

Article

Recursive Sliding Bandwidth-Aware Variational Mode Decomposition for Signal Denoising

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Featured Application

The findings of this research can be directly applied to real-time denoising and valid information extraction of streaming vibration data in structural health monitoring (SHM) systems. By implementing the proposed Recursive Sliding Bandwidth-Aware Variational Mode Decomposition—which adaptively determines decomposition parameters via the weighted spectrum trend to eliminate manual tuning subjectivity, and adopts a recursive sliding Fourier-based online update mechanism—practitioners can achieve real-time denoising of continuous data streams. This method balances denoising accuracy and computational efficiency, providing reliable technical support for online SHM and structural state assessment.

Abstract

Structural health monitoring (SHM) relies heavily on the real-time extraction of valid information from raw streaming vibration data. However, the widely used traditional variational mode decomposition (VMD) method has two critical technical bottlenecks, which include key decomposition parameters that rely on manual empirical setting with unavoidable subjectivity and batch processing mechanisms that cannot support online real-time denoising of continuous monitoring data. To overcome these challenges, this paper develops a novel Recursive Sliding Bandwidth-Aware Variational Mode Decomposition (RSBAVMD) method and carries out a full set of scientific research. Firstly, we introduce the weighted spectrum trend (WST) method to adaptively divide the signal spectrum into sub-bands and automatically solves the optimal number of decomposition modes and the matched penalty factor for each mode, eliminating the dependence on manual parameter tuning. Then, to enable online processing of streaming SHM data, we introduce an online iterative update method based on recursive sliding Fourier transform, thus breaking the offline application limitation of conventional VMD. We conduct validation experiments using numerical simulated signals and measured structural vibration data. The results indicate that the proposed RSBAVMD method achieves adaptive determination of components and penalty factors. Compared with traditional VMD and mainstream online denoising methods, this method has the best denoising effect. Compared with the VMD method, the proposed method reduces analysis time by about 80% and is less affected by the amount of analyzed data. The proposed RSBAVMD method can satisfy the real-time denoising requirements of streaming vibration data while ensuring accuracy, providing reliable support for online SHM and structural state assessment.

Academic Editor: Michel Darmon

Received: 29 March 2026

Revised: 2 June 2026

Accepted: 17 June 2026

Published: 14 July 2026

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Keywords: structural health monitoring; real-time signal denoising; variational mode decomposition; online signal processing; recursive sliding Fourier transform

1. Introduction

Structural health monitoring (SHM) systems, as the core technical means to perceive the operational state of in-service structures, continuously acquire massive streaming vibration data to support structural safety assessments and early fault warnings [1]. However, the raw vibration signals collected by on-site SHM systems are inevitably mixed with heavy environmental noise, harmonic interference, and random excitation components, such as vehicle loads and wind-induced vibrations. The effective structural response signals are often submerged in strong background noise, which directly leads to large deviations in subsequent structural state evaluation and fault diagnosis [2]. Therefore, efficient and high-precision signal denoising to extract clean structural responses from raw noisy signals has become an indispensable core preprocessing step for SHM systems, and puts forward requirements for the denoising method: an adaptivity parameter determination and low-latency real-time processing capability [3].

The adaptive signal decomposition method represented by Empirical Mode Decomposition (EMD) has been widely used in non-stationary vibration signal processing [4]. However, it has inherent defects, such as mode aliasing, severe endpoint effects, and a lack of solid mathematical theoretical support [5]. The variational mode decomposition (VMD) method achieves frequency domain adaptive decomposition of signals by solving constrained variational problems, effectively solving the mode-mixing problem of EMD [6]. It has a strict mathematical theoretical basis and has been widely used in the fields of vibration signal denoising and feature extraction. Wang and Markert [7] first systematically analyzed the filter bank characteristics of VMD in 2016, verifying its superiority in bridge vibration signal denoising through numerical simulation and field monitoring data, which laid the foundation for the application of VMD in the field of BHM. Diao et al. [8] applied VMD to the preprocessing of Leak Detection of Liquid Pipelines and found that VMD can effectively separate noise components, significantly improving the accuracy of subsequent damage identification. However, with the deepening of application research, scholars have found that conventional VMD still has core problems that restrict its application in real-time multi-channel BHM systems.

Conventional VMD requires manual pre-setting of two key parameters, i.e., the number of modes and the penalty factor. Inappropriate parameter settings will directly lead to over-decomposition or under-decomposition of the signal, resulting in mode mixing or loss of effective information [9]. The number of decomposition modes determines the granularity of the decomposition, where a value too small leads to under-decomposition, mixing the desired signal with noise, while a value too large leads to over-decomposition, generating spurious modes. The penalty factor controls the bandwidth of each mode, directly affecting the denoising outcome. To solve this problem, Diao et al. [8] proposed an adaptive VMD method based on particle swarm optimization (PSO), which takes the kurtosis value of the IMF as the fitness function to optimize the parameters, and achieved good results. In recent years, various meta-heuristic optimization methods have been widely applied to VMD parameter tuning; Wu et al. [10] extended the PSO-VMD method to traffic speed deflector measurement denoising, using fuzzy entropy as the fitness function to improve parameter optimization accuracy. Zhong et al. [11] proposed a hippopotamus optimization method (HOA) optimized VMD combined with singular spectrum analysis for bridge dynamic load signal denoising, achieving a 16.22% average reduction in RMSE across different noise levels. Yang et al. [12] developed an adaptive

VMD and principal component analysis-based denoising scheme for borehole radar data, which effectively suppresses high-frequency noise while preserving low-frequency target signals. Jiang et al. [13] used the gray wolf optimization method to optimize VMD parameters, and further improved the optimization efficiency. Chen et al. [14] employed WST to guide parameter selection for Adaptive Chirp Mode Decomposition (ACMD), achieving good results in railway bearing fault diagnosis. Additionally, Liu et al. [15] proposed a PSO-VMD method for ground penetrating radar data denoising. Yao et al. [16] developed an adaptive seismic signal denoising method based on VMD, further verifying the effectiveness of intelligent optimization-based VMD parameter adaptation.

These studies demonstrate the effectiveness of frequency band partitioning methods based on spectral analysis for adaptively determining decomposition parameters. However, these intelligent optimization methods need to search for optimal parameters in a multi-dimensional space, which brings huge computational overhead and is easy to fall into local optimal solutions, making it difficult to meet the real-time processing requirements of on-site BHM systems [17].

Conventional VMD requires repeated Fourier transforms during its iterative process, resulting in high computational cost and operating as a batch processing method. When processing long-term, continuous streaming monitoring data, performing VMD independently on each fixed-length data window would lead to massive redundant calculations, making it difficult to meet the requirements for real-time online processing [18,19]. Regarding the online application of VMD, Soman et al. [20] proposed recursive variational mode decomposition (RVMD), which uses recursive FFT and initializations of the center frequencies to reduce the computational complexity of VMD. Dan et al. [21] proposed the Recursive Sliding Discrete Fourier Transform (RSDFT), which uses the Fourier spectrum of the previous data window to recursively update the spectrum of the current window. Liu et al. [22] proposed a parameter-optimized recursive sliding VMD (PO-RSVMD) method for industrial sensor signal processing, which introduces an iterative termination condition based on modal component error mutation judgment and achieves at least 53% acceleration compared to traditional VMD. Zhang et al. [23] designed a real-time tunnel structure monitoring system based on edge computing and VMD, demonstrating the engineering feasibility of online VMD in structural health monitoring. Additionally, a displacement technique has been employed to handle boundary effects in sliding windows, achieving preliminary online decomposition with VMD.

Although these studies have made progress in either parameter adaptivity or online computation, there is still a lack of comprehensive solutions that effectively combine both aspects to create a system capable of both automatic parameter optimization and real-time streaming data processing [22]. This paper proposes a Recursive Sliding Bandwidth-Aware Variational Mode Decomposition (RSBAVMD) method. The core innovation of this method lies in integrating the adaptive parameter determination capability of WST with the online recursive computational framework of RSDFT. The WST automatically identifies effective frequency bands in the signal spectrum to determine the optimal the number of modes and the penalty factor, providing precise initialization for subsequent online decomposition. Then, during sliding window processing, the RSDFT recursively updates the Fourier spectrum, and parameters from the previous window (such as center frequencies and mode components) are reused as initial values for the current window. Coupled with a displacement technique to eliminate boundary effects, this approach ensures high denoising accuracy while achieving efficient online processing of streaming data.

2. Method Fundamentals

2.1. Variational Mode Decomposition

VMD decomposes a real-valued input signal $f(t)$ into K intrinsic mode functions $u_k(t)$, each modeled as an amplitude-modulated–frequency-modulated (AM-FM) signal [6]:

$$u_k(t) = A_k(t) \cos(\phi_k(t)) \quad (1)$$

where $A_k(t)$ is the instantaneous amplitude with the same physical unit as $u_k(t)$, $\phi_k(t)$ is the instantaneous phase, and the instantaneous frequency is $\omega_k(t) = d\phi_k(t)/dt$. The core objective of VMD is to minimize the sum of estimated bandwidths of all modes while satisfying the perfect reconstruction constraint, which is achieved by constructing the following constrained variational problem:

$$\min_{\{u_k\}, \{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \quad (2)$$

$$\text{s.t.} \quad \sum_k u_k(t) = f(t) \quad (3)$$

where $\{u_k\}$ is the set of decomposed IMFs; $\{\omega_k\}$ is the set of corresponding central frequencies, with the physical unit of Hz; $\delta(t)$ is the Dirac delta function; $*$ is the convolution operation; and ∂_t is the partial derivative operation.

To solve the constrained variational problem above, the quadratic penalty factor α and Lagrange multiplier $\lambda(t)$ are introduced to transform the constrained problem into an unconstrained variational problem. The Alternating Direction Method of Multipliers (ADMM) is used for the iterative solution, and the core iterative formulas are as follows:

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i < k} \hat{u}_i^{n+1}(\omega) - \sum_{i > k} \hat{u}_i^n(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k^n)^2} \quad (4)$$

$$\hat{\lambda}^{n+1}(\omega) = \hat{\lambda}^n(\omega) + \tau \left(\hat{f}(\omega) - \sum_{k=1}^K \hat{u}_k^{n+1}(\omega) \right) \quad (5)$$

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega |\hat{u}_k^{n+1}(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k^{n+1}(\omega)|^2 d\omega} \quad (6)$$

where $\hat{\cdot}$ is the Fourier transform of the signal; n is the number of iterations; α is a dimensionless penalty factor whose physical meaning is the regularization strength controlling the modal bandwidth, with larger values leading to narrower modal bandwidths; and τ is a dimensionless update step of the Lagrange multiplier whose physical meaning is controlling the convergence speed of the reconstruction constraint. The iteration process terminates when the convergence criterion is met, and, finally, K components are obtained.

Traditional VMD requires predefining the number of decomposition modes K and a unified penalty factor α . Improper parameter selection leads to severe mode mixing or

over-decomposition. Its global Fourier transform-based calculation has a complexity of $O(N\log N)$, which cannot meet the real-time processing requirements of streaming signals.

2.2. Weighted Spectrum Trend

The weighted spectrum trend (WST) realizes adaptive and accurate segmentation of signal frequency bands by organically integrating the global spectrum trend information and the local energy density weighting factor of each frequency band, which can effectively reduce the interference of noise on the identification of frequency band boundaries and quantify the effective signal strength of each frequency band [14].

For the discrete Fourier spectrum of the input signal $S(f) = [S(f_0), S(f_1), \dots, S(f_{N-1})]^T$, the Fourier basis function linear fitting model is used to extract the spectrum trend, and its matrix form can be expressed as:

$$s = Kc + h \quad (7)$$

where $K = [1, K_c, K_s]$ is the Fourier basis matrix, in which 1 is the all-ones column vector, while K_c and K_s are the cosine and sine basis function matrices, respectively; c is the Fourier basis coefficient vector; and h is the spectrum high-frequency component vector.

The local minima of the spectrum trend are extracted as the boundary points of frequency band segmentation, denoted as $f_{B_l} (l = 1, 2, \dots, L)$, where $f_{B_1} = f_0$, $f_{B_L} = f_{N-1}$, and a total of $L-1$ initial frequency bands are obtained. The spectral energy density of each frequency band is taken as the weighting factor, and the calculation is:

$$W_l = \frac{\sum_{i=1}^{B_{l+1}-B_l} [S(f_{B_l+i})]^2}{B_{l+1} - B_l}, \quad l = 1, 2, \dots, L-1 \quad (8)$$

where $B_{l+1} - B_l$ is the number of sample points of the l -th frequency band, and W_l reflects the energy intensity of the effective signal in the frequency band.

Finally, the weighted spectrum trend (WST) is obtained as:

$$WST(f) = W_l \times s_l(f), \quad f_{B_l} \leq f < f_{B_{l+1}} \quad (9)$$

WST can effectively highlight the dominant frequency bands containing effective signals, suppress the interference of noise frequency bands, and provide an accurate basis for frequency band segmentation for adaptive parameter determination of VMD.

2.3. Basic Principle of Block-Wise Recursive Sliding DFT

The block-wise recursive sliding DFT uses the DFT calculation results of the previous frame to incrementally update the DFT of the current sliding frame without full recalculation, which can greatly reduce the computational complexity of Fourier transform [21].

For a sliding window of length N , when the window moves forward by k sampling points, the DFT result of the previous frame is denoted as $F_{current}[u]$, the input samples of the current frame are $f_{in(k-s)} = [f[N], f[N+1], \dots, f[N+k-1]]$, and the removed samples are $f_{out(k-s)} = [f[0], f[1], \dots, f[k-1]]$. Then, the DFT of the current frame can be calculated incrementally by the following formula:

$$F_{new}[u] = F_{current}[u] \cdot W_N^{ku} + \sum_{s=0}^{k-1} [f_{in(k-s)}(s) - f_{out(k-s)}(s)] \cdot W_N^{(s+1)u} \quad (10)$$

where $W_N^{ku} = \exp\left(j \frac{2\pi ku}{N}\right)$ is the complex exponential rotation factor, and $u = 0, 1, \dots, N-1$ is the frequency bin index.

The computational complexity of traditional FFT is $O(N \log_2 N)$, while the computational complexity of RSDFT is reduced to $O(N)$, which significantly improves the computational efficiency and provides a core foundation for the recursive sliding update of VMD.

3. The Proposed Recursive Sliding Bandwidth-Aware Variational Mode Decomposition

3.1. Adaptive Parameter Determination Method of VMD Based on WST

Based on the frequency band segmentation results of WST, the adaptive determination of the number of decomposition modes is completed. The number of remaining effective frequency bands is counted and taken as the number of decomposition modes K of VMD, ensuring that each effective signal component corresponds to an independent decomposition mode, which fundamentally avoids the problems of under-decomposition and over-decomposition.

Based on the filter bank characteristics of VMD, the penalty factor α has a negative power-law relationship with the modal bandwidth. Combined with the frequency band's bandwidth segmented by WST, the adaptive calculation formula of the penalty factor corresponding to each mode can be derived.

First, the normalized bandwidth of the l -th effective frequency band is calculated:

$$B_k = f_{D_{k+1}} - f_{D_k} \quad (11)$$

where f_{D_k} and $f_{D_{k+1}}$ are the upper and lower boundary frequencies of the effective frequency band, and F_s is the signal sampling frequency.

Decomposition modes K are determined based on the frequency band's segmented results by WST, and the penalty factor α is calculated corresponding to each mode. The formula for calculating the penalty factors is:

$$\alpha_k = \alpha \left(\frac{F_s - B_k}{F_s} \right) \quad (12)$$

Each mode can be matched with a penalty factor that is precisely adapted to the bandwidth of the corresponding frequency band, replacing the unified penalty factor of traditional VMD, which greatly improves the accuracy of mode separation and avoids mode mixing.

3.2. Recursive Sliding Update Improvement of VMD

We improve the traditional VMD for recursive sliding update using two core dimensions to realize low-latency online processing of streaming vibration signals:

First, we replace the full DFT calculation of each frame in the iterative process of traditional VMD with block-wise RSDFT incremental calculation. For the m -th frame of the sliding window, the DFT result of the $(m-1)$ -th frame is directly used to incrementally calculate the Fourier spectrum of the current frame, without full FFT of the whole frame of data, which fundamentally reduces the single-frame computational time.

Second, we use the convergence result of the previous frame as the initial value of the current frame's ADMM iteration. For the m -th frame of the sliding window, after

completing WST frequency band segmentation and parameter determination, the center frequency of each mode and IMF components, after convergence of the $(m-1)$ -th frame, are used as the initial values of the ADMM iteration of the current frame, replacing the traditional random initialization or fixed frequency initialization. This method can greatly reduce the number of iterations required for convergence while avoiding the iteration process falling into local optimum, effectively solving the mode-mixing problem.

3.3. Multi-Scale Permutation Entropy

Multi-scale permutation entropy (MPE) comprises time series permutation entropy of different scales S . The performance of noisy signals and real signals in permutation entropy is different. Due to the increased randomness of signals caused by noise, the permutation entropy of noise components is higher. On the contrary, signal components containing real information may exhibit more ordered changes in permutation entropy at different scales and may exhibit lower permutation entropy. Adopting multi-scale permutation entropy to distinguish between noisy signals and real signals can help extract characteristic information of seismic signals hidden at different scales.

The time series $X = \{X_i, i = 1, 2, \dots, N\}$ with a length N is processed to obtain a coarse-grained sequence $y_j^{(s)}$, which is expressed as:

$$y_j^{(s)} = \frac{1}{S} \sum_{i=(j-1)S+1}^{jS} x_i, j = 1, 2, \dots, [N/S] \quad (13)$$

where S is the scale factor and N is the length of the vibration signal time series.

The coarse-grained signal is performed by phase space reconstruction to obtain:

$$Y_l^{(s)} = \{y_l^{(s)}, y_{l+\tau}^{(s)}, \dots, y_{l+(m-1)\tau}^{(s)}\} \quad (14)$$

where τ is the delay time and m is the embedding dimension.

Each element is sorted into reconstructed components of ascending order and take the column indices of the element positions to form a set of position sequences:

$$S(r) = (l_1, l_2, \dots, l_m) \quad (15)$$

It can be observed that there are $m!$ possible patterns for the position sequence. We calculate the P_i of each position sequence appearing in Y as its probability:

$$P_i = \frac{N_i}{\sum_{i=1}^{m!} N_i} \quad (16)$$

According to the definition of information entropy, the permutation entropy H_{PK} of the time series $X = \{X_i, i = 1, 2, \dots, N\}$ is:

$$H_{PK} = -\sum_{i=1}^{m!} P_i \ln(P_i) \quad (17)$$

When $p_1 = p_2 = \dots = p_{m!}$, H_{PK} has the maximum value $\ln(m!)$. By compressing the permutation entropy to the range $(0, 1)$, the normalized permutation entropy H_{NPK} is:

$$H_{NPK} = \frac{H_{PK}}{\ln(m!)} \quad (18)$$

3.4. Framework of the Proposed RSBVMD Method

The proposed RSBVMD method constructs an end-to-end online adaptive denoising framework for streaming vibration signals, which fully adaptive parameter matching without manual intervention and incremental recursive calculation for low-latency online processing. The overall execution flow follows a closed-loop sliding recursive logic, which is divided into five sequentially executed and logically connected core stages: (1) initial signal decomposition and denoising, with sliding update of data frames on this basis; (2) spectrum trend extraction based on recursive sliding discrete Fourier transform, with determinations of the number of decomposition modes and the corresponding penalty factor according to the WST results; (3) recursive sliding VMD iterative decomposition stage based on recursive sliding discrete Fourier transform; (4) for each decomposed feature component, the multi-scale permutation entropy of each component is calculated to obtain its entropy value and eliminate noise components; and (5) a final sliding window update and loop execution judgment stage. The complete method logic is shown in Figure 1, and the detailed description of each stage is as follows:

(1) Initial Signal Processing and Data Frame Sliding Update. This stage is the preprocessing link of the method, which completes the preprocessing of the original vibration signal and the construction of the sliding data frame. First, the components and linear trends of the original vibration signal are removed to eliminate the interference of low-frequency trend components on subsequent spectrum analysis; then the initial sliding window is constructed to complete the initial signal decomposition and denoising, and the data frame is updated in a sliding manner according to the set window length and step size, so as to realize the block processing of streaming signals.

(2) Spectrum Trend Extraction and Frequency Band Adaptive Segmentation Based on RSDFT. This stage is the core link to realize the adaptive parameter matching of the method. First, the recursive sliding discrete Fourier transform is used to incrementally calculate the Fourier spectrum of the current data frame, avoiding the full FFT recalculation of each frame and reducing the computational overhead. Then, the spectrum trend of the current frame signal is extracted based on the Fourier model fitting method, and the weighted spectrum trend is constructed by combining the energy density weighting factor of each frequency band. Finally, the number of effective frequency bands is determined according to the WST results, which is used as the number of decomposition modes of VMD, and the corresponding frequency band aware points are determined at the same time.

(3) Recursive Sliding VMD Iterative Decomposition Based on RSDFT. This stage is the core link of signal adaptive decomposition. Based on the number of decomposition modes and penalty factors determined in the previous stage, combined with the bandwidth–penalty factor mapping model, the penalty factor corresponding to each mode is calculated adaptively. Then, the alternating direction method of multipliers (ADMM) is used to carry out the iterative decomposition of recursive sliding VMD, and the convergence results of the previous frame is used as the initial value of the current frame iteration; consequently, this reduces the number of iterations and improves the computational efficiency, and the mode components of the signal are finally obtained.

(4) Noise Component Elimination Based on Multi-Scale Permutation Entropy. This stage is the key link to realize signal denoising and effective feature retention. For each component obtained by decomposition, the multi-scale permutation entropy of each component is calculated to quantify the randomness and complexity of the component. The component with an entropy value higher than the set threshold is judged as a pure noise component and eliminated, whereas the component with an entropy value lower than the threshold is retained as an effective feature component containing the target signal information, thus completing the screening of effective components.

(5) Sliding Window Update. This stage is the closed-loop link to realize continuous online processing of streaming signals. The sliding window is pushed forward according to the set step size to complete the update of the data frame. Then, the method returns to the spectrum calculation and frequency band segmentation stage to execute the processing flow of the next frame.

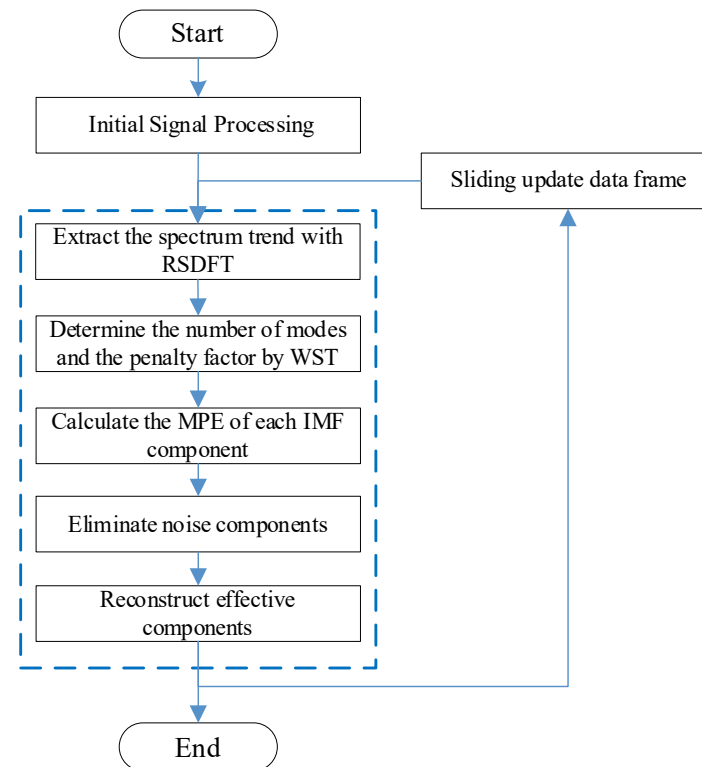


Figure 1. RSBAVMD Method flow chart.

4. Experimental Analysis and Results

To validate the effectiveness of the proposed method for vibration signal denoising, a set of noisy simulation signals containing two components with different frequencies and amplitudes is designed.

The experiments are designed to evaluate the performance of RSBAVMD in comparison to VMD. Simulation experiments demonstrate the signal decomposition capabilities and real-time computational efficiency to evaluate its performance in comparison to VMD. The experiments focused on real-time performance and accuracy of signal decomposition under noise.

The Root Mean Squared Error (RMSE) was used to evaluate the performance indices of the method. The RMSE formula is as follows:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^N |x(n) - \hat{x}(n)|^2} \quad (19)$$

where $x(n)$ is the true component of the signal, $\hat{x}(n)$ is the component obtained after signal decomposition, and N is the length of the signal. The closer the decomposed signal is to the true component, the smaller the RMSE value, indicating better decomposition performance.

4.1. Simulated Signal

To validate the effectiveness of the proposed method for vibration signal denoising, a set of noisy simulation signals of 50 Hz for 60 s, containing components with different frequencies and amplitudes, is designed. The signal is as follows:

$$\begin{cases} s_d(t) = 5e^{-0.1t} \cos(4\pi t + \pi/4) \\ s_1(t) = 1.8 \cos(14\pi t) \\ s_2(t) = 0.8 \cos(30\pi t) \\ x(t) = s_d(t) + s_1(t) + s_2(t) + \text{noise} \end{cases} \quad (20)$$

where $s_d(t)$ is damped vibration mode representing the free vibration response, $s_1(t)$ and $s_2(t)$ are intermodulation components representing the nonlinear intermodulation distortion between different vibration modes, and *noise* represents Gaussian noise. The composite simulation signals are shown in Figure 2.

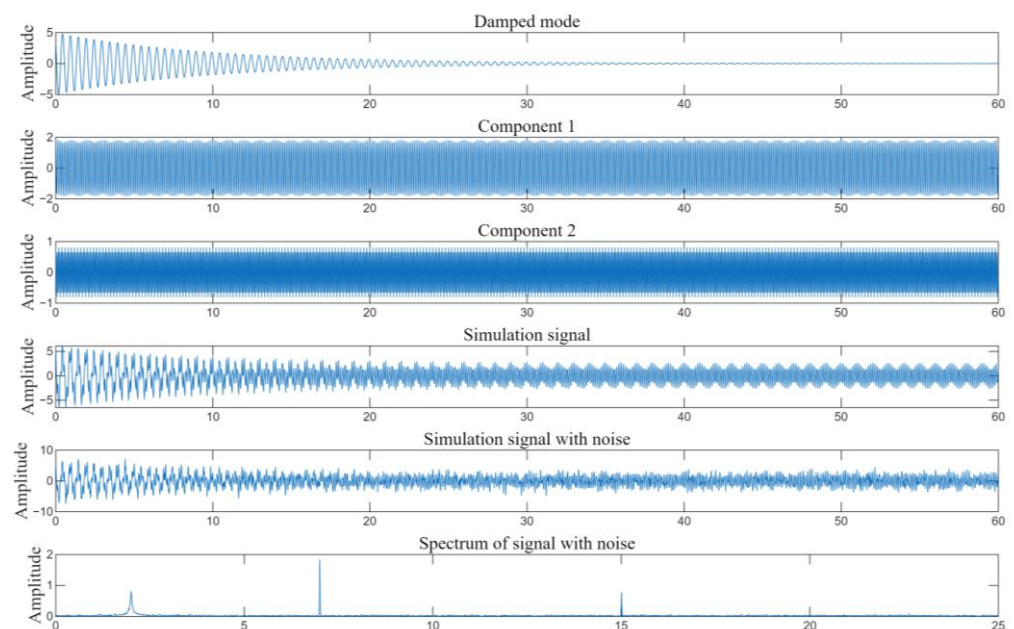


Figure 2. Simulated signals.

From the time domain and frequency domain plots of the simulation signals in Figure 2, it can be seen that, after adding noise to the signal, numerous glitches appear, and the impact of noise on the simulation signals is significant. The simulation signals are decomposed using RSBVMD, and the frequency spectrum of each component is calculated. The results are shown in Figure 3.

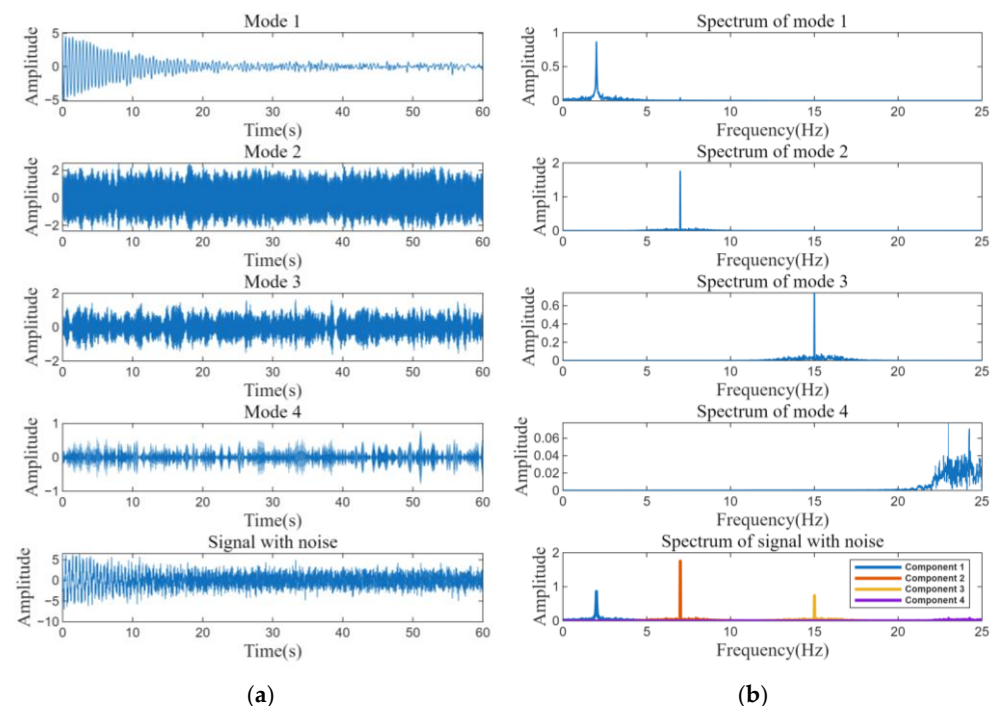


Figure 3. Decomposition results for simulated signals. (a) Time domain plots for decomposition results. (b) Frequency domain plots for decomposition results.

Decomposition mode components of the simulation signals and corresponding spectra are as shown in Figure 3. It can be observed that the noise is mainly concentrated in the high-frequency part of the signals. The simulation signals are decomposed into four feature components. The first three components are low-frequency components, reflecting modal characteristics without mode mixing, while the fourth component is a high-frequency component with apparent noise. The multi-scale permutation entropy method is used to calculate the entropy value of each feature component. The corresponding entropy value and SNR of each feature component are shown in Table 1.

Table 1. Entropy values of components for simulated signal decomposition.

Indicators	Component 1	Component 2	Component 3	Component 4
MPE	0.42	0.48	0.49	0.65
SNR	0.58	0.69	0.36	0.15
CC	5.98	7.23	2.82	0.21

The MPE value of the fourth component in Table 1 fluctuates significantly with high signal complexity, and this component has a low SNR with the noise, indicating that the signal complexity is mainly controlled by noise. This component is identified as the critical noise signal, consistent with the description in Figure 3. Based on the components of multiple signals, the threshold of MPE is selected as 0.6.

After removing the noise component and reconstructing the signal, it was found that the signal denoised had a high degree of similarity with the simulation signal without added noise, with the correlation coefficient of 0.96, proving that this method effectively reduces noise and restores the true signal, as shown in Figure 4.

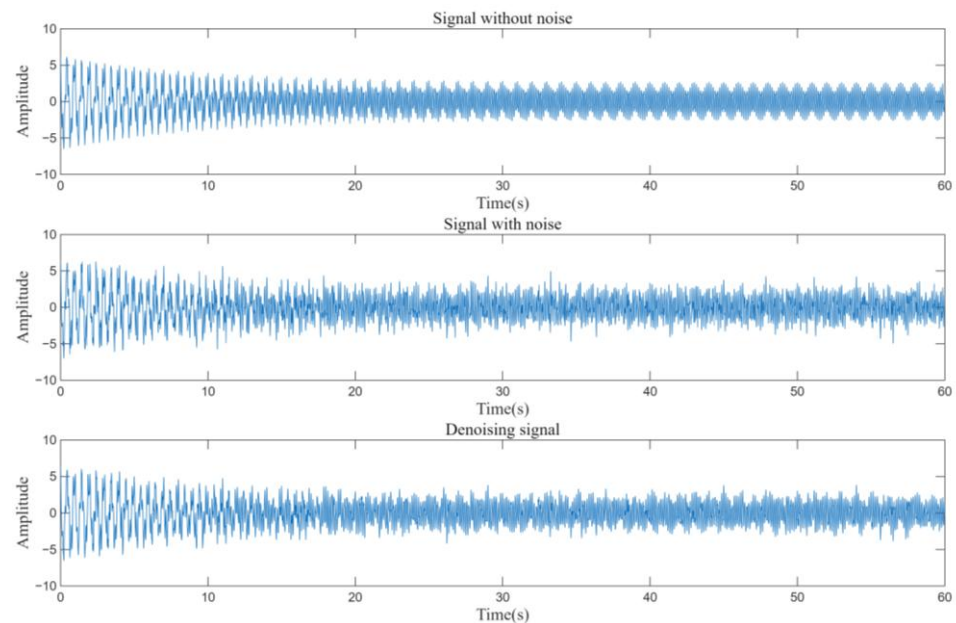


Figure 4. Comparison of simulated signals and denoising signal.

To objectively evaluate the performance of the proposed RSBVMD method, this paper selects mainstream signal denoising methods for comparison, including classical VMD, Empirical Mode Decomposition (EMD), Ensemble Empirical Mode Decomposition (EEMD), and Bandpass Filter (BF). All methods are implemented and used unified evaluation indices. Four core quantitative evaluation indices, including Root Mean Squared Error (RMSE), Signal-to-Noise Ratio (SNR), correlation coefficient (CC), and analysis time, are used to comprehensively evaluate method performance from multiple dimensions.

As shown in Table 2, the proposed RSBVMD method significantly outperforms all other comparative methods on accuracy indices. Compared with the VMD method, the proposed method reduces RMSE by 31.8%, increases SNR by 3.34 dB, and reduces analysis time by 80.2%. These results demonstrate that the RSBVMD method not only achieves superior denoising performance but also has a remarkable advantage in computational efficiency. The visual comparison in Figure 4 shows that the denoised signal has a high correlation with the original signal, indicating that the proposed method can effectively suppress noise. The analysis results are consistent with the actual situation, verifying the effectiveness of the proposed method.

Table 2. Comparison of different methods for simulated Signal.

Method	SNR	RMSE	CC	Time (s)
EMD	5.4	0.80	0.88	0.02
EEMD	5.5	0.79	0.88	0.52
BF	6.5	0.70	0.88	0.03
VMD	8.2	0.69	0.91	0.81
RSBVMD	11.54	0.47	0.96	0.16

The influences of sliding window size and overlap step on the core iteration number of the proposed method are studied for the perspective of computational efficiency, as shown in Figure 5.

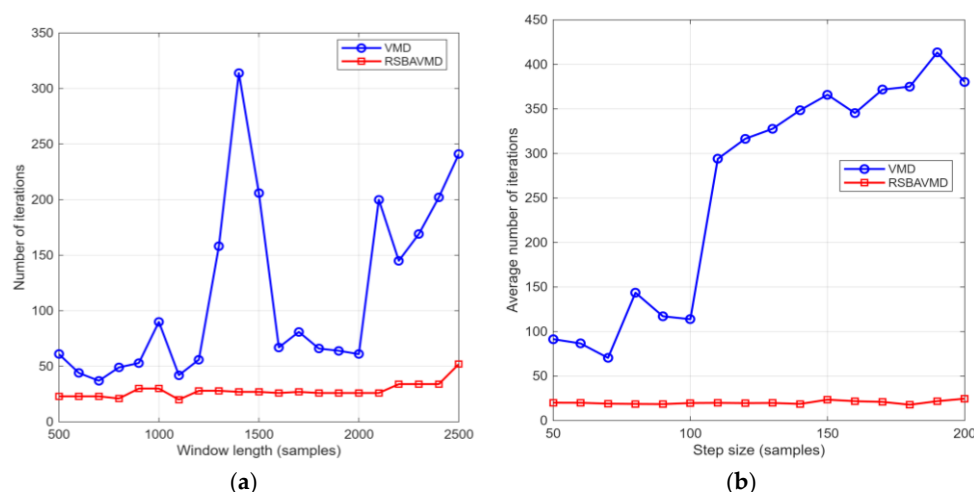


Figure 5. Influence of sliding window size and sliding step size on number of iterations. (a) Variation in iterations with window size; (b) variation in iterations with sliding step size.

As shown in Figure 5, the sliding window size and sliding step size have minimal impact on the convergence speed of the proposed BAACMD method. When the window size increases from 500 points to 2500 points, the average number of iterations only increases from 23 to 52, with a growth rate of about 126%. This is because the recursive sliding update mechanism uses the decomposition result of the previous frame as the initial value, enabling the algorithm to converge to the optimal solution with only a small number of iterations regardless of the window size. In contrast, the number of iterations and analysis time of traditional batch VMD methods are strongly positively correlated with the window size, typically requiring 50–200 iterations to converge. This advantage makes the BAACMD particularly suitable for real-time signal processing applications.

Figure 6 shows the performance of the proposed RSBVMD and VMD. As shown in Figure 6a,b, the deviations of estimated center frequencies by RSBVMD are lower than VMD, which means the results of center frequency recognition of RSBVMD are better than VMD. Figure 6c shows the time taken by the two method types when analyzing different amounts of data. Analysis time required for VMD increases rapidly as the amount of data increases. However, the impact on analysis consumed for RSBVMD as the amount of data increases is not significant through sliding updating. The analysis duration remained at the polar level with almost no change.

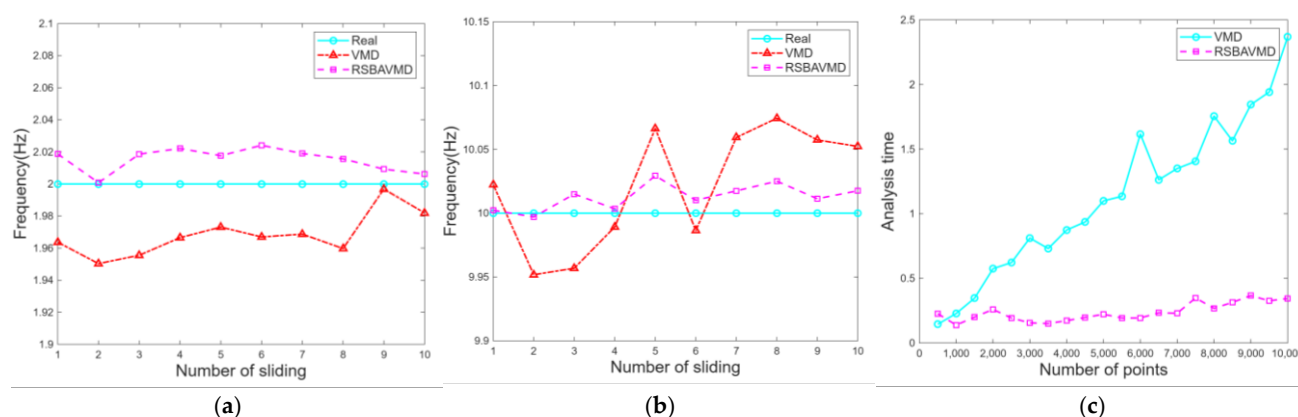


Figure 6. Comparison of performance of the proposed RSBVMD and VMD methods. (a) The center frequency of component 1. (b) The center frequency of component 2. (c) Analysis time required for RSBVMD and VMD as the amount of data increases.

4.2. Vibration Signal

The prototype bridge, located in China, is a three-tower cable-stayed bridge, with the spans of 110 + 129 + 258 + 258 + 129 + 110 m. A total of 24 accelerometers with a sampling frequency of 50 Hz were installed at the stay cables to monitor the acceleration of the stay cables, as shown in Figure 7. Stay cables are lightweight and highly flexible, making them highly susceptible to abnormal vibrations caused by wind and vortex-induced vibration (VIV) under various external forces, which can potentially damage the cable stays and affect the service life [24]. It is of great significance to accurately remove noise signals during vortex vibration and extract effective vortex vibration modes of inclined cables.

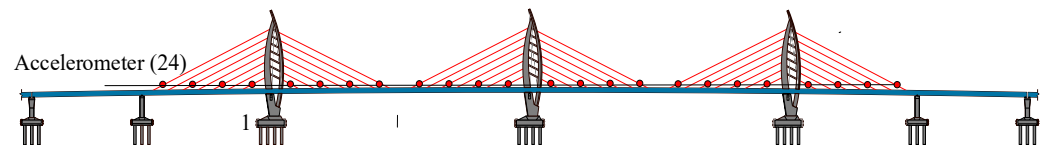


Figure 7. The layout of accelerometers.

According to the analysis time of data with different window lengths and step sizes in the proposed method, it can be seen that different window lengths and step sizes have no significant effect on the analysis time. Therefore, the window length can be selected based on other subsequent evaluation requirements of the signal. The step size can be selected based on the real-time data reception situation. The real-time interface using the most sensor data pushes the latest data every second. Therefore, this paper chooses a window length of 1 min and a step length of 1 s.

To validate the feasibility of the proposed method, the monitoring data of acceleration are analyzed during VIV are selected for analysis, as shown in Figure 8.

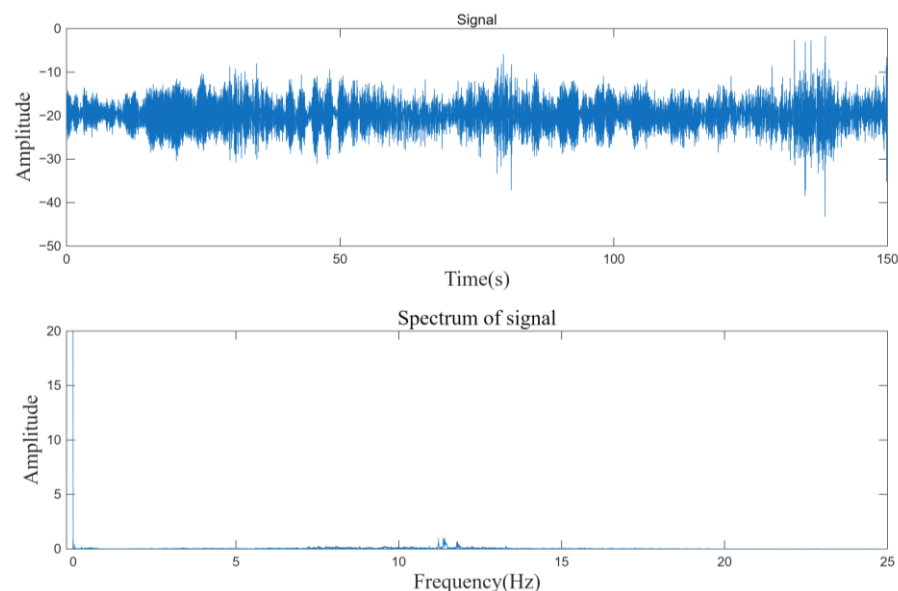


Figure 8. Vibration signal in VIV.

As shown in Figure 8, it can be seen that the signal is significantly affected by noise. The VIV signal is not obvious, which is similar to that of the noise signal. The signal is decomposed, and the frequency spectrum of each component is calculated. The results are shown in Figure 9.

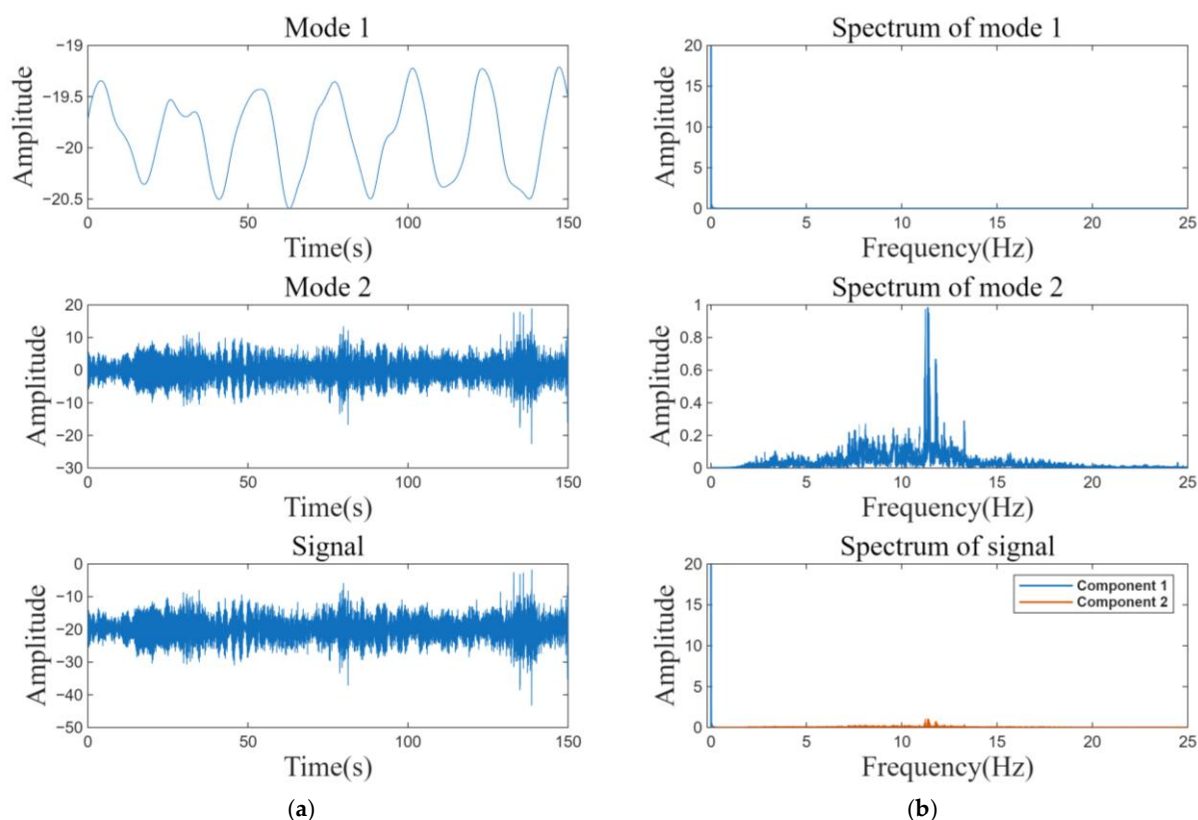


Figure 9. Decomposition results for vibration signal in VIV. (a) Time domain plots for decomposition results. (b) Frequency domain plots for decomposition results.

As shown in Figure 9, it can be observed that the signal is decomposed into two feature components. The first component constitutes the main body of the VIV signal, and the amplitude of the second component is relatively low, with a relatively high proportion of high-frequency noise components. The entropy values and SNRs of each component are shown in Table 3.

Table 3. Entropy values of components for VIV signal decomposition.

Indicators	Component 1	Component 2
MPE	0.46	0.69
SNR	12.86	1.15

It can be seen that the entropy values of the two components are relatively high in Table 2. The first component has a relatively low amplitude and a high SNR value. The MPE value is relatively low, which means its complexity is relatively low. This component is identified as the VIV component. The second component has a low SNR. This is because the second signal is a non-stationary signal. As the main component of the original signal, the second component carries a large amount of high-frequency noise, resulting in high complexity. This component is identified as the critical noise signal.

As shown in Figure 10, after removing the noise component and reconstructing the signal, the noise in the original is effectively suppressed. The amplitude of the VIV part is relatively clear, proving that this method effectively reduces noise and restores the effective signal.

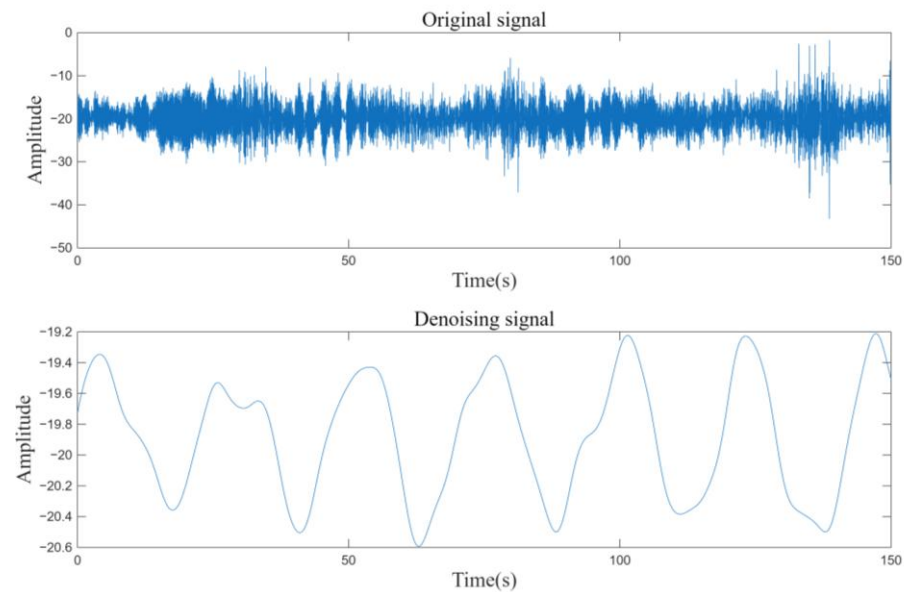


Figure 10. Comparison of VIV signal and denoising signal.

As shown in Table 4, the proposed RSBVMD method significantly outperforms all other methods on accuracy indices. Compared with the VMD, the proposed method reduces RMSE by 82.3% and increases SNR by 10.71 dB, indicating its slight advantage in vibration signal denoising. Meanwhile, the proposed method reduces analysis time by 79.3%. These results demonstrate that the RSBVMD method not only achieves exceptional denoising performance in strong noise environments but also has a remarkable advantage in computational efficiency.

Table 4. Comparison of different methods for vibration Signal.

Method	SNR	RMSE	CC	Time(s)
EMD	1.53	19.63	0.98	0.02
EEMD	1.44	19.88	0.98	0.68
BF	1.47	19.81	0.99	0.02
VMD	2.15	18.27	0.92	0.87
RSBVMD	12.86	3.53	0.38	0.18

5. Conclusions

This paper proposes a Recursive Sliding Bandwidth-Aware Variational Mode Decomposition (RSBVMD) method for adaptive online denoising of engineering streaming vibration signals. The core parameters of VMD, i.e., the number of decomposition modes and penalty factors, are fully and automatically determined by the weighted spectrum trend (WST). The recursive sliding discrete Fourier transform (RSDFT) is introduced to replace the traditional full fast Fourier transform, to realize the recursive sliding update improvement of VMD. A self-adaptive screening strategy for effective components is constructed by introducing multi-scale permutation entropy to quantify the complexity of each decomposed intrinsic mode function, which further enhances the denoising performance and feature retention ability of the method in strong noise environments.

Validations through simulation signals and acceleration data fully demonstrate the superiority and engineering applicability of the proposed method. The results indicate that the proposed RSBVMD method achieves adaptive determination of components and penalty factors. Compared with traditional VMD and mainstream online denoising

methods, this method has the best denoising effect. Compared with the VMD method, the proposed method reduces analysis time by about 80% and is less affected by the amount of analyzed data. The proposed RSBVMD method can satisfy the real-time denoising requirements of streaming vibration data while ensuring accuracy, providing reliable support for online SHM and structural state assessment.

The proposed method has broad generalizability beyond bridge vortex-induced vibration signal processing, which can be applied to fields where signals exhibit non-stationarity, multi-scale features, and complex noise interference, such as rotating machinery fault diagnosis and seismic signal processing.

From the perspective of methodological integration, the RSBVMD method has natural complementarity with existing stochastic process and time series modeling frameworks. The intrinsic mode components with clear physical meanings output by this method can be used as inputs to more advanced stochastic models, thereby enabling deeper modeling and prediction of system dynamic characteristics.

Despite its advantages, the proposed method still has certain limitations. The current MPE-based noise screening strategy relies on the statistical difference in complexity between signals and noise, which may fail when the noise itself exhibits structured or multi-scale characteristics.

Author Contributions: Conceptualization, J.L. and P.L.; methodology, J.L.; software, J.L. and F.Y.; analysis, J.L. and Y.Z.; writing—original draft preparation, J.L.; writing—review and editing, F.Y. and J.L.; supervision, J.L.; funding acquisition, J.L. and P.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Key R&D Program of China (Grant numbers: 2019YFB1600702).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are openly available in GitHub at <https://github.com/L693/Recursive-Sliding-Bandwidth-Aware-Variational-Mode-De-composition>.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Farrar, C.R.; Worden, K. *Structural Health Monitoring: A Machine Learning Perspective*; John Wiley & Sons: Hoboken, NJ, USA, 2012.
2. Ou, J.; Li, H. Structural Health Monitoring Research in China: Trends and Applications. In *Structural Health Monitoring of Civil Infrastructure Systems*; Woodhead Publishing: Oxford, UK, 2009.
3. Li, S.; Xu, H.; Zhang, X.; Cao, M.; Sumarac, D.; Novák, D. Automatic Uncoupling of Massive Dynamic Strains Induced by Vehicle- and Temperature-Loads for Monitoring of Operating Bridges. *Mech. Syst. Signal Process.* **2022**, *166*, 108332. <https://doi.org/10.1016/j.ymssp.2021.108332>.
4. Huang, N.E.; Shen, Z.; Long, S.R.; Wu, M.C.; Shih, H.H.; Yen, N.; Tung, C.C.; Liu, H.H. The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis. *Proc. R. Soc. A* **1996**, *454*, 903–995.
5. Yang, J.N.; Lei, Y.; Lin, S.; Huang, N. Hilbert-Huang Based Approach for Structural Damage Detection. *J. Eng. Mech.* **2004**, *130*, 85–95. [https://doi.org/10.1061/\(asce\)0733-9399\(2004\)130:1\(85\)](https://doi.org/10.1061/(asce)0733-9399(2004)130:1(85)).
6. Dragomiretskiy, K.; Zosso, D. Variational Mode Decomposition. *IEEE Trans. Signal Process.* **2014**, *62*, 531–544. <https://doi.org/10.1109/TSP.2013.2288675>.
7. Wang, Y.; Markert, R. Filter Bank Property of Variational Mode Decomposition and Its Applications. *Signal Process.* **2016**, *120*, 509–521. <https://doi.org/10.1016/j.sigpro.2015.09.041>.
8. Diao, Y.; Lv, J.; Wang, Q.; Li, X.; Xu, J. Structural Damage Identification Based on Variational Mode Decomposition–Hilbert Transform and CNN. *J. Civ. Struct. Heal. Monit.* **2023**, *13*, 1415–1429. <https://doi.org/10.1007/s13349-023-00715-3>.

9. Mohanty, S.; Gupta, K.K.; Raju, K.S. Comparative Study between VMD and EMD in Bearing Fault Diagnosis. In Proceedings of the 9th International Conference on Industrial and Information Systems, ICIIS 2014, Gwalior, India, 15–17 December 2014.
10. Wu, C.; Duan, Y.; Wang, H. Signal Denoising of Traffic Speed Deflectometer Measurement Based on Partial Swarm Optimization—Variational Mode Decomposition Method. *Sensors* **2024**, *24*, 3708.
11. Zhong, Z.; Li, Z.; Wang, J.; Tang, C.; Liu, Y.; Guo, K. Research on Denoising of Bridge Dynamic Load Signal Based on Hippopotamus Optimization Method—Variational Mode Decomposition—Singular Spectrum Analysis Method. *Buildings* **2025**, *15*, 1390.
12. Yang, D.; Guo, C.; Persico, R.; Liu, Y.; Liu, H.; Bai, C.; Lian, C.; Zhao, Q. Adaptive Variational Mode Decomposition and Principal Component Analysis-Based Denoising Scheme for Borehole Radar Data. *Remote Sens.* **2025**, *17*, 525.
13. Jiang, X.; Shen, C.; Shi, J.; Zhu, Z. Initial Center Frequency-Guided VMD for Fault Diagnosis of Rotating Machines. *J. Sound Vib.* **2018**, *435*, 36–55. <https://doi.org/10.1016/j.jsv.2018.07.039>.
14. Chen, S.; Guo, L.; Fan, J.; Yi, C.; Wang, K.; Zhai, W. Bandwidth-Aware Adaptive Chirp Mode Decomposition for Railway Bearing Fault Diagnosis. *Struct. Health Monit.* **2024**, *23*, 876–902. <https://doi.org/10.1177/14759217231174699>.
15. Liu, S.; Chen, Y.; Luo, C.; Jiang, H.; Li, H.; Li, H.; Lu, Q. Particle Swarm Optimization-Based Variational Mode Decomposition for Ground Penetrating Radar Data Denoising. *Remote Sens.* **2022**, *14*, 2973.
16. Yao, X.; Zhou, Q.; Wang, C.; Hu, J.; Liu, P. An Adaptive Seismic Signal Denoising Method Based on Variational Mode Decomposition. *Measurement* **2021**, *177*, 109277. <https://doi.org/10.1016/j.measurement.2021.109277>.
17. Li, Z.; Chen, J.; Zi, Y.; Pan, J. Independence-Oriented VMD to Identify Fault Feature for Wheel Set Bearing Fault Diagnosis of High Speed Locomotive. *Mech. Syst. Signal Process.* **2017**, *85*, 512–529. <https://doi.org/10.1016/j.ymssp.2016.08.042>.
18. Van Der Byl, A.; Inggs, M.R. Recursive Sliding Discrete Fourier Transform with Oversampled Data. *Digit. Signal Process. A Rev. J.* **2014**, *25*, 275–279. <https://doi.org/10.1016/j.dsp.2013.10.008>.
19. Lyons, R. Dsp Tips & Tricks—The Sliding DFT. *IEEE Signal Process. Mag.* **2003**, *20*, 74–80. <https://doi.org/10.1109/msp.2003.1184347>.
20. Soman, K.P.; Poornachandran, P.; Athira, S.; Harikumar, K. Recursive Variational Mode Decomposition Method for Real Time Power Signal Decomposition. *Procedia Technol.* **2015**, *21*, 540–546. <https://doi.org/10.1016/j.protcy.2015.10.048>.
21. Dan, D.; Zeng, G.; Pan, R.; Yin, P. Block-Wise Recursive Sliding Variational Mode Decomposition Method and Its Application on Online Separating of Bridge Vehicle-Induced Strain Monitoring Signals. *Mech. Syst. Signal Process.* **2023**, *198*, 110389. <https://doi.org/10.1016/j.ymssp.2023.110389>.
22. Liu, Y.; He, W.; Pan, T.; Qin, S.; Ruan, Z.; Li, X. The Parameter-Optimized Recursive Sliding Variational Mode Decomposition Method and Its Application in Sensor Signal Processing. *Sensors* **2025**, *25*, 1944. <https://doi.org/10.3390/s25061944>.
23. Zhang, D.; Liang, L.; Wang, W.; Song X.; Ren X. Real Time Tunnel Structure Monitoring System Based on Edge Computing. In Proceedings of the 2021 IEEE International Conference on Artificial Intelligence and Computer Applications (ICAICA), Dalian, China, 28–30 June 2021; pp. 791–795. <https://doi.org/10.1109/ICAICA52286.2021.9498000>.
24. Gao, D.; Chen, W.L.; Zhang, R.T.; Huang, Y.W.; Li, H. Multi-Modal Vortex- and Rain-Wind- Induced Vibrations of an Inclined Flexible Cable. *Mech. Syst. Signal Process.* **2019**, *118*, 245–258. <https://doi.org/10.1016/j.ymssp.2018.08.057>.

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