

Article

Two Case Studies on the Evaluation of Harmonic Impedance Using a Linearized Mathematical Model of a High-Voltage Distribution Network

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Abstract

One of the negative effects of the rapid increase in the number and capacity of photovoltaic power sources distributed in consumption areas by connecting to existing low-, medium-, and high-voltage distribution networks is the increased risk of amplifying the non-sinusoidal steady-state condition caused by high-power inverters. This amplification occurs if the peak values of harmonic impedance in the sections of the respective grid zone, which depend on the values of the equivalent shunt capacitances present in the grid, correspond to frequency values that coincide with or are close to the frequencies of the harmonic currents injected by the distributed sources. The risk is particularly high in cases of malfunction or failure of the filters within the installations associated with these sources. High shunt capacitance values are not only caused by the capacitive compensators used to increase the power factor of consumers or to improve the voltage level in the grid, but also by long power lines, particularly long underground lines. Risks increase as the proportion of underground power lines in the network area with sources of harmonic currents rises. This article refers to a real high-voltage grid area where the distribution operator plans to install high-power photovoltaic sources, the connection of which requires grid expansion through the construction of new substations and high-voltage power lines (110 kV). Based on the designer's intention for the new high-voltage power lines to be underground and of relatively long lengths—which implies the presence of high natural capacitances—the authors conduct two case studies to predictively evaluate the harmonic impedance in the network sections resulting from the expansion, with the aim of identifying the frequencies at which parallel resonances may occur. The authors use two software tools for numerical analysis, Mathcad and MATLAB–Simulink, and compare the results obtained for two design variants of the new high-voltage lines: overhead and underground, respectively. Using the mathematical model of long lines in steady state with uniformly distributed parameters, the analysis highlights that the highest values of harmonic impedance correspond to the overhead line design variant. The use of this design variant increases the risk of parallel resonances, not only by increasing the harmonic impedance values but also by widening the frequency ranges for which the impedance has high values. In both the overhead line and underground line variants, increasing the load leads to a reduction in the maximum values. Compared to the design variant using overhead lines, the variant with underground lines increases the risk of parallel resonances by increasing the number of frequency intervals for which the harmonic impedance has high values, but this increase is limited by narrowing these frequency intervals and reducing

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the harmonic impedance values. An important contribution of the article lies in arguing the need to extend the conventional node-based evaluation of harmonic impedance by treating harmonic impedance as a continuous spatial frequency characteristic of the transmission network. The impedances seen at nodes become the impedances seen in particular sections/sections of the network, identified by the value of the variable that specifies their position in space, more precisely the distance from the end of one of the feeders on which they are located.

Keywords: harmonic impedance; frequency characteristics; three-phase distribution HV network; distributed energy resources; lumped-parameter modeling

1. Introduction

In a major paper published more than ten years ago, the authors presented a conclusion based on forecasts at the time, according to which, over the next 10 years, more than 60% of the loads on public utility power grids in the U.S. would be nonlinear [1]. The negative impact of conditions caused by the presence of nonlinear loads on distribution networks and especially on the consumers they serve was anticipated to increase exponentially. For example, in the industrial sector of the U.S. economy, the projected damages caused by the effects of poor power quality were estimated at tens of billions of dollars per year [1].

Meanwhile, major changes in the energy sector have begun worldwide, marking the start of an era of the so-called “energy transition,” which is currently in full swing and whose ultimate goal is to phase out fossil fuels. In this context, a large number of thermal power plants have been shut down, while distributed renewable energy sources have multiplied exponentially. With installed capacities ranging from a few kilowatts to tens (or hundreds) of megawatts, renewable sources have flooded the electricity distribution grids at all voltage levels: low, medium, and high.

However, as is often the case, the benefits have come with their share of challenges. In this case, the latter has been and continues to be felt, increasingly so, primarily by distribution network operators but also by transmission network operators. The main problems are caused by the effects of the interaction between the high-power electronic converters associated with renewable sources and the power grid to which these sources have been connected. Among these effects, the most significant are those associated with the resulting steady-state or dynamic non-sinusoidal waveforms, which affect the technical and economic performance of the grids and the consumers they supply.

In other words, the impact of nonlinear loads on power grids, mentioned at the beginning of this chapter, is now compounded by that of nonlinear sources, which requires a commensurate response from grid operators. This article fits into this context, addressing the frequency-dependent modeling of long power lines, which, as part of high-voltage networks, are increasingly involved in the propagation of high-frequency disturbances emitted by renewable sources.

The modeling of long AC power lines, by considering the frequency variation of the equivalent phase, longitudinal, and transverse impedances, is not a new concern for specialists. Among the studies conducted in the early stages are those aimed at investigating the propagation of non-sinusoidal disturbances on power lines used to supply railway traction [2–5].

Another application that required the construction of frequency-dependent models for high-voltage alternating current overhead power lines (HVAC OHL), frequently encountered in the technical literature, is that used to study the corona effect [6–8].

In recent years, the main issues arising from the interaction between direct current (DC) installations based on power electronics equipment within renewable energy sources and high-voltage alternating current (AC) grids have become those related to broadband resonance problems [9].

High-voltage AC transmission lines, which have begun to play an increasingly important role in the integration of renewable energy sources, due to their relatively high, uniformly distributed natural equivalent electrical capacitances, can also increase the risk of broadband resonance when exposed to the propagation of high-frequency disturbances. For this reason, high-accuracy modeling of AC transmission lines over as wide a range of harmonic frequencies as possible has become a matter of major importance [10].

The two main methods for the mathematical modeling of alternating current transmission lines (long lines) are well known: lumped-parameter modeling and distributed-parameter modeling. In some studies, to identify high-frequency resonances in the system, the first method is applied, using an equivalent electrical circuit consisting of a chain of several π -type quadripoles [11,12]. However, this simplified equivalence reduces the efficiency of the simulation in the time domain, and in the frequency domain, the simulation may lead to resonances that would not occur in the real system.

In order for the time-domain and frequency-domain analysis of the propagation of high-frequency electromagnetic wave components to yield highly accurate results, both for steady-state and transient conditions, long AC power lines must be modeled using the method of uniform distribution along the line of equivalent phase, eigen, and mutual parameters—both longitudinal and transverse (shunt)—and by considering the frequency-dependent characteristics of these parameters. Accounting for these characteristics is very difficult to implement in mathematical models, and this has been a constant concern for researchers over the past five decades and even longer [13–21].

The enormous advances in computing performance in recent years have enabled the development of advanced specialized modeling software tools, as well as the implementation of complex methods for analyzing the transient regimes of current power system structures in general, and transmission lines in particular [22–28].

In this context, in the specific case of steady-state non-sinusoidal regimes, one of the most important analytical tools is harmonic impedance. According to the universally accepted definition, the harmonic impedance seen at a node of an alternating voltage network is the positive-sequence complex impedance seen at that node, expressed as a function of frequency (or harmonic order) [29]. Harmonic impedance is a state variable that provides information regarding the state of an electrical network operating in a non-sinusoidal steady-state regime. It is also one of the most useful analytical tools for addressing the challenges posed by non-sinusoidal steady-state operation, particularly regarding the generation and propagation of harmonic disturbances through the electrical network, quantifying contributions to harmonic pollution, locating sources of distortion, assessing the risks of parallel resonances, and adopting the most effective methods and means of intervention to mitigate or limit the effects of harmonic pollution [30–33]. It can be said that determining the harmonic impedance at network nodes with high precision is not only an essential requirement but also a complex and difficult problem.

This article presents analytical elements regarding the calculation of harmonic impedance seen in the nodes of a real three-phase high-voltage distribution network, with particular attention to the influence of relatively long underground power lines. This analysis is driven by the need to identify in advance potential risks of harmonic resonance in the analyzed network area, resulting from the future connection of photovoltaic plants.

The frequency-domain analysis of three-phase high-voltage power lines in general, and underground/submarine lines in particular, has received special attention from experts over the years. Among the works developed with the aim of generalizing methods

for modeling the elements of a transmission network for frequency-domain analysis are [34–39].

Among the articles presenting case studies in which the harmonic impedance seen at the nodes of a high-voltage three-phase network containing long cables is calculated is [40]. To model the underground line, the authors use a two-port (four-pole) model with concentrated parameters. The main finding of the study is that when underground lines are used, parallel resonances can occur at network nodes at frequencies much lower than when only overhead lines are used.

The increasing prevalence of non-sinusoidal operating conditions in transmission networks has led to a growing need for more accurate results, prompting most case studies to implement frequency-dependent models of transmission network components. These models were used, for example, in [41], where the calculation of equivalent parameters is a function of frequency.

The determination of impedances at network nodes was performed by applying two well-known methods for obtaining the frequency characteristics of AC power systems: the frequency scan method (FSM) and harmonic resonance mode analysis (HRMA). The two methods are based on the calculation of the nodal impedance matrix and the modal impedance matrix, respectively.

The first method allows the frequency characteristics of the impedance seen at a network node to be obtained by performing a frequency scan of the corresponding diagonal element in the nodal impedance matrix [1]. The second method uses eigenvector matrices to calculate the values of the nodes' participation factors in the critical mode. Based on these values, it is possible to identify the lines that, by replacing the overhead line with an underground one, may or may not result in a decrease in parallel resonance frequencies [42].

The two methods mentioned above (FSM and HRMA, respectively) can also be found in other studies regarding the frequency characteristics of AC networks.

Thus, in [43], two types of grids are analyzed to highlight the specific characteristics of each grid level operating under non-sinusoidal conditions, amplified due to the increase in the number and unit power of the inverters comprising distributed renewable energy sources. The first type is high/extra-high voltage (HV/EHV) networks, in which there is a growing trend toward an increased proportion of underground lines, which negatively affects the network's resonance frequencies. The second type is medium-voltage/low-voltage (MV/LV) networks, where the increasingly frequent use of decentralized generation induces new harmonic pollution, which could interfere with power line communications.

The HRMA method is also applied in [44] to analyze the resonance behavior of an EHV transmission network for different cable penetration levels. The results show a decrease in parallel resonance frequencies as cable penetration increases. Furthermore, for the same frequency bandwidth considered, the number of parallel resonance frequencies increases.

The issue of the effect of underground HVAC power lines on the harmonic impedance values observed at network nodes has become relevant again, given the reconsideration of long-distance submarine connections made via high-voltage three-phase lines. In [45], it is shown that although the HVDC solution for long submarine links intended to connect offshore oil and gas platforms or wind farms to onshore grids, or to interconnect onshore grids, has long been considered the default option, the HVAC solution must be reconsidered, as it is often less expensive and simpler. This solution has good potential for improvement in terms of extending the maximum acceptable lengths of HVAC cables. The main obstacle to increasing the distances over which electricity is transmitted via HVAC cables is the very high values of equivalent electrical capacitances, which cause a

decrease in the frequencies of parallel resonances resulting from interaction with the electrical grid operating in a non-sinusoidal mode and thus an increase in the risks of amplifying this mode, with all the negative consequences resulting from this [45].

Thus, the evaluation of harmonic impedances at the nodes of an HVAC network containing long cables and operating in a non-sinusoidal mode becomes a mandatory step.

This article presents analytical contributions and numerical results pertaining to a real three-phase high-voltage network area, relating to the same subject.

The structure of this paper is as follows: Chapter 2 contains the main information regarding the studied grid area. In this area, following the installation of several photovoltaic parks, the grid is set to expand through the construction of two relatively long three-phase underground lines and two new substations. The next chapter presents the procedure for the numerical calculation of the harmonic impedance seen along a three-phase power line, as applied in this paper. Then, to evaluate in advance the harmonic impedances seen in the nodes of the network area following the introduction of the new lines, with the aim of assessing the risks of increased harmonic pollution in the network, an initial analysis is performed for the harmonic impedance seen along the lines operating individually, for two construction variants (overhead and underground, respectively) and for various load conditions. Chapter 4 is dedicated to this analysis. The second case study is included in Chapter 5, in which the new lines are integrated into the network area containing photovoltaic parks. The harmonic impedances seen in the network area nodes are analyzed for both construction variants under forecasted low-load conditions, which are considered critical. The numerical analysis uses the MathCAD tool, with the results then confirmed via MATLAB–Simulink simulation. (Text 1-1) Mathcad and MATLAB/Simulink were employed as complementary rather than alternative computational environments. Mathcad was used for the analytical implementation of the proposed mathematical methodology, allowing each computational step and the influence of the governing parameters to be examined explicitly. MATLAB/Simulink was subsequently used as an independent numerical validation environment based on a detailed distributed-parameter network model and the standard Impedance Measurement tool available in Simscape Electrical. The purpose of employing two independent computational environments was to validate the proposed methodology rather than the software implementation itself. The excellent agreement obtained between the analytical and numerical results provides independent validation of the proposed methodology and supports the correctness of the reported results within the adopted modeling assumptions.

Chapter 6 presents a method that allows the evaluation of the harmonic impedance seen in nodes to be extended to an evaluation across all relevant sections of the network. Applying this method transforms the impedance seen at a given node into the impedance seen in that particular section of the network, corresponding to the specific value of the variable that specifies its position along the network.

The main novelty of the proposed methodology does not consist simply in increasing the number of harmonic impedance calculations. Rather, it introduces a different representation of harmonic impedance in high-voltage transmission networks. Instead of considering harmonic impedance only at discrete network nodes, the proposed distributed-parameter formulation allows harmonic impedance to be treated as a continuous function of both frequency and position along the transmission corridors. Consequently, the impedance values traditionally evaluated at substations or network buses become particular cases of a continuous spatial-frequency distribution. This representation provides additional engineering information regarding the location and severity of potential resonance phenomena, thereby extending the capabilities of conventional node-based harmonic studies.

2. Overview of the High-Voltage Distribution Network Area Where the Numerical Analysis Is Conducted

For the numerical analysis presented in the case studies in this paper, we consider a real-world high-voltage distribution network area managed by the distribution operator in the Banat area, Romania, where medium-power photovoltaic sources are installed and are set to expand, through the construction of two new three-phase lines and two new substations, which are necessary due to the continued increase in the number of photovoltaic parks as well as in electricity consumption in that area.

The distribution network has a nominal voltage of 110 kV and is connected at several points to the transmission network with higher nominal voltages of 220–400 kV in a geographical area located in western Romania. Figure 1 shows a section of the single-line diagram of the transmission and distribution network that includes the analyzed area.

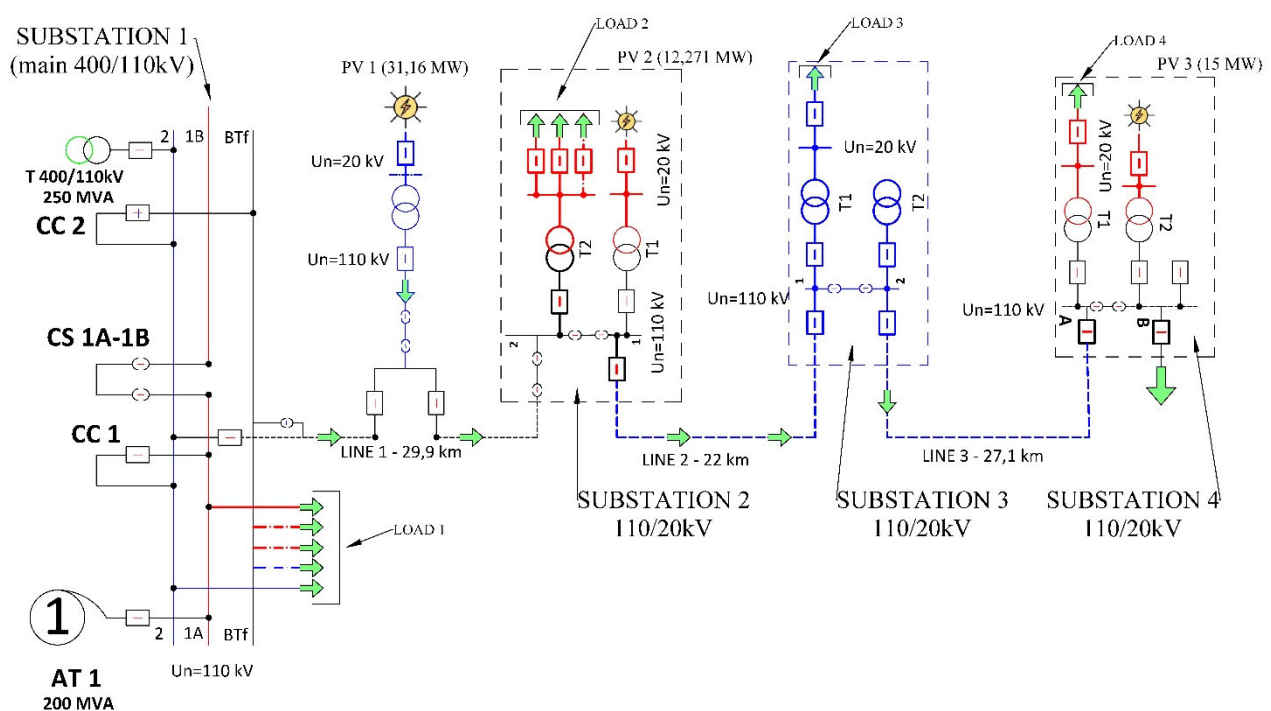


Figure 1. Single-line diagram of the distribution network area under analysis.

The figure shows, in blue, the grid components (two substations and two power lines) that are to be added in the near future to enable the connection of new photovoltaic parks, the redistribution of power to loads in the consumption area, and the reconfiguration of the medium-voltage distribution network. At the end of this phase of the distribution network's development, X photovoltaic parks with a total installed capacity of 58.43 MWp will be connected in the area in question.

Specific restrictions regarding the use of land along the routes of the two power lines necessitate their underground installation. Since the two new underground lines will have a total length of approximately 49 km, and the area will contain a high density of high-power static converters associated with the connection points of the photovoltaic sources, the analysis of harmonic impedance at the network nodes is a very important step in studying the interaction, under steady-state operating conditions, between the new sources and the grid, in terms of the risk of parallel harmonic resonance.

The two new three-phase high-voltage lines will be constructed entirely underground (UGL—underground line) and will have the characteristics specified in Table 1. By definition, the harmonic impedance is determined for symmetrical and balanced

operating conditions, which is why the analytical modeling considers only positive-sequence equivalent circuits.

Table 1. The design characteristics and equivalent unit parameters of the positive sequence for the two UGLs.

Name	Cable Type	Length	Equivalent Resistance	Equivalent Inductance	Equivalent Capacitance
		L [km]	R_{u^+} [Ω /km]	L_{u^+} [mH/km]	C_{u^+} [μ F/km]
UGL 1	$3 \times 1 \times 1600 \text{ mm}^2$ Al 76/138(145) kV	22	0.0211	0.362	0.3636
UGL 2	$3 \times 1 \times 1600 \text{ mm}^2$ Al 76/138(145) kV	27.1	0.0185	0.317	0.3188

The underground construction of the two high-voltage lines was identified as the optimal solution in the feasibility study, due to the characteristics of the route, which is set to cross numerous private properties. However, this solution did not consider the impact regarding the values of the equivalent harmonic impedance and their effects on the non-sinusoidal steady state of the network zone, once all the photovoltaic parks to be connected here are fully operational. This impact will be anticipated based on the harmonic impedance values observed at the nodes of the grid zone, which is the purpose of the analysis presented in this paper.

A first step in this analysis is to compare the proposed solution, in terms of the harmonic impedance values observed along the line, with the alternative solution of using overhead lines (OHL) for the two power lines, which, of course, have the same lengths. This alternative involves the use of the positive-sequence equivalent unit parameters of the overhead lines, whose values are presented in Table 2.

Table 2. The design characteristics and equivalent unit parameters of the positive sequence for the two OHLs.

Name	Cable Type	Length	Equivalent Resistance	Equivalent Inductance	Equivalent Capacitance
		L [km]	R_{u^+} [Ω /km]	L_{u^+} [mH/km]	C_{u^+} [μ F/km]
OHL 1	$3 \times 185/32 \text{ mm}^2$ ACSR	22	0.1571	1.346	8.675
OHL 2	$3 \times 185/32 \text{ mm}^2$ ACSR	27.1	0.1571	1.346	8.675

3. Procedure for the Numerical Calculation of the Harmonic Impedance Seen Along a Three-Phase Power Line

The present study adopts a linear distributed-parameter representation of the transmission lines, which captures the dominant physical behavior within the investigated frequency range while maintaining analytical tractability. It is recognized that several equivalent parameters of transmission lines exhibit frequency dependence owing to phenomena such as the skin effect, corona losses and dielectric losses. These effects mainly influence the damping characteristics of resonance phenomena and, to a lesser extent, their exact resonance frequencies, particularly at higher frequencies.

Since different frequency-dependent line models have been proposed in the literature, each with specific assumptions and ranges of applicability, no universally accepted model is currently available. The principal objective of the present work is therefore not the development of a new transmission-line model but the extension of harmonic

impedance assessment from discrete network nodes to arbitrary sections along transmission lines. Consequently, the proposed methodology may also be applied together with more sophisticated frequency-dependent models, representing a natural direction for future developments.

First, the values of the harmonic impedance seen are determined at selected sections along the power line, assuming it is part of a three-phase power system with a simplified structure consisting of: the upstream power system—the power line—the downstream network (Figure 2). This structure is obtained by reducing the upstream and downstream network sections to equivalent elements, which allows for a simplification of the modeling. Thus, the upstream power system is reduced to an equivalent source and an equivalent load, the latter allowing the modeling of the network connected to the same busbars as the line under study. Equivalence with a three-phase load also applies to the modeling of the network connected to the same busbars as the end of the line. At the same time, this simplified representation of the loads allows for a linearization of the corresponding mathematical model. The loads are represented by equivalent impedances derived from the specified active and reactive powers at the operating point. This representation is intended to capture the influence of the loading level on the network frequency response rather than the detailed frequency-dependent behavior of individual loads. The loading level acts mainly as a damping factor rather than as a parameter governing resonance frequencies.

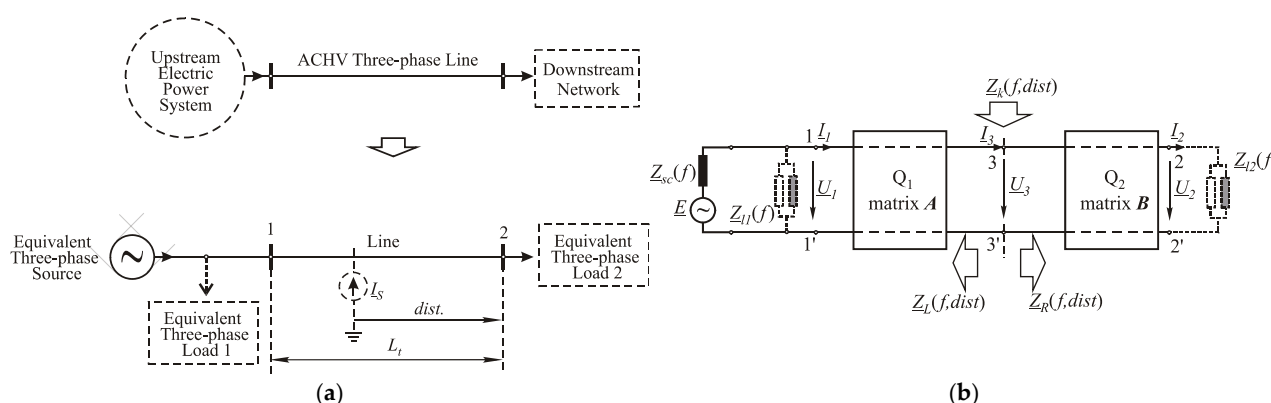


Figure 2. The single-line diagram (a) and the equivalent four-pole circuit diagram (b) were used to determine the analytical relationship of the harmonic impedance “seen” in sections along a power line.

To calculate the value of the harmonic impedance seen in any given section of the line, it is modeled using an equivalent electrical circuit with uniformly distributed parameters. The analytical form of the harmonic impedance is obtained by connecting the two equivalent impedances in parallel, viewed upstream and downstream, respectively, relative to the section under consideration [2].

The study is conducted for steady-state operating conditions; therefore, the single-phase, four-pole, positive-sequence equivalent circuit will be used for numerical modeling, which is also required for calculating harmonic impedances. The values of the electrical quantities, including the equivalent impedances and the other quantities involved in the mathematical model, are calculated using known relationships and methods. The table in Appendix A identifies these quantities and lists and explains the mathematical relationships for their calculation, in the order of application. The numerical calculation was performed by implementing these equations in MathCAD.

Thus, the upstream power system is reduced to an equivalent three-phase source and an upstream equivalent load, corresponding to the network area connected to the same

busbars as the line's starting point. The four-pole equivalent circuit of the source contains an ideal voltage source (with zero internal impedance) and the short-circuit impedance of the respective network node, determined by equations (A1–A5).

The upstream equivalent load is introduced into the equivalent circuit to model the sum of the active and reactive powers drawn by the lines connected to the same busbars as the beginning of the line under study. The modeling is performed using the active and reactive values corresponding to the resulting impedance at the fundamental frequency and rated voltage, for each normal operating condition considered (A6–A9). Similarly, the downstream network is modeled, with the equivalent load used to replace it corresponding to the active and reactive powers drawn by the lines connected to the same busbars as the end of the line (A11–A14).

The reactive components of the impedances Z_{cc} , Z_{J1} and Z_{J2} are expressed as functions of frequency, the variable f being necessary for the frequency-domain analysis of the network.

For the mathematical modeling of the line's operation under sinusoidal steady state, the solutions of the telegraph equations in phasor form are used, which allow for their implementation as a quadripole equivalent circuit. The mathematical model uses uniformly distributed longitudinal and transverse equivalent parameters, with the quadripole equivalent circuit allowing for the phasor representation of electromagnetic wave propagation along the line. The relationships between the quantities at the beginning and end of the line (equivalent quadripole) can be expressed using chain matrices, which allows for the study of the line's operating modes using quadripole theory. The beginning of the line becomes the input port, and the end of the line becomes the output port of the equivalent quadripole.

Thus, to determine the harmonic impedance seen along the line, it will be divided into two sections, one of length l (dist) and the other of length $L-l$ (where L is the total length of the line—Figure 2a). The quantity l , which defines the position (distance) of the section of interest relative to the end of the line, becomes the variable in the mathematical relations and takes values in the interval $[0-L]$. The two sections are modeled using two symmetric passive quadripoles, denoted Q_2 and Q_1 , respectively (Figure 2b), to which the chain matrices **B** and **A** are associated.

By assigning the indices 1, 2, and 3 to the beginning, end, and middle sections of the line, respectively, equations (A15–A19) can be written.

The analytical relations for calculating the components of the two chain matrices show the dependence of their values on both the position of the section of interest along the line (variable l (dist)) and the frequency (variable f) (A20–A25).

The characteristic quantities of the waveguide (power line with uniformly distributed parameters) are involved in the calculation relations: the characteristic impedance (wave impedance) and the propagation constant, respectively. These characteristic quantities depend on the geometric and material properties of the line and are calculated using the equivalent positive-sequence unit parameters. Their analytical expressions show the dependence on the variable f (A26–A29).

The value of the harmonic impedance seen at any given section along the power line is determined by connecting in parallel the impedances seen from that section upstream (to the left Z_L) and downstream (to the right Z_R). Their analytical expressions are determined by applying methods from quadripole theory.

Thus, the impedance Z_L can be determined as the input impedance of the quadripole Q_1 , in a regime where it would be fed from a fictitious single-phase source at terminals 3-3', with the load being the impedance seen upstream, at the beginning of the line. Similarly, the impedance Z_R can be determined as the input impedance of the quadripole Q_2 , in a configuration where it would be fed from a fictitious single-phase source at terminals

3-3', with the load being the impedance seen downstream at the end of the line. This yields relations (A30–A31) and then (A32–A33). The final relation, to be applied in the numerical calculation of the harmonic impedance seen in any given section of a power line, is (A34).

4. First Case Study—Small Network

The first numerical case study pertained to a high-voltage power line belonging to a radial network configuration (Figure 1) and situated between the upstream and downstream sections, which correspond to the beginning and end of the line, respectively (Figure 2). The numerical modeling was performed by applying the equivalences and calculation relationships explained above and detailed in Appendix A.

The two design variants, UGL1 (Table 1) and OHL1 (Table 2), were considered, operating in two groups of operating conditions, with the aim of analyzing, on the one hand, the influence of the upstream equivalent load (Load 1) and the downstream equivalent load (Load 2) values, and on the other hand, the influence of the equivalent short-circuit apparent power value corresponding to the busbars of the substation at the beginning of the line.

The two groups of operating conditions correspond to a reduced value and an average value, respectively, for Load 1. In each case, the modeling is performed with and then without Load 2, considering two variants for the values of the equivalent short-circuit apparent power on the busbars at the line's starting point. This resulted in 8 sub-regimes, with the corresponding notations and power values specified in Table 3.

Table 3. The values of the active and reactive powers of the equivalent loads and of the equivalent short-circuit apparent powers considered in the operating conditions of the first case study.

Regime/Sub-Regime		P_{l2} [MW]	Q_{l2} [Mvar]	P_{l1} [MW]	Q_{l1} [Mvar]	S_{cc} [MVA]
1 (low load)	1.1	1	0.2	-	-	2000
	1.2			5	1	2000
	1.3			-	-	4000
	1.4			5	1	4000
2 (average load)	2.1	10	2	-	-	2000
	2.2			50	10	2000
	2.3			-	-	4000
	2.4			50	10	4000

4.1. OHL1 Model

An initial way of presenting the results of the numerical analysis performed using MATLAB Simulink modeling is through graphs—see Figures 3–6 for OHL1 and Figure 7 and Figure 8 for UGL1. First, the graphs are presented as 3D surfaces of the harmonic impedance observed along the line. The coordinates of the points on this surface correspond to the values in the matrix $Z_k(f, dist)$, calculated for all integer values of the variables $dist \in [0 \dots 22]$ km and $f \in [0 \dots 5000]$ Hz. The maximum frequency considered in this analysis (corresponding to the $k = 100$ th harmonic) was set to be sufficiently low to allow the frequency variation of the equivalent R (G), L , and C parameters to be neglected, and sufficiently high to cover the main effects of the non-sinusoidal steady-state regime.

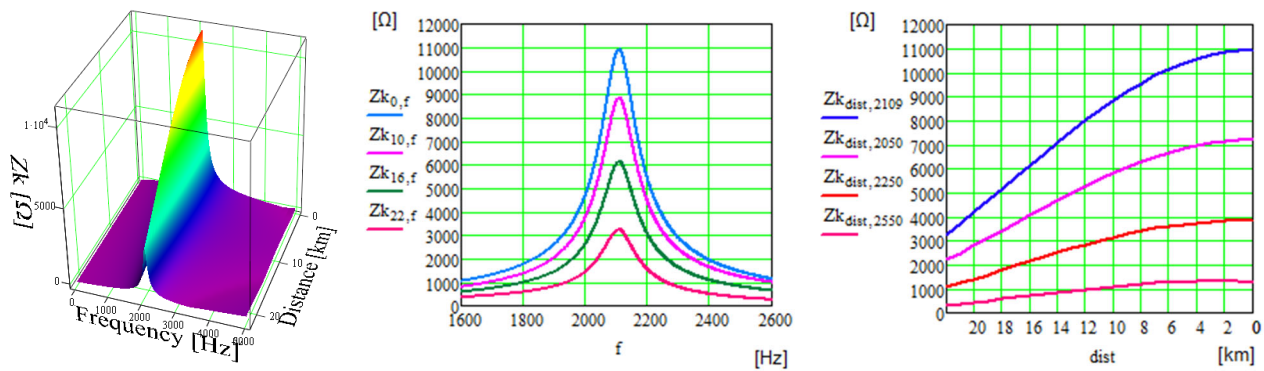


Figure 3. Graphical representation of the values of the function $Z_k(f, dist)$ for OHL1, sub-regime 1.1.

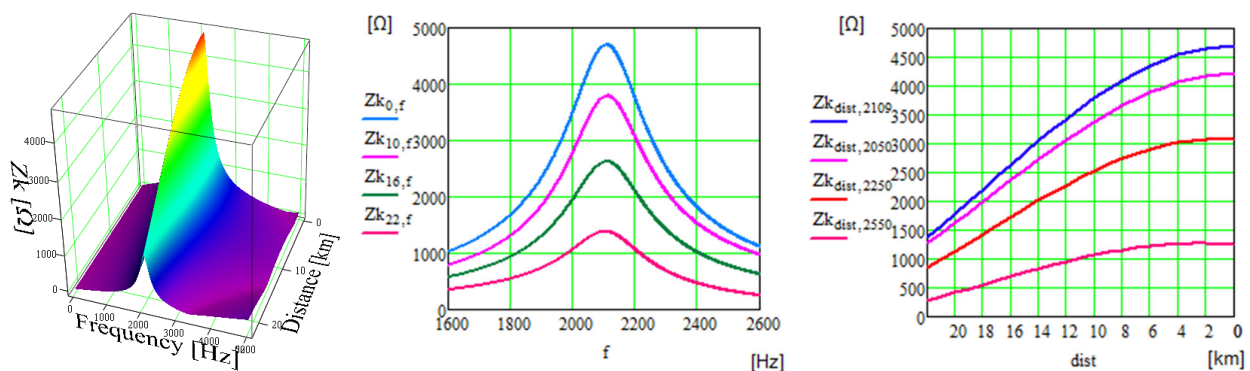


Figure 4. Graphical representation of the values of the function $Z_k(f, dist)$ for OHL1, sub-regime 1.2.

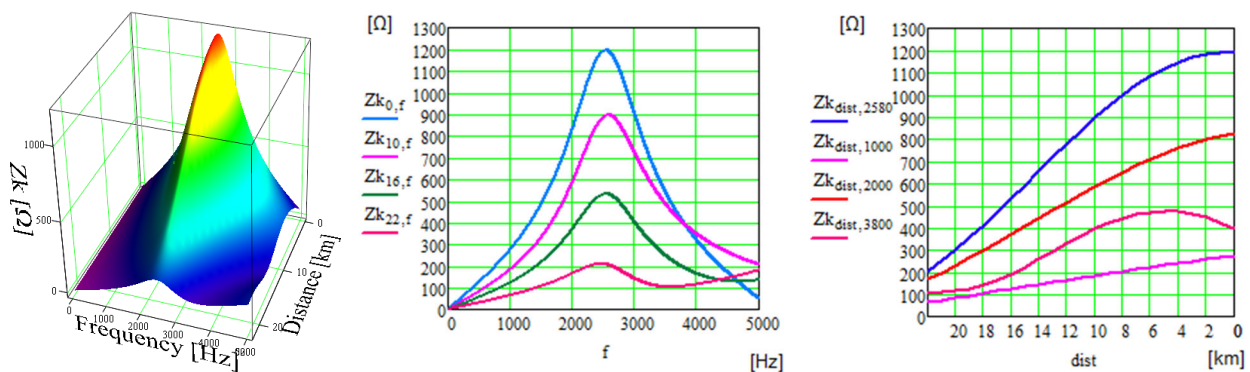


Figure 5. Graphical representation of the values of the function $Z_k(f, dist)$ for OHL1, sub-regime 2.3.

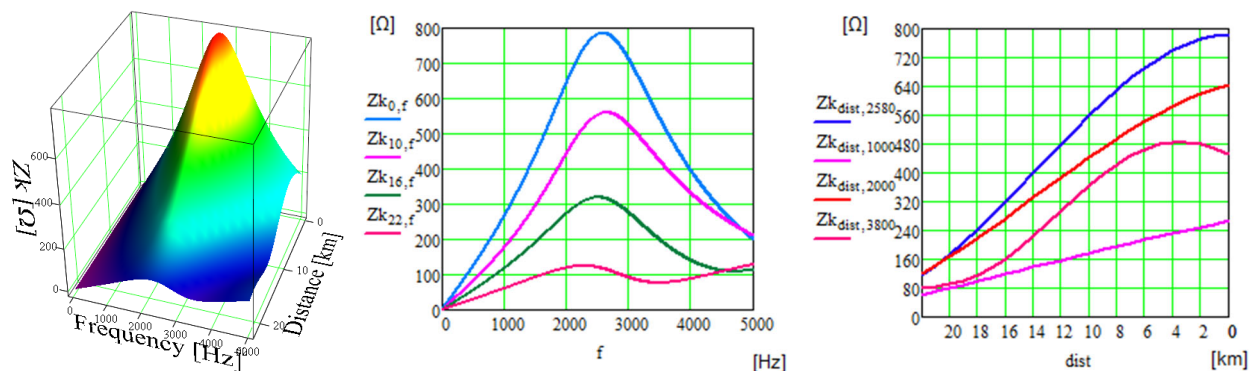


Figure 6. Graphical representation of the values of the function $Z_k(f, dist)$ for OHL1, sub-regime 2.4.

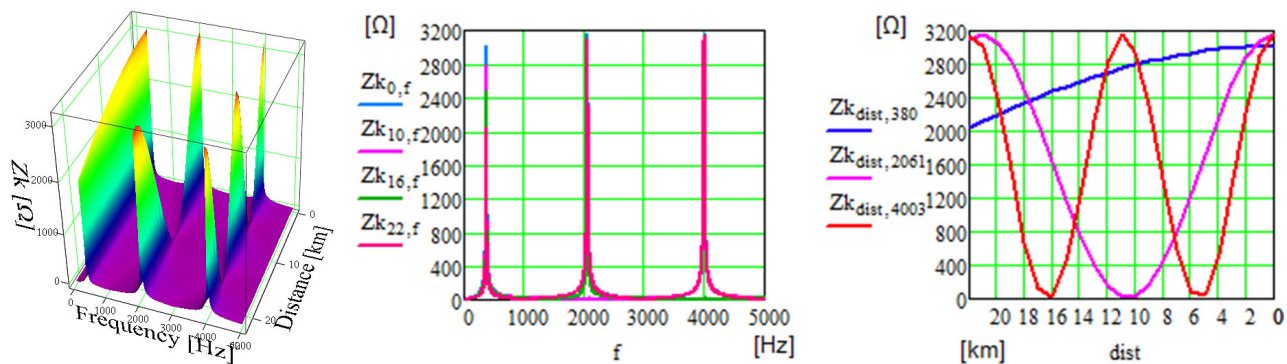


Figure 7. Graphical representation of the values of the function $Z_k(f, dist)$ for UGL1, sub-regime 1.1.

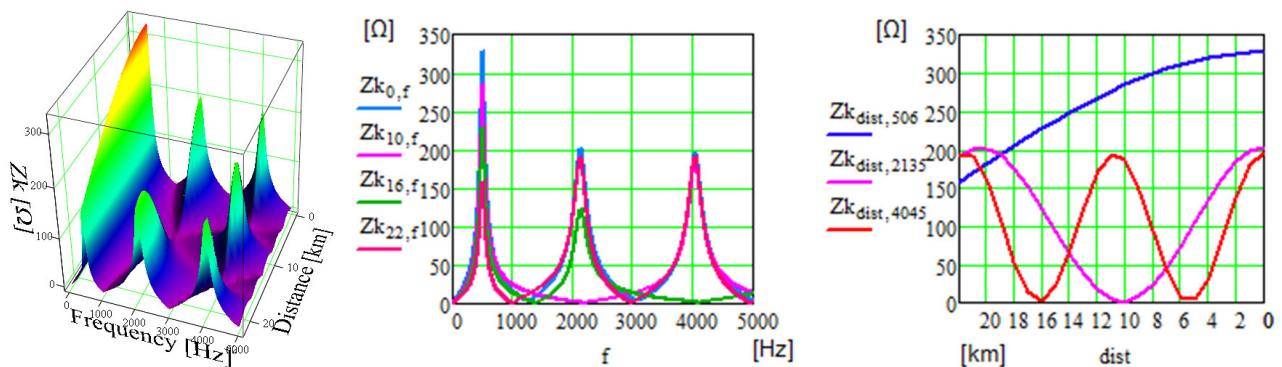


Figure 8. Graphical representation of the values of the function $Z_k(f, dist)$ for UGL1, sub-regime 2.4.

The graphical representation of the results is supplemented with 2D plots, obtained as orthogonal, longitudinal ($f = \text{constant}$), and transverse ($dist = \text{constant}$) cross-sections of the 3D surface, for several of the 8 sub-modes. The frequency values corresponding to the longitudinal sections and the distance values (from the end of the line) corresponding to the transverse sections were chosen arbitrarily and can be seen in the respective graphs.

The main information derived from the results of the numerical analysis concerns the identification of the pole coordinates, specifically the frequencies and distances corresponding to the maximum impedance values. The poles and their surrounding regions correspond to high values of the risk of parallel resonance and, consequently, of amplification of the distortion mode in voltages and/or currents, with all the negative effects that may result from them. Therefore, the pole coordinate values were extracted and listed in Table 4 for all 8 sub-modes to facilitate qualitative and quantitative evaluation.

Table 4. The coordinates of the poles of the harmonic impedance seen in the OHL1 sections.

Re-gime/Pols	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]
Regime 1		1.1			1.2			1.3			1.4	
Pole 1	0	2109	10,938	0	2109	4695	0	2538	10,831	0	2538	6816
Regime 2		2.1			2.2			2.3			2.4	
Pole 1	0	2109	10,938	0	2109	4695	0	2538	10,831	0	2538	6816

The main findings from the assessment of OHL1 are as follows:

- For all sub-modes, a single 3D pole appears in the harmonic impedance values observed along the line, with an axis corresponding to a frequency range approximately between (2100...2600) Hz, ($k = 42...52$).

- Since they are determined primarily by the values of the equivalent transverse resistances, the harmonic impedance values corresponding to the peak of the 3D pole (at the parallel resonance frequencies) depend largely on the values of the equivalent active loads. An increase in load means a decrease in the value of the respective equivalent transverse resistance and thus a decrease in the value of Z_k . Thus, comparing sub-regimes 1.1 and 1.2, it can be observed that if Load 2 (five times larger than Load 1) is also considered, this results in a 57% decrease in the maximum value of Z_k . Or, comparing sub-regimes 1.3 and 1.4, a 37% decrease in the maximum value of Z_k is observed in sub-regime 1.4. Thus, the most unfavorable situations in terms of the risk of non-sinusoidal regime amplification correspond to the regimes with the lowest loads.
- At the same value of equivalent short-circuit apparent power, the load variation does not produce a significant change in the pole frequency. For example, comparing the low-load operating conditions 1.1 and 1.2, we find the same value for the maximum pole frequency. At the average load, there is a +1.4% increase in the frequency value in regime 2.4 compared to regime 2.3.
- Regarding the surface profile represented graphically by the function $Z_k(f, dist)$, we can say, without much error, that it resembles a “shark’s tooth.” In the specific case of the analysis performed for OHL1, the shape of a half “tooth” appears, which can be described as long and thin in low-load modes, but short and thick in high-load modes. For all eight sub-regimes, the most unfavorable situations in terms of the risk of non-sinusoidal mode amplification correspond to the sections toward the end of the line.
- The number of “teeth” (3D poles), their height (the Z_k value), and their density (periodicity in the frequency domain) are greatly influenced by the line’s length. Additional information can be found in Appendix B.

In the specific case of OHL1.2, a double periodicity in the arrangement of the “teeth” (3D poles) can be observed, both in space and in frequency. Note that the poles are arranged in three “rows,” each of which corresponds to virtually the same frequency value. It can therefore be stated that the period of the poles’ appearance in the frequency domain decreases as the line length increases (the number of rows of “teeth” is greater the longer the line is). As the frequency increases, the number of 3D poles increases, due to the decrease in the periodicity intervals of their occurrence, both in the spatial domain and in the frequency domain. The maximum values of Z_k corresponding to the pole peaks increase substantially for low-load conditions (condition 1) and decrease for medium-load conditions (condition 2).

As a concluding remark, it can be stated that line length is an essential parameter in evaluating the function $Z_k(f, dist)$, which makes analysis necessary for each individual case, with quantitative conclusions having a limited degree of generalizability.

4.2. UGL1 Model

The same operating conditions, defined in Table 3, are used to study the frequency response of the underground line, constructed with the characteristics from Table 1.

The design differences compared to OHL1 are reflected in the differences between the values of the positive-sequence unit parameters. The main influence on the Z_k function is exerted by the value of the equivalent transverse reactive parameter, more specifically the value of the capacitance (capacitive reactance).

The much higher value of the unit capacitance (per km) of an underground line compared to that of an overhead line causes a short UGL to exhibit effects similar to those of a long OHL. More specifically, we refer to the effect of multiplying the number of “teeth”

and the number of “rows of teeth,” which, at first glance, is equivalent to an increased risk of parallel resonances.

The numerical results obtained allow for a qualitative and quantitative assessment of the variation in the values of the $Z_k(f, dist)$ function for the design variant using UGL1, utilizing 2D and 3D graphical representations (Figures 7 and 8) and tabular organization (Table 5).

Table 5. The l , f and Z coordinates corresponding to the poles of the harmonic impedance seen in sections of the UGL1.

Re-gime/Pols	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]	l [km]	f_k [Hz]	Z_k [Ω]
<i>Regime 1</i>		<i>1.1</i>			<i>1.2</i>			<i>1.3</i>			<i>1.4</i>	
Pole 1	0	380	3015	0	380	1637	0	505	3050	0	505	1895
Pole 2	0	2061	3152	0	2061	1383	0	2132	3144	0	2133	1414
Pole 3	20	2061	3061	21	2061	1342	20	2132	3130	20	2133	1407
Pole 4	0	4003	3157	0	4003	1374	0	4043	3159	0	4043	1384
Pole 5	11	4003	3153	11	4003	1372	11	4043	3146	11	4043	1378
Pole 6	22	4003	3144	22	4003	1368	22	4043	3107	22	4043	1361
<i>Regime 2</i>		<i>2.1</i>			<i>2.2</i>			<i>2.3</i>			<i>2.4</i>	
Pole 1	0	380	932	0	381	259	0	506	933	0	506	327
Pole 2	0	2061	943	0	2062	195	0	2133	942	0	2135	202
Pole 3	20	2061	916	20	2064	190	20	2133	938	20	2136	201
Pole 4	0	4003	943	0	4004	193	0	4043	943	0	4045	195
Pole 5	11	4003	942	11	4004	193	11	4043	939	11	4045	194
Pole 6	22	4003	940	22	4003	192	22	4043	928	22	4043	192

These allow the following observations to be drawn:

- For the given length, we obtained a result of 6 poles (“teeth”), arranged in three “rows,” containing one (1), two (2 and 3), and three (4, 5, and 6) poles, respectively (Table 5).
- The rows of “teeth” correspond to the values of the parallel resonance frequencies; these values change very little with the load condition or with the equivalent short-circuit power of the upstream network, falling within very narrow ranges that narrow as the frequency increases: $f_1 = 380\text{--}506$ Hz ($k_1 \approx 7\text{--}10$), $f_{2,3} = 2061\text{--}2135$ Hz ($k_{2,3} \approx 42\text{--}43$), and $f_{4,5,6} = 4003\text{--}4045$ Hz ($k_{4,5,6} \approx 80\text{--}81$).
- The maximum values of the function $Z_k(f, dist)$ are strongly influenced by the load regime and only slightly by the short-circuit power of the equivalent upstream source. To support this claim, the Z_k values for the poles in sub-regime 2.2 are compared with those of the poles in sub-regime 2.4. In the two sub-regimes, the loads are the same, but the value of the equivalent short-circuit power applied to the modeling of the upstream network increases from 2000 MVA (sub-regime 2.2) to 4000 MVA (sub-regime 2.4). The differences between the Z_k values fall within the range ($0 \div 5.79$) %.
- The Z_k values corresponding to the poles are much higher at low loads (regime 1) than at medium loads (regime 2). For example, comparing sub-regime 1.1 with sub-regime 2.1, at the same short-circuit power value of the equivalent upstream source, a tenfold increase in load results in a 3.5-fold decrease in the Z_k values corresponding to the poles; the increase in the attenuation effect with increasing load is confirmed once again, and thus the conclusion that low-load conditions (or even no-load operation) correspond to increased risks of amplification of the non-sinusoidal regime under conditions of parallel resonances.

The number of “teeth” and rows of “teeth” is influenced by the line’s design characteristics (geometric and material). For the same design type, the frequency characteristics are largely influenced by the line’s length. Additional information can be found in Appendix B.

5. Second Case Study—Network Area

The second case study aims to evaluate the influence on the frequency characteristics of a network zone by integrating power lines modeled with distributed parameters. For this purpose, the actual network zone presented in the second chapter of the article was considered. This zone was “extracted” from a high-voltage distribution network ($U_r = 110$ kV), connected to the busbars of the main substation (Substation 1) 220/110 kV. For the numerical calculation, performed in Mathcad and then in MATLAB–Simulink, this network area was simplified by reducing it to its main elements (Figure 9): the equivalent source of the upstream network connected to the main substation, the three high-voltage lines, and the four equivalent loads connected to the four substations.

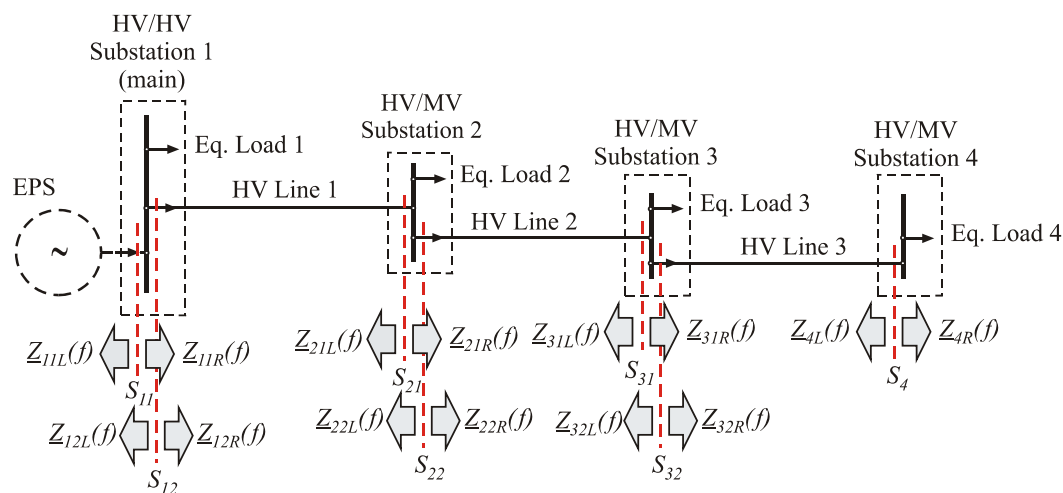


Figure 9. Simplified diagram of the network section under consideration, used to evaluate harmonic impedance values.

The equivalent source was modeled such that at the rated voltage and rated frequency, it corresponds to the actual value of the short-circuit apparent power on the 110 kV busbars of Substation 1—4070 MVA. The interconnection line between Substation 1 and Substation 2 (Line 1) is of the same construction type as OHL1 and OHL2, with a length of 29.9 km.

The equivalent load connected to the 110 kV busbars of Substation 1 HV/HV (Load 1) models a load that consumes active and inductive reactive power, each equal to the sum of the active and reactive powers transmitted on all other power lines connected to the same busbars as Line 1.

The equivalent loads connected to the 110 kV busbars of the other three substations (Load 2, 3, 4) correspond to the values of the active and reactive power transferred to the respective distribution networks via the HV/MV transformers. It can be said that the total consumption of each of the respective distribution networks was equivalent to the load connected to the HV busbars of the respective substation.

The operating modes were identified as posing the highest risk of amplifying the distortion caused by the photovoltaic sources installed in the studied grid area, specifically the “Peak Summer Morning” (PSM) and the “Low Summer Morning” (LSM) regime. These regimes correspond to situations that can become critical due to low electricity

consumption, under conditions where power generated by photovoltaic sources is close to maximum levels.

The values of the active and reactive power consumed by the four equivalent loads (Figure 9), for the two specific modes considered, were provided by the distribution operator and were obtained based on a forecast for the period immediately following the commissioning of the photovoltaic sources to be connected to the studied grid area. These values are presented in Table 6.

Table 6. The values of the active and reactive power consumed by the equivalent loads in the two operating modes under consideration.

Equivalent Load	PSM		LSM	
	P	Q	P	Q
	[MW]	[Mvar]	[MW]	[Mvar]
Load 1	59.45	13.4	17.83	4.02
Load 2	11.65	2.25	3.50	0.67
Load 3	8.50	1.80	2.55	0.54
Load 4	5.09	0.97	1.53	0.29

For both specific operating modes, the harmonic impedance seen in the network nodes is determined, considering the two design options for a new substation—Substation 3, located between Substation 2 and Substation 4, respectively—and two new connecting power lines—Line 2 and Line 3, respectively:

- Option 1—overhead power lines: OHL1 (Line 2) and OHL2 (Line 3)—as shown in Table 2;
- Option 2—underground power lines: UGL1 (Line 2) and UGL2 (Line 3)—as shown in Table 1.

To increase confidence in the accuracy of the results, the harmonic impedance values were determined using two numerical calculation tools: Mathcad and MATLAB–Simulink.

The proposed methodology has been validated through an independent comparison between the analytical results obtained from the developed mathematical model and MATLAB/Simulink simulations performed using the Impedance Measurement tool, showing excellent agreement. Although this comparison provides confidence in the correctness of the proposed approach within the adopted modeling assumptions, further validation through field measurements, benchmark systems and additional commercial software constitutes an important direction for future research and practical implementation.

The calculation in Mathcad was based on the mathematical relationships explained in the first case study (A1–A34). These were applied to each element in the network area under consideration, and for each relevant section, the harmonic impedance was calculated by placing the two components “visible” to the left and to the right, respectively, in parallel. For the analyzed network area (Figure 9), consisting of four nodes and three sides with a radial structure, the calculation relationships for the harmonic impedances “seen” to the left are obtained by traversing the diagram from the left end to the right end, and the harmonic impedances “seen” to the right are obtained by traversing the diagram from the right end to the left end. These mathematical relationships are written out and explained in Appendix A (A35–A55).

The application of the second calculation tool involves building a Simulink model for the same network section, using components from the Simscape Electrical/Specialized Power Systems (SPS) library, based on the same characteristic values of the component elements. To calculate the harmonic impedance in the same sections of the three-phase

circuit, the specialized virtual tool (Impedance Measurement), located in the same Simulink library, was installed. Figure 10 shows the model and the power flow results for the LSM mode—Variant 2.

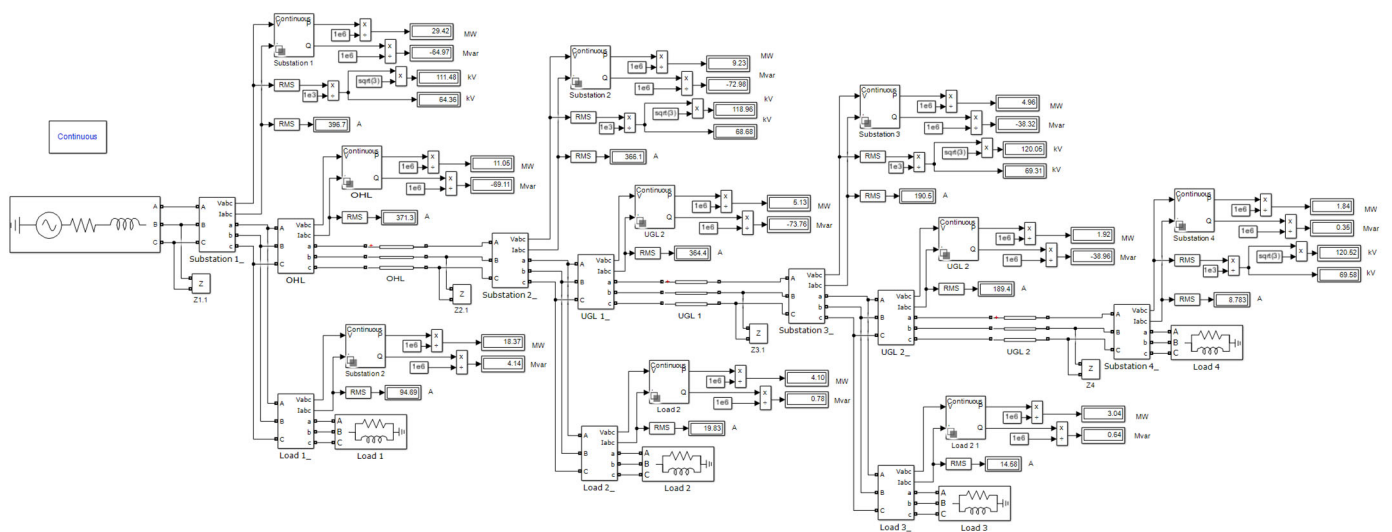


Figure 10. Simulink model for LSM mode—version 2.

According to the MATLAB/Simulink R2024a Documentation, the Impedance Measurement tool computes impedance using a state-space linearization method. It injects a small test current between two nodes, measures the resulting voltage, and defines the impedance as the transfer function: $Z(s) = V(s)/I(s)$. The network is represented in state-space form, the transfer function is derived from the system matrices, and the impedance is obtained by evaluating the transfer function at $s = j\omega$, over the specified frequency range. The method is valid only for network elements that the Powergui central block can linearize. Modeling lines with distributed parameters cannot be linearized, but the Distributed Parameter Line block is one of the exceptions, so it could be used in the case study.

The graphical representation of the frequency variation of the harmonic impedance values for the two operating modes and the two design variants applicable to the grid expansion, as “observed” on the four busbars of the stations under consideration, can be seen in Figures 11–14.

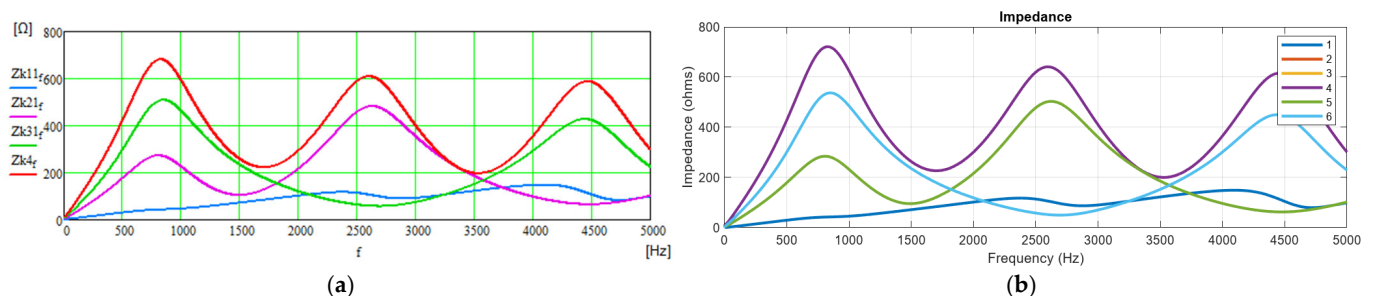


Figure 11. Harmonic impedance for PSM mode, variant 1: (a) Mathcad; (b) MATLAB–Simulink.

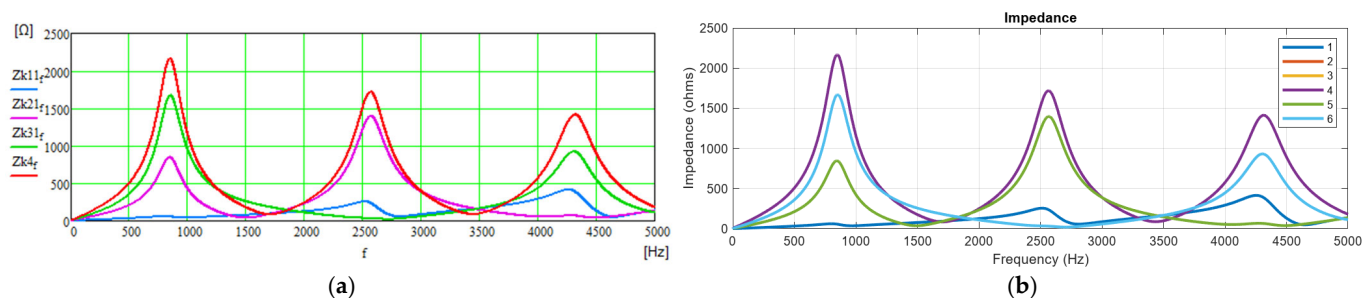


Figure 12. Harmonic impedance for LSM mode, variant 1: (a) Mathcad; (b) MATLAB–Simulink.

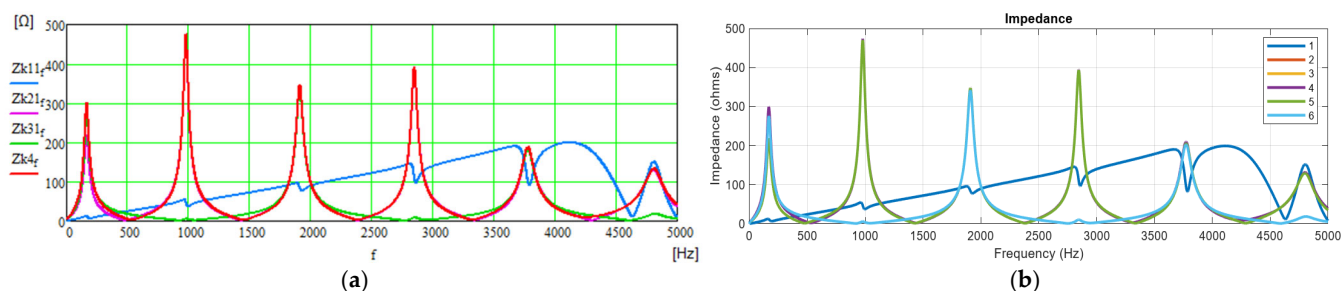


Figure 13. Harmonic impedance for PSM operation, variant 2: (a) Mathcad; (b) MATLAB–Simulink.

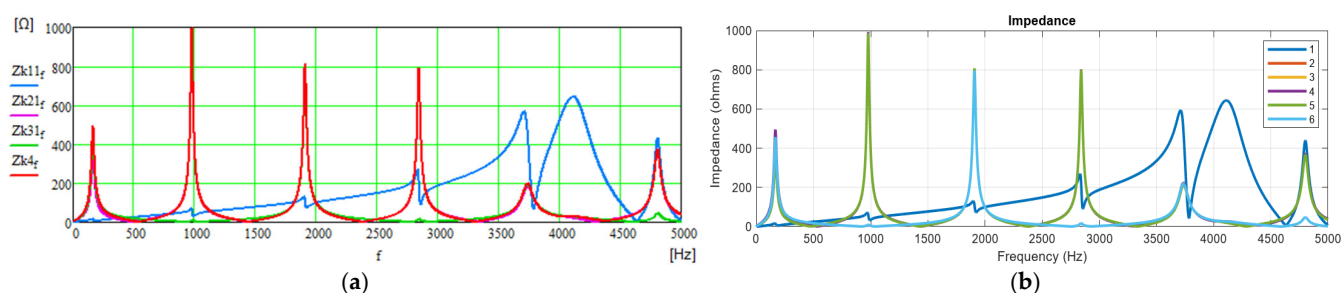


Figure 14. Harmonic impedance for LSM mode, variant 2: (a) Mathcad; (b) MATLAB–Simulink.

For the same operating mode and the same design variant, the graphical representation of the values obtained using the two numerical calculation tools has been placed side by side to facilitate a quick comparison. It can be observed that the values obtained by the two methods are identical. It should be noted here that the graphs plotted using the Impedance Measurement tool include the representation of the impedance values seen on either side of the nodes corresponding to the substations. Because these values are equal, the corresponding graphs overlap, so that 6 representations appear in the legend, but only 4 of them are visible.

The most significant values, corresponding to the poles, represented by the coordinates (f_k, Z_k) , are extracted in Table 7 and Table 8.

Table 7. Coordinates of the poles of harmonic impedances—design variant 1 (OHL).

Poles		Pole 1		Pole 2		Pole 3	
Regime	Node	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]
LSM	Subst. 1	804	61	2516	256	4258	416
	Subst. 2	847	846	2570	1397	4280	69
	Subst. 3	853	1670	-	-	4308	924
	Subst. 4	850	2164	2568	1722	4318	1416
	k	16–17	-	50–51	-	85–86	-

PSM	Subst. 1	-	-	2376	118	4097	149
	Subst. 2	807	285	2625	503	-	-
	Subst. 3	849	537	-	-	4442	448
	Subst. 4	829	721	2599	642	4463	617
	k	16–17	-	48–53	-	82–89	-

Table 8. Coordinates of the poles of harmonic impedances—design variant 2 (UGL).

Poles		Pole 1		Pole 2		Pole 3		Pole 4		Pole 5		Pole 5.2		Pole 6	
Regime	Node	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]	f_k [Hz]	Z_k [Ω]
LSM	Subst. 1	162	17	973	73	1901	131	2837	270	3711	566	4112	644	4810	432
	Subst. 2	166	363	979	989	1911	811	2846	790	3741	186	-	-	4810	370
	Subst. 3	166	458	979	10	1911	799	2846	16	3740	195	-	-	4810	45
	Subst. 4	166	496	979	995	1911	811	2846	792	3740	198	4092	27	4810	376
	k	3	-	19–20	-	38	-	57	-	74–75	-	82	-	96	-
PSM	Subst. 1	159	13	962	55	1880	96	2812	146	3667	190	4116	199	4811	151
	Subst. 2	167	219	980	471	1912	348	2850	391	3780	186	-	-	4809	131
	Subst. 3	167	277	981	5	1912	343	2851	9	3779	183	-	-	4817	17
	Subst. 4	167	300	980	474	1912	348	2850	392	3779	189	-	-	4811	133
	k	3	-	19–20	-	38	-	56–57	-	73–76	-	82	-	96	-

Analysis of these results leads to the following conclusions:

- For the frequency range under consideration, the number of poles observed at the four nodes of the network area is:
- Option 1—OHL: 2 or 3;
- Option 2—UGL: 6 or 7.
- The observation from the first case study is confirmed, namely that the use of UGL results in a higher number of “teeth”.
- The pole frequency values are grouped into relatively narrow ranges, which are relatively unaffected by operating conditions. The harmonic orders corresponding to these frequency ranges fall within the following intervals:
- Option 1—OHL: 16÷17 (Pole 1); 48÷53 (Pole 2); 82÷89 (Pole 3);
- Variant 2—UGL: 3 (Pole 1); 19÷20 (Pole 2); 38 (Pole 3); 56÷57 (Pole 4); 73÷76 (Pole 5); 82 (Pole 5.2); 96 (Pole 6).
- The pole amplitudes are largely influenced by the load. Increasing the load from the values in the LSM regime to those in the PSM regime results in values that are (2÷3.11) times smaller for Option 1, and (1.65÷2.86) times smaller for Option 2.
- The highest values correspond to Variant 1. Using this design variant increases the risk of parallel resonance, not only by increasing the values of but also by increasing the width of the “teeth.” In both variants, increasing the load leads to the widening of the “teeth” but also to their shortening.
- The use of Variant 2 increases the risk of parallel resonances by increasing the number of “teeth,” but this increase is limited due to the small widths of the “teeth” and the lower values of the peaks.

The different resonance behavior of overhead transmission lines and underground cables originates from the significantly different values of their distributed series and shunt parameters, determined by the geometrical configuration and material properties of the respective transmission-line technologies. Under parallel resonance conditions, the equivalent inductive and capacitive reactances compensate each other, so that the harmonic impedance becomes predominantly determined by the equivalent resistive component. Consequently, the considerably higher distributed series resistance of overhead

transmission lines leads to higher harmonic impedance peaks under comparable operating conditions.

In contrast, the resonance frequencies are governed mainly by the distributed inductance and capacitance. For the transmission lines investigated in this work, the much larger distributed capacitance of underground cables constitutes the dominant factor responsible for reducing the spacing between successive resonance frequencies within the investigated frequency range. At the same time, it reduces the spatial spacing between successive harmonic impedance maxima, producing a higher spatial density of resonance locations along the transmission line. Consequently, for identical transmission-line lengths, identical operating conditions and the same investigated frequency range, underground cables exhibit both a larger number of resonance frequencies and a larger number of line sections where the harmonic impedance reaches local maxima.

Unlike conventional node-based harmonic impedance assessment, the proposed methodology enables the identification not only of resonance frequencies but also of the corresponding line sections susceptible to harmonic amplification. This provides transmission and distribution system operators with information on where resonance-induced harmonic amplification is most likely to occur, thereby supporting insulation coordination, monitoring strategies and future harmonic mitigation measures.

6. Harmonic Impedance Along the Network

In the first case study, we found that the value of harmonic impedance varies significantly across the sections considered along a power line, and in the second case study, we observed the variation of harmonic impedance at the nodes of a network zone. As will be shown in this chapter, by modeling lines with distributed parameters, each section along a line can be treated as a network node.

This chapter applies the methods from the previous chapters to determine the harmonic impedance seen in the sections considered along the entire network zone (Figure 9). To this end, using the distance from the end of the line as a variable, expressions (A32–A34) are applied, in which, for the equivalent impedances connected at the ends, the values calculated at the respective nodes of the network area are substituted using expressions (A35–A55).

For example, for line 1, connected between nodes 1 and 2, the impedance seen along the line, $Z_{line1}(f, l_1)$, (A58) is written using the calculation expressions for the equivalent impedance seen upstream (in the section at the beginning of the line) $Z_{12L}(f)$, (A56), and for the equivalent impedance seen downstream (in the section at the end of the line) $Z_{21R}(f)$, (A57).

It can be observed that the values of the impedances seen in the sections on the sides connected to a node, in its immediate vicinity, are equal (A45, A49, A53). The value corresponding to these is the equivalent impedance obtained by connecting in parallel all the transverse equivalent impedances seen from that node (connected to the node). In other words, it is the impedance seen at that node.

Thus, each node of the network, both the end of one (or more) branches and the beginning of another (or others) branches of the network, can be associated with a transfer section between those branches, which ensures the continuity of the harmonic impedance values throughout the network.

Based on this property, a unified analysis of the variation in harmonic impedance across the entire network can be performed, with the nodes corresponding to specific sections of the circuits and thus being characterized, like the other sections, by the coordinate that positions them in space—the distance from the end of the network.

This approach allows the 3D representation of the variation in harmonic impedance, $Z_k(f, dist)$, to be created for all sections of the network, so that the 2D representations of

the variations, corresponding to the nodes, $Z_k(f)$, can be obtained by making longitudinal “slices” in the 3D surface, corresponding to the associated values of the coordinate.

This method was applied in the continuation of the case study for the network area presented earlier. The three matrices corresponding to the harmonic impedance seen in the cross-sections of the three lines of dimensions $L_1 \times f_{max}$, $L_2 \times f_{max}$, $L_3 \times f_{max}$, were concatenated, unifying the variable representing the distance from the end of the line. The distance variable (dist) is measured relative to the end of line 3, resulting in a matrix of dimensions $L \times f_{max}$ (where $L = L_1 + L_2 + L_3 = 79$ km, $f_{max} = 5000$ Hz).

For the analyzed network area, the same construction variants were considered for Line 1 and Line 2 (OHL and UGL, respectively) and the same operating modes (LSM and PSM, respectively). The graphical representation of the obtained values is shown in Figure 15, and the pole coordinates were extracted in Table 9 and Table 10.

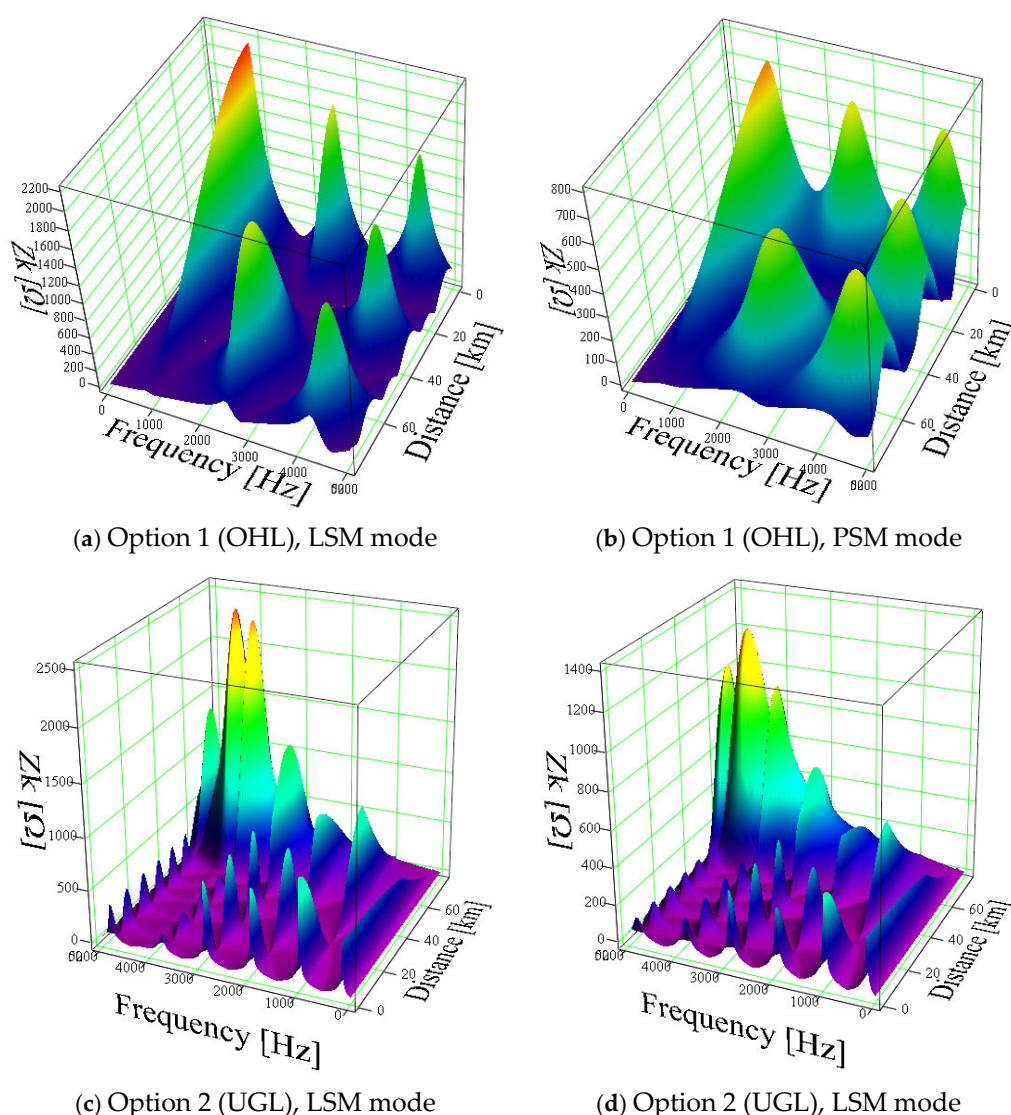


Figure 15. Graphical representation of the harmonic impedance variation across the network area.

Table 9. Coordinates of the poles of the harmonic impedances seen along the network section—Option 1 (OHL).

Regime	Coordinates	Row 1	Row 2		Row 3		
		Pole 1	Pole 2	Pole 3	Pole 4	Pole 5	Pole 6
LSM	f_k [Hz]	850	2568	2565	4318	4318	4312

	k	17	51	51	86	86	86
	$dist$ [km]	0	0	57	0	34	68
	Z_k [Ω]	2164	1722	1706	1416	1410	1402
PSM	f_k [Hz]	829	2599	2601	4463	4473	4468
	k	17	52	52	89	89	89
	$dist$ [km]	0	0	57	0	33	66
	Z_k [Ω]	721	642	590	617	613	624

Table 10. Coordinates of the poles of the harmonic impedance seen along the network section—Option 2 (UGL).

Regime Coordinates													
LSM		Row 1		Row 2		Row 3		Row 4		Row 5			
		Pole 1	Pole 2	Pole 3	Pole 4	Pole 5	Pole 6	Pole 7	Pole 8	Pole 9	Pole 10	Pole 11	Pole 12
	f_k [Hz]	166	979	979	1911	1911	1911	2846	2846	2846	2843	3742	3740
	k	3	20	20	38	38	38	57	57	57	57	75	75
	$dist$ [km]	0	0	48	0	26	49	0	17	34	60	0	13
	Z_k [Ω]	496	991	996	799	811	810	767	786	792	1285	186	197
		Row 5					Row 6						
		Pole 13	Pole 14	Pole 15	Pole 16	Pole 17	Pole 18	Pole 19	Pole 20	Pole 21	Pole 22	Pole 23	
	f_k [Hz]	3740	3740	3722	4137	4810	4810	4810	4810	4810	4809	4819	-
	k	75	75	74	83	96	96	96	96	96	96	96	-
	$dist$ [km]	27	38	66	67	0	10	21	31	40	49	69	-
	Z_k [Ω]	195	196	2386	2458	376	372	372	366	368	377	1418	-
PSM		Row 1		Row 2		Row 3		Row 4		Row 5			
		Pole 1	Pole 2	Pole 3	Pole 4	Pole 5	Pole 6	Pole 7	Pole 8	Pole 9	Pole 10	Pole 11	Pole 12
	f_k [Hz]	167	980	980	1912	1912	1912	2851	2850	2850	2839	3781	3779
	k	3	20	20	38	38	38	57	57	57	57	76	76
	$dist$ [km]	0	0	48	0	26	49	0	17	34	60	0	13
	Z_k [Ω]	300	472	474	343	348	348	380	390	392	596	178	189
		Row 5					Row 6						
		Pole 13	Pole 14	Pole 15	Pole 16	Pole 17	Pole 18	Pole 19	Pole 20	Pole 21	Pole 22	Pole 23	
	f_k [Hz]	3779	3779	3741	4387	4811	4810	4811	4810	4810	4809	4887	-
	k	76	76	75	88	96	96	96	96	96	96	98	-
	$dist$ [km]	26	38	64	65	0	10	21	31	40	49	66	-
	Z_k [Ω]	188	188	993	1277	133	132	132	130	130	134	1062	-

The main observations that can be drawn from the analysis of the results are as follows:

- The values obtained for harmonic impedance, for the same design variants and operating conditions, reproduce the results obtained in the second case study. More accurately, they include them, because the values of the function $Z_k(f)$ corresponding to nodes Substation 1, 2, 3, 4 in the second case study are practically identical to the values of the function $Z_k(f, dist)$ for the sections located near the end of the network zone, at distances $dist = 0, 27, 49, 79$ km, respectively.
- The poles of the harmonic impedance with the highest amplitudes, in the case of Variant 1, are grouped in sections located at the end of the network, and in the case of Variant 2, in sections located toward the beginning of the network.
- The maximum values of the poles in the case of Variant 1 belong to more restricted domains than in the case of Variant 2:
- OHL, LSM: $Z_k \in (1402 \div 2164) \Omega$;
- OHL, PSM: $Z_k \in (624 \div 721) \Omega$;

- UGL, LSM: $Z_k \in (195 \div 2458) \Omega$;
- UGL, PSM: $Z_k \in (130 \div 1277) \Omega$.
- The maximum poles have the following coordinates and values:
- OHL, LSM: $Z_k(f, dist) = MAX = Z_k(850, 0) = 2164 \Omega$;
- OHL, PSM: $Z_k(f, dist) = MAX = Z_k(829, 0) = 721 \Omega$;
- UGL, LSM: $Z_k(f, dist) = MAX = Z_k(4137, 67) = 2458 \Omega$;
- UGL, PSM: $Z_k(f, dist) = MAX = Z_k(4385, 65) = 1277 \Omega$.
- Most of the sections (apart from the section at the end of the network) in which harmonic impedance poles have been identified do not correspond to network nodes, but are located along the lines, at varying distances from the nodes. This leads to the conclusion that harmonic impedance must be evaluated in all sections of the network in order to more accurately assess the level of distortion amplification and its effects.

The proposed methodology makes it possible to identify not only the resonance frequencies but also the transmission line sections where the harmonic impedance reaches high values and harmonic voltage amplification is most likely to occur under harmonic excitation. Consequently, the network operator can identify the locations and frequencies associated with an increased risk of harmonic overvoltages, evaluate their possible impact on insulation stress and corona phenomena, and adopt appropriate mitigation measures during network planning and operation. Therefore, the proposed approach provides a practical tool for assessing harmonic resonance risk and supporting engineering decisions aimed at improving the reliability and power quality of high-voltage AC transmission networks.

7. Conclusions

The evaluation of harmonic impedance at the nodes of a three-phase distribution network is becoming increasingly necessary as the concentration of distributed renewable sources and their individual power levels continues to grow. Inverters, as interface devices between the grid and these sources, can cause the amplification of the distortion regime caused by nonlinear loads, especially when harmonic current filters are malfunctioning. If the frequencies of the harmonic currents originating from the inverters are close to or identical to those of the poles of the harmonic impedances present at the respective grid nodes, parallel resonances result in the amplification of the corresponding harmonic currents and voltages. The risk of such a phenomenon is all the greater as the number of poles of the harmonic impedances in the nodes increases (across a wider range of frequencies) and as the amplitude of these poles increases.

The poles correspond to the parallel resonance frequencies produced between the equivalent transverse capacitive reactances, whether natural or artificial, and the equivalent transverse inductive reactances of the network elements. They are more numerous (appearing across a wider range of frequency values) as the number of capacitive elements in the network increases. To numerically determine the harmonic impedances and thus their poles, long power lines are modeled with uniformly distributed parameters, with the aim of reducing modeling errors.

This article presents two case studies showing the results of known methods for the numerical determination of harmonic impedance, applied to a real high-voltage (110 kV) distribution network area comprising overhead and underground power lines, respectively.

In the first case study, which utilized Mathcad, the variation in harmonic impedance is determined at selected sections along the power line under various load conditions. The main observations that can be drawn from this study are as follows:

- Along a power line, the variation in harmonic impedance values is very large.
- There is a dual periodicity of poles: in the spatial (distance) domain and in the frequency domain.
- For a given frequency range, the period of pole occurrence in the frequency domain decreases as the line length increases (the number of poles and pole groups is greater the longer the line).
- For a given line length, the period of pole occurrence in the length domain decreases as the frequency increases (the number of poles and pole orders is greater the wider the frequency range considered).
- The maximum values of harmonic impedance (corresponding to the pole peaks) increase as the active load decreases, and they decrease as the active load increases, which confirms the attenuation effect of the active load in the case of parallel resonances.
- For the same length and frequency range, the number of poles and pole orders is higher in an OHL than in a UGL.
- Modeling the line using uniformly distributed parameters allows for the identification of critical sections and critical frequencies, for which amplifications of harmonic voltages—and thus of the rms voltage values—may occur, resulting in insulation overloading and an increased corona effect, and the greatest risks are associated with the lowest load conditions (no-load operation).

The second case study focuses on the numerical evaluation of harmonic impedance observed at the nodes of a real 110 kV distribution network, where new lines are to be integrated—overhead lines (Variant 1) and underground cables (Variant 2), having the same construction characteristics as those in the first study.

Considering two forecasted low-load operating scenarios associated with maximum generation from future distributed energy sources, the main conclusions are as follows:

- Variant 2 (UGL) results in a higher number of impedance poles at network nodes compared to Variant 1.
- Pole frequency values are grouped within relatively narrow bands, only being slightly influenced by operating conditions (load) but strongly dependent on online construction characteristics (especially natural equivalent capacitances).
- Pole amplitudes are significantly influenced by load: minimum amplitudes correspond to maximum load, and vice versa.
- The highest pole amplitudes are associated with Variant 1 (OHL), which increases the risk of parallel resonance through both higher amplitudes and wider pole bandwidths.
- In both variants, increasing the load leads to widening of the poles (frequency intervals) while also reducing their sharpness.
- Variant 2 increases the risk of resonance through a higher number of poles, but this effect is partially mitigated by the reduced pole amplitudes and narrower bandwidths.

The reliability of the results obtained using Mathcad modeling is confirmed through validation with MATLAB–Simulink simulations, which yielded consistent results.

The most important conclusions of the study were derived from the analysis of harmonic impedance along the entire network area:

- For a radially configured network, a unified analysis of harmonic impedance variation can be performed along any supply path formed by series-connected lines between the source node and terminal nodes. Network nodes correspond to particular sections of this path and are characterized by their spatial coordinate, defined as the distance from the network end.

- Evaluating impedances along an entire supply path allows for a 3D representation of the function $Z_k(f, dist)$, covering all network sections. The traditional 2D plots of $Z_k(f)$ at specific nodes can be obtained as longitudinal cross-sections of this 3D surface.
- Most sections where harmonic impedance poles are identified (except at the end-node section) do not coincide with actual network nodes but are located at varying distances from them. This leads to the conclusion that harmonic impedances must be evaluated continuously along all network sections to accurately predict harmonic amplification caused by transverse equivalent capacitances.
- This supports the need to shift from the traditional paradigm of harmonic impedance evaluation at discrete nodes to a continuous, section-based evaluation approach across the entire network.

The principal scientific contribution of the present work is the reformulation of harmonic impedance assessment from a discrete node-based quantity into a continuous spatial-frequency characteristic of high-voltage transmission networks, enabled by distributed-parameter transmission-line modeling. Consequently, the harmonic impedance evaluated at network buses is no longer the only available representation of the network behavior but rather a particular case of a continuous harmonic impedance distribution along the transmission lines.

Beyond the identification of resonance frequencies, the proposed methodology enables the localization of line sections where harmonic voltage amplification is most likely to occur. This information allows transmission and distribution system operators to identify critical sections that may experience stationary harmonic overvoltages, resulting in increased insulation stress and, for overhead transmission lines, intensified corona-related phenomena. Consequently, the proposed approach provides valuable support for insulation coordination, condition assessment, monitoring strategies and the future implementation of harmonic mitigation measures.

The highest risk of harmonic amplification generally occurs under lightly loaded operating conditions, where the natural capacitive behavior of long transmission lines becomes more pronounced. Under abnormal operating conditions, such as malfunction or disconnection of harmonic filters associated with renewable-energy installations, the resonance phenomena may become considerably more severe. Experimental validation of these aspects and the evaluation of possible mitigation strategies constitute important directions for future research.

For larger or meshed transmission networks, to apply the results obtained for a radial network, a methodology consisting of two successive computational stages can be outlined. First, the harmonic impedances at the network buses are determined using any conventional harmonic impedance assessment technique. Subsequently, the harmonic impedance distribution along each transmission-line section is evaluated from the harmonic impedances at its terminal nodes together with the distributed electrical parameters of the corresponding transmission line. This computational framework suggests that the proposed methodology has the potential to be extended to larger transmission systems without fundamentally modifying its computational procedure. However, this potential has not yet been systematically investigated. In particular, the applicability of the proposed methodology to large interconnected and closed-loop (meshed) transmission networks remains to be verified and will constitute an important direction for future research.

Although the proposed methodology has demonstrated its capability to evaluate harmonic impedance continuously along transmission lines within the adopted modeling framework, a few other aspects deserve further investigation. Future research will focus on incorporating more advanced frequency-dependent transmission-line models, performing systematic experimental validation through field measurements, benchmarking

the methodology against additional commercial simulation tools, extending the analysis to large interconnected and closed-loop (meshed) transmission networks, and investigating practical resonance mitigation techniques based on the identification of line sections susceptible to harmonic voltage amplification. These developments are expected to further enhance the applicability of the proposed methodology under practical operating conditions while preserving its fundamental analytical framework.

The present study compares two limiting transmission-line design alternatives in order to emphasize the influence of line construction technology on harmonic impedance and resonance behavior. Nevertheless, the proposed methodology is equally applicable to hybrid overhead–underground transmission corridors by assigning the corresponding distributed electrical parameters to each line section while preserving the same analytical framework. Furthermore, although the methodology is directly applicable to radial transmission corridors, it can also be extended to more complex branched networks through the equivalent-impedance approach. If the methodology is applied to networks with complex configurations, the computational effort required to determine the profile of harmonic impedance throughout the network will inevitably increase, and it may be necessary to perform additional modeling of multiple harmonic sources, frequency-dependent load characteristics, converter dynamics, and switching events. These aspects are beyond the scope of the present paper and constitute important directions for future research.

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Appendix A

Electrical quantities, characteristic quantities, and calculation relationships used in the construction of the mathematical models implemented in MathCAD:

A1	$Z_{cc}(f) = R_{cc} + j2\pi f L_{cc}$	The equivalent short-circuit impedance for modeling the upstream three-phase power system (as seen at the beginning of the power line), as a function of frequency;
A2	$L_{cc} = \frac{X_{cc}}{2\pi f_1}$	The equivalent inductance corresponding to the equivalent short-circuit reactance at the beginning of the line. This is determined by the fundamental frequency;
A3	$X_{cc} = \frac{U_n^2}{S_{cc}}$	The equivalent short-circuit reactance corresponding to the equivalent short-circuit apparent power at the node at the beginning of the line, of known value at the fundamental frequency; It is calculated using the rated voltage U_r ;

A4	$f_1 = 50 \text{ Hz}$	The value of the fundamental frequency;
A5	$R_{cc} = 0.1 \cdot X_{cc}$	The typical value of the resistive component of the short-circuit impedance, calculated in relation to the value of the equivalent short-circuit reactance. It is considered constant with frequency;
A6	$Z_{l1}(f) = \frac{1}{Y_{l1}(f)}$	The impedance of the equivalent load connected at the beginning of the line (upstream). Lines connected to the same busbars as the line under study are modeled by the sum of the active and reactive powers drawn from the busbars, which are attributed to an equivalent load. Its equivalent impedance (admittance) is expressed as a function of frequency;
A7	$Y_{l1}(f) = G_{l1} - j \frac{1}{2\pi f L_{l1}}$	The equivalent admittance of the upstream load, expressed as a function of frequency;
A8	$G_{l1} = \frac{P_{l1}}{U_n^2}$	The equivalent conductance corresponding to the active power transferred to the equivalent upstream load, calculated at the rated voltage. It is considered constant with frequency;
A9	$L_{l1} = \frac{U_n^2}{2\pi f_1 Q_{l1}}$	The inductance corresponding to the equivalent susceptance used to model the inductive reactive power transferred to the equivalent upstream load; The determination is made for the fundamental frequency;
A10	$Z_1(f) = Z_{l1}(f) \parallel Z_{cc}(f) = \frac{Z_{l1}(f) \cdot Z_{cc}(f)}{Z_{l1}(f) + Z_{cc}(f)}$	The equivalent impedance at the beginning of the line, as viewed from upstream, is calculated using the formula for the impedance resulting from connecting the impedances in parallel $Z_{cc}(f)$ and $Z_{l1}(f)$;
A11	$Z_{l2}(f) = \frac{1}{Y_{l2}(f)}$	The impedance of the equivalent load connected at the end of the line used to model the downstream electrical network, expressed as a function of frequency;
A12	$Y_{l2}(f) = G_{l2} - j \frac{1}{2\pi f L_{l2}}$	The admittance of the equivalent downstream load, expressed as a function of frequency;
A13	$G_{l2} = \frac{P_{l2}}{U_n^2}$	The equivalent conductance corresponding to the active power transferred to the downstream network, calculated at the rated voltage. It is considered constant with frequency;
A14	$L_{l2} = \frac{U_n^2}{2\pi f_1 Q_{l2}}$	The inductance corresponding to the equivalent susceptance used to model the inductive reactive power transferred to the downstream network; The determination is made for the fundamental frequency;
A15	$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} U_3 \\ I_3 \end{bmatrix}$	The matrix relationship between the electrical quantities (voltage, current) in the section at the beginning of the line (Section 1) and those in any section along the line (Section 3), corresponding to the positive-sequence four-pole equivalent circuit;
A16	$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$	The matrix of coupling coefficients (chain coefficients) between the electrical quantities (voltage, current) in the section at the beginning of the line and those in any section along the line;
A17	$\begin{bmatrix} U_3 \\ I_3 \end{bmatrix} = \mathbf{B} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$	The matrix relationship between the electrical quantities (voltage, current) in any section along the line and those in the section at the end of the line (Section 2), corresponding to the positive-sequence four-pole equivalent circuit;
A18	$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$	The matrix of coupling coefficients (chain coefficients) between the electrical quantities (voltage, current) in any section along the line and those in the section at the end of the line;
A19	$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \mathbf{A} \cdot \mathbf{B} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$	The matrix relationship between the electrical quantities (voltage, current) in the section at the beginning of the line and those in the section at the end of the line;
A20	$\underline{A}_{11}(f, l) = \underline{A}_{22}(f, l) = \cosh[\underline{\gamma}(f) \cdot (L - l)]$	The expressions for the coefficients in matrix $\mathbf{A}$, valid for a line of total length L , modeled with uniformly distributed equivalent parameters, which forms a waveguide with characteristic impedance L , for the propagation of an electromagnetic wave with propagation constant $\underline{Z}_c$ that transfers energy between the ends of the line. The values of the coefficients depend on the frequency and the distance $(L - l)$ between Section 3 and the beginning of the line (Section 1);
A21	$\underline{A}_{12}(f, l) = \underline{Z}_c(f) \cdot \sinh[\underline{\gamma}(f) \cdot (L - l)]$	
A22	$\underline{A}_{21}(f, l) = \frac{\sinh[\underline{\gamma}(f) \cdot (L - l)]}{\underline{Z}_c(f)}$	
A23	$\underline{B}_{11}(f, l) = \underline{B}_{22}(f, l) = \cosh[\underline{\gamma}(f) \cdot l]$	The expressions for the coefficients in matrix $\mathbf{B}$. The values of the coefficients depend on the frequency and the distance $\mathbf{B}$ between Section 3 and the end of the line (Section 2);
A24	$\underline{B}_{12}(f, l) = \underline{Z}_c(f) \cdot \sinh[\underline{\gamma}(f) \cdot l]$	
A25	$\underline{B}_{21}(f, l) = \frac{\sinh[\underline{\gamma}(f) \cdot l]}{\underline{Z}_c(f)}$	

A26	$\underline{Z}_c(f) = \sqrt{\frac{\underline{Z}_u(f)}{\underline{Y}_u(f)}}$	The characteristic impedance of the line (wave impedance), as a function of frequency. It does not depend on the length of the line;
A27	$\underline{\gamma}(f) = \sqrt{\underline{Z}_u(f) \cdot \underline{Y}_u(f)}$	The propagation constant of the electromagnetic wave, as a function of frequency. It does not depend on the length of the line;
A28	$\underline{Z}_u(f) = R_u^+ + j2\pi f L_u^+$	The positive-sequence equivalent longitudinal impedance, used in the quadripole equivalent circuit of the line, per unit length, expressed as a function of frequency and using the known values of the positive-sequence equivalent resistance and inductance for the fundamental frequency;
A29	$\underline{Y}_u(f) = G_u^+ + j2\pi f C_u^+$	The equivalent positive-sequence transverse admittance, used in the line's quadripole equivalent circuit, per unit length, expressed as a function of frequency and using the known values of the equivalent positive-sequence conductance and capacitance for the fundamental frequency;
A30	$\underline{Z}_L = \frac{\underline{A}_{22} \cdot \underline{Z}_1 + \underline{A}_{12}}{\underline{A}_{21} \cdot \underline{Z}_1 + \underline{A}_{11}}$	The general relationship for calculating the equivalent impedance seen at the junction of quadripoles Q ₁ and Q ₂ , viewed in the upstream direction (to the left in Figure 2), determined as the input impedance of quadripole Q ₁ fed across terminals 3-3' and having impedance $\underline{Z}_1$ as its load;
A31	$\underline{Z}_R = \frac{\underline{B}_{11} \cdot \underline{Z}_2 + \underline{B}_{12}}{\underline{B}_{21} \cdot \underline{Z}_2 + \underline{B}_{22}}$	The general equation for calculating the equivalent impedance seen in the section at the boundary between quadripoles Q ₁ and Q ₂ , as viewed in the downstream direction (to the right in Figure 2), is defined as the input impedance of quadripole Q ₂ , which is fed across terminals 3-3' and has impedance $\underline{Z}_2$ as its load;
A32	$\underline{Z}_L(f, l) = \underline{Z}_c(f) \cdot \frac{\tanh[\underline{\gamma}(f) \cdot (L - l)] \cdot \underline{Z}_c(f) + \underline{Z}_1(f)}{\tanh[\underline{\gamma}(f) \cdot (L - l)] \cdot \underline{Z}_1(f) + \underline{Z}_c(f)}$	The specific relationship for calculating the equivalent impedance seen from the left (upstream) at any given section of the line located at a distance $(L - l)$ from the line's starting point, expressed as a function of frequency [46,47];
A33	$\underline{Z}_R(f, l) = \underline{Z}_c(f) \cdot \frac{\tanh[\underline{\gamma}(f) \cdot l] \cdot \underline{Z}_c(f) + \underline{Z}_2(f)}{\tanh[\underline{\gamma}(f) \cdot l] \cdot \underline{Z}_2(f) + \underline{Z}_c(f)}$	The specific relationship for calculating the equivalent impedance seen in the forward direction (downstream) at any point along the line located at a distance l from the end of the line, expressed as a function of frequency;
A34	$\underline{Z}_k(f, l) = \underline{Z}_L(f, l) \parallel \underline{Z}_R(f, l) = \frac{\underline{Z}_L(f, l) \cdot \underline{Z}_R(f, l)}{\underline{Z}_L(f, l) + \underline{Z}_R(f, l)}$	The equation for numerically determining the value of the harmonic impedance seen along the power line (at any given section of the line), corresponding to the equation for the equivalent impedance resulting from the parallel connection of the impedances $\underline{Z}_L$ and $\underline{Z}_R$. The value depends both on the position of the calculation section along the line and on the frequency;
A35	$\underline{Z}_{AR}(f) = \underline{Z}_{load4}(f)$	The impedance shown on the right side of Section 4 is the impedance of load 4;
A36	$\underline{Z}_{32R}(f) = \underline{Z}_{c3}(f) \cdot \frac{\underline{Z}_{AR}(f) + \underline{Z}_{c3}(f) \cdot \tanh(\underline{\gamma}_3(f) \cdot L_3)}{\underline{Z}_{c3}(f) + \underline{Z}_{AR}(f) \cdot \tanh(\underline{\gamma}_3(f) \cdot L_3)}$	The impedance shown on the right side of circuit section S ₃₂ (Figure 9) is the impedance at the beginning of line 3, with characteristic values $\underline{Z}_{c3}(f)$, $\underline{\gamma}_3(f)$, L_3 and the equivalent impedance connected at the end equal to $\underline{Z}_{AR}(f)$;
A37	$\underline{Z}_{31R}(f) = \frac{1}{\frac{1}{\underline{Z}_{load3}(f)} + \frac{1}{\underline{Z}_{32R}(f)}}$	The impedance shown on the right side of circuit section S ₃₁ (Figure 9) is obtained by connecting the impedances $\underline{Z}_{load3}(f)$ and $\underline{Z}_{32R}(f)$ in parallel;
A38	$\underline{Z}_{22R}(f) = \underline{Z}_{c2}(f) \cdot \frac{\underline{Z}_{31R}(f) + \underline{Z}_{c2}(f) \cdot \tanh(\underline{\gamma}_2(f) \cdot L_2)}{\underline{Z}_{c2}(f) + \underline{Z}_{31R}(f) \cdot \tanh(\underline{\gamma}_2(f) \cdot L_2)}$	The impedance shown on the right side of circuit section S ₂₂ (Figure 9) is the impedance at the beginning of line 2, with characteristic values $\underline{Z}_{c2}(f)$, $\underline{\gamma}_2(f)$, L_2 and the equivalent impedance connected at the end equal to $\underline{Z}_{31R}(f)$;
A39	$\underline{Z}_{21R}(f) = \frac{1}{\frac{1}{\underline{Z}_{load2}(f)} + \frac{1}{\underline{Z}_{22R}(f)}}$	The impedance shown on the right side of circuit section S ₂₁ (Figure 9) is obtained by connecting the impedances $\underline{Z}_{load2}(f)$ and $\underline{Z}_{22R}(f)$ in parallel;
A40	$\underline{Z}_{12R}(f) = \underline{Z}_{c1}(f) \cdot \frac{\underline{Z}_{21R}(f) + \underline{Z}_{c1}(f) \cdot \tanh(\underline{\gamma}_1(f) \cdot L_1)}{\underline{Z}_{c1}(f) + \underline{Z}_{21R}(f) \cdot \tanh(\underline{\gamma}_1(f) \cdot L_1)}$	The impedance shown on the right side of circuit section S ₁₂ (Figure 9) is the impedance at the beginning of Line 1, with characteristic values $\underline{Z}_{c1}(f)$, $\underline{\gamma}_1(f)$, L_1 and the equivalent impedance connected at the end equal to $\underline{Z}_{21R}(f)$;
A41	$\underline{Z}_{11R}(f) = \frac{1}{\frac{1}{\underline{Z}_{load1}(f)} + \frac{1}{\underline{Z}_{12R}(f)}}$	The impedance shown on the right side of circuit section S ₁₁ (Figure 9) is obtained by connecting in parallel the impedances $\underline{Z}_{load1}(f)$ and $\underline{Z}_{12R}(f)$;
A42	$\underline{Z}_{11L}(f) = \underline{Z}_{cc}(f)$	The impedance shown on the left in S ₁₁ is the equivalent short-circuit impedance of the upstream network ($\underline{Z}_{cc}(f)$);
A43	$\underline{Z}_{12L}(f) = \frac{1}{\frac{1}{\underline{Z}_{load1}(f)} + \frac{1}{\underline{Z}_{11L}(f)}}$	The impedance shown on the left in S ₁₂ is obtained by connecting the impedances ($\underline{Z}_{load1}(f)$ and $\underline{Z}_{11L}(f)$) in parallel;

A44	$\underline{Z}_{k11}(f) = \frac{1}{\frac{1}{\underline{Z}_{11L}(f)} + \frac{1}{\underline{Z}_{11R}(f)}}$ $\underline{Z}_{k12}(f) = \frac{1}{\frac{1}{\underline{Z}_{12L}(f)} + \frac{1}{\underline{Z}_{12R}(f)}}$	The harmonic impedances shown in S_{11} and S_{12} , respectively, are obtained by connecting the impedances shown on the left and right sides of those sections in parallel;
A45	$\underline{Z}_{k1}(f) = \underline{Z}_{k11}(f) = \underline{Z}_{k12}(f)$	The harmonic impedances seen in the sections on the sides leading to node 1 are equal and correspond to the harmonic impedance seen in node 1;
A46	$\underline{Z}_{21L}(f) = \underline{Z}_{c1}(f) \cdot \frac{\underline{Z}_{12L}(f) + \underline{Z}_{c1}(f) \cdot \tanh(\underline{\gamma}_1(f) \cdot L_1)}{\underline{Z}_{c1}(f) + \underline{Z}_{12L}(f) \cdot \tanh(\underline{\gamma}_1(f) \cdot L_1)}$	The impedance seen to the left of S_{21} is the impedance seen at the end of line 1, having the characteristic quantities $\underline{Z}_{c1}(f)$, $\underline{\gamma}_1(f)$, L_1 and the equivalent upstream network impedance equal to $\underline{Z}_{12L}(f)$;
A47	$\underline{Z}_{22L}(f) = \frac{1}{\frac{1}{\underline{Z}_{load2}(f)} + \frac{1}{\underline{Z}_{21L}(f)}}$	The impedance seen to the left of S_{22} is obtained by placing in parallel the impedances $\underline{Z}_{load2}(f)$ and $\underline{Z}_{21L}(f)$;
A48	$\underline{Z}_{k21}(f) = \frac{1}{\frac{1}{\underline{Z}_{21L}(f)} + \frac{1}{\underline{Z}_{21R}(f)}}$ $\underline{Z}_{k22}(f) = \frac{1}{\frac{1}{\underline{Z}_{22L}(f)} + \frac{1}{\underline{Z}_{22R}(f)}}$	The harmonic impedances seen in S_{21} and S_{22} are obtained by placing in parallel the impedances seen to the left and to the right of the respective sections;
A49	$\underline{Z}_{k2}(f) = \underline{Z}_{k21}(f) = \underline{Z}_{k22}(f)$	The harmonic impedances seen in the sections on the sides incident to node 2 are equal and correspond to the harmonic impedance seen at node 2;
A50	$\underline{Z}_{31L}(f) = \underline{Z}_{c2}(f) \cdot \frac{\underline{Z}_{22L}(f) + \underline{Z}_{c2}(f) \cdot \tanh(\underline{\gamma}_2(f) \cdot L_2)}{\underline{Z}_{c2}(f) + \underline{Z}_{22L}(f) \cdot \tanh(\underline{\gamma}_2(f) \cdot L_2)}$	The impedance shown on the left in S_{31} is the impedance at the end of line 1, with characteristic values $\underline{Z}_{c2}(f)$, $\underline{\gamma}_2(f)$, L_2 and the equivalent impedance of the upstream network equal to $\underline{Z}_{22L}(f)$;
A51	$\underline{Z}_{32L}(f) = \frac{1}{\frac{1}{\underline{Z}_{load3}(f)} + \frac{1}{\underline{Z}_{31L}(f)}}$	The impedance shown on the left side of S_{32} is obtained by connecting the impedances $\underline{Z}_{load3}(f)$ and $\underline{Z}_{31L}(f)$ in parallel;
A52	$\underline{Z}_{k31}(f) = \frac{1}{\frac{1}{\underline{Z}_{31L}(f)} + \frac{1}{\underline{Z}_{31R}(f)}}$ $\underline{Z}_{k32}(f) = \frac{1}{\frac{1}{\underline{Z}_{32L}(f)} + \frac{1}{\underline{Z}_{32R}(f)}}$	The harmonic impedances shown in S_{31} and S_{32} , respectively, are obtained by connecting the impedances shown on the left and right sides of those sections in parallel;
A53	$\underline{Z}_{k3}(f) = \underline{Z}_{k31}(f) = \underline{Z}_{k32}(f)$	The harmonic impedances seen in the sections on the sides leading to node 3 are equal and correspond to the harmonic impedance seen in node 3;
A54	$\underline{Z}_{4L}(f) = \underline{Z}_{c3}(f) \cdot \frac{\underline{Z}_{32L}(f) + \underline{Z}_{c3}(f) \cdot \tanh(\underline{\gamma}_3(f) \cdot L_3)}{\underline{Z}_{c3}(f) + \underline{Z}_{32L}(f) \cdot \tanh(\underline{\gamma}_3(f) \cdot L_3)}$	The impedance shown on the left side of Section 4 is the impedance at the end of Line 3, with characteristic values $\underline{Z}_{c3}(f)$, $\underline{\gamma}_3(f)$, L_3 and the equivalent impedance of the upstream network equal to $\underline{Z}_{32L}(f)$;
A55	$\underline{Z}_{k4}(f) = \frac{1}{\frac{1}{\underline{Z}_{4L}(f)} + \frac{1}{\underline{Z}_{4R}(f)}}$	The impedance seen at Section 4 (node 4) is obtained by connecting the impedances seen to the left and right of Section 4 in parallel;
A56	$\underline{Z}_{Lline1}(f, l_1) = \underline{Z}_{c1}(f) \cdot \frac{\tanh[\underline{\gamma}_1(f) \cdot (L_1 - l_1)] \cdot \underline{Z}_{c1}(f) + \underline{Z}_{12L}(f)}{\tanh[\underline{\gamma}_1(f) \cdot (L_1 - l_1)] \cdot \underline{Z}_{12L}(f) + \underline{Z}_{c1}(f)}$	The harmonic impedance seen to the left of a given section on line 1, located at a distance l_1 from the end of the line;
A57	$\underline{Z}_{Rline1}(f, l_1) = \underline{Z}_{c1}(f) \cdot \frac{\tanh[\underline{\gamma}_1(f) \cdot l_1] \cdot \underline{Z}_{c1}(f) + \underline{Z}_{21R}(f)}{\tanh[\underline{\gamma}_1(f) \cdot l_1] \cdot \underline{Z}_{21R}(f) + \underline{Z}_{c1}(f)}$	The harmonic impedance seen to the right of a given section on line 1, located at a distance l_1 from the end of the line;
A58	$\underline{Z}_{kline1}(f, l_1) = \frac{\underline{Z}_{Lline1}(f, l_1) \cdot \underline{Z}_{Rline1}(f, l_1)}{\underline{Z}_{Lline1}(f, l_1) + \underline{Z}_{Rline1}(f, l_1)}$	The harmonic impedance seen in a given point on Line 1, located at a distance l_1 from the end of the line;
A59	$\underline{Z}_{kline1}(f, 0) = \underline{Z}_{k12}(f) = \underline{Z}_{k1}(f)$	The harmonic impedance seen in the section at the beginning of line 1 is equal to the harmonic impedance seen in node 1;
A60	$\underline{Z}_{Lline2}(f, l_2) = \underline{Z}_{c2}(f) \cdot \frac{\tanh[\underline{\gamma}_2(f) \cdot (L_2 - l_2)] \cdot \underline{Z}_{c2}(f) + \underline{Z}_{22L}(f)}{\tanh[\underline{\gamma}_2(f) \cdot (L_2 - l_2)] \cdot \underline{Z}_{22L}(f) + \underline{Z}_{c2}(f)}$	The harmonic impedance seen to the left of a given section on line 2, located at a distance l_2 from the end of the line;
A61	$\underline{Z}_{Rline2}(f, l_2) = \underline{Z}_{c2}(f) \cdot \frac{\tanh[\underline{\gamma}_2(f) \cdot l_2] \cdot \underline{Z}_{c2}(f) + \underline{Z}_{31R}(f)}{\tanh[\underline{\gamma}_2(f) \cdot l_2] \cdot \underline{Z}_{31R}(f) + \underline{Z}_{c2}(f)}$	The harmonic impedance seen to the right of a given section on line 2, located at a distance l_2 from the end of the line;
A62	$\underline{Z}_{kline2}(f, l_2) = \frac{\underline{Z}_{Lline2}(f, l_2) \cdot \underline{Z}_{Rline2}(f, l_2)}{\underline{Z}_{Lline2}(f, l_2) + \underline{Z}_{Rline2}(f, l_2)}$	The harmonic impedance seen in a given point on line 2, located at a distance l_2 from the end of the line;

A63	$\begin{aligned} Z_{kline2}(f, 0) &= Z_{k22}(f) = Z_{k21}(f) = Z_{kline1}(f, L_1) \\ &= Z_{k2}(f) \end{aligned}$	The harmonic impedance seen in the section at the beginning of line 2 is equal to the harmonic impedance seen in the section at the end of line 1 and equal to the harmonic impedance seen in node 2;
A64	$\begin{aligned} Z_{Lline3}(f, l_3) \\ = Z_{c3}(f) \cdot \frac{\tanh\left[\gamma_3(f) \cdot (L_3 - l_3)\right] \cdot Z_{c3}(f) + Z_{32L}(f)}{\tanh\left[\gamma_3(f) \cdot (L_3 - l_3)\right] \cdot Z_{32L}(f) + Z_{c3}(f)} \end{aligned}$	The harmonic impedance seen on the left side of any given section of Line 3, located at a distance l_3 from the end of the line;
A65	$\begin{aligned} Z_{Rline3}(f, l_3) = Z_{c3}(f) \\ \cdot \frac{\tanh\left[\gamma_3(f) \cdot l_3\right] \cdot Z_{c3}(f) + Z_{4R}(f)}{\tanh\left[\gamma_3(f) \cdot l_3\right] \cdot Z_{4R}(f) + Z_{c3}(f)} \end{aligned}$	The harmonic impedance seen on the right side of any given section of Line 3, located at a distance l_3 from the end of the line;
A66	$Z_{kline3}(f, l_3) = \frac{Z_{Lline3}(f, l_3) \cdot Z_{Rline2}(f, l_3)}{Z_{Lline3}(f, l_3) + Z_{Rline2}(f, l_3)}$	The harmonic impedance seen in a given point on Line 3, located at a distance l_3 from the end of the line;
A67	$\begin{aligned} Z_{kline3}(f, 0) &= Z_{k32}(f) = Z_{k31}(f) = Z_{kline2}(f, L_2) \\ &= Z_{k3}(f) \end{aligned}$	The harmonic impedance seen in the section at the beginning of line 3 is equal to the harmonic impedance seen in the section at the end of line 2 and equal to the harmonic impedance seen in node 3.

Appendix B

Simplified theoretical extrapolation for OHL1 and UGL1 with fictitious lengths three times greater than their actual lengths

The number of “teeth” (3D poles), their height (the Z_k value), and their density (periodicity in the frequency domain) are greatly influenced by the line’s length. Although the line under study (OHL1) has a well-defined length, discussing the influence of length on the Z_k values can complement the analysis. To illustrate this influence, we consider the specific case of an OHL with the same design as OHL1, but with a length three times greater (66 km). We will refer to this line as OHL1.2. Figure A1 shows the 3D plots of Z_k along the line, for the same frequency range as above, corresponding to sub-regimes 1.1 and 2.4, respectively.

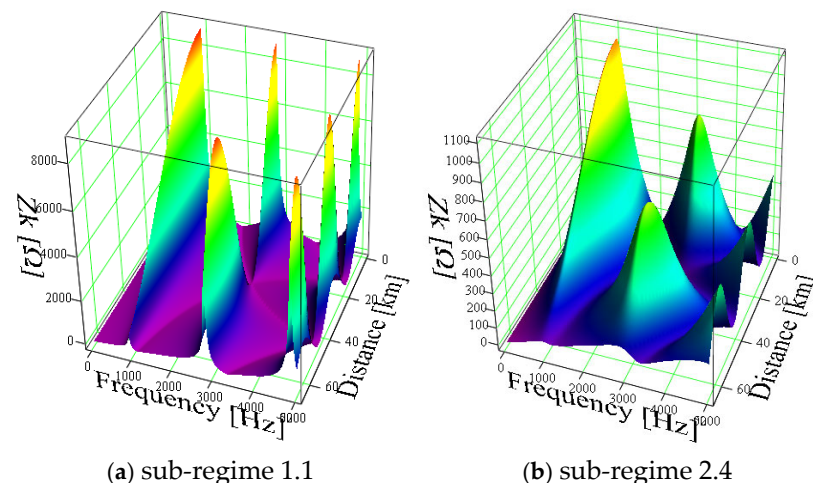


Figure A1. Graphical representation of the values of the function $Z_k(f, dist)$ for OHL1.2.

Similar observations can be made for an underground line as well. The number of “teeth” and rows of “teeth” is influenced by the line’s design characteristics (geometric and material). For the same design type, the frequency characteristics are largely influenced by the line’s length. This statement can be supported by considering an underground line, of the same construction type as UGL1, but three times longer, which we will refer to as UGL1.2. For the same sub-modes 1.1 and 2.4, respectively, the graphs in Figure A2 are shown. As can be seen, this resulted in 36 “teeth,” arranged in 8 rows.

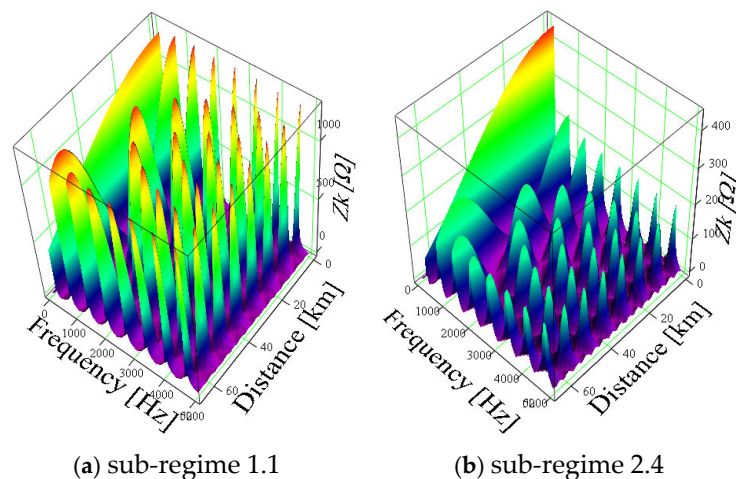


Figure A2. Graphical representation of the values of the function $Z_k(f, dist)$ for UGL1.2.

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