land dominating at roughly 64%, followed by spin ($\approx 27\%$) and sliding traction ($\approx 9\%$). Each channel removes energy monotonically at every step (no second-law violation) and the rigid-body gyroscopic power vanishes to machine precision, while the instantaneous power-residual oracle stays bounded (Figure 5b)—its peak is a warm-start transient that does not accumulate.

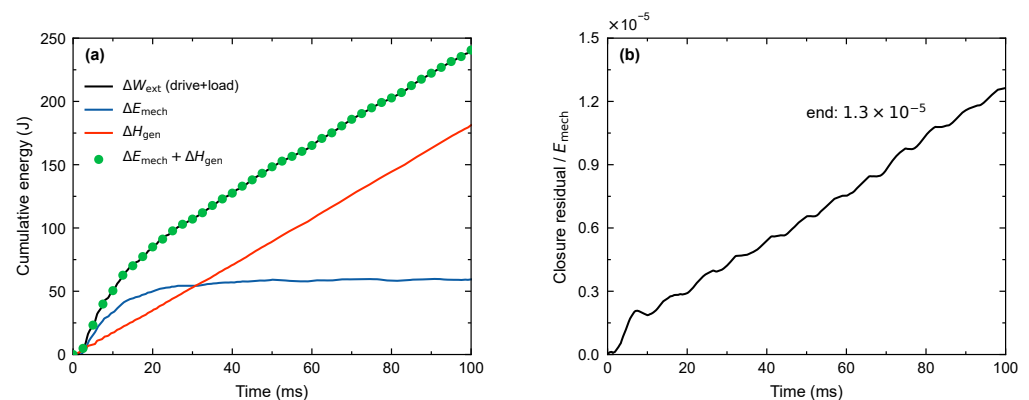


Figure 4. First-law energy balance over the 100 ms transient of the caged 140 mm BallBrg bearing (ball-windage drag disabled). (a) Energy ledger: the sum $\Delta E_{\text{mech}} + \Delta H_{\text{gen}}$ (green markers) coincides with ΔW_{ext} to within the line width, confirming first-law closure (cf. the partitioning scheme in Figure 3). (b) The resulting closure residual, as a fraction of the mechanical energy, grows slowly to 1.3×10^{-5} over 100 ms.

3.2. Cross-Verification Against Published Benchmarks

The two ADORE references probe complementary regimes: the NASA bearing tests the quasi-static equilibrium under pure axisymmetric thrust, while the BallBearingTestCase adds a symmetry-breaking radial load and a cage, and is the case carried into the transient regime. The tabulated quasi-static and transient benchmark comparisons use independently published, peer-reviewed values, so agreement reflects external consistency rather than numerical repeatability. The auxiliary speed-envelope check in Section 2.6 is a separate

quasi-static range check against ADORE Evaluation/AdoreQS output generated from the distributed example, not an additional published transient data set.

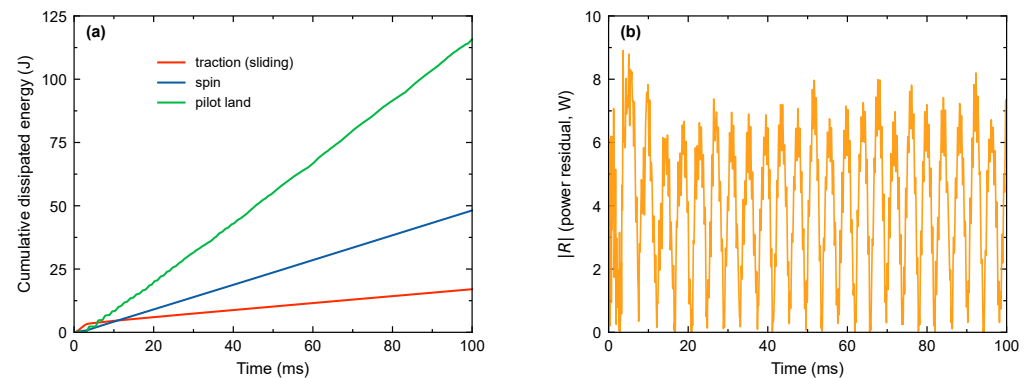


Figure 5. Dissipation channels and frame consistency of the caged 140 mm BallBrg bearing. (a) Cumulative dissipated energy over 100 ms in the leading contact channels—sliding traction, spin, and the cage pilot land—each monotonically non-negative, as required by the per-channel second-law check; all three rates converge to constant slopes after ~ 20 ms. (b) The instantaneous power-residual oracle $|R|$ over 100 ms stays within a bounded band below 9 W with no secular growth, confirming that the rotating-frame formulation carries no false work.

3.2.1. Quasi-Static Equilibrium

Table 3 compares the OpenBEARD quasi-static solver against both published ADORE references. For the NASA thrust bearing, with the M-50 elastic modulus at operating temperature taken from the reference (166.8 GPa), the contact loads, angles, stresses, and centrifugal force all agree to within 0.3%; the ball spin and orbital velocities and the spin-axis orientation to $\leq 0.1\%$; and the inner-race spin-to-roll ratio (with the outer ratio at ≈ 0 , the signature of outer-race control) agrees to 8.8%. For the caged 140 mm bearing under combined 4500 N thrust + 2000 N radial load at 20,000 rpm, all contact angles, loads, stresses, kinematics, and the cage speed ratio agree to within 0.31% RMS per ball; the inner spin-to-roll ratio agrees to 3.2%. Agreement at this level—sub-percent across twenty-four independent comparisons—establishes that the equilibrium solver is consistent with the benchmark.

Table 3. Quasi-static cross-verification of the two benchmark bearings (Table 1) against published ADORE references. NASA: 25,000 rpm, 22,240 N thrust, vs. Gupta [40]; BallBrg: 20,000 rpm, combined 4500 N thrust + 2000 N radial, vs. the published ADORE BallBearingTestCase [23]. Combined-load values vary around the azimuth and are listed as min–max envelopes; deviations are per-ball RMS.

Quantity	NASA Bearing			BallBrg Bearing		
	Ref.	OB	Dev.	Ref.	OB	Dev.
Outer contact load (N)	7257	7241	0.2%	2139–2502	2143–2507	0.20%
Inner contact load (N)	3525	3523	0.1%	295–792	295–791	0.15%
Outer contact angle ($^\circ$)	11.79	11.82	0.2%	3.98–8.39	3.97–8.38	0.23%
Inner contact angle ($^\circ$)	24.87	24.89	0.1%	27.46–30.22	27.50–30.27	0.13%
Centrifugal force (N)	3906	3893	0.3%	1772–1878	1778–1884	0.31%
Outer contact stress (GPa)	1.922	1.925	0.2%	1.509–1.590	1.510–1.592	0.08%
Inner contact stress (GPa)	1.951	1.955	0.2%	1.008–1.402	1.007–1.402	0.06%
Ball angular velocity (krpm)	94.63	94.55	0.1%	75.55–77.82	75.41–77.68	0.18%
Spin-axis orientation ($^\circ$)	169.59	169.57	0.0%	172.6–176.5	172.6–176.5	0.02%
Ball orbital velocity (krpm)	11.14	11.14	0.0%	9.02–9.29	9.04–9.31	0.18%
Outer spin-to-roll ratio	0	0.024	—	≈ 0	≈ 0	—
Inner spin-to-roll ratio	0.283	0.258	8.8%	0.38–0.51	0.37–0.50	3.2%
Cage speed ratio	—	—	—	0.4577	0.4585	0.17%

3.2.2. Transient Dynamics

The caged BallBrg bearing is carried into the transient regime to verify the dynamic kernel. OpenBEARD integrates the full 274-state system (inner ring, cage, and 18 balls $\times$ 13 states each) with a rigid inner-race mount matching the reference boundary condition and ball-windage drag disabled.

Figure 6 shows the OpenBEARD time history of ball #1 over the first 13 ms, overlaid with the published ADORE step-100 per-ball values mapped to the same azimuthal trajectory. The contact loads and the inner contact angle track the reference closely, while the small outer contact angle shows a systematic offset. Figure 7 compares the steady-window azimuthal distribution against the published ADORE step-100 values. The contact loads and the inner contact angle agree to within a few percent, and the cage speed ratio is within 0.17% in Table 3. The small outer contact angle ($3\text{--}8^\circ$) runs about 0.7° below the reference but preserves the same azimuthal shape (Figure 7c). Thus, the visible amplitude difference is treated as a repeatable model-dependent offset rather than uncorrelated numerical scatter. The related spin-derived offset is discussed in Section 4.4.

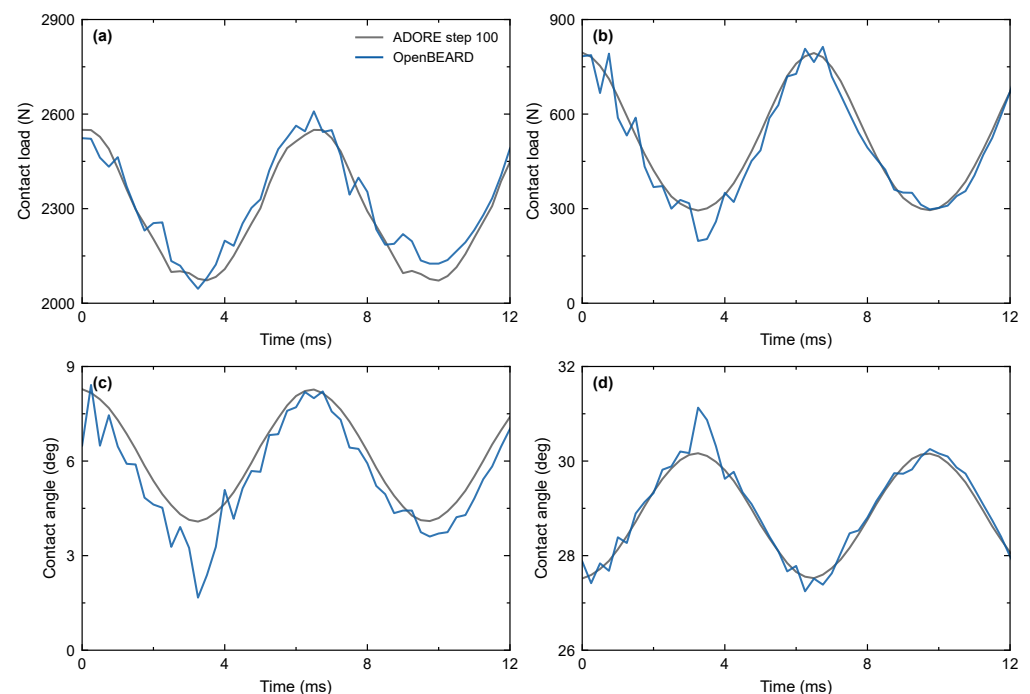


Figure 6. Transient time history of ball #1 on the caged 140 mm bearing (20,000 rpm, combined load) over the first 13 ms: OpenBEARD (blue) against the published ADORE step-100 per-ball values mapped to the same azimuthal trajectory (grey). Panels show (a) outer and (b) inner contact load, (c) outer and (d) inner contact angle.

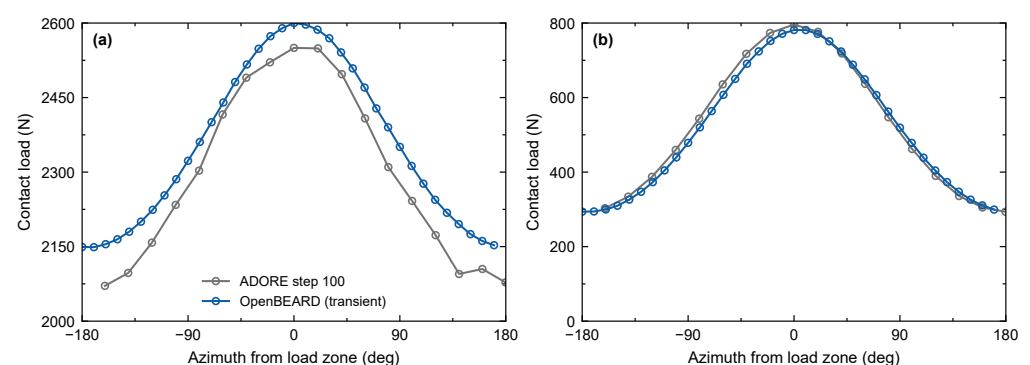


Figure 7. Cont.

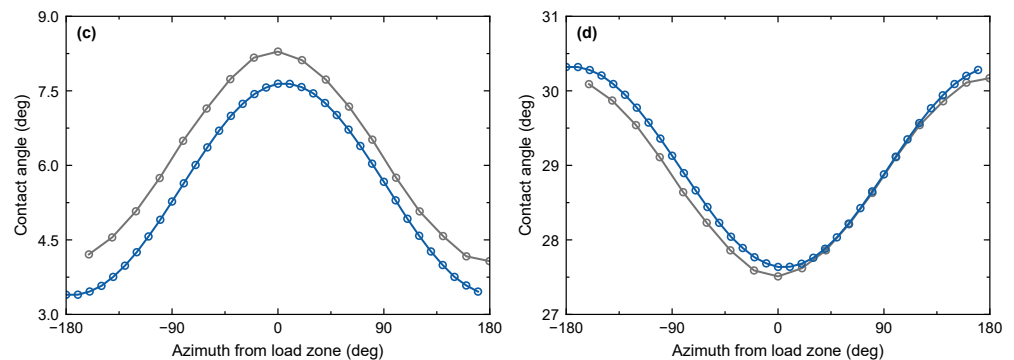


Figure 7. Azimuthal distribution of the contact loads and angles around the caged 140 mm bearing under combined load: OpenBEARD transient steady state (blue markers) against the published ADORE step-100 values (grey markers), aligned at the load zone. Panels show (a) outer contact load, (b) inner contact load, (c) outer contact angle, and (d) inner contact angle.

4. Discussion

Section 3 established that OpenBEARD reproduces the two published ADORE benchmarks to sub-percent accuracy on every primary contact quantity, with a self-audited energy-closure residual of order 10^{-5} and no second-law violations. Rather than competing on the underlying quasi-static physics—which the mature benchmark and the commercial packages that followed it [26] already capture well—OpenBEARD is offered as an open, extensible, and verifiable complement. This section turns from that agreement to what its differentiable and self-auditing formulation adds beyond the reference—inspectable constitutive models, an exact differentiable Jacobian, and the fully general six-degree-of-freedom dynamics across the speed envelope—and finally to the single quantity, the transient ball spin-to-roll ratio, in which the dynamic solution departs from the reference, together with the emergent race control that this departure reveals.

4.1. Capabilities Beyond the Benchmark: Exposed Models, Differentiability, and General Dynamics

The capabilities that set OpenBEARD apart from the benchmark are shown directly in Figures 8 and 9, on the caged 140 mm bearing. Figure 8 shows two constitutive models that a closed solver exposes only as fixed internal correlations. Figure 8a is the thermal-elastohydrodynamic traction curve: the isothermal four-parameter SKF law and its flash-temperature-corrected form at three contact pressures, the correction lowering the traction by up to $\sim 30\%$ at high slip as the pressure—and hence the frictional flash heating in the conjunction—rises. Figure 8b,c are the Hamrock–Dowson central and minimum film thickness versus entrainment speed (growing as $\sim U^{0.67}$) and versus contact load (only weakly load-dependent, $\sim W^{-0.07}$, the classical elastohydrodynamic result). In OpenBEARD each of these is a plain, inspectable, replaceable function rather than a black box.

Figure 9 shows the differentiability and the general dynamics. The entire $40 + 13Z$ -component right-hand side (274 state components for the $Z = 18$ caged bearing) is differentiated by forward-mode automatic differentiation. Figure 9a confirms that the AD-assembled Jacobian matches a central finite-difference Jacobian entry-for-entry across three decades (the line is $y = x$; the residual scatter is the finite-difference truncation error, not an AD error), and Figure 9b shows the resulting sparsity pattern. The top rows and left columns are dense because the inner-ring and cage equations collect contact forces from every ball, and every ball in turn feels the raceway and guide-land geometry set by the ring and cage; the interior is a block diagonal of 18 identical 13×13 blocks because each ball interacts with the raceways and the cage pocket but not directly with any other ball. This arrowhead structure—dense border, sparse interior—allows sparse-colouring techniques to evaluate the full 274×274 Jacobian in far fewer right-hand-side evaluations than the 274 required

by a dense finite difference, an efficiency the stiff integrator exploits at every step. This exact Jacobian is the same object that enables the gradient-based sensitivity analysis, design optimization, and inverse identification that a non-differentiable benchmark cannot provide. The practical advantage is not only computational speed: the AD Jacobian avoids the step-size ambiguity of finite differences near regularized contact, traction, and cage-impact transitions, and it exposes the same derivative information to the time integrator, sensitivity calculations, and future inverse problems. This is why differentiability is treated here as a solver capability rather than as a software convenience. Figure 9c shows a typical output of the fully general dynamics that a quasi-static analysis cannot produce: the cage mass-center whirl orbit over the full 0–100 ms transient, colour-coded from light to dark by time; the trajectory spirals inward from its initial condition and converges within ~ 20 ms to a closed ring of radius ≈ 0.8 mm bounded by the guidance clearance.

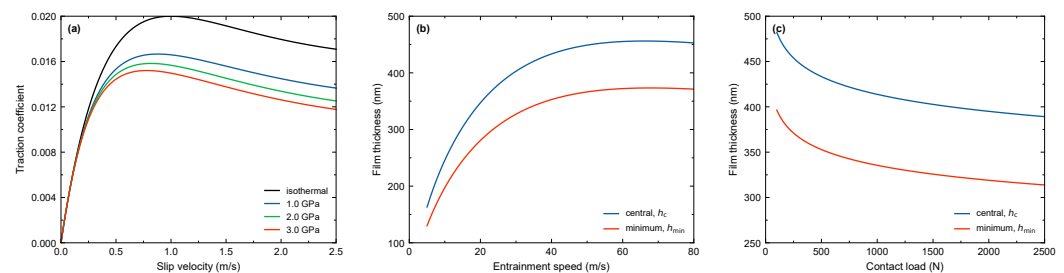


Figure 8. Contact-scale constitutive models exposed by OpenBEARD, for the caged 140 mm bearing. (a) Thermal–elastohydrodynamic traction coefficient versus slip velocity at increasing contact pressure: the isothermal SKF law (black) and its flash-temperature-corrected form at 1.0, 2.0 and 3.0 GPa, the flash heating—and hence the traction reduction—growing with pressure. (b,c) Hamrock–Dowson central and minimum film thickness versus (b) entrainment speed (growing as $\sim U^{0.67}$) and (c) contact load (nearly load-independent, $\sim W^{-0.07}$: a 19% change over a $25\times$ load range).

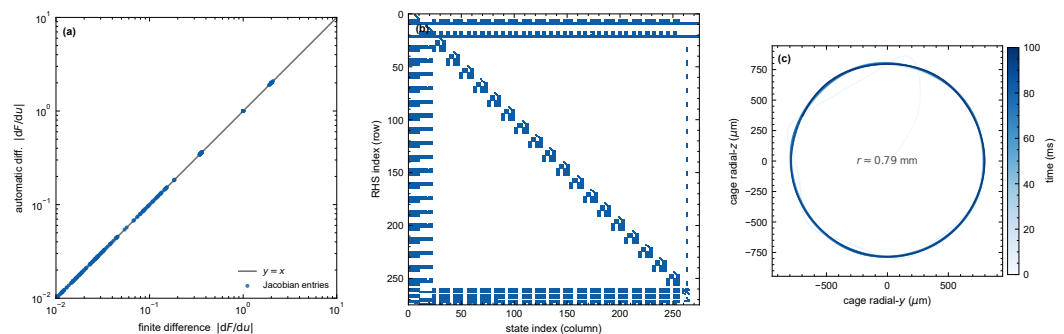


Figure 9. Differentiability and generalized dynamics. (a) The forward-mode automatic-differentiation Jacobian of the full 274-state transient right-hand side reproduces a central finite-difference Jacobian entry-for-entry (log–log; line is $y = x$). (b) Sparsity pattern of that Jacobian (7344 nonzeros): dense rows/columns for the globally-coupled ring and cage states, and a block diagonal of 13×13 per-ball blocks reflecting the absence of direct ball-to-ball contact; this arrowhead structure is exploited by sparse colouring in the stiff integrator. (c) Cage mass-center whirl orbit over the full 0–100 ms transient (colour gradient: light = early, dark = late); the trajectory converges from the initial condition to a closed ring bounded by the guidance clearance within ~ 20 ms.

4.2. Sensitivity, Uncertainty, and Robustness Scope

The present paper does not claim a full probabilistic uncertainty quantification of all model parameters, because that would require independent experimental distributions for contact, traction, cage-guidance, thermal-boundary, lubricant-flow, and manufacturing-tolerance parameters. To make the robustness scope quantitative nevertheless, we performed a deterministic screening sensitivity analysis around the caged BallBrg benchmark. A 1024-point Latin-hypercube sample varied 15 inputs: inner-ring speed

(18,000–22,000 rpm), axial and radial load ($\pm 10\%$), inner-ring diametral fit (40–60 μm), thermal radial offset ($\pm 5 \mu\text{m}$), elastic modulus ($\pm 5\%$), Poisson ratio (± 0.01), ball density ($\pm 2\%$), ball diameter ($\pm 0.5\%$), pitch diameter ($\pm 0.2\%$), groove conformities f_i and f_o (± 0.005), additional diametral clearance (0–20 μm), free contact angle ($\pm 0.25^\circ$), and the shaft-constraint factor in the centrifugal-growth closure (0.8–1.0). All 1024 samples and the accompanying one-at-a-time perturbation cases converged. The resulting 5th–95th percentile output envelopes and dominant linear main effects are listed in Table 4.

Table 4. Deterministic sensitivity screening around the caged BallBrg benchmark. The output envelope is the 5th–95th percentile range from 1024 Latin-hypercube samples. Dominant effects are the fitted low-to-high main-effect changes over the parameter ranges stated in the text.

Output	Baseline	5–95% Envelope	Dominant Low-to-High Effects
Mean outer contact angle $\bar{\alpha}_o$ ($^\circ$)	6.15	5.18–7.33	speed -2.01° ; axial load $+1.06^\circ$
Mean inner contact angle $\bar{\alpha}_i$ ($^\circ$)	28.89	28.31–31.91	$f_o + 2.70^\circ$; $P_d + 1.47^\circ$; $\delta r_{\text{thermal}} - 1.46^\circ$
Mean outer contact load $\bar{Q}_o$ (N)	2303	1977–2664	speed $+759$ N; axial load $+76$ N; $\rho_b + 76$ N
Mean inner contact load $\bar{Q}_i$ (N)	523	455–558	axial load $+99$ N; $f_o - 38$ N
Mean centrifugal force $\bar{F}_c$ (N)	1829	1528–2210	speed $+750$ N; $\rho_b + 75$ N; ball diameter $+54$ N
Cage speed ratio $\bar{\omega}_m / \omega$	0.4585	0.4570–0.4659	$f_o + 0.0065$; $P_d + 0.0035$
Operating radial offset δr_{op} (μm)	30.7	20.3–35.6	speed $+10.75 \mu\text{m}$; $\delta r_{\text{thermal}} + 9.95 \mu\text{m}$; $k_s + 5.95 \mu\text{m}$

The linear main-effect fits for these screened outputs had $R^2 = 0.989$ – 0.999 , so the ranking is controlled primarily by first-order trends over these local engineering ranges. The screen identifies two distinct sensitivity groups: speed and ball-density terms dominate the centrifugal force and outer-race load, whereas groove conformity, clearance, thermal radial offset, and shaft constraint dominate the contact-angle split and cage-speed ratio. This is consistent with the speed-envelope analysis of Section 2.6, where the press-fit/centrifugal clearance closure is the speed-sensitive approximation.

In addition to this parameter screen, the robustness checks tied to the solver claims are as follows. First, the AD Jacobian is checked directly against central finite differences over the full 274-state transient right-hand side (Figure 9a), bounding derivative-assembly error. Second, the first-law, second-law, and gyroscopic-power audits are run over 100 ms on both benchmark geometries (Table 2), bounding numerical energy drift and framework bookkeeping error. Third, the dynamically selected race-control state is tested by perturbing the initial ball spin by $\pm 12\%$ and integrating five independent initial conditions to the same steady branch. This robustness check is discussed further in Section 4.4 and bounds sensitivity to the warm-start spin assumption. Fourth, the operating-clearance speed scan in Section 2.6 bounds the smooth press-fit/centrifugal closure: the maximum mean error over the primary outputs stays below the 5% criterion through ≈ 36 krpm for the caged ADORE geometry. Finally, the spin-moment closure is perturbed deliberately in Section 4.4; replacing the integrated traction moment by a constant-coefficient drilling moment increases the energy-closure residual by more than three orders of magnitude. These tests identify which numerical and constitutive uncertainties affect the present conclusions and which do not.

4.3. Internal Mechanics Across the Speed Envelope

The azimuthal load distribution of this caged combined-load bearing was cross-verified against the published reference in Section 3. Beyond that single operating point, OpenBEARD resolves how the internal mechanics evolve with speed and, because the contact angles are solved rather than prescribed, exposes the mechanism that sets them.

The race-control contact-angle split is a traceable consequence of the coupled $4Z + 5$ equilibrium (Section 2). Each ball orbits at the cage speed ω_m , which tracks the shaft speed ω through the nearly constant cage ratio ($\omega_m/\omega \approx 0.45$, Table 3), and is thrown outward by the centrifugal force $F_c = \frac{1}{2}m_b d_m \omega_m^2 \propto \omega^2$, which enters its radial force balance. With the gyroscopic moment neglected in the benchmark quasi-static convention, the ball's axial and radial balances reduce to

$$Q_i \sin \alpha_i = Q_o \sin \alpha_o, \quad Q_o \cos \alpha_o - Q_i \cos \alpha_i = F_c, \quad (15)$$

so the outer contact must react the centrifugal load on top of the inner radial component. Eliminating the two contact loads gives the split in closed form,

$$\sin(\alpha_i - \alpha_o) = \frac{F_c}{Q_o} \sin \alpha_i, \quad (16)$$

which vanishes at rest—where both angles equal the free contact angle—and opens as $F_c \propto \omega^2$ grows. The outer load Q_o and the inner angle α_i in Equation (16) are not free parameters: they are fixed by the global ring equilibrium, in which the five inner-ring displacements (one axial, two radial, two tilt) shift until the azimuthal sum of the Z ball contact forces balances the applied thrust and radial load. Every ball is therefore coupled through the shared ring position. Raising the speed raises F_c on all balls at once; the rings re-seat to preserve the load balance; and the split opens on every ball—uniformly under axisymmetric thrust, and azimuthally modulated through the speed-dependent $Q_o(\psi)$ once the radial load breaks the symmetry. Figure 10 traces this split and the per-ball centrifugal force across a 1909–20,000 rpm sweep: F_c follows the ω^2 scaling that drives the monotonic growth of the split through Equation (16).

The same centrifugal growth governs the operating clearance. Figure 11 isolates the interference-fit model across the envelope. The inner-ring press fit (Lamé thick-ring solution, Equation (10)) expands the raceway, consumes part of the operating clearance, and lowers the low-speed inner contact angle relative to a fit-free assembly (Figure 11a). This offset is largest at 1909 rpm, where the fit dominates the clearance; it then decays monotonically and, by 20,000 rpm, falls to zero—the fit-free and fitted curves merge (Figure 11b). The cause is the smooth-maximum blend of Equation (11): the centrifugal ring growth ($\propto \omega^2$, Equation (9)) overtakes the fixed interference near 14–15,000 rpm, so the spinning ring grows off the shaft and the fit is relieved. The same ω^2 loading thus leaves two distinct high-speed signatures: it opens the contact-angle split through Equation (16) and erases the press-fit clearance offset.

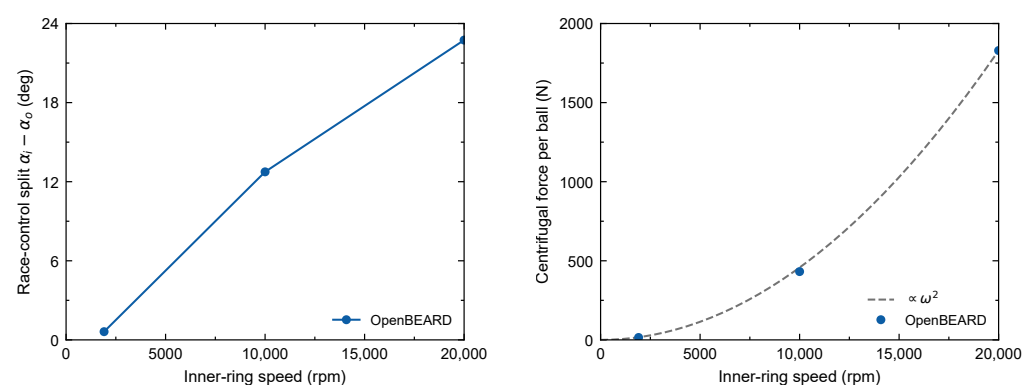


Figure 10. Speed dependence of (a) the inner/outer race-control contact-angle split and (b) the per-ball centrifugal force, predicted by OpenBEARD on the caged bearing across the 1909–20,000 rpm sweep. The centrifugal force follows the ω^2 scaling and drives the monotonic growth of the split.

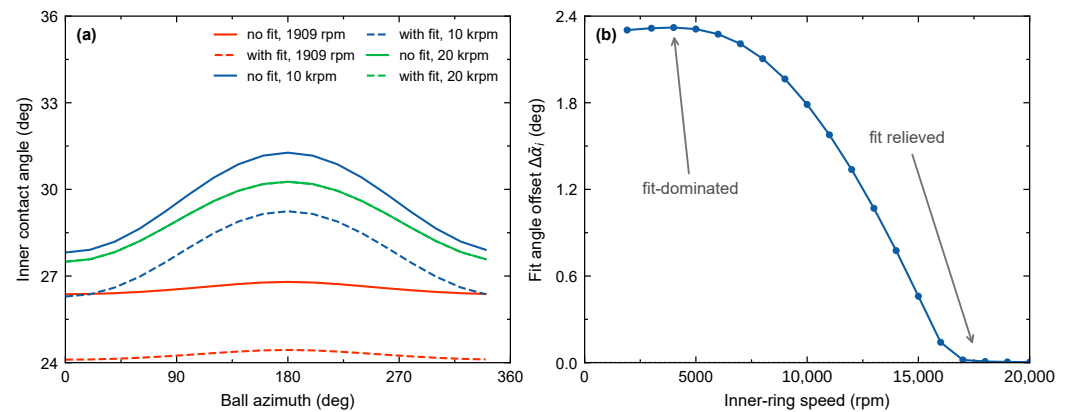


Figure 11. Interference-fit clearance model and its high-speed relief, on the caged 140 mm bearing. (a) Inner contact angle around the bearing, with the inner-ring press fit (dashed) and without it (solid), at 1909 rpm (red), 10,000 rpm (blue), and 20,000 rpm (green). The fit (Lamé thick-ring growth from a 50 μm diametral interference, Equation (10)) consumes part of the operating clearance and lowers the inner contact angle; the offset is largest at low speed and vanishes at 20,000 rpm, where the solid and dashed green curves coincide. (b) The azimuth-mean fit offset $\Delta\bar{\alpha}_i = \bar{\alpha}_i^{\text{no fit}} - \bar{\alpha}_i^{\text{fit}}$ versus speed: it holds near the static fit value below ~ 5000 rpm, then decays through a $\sim 14\text{--}15,000$ rpm crossover to zero as the centrifugal ring growth ($\propto \omega^2$, Equation (9)) overtakes the fixed interference through the smooth-maximum blend of Equation (11)—the spinning ring grows off the shaft and the fit is fully relieved.

4.4. Transient Spin-Axis Dynamics: A Bounded Model Difference and Emergent Race Control

The one quantity that departs most from the reference is the transient ball spin-to-roll ratio (Figure 12). From its race-control initial condition—which matches the ADORE step-0 envelope—the inner ratio overshoots to ≈ 0.64 during the first few milliseconds, then relaxes to a bounded, converged steady state by ≈ 25 ms, with its per-ball mean settling about 16% above ADORE’s converged step-100 envelope (0.41–0.49); the outer ratio remains small (< 0.11), consistent with outer-race control.

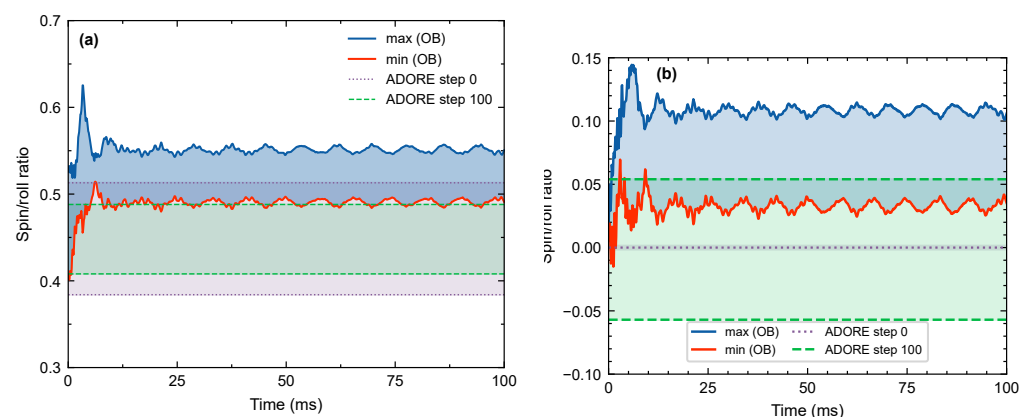


Figure 12. Transient ball spin-to-roll ratio on the caged 140 mm bearing, integrated to 100 ms: the per-ball maximum (blue) and minimum (red) OpenBEARD envelope for (a) the inner and (b) the outer raceway, against the published ADORE BallBrg reference at step 0 (magenta, race-control initial state) and the converged step 100 (green). ADORE’s spin-to-roll ratio is essentially stationary near 0.45 between its initial and converged states, whereas OpenBEARD’s fully dynamic spin axis relaxes to a bounded steady state $\approx 16\%$ above the inner-race reference—a constitutive (spin-moment) model difference.

This is a real model difference rather than a numerical artifact. For the same caged bearing carried into the transient, the quasi-static spin-axis orientation and inner-race spin-to-roll ratio already agree with the published values (Table 3) to 0.02% and 3.2% (the high-

speed NASA bearing, to 0.0% and 8.8%), and in the fully general transient the spin axis is not fixed but evolves under the contact friction moments. The quasi-static ratio sits just below the reference and the transient ratio just above it, because the former is fixed by the race-control kinematic closure while the latter is set by the dynamic friction-moment balance—two separate computations rather than a single biased one. Because a homogeneous sphere carries no gyroscopic restoring torque, the steady tilt is set entirely by the balance of those moments, so the difference depends on how they are modeled. As a direct test on this bearing, replacing the integrated thermal-elastohydrodynamic spin moment by a constant-coefficient (Harris race-control) drilling moment with a representative spin-friction coefficient ($\mu = 0.06$) over-predicts the per-contact drilling torque about fourfold and inflates the energy-closure residual from the audited 1.3×10^{-5} (Table 2) to 1.9×10^{-2} —a self-consistency failure of more than three orders of magnitude that the built-in audit flags directly; the more complete integrated-traction moment, which is energetically consistent, is therefore kept. The remaining difference reflects ADORE's stronger constant-coefficient race-control spin moment and affects only the slip-derived secondary quantities, leaving all primary contact quantities in agreement. This pattern is consistent with Gupta's own assessment of the minimum-energy hypothesis [40]: the ball velocities computed from the minimum-energy closure differ from those of the conventional race-control theory, but “these differences . . . are too small to influence the contact angles and overall contact configuration”—precisely the sub-percent primary-quantity agreement reported in Table 3—while remaining “significant [only] in the computation of frictional dissipation,” i.e., in the spin-derived secondary quantities where our 16% offset appears. Gupta [40] further notes, following the Hirano high-speed experiments, that only the outer-race-control prediction of ball orbital angular velocity matched the experimental data, and that reproducing the experimentally observed tilt of the ball spin axis required the inclusion of gyroscopic moments. The present solver treats a homogeneous ball, for which the gyroscopic term $\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega}$ vanishes identically (Equation (2)), so the steady-state tilt is set entirely by the traction-moment balance; differences in how that balance is closed—integrated thermal-elastohydrodynamic traction here vs. constant-coefficient race-control drilling in ADORE—therefore propagate into the spin-to-roll ratio but not into the contact geometry.

Which of the two spin-moment treatments is closer to the true steady ratio cannot be settled without dedicated dynamic experiment (Section 4.5); what the present evidence does establish is narrower but firm—that the 16% offset is a bounded, energetically self-consistent consequence of the spin-moment constitutive choice rather than a numerical defect, and that the reference solver's own constitutive model carries acknowledged simplifications in the same quantity.

To confirm that this dynamically computed spin axis is determined by the contact physics rather than inherited from its quasi-static initial condition, we perturbed the initial ball spin by $\pm 12\%$ about the race-control value—a 1.8×10^4 rpm spread that brackets both the inner- and the outer-race-control kinematic limits—and integrated each of the five cases independently (Figure 13). All trajectories relaxed to the same outer-race-control steady state within a few shaft revolutions, with the inner-race-control branch acting as a repeller rather than an attractor. For this bearing and operating point, the race-control regime is therefore an emergent attractor of the coupled traction-dynamics balance, computed from the contact friction moments, rather than the kinematic hypothesis imposed in advance by quasi-static theory [2]. This perturbation test establishes the controlling race as an initial-condition-independent attractor and the elevated steady ratio as a robust property of the present model; it does not, by itself, adjudicate that ratio against ADORE, for which only two published snapshots (steps 0 and 100) are available rather than a full trajectory.

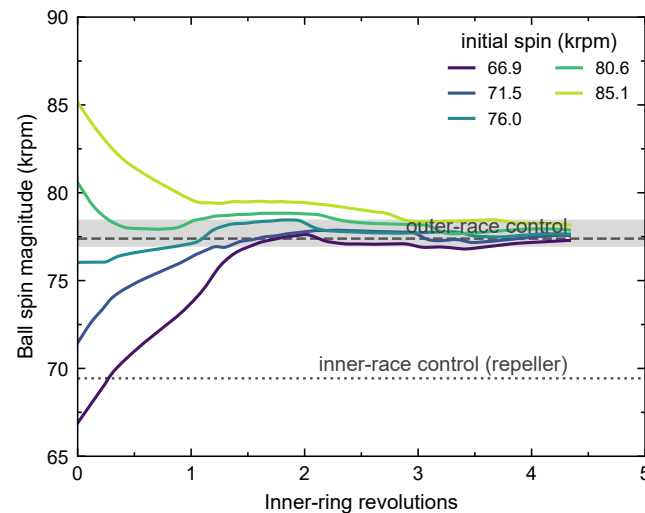


Figure 13. Race control as an emergent dynamical attractor, on the caged 140 mm bearing. The ball spin magnitude is integrated from five initial conditions spanning $\pm 12\%$ about the race-control value (66.9–85.1 krpm), bracketing the inner- and outer-race-control kinematic limits (dotted and dashed). All five relax to the same outer-race-control steady band (grey) within a few inner-ring revolutions, while the inner-race-control branch acts as a repeller. The transient solver therefore computes the controlling race from the contact friction moments rather than imposing it as a kinematic hypothesis.

4.5. Limitations and Future Work

Five limitations apply to the present study. First, OpenBEARD models only ball bearings; cylindrical and tapered rollers are out of the present scope. Second, the current evidence is verification and code-to-code cross-verification (Section 2.10); broader validation against dedicated experiments remains to be done. Third, the transient comparison rests on a single published dynamic snapshot (ADORE step 100); the azimuthal comparison of Section 3.2.2 shows a repeatable load-zone pattern, but the ball spin-to-roll ratio settles about 16% high (Section 4.4), so time-resolved dynamic validation against experiment remains outstanding. Fourth, the Ranz–Marshall convective closure and the smooth press-fit/centrifugal clearance blend are engineering closures with explicit validity bounds (Sections 2.2, 2.3, 2.5 and 2.6); high-speed oil-jet, starved, or strongly thermally stratified regimes require calibration or higher-fidelity flow/structural models. Finally, the robustness checks of Section 4.2 do not constitute a complete global sensitivity or uncertainty analysis. These limitations set a clear development path: extending the contact and inertia models to line-contact roller bearings, validating the transient cage and ball-attitude dynamics against experimental cage-motion and skid data, replacing or calibrating the thermal closure for bearing-specific flow regimes, and using the differentiable kernel for gradient-based inverse identification of internal clearance and localized defects with full uncertainty propagation.

5. Conclusions

We presented OpenBEARD, an open-source, fully differentiable, and self-auditing transient multibody dynamics solver for ball bearings, and cross-verified it against two published ADORE references of Gupta. For a high-speed NASA angular-contact ball bearing the solver matches the published contact loads, angles, stresses, and centrifugal force to within 0.3%, and the ball spin and orbital velocities and spin-axis orientation to $\leq 0.1\%$; for the caged BallBearingTestCase bearing under combined load it matches the per-ball contact angles and loads to 0.23% root-mean-square. Its built-in conservation audit reports an energy-closure residual of order 10^{-5} with no second-law violations. The differentiable kernel, the conservation audit, and the fully general transient dynamics—including thermal-

elastohydrodynamic traction, elastohydrodynamic film, and cage whirl—are shown on the caged bearing. In the transient regime the solver resolves race control as an emergent dynamical attractor rather than an imposed kinematic assumption, a behavior accessible only to a fully general formulation. By combining benchmark-consistent fidelity with open source, automatic differentiability, and a built-in conservation audit, OpenBEARD provides a research platform on which rolling-bearing dynamics models can be verified, differentiated, and extended. Future work will extend the contact and inertia models to cylindrical and tapered rollers and use the differentiable kernel for gradient-based inverse identification of internal clearance and localized defects.

Author Contributions: Conceptualization, X.Y.; methodology, X.Y.; software, X.Y.; validation, X.Y.; writing—original draft preparation, X.Y.; writing—review and editing, K.W., Y.H. and Y.F.; visualization, K.W. and Y.H.; data curation, K.W. and Y.H.; supervision, X.Y.; project administration, X.Y.; funding acquisition, Y.F. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Key Laboratory of Impact and Safety Engineering (Ningbo University), Ministry of Education, grant number CJ202501.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: OpenBEARD is released as open-source software under the MIT license at <https://github.com/yxlnbu/OpenBEARD-solver> (accessed on 23 June 2026) (version v0.9.0; the tagged release is archived at Zenodo, DOI <https://doi.org/10.5281/zenodo.20805936>). The repository includes the benchmark input deck and the scripts that regenerate every figure and table in this paper. The ADORE reference data used for comparison are the published quasi-static solution of Gupta [40] and the BallBearingTestCase example distributed with ADORE [23] (Table 3).

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Harris, T.A.; Kotzalas, M.N. *Rolling Bearing Analysis*, 5th ed.; CRC Press: Boca Raton, FL, USA, 2007.
2. Jones, A.B. A General Theory for Elastically Constrained Ball and Radial Roller Bearings under Arbitrary Load and Speed Conditions. *J. Basic Eng.* **1960**, *82*, 309–320.
3. Bai, X.; Zou, R.; Shi, H.; Wang, Z.; Zhang, X.; Zheng, H. Influence of Lubrication State Transition on Dynamic Characteristics of Deep Groove Ball Bearing with Localized Defect in Thermal Environment. *Int. J. Non-Linear Mech.* **2024**, *167*, 104879. <https://doi.org/10.1016/j.ijnonlinmec.2024.104879>.
4. Shah, D.S.; Patel, V.N. A Dynamic Model for Vibration Studies of Dry and Lubricated Deep Groove Ball Bearings Considering Local Defects on Races. *Measurement* **2019**, *137*, 535–555. <https://doi.org/10.1016/j.measurement.2019.01.097>.
5. Singh, S.; Köpke, U.G.; Howard, C.Q.; Petersen, D. Analyses of Contact Forces and Vibration Response for a Defective Rolling Element Bearing Using an Explicit Dynamics Finite Element Model. *J. Sound Vib.* **2014**, *333*, 5356–5377. <https://doi.org/10.1016/j.jsv.2014.05.011>.
6. Xu, L.; Yang, Y.; Li, Y.; Li, C.; Wang, S. Modeling and Analysis of Planar Multibody Systems Containing Deep Groove Ball Bearing with Clearance. *Mech. Mach. Theory* **2012**, *56*, 69–88. <https://doi.org/10.1016/j.mechmachtheory.2012.05.009>.
7. Harsha, S.P. Nonlinear Dynamic Response of a Balanced Rotor Supported on Rolling Element Bearings. *Mech. Syst. Signal Process.* **2005**, *19*, 551–578. <https://doi.org/10.1016/j.ymsp.2004.04.002>.
8. Wang, H.; Han, Q.; Zhou, D. Nonlinear Dynamic Modeling of Rotor System Supported by Angular Contact Ball Bearings. *Mech. Syst. Signal Process.* **2017**, *85*, 16–40. <https://doi.org/10.1016/j.ymsp.2016.07.049>.
9. Jin, H.; Wang, Y.; Wang, X.; Zhang, X.; Wang, K.; Li, G. Nonlinear Vibrations of a Dual-Rotor System with Aerodynamic Excitation. *Int. J. Mech. Sci.* **2025**, *305*, 110738. <https://doi.org/10.1016/j.ijmecsci.2025.110738>.
10. He, X.; Su, J.; Zhao, T.; Yu, Y. Nonlinear Dynamics of Shaft-Blade Coupling System with Rolling Bearing Support and Blade-Casing Rubbing. *Thin-Walled Struct.* **2026**, *227*, 115024. <https://doi.org/10.1016/j.tws.2026.115024>.
11. Tandon, N.; Choudhury, A. An Analytical Model for the Prediction of the Vibration Response of Rolling Element Bearings Due to a Localized Defect. *J. Sound Vib.* **1997**, *205*, 275–292.

12. Patil, M.S.; Mathew, J.; Rajendrakumar, P.K.; Desai, S. A Theoretical Model to Predict the Effect of Localized Defect on Vibrations Associated with Ball Bearing. *Int. J. Mech. Sci.* **2010**, *52*, 1193–1201. <https://doi.org/10.1016/j.jimecs.2010.05.005>.
13. Tadina, M.; Boltežar, M. Improved Model of a Ball Bearing for the Simulation of Vibration Signals Due to Faults During Run-Up. *J. Sound Vib.* **2011**, *330*, 4287–4301. <https://doi.org/10.1016/j.jsv.2011.03.031>.
14. Cui, L.; Zhang, Y.; Zhang, F.; Zhang, J.; Lee, S. Vibration Response Mechanism of Faulty Outer Race Rolling Element Bearings for Quantitative Analysis. *J. Sound Vib.* **2016**, *364*, 67–76. <https://doi.org/10.1016/j.jsv.2015.10.015>.
15. Petersen, D.; Howard, C.; Sawalhi, N.; Moazen Ahmadi, A.; Singh, S. Analysis of Bearing Stiffness Variations, Contact Forces and Vibrations in Radially Loaded Double Row Rolling Element Bearings with Raceway Defects. *Mech. Syst. Signal Process.* **2015**, *50–51*, 139–160. <https://doi.org/10.1016/j.ymssp.2014.04.014>.
16. Singh, S.; Howard, C.Q.; Hansen, C.H.; Köpke, U.G. Analytical Validation of an Explicit Finite Element Model of a Rolling Element Bearing with a Localised Line Spall. *J. Sound Vib.* **2018**, *416*, 94–110. <https://doi.org/10.1016/j.jsv.2017.09.007>.
17. Qin, Y.; Li, C.; Cao, F.; Chen, H. A Fault Dynamic Model of High-Speed Angular Contact Ball Bearings. *Mech. Mach. Theory* **2020**, *143*, 103627. <https://doi.org/10.1016/j.mechmachtheory.2019.103627>.
18. Li, Y.; Li, Z.; He, D.; Wang, F. Effect of Clearance on the Dynamic Characteristics of High-Speed Rolling Bearings with Multiple Defects. *Mech. Syst. Signal Process.* **2023**, *200*, 110512. <https://doi.org/10.1016/j.ymssp.2023.110512>.
19. Fang, C.; Hu, X.; Chang, Q.; Zhou, W.; Liu, X. Fault Dynamic Modeling for Cylindrical Roller Bearings Considering Cage Fracture and Raceway Defects. *Eng. Fail. Anal.* **2025**, *169*, 109152. <https://doi.org/10.1016/j.engfailanal.2024.109152>.
20. Bai, X.; Chen, G.; Ding, X.; Shao, Z.; Yu, D.; Ma, Y.; Chi, Y. A Nonlinear Vibration Model for Coupled Faults in Aero-Engine Rolling Bearings and Its Experimental Validation. *Mech. Syst. Signal Process.* **2026**, *252*, 114085. <https://doi.org/10.1016/j.ymssp.2026.114085>.
21. Gao, P.; Hou, L.; Yang, R.; Chen, Y. Local Defect Modelling and Nonlinear Dynamic Analysis for the Inter-Shaft Bearing in a Dual-Rotor System. *Appl. Math. Model.* **2019**, *68*, 29–47. <https://doi.org/10.1016/j.apm.2018.11.014>.
22. Gupta, P.K. Dynamics of Rolling-Element Bearings, Parts I–IV. *ASME J. Lubr. Technol.* **1979**, *101*, 293–326.
23. Gupta, P.K. *Advanced Dynamics of Rolling Elements*; Springer: New York, NY, USA, 1984.
24. Gupta, P.K. Frictional Instabilities in Ball Bearings. *Tribol. Trans.* **1988**, *31*, 258–268.
25. Gupta, P.K. Modeling of Instabilities Induced by Cage Clearances in Ball Bearings. *Tribol. Trans.* **1991**, *34*, 93–99.
26. Stacked, L.-E.; Fritzson, D.; Nordling, P. BEAST—A Rolling Bearing Simulation Tool. *Proc. IMechE Part K* **1999**, *213*, 63–71.
27. Gupta, P.K.; Oswald, F.B.; Zaretsky, E.V. Comparison of Models for Ball Bearing Dynamic Capacity and Life. *Tribol. Trans.* **2015**, *58*, 1039–1053.
28. Bezanson, J.; Edelman, A.; Karpinski, S.; Shah, V.B. Julia: A Fresh Approach to Numerical Computing. *SIAM Rev.* **2017**, *59*, 65–98.
29. Rackauckas, C.; Nie, Q. DifferentialEquations.jl. *J. Open Res. Softw.* **2017**, *5*, 15.
30. Revels, J.; Lubin, M.; Papamarkou, T. Forward-Mode Automatic Differentiation in Julia. *arXiv* **2016**, arXiv:1607.07892.
31. Venner, C.H.; Lubrecht, A.A. *Multilevel Methods in Lubrication*; Elsevier: Amsterdam, The Netherlands, 2000.
32. Hamrock, B.J.; Dowson, D. Isothermal Elastohydrodynamic Lubrication of Point Contacts: Part III. *J. Lubr. Technol.* **1977**, *99*, 264–275.
33. Baumgarte, J. Stabilization of Constraints and Integrals of Motion in Dynamical Systems. *Comput. Methods Appl. Mech. Eng.* **1972**, *1*, 1–16.
34. Ranz, W.E.; Marshall, W.R. Evaporation from Drops. Parts I and II. *Chem. Eng. Prog.* **1952**, *48*, 141–146, 173–180.
35. Whitaker, S. Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, Single Spheres, and for Flow in Packed Beds and Tube Bundles. *AIChE J.* **1972**, *18*, 361–371. <https://doi.org/10.1002/aic.690180219>.
36. Tevaarwerk, J.L.; Johnson, K.L. The Influence of Fluid Rheology on the Performance of Traction Drives. *ASME J. Lubr. Technol.* **1979**, *101*, 266–274.
37. Archard, J.F. The Temperature of Rubbing Surfaces. *Wear* **1959**, *2*, 438–455.
38. Morrison, P.J. A Paradigm for Joined Hamiltonian and Dissipative Systems. *Phys. D* **1986**, *18*, 410–419.
39. Bamberger, E.N.; Zaretsky, E.V.; Signer, H. *Effect of Speed and Load on Ultra-High-Speed Ball Bearings*. NASA Technical Note D-7870; NASA Lewis Research Center: Cleveland, OH, USA, 1975.
40. Gupta, P.K. Minimum Energy Hypothesis in Quasi-Static Equilibrium Solutions for Angular Contact Ball Bearings. *Tribol. Trans.* **2020**, *63*, 1051–1066.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.