

Article

Energy-Efficient Hydrodynamic Treatment of High-Solid Water-Based Spent Drilling Fluids from Uranium Technological Wells: Experimental Validation, Response-Surface Modeling, and Process Stability

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Abstract

Spent water-based drilling fluids generated during technological-well drilling for uranium production are high-solid, chemically heterogeneous suspensions whose treatment is constrained by water scarcity, waste-handling requirements, reagent demand and electrical-energy consumption. This study evaluated a recirculating flow-through disperser that converts pressure-driven flow into controlled nominal velocity gradients and localized contact zones. The laboratory-pilot dataset comprised five high-solid water-based spent-fluid samples, 120 hydrodynamic treatment runs, 216 particle-size-bin observations corresponding to 12 complete distributions, nine mechanical-mixing baseline tests and ten repeated-cycle runs. The investigated range covered nominal Camp–Stein-equivalent velocity gradients of 500–1500 s^{−1}, treatment times of 60–180 s and flocculant dosages of 0–100 g/t dry solids. A 50 L batch was recirculated through a disperser with a cylindrical geometric envelope of approximately 4.75 L. The maximum separation efficiency was 94.20%. The recommended compromise condition ($G = 1300 \text{ s}^{-1}$, $\tau = 150 \text{ s}$ and 50 g/t dry solids) achieved $92.15 \pm 1.81\%$ separation efficiency, residual aqueous-phase TSS of $125.2 \pm 30.0 \text{ mg/L}$, water recovery of $83.84 \pm 0.28\%$, sludge-volume reduction of $59.02 \pm 2.11\%$ and wall-plug specific energy consumption of $0.1227 \pm 0.0062 \text{ kWh/m}^3$. A separate ten-cycle series yielded $0.1245 \pm 0.0065 \text{ kWh/m}^3$, consistent with the triplicate compromise-mode result. Two-factor response-surface models reproduced the measured G – τ trends and provided prediction intervals for operating-window screening within the investigated fluid class. The findings support coupled energy–separation optimization, rather than selection of the maximum nominal mixing intensity.



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Keywords: spent drilling fluids; hydrodynamic treatment; phase separation; velocity gradient; flocculation; specific energy consumption; pressure drop; water recovery; sludge reduction; response-surface model; process stability

1. Introduction and Literature Background

Spent water-based drilling fluids from technological wells are difficult to treat because they combine a high bulk solid inventory with a fine aqueous-phase fraction, residual polymers and salts, and a rheology intentionally designed to resist sedimentation. Existing research addresses drilling-waste management, flocculation, hydrodynamic cavitation or energy analysis largely as separate tasks [1–3]. The unresolved problem for the investigated fluid class is therefore not the generic use of hydraulic mixing, but the experimentally supported selection of an operating window that simultaneously constrains clarification, recovered-water yield, sludge compaction, reagent dose and wall-plug energy under repeated recirculation.

The main difficulty is that the treatment target is multi-objective. A regime that maximizes clarification may be unattractive if it consumes excessive energy, produces a large volume of loose sludge or requires overdosing of polymeric reagent. Conversely, a low-energy regime can be operationally attractive only if the recovered water satisfies the residual suspended-solid threshold and if the sludge phase is sufficiently compact for transport or further handling [4–6].

The literature relevant to this problem can be organized into three connected groups. Drilling-waste studies define the composition, handling and water-recovery constraints [7–9]. Coagulation and flocculation studies establish that polymer distribution, collision frequency and floc strength depend non-monotonically on hydrodynamic exposure [10–12]. Hydrodynamic-cavitation and flow-through-reactor studies demonstrate that constrictions, rotating or moving elements and pressure loss can intensify contact, but their performance remains strongly geometry- and matrix-dependent [13–15]. Accordingly, the present apparatus is not claimed to introduce the general concept of flow-through hydrodynamic treatment; its contribution is evaluated at the level of the specific device, fluid class, measurement boundary and coupled operating criterion.

Related well-engineering studies further show that particle transport and fluid behavior depend on viscosity, pressure, surface interaction, formation properties and thermal-mechanical boundary conditions. Recent analyses of particle sedimentation in CO₂ fracturing fluids, injection filtration of fracturing fluids and hydrate-wellhead stability illustrate this broader sensitivity of well-fluid systems to coupled fluid, particle and geological variables [16–18]. These studies do not investigate spent-fluid clarification, but they reinforce the need to state the material scope and boundary conditions explicitly, rather than transferring an operating regime between dissimilar well-fluid systems.

The engineering difficulty of spent drilling fluid treatment is related to the physical structure of the suspension. Drilling fluids are designed to be stable during circulation; consequently, their spent forms often resist rapid sedimentation after use. Their rheology, density and particle-size distribution depend on mineral composition, formation interaction, additives and solid-loading history [19–21]. For this reason, conventional wastewater metrics alone are insufficient. A process that is effective for dilute wastewater can be inefficient or unstable when applied to dense drilling suspensions, particularly if the dispersed phase contains fine clay and colloidal fractions.

Chemical conditioning by coagulation and flocculation is a standard route for destabilizing fine particles and increasing settling velocity. Polymer flocculants promote bridge for-

mation and aggregate growth, but their action is strongly dependent on mixing conditions, contact time and particle collision frequency [22–24]. The structure and strength of flocs are hydrodynamically controlled: insufficient mixing leads to poor reagent distribution, while excessive shear can break aggregates and reduce the benefit of flocculation [25–27]. This non-monotonic behavior is central for the present study because the treatment process must be optimized within a favorable hydrodynamic window, rather than by simply increasing mixing intensity.

Hydrodynamic intensification methods have received increasing attention because pressure-driven devices can convert hydraulic energy into localized velocity gradients, cavitation, turbulence, shear, and micro-mixing effects [28–30]. In contrast to conventional impeller systems, flow-through hydrodynamic devices can be integrated into continuous treatment lines and controlled through flow rate and pressure drop. Studies on hydrodynamic cavitation and related reactors show that the geometry of constrictions, orifices, Venturi zones and rotating elements strongly affects pressure loss, local shear and treatment efficiency [31–33]. Although the present disperser is not interpreted solely as a cavitation reactor, these works are relevant because they provide the broader process-intensification framework for hydraulic-energy conversion into useful treatment effects [34–36].

Energy analysis is essential for evaluating drilling-fluid treatment technologies. High clarification efficiency alone is insufficient if the result requires excessive electrical input, long residence time or high reagent consumption. The relevant engineering indicators are therefore specific wall-plug energy per treated volume, energy per recovered-water volume, residual suspended solids, water recovery, sludge reduction and repeated-cycle stability. Response-surface modeling is used here as an interpolation tool within the tested domain, with calibration, internal validation and prediction uncertainty reported separately [37–39]. The process is also considered in the broader context of water reuse and sustainable treatment [40–42].

The resulting research gap is specific and experimentally testable. Previous studies do not provide a directly transferable operating map for the investigated high-solid water-based spent drilling fluids treated in a recirculating perforated-disk/moving-ball disperser while simultaneously reporting aqueous-phase TSS, water recovery, sludge reduction, pressure drop, wall-plug energy and repeated-cycle variability. The study, therefore, does not claim novelty for hydrodynamic flocculation or energy-separation optimization as general concepts. It tests whether this particular configuration can define a reproducible compromise region and quantifies the limits of that inference relative to an internal mechanical-mixing baseline and an internally validated response model [43–45].

The objective of this study is to determine an energy-rational operating window for high-solid water-based spent drilling fluids from uranium technological wells. The contribution is limited to four verifiable elements: a laboratory-pilot dataset linking nominal Camp–Stein-equivalent G , treatment time and a separate flocculant-dose series to clarification, water and sludge responses; an explicit wall-plug energy boundary, together with pressure–flow accounting; statistically identifiable two-factor G – τ response models with internal validation and prediction intervals; and repeated-cycle evidence and a bounded comparison with a non-hydrodynamically matched mechanical-mixing baseline. These elements constitute a device- and fluid-specific engineering characterization, rather than a claim of a new universal separation mechanism.

2. Materials and Methods

2.1. Spent Drilling Fluid Samples

The object of the study was high-solid water-based spent drilling fluid generated during technological-well drilling for uranium production in the Republic of Kaza-

khstan [46,47]. The experimental dataset included five laboratory-pilot samples. Each treatment batch was homogenized in the 50 L feed tank before testing to limit stratification and standardize the initial suspension state [48]. Two different solids' descriptors were measured and are kept explicitly separate. Bulk dry-solid content is the gravimetric mass fraction of oven-dried residue in the homogenized whole suspension, whereas TSS is the concentration of non-settled solids in the aqueous phase sampled from the clarified zone using the same sampling and filtration protocol for untreated and treated conditions. The treatment batches themselves were not diluted. When an analytical aliquot required dilution for filtration, the reported concentration was corrected by the recorded dilution factor.

Table 1 summarizes the bulk suspension properties and the initial aqueous-phase TSS basis used for calculation of separation efficiency.

Table 1. Physicochemical properties of spent drilling fluid samples.

Sample ID	Source Site	Bulk Dry-Solid Content, wt.%	Density, g/cm ³	Viscosity, mPa·s	pH	Initial Aqueous-Phase TSS, mg/L	d10, µm	d50, µm	d90, µm
SDF-01	Well cluster KZ-UR-12	31.6	1.123	25.8	7.62	1540	5.8	24.5	88.0
SDF-02	Well cluster KZ-UR-14	34.1	1.139	28.6	7.78	1625	6.4	29.8	96.5
SDF-03	Well cluster KZ-UR-17	36.8	1.157	31.9	7.91	1710	7.1	34.6	112.0
SDF-04	Well cluster KZ-UR-21	38.2	1.168	34.4	8.08	1785	8.2	40.5	128.0
SDF-05	Composite sample KZ-UR	33.4	1.134	27.9	7.71	1598	6.0	27.2	92.4

The samples were concentrated water-based suspensions with bulk dry-solid contents of 31.6–38.2 wt.%, density of 1.123–1.168 g/cm³, and apparent viscosity of 25.8–34.4 mPa·s. The initial aqueous-phase TSS values of 1540–1785 mg/L, therefore, do not represent the total solid mass in the whole suspension, and should not be converted directly into the bulk wt.% values. Separation efficiency was calculated from C0 and Cres measured on this same aqueous-phase basis, while flocculant dosage was normalized to the total dry-solid mass in the 50 L treatment batch. The d50 range of 24.5–40.5 µm confirms the presence of fine mineral fractions requiring hydrodynamic and reagent-assisted conditioning.

2.2. Hydrodynamic Disperser and Experimental Line

The hydrodynamic disperser was operated as the in-line contact element of a closed-loop laboratory-pilot system. During each test, suspension was withdrawn from the 50 L homogenization tank, passed through the variable-flow pump, flowmeter, and disperser, and returned to the tank. At the end of the prescribed treatment interval, a three-way valve directed the conditioned batch to the clarification vessel. The disperser contains perforated cascade disks and movable spherical elements that create repeated contraction, redirection and particle–reagent contact without a separate high-speed agitator. Table 2 distinguishes the disperser dimensions from the batch volume and defines the hydraulic and energy-accounting boundaries.

Table 2. Geometry and operating parameters of the hydrodynamic disperser.

Parameter	Value or Definition
Apparatus ID	HD-01
Body material	Stainless steel AISI 304
Body inner diameter	120 mm
Body height	420 mm
Cylindrical body-envelope volume	4.75 L, calculated from the reported diameter and height
Treated batch/feed-tank volume, V_0	50 L
Effective dissipation volume used for nominal G , V_{eff}	0.50 L; combined perforated-disk contact-zone estimate, not total body hold-up
Inlet/outlet diameter	18/18 mm
Number of perforated disks	3
Disk spacing	65 mm
Holes per disk	24
Hole diameter	3 mm
Hole inclination angle	25°
Ball material and diameter	Stainless steel; 6 mm
Number of balls	120
Pump type	Variable-frequency centrifugal recirculation pump
Measured flow-rate range	4.64–16.23 L/min for the reported G – τ matrix; equipment capability 5–22 L/min
Pressure- and flow-sensor accuracy	Pressure: ± 1.0 kPa; flow: $\pm 1.5\%$
Flow configuration	Closed-loop recirculation during treatment; diversion to clarification after τ
Treatment time, τ	Elapsed closed-loop recirculation time
Pass-equivalent exposure, N_p	$N_p = Q\tau / V_0$; approximately 0.09–0.97 for the reported matrix
Electrical-energy boundary	Integrated wall-plug input to the complete active treatment line

The previously stated “50 L working volume” referred to the treatment batch, and not to the disperser body. A 120 mm inner diameter and 420 mm body height correspond to a cylindrical envelope of 4.75 L before displacement by internal components. The nominal Camp–Stein-equivalent gradient was calculated from measured hydraulic dissipation as

$$G = \left(\frac{\Delta P \cdot Q}{\mu \cdot V_{eff}} \right)^{1/2},$$

where ΔP is the pressure difference across the disperser, Q is the measured volumetric flow rate, μ is the run-specific apparent viscosity and $V_{eff} = 0.50$ L is the operational estimate of the combined high-dissipation contact-zone volume. Thus, G is a volume-averaged operational estimate for the contact zones, not a directly measured local velocity derivative, and not an average over the 50 L tank.

For the six nominal G levels of 500, 700, 900, 1100, 1300 and 1500 s^{-1} , the representative mean flow rates were approximately 4.64, 6.65, 9.16, 11.66, 13.68 and 16.23 L/min, and the corresponding mean pressure drops were approximately 60.6, 72.8, 86.2, 99.0, 113.6 and 125.2 kPa. Run-specific viscosity values from 25.8 to 34.4 mPa·s were used; consequently, the displayed G values are rounded nominal bands, rather than exact local gradients. The complete run-level ΔP , Q and viscosity records are included in the processed dataset available from the corresponding author.

The 18 mm inlet-pipe Reynolds number calculated from the measured density, viscosity and flow range is approximately 180–870, so the manuscript does not infer a fully turbulent bulk pipe flow. The terms “localized shear” and “contact zones” refer to contraction, jet interaction, moving-element disturbance and repeated redirection inside the perforated cascade. Local turbulence intensity and ball-collision efficiency were not measured, and remain subjects for CFD and residence-time-distribution studies [14,33,34].

The internal mechanism of the hydrodynamic disperser is shown in Figure 1, which identifies the perforated cascade, moving elements, pressure taps, and localized contact zones used in the operational interpretation of G .

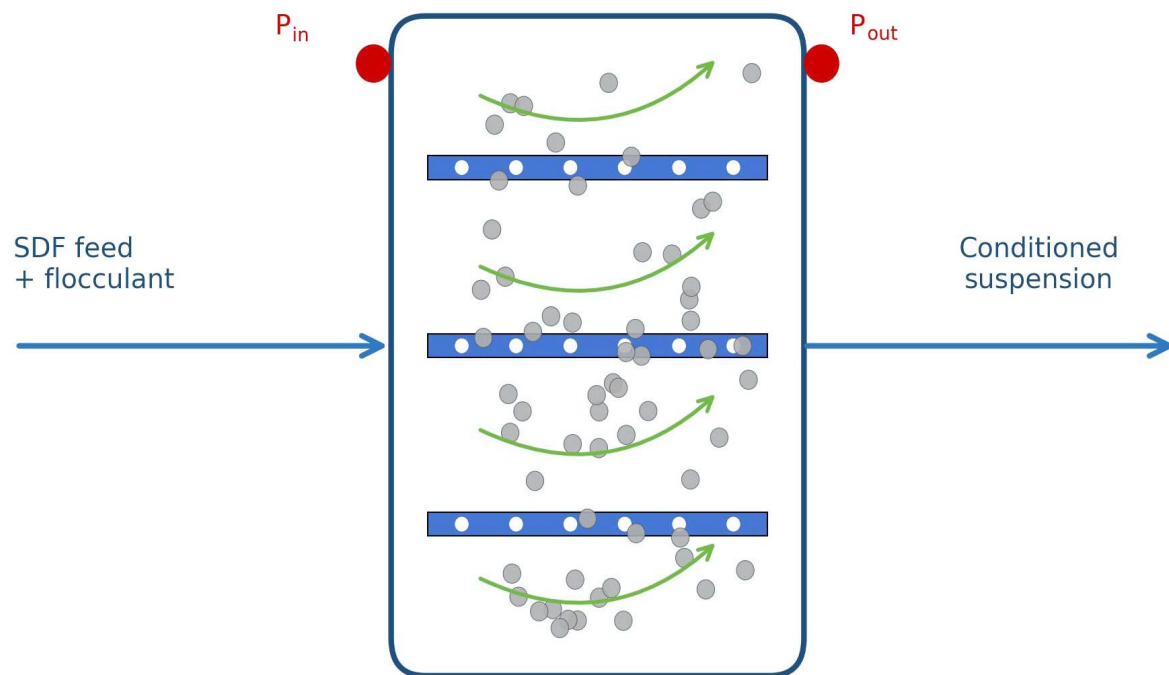
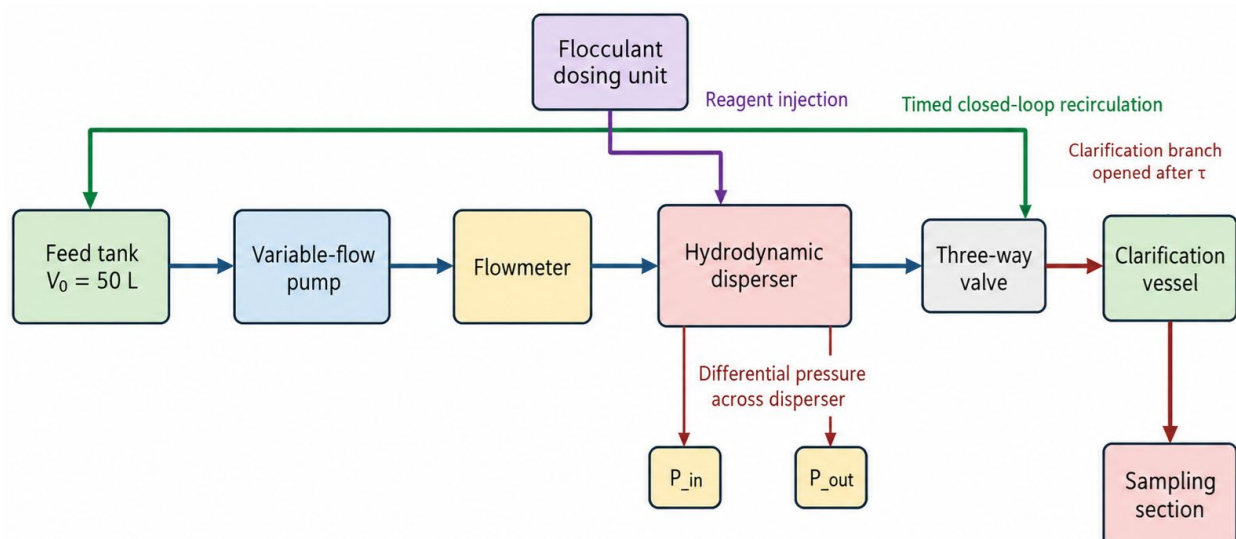


Figure 1. Schematic diagram of the hydrodynamic disperser and principal flow zones.

Figure 1 shows how pump-induced pressure loss is converted into flow acceleration, redirection, and localized shear within the perforated cascade. The figure is a mechanistic schematic; it does not constitute a CFD-resolved map of local gradients or cavitation intensity.

The recirculating laboratory-pilot line is shown in Figure 2. The revised diagram distinguishes timed closed-loop treatment from the subsequent diversion to clarification.



Treatment time τ is the elapsed recirculation interval; pass-equivalent exposure $N_p = Q\tau/V_0$.

Specific energy is calculated from integrated electrical input to the complete laboratory line, not from $\Delta P \cdot Q$ alone.

Figure 2. Recirculating experimental setup for hydrodynamic conditioning and subsequent clarification.

Figure 2 shows that the 50 L batch was recirculated through the pump, flowmeter, and hydrodynamic disperser for the prescribed elapsed time τ . Pressure was measured immediately upstream and downstream of the disperser, flocculant was injected before the contact zone, and the outlet was switched to the clarification vessel only after completion of the timed treatment interval. The pass-equivalent exposure was calculated as $N_p = Q_\tau/V_0$; it is an exposure descriptor, rather than the plug-flow residence time of an individual particle.

2.3. Experimental Design

The experimental program was organized as linked, but statistically distinct, blocks. The main matrix crossed six levels of mean velocity gradient with five treatment times at a fixed flocculant dosage of 50 g/t dry solids and included three physical repetitions per condition. The dosage–response series was conducted only at $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$; consequently, it was analyzed as a separate one-factor experiment, and was not used to estimate $G \times \text{dose}$ or $\tau \times \text{dose}$ interactions. Five additional G – τ regimes, each repeated three times at the reference dose, were reserved for internal validation. This separation of calibration, dose–response, and validation data follows standard response-surface practice and prevents non-identifiable interaction terms from being inferred from a non-factorial dose series [49–51].

Table 3 defines the factor combinations, replication structure and statistical role of each experimental block.

Table 3. Experimental design matrix.

Experimental Block	Structure	Number of Runs	Purpose
G – τ calibration matrix at fixed dose	6 G levels $\times$ 5 τ levels $\times$ 3 physical repetitions; Dfl = 50 g/t dry solids	90	Calibration of two-factor G – τ response surfaces
Separate dose–response series	5 flocculant doses at $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$ $\times$ 3 physical repetitions	15	One-factor reagent-dose assessment; no $G \times \text{dose}$ or $\tau \times \text{dose}$ estimation
Internal validation set	5 additional G – τ regimes at Dfl = 50 g/t dry solids $\times$ 3 physical repetitions	15	Internal validation of the two-factor G – τ models
Baseline mechanical mixing	3 operating modes $\times$ 3 repetitions	9	Comparison with conventional treatment
Repeated-cycle stability tests	10 repeated cycles	10	Process stability assessment
Particle-size analysis	4 conditions $\times$ 3 repetitions $\times$ 18 size classes	216 bin-level observations	Descriptive endpoint granulometric redistribution; 12 complete PSDs

The two-factor models were calibrated on the 30 condition means from the G – τ matrix, with each mean calculated from three physical repetitions; this avoided treating replicated measurements at the same factor setting as additional design points. The 15-run evaluation set was retained as an internal independent-regime check. Because all samples belong to the same drilling-fluid class and experimental facility, this evaluation is not described as external validation, and transfer to independent fluid batches requires additional testing.

2.4. Performance Indicators, Energy Accounting and Modeling

The indicator system was formulated to keep the hydraulic, electrical and separation quantities distinct. Separation efficiency, residual aqueous-phase TSS, water recovery and sludge reduction describe treatment output. Hydraulic power $\text{Phyd} = \Delta P \cdot Q$ describes the recoverable pressure–flow term across the instrumented disperser only. Total treatment energy was obtained independently by integrating electrical input at the supply boundary

of the active laboratory line. Therefore, pump and motor inefficiency, recirculation losses, and the non-instrumented pipeline load are included in E_{el} , rather than being omitted [52].

The response-surface formulation was applied only to the crossed G – τ calibration matrix at the reference flocculant dosage. A hierarchical second-order model was retained because the measured surfaces showed curvature and because its six coefficients can be estimated from 30 distinct factor combinations without invoking unsupported dose interactions [37,39,49]. The separate dosage series was interpreted independently, through its measured response curve.

Table 4 defines the hydraulic-intensity, separation, and wall-plug-energy metrics. In particular, $\Delta P \cdot Q$ is treated as the hydraulic pressure–flow term across the instrumented disperser, not as total electrical input or as the fraction of energy that directly produces flocculation.

Table 4. Performance indicators and calculation formulas.

Indicator	Symbol	Formula	Unit	Purpose
Separation efficiency	η_{sep}	$(C_0 - C_{res})/C_0 \times 100$	%	Aqueous-phase TSS removal on an identical sampling basis
Water recovery	R_w	$V_w/V_0 \times 100$	%	Recoverable process water
Sludge reduction	R_s	$(V_{s,ref} - V_s)/V_{s,ref} \times 100$	%	Waste-volume reduction
Hydraulic power across disperser	P_{hyd}	$\Delta P \times Q$	W	Measured pressure–flow term
Nominal Camp–Stein-equivalent gradient	G	$[\Delta P \cdot Q / (\mu \cdot V_{eff})]^{1/2}$	s^{-1}	Operational contact-zone intensity estimate
Pass-equivalent exposure	N_p	$Q\tau/V_0$	—	Recirculation exposure of the batch
Measured electrical energy	E_{el}	$\int P_{el}(t) dt$	kWh	Wall-plug energy of the active treatment line
Specific electrical energy	E_v	E_{el}/V_0	kWh/m ³	Energy per treated-batch volume
Energy per recovered water	E_w	E_{el}/V_w	kWh/m ³	Energy per useful-water output
Response model	$\hat{y}$	$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{12} x_1 x_2 + \beta_{22} x_2^2$	various	Two-factor operating-window model
Validation error	MAPE	$\frac{100}{n} \sum \left \frac{y_{exp} - y_{pred}}{y_{exp}} \right $	%	Prediction accuracy

The same denominator $V_0 = 0.050 \text{ m}^3$ and the same wall-plug integration principle were used for the hydrodynamic and mechanical-baseline E_v calculations. The comparison, therefore, uses a consistent electrical-energy boundary, although it is not a hydrodynamically matched comparison because the archived mechanical-mixing records do not support conversion to the same G or $G\tau$ basis.

For each response, the two-factor model was formulated as

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{12} x_1 x_2 + \beta_{22} x_2^2, \quad (1)$$

where $x_1 = \frac{G - 1000}{500}$ and $x_2 = \frac{\tau - 120}{60}$.

The residual suspended-solid model was fitted as $\ln(C_{res})$ to reduce heteroscedasticity and keep back-transformed concentrations positive [49,53]. The dosage variable D_{fl} was not included in this regression because it was not crossed with G and τ . Accordingly, no coefficients for x_3 , $x_1 x_3$, $x_2 x_3$, or x_3^2 are reported or interpreted.

Model uncertainty was quantified from the residual variance of the 30-condition calibration fit. For a factor vector x_0 , the 95% confidence interval for the mean response was calculated as $\hat{y}_0 \pm t_{0.975, n-p} s \sqrt{h_0}$, and the 95% prediction interval as $\hat{y}_0 \pm t_{0.975, n-p} s \sqrt{1 + h_0}$, where $h_0 = x_0^T (X^T X)^{-1} x_0$, $n = 30$ and $p = 6$. For C_{res} , limits were calculated in the logarithmic domain and then back-transformed. Experimental results are reported as arithmetic mean $\pm$ sample standard deviation for $n = 3$ independent physical repetitions, except for the repeated-cycle series ($n = 10$); no pooled standard deviation or post hoc variance equalization was applied [39,49,53].

3. Results

3.1. Overall Treatment Performance

The verified dataset shows a transition from incomplete separation at low hydrodynamic intensity to high-efficiency clarification within the optimized region. Table 5 reports representative operating modes across the measured process range, including low-intensity, compromise, dose-optimized and maximum-efficiency conditions.

Table 5. Separation efficiency, residual solids, water recovery and sludge reduction under representative treatment regimes.

Treatment Mode	G, s^{-1}	τ, s	Dose, g/t Dry Solids	$\eta_{sep}, \%$	Residual TSS, mg/L	Water Recovery, %	Sludge Reduction, %	$E_v, kWh/m^3$
Dose-optimized high-performance mode	1300	150	75	93.23 ± 1.07	107.7 ± 14.2	83.95 ± 0.08	60.41 ± 1.57	0.1203 ± 0.0085
Energy-saving acceptable mode	1100	120	50	86.02 ± 0.43	238.9 ± 14.2	80.96 ± 1.51	56.81 ± 2.16	0.0881 ± 0.0101
Compromise operating mode	1300	150	50	92.15 ± 1.81	125.2 ± 30.0	83.84 ± 0.28	59.02 ± 2.11	0.1227 ± 0.0062
Maximum-efficiency mode	1500	180	50	94.20 ± 0.00	98.4 ± 4.8	83.79 ± 0.36	55.29 ± 0.59	0.1681 ± 0.0075
Low-intensity hydrodynamic treatment	500	60	50	58.26 ± 0.43	683.2 ± 46.3	70.54 ± 1.15	48.01 ± 1.62	0.0800 ± 0.0000

The compromise mode at $G = 1300 s^{-1}$, $\tau = 150 s$ and 50 g/t dry solids increased separation efficiency to $92.15 \pm 1.81\%$ and reduced residual TSS to $125.2 \pm 30.0 mg/L$. The dose-optimized mode at 75 g/t dry solids further reduced residual TSS to $107.7 \pm 14.2 mg/L$, but the improvement over 50 g/t was smaller than the initial improvement produced by moving from no reagent to moderate dosage.

3.2. Hydrodynamic Response: Velocity Gradient and Treatment Time

The velocity-gradient response should be interpreted as the macroscopic manifestation of microscale collision and breakage processes [54–56]. At low G , particle–polymer collisions are insufficient and the flocculant cannot be distributed uniformly through the high-solid matrix. At high G , the rate of productive collisions increases, but the process progressively approaches an energy penalty region where additional shear produces diminishing returns [52]. This is why the optimum is better represented as an operating window, rather than a single maximum point.

The granulometric measurements were used as a mechanistic consistency check for the macroscopic separation response. The 216 values reported in Table 3 are 18 size-class observations for four operating conditions measured in triplicate, i.e., 12 particle-size distributions rather than 216 independent treatment experiments. This distinction is important because treating the bin values as independent samples would create pseudo replication, and artificially inflate statistical significance.

Figure 3 compares the distributions before treatment, after low-intensity treatment, at the recommended regime, and after excessive hydrodynamic exposure. Across the measured size classes, the modal region shifts from the fine-particle domain before treatment toward larger aggregates under the recommended condition; excessive treatment shifts part of the volume fraction back toward smaller classes. The plotted transition is therefore consistent with aggregate growth followed by shear-limited breakage, although it does not identify kinetic rate constants.

The recommended treatment produced the clearest displacement of volume fraction toward the coarse flocculated classes, whereas the excessive condition showed a partial return toward intermediate sizes. This non-monotonic granulometric response agrees with the simultaneous improvement and subsequent saturation of separation efficiency.

A kinetic floc-growth model and regressions such as $d_{50}-\eta_{\text{sep}}$ or equilibrium floc size- E_v were not fitted because only four granulometric operating conditions were sampled; such regressions would be underdetermined, and would overstate the information content of the dataset. The particle-size data are therefore used to corroborate, not independently calibrate, the proposed aggregation-breakage interpretation [25,48,54].

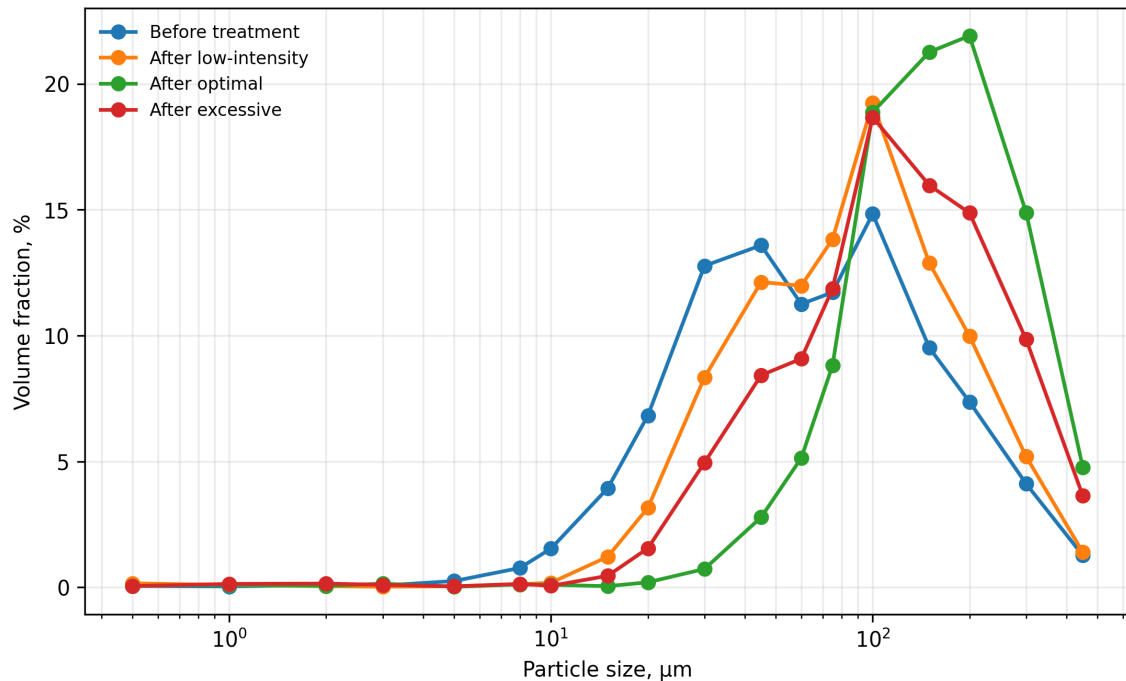


Figure 3. Endpoint particle-size distributions before and after treatment (four conditions, $n = 3$ per condition, 18 size classes; 12 complete distributions).

Figure 4 compares separation efficiency across the investigated velocity-gradient and treatment-time levels.

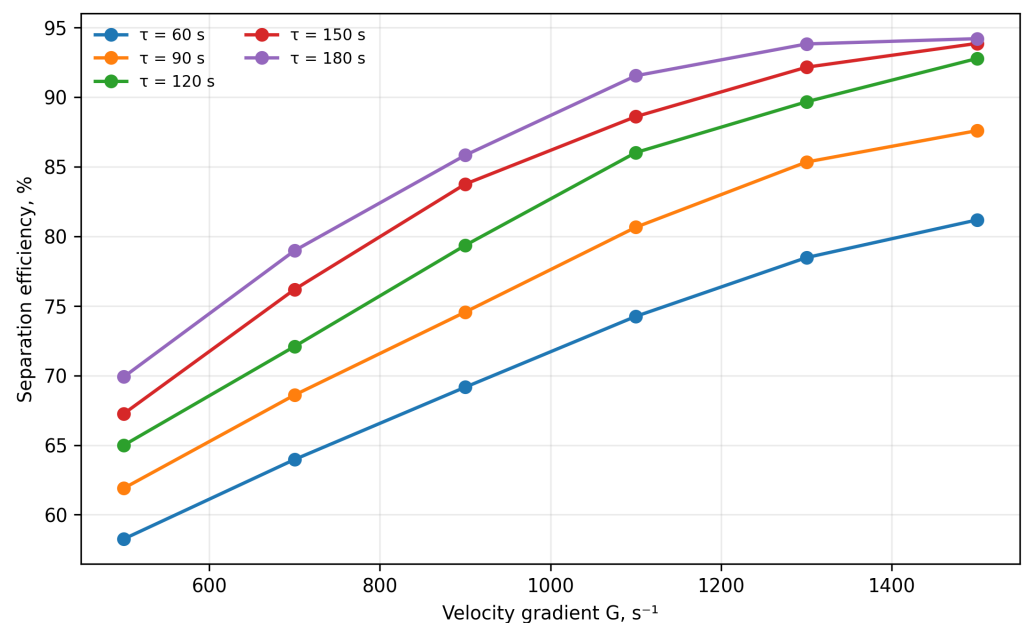


Figure 4. Effect of velocity gradient and treatment time on separation efficiency.

Figure 4 demonstrates that increasing G and τ improves separation efficiency, but the slope decreases at high intensity. At low G , longer treatment times still provide limited

gains because particle collision and reagent distribution remain insufficient. At high G , additional time produces smaller marginal improvement and begins to compete with the energy penalty [39,52].

Figure 5 presents the same response as a three-dimensional surface, and is used to identify the location of the high-efficiency zone.

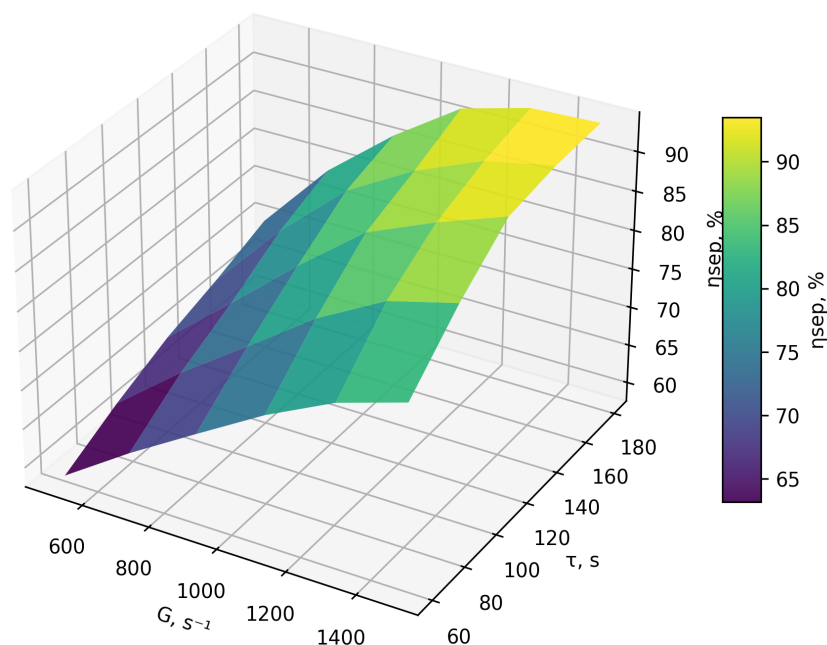


Figure 5. Three-dimensional response surface of separation efficiency as a function of G and τ .

The surface shows that the main high-efficiency domain begins near $G = 1300 s^{-1}$ and $\tau = 150 s$. The maximum measured separation efficiency of 94.20% was obtained at $G = 1500 s^{-1}$ and $\tau = 180 s$, but this regime did not provide the best energy balance.

Residual suspended-solid concentration is shown in Figure 6 because it is the direct water-quality counterpart of separation efficiency.

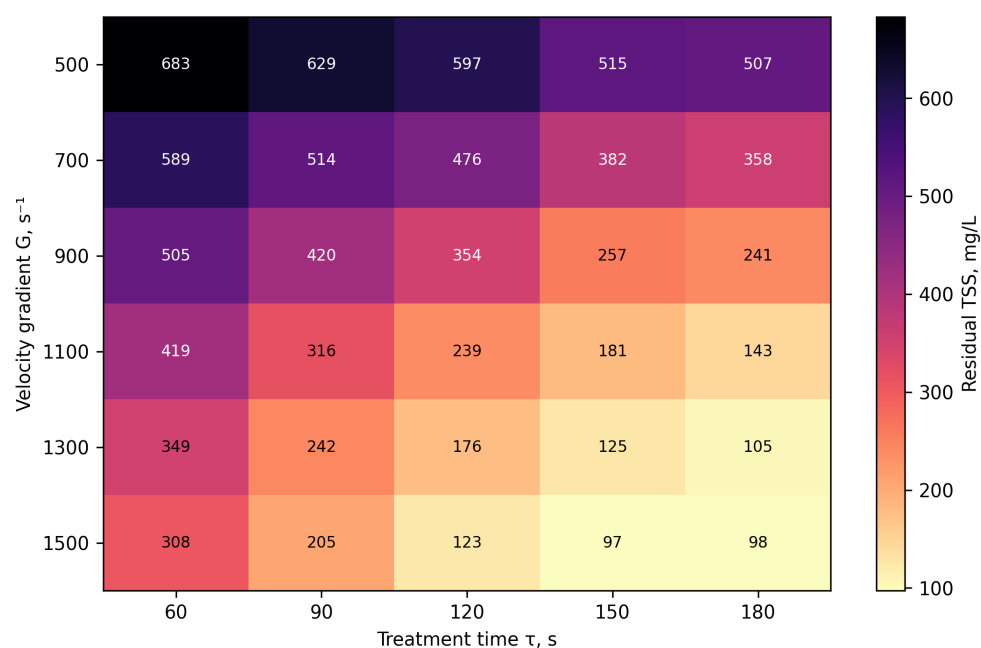


Figure 6. Residual suspended-solid concentration under different hydrodynamic regimes.

Figure 6 confirms the technological significance of the optimized hydrodynamic region. Residual TSS decreased from 683.2 mg/L at $G = 500 \text{ s}^{-1}$ and $\tau = 60 \text{ s}$ to 125.2 mg/L at $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$. At very high intensity, residual TSS decreased further, but the additional reduction required substantially higher specific energy [40,52].

3.3. Reagent Dose and Granulometric Mechanism

The flocculant dosage series was performed at $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$ because this hydrodynamic regime had already reached the high-efficiency region. The purpose was to determine the one-factor dosage response at this fixed hydrodynamic condition. Since dose was not crossed with the remaining G – τ combinations, these observations were not used to estimate multivariable interaction coefficients.

Figure 7 shows the dose–response behavior of the hydrodynamic treatment process.

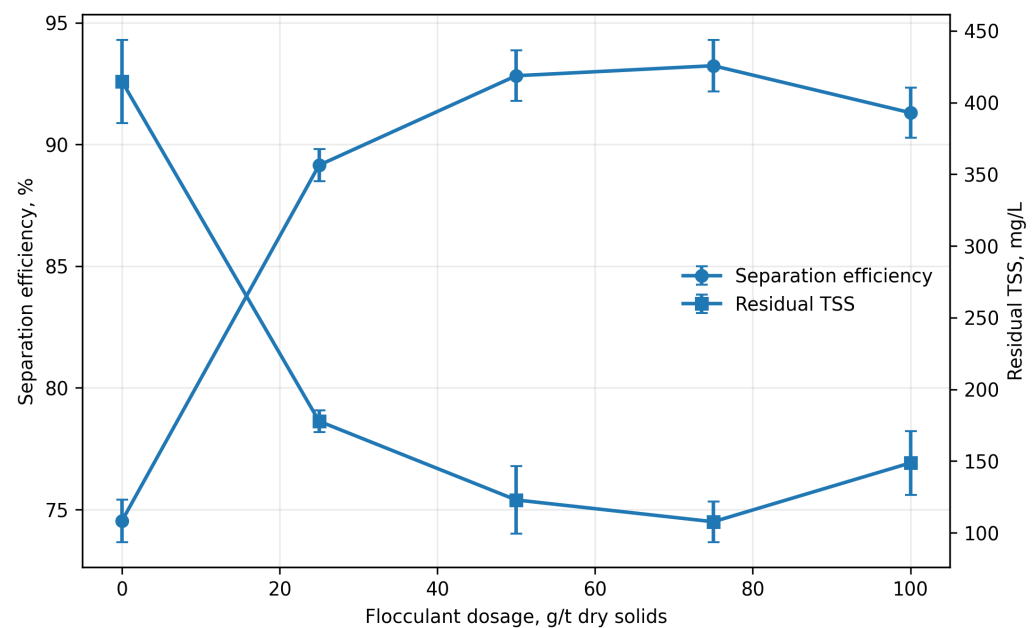


Figure 7. Effect of flocculant dosage on separation efficiency and residual suspended solids.

The dosage series shows a sharp improvement between 0 and 50 g/t dry solids, and a plateau thereafter. The slight deterioration at 100 g/t dry solids indicates that overdosing can reduce separation quality, probably through restabilization, formation of weaker aggregates, or excess polymer remaining in the clarified phase [10,12,26].

3.4. Water Recovery, Sludge Reduction and Pressure Drop

The value of the hydrodynamic disperser is not restricted to suspended-solid removal. For field application, the water and sludge balances are equally important because they determine the volume of reusable process water and the amount of residual waste requiring transportation or disposal [57,58]. Figures 8 and 9 present these secondary, but practically decisive, responses.

Water recovery increased from approximately 70.5% in the low-intensity regime to more than 83% in the optimized region. The response plateau indicates that the clarified-water fraction becomes limited by suspension composition and sludge compaction, rather than by hydrodynamic intensity only.

Figure 9 shows the sludge-volume reduction response, which complements the water recovery analysis.

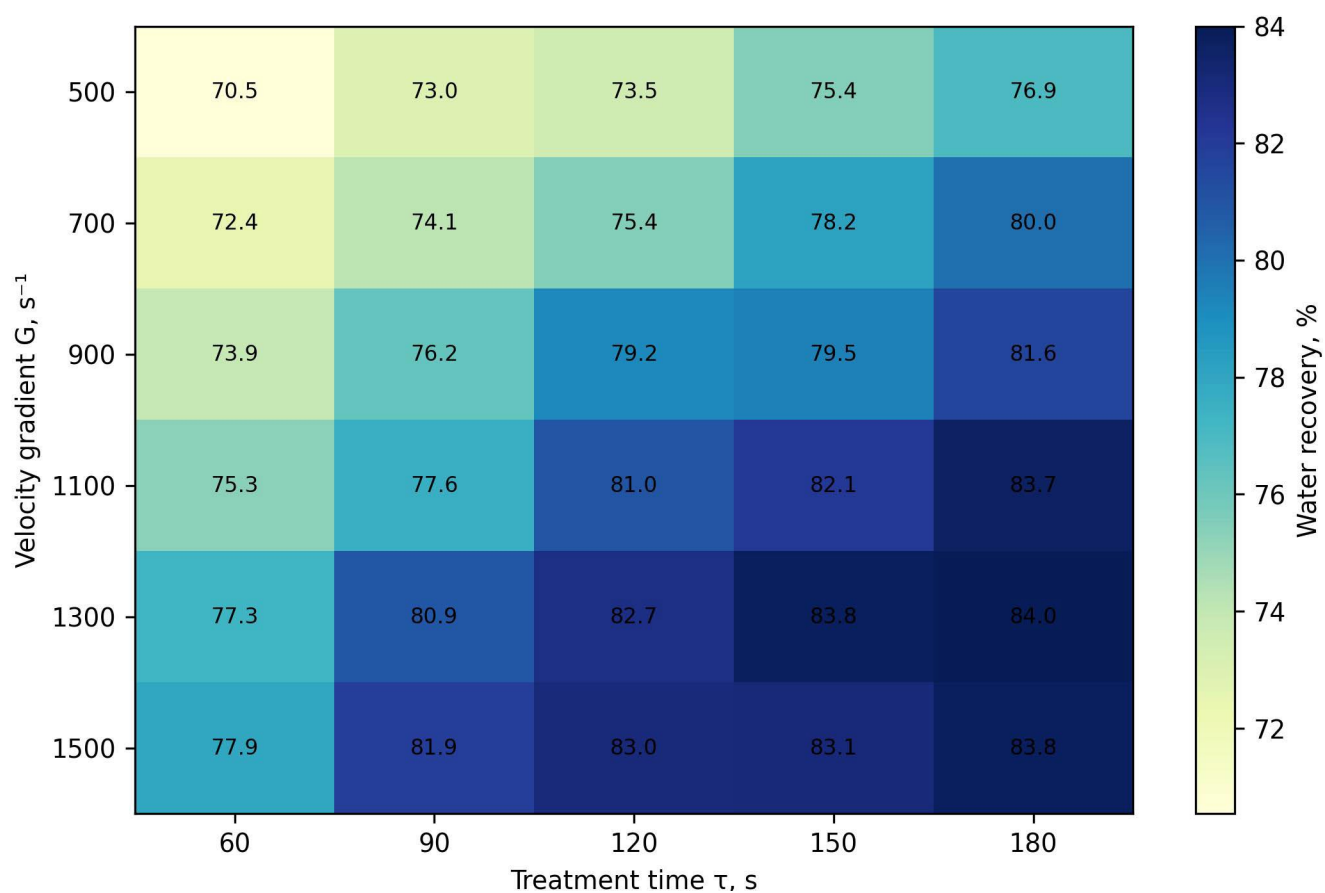


Figure 8. Water-recovery response surface as a function of velocity gradient and treatment time.

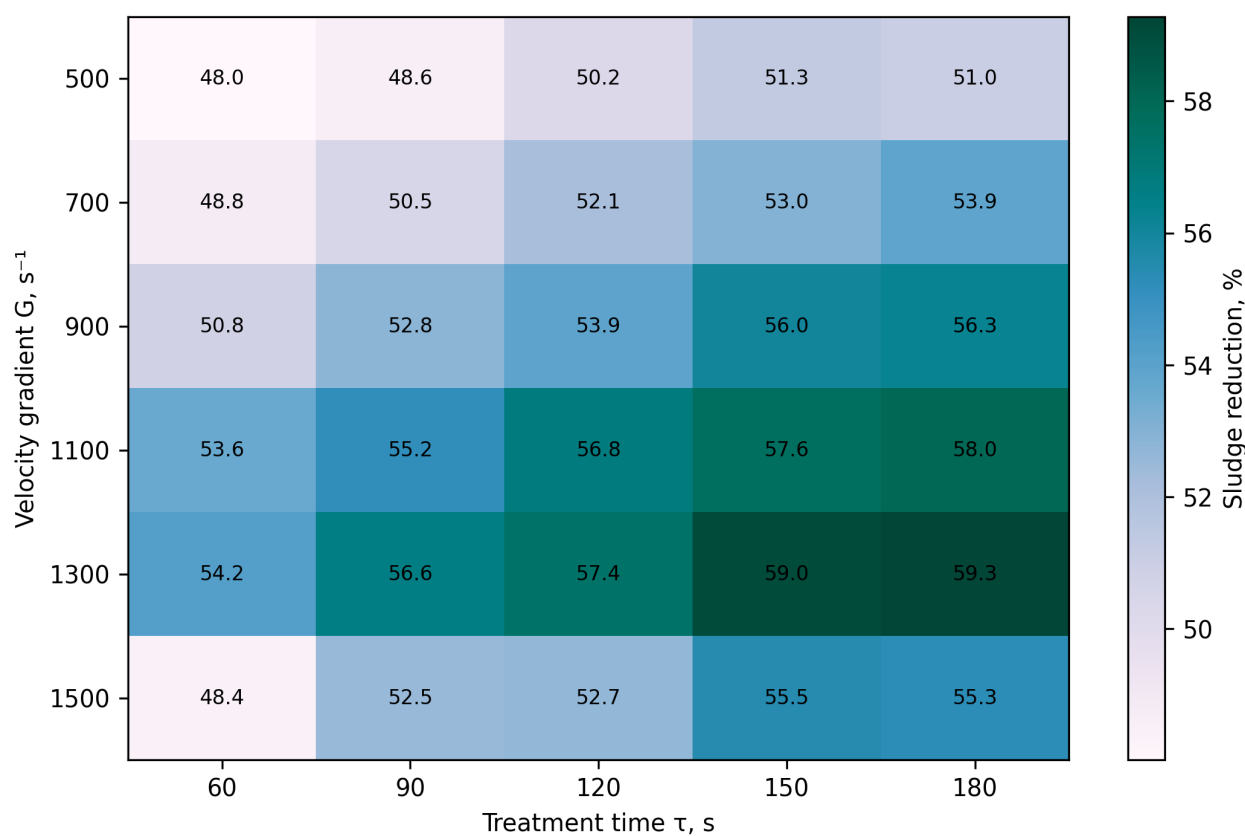


Figure 9. Sludge-volume reduction response surface.

The sludge reduction surface indicates that hydrodynamic conditioning improves both clarification and solid-phase compaction. The highest sludge reduction was obtained near the same region that provided high separation efficiency, supporting the interpretation that aggregate formation and sediment compaction are coupled process outcomes.

The hydraulic cost of increasing treatment intensity is represented by the pressure drop, in Figure 10.

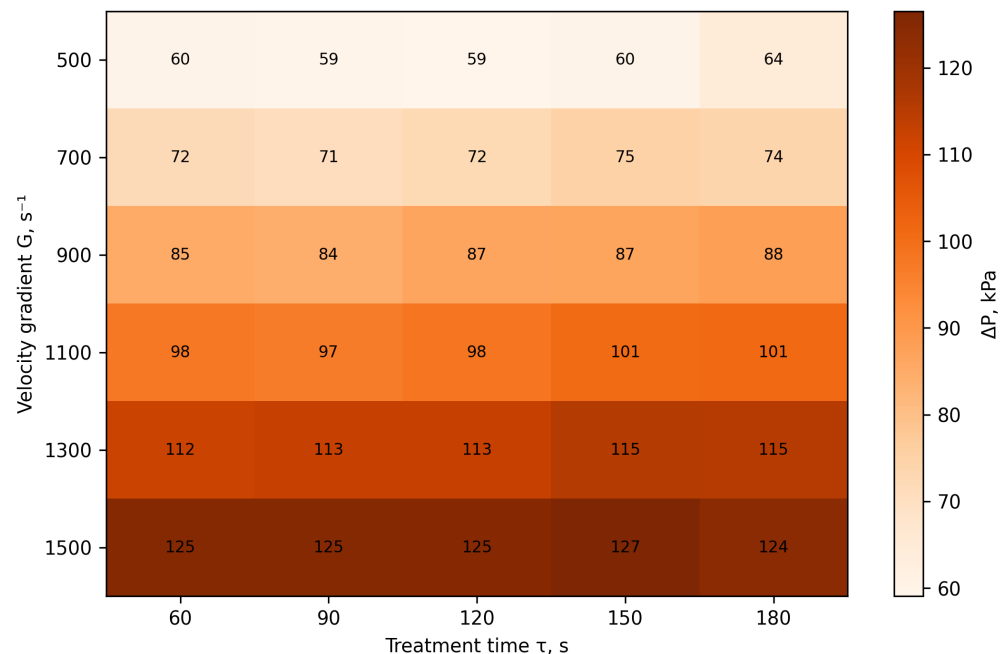


Figure 10. Pressure drop across the hydrodynamic disperser under different G – τ regimes.

The pressure drop increased with the imposed hydrodynamic intensity, indicating that the energy benefit of the disperser cannot be evaluated from separation efficiency alone. Pressure drop is a control variable for field operation because it determines pump selection, equipment loading, and the risk of excessive hydraulic resistance.

3.5. Energy Consumption and Trade-Off Analysis

The energy analysis evaluates how much separation is obtained per unit of measured electrical input. The final TSS alone is insufficient for comparing regimes because higher pressure drop, longer operation, and additional reagent contact can increase both clarification and energy demand [40,41,59].

The specific energy curves must therefore be read together with the separation-efficiency curves. A regime with the highest η_{sep} is not automatically the recommended regime. The recommended regime is the one that satisfies the residual TSS and water recovery constraints, while keeping E_v and reagent dosage within a rational operating range.

Figures 11–14 show the resulting energy-separation trade-off: higher nominal G and longer treatment time generally improve clarification, but increase wall-plug energy per treated volume and per recovered-water volume [40,52,59].

Figure 11 shows the effect of velocity gradient on specific energy consumption at different treatment times.

The curves demonstrate that the energy penalty becomes substantial at high G and long τ . The specific energy increased from 0.0800 kWh/m³ in low-intensity regimes to 0.1681 kWh/m³ in the maximum-efficiency regime. This nonlinear response justifies using a compromise operating point, rather than a maximum-intensity setting.

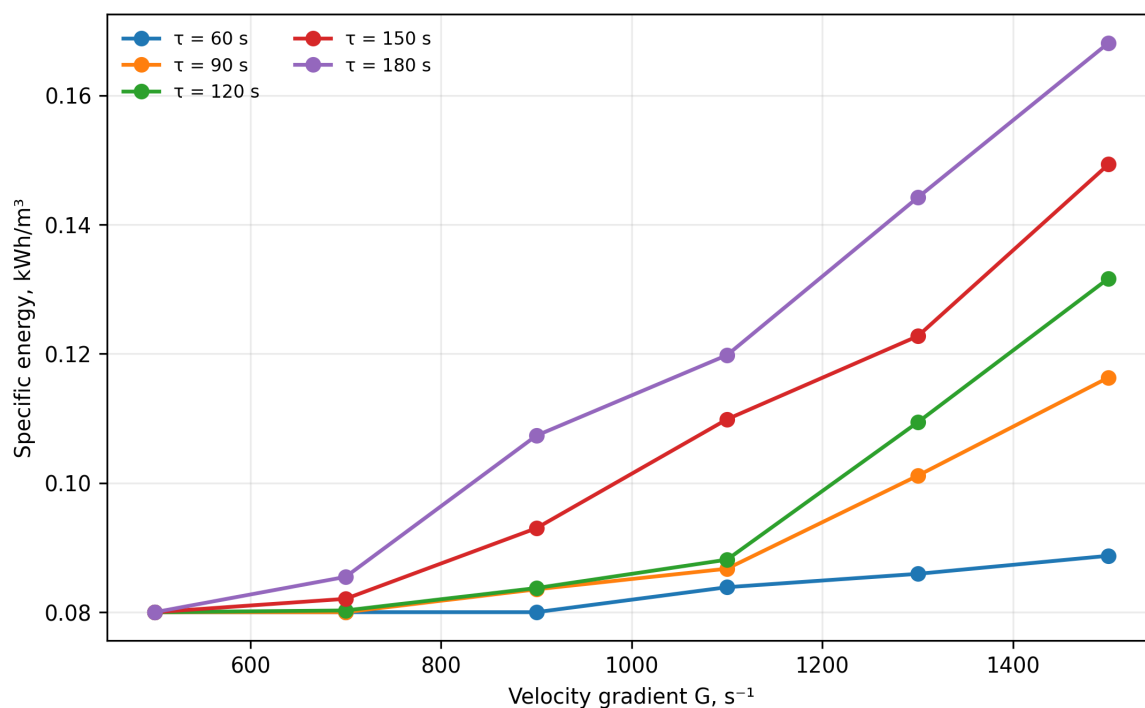


Figure 11. Specific energy consumption as a function of velocity gradient.

Figure 12 combines separation efficiency and specific energy consumption in a single plane.

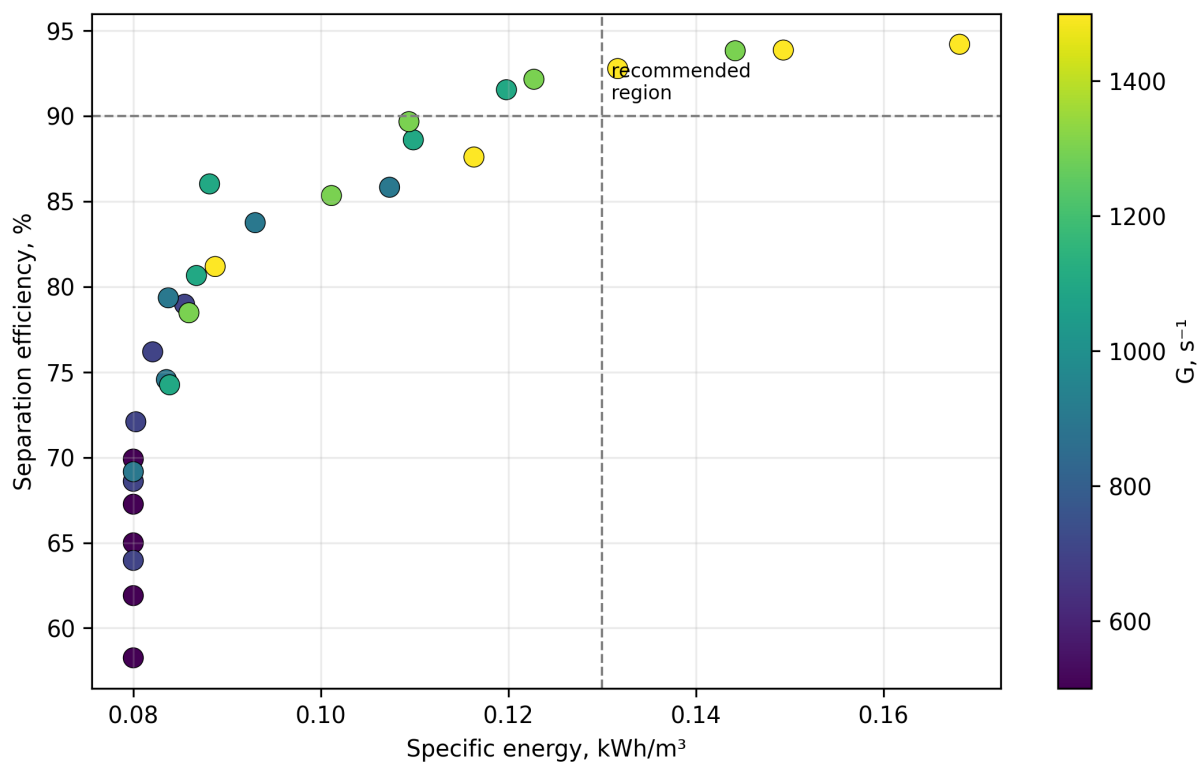


Figure 12. Energy-separation trade-off curve.

The recommended operating window appears near the high-efficiency region but before the steepest energy increase. This representation is useful for engineering decision-making because it directly shows which regimes are technically effective and which are energy-excessive.

Table 6 separates the directly measured wall-plug energy from the hydraulic pressure–flow term across the disperser. No residual term is labelled as useful shear or as a measured dissipation mechanism.

Table 6. Hydraulic pressure–flow term and wall-plug energy under representative operating modes.

Mode	G, s^{-1}	τ, s	$\Delta P, \text{kPa}$	$Q, \text{L/min}$	P_{hyd}, W	$E_{\text{hyd}}, \text{kWh}$	$E_{\text{el}}, \text{kWh}$	$E_{\text{hyd}}/E_{\text{el}}, \%$	$E_v, \text{kWh/m}^3$	$E_w, \text{kWh/m}^3$
High-efficiency moderate-energy mode	1100	180	101.17	11.66	19.66	0.00098	0.00599	16.4	0.1198	0.1432
Compromise operating mode	1300	150	115.47	13.68	26.33	0.00110	0.00614	17.9	0.1227	0.1464
Maximum-efficiency mode	1500	180	124.00	16.23	33.55	0.00168	0.00841	19.9	0.1681	0.2007
Low-intensity hydrodynamic treatment	500	60	59.87	4.64	4.63	0.00008	0.00400	1.9	0.0800	0.1134

Table 6 shows that $E_{\text{hyd}} = \Delta P \cdot Q \cdot \tau$ is only 1.9–19.9% of the measured wall-plug energy for the representative modes. The remainder is not assigned to a single physical mechanism: it includes pump and motor losses, recirculation and line losses, tank-scale motion, and other active-system loads. Consequently, the manuscript bases E_v on E_{el} , and does not treat P_{hyd} as total energy or as useful flocculation work. The maximum-efficiency mode still requires substantially more wall-plug energy per recovered-water volume than the compromise mode.

Figure 13 visualizes the previously reported power components; its interpretation is restricted to the measured hydraulic term and the unresolved balance to total input.

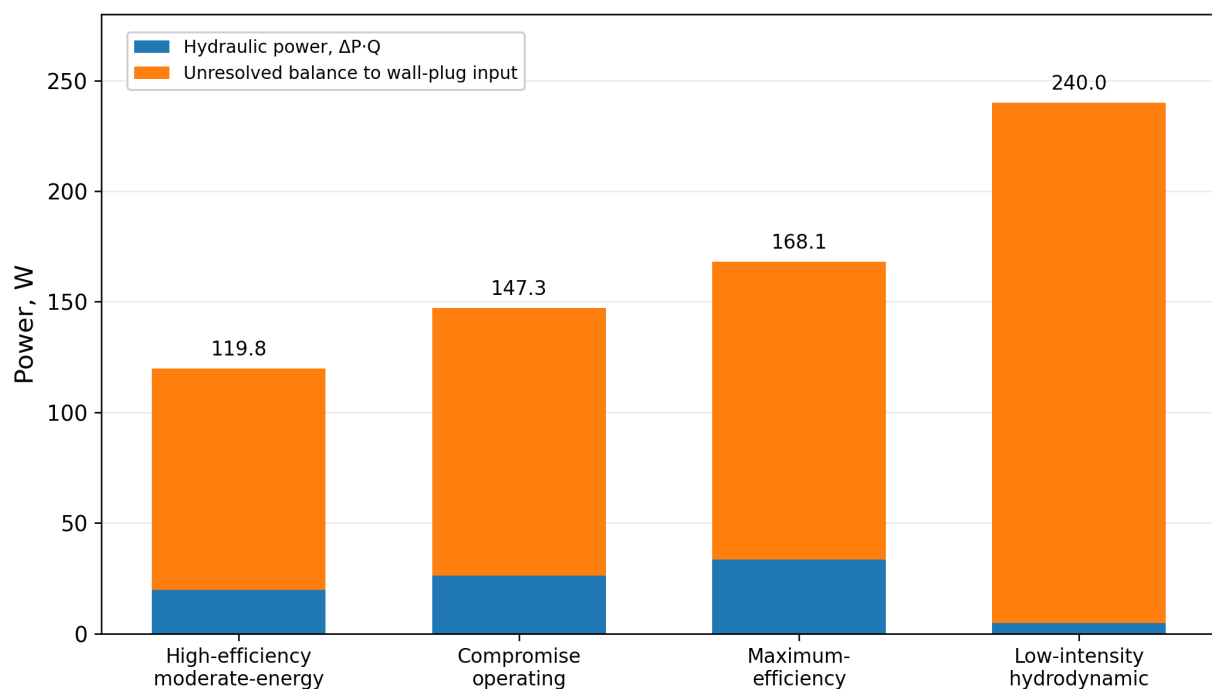


Figure 13. Measured hydraulic power and unresolved balance to wall-plug input under representative operating modes.

Figure 13 shows that the measured hydraulic term increases with pressure drop and flow rate. The complementary portion should be read as an unresolved balance to wall-plug input, not as a direct measurement of useful turbulent dissipation. Table 6 provides the corrected quantitative energy boundary used for all subsequent comparisons.

Figure 14 presents residual TSS against specific energy consumption, and provides a Pareto-style view of the treatment matrix.

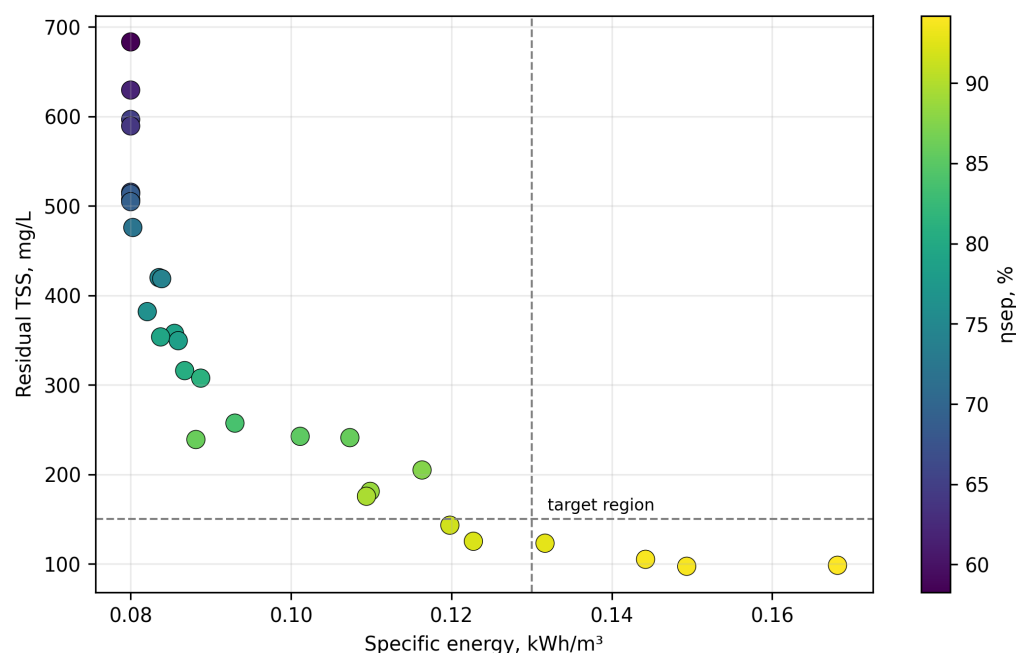


Figure 14. Pareto interpretation of residual suspended solids versus specific energy consumption.

The target region combines low residual solids and moderate energy consumption. The compromise regime lies close to this region, while the maximum-efficiency regime improves residual solids only at the cost of higher specific energy.

3.6. Response-Surface Modeling and Validation

The G - τ calibration responses at $D_{fl} = 50$ g/t dry solids were approximated with hierarchical second-order models in the coded variables x_1 and x_2 . The dosage series was excluded from these regressions and retained as the separate analysis in Section 3.3. Table 7 reports the six refitted coefficients for each statistically identifiable two-factor model; unsupported dose and dose-interaction terms are not included.

Table 7. Two-factor G - τ response-surface coefficients at $D_{fl} = 50$ g/t dry solids.

Parameter	η_{sep} -Model Coefficient	$\ln(C_{res})$ -Model Coefficient	E_v -Model Coefficient
Constant	82.7511	5.6152	0.09097
x_1	12.8686	−0.7091	0.02544
x_2	7.3000	−0.4289	0.01670
x_1^2	−4.3259	0.0289	0.01250
x_1x_2	0.1829	−0.2466	0.01947
x_2^2	−2.5048	0.0805	0.00367

The coefficient signs are physically interpretable within the investigated domain. Positive linear coefficients in the η_{sep} model and negative linear coefficients in the $\ln(C_{res})$ model indicate improved separation with increasing G and τ , whereas the quadratic terms represent diminishing returns. The positive G - τ interaction in the E_v model quantifies the accelerating energy penalty when high velocity gradient is combined with long treatment time. Dosage-related curvature is discussed only from the separate measured series in Figure 7, and is not attributed to the response-surface coefficients.

Table 8 summarizes the refitted calibration statistics and the 15-run internal validation results. For residual TSS, R^2 refers to the logarithmic fit, whereas RMSE, MAPE and maximum relative error are reported after back-transformation to mg/L.

Table 8. Calibration and internal-validation metrics for the two-factor G– τ models.

Indicator	Dataset Role	R^2	RMSE	MAPE, %	Maximum Relative Error, %
Separation efficiency	Calibration	0.991	0.98	1.07	4.06
Separation efficiency	Internal validation	0.600	1.41	1.07	2.86
Residual TSS	Calibration	0.985	20.64	5.19	21.72
Residual TSS	Internal validation	0.721	19.17	13.76	43.16
Specific energy consumption	Calibration	0.983	0.0031	2.56	9.72
Specific energy consumption	Internal validation	0.880	0.0100	6.07	12.33

The calibration statistics confirm that the reduced two-factor specification reproduces the measured G– τ surfaces without unsupported dosage interactions. The validation R^2 of 0.600 for separation efficiency is not interpreted as strong generalization: the 15 validation observations occupy a compressed high-efficiency range, so R^2 is sensitive to small deviations, relative to the limited between-point variance. The corresponding RMSE of 1.41 percentage points and MAPE of 1.07% indicate small absolute errors, but the validation remains an internal interpolation check, rather than evidence of transferability to independent drilling-fluid batches. Residual TSS has wider relative uncertainty, including a maximum relative error of 43.16%; therefore, model predictions near a water-quality threshold require direct experimental confirmation.

At the recommended condition ($G = 1300 \text{ s}^{-1}$, $\tau = 150 \text{ s}$ and $D_{fl} = 50 \text{ g/t}$ dry solids), the two-factor models gave point predictions of approximately 92.0% separation efficiency, 139 mg/L residual TSS, and 0.126 kWh/m^3 specific energy. The corresponding 95% prediction intervals were 89.6–94.4%, 116–165 mg/L, and 0.118 – 0.133 kWh/m^3 , respectively. These intervals are used to express local model uncertainty; they do not replace the measured triplicate results reported in Table 5.

The condition-specific standard deviations were also retained without pooling. For example, the residual-TSS relative standard deviation was 24.0% at the 50 g/t compromise condition and 13.2% at 75 g/t. This difference is consistent with greater sampling sensitivity near the transition between incompletely captured fines and more fully developed flocs. By contrast, the specific-energy relative standard deviation was 2.3% for short mechanical mixing and 4.5% for the maximum-intensity hydrodynamic mode; the latter includes run-to-run pressure and flow fluctuations. These differences reflect response-specific experimental variability, and were not produced by different statistical formulas.

Figure 15 provides the visual counterpart to the internal validation metrics in Table 8.

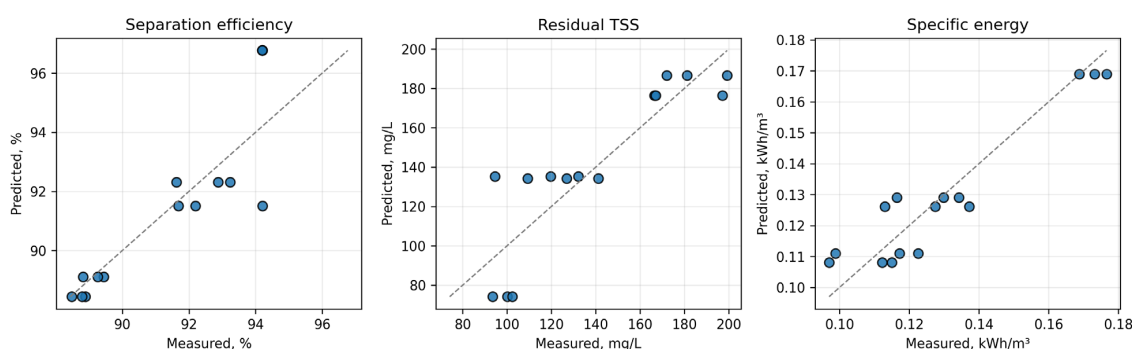


Figure 15. Predicted versus measured values for the internal independent-regime validation set.

The points remain close to the 1:1 line for separation efficiency and specific energy, whereas residual TSS shows greater scatter. Together with Table 8 and the prediction intervals, this pattern supports the use of the model for interpolation, regime ranking, and operating-window screening within the investigated domain. It does not support replacing experimental verification for a new drilling-fluid composition or an independent production batch.

The combined optimization constraints are summarized in Figure 16.

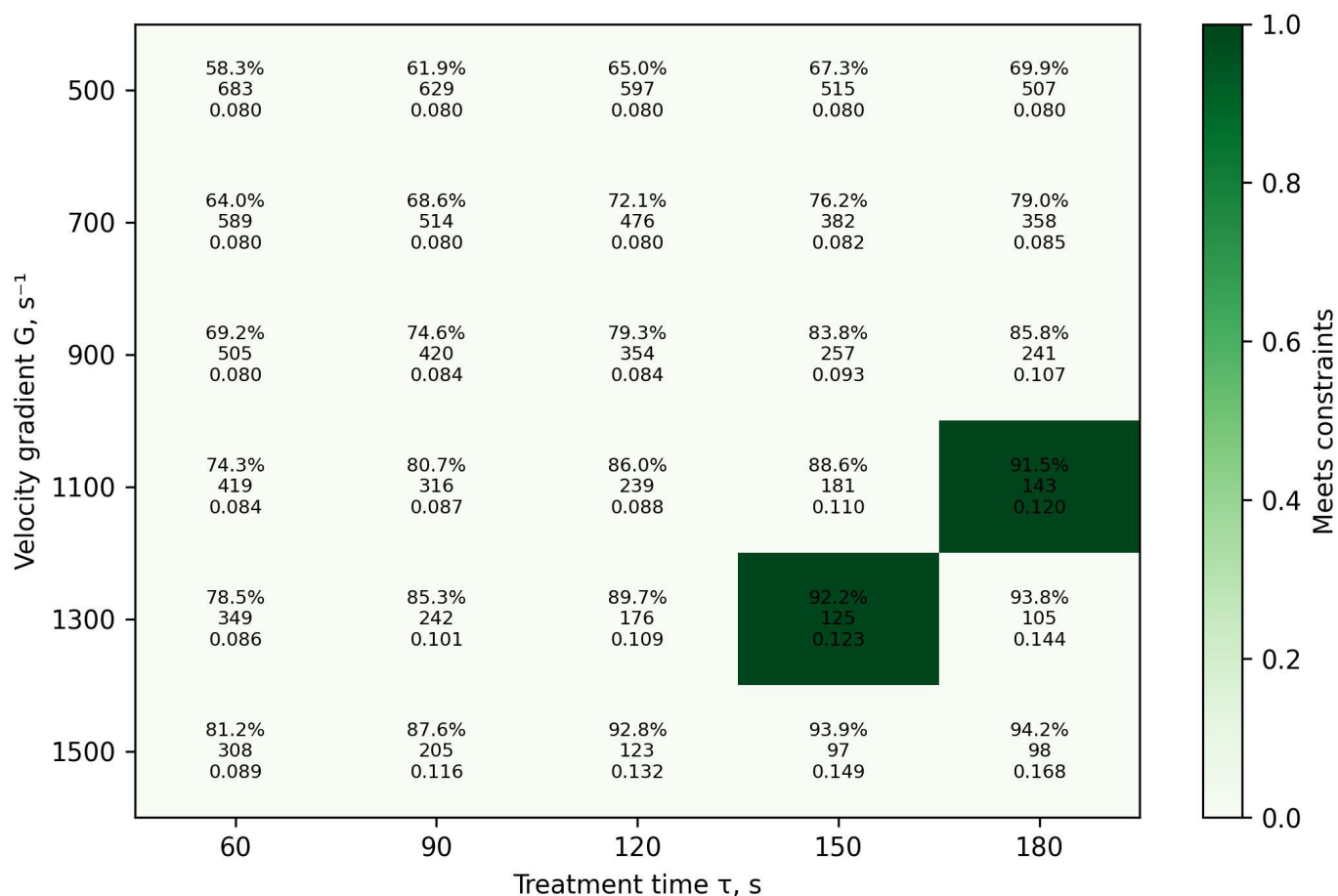


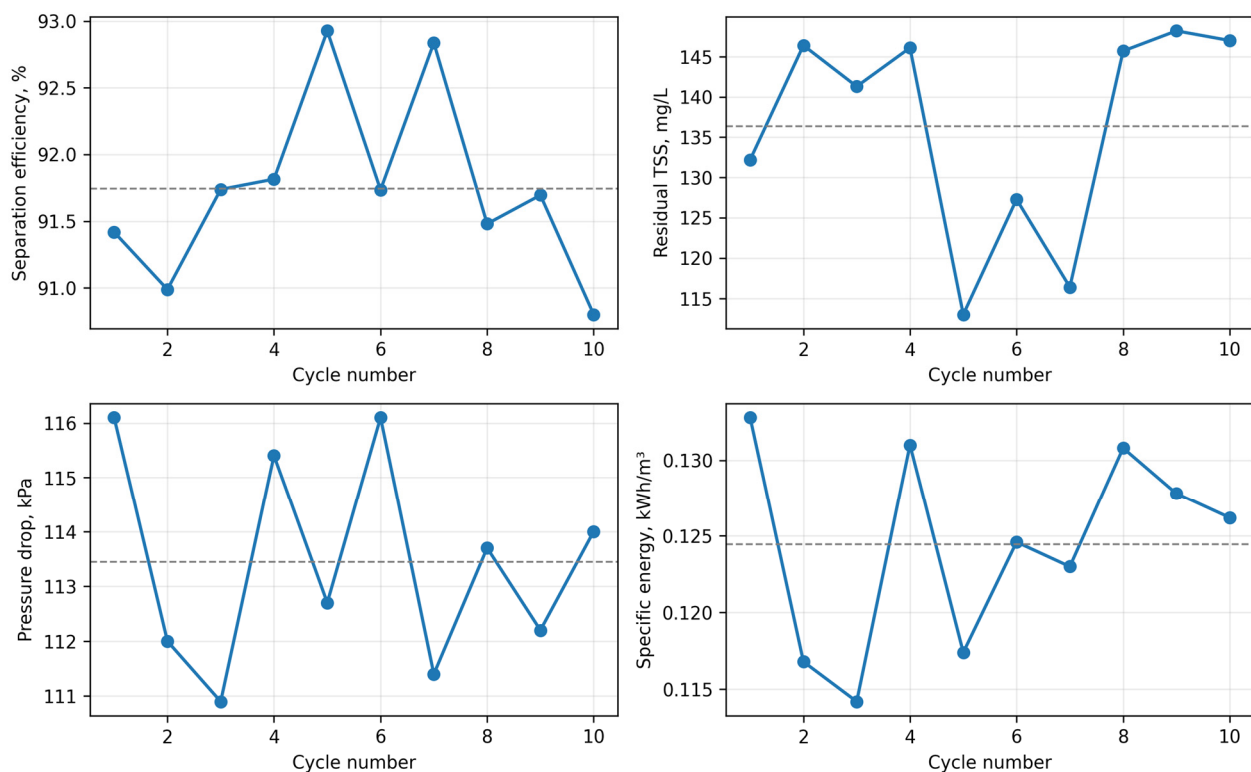
Figure 16. Optimization map showing the recommended operating window.

The map identifies regimes satisfying simultaneous point-prediction constraints on separation efficiency, residual TSS, and specific energy. The recommended region is located around $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$, where water quality and energy demand are balanced. Because the map is based on point predictions, regimes close to a constraint boundary should be checked against the 95% prediction intervals and confirmed experimentally.

3.7. Repeated-Cycle Stability and Comparison with Mechanical Mixing

Process stability was evaluated under repeated operation at $G = 1300 \text{ s}^{-1}$, $\tau = 150 \text{ s}$, and 50 g/t dry solids. This test is important because a field treatment unit must reproduce the target response over multiple cycles, rather than only under a single optimized laboratory run [60–62].

Figure 17 shows repeated-cycle stability for separation efficiency, residual suspended solids, pressure drop, and specific energy consumption.

Repeated-cycle stability at $G = 1300 \text{ s}^{-1}$, $\tau = 150 \text{ s}$, $D_{fl} = 50 \text{ g/t dry solids}$ **Figure 17.** Repeated-cycle stability at the recommended operating mode.

The average repeated-cycle separation efficiency was $91.74 \pm 0.69\%$, residual aqueous-phase TSS was $136.4 \pm 13.4 \text{ mg/L}$, pressure drop was $113.45 \pm 1.92 \text{ kPa}$, and wall-plug specific energy was $0.1245 \pm 0.0065 \text{ kWh/m}^3$. The value $0.1227 \pm 0.0062 \text{ kWh/m}^3$ in the Abstract and Table 5 is the mean of the three compromise-mode runs used in the G – τ matrix, whereas $0.1245 \pm 0.0065 \text{ kWh/m}^3$ is the mean of a separate ten-cycle stability series. Their difference is 1.5% and is smaller than the uncertainty of either series; therefore, the values are consistent, rather than contradictory.

Table 9 compares the hydrodynamic results with the archived mechanical-mixing baseline using the same 50 L batch basis and the same wall-plug specific-energy calculation.

Table 9. Internal baseline comparison with laboratory mechanical-mixing and alternative hydrodynamic modes.

Treatment Mode	Time, s	η_{sep} , %	Residual TSS, mg/L	Water Recovery, %	Sludge Volume, L	E_v , kWh/m ³
Mechanical mixing, short mixing	120	68.15 ± 0.42	519.7 ± 19.1	73.39 ± 0.75	15.40 ± 0.21	0.1204 ± 0.0028
Mechanical mixing, medium mixing	300	74.02 ± 0.87	426.7 ± 24.6	74.65 ± 0.43	15.22 ± 0.27	0.3677 ± 0.0076
Mechanical mixing, long mixing	600	78.57 ± 0.29	365.9 ± 18.5	75.48 ± 1.02	15.26 ± 0.27	0.8377 ± 0.0028
Low-intensity hydrodynamic treatment	60	58.26 ± 0.43	683.2 ± 46.3	70.54 ± 1.15	15.93	0.0800 ± 0.0000
Compromise operating mode	150	92.15 ± 1.81	125.2 ± 30.0	83.84 ± 0.28	12.00	0.1227 ± 0.0062
Maximum-efficiency mode	180	94.20 ± 0.00	98.4 ± 4.8	83.79 ± 0.36	14.21	0.1681 ± 0.0075

For all mechanical-baseline runs, E_v was calculated as integrated-mixer electrical input divided by the same 0.050 m^3 batch volume used for hydrodynamic treatment. The

archived baseline records preserve elapsed time, wall-plug energy, and treatment responses, but do not contain sufficiently complete impeller geometry, rotational-speed and local-mixing data to convert those runs to an equivalent G or $G\tau$. The comparison, therefore, demonstrates performance relative to the available laboratory baseline, not superiority over an optimized industrial impeller system [2,58,59].

Figure 18 visualizes the comparison with mechanical mixing using separation efficiency and residual TSS.

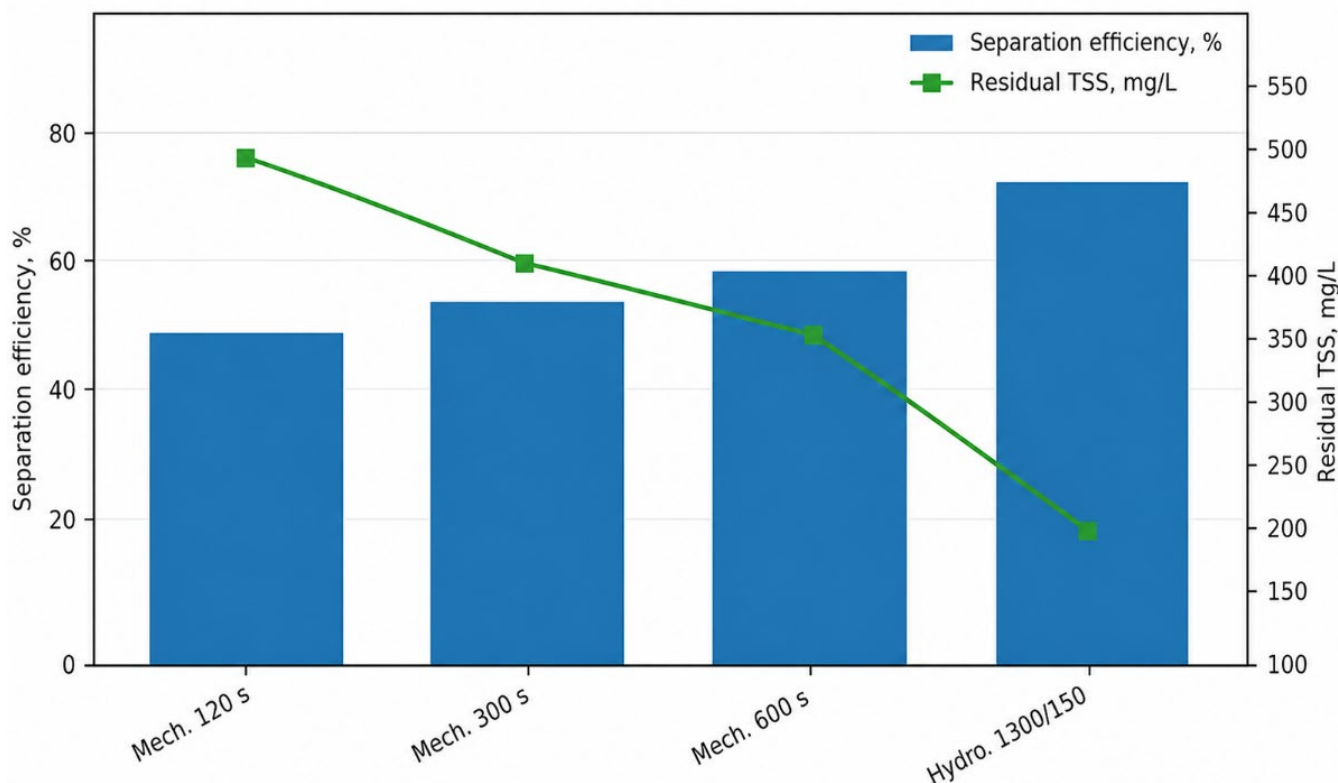


Figure 18. Hydrodynamic treatment versus baseline mechanical mixing.

Figure 18 shows a larger reduction in residual solids for the hydrodynamic compromise mode than for the recorded mechanical-baseline runs. Because the mechanical apparatus was neither optimized nor hydrodynamically matched, the result is interpreted as an internal screening benchmark, rather than as a fully matched technology comparison.

3.8. Recommended Operating Modes

The experimental and modeling results define several objective-dependent operating modes. Table 10 summarizes minimum-energy, compromise, reagent-optimized, maximum-separation and repeated-cycle conditions for the investigated high-solid water-based fluid class; these values are not a universal prescription, and require recalibration for materially different compositions.

The table distinguishes between minimum-energy operation, maximum-quality operation, and compromise operation. For field deployment, the compromise and repeated-cycle modes are the most relevant because they combine high separation efficiency with manageable pressure drop and stable specific-energy consumption.

Table 10. Recommended operating modes for laboratory and pilot-scale conditions.

Objective	G, s^{-1}	τ, s	Dose, g/t Dry Solids	$\eta_{sep}, \%$	Residual TSS, mg/L	Water Recovery, %	Sludge Reduction, %	$E_v, kWh/m^3$	Recommendation
Minimum-energy acceptable treatment	1100	120	50	86.02	238.9	80.96	56.81	0.0881	For moderate pretreatment
Compromise operating mode	1300	150	50	92.15	125.2	83.84	59.02	0.1227	Recommended main mode
Reagent-optimized mode	1300	150	75	93.23	107.7	83.95	60.41	0.1203	Best combined result
Maximum-separation mode	1500	180	50	94.20	98.4	83.79	55.29	0.1681	Use when water-quality target dominates
Stable repeated-cycle mode	1300	150	50	91.74 ± 0.69	136.4 ± 13.4	79.79 ± 0.70	—	0.1245 ± 0.0065	Candidate for pilot-scale continuity tests

3.9. Additional Hydraulic and Multivariate Diagnostic Analysis

Flow rate, pressure drop, separation response, and specific energy are mutually coupled. Figures 19 and 20, therefore, provide complementary hydraulic and multivariate diagnostics for checking the internal physical consistency of the experimental dataset.

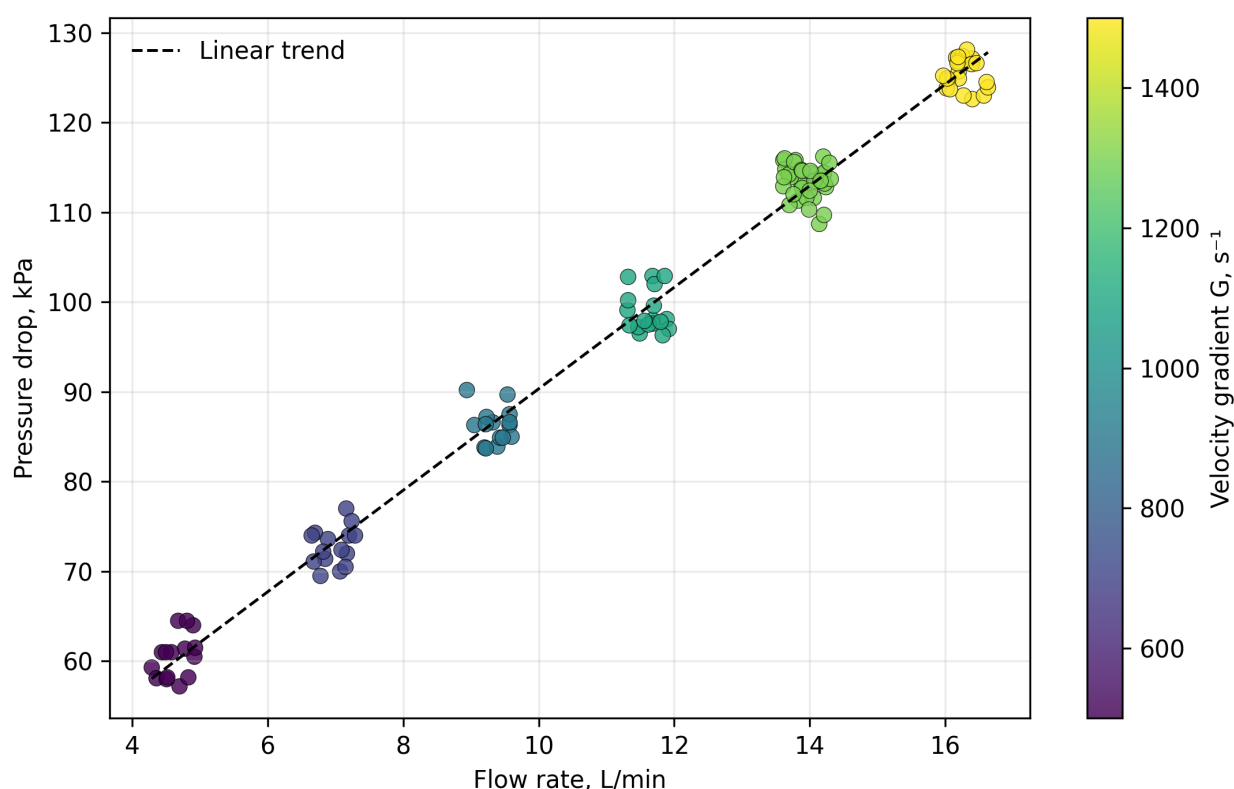
**Figure 19.** Pressure drop versus suspension flow rate for the complete hydrodynamic experimental dataset.

Figure 19 links measured flow rate and pressure drop. This relationship is important because pressure drop is both a control variable and an energy-related constraint. A stable positive hydraulic trend indicates that the experimental regimes were physically consistent: higher flow rates and velocity gradients produced higher hydraulic loading of the disperser, rather than isolated, non-systematic pressure fluctuations.

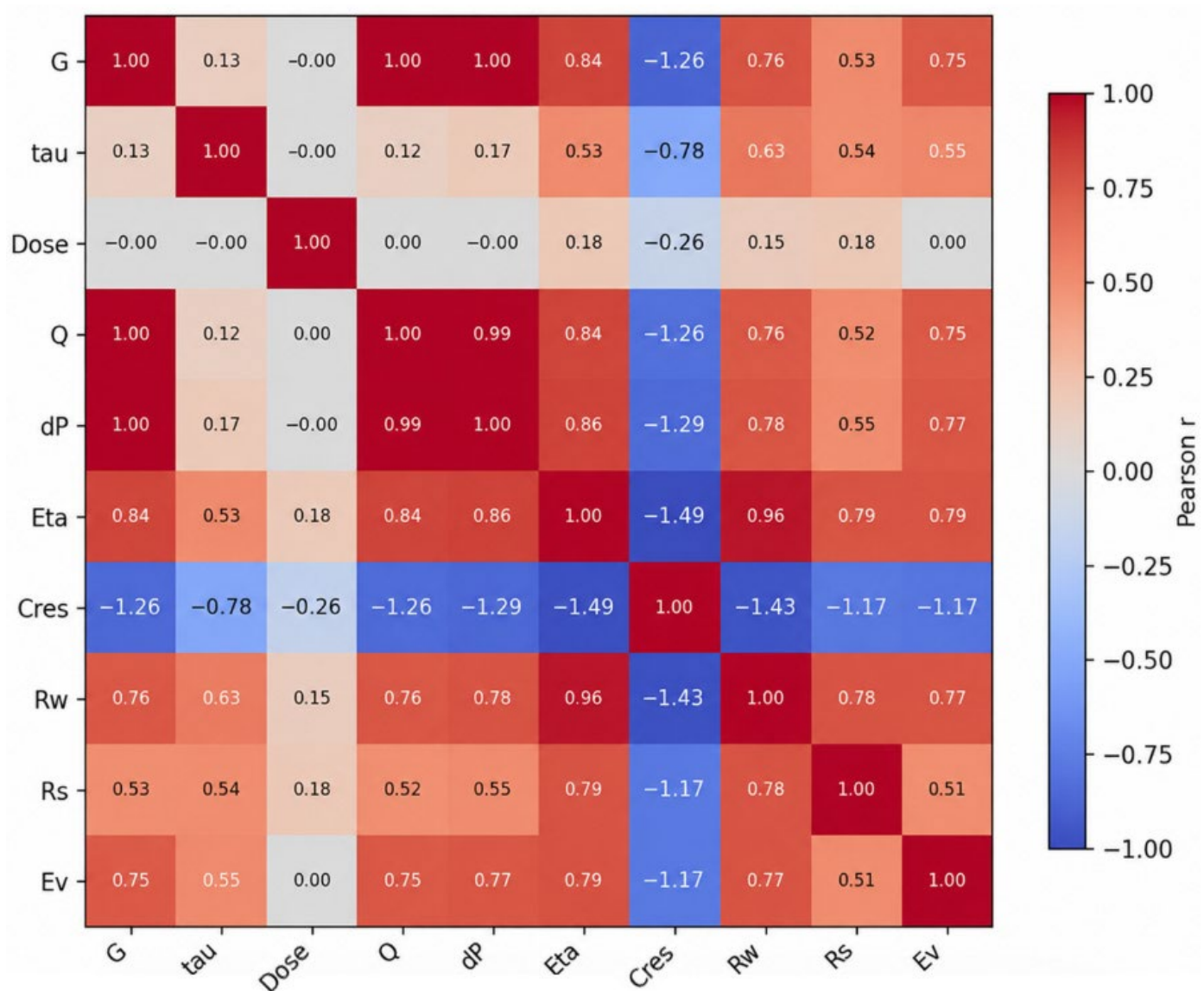


Figure 20. Correlation matrix for operating parameters and treatment-response indicators.

The pressure–flow diagram shows a coherent hydraulic response of the disperser. The scatter is expected because flow rate, velocity gradient, treatment time, and suspension properties were varied simultaneously; nevertheless, the cloud of points follows a physically plausible trend. This confirms that the pressure–drop values used for energy accounting are not arbitrary measurements, but belong to a consistent operating domain.

Figure 20 summarizes the correlation structure among the main operating and response variables. The matrix is useful for identifying which variables are directly coupled and which effects must be interpreted through intermediate mechanisms. For example, separation efficiency is positively associated with hydrodynamic intensity and water recovery, whereas residual suspended solids are negatively associated with the same treatment-intensification variables. Specific energy consumption is correlated with hydraulic loading and treatment severity, confirming the need for a trade-off criterion.

The correlation matrix should not be interpreted as a substitute for the response-surface model, because correlation coefficients do not capture nonlinear saturation and interaction effects. Its role is diagnostic: it verifies that the dataset has the expected physical structure, and supports the subsequent discussion of hydrodynamic-reagent coupling, energy penalty, and model limits.

4. Discussion

The experimental results support a bounded aggregation–breakage–energy interpretation. The measured evidence comprises nominal G and $G\tau$, pressure–flow behavior, particle-size redistribution, clarification responses, and wall-plug energy. The apparatus was not instrumented to resolve the three-dimensional local shear field, moving-ball collision statistics, polymer adsorption kinetics, or floc-strength parameters. The discussion therefore separates direct observations from plausible mechanisms, and avoids presenting the flow-through concept itself as novel.

The practical consequence is that scale-up should not be based only on increasing throughput. If flow rate is increased without preserving residence time, pressure-drop characteristics and the effective velocity-gradient window, the industrial unit may move away from the experimentally verified region. Thus, future pilot design should combine hydraulic similarity, residence-time distribution analysis, and online monitoring of pressure drop and clarified-water quality [14,33,34].

4.1. Why the Optimum Is a Window, Rather than a Point

The experimental optimum is not a single mathematical maximum because field treatment is constrained by several simultaneous requirements. A high separation efficiency is useful only if the clarified-water volume remains high, the sludge phase is compact, and the specific energy consumption remains acceptable. In this sense, the recommended window represents a robust process zone, rather than a narrowly tuned laboratory point.

This interpretation is important for industrial operation. In a field unit, feed composition, suspended-solid concentration, and temperature will vary. If the selected regime is too close to the edge of the feasible domain, small disturbances may move the process into an under-mixed or over-sheared state. A window-based recommendation provides more operational tolerance, and is more realistic for mobile or semi-continuous treatment systems.

The nominal $G\tau$ exposure ranged from 3.0×10^4 to 2.7×10^5 , with 1.95×10^5 at the recommended mode. Using the measured kinematic-viscosity range and the Camp–Stein-equivalent relation gives an order-of-magnitude microscale estimate $[\nu/G]^{1/2}$ of approximately 121–248 μm over the investigated domain, and approximately 130–154 μm near $G = 1300 \text{ s}^{-1}$. This scale overlaps with the coarser aggregate range observed after treatment in Figure 3, which is consistent with shear-limited aggregate growth but does not uniquely identify turbulent breakage kinetics [11,12,26].

This behavior explains why the recommended regime is not the maximum-intensity point. The maximum-efficiency regime reached 94.20% separation efficiency, but its specific energy was 0.1681 kWh/m^3 . The compromise mode at $G = 1300 \text{ s}^{-1}$ and $\tau = 150 \text{ s}$ delivered $92.15 \pm 1.81\%$ efficiency at $0.1227 \pm 0.0062 \text{ kWh/m}^3$. From an engineering standpoint, the small loss in separation efficiency is justified by the energy saving and lower hydraulic loading.

4.2. Hydrodynamic–Reagent Coupling

The dose–response series confirms that the hydrodynamic field and flocculant action are inseparable. Hydrodynamic treatment without flocculant provided only partial separation, whereas the transition to 25–50 g/t dry solids produced a strong decrease in residual solids. The response plateau above 50–75 g/t dry solids indicates that reagent addition becomes less effective once particle surfaces and bridging sites are sufficiently conditioned.

The slight deterioration at 100 g/t dry solids prevents interpretation of reagent addition as a monotonic improvement factor. Restabilization, weak-floc formation and residual polymer are plausible explanations [10,12,26], but ζ -potential and residual-polymer con-

centration were not measured. Overdosing is therefore reported as an observed response trend, rather than as a chemically proven mechanism. Capillary and Damköhler numbers, floc breakage and re-flocculation rate constants, and moving-ball collision efficiency cannot be calculated reliably from the available dataset because floc strength, polymer-reaction kinetics and local velocity fields were not measured.

The particle-size dataset provides an additional physical cross-check. The 12 replicated distributions show that the recommended regime shifts the volume fraction toward coarser aggregates, whereas excessive exposure partly restores intermediate-size fractions. This trend is consistent with the observed TSS minimum and the diminishing marginal gain in separation efficiency. However, the four sampled granulometric conditions are insufficient for estimating breakage and re-flocculation rate constants or for fitting a transferable d_{50} – $G\tau$ – Ev surface. A future kinetic design should measure size distributions at several intermediate times for each G level and pair them with floc-strength, ζ -potential and residual-polymer measurements.

4.3. Energy Interpretation and Mechanical-Mixing Comparison

Treatment time alone is an inadequate basis for comparing the two devices. The revised energy accounting uses integrated wall-plug input for both systems, but the archived mechanical tests are not matched to the hydrodynamic disperser by impeller power number, Camp–Stein G , $G\tau$, or residence-time distribution. They are therefore retained as a laboratory baseline, rather than as an optimized competing process.

In the disperser, the measured pressure–flow term is introduced through constrictions, inclined holes and moving-element contact zones. The observed combination of lower residual aqueous-phase TSS and moderate wall-plug Ev is consistent with improved reagent–particle contact [63]; however, the experiment does not partition electrical input into useful aggregation work, viscous heating, pump/motor losses and pipe friction.

4.4. Model Use and Limits

Comparison with published technologies must account for differences in feed matrices, system boundaries, and energy denominators. Conventional coagulation–flocculation studies commonly optimize reagent dose and mixing sequence, whereas hydrodynamic-cavitation studies emphasize reactor geometry, pressure drop, and local energy density [15,30,64]. Published specific-energy values are not directly comparable when one study uses hydraulic power, another uses motor nameplate power, and another integrates wall-plug energy over a different treated volume. The present range of 0.0800–0.1681 kWh/m³ is reported on a measured wall-plug basis for a 50 L recirculated batch. Its principal comparative meaning is internal: the recommended regime satisfies the clarification objective before the steep energy increase. Relative to the literature, the apparatus offers compact recirculating operation and simultaneous water/sludge accounting, but matched cross-technology experiments are still required before any claim of general energy superiority can be made [65–67].

The main value of the models is simultaneous screening of separation efficiency, residual TSS, and specific energy. Nevertheless, the 15-run validation set is internal, and drawn from the same fluid class and experimental system. The low validation R^2 for separation efficiency and the large maximum relative error for residual TSS preclude claims of broad predictive generalization. External validation should use independent drilling-fluid batches, preferably with formation- and season-related compositional variation, before the models are used for design decisions beyond the present experimental domain.

4.5. Scale-Up Implications

No direct industrial scale-up is claimed from the present 50 L batch system. A larger unit would have to preserve dynamic, rather than merely geometric, similarity. At minimum, scale-up should match the operational G and $G\tau$ ranges, pressure-drop/flow relationship, V_{eff}/V_0 ratio, solid loading, recirculation exposure N_p , and residence-time distribution. The number and spacing of perforated elements, hole diameter, and moving-ball inventory must then be selected to reproduce these quantities, rather than them being obtained by simple linear enlargement.

Moving-ball behavior introduces a specific scale dependence. Ball suspension, collision frequency, and the probability of settling or jamming depend on particle loading, fluid viscosity, local velocity, and the ratio of ball size to hydraulic passage size. These effects were not measured in the present apparatus, and require transparent-wall tests or validated CFD–DEM analysis before industrial dimensions are fixed.

Residence-time distribution is another unresolved scale-up variable. The present τ is elapsed recirculation time, not single-pass residence time. A pilot unit should therefore be characterized by tracer testing, and should report mean residence time, dispersion, and short-circuiting, together with ΔP , Q , G and N_p . Direct enlargement without this analysis can change local shear, create stagnant zones, or increase abrasive wear [68].

Industrial deployment would require feed homogenization, screened removal of coarse inclusions, reagent preparation and controlled dosing, pressure and electrical-energy monitoring, clarification, sludge discharge, and recovered-water quality control. Long-duration pilot operation is required to quantify pressure-drop drift, erosion of perforated disks, motion and wear of spherical inserts, clogging, and maintenance intervals [69,70].

4.6. Limitations and Future Research

The applicability of the findings is restricted to high-solid water-based spent drilling fluids from uranium technological wells in Kazakhstan. Oil-based, low-solid, high-salinity, and compositionally different polymer drilling fluids were not tested; transfer to those systems requires new experiments and model recalibration.

Bulk dry-solid content and aqueous-phase TSS describe different measurement compartments; the present study does not provide a complete component-by-component solid mass balance. Future work should add wet- and dry-sludge mass, dissolved-solid and polymer balances, together with conductivity, salinity, alkalinity, dissolved organics and reuse-compatibility measurements.

Formation- and season-related changes in clay mineralogy, dissolved salts, temperature, polymer carryover and particle-size distribution may shift the optimal dose and G – τ window. One commercial polymer product was used throughout the experimental program; detailed supplier formulation, molecular-weight distribution, and charge-density data were not available for disclosure. Reagent transferability is therefore an explicit limitation, and each new fluid–floculant combination requires recalibration.

Future work should include independent fluid batches, continuous pilot testing, tracer-based residence-time analysis, CFD or CFD–DEM characterization of internal flow and ball motion, direct floc-strength and breakage measurements, ζ -potential and residual-polymer measurements, abrasive-wear assessment, and techno-economic comparison with centrifugation, hydrocycloning, gravitational settling, and combined-treatment schemes [71].

5. Conclusions

For the investigated fluid class, the principal practical outcome is a reproducible operating window that jointly constrains residual aqueous-phase solids and wall-plug

energy. The results support pilot-scale design and control-point selection, but they do not yet justify direct industrial scale-up or transfer to other drilling-fluid chemistries.

1. A recirculating 50 L laboratory-pilot line was used to evaluate a flow-through disperser with a 4.75 L cylindrical geometric envelope, perforated cascade disks, and movable spherical elements for treatment of high-solid water-based spent drilling fluids from uranium technological wells.
2. The verified laboratory-pilot dataset covered 120 hydrodynamic treatment experiments, five spent drilling fluid samples, 216 particle-size bin observations (12 complete distributions), nine baseline mechanical mixing tests, and ten repeated-cycle runs.
3. The recommended compromise operating mode was $G = 1300 \text{ s}^{-1}$, $\tau = 150 \text{ s}$ and 50 g/t dry solids. This regime achieved $92.15 \pm 1.81\%$ separation efficiency, residual aqueous-phase TSS of $125.2 \pm 30.0 \text{ mg/L}$, water recovery of $83.84 \pm 0.28\%$, sludge reduction of $59.02 \pm 2.11\%$ and wall-plug specific energy of $0.1227 \pm 0.0062 \text{ kWh/m}^3$.
4. The reagent-optimized regime at 75 g/t dry solids achieved $93.23 \pm 1.07\%$ separation efficiency and residual TSS of $107.7 \pm 14.2 \text{ mg/L}$, but the additional benefit over 50 g/t was limited, indicating a dosage plateau.
5. The maximum-efficiency regime reached 94.20% separation efficiency, but its specific energy consumption increased to 0.1681 kWh/m^3 , confirming that maximum hydrodynamic intensity is not the most energy-rational operating point.
6. Repeated-cycle testing confirmed process stability, with average separation efficiency of $91.74 \pm 0.69\%$, residual TSS of $136.4 \pm 13.4 \text{ mg/L}$, and specific energy of $0.1245 \pm 0.0065 \text{ kWh/m}^3$.
7. Relative to the available laboratory mechanical-mixing baseline, the hydrodynamic process produced higher separation and lower residual aqueous-phase TSS at lower E_v than the medium- and long-duration baseline runs. Because impeller hydrodynamics were not fully recorded or optimized, this comparison is an internal benchmark, rather than a universal technology ranking.
8. The nominal G values are Camp–Stein-equivalent contact-zone estimates derived from measured ΔP , Q , run-specific viscosity, and an explicitly stated effective dissipation volume. The response models and recommended window are restricted to the investigated fluid class. Independent-batch validation, tracer-based residence-time characterization, long-duration wear and clogging tests, and scale-dependent assessment of moving-ball dynamics are required before predictive control or industrial deployment.

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