

Article

Battery State-of-Charge Estimation Using Factor Graph Optimization with Linear and Nonlinear Voltage Observation Models

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Abstract

Accurate battery state-of-charge (SOC) estimation is essential for battery management systems in electric vehicles and energy storage applications. Traditional estimation methods often rely on recursive filtering techniques and may experience performance degradation under nonlinear operating conditions and measurement uncertainty. This study proposes a factor graph optimization (FGO)-based SOC estimation framework incorporating voltage observation constraints. Three estimation configurations are investigated, including Current-Only FGO, Linear Voltage FGO, and Nonlinear Voltage FGO. A custom 3 Ah lithium-ion battery simulation model implemented in C++ was developed to generate benchmark datasets under both constant-current and random-current operating conditions. The results indicate that incorporating voltage information can improve SOC estimation accuracy within the investigated simulation benchmark. Under constant-current conditions, the Current-Only FGO achieved an RMSE of 0.128303, while the Linear Voltage FGO reduced the RMSE to 0.067395. The Nonlinear Voltage FGO further reduced the RMSE to 0.013604. Under random-current conditions, the nonlinear voltage formulation achieved an RMSE of 0.009022. Noise sensitivity analysis was conducted by injecting zero-mean Gaussian voltage measurement noise ranging from 0.01 V to 0.10 V. Additional validation experiments were conducted by including an EKF baseline, practical uncertainty cases, OCV–SOC model mismatch, and covariance sensitivity analysis. The results suggest that the nonlinear voltage formulation maintained relatively low estimation errors under the investigated voltage noise levels, while model mismatch and covariance settings can influence estimation accuracy. Since the present study is based on a simplified simulation model, the proposed framework should be regarded as a simulation-based proof-of-concept for voltage-constrained FGO-based SOC estimation. Future work will focus on validation using real battery datasets and comparison with conventional battery state estimation methods.



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1. Introduction

Accurate battery State-of-Charge (SOC) estimation is one of the most important functions of battery management systems (BMSs) [1,2] and is widely used in electric vehicles, energy storage systems, and portable electronic devices.

Reliable SOC estimation enables safe battery operation, improves energy utilization efficiency, and provides accurate information for remaining driving range prediction.

Recent studies on advanced battery management systems for electric vehicles have emphasized that reliable sensing, communication, and state estimation are essential for battery safety and operational performance [3].

Various SOC estimation methods have been proposed in the literature [4–6]. Coulomb counting remains one of the most widely used techniques because of its simplicity and low computational cost [7]. However, its estimation accuracy deteriorates over time due to accumulated integration errors and sensor bias. To overcome these limitations, model-based approaches such as the Kalman Filter (KF), Extended Kalman Filter (EKF), and Unscented Kalman Filter (UKF) have been extensively investigated [4,5,8]. Although these recursive filtering methods improve estimation accuracy, they are sensitive to model uncertainty and measurement noise.

In practical battery management systems, SOC estimation is affected by multiple uncertainty sources, including voltage measurement noise, current sensor bias, nominal capacity uncertainty, initial SOC error, temperature variation, aging effects, and battery model mismatch. Therefore, in addition to estimation accuracy under ideal conditions, the robustness of SOC estimation methods under practical uncertainty conditions is also an important evaluation aspect. Conventional methods such as Coulomb counting, EKF, UKF, particle filtering, and moving horizon estimation have been widely investigated for battery SOC estimation. These methods provide important reference points for evaluating newly developed estimation frameworks.

From a broader modeling perspective, practical battery management also requires a balance between model fidelity and computational efficiency. Detailed electrochemical and diffusion-related processes can provide rich physical information, but they are often difficult to incorporate directly into lightweight BMS algorithms. Recent work on characteristic-value techniques for approximating Warburg diffusion devices has highlighted the importance of reduced-order and computationally efficient modeling for energy-related systems [9]. In this context, the simplified voltage–SOC model adopted in the present study is used to provide a controlled benchmark for investigating voltage observation factor design in FGO-based SOC estimation.

Factor Graph Optimization (FGO) has recently emerged as a powerful state estimation framework in robotics, autonomous navigation, and simultaneous localization and mapping (SLAM) [10–12].

Unlike recursive filtering approaches, FGO formulates estimation problems as graph-based optimization problems and jointly optimizes all states using the complete measurement set. This global optimization capability enables the exploitation of temporal correlations among measurements and reduces accumulated estimation errors.

The application of FGO to battery state estimation has attracted increasing attention in recent years. In our previous work, a review framework integrating factor graph optimization and state-space models for battery management systems was presented [13]. Building upon that study, the present work focuses on the implementation and experimental validation of voltage-constrained factor graph optimization for battery SOC estimation. However, most existing studies primarily focus on current integration constraints and do not fully utilize battery voltage information. Furthermore, the nonlinear relationship between battery voltage and SOC is often simplified using linear approximations, which may limit estimation accuracy.

Moreover, there is still limited literature systematically comparing different voltage observation formulations within a unified factor graph framework. In particular, the impact

of linear and nonlinear voltage observation models on SOC estimation accuracy under varying operating conditions has not been sufficiently investigated.

Therefore, this study proposes a battery SOC estimation framework based on factor graph optimization incorporating both current process factors and voltage observation factors. In addition to a conventional linear voltage model, a nonlinear voltage observation model is introduced to better represent battery behavior. The proposed framework is evaluated using a 3 Ah lithium-ion battery model under constant-current, random-current, and noisy measurement conditions.

Compared with our previous review-oriented work on factor graph optimization for battery state estimation, the present study focuses on the implementation and validation of a complete FGO-based SOC estimation framework. In particular, linear and nonlinear voltage observation factors are integrated into the optimization graph and evaluated under multiple operating conditions and noise levels.

The main contributions of this paper are summarized as follows:

- (1) A simulation-based FGO framework for battery SOC estimation is constructed by integrating current process factors and voltage observation factors.
- (2) Linear and nonlinear voltage observation factors are systematically compared within the same FGO framework to investigate the influence of voltage model design on SOC estimation accuracy.
- (3) A conventional Coulomb counting baseline and an EKF-based baseline are introduced to provide reference comparisons with the proposed FGO-based methods.
- (4) Additional robustness analyses are conducted under practical uncertainty conditions, including current sensor bias, nominal capacity uncertainty, initial SOC error, voltage model mismatch, and covariance parameter variation.
- (5) The scope and limitations of the proposed framework are clarified, and the present study is positioned as a simulation-based methodological investigation before comprehensive real-battery-data validation.

2. Methodology

2.1. Battery Model

A lithium-ion battery model with a nominal capacity of 3 Ah was developed to generate reference data for the SOC estimation experiments. The model represents a representative 18,650-type lithium-ion cell. In the present study, the battery is described using a simplified equivalent-circuit representation consisting of an open-circuit voltage (OCV) term and an ohmic internal resistance. The terminal voltage is expressed as

$$V_k = \text{OCV}(\text{SOC}_k) - I_k R_0, \quad (1)$$

where V_k is the terminal voltage, SOC_k is the state of charge, I_k is the discharge current, and R_0 is the ohmic internal resistance. In this study, positive current is defined as discharge current.

The SOC evolution is calculated by Coulomb counting as

$$\text{SOC}_{k+1} = \text{SOC}_k - \frac{\Delta t}{Q_n} I_k, \quad (2)$$

where Δt is the sampling interval and Q_n is the nominal battery capacity expressed in ampere-seconds. The SOC value is constrained within the physical range of $[0, 1]$.

The nonlinear OCV–SOC relationship used in the simulation is defined as

$$\text{OCV}(\text{SOC}_k) = 3.0 + 1.2\text{SOC}_k - 0.1 \sin(2\pi\text{SOC}_k). \quad (3)$$

This function was used to represent a simplified nonlinear voltage–SOC relationship for the simulated lithium-ion cell. The linear voltage model used later in the factor graph is a first-order approximation of this nonlinear relationship.

The generated dataset serves as the reference trajectory for evaluating the performance of the proposed factor graph optimization framework. Two operating scenarios were considered in this study:

1. constant-current discharge conditions;
2. random-current discharge conditions simulating dynamic load variations.

For each simulation step, the battery current, terminal voltage, and reference SOC were recorded and exported for subsequent factor graph optimization. Since the dataset is generated from a simplified simulation model, the present validation should be regarded as a simulation-based proof-of-concept study rather than a complete validation using real battery experimental data.

2.2. Factor Graph Formulation

Factor Graph Optimization (FGO) formulates the SOC estimation problem as a graph-based optimization problem [10,12,14]. Let the battery state at time step k be represented by the SOC variable SOC_k .

The complete battery trajectory is represented as a sequence of SOC states over time. The factor graph consists of variable nodes representing SOC states and factor nodes representing measurement constraints.

The overall optimization objective is to find the SOC trajectory that minimizes the residual errors associated with all measurement factors.

$$X^* = \arg \min_X \sum_i \|e_i(X)\|_{\Sigma_i}^2 \quad (4)$$

2.2.1. Current Process Factor

The current process factor describes the SOC transition between two consecutive time steps according to Coulomb counting. The residual function is defined as

$$e_k^I = SOC_{k+1} - \left(SOC_k - \frac{\Delta t}{Q_n} I_k \right) \quad (5)$$

where I_k denotes the measured current, Δt is the sampling interval, and Q_n is the nominal battery capacity.

The corresponding process noise covariance is denoted by

$$\Sigma_I = \sigma_I^2 \quad (6)$$

where σ_I represents the standard deviation of the current process uncertainty.

In this study, the covariance values were selected empirically to provide stable optimization convergence and consistent estimation performance. The same covariance settings were applied to all experiments to ensure fair comparison among the investigated methods.

2.2.2. Linear Voltage Observation Factor

The linear voltage observation factor assumes a linear relationship between terminal voltage and SOC,

$$V_k = aSOC_k + b \quad (7)$$

The corresponding residual is

$$e_k^{V_L} = V_k^{meas} - (aSOC_k + b) \quad (8)$$

2.2.3. Nonlinear Voltage Observation Factor

The nonlinear voltage observation factor uses the nonlinear OCV-SOC model introduced in Section 2.1. The residual is defined as

$$e_k^{V_N} = V_k^{meas} - OCV(SOC_k) \quad (9)$$

where

$$OCV(SOC_k) = 3.0 + 1.2SOC_k - 0.1 \sin(2\pi SOC_k) \quad (10)$$

The voltage observation covariance is represented by

$$\Sigma_V = \sigma_V^2 \quad (11)$$

where σ_V denotes the voltage measurement uncertainty.

Unlike recursive filtering approaches, which update states sequentially, factor graph optimization jointly estimates all states using the complete measurement set. This allows the optimizer to exploit temporal correlations among measurements and reduce accumulated estimation errors.

2.3. Optimization Procedure

The overall estimation procedure consists of several sequential steps.

First, battery operation datasets are generated using a custom lithium-ion battery simulation model developed in C++. The resulting current, voltage, and reference SOC measurements are exported as CSV files for subsequent processing.

Second, SOC variable nodes are created for all time steps in the factor graph. Current process factors are inserted between neighboring SOC states to represent battery charge balance constraints.

Third, voltage observation factors are added at each measurement instant. Depending on the investigated estimation configuration, either linear or nonlinear voltage factors are incorporated into the graph.

Fourth, the SOC trajectory is initialized using Coulomb counting. This initial estimate serves as the starting point for the optimization process.

Finally, the complete optimization problem is solved using the Levenberg–Marquardt optimizer provided by the GTSAM library. The optimized SOC trajectory is then extracted and compared with the reference SOC generated by the battery simulation model.

3. Experimental Setup

3.1. Simulation Environment

All experiments were conducted using datasets generated from a custom lithium-ion battery model. A representative 3 Ah 18,650-type lithium-ion cell was selected as the benchmark battery for all experiments.

The battery model provides current, terminal voltage, and reference state-of-charge (SOC) outputs throughout the discharge process. Equivalent-circuit battery models are widely used for SOC estimation because they provide a practical balance between physical interpretability and computational efficiency [15,16], while broader reviews summarize the development of model-based SOC estimation methods [17]. These data were exported

as CSV files and subsequently processed using a factor graph optimization framework implemented in C++.

The optimization framework was developed using the Georgia Tech Smoothing and Mapping (GTSAM) library [18]. All optimization problems were solved using the Levenberg–Marquardt nonlinear optimization algorithm.

To ensure a fair comparison, identical datasets, initialization conditions, sampling intervals, and optimization settings were used for all investigated estimation methods. The battery capacity was set to 3 Ah. The initial SOC was set to 100%. The sampling interval was 1 s. The simulation duration was 12,000 s. For the random-current experiment, the discharge current was randomly varied within the predefined operating range of the simulation model. The exact current profile used for the experiment is included in the public dataset repository to support reproducibility.

The noise models used in the factor graph were implemented as isotropic Gaussian noise models. The standard deviation of the prior SOC factor was set to $\sigma_{\text{prior}} = 1.0 \times 10^{-4}$, the standard deviation of the current process factor was set to $\sigma_{\text{process}} = 1.0 \times 10^{-3}$, and the standard deviation of the voltage observation factor was set to $\sigma_{\text{voltage}} = 2.0 \times 10^{-2}$. These values were selected empirically to provide stable optimization behavior under the simulated measurement-noise conditions considered in this study. To further evaluate the influence of these empirical covariance settings, an additional covariance sensitivity analysis was conducted.

Coulomb counting was additionally included as a conventional baseline method. In this baseline, SOC was updated by integrating the measured current over time using the same sampling interval, initial SOC, and nominal battery capacity as those used in the FGO experiments. Since the reference SOC in the present simulation framework was generated from current integration without current sensor bias or capacity uncertainty, Coulomb counting represents an idealized baseline under the present conditions.

For the 12,001-sample dataset, the current-only FGO problem contained 12,001 SOC variables and 12,000 current-process factors. In the present C++/GTSAM implementation, this batch optimization was completed in approximately 1.205 s in the tested computational environment. This result provides a basic indication of the computational scale of the implemented factor graph. However, the present implementation was evaluated in batch mode, and online BMS implementation would require additional development, such as sliding-window optimization or incremental smoothing.

3.2. Constant-Current Experiment

The first experiment was conducted under constant-current discharge conditions to evaluate the baseline performance of different factor graph configurations.

Three estimation methods were investigated:

1. Current-Only FGO;
2. Linear Voltage FGO;
3. Nonlinear Voltage FGO.

The objective of this experiment was to quantify the influence of voltage observation factors on SOC estimation accuracy under a controlled operating environment.

The estimation performance of each method was evaluated by comparing the estimated SOC trajectory with the reference SOC generated by the battery model.

3.3. Random-Current Experiment

To simulate more realistic battery operating conditions, a random-current discharge experiment was performed.

The battery current profile was varied over time to introduce dynamic load changes. Compared with the constant-current scenario, this experiment presents a more challenging estimation problem due to current fluctuations and varying operating conditions.

The objective of this experiment was to evaluate the estimation performance of the Linear Voltage FGO and Nonlinear Voltage FGO methods under dynamic load conditions.

The estimation accuracy was assessed using the root mean square error (RMSE) between the estimated SOC and the reference SOC.

For the random-current dataset, Coulomb counting achieved an RMSE of 0.000021, an MAE of 0.000019, and a maximum absolute error of 0.000039. This very low error is expected because the reference SOC was generated by current integration in the absence of current sensor bias, capacity uncertainty, and initial SOC error. Therefore, this baseline mainly serves to confirm the internal consistency of the simulated dataset. The comparison between the Linear Voltage FGO and Nonlinear Voltage FGO methods should be interpreted as an evaluation of the influence of voltage observation factors within the FGO framework.

3.4. Noise Sensitivity Experiment

To investigate the influence of measurement uncertainty, zero-mean Gaussian voltage noise was artificially injected into the voltage measurements.

Four voltage noise levels were considered:

1. 0.01 V;
2. 0.03 V;
3. 0.05 V;
4. 0.10 V.

For each noise level, SOC estimation was performed using both the Linear Voltage FGO and Nonlinear Voltage FGO methods.

The resulting RMSE values were used to evaluate the sensitivity of each estimation approach to voltage measurement uncertainty.

This experiment provides a preliminary assessment of voltage-noise sensitivity within the proposed simulation framework. Additional practical uncertainty factors, including current sensor bias, nominal capacity uncertainty, initial SOC error, and OCV–SOC model mismatch, are further investigated in the additional validation experiments. Temperature variation and aging effects remain important topics for future investigation.

3.5. Additional Validation Experiments

To further address the influence of practical uncertainty factors and to avoid relying only on an idealized matched-model simulation condition, additional validation experiments were conducted in the revised study. These experiments were designed to evaluate the proposed FGO-based SOC estimation framework under less idealized conditions while maintaining the controlled simulation-based scope of the present manuscript.

First, an EKF-based SOC estimator was implemented as a representative conventional recursive filtering baseline. The EKF baseline was evaluated using the same battery simulation dataset and the same RMSE metric as the FGO-based methods.

Second, practical uncertainty conditions were introduced into the simulation-based evaluation. These conditions included current sensor bias, nominal capacity uncertainty, and initial SOC error. The current sensor bias experiment was used to evaluate the influence of systematic current measurement error. The capacity uncertainty experiment was used to evaluate the influence of nominal battery capacity mismatch. The initial SOC error experiment was used to evaluate the ability of the estimator to correct an inaccurate initial SOC assumption.

Third, an OCV-SOC model mismatch experiment was conducted. In the original simulation benchmark, the nonlinear voltage observation factor used the same OCV-SOC relationship as the data-generation model. This represents an idealized matched-model condition. To evaluate the influence of model mismatch, the voltage observation model used in the estimator was intentionally perturbed from the OCV-SOC relationship used for data generation.

Finally, a covariance sensitivity analysis was conducted by varying the standard deviations of the current process factor and voltage observation factor. This analysis was used to evaluate how the empirical covariance settings influence SOC estimation accuracy.

4. Results and Discussion

This section presents the experimental results obtained using the proposed factor graph optimization framework. The estimation performance of Current-Only FGO, Linear Voltage FGO, and Nonlinear Voltage FGO is evaluated under constant-current and random-current operating conditions. In addition, the sensitivity of the voltage-constrained FGO methods to voltage measurement noise is investigated.

The estimation accuracy is assessed using the root mean square error (RMSE) between the estimated SOC and the reference SOC generated by the battery simulation model. The corresponding SOC trajectories, estimation errors, and noise sensitivity results are discussed in the following subsections. Since all reference data are generated from a simulation model, the results should be interpreted as a simulation-based benchmark evaluation rather than a complete validation using real battery experimental data.

4.1. Constant-Current Results

The first experiment was conducted under constant-current discharge conditions to evaluate the baseline performance of different factor graph configurations. Table 1 summarizes the RMSE values obtained by the investigated estimation methods.

Table 1. SOC estimation accuracy under constant-current conditions.

Method	RMSE
Current-Only FGO	0.128303
Linear Voltage FGO	0.067395
Nonlinear Voltage FGO	0.013604

The Current-Only FGO achieved an RMSE of 0.128303. Although the current process factor captured the general SOC decreasing trend, the estimation error accumulated during the discharge process because no voltage-based correction information was used.

By introducing linear voltage observation factors, the estimation accuracy was improved. The Linear Voltage FGO reduced the RMSE to 0.067395, indicating that voltage measurements provide useful correction information for reducing current integration error.

The Nonlinear Voltage FGO obtained the lowest RMSE among the three investigated FGO configurations, with an RMSE of 0.013604. This result suggests that the nonlinear voltage observation model can better represent the simulated voltage–SOC relationship than the linear approximation under the present constant-current benchmark condition.

Figure 1 presents the SOC estimation trajectories obtained by the three investigated methods.

The Nonlinear Voltage FGO produced trajectories that closely followed the reference SOC in the simulation benchmark, whereas larger deviations were observed in the Current-Only and Linear Voltage approaches. This trend is consistent with the RMSE values reported in Table 1.

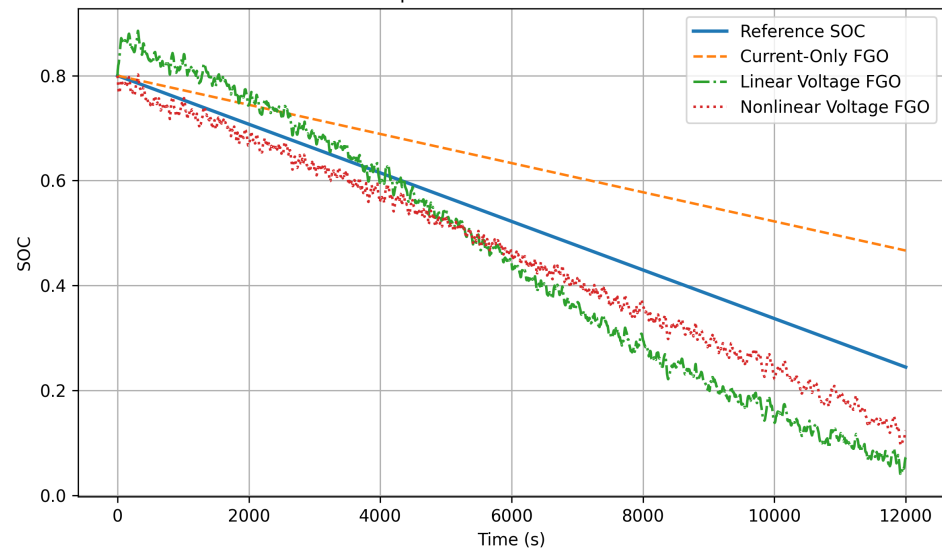


Figure 1. SOC estimation trajectories under constant-current conditions.

Figure 2 further shows the absolute SOC estimation errors under constant-current conditions. The results indicate that voltage observation factors help correct accumulated current integration errors in the simulated dataset. In particular, the nonlinear voltage factor provides a closer match to the nonlinear voltage–SOC relationship used in the benchmark model.

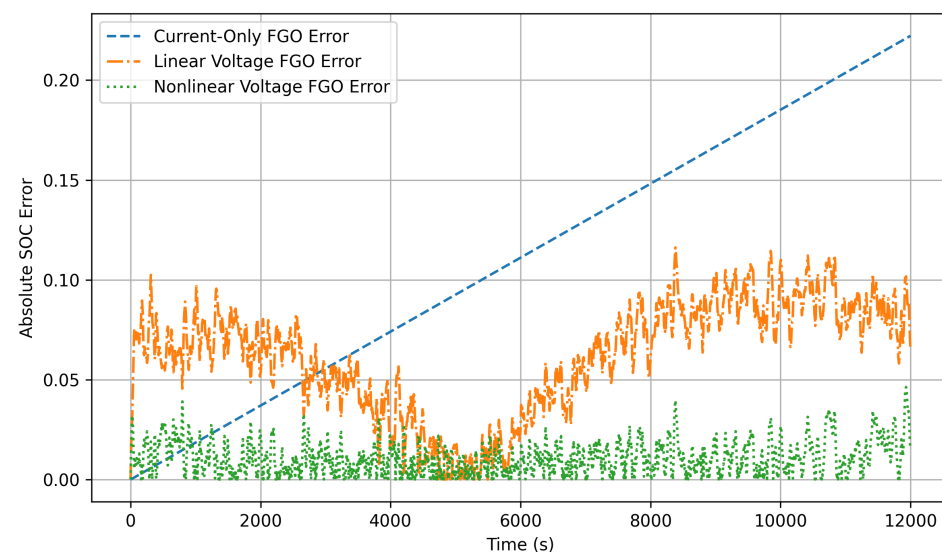


Figure 2. Absolute SOC estimation errors under constant-current conditions.

4.2. Random-Current Results

The second experiment was conducted under random-current discharge conditions to evaluate the estimation performance of the proposed framework under dynamic operating conditions. Compared with the constant-current case, the varying current profile introduces additional estimation challenges due to continuous changes in battery operating states.

Table 2 summarizes the RMSE values obtained under random-current conditions. In addition to the FGO-based methods, Coulomb counting was included as a conventional baseline. The Coulomb counting baseline achieved an RMSE of 0.000021, an MAE of 0.000019, and a maximum absolute error of 0.000039. This very low error is expected because the reference SOC in the present simulation framework was generated by current

integration without current sensor bias, capacity uncertainty, or initial SOC error. Therefore, this baseline mainly confirms the internal consistency of the simulated dataset.

Table 2. SOC estimation accuracy under random-current conditions.

Method	RMSE
Coulomb Counting	0.000021
Linear Voltage FGO	0.066174
Nonlinear Voltage FGO	0.009022

The Linear Voltage FGO achieved an RMSE of 0.066174. Although the voltage observation factors improved estimation performance compared with the current-only FGO formulation, the linear voltage model could not fully represent the nonlinear voltage–SOC relationship used in the battery simulation model.

The Nonlinear Voltage FGO achieved an RMSE of 0.009022, which was lower than that of the Linear Voltage FGO. This improvement suggests that incorporating nonlinear voltage information can provide more effective measurement constraints for SOC estimation within the present simulation framework.

The nonlinear voltage model consistently produced SOC estimates that closely followed the reference trajectory, whereas larger deviations were observed in the linear voltage model during periods of current variation.

It should be noted that the random-current profile used in this study is a synthetic simulation profile rather than a standardized electric-vehicle driving cycle. Therefore, the results should be interpreted as a simulation-based evaluation of dynamic-load performance rather than a complete validation under real vehicle operating conditions.

Overall, the Coulomb counting result represents an idealized baseline under the present simulation conditions, while the comparison between the Linear Voltage FGO and Nonlinear Voltage FGO methods demonstrates the influence of voltage observation factor design within the FGO framework. The results indicate that the nonlinear voltage observation factor improves estimation accuracy over the linear voltage observation factor under the investigated dynamic operating conditions.

4.3. Effect of Voltage Measurement Noise

To investigate the influence of measurement uncertainty, zero-mean Gaussian voltage noise was injected into the voltage measurements at four different noise levels: 0.01 V, 0.03 V, 0.05 V, and 0.10 V.

Table 3 summarizes the RMSE values obtained by the Linear Voltage FGO and Nonlinear Voltage FGO methods under different voltage noise levels.

Table 3. Noise sensitivity analysis.

Noise (V)	Linear FGO	Nonlinear FGO
0.01	0.066174	0.009022
0.03	0.066339	0.009565
0.05	0.066565	0.010440
0.10	0.067395	0.013604

For the Linear Voltage FGO, the RMSE remained relatively stable, increasing only slightly from 0.066174 at 0.01 V noise to 0.067395 at 0.10 V noise.

For the proposed Nonlinear Voltage FGO, the RMSE increased from 0.009022 at 0.01 V noise to 0.013604 at 0.10 V noise. Although estimation accuracy gradually decreased as the

noise level increased, the nonlinear voltage formulation consistently achieved lower RMSE values than the linear formulation across all investigated noise conditions.

Figure 3 illustrates the variation of RMSE with increasing voltage measurement noise. Both methods exhibited relatively stable estimation performance throughout the investigated noise range. The nonlinear voltage formulation maintained lower estimation errors and showed only moderate degradation as the noise level increased.

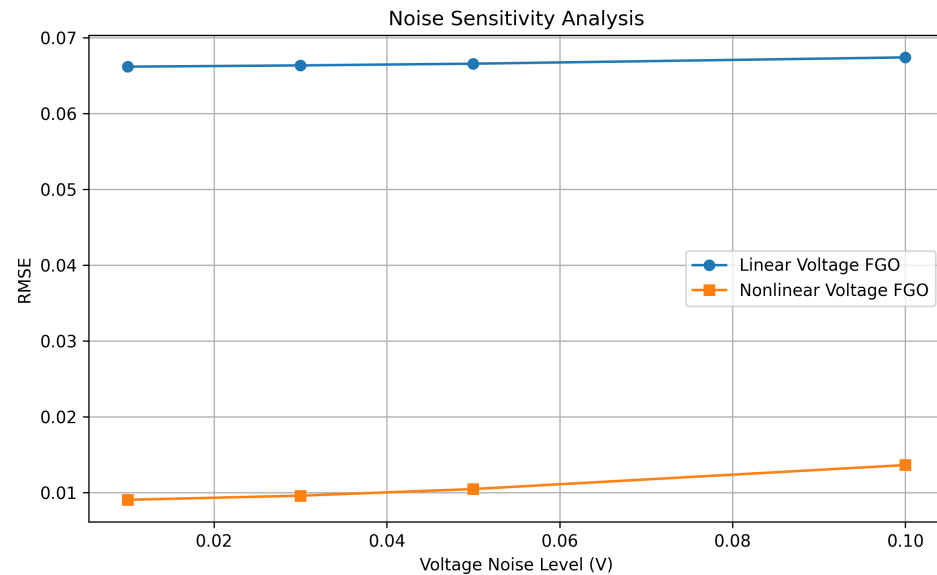


Figure 3. Noise sensitivity analysis of Linear Voltage FGO and Nonlinear Voltage FGO.

The results suggest that incorporating nonlinear voltage information can improve both estimation accuracy and resistance to voltage measurement uncertainty within the present simulation framework. These findings indicate that nonlinear voltage-constrained factor graph optimization may provide a useful approach for battery SOC estimation in the presence of sensor noise.

It should be noted that this subsection focuses only on voltage measurement noise. Additional practical uncertainty factors, including current sensor bias, nominal capacity uncertainty, initial SOC error, and OCV–SOC model mismatch, are further evaluated in Section 4.4. Temperature effects, aging mechanisms, parameter drift, and real battery experimental validation remain important topics for future research.

4.4. Additional Validation Results

To further evaluate the proposed FGO-based SOC estimation framework under less idealized simulation conditions, additional validation experiments were conducted. These experiments included an EKF baseline comparison, practical uncertainty analysis, OCV–SOC model mismatch analysis, and covariance sensitivity analysis.

Table 4 summarizes the comparison between the conventional baseline methods and the FGO-based methods under the random-current condition. Coulomb counting was included as an idealized current-integration baseline, while EKF was included as a representative recursive filtering-based SOC estimation method. The Linear Voltage FGO and Nonlinear Voltage FGO methods were evaluated using the same dataset and RMSE metric.

The EKF baseline provides a reference comparison with a conventional recursive filtering approach. In this simulation benchmark, the EKF baseline achieved an RMSE of 0.012743, which was lower than that of the Linear Voltage FGO and close to that of the Nonlinear Voltage FGO. The comparison helps clarify the relative performance of the pro-

posed FGO-based methods under the investigated simulation conditions. Since the present benchmark remains simulation-based, the results should be interpreted as a controlled comparison rather than a complete validation under real battery operating conditions.

Table 4. Comparison with conventional SOC estimation baselines under random-current conditions.

Method	RMSE
Coulomb Counting	0.000021
EKF	0.012743
Linear Voltage FGO	0.066174
Nonlinear Voltage FGO	0.009022

Table 5 presents the results of the practical uncertainty analysis. The investigated uncertainty conditions included current sensor bias, nominal capacity uncertainty, initial SOC error, and OCV–SOC model mismatch. These cases were selected because they represent common sources of error in practical battery SOC estimation.

Table 5. SOC estimation accuracy under practical uncertainty conditions.

Condition	Linear FGO	Nonlinear FGO
Current sensor bias (+1%)	0.067332	0.013446
Capacity uncertainty (+5%)	0.067395	0.013603
Initial SOC error (−10%)	0.067498	0.013723
OCV–SOC model mismatch	0.067395	0.027196

The RMSE values of the Linear Voltage FGO and Nonlinear Voltage FGO methods under practical uncertainty conditions are summarized in Table 5.

The uncertainty analysis shows how different practical error sources influence the SOC estimation accuracy of the FGO-based methods. In particular, the OCV–SOC model mismatch case was added to address the possibility that using the same voltage model for both data generation and estimation may lead to overly optimistic results. In this experiment, the voltage observation model used in the estimator was intentionally perturbed from the OCV–SOC relationship used for data generation.

Table 6 summarizes the covariance sensitivity analysis. The standard deviations of the current process factor and voltage observation factor were varied to evaluate the influence of the empirical covariance settings on SOC estimation accuracy.

Table 6. Covariance sensitivity analysis of the nonlinear voltage FGO method.

σ_{process}	σ_{voltage}	RMSE
1.0×10^{-3}	1.0×10^{-2}	0.016968
1.0×10^{-3}	2.0×10^{-2}	0.013604
1.0×10^{-3}	5.0×10^{-2}	0.011153
5.0×10^{-3}	2.0×10^{-2}	0.024089

The covariance sensitivity analysis provides additional information on how the factor graph noise settings affect estimation accuracy. When σ_{process} was fixed at 1.0×10^{-3} , changing σ_{voltage} from 1.0×10^{-2} to 5.0×10^{-2} changed the RMSE from 0.016968 to 0.011153. When σ_{process} was increased to 5.0×10^{-3} with $\sigma_{\text{voltage}} = 2.0 \times 10^{-2}$, the RMSE increased to 0.024089. These results indicate that the covariance parameters influence the balance between the current process constraint and the voltage observation constraint. Therefore, covariance selection is an important practical issue for FGO-based battery SOC estima-

tion. In the present study, the covariance parameters were selected empirically, and more systematic adaptive covariance tuning will be considered in future work.

4.5. Discussion

The primary contribution of this work is not the introduction of factor graph optimization itself, but the systematic investigation of different voltage observation formulations within a unified FGO-based SOC estimation framework.

Compared with the Current-Only FGO, both Linear Voltage FGO and Nonlinear Voltage FGO improved SOC estimation accuracy within the investigated simulation benchmark. This improvement can be attributed to the additional measurement constraints introduced by battery voltage observations, which provide correction information for accumulated current integration errors.

The comparison between Linear Voltage FGO and Nonlinear Voltage FGO further highlights the importance of voltage model selection. Although the linear voltage model improved estimation performance, its simplified representation was unable to fully capture the nonlinear voltage–SOC relationship used in the benchmark model.

By contrast, the nonlinear voltage observation factor achieved lower RMSE values under both constant-current and random-current operating conditions. The nonlinear formulation allows the optimization framework to better exploit voltage information across the investigated SOC range, resulting in improved estimation accuracy within the present simulation framework.

The noise sensitivity analysis also showed that the Nonlinear Voltage FGO maintained relatively low estimation errors under the investigated voltage noise levels. This suggests that the factor graph formulation can integrate voltage measurement information and reduce the influence of voltage measurement uncertainty in the present benchmark.

The comparison with Coulomb counting further clarifies the scope of the present simulation study. Under the idealized simulation conditions considered here, Coulomb counting achieved the lowest error because the reference SOC was generated by current integration without current sensor bias, capacity uncertainty, or initial SOC error. Therefore, the Coulomb counting result should be interpreted as an ideal baseline that confirms the internal consistency of the simulated dataset, rather than as a practical benchmark under realistic BMS uncertainties. In real battery applications, Coulomb counting is sensitive to sensor bias, capacity uncertainty, initial SOC uncertainty, and accumulated integration errors. The present FGO-based comparison therefore focuses on how different voltage observation factors influence SOC estimation within a controlled factor-graph framework.

The additional validation results further clarify the scope and robustness of the proposed framework. The EKF baseline achieved an RMSE of 0.012743 under the random-current simulation condition, providing a reference comparison with a conventional recursive filtering method. The Nonlinear Voltage FGO achieved a comparable RMSE of 0.009022 under the same simulation benchmark. This result suggests that the nonlinear voltage-constrained FGO formulation can achieve competitive estimation accuracy within the investigated simulation framework.

The practical uncertainty analysis shows that the proposed framework is affected by realistic error sources, but the nonlinear voltage formulation maintained lower RMSE values than the linear voltage formulation under the investigated current sensor bias, nominal capacity uncertainty, and initial SOC error conditions. In particular, under the OCV–SOC model mismatch condition, the RMSE of the Nonlinear Voltage FGO increased to 0.027196, indicating that the original matched-model condition produced a more optimistic result. This finding directly shows that OCV–SOC model mismatch can degrade estimation accuracy and should be considered in practical applications. Nevertheless, the nonlinear

voltage formulation still outperformed the linear voltage formulation under the tested model-mismatch condition.

The covariance sensitivity analysis indicates that the factor graph covariance settings influence SOC estimation accuracy. Changing the voltage observation factor standard deviation and current process factor standard deviation altered the RMSE of the Nonlinear Voltage FGO. This result confirms that covariance selection affects the balance between current-process constraints and voltage-observation constraints. Therefore, systematic covariance tuning or adaptive noise estimation should be considered in future FGO-based battery SOC estimation studies. The use of a simplified voltage–SOC model also makes the present results more accessible from a methodological perspective. Rather than attempting to reproduce all electrochemical diffusion phenomena, the present benchmark isolates the effect of voltage observation factor design within the FGO framework. This is consistent with the broader need for reduced-order and computationally efficient models in energy-related systems. Nevertheless, more detailed electrochemical models, including diffusion-related impedance behavior, should be considered in future extensions to improve physical representativeness.

Nevertheless, several limitations should be acknowledged. First, the present study still relies on a simplified battery simulation model rather than experimental battery data. Second, although current sensor bias, nominal capacity uncertainty, initial SOC error, OCV–SOC model mismatch, voltage noise, and covariance sensitivity were considered in the revised manuscript, other important practical factors such as temperature variation, aging effects, parameter drift, and cell-to-cell variation were not fully modeled. Third, the investigated nonlinear OCV–SOC relationship represents a simplified battery characteristic and may not fully reflect real lithium-ion battery behavior. Fourth, the present EKF comparison provides only an initial reference comparison with a conventional recursive filtering method, and more comprehensive comparisons with UKF, particle filtering, and moving horizon estimation remain necessary.

Although real battery data validation is essential for practical applicability, a full real-data validation study would substantially expand the scope of the present manuscript from voltage observation factor design to complete experimental battery validation. Therefore, the present paper focuses on a controlled simulation-based methodological evaluation, while comprehensive validation using public and experimental real battery datasets is being prepared as a separate follow-up study.

Future work will focus on validating the proposed framework using publicly available battery datasets and real battery experimental measurements. In addition, further comparisons with conventional battery state estimation methods, including UKF, particle filtering, and moving horizon estimation, will be conducted under more realistic uncertainty conditions.

Although computational complexity was not the primary focus of this study, the current-only FGO problem with 12,001 SOC variables and 12,000 current-process factors was completed in approximately 1.205 s in the present C++/GTSAM implementation. This provides a basic indication of the computational scale of the implemented factor graph. However, the present implementation was evaluated in batch mode. A detailed computational complexity analysis and online implementation study, including sliding-window optimization or incremental smoothing, will be conducted in future work.

5. Conclusions

This paper presented a factor graph optimization framework for battery state-of-charge estimation incorporating both current process factors and voltage observation factors.

A lithium-ion battery simulation model was developed to generate benchmark datasets under constant-current and random-current operating conditions. Three FGO-based estimation approaches, namely Current-Only FGO, Linear Voltage FGO, and Nonlinear Voltage FGO, were investigated and compared. In addition, Coulomb counting and EKF were included as conventional baseline methods. The Coulomb counting baseline achieved the lowest error under the idealized simulation conditions because the reference SOC was generated by current integration without current sensor bias, capacity uncertainty, or initial SOC error.

The experimental results showed that incorporating voltage observation factors improves SOC estimation accuracy within the investigated simulation framework. Among the three investigated FGO configurations, the Nonlinear Voltage FGO achieved the lowest RMSE values, reducing RMSE from 0.128303 to 0.013604 under constant-current conditions and from 0.066174 to 0.009022 under random-current conditions. The EKF baseline achieved an RMSE of 0.012743 under the random-current condition, providing a reference comparison with a conventional recursive filtering method.

The additional uncertainty analysis showed that current sensor bias, nominal capacity uncertainty, initial SOC error, and OCV–SOC model mismatch can affect SOC estimation accuracy. In particular, the OCV–SOC model mismatch experiment showed that the RMSE of the Nonlinear Voltage FGO increased to 0.027196, indicating that the matched-model condition produced a more optimistic result. The covariance sensitivity analysis further confirmed that factor graph noise settings influence estimation performance.

Although the present study is based on simulation data, the proposed framework provides a useful methodological basis for investigating voltage observation factor design in FGO-based battery SOC estimation. The present paper should therefore be regarded as a simulation-based methodological study rather than a complete real-battery validation. Future work will focus on validation using publicly available battery datasets and real battery experimental measurements, as well as more comprehensive comparisons with conventional battery state estimation approaches such as UKF, particle filtering, and moving horizon estimation under realistic operating conditions.

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