

Article

Three-Dimensional Temperature Distribution Reconstruction via an Ellipsoidal Reflector: Algorithm Development and Numerical Validation

Yaofang Zhang ¹ and Shi Liu ^{2,*}

¹ School of Energy, Power and Mechanical Engineering, North China Electric Power University, Beijing 102206, China; zhangyaofang@ncepu.edu.cn

² School of Control and Computer Engineering, North China Electric Power University, Beijing 102206, China

* Correspondence: liushi@ncepu.edu.cn

Abstract

Accurate measurement of three-dimensional flame temperature distribution is crucial for combustion diagnosis. Traditional optical methods rely on observation windows or multi-camera arrays, which can be difficult to deploy in environments with restricted optical access, such as aviation engine burners. This paper proposes a passive radiation temperature measurement method based on ellipsoidal reflector (ER). By using the confocal characteristics of the ER, the radiation inside the flame is collected through a single aperture at one focus and guided to the Charge-Coupled Device (CCD) at another focus. A complete measurement model has been developed, combining the ray tracing of each detector element through the ER with the discretized Radiative Transfer Equation (RTE) under the pure absorption approximation. Temperature reconstruction is performed using the Least Squares QR (LSQR) algorithm. The method is validated numerically on a cylindrical flame under three configurations with identical grid and detection settings, differing only in the temperature field and absorption coefficient: symmetric with uniform absorption, asymmetric with uniform absorption, and asymmetric with non-uniform absorption. Gaussian noise (0–5%) is added to the simulated measurements. Noise-free reconstruction converges to near machine precision. Under 5% noise, the mean relative errors are 3.1%, 8.7%, and 6.1% for the three cases, with errors concentrated at the flame periphery while the core remains accurately reconstructed. The proposed method provides a practical solution for temperature monitoring where optical access is severely limited.

Keywords: flame temperature measurement; ellipsoidal reflector; radiation thermometry; LSQR algorithm; three-dimensional reconstruction

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1. Introduction

Combustion processes play a critical role in industrial production, including coal-fired power plants, steel-making furnaces, aviation engines, and other high-temperature industrial systems. The combustion reaction involves multiphase fluid flow, high-temperature chemical kinetics, and radiative heat transfer [1], making online monitoring of the combustion state and its characteristics a significant challenge. Among combustion diagnostic indicators, the temperature field is of particular importance: it reflects the

progress of chemical reactions, the degree of combustion completion, and the formation of particulate products [2]. In addition, precise temperature control is directly related to the safe operation of industrial equipment and the control of pollutant emissions [2]. These needs have prompted extensive research on flame temperature measurement.

Generally, flame temperature measurement methods can be divided into two categories: contact and non-contact [3]. Thermocouples, the most widely used contact method, provide simple and direct readings. But inserting a probe into the flame will disturb the flow field and the combustion medium, introduce the measurement error, and be limited to point measurements [4]. Non-contact methods avoid these shortcomings and have therefore attracted sustained research interest. They can be further categorized into active and passive technologies. Active optical diagnostics such as Rayleigh scattering spectroscopy, coherent anti-Stokes Raman spectroscopy (CARS), and laser-induced fluorescence (LIF) achieve high spatial and temporal resolution [5]. These techniques, however, require laser sources and usually require multi-beam optical access around the flame, resulting in complex experimental settings with limited robustness in actual engineering environment. In contrast, passive radiation temperature measurement depends on the self-emission of the flame without external excitation [6]. Its simpler setting and non-invasiveness make it an increasingly attractive alternative.

Guo et al. [7] used a high-pixel CMOS camera with three narrow-band filters to image symmetrical ethylene diffuse flames of multiple wavelengths. After radiometric calibration, the projection radiation intensity distribution was obtained. Combined with Abel inversion, the radial distribution of temperature and soot concentration was restored by the two-color method. Wang et al. [8] established a geometric mapping between points in the three-dimensional flame and pixels on the Charge-Coupled Device (CCD) camera, and formulated a non-scattered radiation transmission model. They solved the RTE through the simulated annealing algorithm, deployed four CCD cameras around the flame to reconstruct two-dimensional temperature and particle concentration distributions on several cross-sections, and verified the thermocouple measurement.

Despite these advances, a fundamental limitation remains: images recorded by traditional cameras capture line-of-sight integrals [9], which represent projections rather than local temperatures at specific positions within the flame.

To overcome this, Hossain et al. [10] developed a tomographic system that uses eight optical fiber bundles arranged around the flame at equal angles and connected to two CCD cameras. By obtaining images from eight views and applying filtered back-projection and Simultaneous Algebraic Reconstruction Technique (SART) algorithms, they demonstrated the three-dimensional temperature reconstruction of the burner flame. As reviewed by Grauer et al. [9], this multi-view tomography strategy has been extended through various reconstruction algorithms.

However, multi-camera and multi-optic tomography methods have a common practical limitation: they require observation windows of a certain size or multiple access ports distributed around the flame. This channel is usually not available in confined combustion environments such as aviation engine combustion chambers, carbon black reactors and industrial furnaces [11]. There may be only a small observation port, under which traditional multi-view method becomes unfeasible.

Encouraged by this constraint, researchers have been looking for a single-camera solution. Sun et al. [12] demonstrated the use of a light field camera, which integrates a microlens array between the main lens and the sensor. This design enables each pixel to record the intensity and direction of incident radiation. Through ray tracing, the path of each light through the flame can be restored. They established a pure-absorption radiation transmission model and used the LSQR algorithm to solve the inverse problem,

reconstructing the three-dimensional temperature field of an ethylene diffusion flame from a single exposure.

On this basis, Huang et al. [13] extended the light-field approach to multispectral imaging, and used the LSQR-CG method to reconstruct the temperature and absorption coefficient distributions in non-axisymmetric flames simultaneously. Shi et al. [14] further improved the forward model by incorporating wave optics into the imaging framework, proving that LSQR improves the reconstruction accuracy.

The light-field technique has since been advanced as a single-camera diagnostic approach. It has been applied to flames with wall radiation effect [11], extended to Bayesian reasoning framework to enhance noise robustness [15], and is suitable for real-time dynamic temperature monitoring using Kalman filters [16]. These developments confirm that single-camera light-field imaging can recover three-dimensional flame temperature distributions under increasingly realistic conditions.

Despite the progress made with light-field imaging, a residual limitation remains. Although the light field camera only needs one point of view, it still needs an observation window in a specific area to collect sufficient samples of radiation field angles. When only a pinhole-sized access port is available, even a light-field camera may not be able to capture sufficient signals. This paper therefore proposes a fundamentally different optical configuration: a temperature measurement method based on an ER.

ER has a well-known confocal characteristic: any light passing through the first focus will be reflected by the ellipsoid and converge at the second focus. This principle has been applied in thermal photovoltaic systems, in which an elliptical optical cavity with confocal geometry is used to enhance photon recovery and spectral efficiency [17]. However, its application in flame radiation temperature measurement has not been developed before. In the proposed system, the flame is located at the first focus and the CCD detector is placed near the second focal point, behind the small aperture. Radiation from the first focal region is directed to the detector through the aperture, which also acts as a spatial filter: only rays originating from the vicinity of the first focus can pass through, while off-focus radiation is blocked. The spatial trajectory of each detected light is determined by ray tracing from each detector element through ER to the flame volume. This makes it possible to reconstruct the temperature from the intensity measurement of directional resolution without the need for large windows or multi-camera arrays.

This paper develops a complete theoretical framework for this method: through the ray tracing model of ER, the discrete RTE under pure absorption approximation, and the inversion strategy based on LSQR. The method is numerically verified on the cylindrical flame under three progressive challenge configurations, which are symmetric uniform absorption, asymmetric uniform absorption and asymmetric non-uniform absorption, and the noise levels are 0%, 1%, 3% and 5% respectively. The results demonstrate that the proposed method accurately and stably reconstructs the temperature field, confirming its feasibility of deployment in a combustion environment with severely limited optical access.

2. Theory and Methods

2.1. System Description

The proposed passive radiation thermometry system is built upon an optical system centered on a rotationally symmetric ER as its core component, as illustrated in Figure 1. The optical train consists of three principal components—the ER, a small-aperture shield, and a CCD image detector—mounted on an optical bench. The detector is connected to a computer for data acquisition and processing.

The measurement principle relies on the two-focus optical property of the ER: any ray passing through one focus of the ellipsoid reflects off the surface and converges strictly

at the other focus. During measurement, the flame is described in coordinate system O - XYZ , and the optical measurement system in O' - $X'Y'Z'$, as illustrated in Figure 2. The flame is positioned near the first focal region of the ER; the specific position can be adjusted according to the zone of diagnostic interest. The ER and the detection optics are placed outside the measurement domain; only the access port at F_1 interfaces with the flame side.

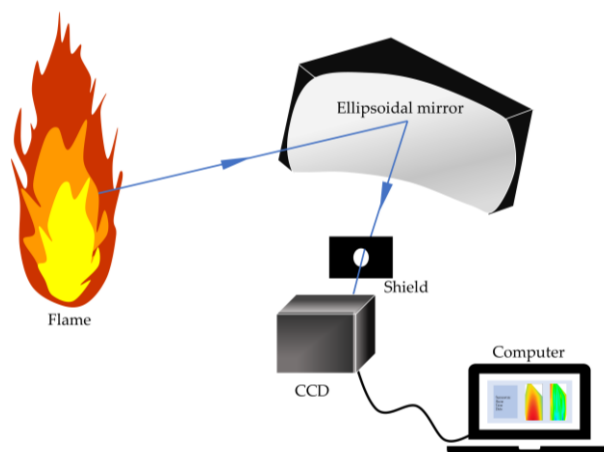


Figure 1. Schematic of the ellipsoidal-reflector radiation thermometry system.

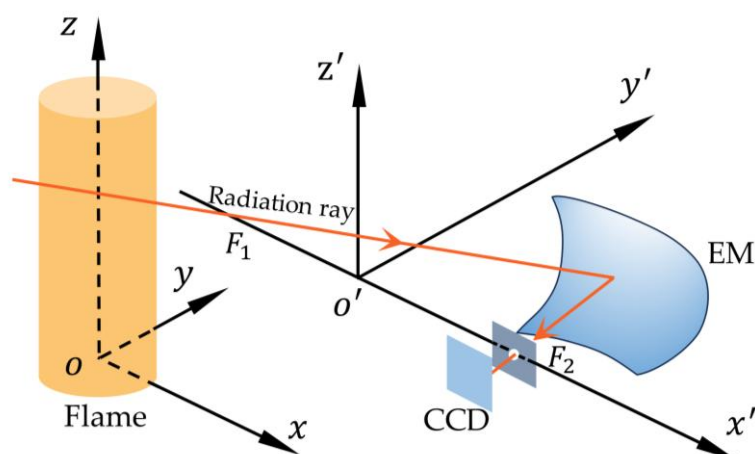


Figure 2. Coordinate systems of the flame and the ER measurement system.

Denoting the two foci of the ellipsoid as F_1 and F_2 , radiation emitted from the vicinity of F_1 reflects off the ellipsoidal surface and converges at F_2 . It then passes through the small aperture in the shield—which is precisely aligned with F_2 —and continues to the CCD detector. Therefore, small aperture shielding acts as a space filter: only radiation from F_1 can pass through the aperture, while out-of-focus radiation is blocked. This confocal filtering scheme greatly improves the signal-to-noise ratio.

Beyond spatial filtering, the ER also provides spatial positioning and directional discrimination through its well-defined optical path. In conventional CCD-based temperature measurement, each pixel records the line-of-sight integral of all radiation along its optical path, which is a projection of discarding directional information. By introducing ER, the complete three-dimensional trajectory of each ray is determined by ray tracing (Section 2.2). These features, single-port access, spatial filtering and direction identification, make the deployment of the proposed system in a confined combustion environment more practical than the traditional multi-camera or large window method.

2.2. Forward Model: Ray Tracing and Radiative Transfer

The forward model describes how radiation emitted by the flame reaches the detector. It includes two stages: geometric ray tracing to determine the path of each detection ray through the flame, and the radiation transmission equation to calculate the accumulated intensity along each path.

2.2.1. Ray Tracing

As noted in Section 2.1, the reflector geometry and ray tracing are formulated in the ellipsoid-centered coordinate system (X', Y', Z') , while the flame is described in the flame-centered system (X, Y, Z) . When a ray crosses from the measurement domain into the flame domain, its coordinates are transformed by a simple translation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}, \quad (1)$$

where $(\Delta x, \Delta y, \Delta z)$ are the known offsets between the two coordinate origins. Since the relative positions of the two systems are fixed a priori, this translation is exact and does not introduce additional uncertainty.

The ellipsoidal reflector is a rotationally symmetric surface formed by revolving an elliptical generatrix $\widehat{AB}$ in the $x-y$ plane about the x -axis by $\pm\omega$, as illustrated in Figure 3. The CCD detector plane is parallel to the $x-z$ plane, located at $y = -d$.

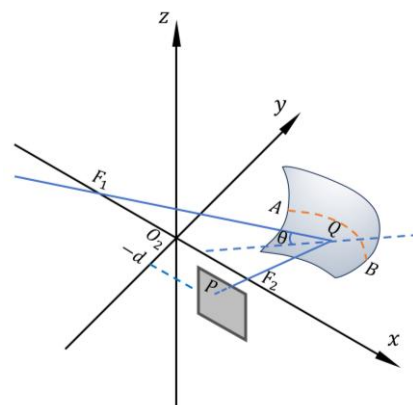


Figure 3. Geometry of the ellipsoidal reflector and ray-tracing parameters.

In the centered coordinate system of the ellipsoid, the two foci are located at $F_1(-c, 0, 0)$ and $F_2(c, 0, 0)$, where $c = \sqrt{a^2 - b^2}$. The ellipsoid equation is:

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1, \quad x \in (x_a, x_b) \quad (2)$$

The CCD position, the coordinates of F_2 , and the geometric parameters of the ER (a , b , x_a , x_b) are all known a priori. For each pixel on the CCD, connecting its center $P(x_p, y_p, z_p)$ to $F_2(c, 0, 0)$ gives the direction of the reflected ray. The parametric equation of the reflected ray is:

$$\frac{x-c}{x_p-c} = \frac{y}{y_p} = \frac{z}{z_p} = t, \quad (3)$$

with direction cosines:

$$\begin{aligned} \cos\alpha_2 &= \frac{x_p - c}{\sqrt{(x_p - c)^2 + y_p^2 + z_p^2}} \\ \cos\beta_2 &= \frac{y_p}{\sqrt{(x_p - c)^2 + y_p^2 + z_p^2}} \end{aligned} \quad (4)$$

$$\cos\gamma_2 = \frac{z_p}{\sqrt{(x_p - c)^2 + y_p^2 + z_p^2}}$$

The parametric equations of the reflected ray are:

$$\begin{cases} x = t\cos\alpha_2 + c \\ y = t\cos\beta_2 \\ z = t\cos\gamma_2 \end{cases}, \quad (5)$$

Substituting Equation (5) into Equation (2) produces a quadratic equation in t :

$$(b^2\cos^2\alpha_2 + a^2(\cos^2\beta_2 + \cos^2\gamma_2))t^2 + 2b^2c \cdot \cos\alpha_2 \cdot t + b^2c^2 - a^2b^2 = 0, \quad (6)$$

This quadratic has two roots in general, corresponding to the two intersections of a line through F_2 with the full ellipsoid. Because the mirror spans only a segment of the full ellipsoid, the physically relevant root is selected using the boundary condition defined by the arc endpoints. Substituting the selected t back into Equation (5) yields the coordinates of the reflection point $Q(x_q, y_q, z_q)$.

The unit normal vector to the ellipsoidal surface at Q is obtained from the gradient of the ellipsoid equation:

$$\frac{x - x_q}{\left. \frac{df(x, y, z)}{dx} \right|_{x_q}} = \frac{y - y_q}{\left. \frac{df(x, y, z)}{dy} \right|_{y_q}} = \frac{z - z_q}{\left. \frac{df(x, y, z)}{dz} \right|_{z_q}} \quad (7)$$

After normalization, the unit normal is $n = (\cos\alpha_n, \cos\beta_n, \cos\gamma_n)$. The cosine of the angle between the incident ray and the surface normal is

$$\cos\theta = \frac{\langle \vec{n}, \overrightarrow{PF_2} \rangle}{\|\vec{n}\| \|\overrightarrow{PF_2}\|} \quad (8)$$

The law of reflection relates the direction cosines of the incident and reflected rays as follows [18]:

$$\begin{cases} \cos\alpha_2 = \cos\alpha_1 - 2\cos\theta\cos\alpha_n, \\ \cos\beta_2 = \cos\beta_1 - 2\cos\theta\cos\beta_n, \\ \cos\gamma_2 = \cos\gamma_1 - 2\cos\theta\cos\gamma_n, \end{cases} \quad (9)$$

From these equations, the direction cosines of the incident ray are determined. The spatial line equation of the incident ray is then:

$$\frac{x - x_q}{\cos\alpha_1} = \frac{y - y_q}{\cos\beta_1} = \frac{z - z_q}{\cos\gamma_1} = s, \quad (10)$$

At this stage, both the direction cosines of the incident ray and the coordinates of the reflection point Q are known. The complete ray path from the detector pixel, via the ellipsoidal reflection, into the flame volume is fully determined. Repeating this procedure for every CCD pixel constitutes the complete ray-tracing algorithm, yielding the spatial trajectory of each detection ray.

2.2.2. Radiative Transfer Model for the Flame

A flame is a participating medium. Radiative energy transport within it is governed by the Radiative Transfer Equation (RTE). Under steady-state conditions, the spectral radiative intensity I_λ along direction Ω satisfies:

$$\frac{dI_\lambda(s, \Omega)}{ds} = -\kappa_{e\lambda}(s)I_\lambda(s, \Omega) + \kappa_{a\lambda}(s)I_{b\lambda}(s) + \frac{\kappa_{s\lambda}(s)}{4\pi} \int_{4\pi} I_\lambda(s, \Omega') \phi(\Omega', \Omega) d\Omega', \quad (11)$$

where $\kappa_{e\lambda} = \kappa_{a\lambda} + \kappa_{s\lambda}$ is the extinction coefficient (sum of the absorption and scattering coefficients), $I_{b\lambda}$ is the blackbody spectral intensity of the medium, and $\phi(\Omega', \Omega)$ is the scattering phase function.

The RTE in its full integro-differential form rarely admits an analytic solution and must be simplified for practical computation. Scattering integral is the main source of difficulty. According to Mie theory, the absorption cross-section of soot particles in flames greatly exceeds their scattering cross-section [19]. The flame can therefore be treated as a purely absorbing medium ($\kappa_{s\lambda} \approx 0$), eliminating the scattering term. Under this simplification, the RTE reduces to:

$$\frac{dI_{\lambda}(r, \Omega)}{d\tau_{\lambda}} = -I_{\lambda}(r, \Omega) + I_{b\lambda}(r), \quad (12)$$

where τ_{λ} is the optical depth. This is now a first-order ordinary differential equation that can be integrated directly.

To apply this equation to the discretized flame domain, the cylindrical flame is divided into $l \times m \times n$ grid cells along the circumferential (θ), radial (r), and axial (z) directions, respectively. The optical properties are assumed uniform within each cell. For a ray traversing a sequence of grid cells, integrating along the ray path yields the recurrence relation for the spectral intensity exiting the i -th cell:

$$I_{\lambda,i}(\Omega) = I_{\lambda,i-1}(\Omega)e^{-\tau_{\lambda,i}} + I_{b\lambda,i}(1 - e^{-\tau_{\lambda,i}}), \quad (13)$$

where $\tau_{\lambda,i} = \kappa_{a\lambda,i} \cdot \Delta s_i$ is the optical thickness of the i -th cell, and Δs_i is the path length of the ray within that cell. Propagating this recurrence from the first cell intersected by the ray to the last gives the outgoing spectral intensity at the detector:

$$I_{\lambda}(\Omega) = I_{b\lambda n}[1 - \exp(-\tau_{\lambda n})] + \sum_{i=1}^{n-1} [\exp(-\sum_{j=i+1}^n \tau_{\lambda j}) - \exp(-\sum_{j=i}^n \tau_{\lambda j})] I_{b\lambda i}, \quad (14)$$

where n is the total number of grid cells traversed by the ray. Equation (14) shows that the outgoing intensity can be expressed as a linear combination of the blackbody intensities of the individual grid cells; the weighting coefficients depend solely on the optical thicknesses of the cells along the ray path.

Assembling this relationship for all M effective detection rays yields the linear system:

$$I_{\lambda} = A_{\lambda} I_{b\lambda}, \quad (15)$$

where $I_{\lambda} \in \mathbb{R}^M$ is the vector of outgoing spectral intensities measured at the detector, $I_{b\lambda} \in \mathbb{R}^N$ is the vector of unknown blackbody intensities in the N effective grid cells, and $A_{\lambda} \in \mathbb{R}^{M \times N}$ is the coefficient matrix whose element a_{ij} is the weighting factor of the j -th cell for the i -th ray.

Once the blackbody intensity $I_{b\lambda,i}$ is recovered for each grid cell, the temperature is obtained from Planck's law:

$$I_{b\lambda,i} = \frac{c_1 \lambda^{-5}}{\pi \left[\exp\left(\frac{c_2}{\lambda T_i}\right) - 1 \right]} \Rightarrow T_i = \frac{c_2}{\lambda \ln \left[c_1 / (\lambda^5 \pi I_{b\lambda,i}) + 1 \right]} \quad (16)$$

where $c_1 = 3.7418 \times 10^{-16} \text{ W} \cdot \text{m}^2$ is the first radiation constant, $c_2 = 1.4388 \times 10^{-2} \text{ m} \cdot \text{K}$ is the second radiation constant, and $\lambda = 650 \text{ nm}$ in this article.

2.3. LSQR Algorithm

Solving the linear system $I_{\lambda} = A_{\lambda} I_{b\lambda}$ for the unknown blackbody intensity vector constitutes a linear inverse problem. The coefficient matrix A_{λ} is generally ill-conditioned, so that a direct solution is highly sensitive to measurement noise—even small perturbations in the data can lead to disproportionately large errors in the solution. Regularization is therefore required to stabilize the inversion.

The inverse problem is cast as the following least-squares minimization:

$$\min \|Ax - b\|_2, \quad (17)$$

where $A \in \mathbb{R}^{M \times N}$ is the coefficient matrix, $b \in \mathbb{R}^M$ is the noise-corrupted measurement vector, and $x \in \mathbb{R}^N$ is the unknown blackbody intensity distribution to be recovered.

The LSQR algorithm proposed by Paige and Saunders [20] is based on the double diagonalization process of Lanczos and is specially designed for large-scale sparse least squares problems. The algorithm has good numerical stability, effectively suppresses the amplification of measurement noise, and the convergence speed is fast. Its core idea is to transform the primitive matrix equation into an equivalent double diagonal system, and construct the solution incrementally through Lanczos iteration.

The algorithm proceeds in three phases:

(1) Initialization. Set the initial estimate $x_0 = 0$ and compute the initial residual and Lanczos vectors:

$$\beta_0 = \|b\|_2, \quad u_0 = b/\beta_0, \quad v_0 = A^T u_0, \quad \alpha_0 = \|v_0\|_2, \quad w_0 = v_0/\alpha_0, \quad \bar{\varphi}_0 = \beta_0 \quad (18)$$

(2) Lanczos bidiagonalization. At iteration step i , new orthogonal vectors u_i and v_i are constructed alternately:

$$\begin{aligned} u_{i+1} &= Av_i - \alpha_i u_i, \quad \beta_{i+1} = \|u_{i+1}\|_2, \quad u_{i+1} = \frac{u_{i+1}}{\beta_{i+1}} \\ v_{i+1} &= A^T u_{i+1} - \beta_{i+1} v_i, \quad \alpha_{i+1} = \|v_{i+1}\|_2, \quad v_{i+1} = \frac{v_{i+1}}{\alpha_{i+1}} \end{aligned} \quad (19)$$

(3) Solution update. Apply Givens rotation to eliminate the sub-diagonal elements of the double diagonal system, and refine the approximate solution in each step:

$$\begin{aligned} \rho_i &= \sqrt{\bar{\rho}_i^2 + \beta_{i+1}^2} \\ x_{i+1} &= x_i + w_i \frac{\bar{\rho}_i \bar{\varphi}_i}{\rho_i^2} \\ w_{i+1} &= v_{i+1} - w_i \frac{\beta_{i+1} \alpha_{i+1}}{\rho_i^2} \\ \bar{\rho}_{i+1} &= -\frac{\bar{\rho}_i}{\rho_i} \alpha_{i+1}, \quad \bar{\varphi}_{i+1} = \frac{\beta_{i+1}}{\rho_i} \bar{\varphi}_i \end{aligned} \quad (20)$$

In LSQR, the iteration count itself serves as an implicit regularization parameter. Early iteration captures the solution components related to the large singular value; as the iteration progresses, the components related to the small singular value (and therefore dominated by noise) gradually enter the solution. In practical application, the iteration proceeds until a prescribed convergence criterion is satisfied or a maximum iteration limit is reached.

Summary of the Reconstruction Workflow. The complete procedure for reconstructing the three-dimensional flame temperature field is summarized in the following six steps:

Step 1. Specify the system parameters: ellipsoidal mirror geometry (semi-major axis a , semi-minor axis b , arc extent $[x_a, x_b]$, rotation angle $\pm\omega$), flame dimensions (R , H) and temperature model, coordinate translation offsets between the measurement and flame systems (Δx , Δy , Δz), CCD detector position and pixel partition ($n_x \times n_z$), and the grid discretization scheme $l \times m \times n$ divisions in the circumferential, radial, and axial directions.

Step 2. Determine the measurement volume by ray tracing (Section 2.2.1). The set of grid cells intersected by at least one detection ray constitutes the measurable sub-volume of the flame.

Step 3. Perform ray tracing for each detection ray (one ray per $n_x \times n_z$ partition, with the center of each partition as the starting point). For each effective ray, record the indices of the cells it traverses and the segment length Δs_j within each cell.

Step 4. Assemble the coefficient matrix A from the ray-tracing results. Compute the forward model (Section 2.2.2) to obtain the ideal (noise-free) outgoing intensity distribution at the detector.

Step 5. Add Gaussian white noise to the computed intensities to simulate realistic measurement signals. The noise-corrupted measurement is $I_m = I_e \cdot (1 + \sigma\zeta)$, where $\sigma = \delta\%/2.576$ (corresponding to a 99% confidence interval) and $\zeta \sim \mathcal{N}(0,1)$.

Step 6. Solve the noisy linear system $Ax = b$ using the LSQR algorithm. Convert the recovered blackbody intensities $I_{b\lambda,i}$ to temperature values T_i via Planck's law (Equation (16)).

The quality of the reconstruction is assessed by comparing the recovered temperature field with the true temperature distribution. The relative error for the i -th grid cell is:

$$\varepsilon_{r,i} = \frac{|T_{\text{rec},i} - T_{\text{true},i}|}{T_{\text{true},i}} \quad (21)$$

where $T_{\text{rec},i}$ and $T_{\text{true},i}$ are the reconstructed and true temperatures in the i -th grid cell, respectively. The minimum, maximum, and mean relative errors over the effective domain serve as overall performance metrics.

3. Results

3.1. System Configuration and Simulation Setup

The numerical simulations were conducted using the ER-based radiation thermometry model described in Section 2. All calculations were implemented in MATLAB (The MathWorks, Inc., Natick, MA, USA) using base MATLAB functions. The geometric parameters of the ellipsoidal mirror were set as follows: semi-major axis $a = 0.05$ m, semi-minor axis $b = 0.04$ m, with the elliptical arc spanning the range $0 \leq x \leq 11a/12$ along the x -axis, and revolved about the x -axis by $\pm\omega = \pi/5$. This rotation angle is an illustrative choice, a sensitivity analysis of ω is provided in Section 3.5.2. Any value not exceeding $\pi/2$ (90°) is theoretically allowed, a larger angle will cause the total span to exceed 180° , thus preventing some reflected rays from reaching the CCD plane. The resulting focal points are located at $F_1(-c, 0, 0)$ and $F_2(c, 0, 0)$, where $c = \sqrt{a^2 - b^2} = 0.03$ m.

The measurement target is a cylindrical flame with radius $R = 0.05$ m and height $H = 0.4$ m. The plane of the CCD detector is placed parallel to the x - z plane at $y = -d$ ($d = 0.01$ m), near the second focus. The detector plane is conceptually divided into rectangular areas equal to $n_x \times n_z = 40 \times 80$; the center point of each region is used as the starting point for reverse ray tracing through the optical system. After tracking from CCD through ER, the light that intersects with the flame volume is classified as effective detection light; the light that misses the flame is discarded. The resulting detection plane spans approximately 21.5 mm in the x -direction and 15.0 mm in the z -direction at the plane $y = -0.01$ m.

The flame domain was discretized in cylindrical coordinates with $l \times m \times n = 4 \times 8 \times 12$. All three test cases share the same grid resolution to facilitate direct comparison. The translation offsets between the ellipsoid-centered coordinate system (X', Y', Z') and the flame-centered system (X, Y, Z) were set to $\Delta x = 2R$, $\Delta y = R$, and $\Delta z = (2/5)H$. These offsets place the flame such that detection rays traverse all four angular grid positions, improving coverage of the flame core—the region of primary diagnostic interest. These offsets were selected as a representative configuration that places the flame within the detection zone of the reflector. They are not unique; any

placement that positions the flame within the accessible detection volume of the reflector is in principle admissible for the method. Only grid cells intersected by at least one effective detection ray contribute to the reconstruction; these form the effective grid set, representing a sub-volume of the entire flame domain. After removing zero columns, the coefficient matrix A has dimensions of 2808×93 . The matrix is highly sparse: only 11.6% of its entries are non-zero, reflecting the fact that each detection ray traverses a small fraction of the total flame volume. The condition number of A is 462 for uniform absorption (Cases 1 and 2) and 407 for non-uniform absorption (Case 3).

Three test cases were designed to evaluate the method under progressively more challenging conditions, as summarized in Table 1.

Table 1. Overview of the three test cases.

	Temperature Field	Absorption Coefficient
Case 1	Axisymmetric, $T(z, r)$	Uniform, $\kappa_a = 0.5$
Case 2	Asymmetric, $T(x, y, z)$	Uniform, $\kappa_a = 0.5$
Case 3	Asymmetric, same as Case 2	Non-uniform, $\kappa_a(z, r)$

All three cases use identical grid resolution ($4 \times 8 \times 12$) and CCD partition (40×80), producing the same total number of detection rays (3200). The four angular positions (45° , 135° , 225° , 315°) are accessible in all cases. For each case, the reconstructed temperature field is presented at two representative cross-sections— 45° – 225° and 135° – 315° —together with the corresponding relative error distributions at $\delta = 3\%$ and $\delta = 5\%$.

For each case, the forward model was evaluated to compute the noise-free outgoing radiative intensity at the detector. Gaussian white noise was then added following the procedure described in Section 2.3 (Step 5) at four noise levels: $\delta = 0\%$, 1% , 3% , and 5% . The LSQR algorithm was applied to reconstruct the blackbody intensity field from the noisy measurements. All reconstructions used the same settings: a convergence tolerance of 10^{-8} and a maximum of 500 iterations. The recovered intensities were converted to temperature via Planck's law (Equation (16)).

Reconstruction quality was quantified using the relative error for each effective grid cell in Equation (21), together with the minimum, maximum, and mean relative errors over the effective domain.

3.2. Case 1: Symmetric Flame with Uniform Absorption Coefficient

The axisymmetric temperature distribution and constant absorption coefficient for Case 1 are given by:

$$T(z, r) = -600 \times \frac{2600}{17} (z - 0.4)(0.05 - 12r^2 + 8r^3) + 800, \quad \kappa_a = 0.5, \quad (22)$$

Figure 4 shows the true temperature distribution over the measured domain at the two cross-sections. The reconstructed temperature fields under four noise levels are presented in Figure 5. Figure 6 gives the relative error distributions at $\delta = 3\%$ and $\delta = 5\%$ for each cross-section.

In the noise-free case ($\delta = 0\%$), the LSQR algorithm recovered the temperature field to an accuracy on the order of 10^{-11} , confirming that the algorithm is capable of solving this class of inverse radiation problem to within machine precision.

When 1% Gaussian noise was added to the measurement signal, the reconstructed temperature distribution remained nearly indistinguishable from the true field. The mean relative error was 1.5%. At $\delta = 3\%$, minor oscillations appeared at the flame periphery, while the central region continued to track the true distribution closely; the mean relative error increased to 2.7%.

At $\delta = 3\%$ and $\delta = 5\%$, the error distribution maps reveal that the reconstruction error is concentrated at the flame boundary. This is attributable to two factors: the boundary region has lower temperature, hence weaker radiative intensity and reduced signal-to-noise ratio; and fewer effective detection rays traverse the peripheral cells compared to the core region. At the highest noise level $\delta = 5\%$, the edge oscillations became more obvious, with an average relative error of 3.1%. However, the spatial distribution of the error was highly non-uniform: reconstruction errors in the flame core were consistently lower than at the periphery, while the boundary layer oscillations were confined to a small fraction of the effective domain.

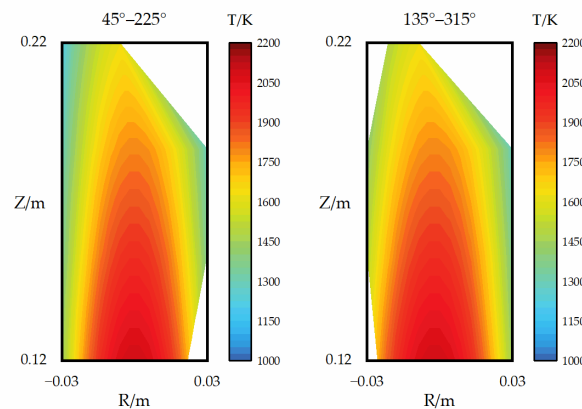


Figure 4. Case 1: True temperature distribution at the two cross-sections.

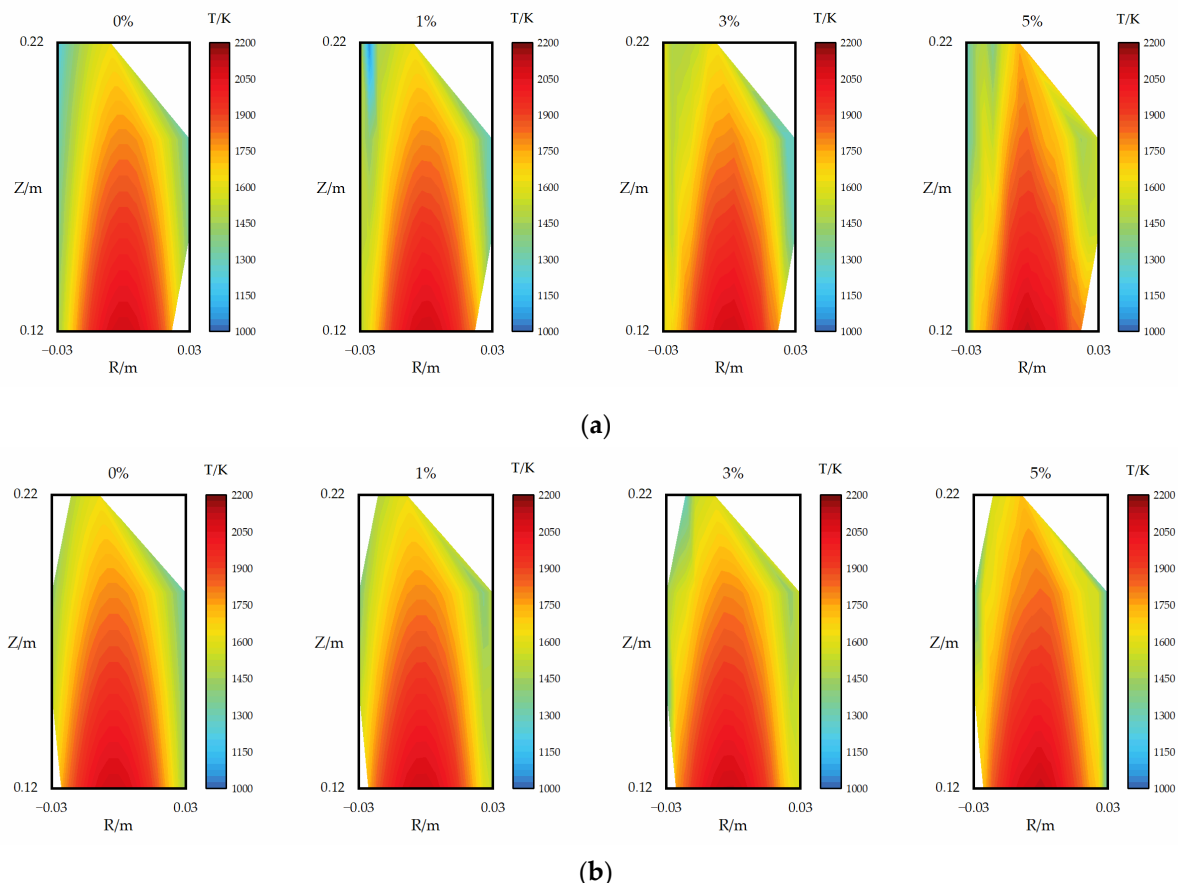


Figure 5. Case 1: Reconstructed temperature at the two cross-sections ($\delta = 0\%$, 1% , 3% , 5%). (a) 45–225° cross-section; (b) 135–315° cross-section.

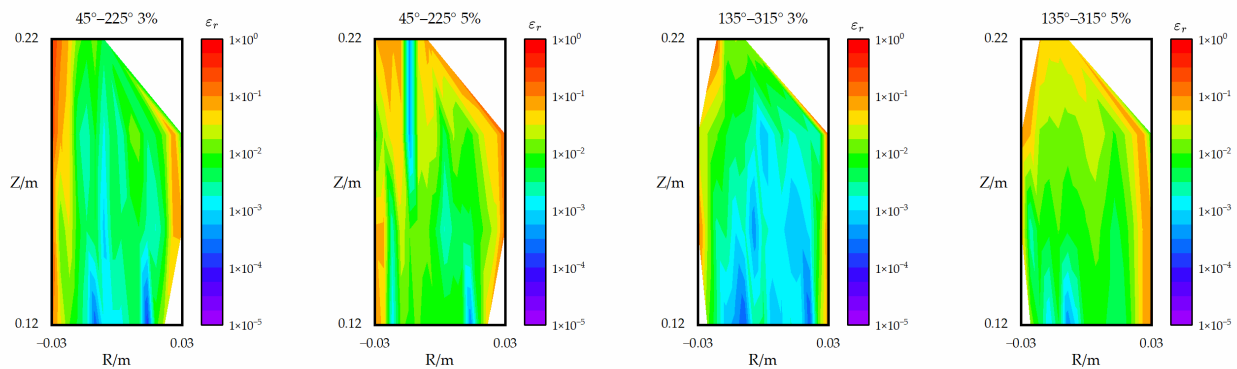


Figure 6. Case 1: Relative error at $\delta = 3\%$ and 5% for both cross-sections.

Table 2 summarizes the reconstruction error statistics for Case 1.

These results establish two important baselines. First of all, the ER measurement model combined with LSQR inversion is fundamentally reasonable: it reconstructs the correct temperature field that is close to the accuracy of the machine without noise. Secondly, the quality of reconstruction declines gracefully with the increase in measurement noise, and this decline is spatially structured—it is mainly manifested as boundary layer oscillation, and the flame core can still be recovered steadily.

Table 2. Reconstruction error statistics for Case 1.

Noise	$\epsilon_{r,\min}$	$\epsilon_{r,\max}$	$\epsilon_{r,\text{mean}}$
0%	4.49×10^{-16}	8.97×10^{-11}	3.53×10^{-12}
1%	2.16×10^{-5}	0.232	0.015
3%	1.24×10^{-4}	0.338	0.027
5%	3.62×10^{-4}	0.310	0.031

3.3. Case 2: Asymmetric Flame with Uniform Absorption Coefficient

Case 2 replaces the axisymmetric field of Case 1 with a fully three-dimensional temperature distribution, while keeping the absorption coefficient uniform:

$$T(x, y, z) = 1000 \left\{ \exp \left[-40 \left(\frac{120x+7.5}{9} - 1.1 \right)^2 - 25 \left(\frac{120y+8.5}{9} - 0.8 \right)^2 \right] + 0.8 \exp \left[-25 \left(\frac{120x+7.5}{9} - 0.8 \right)^2 - 35 \left(\frac{120y+8.5}{9} - 1.2 \right)^2 \right] \right\} + 3000(0.4 - z) + 800, \quad \kappa_a = 0.5, \quad (23)$$

The true temperature field over the measured domain is given in Figure 7. Figure 8 displays the reconstructed temperature at both cross-sections across all four noise levels, and the corresponding error distributions at $\delta = 3\%$ and $\delta = 5\%$ are provided in Figure 9.

At $\delta = 0\%$, the reconstruction accuracy was again effectively perfect ($\epsilon_{r,\text{mean}} \sim 10^{-12}$), confirming that the LSQR algorithm handles the non-axisymmetric geometry without loss of precision.

At $\delta = 1\%$ and $\delta = 3\%$, the mean relative errors were 2.3% and 7.1% respectively. The reconstructed temperature distributions in the flame core region remained in good agreement with the true field.

A clear difference is observed between the two cross-sections. The 45° – 225° cross-section exhibits more pronounced boundary-layer oscillations than the 135° – 315° cross-section. At the 45° – 225° view under 5% noise, the high-temperature core region appears slightly expanded; the 135° – 315° cross-section, by contrast, maintains good reconstruction quality across all noise levels. The two cross-sections correspond to

different temperature distributions and the reconstruction is consistently more accurate at the 135°–315° cross-section. The mean error at $\delta = 5\%$ reached 8.7%. In the error maps, the core region shows low error while elevated error is confined to the low-temperature periphery, consistent with the spatial pattern observed in Case 1: errors are concentrated at the boundary, where both the radiative intensity and the ray coverage density are lower.

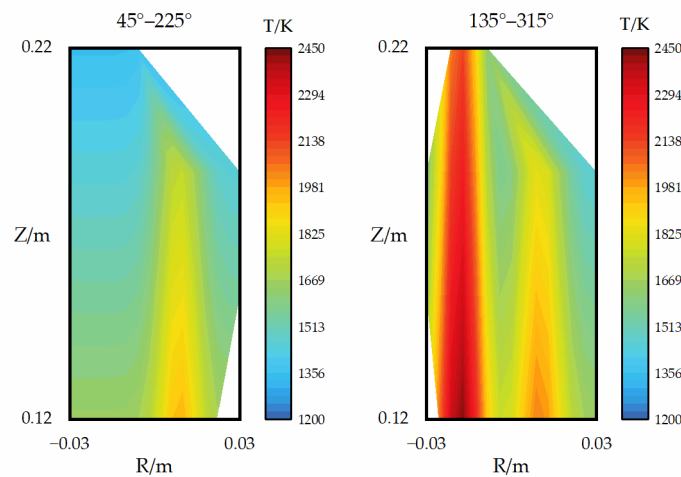


Figure 7. Case 2: True temperature distribution at the two cross-sections.

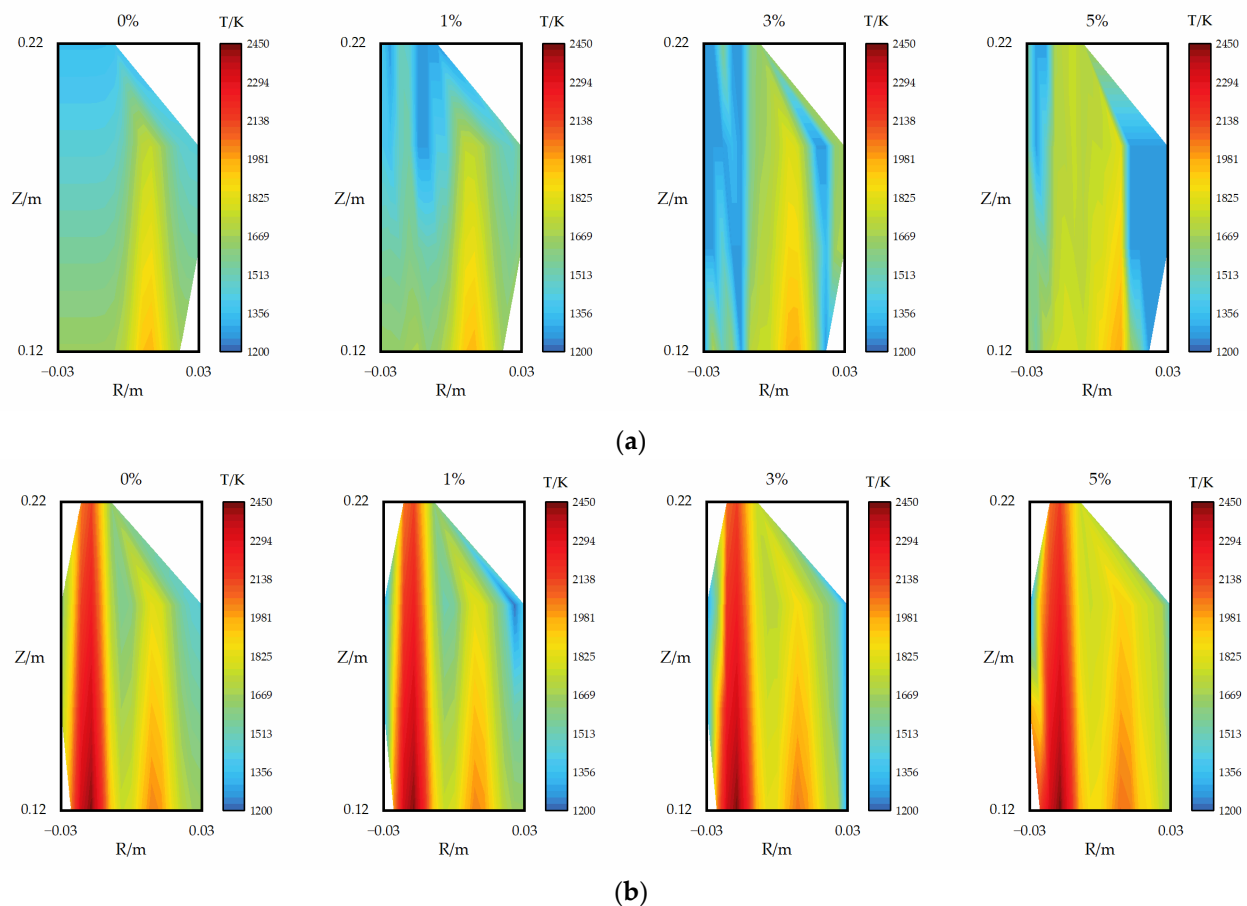


Figure 8. Case 2: Reconstructed temperature at the two cross-sections ($\delta = 0\%$, 1% , 3% , 5%). (a) 45–225° cross-section; (b) 135–315° cross-section.

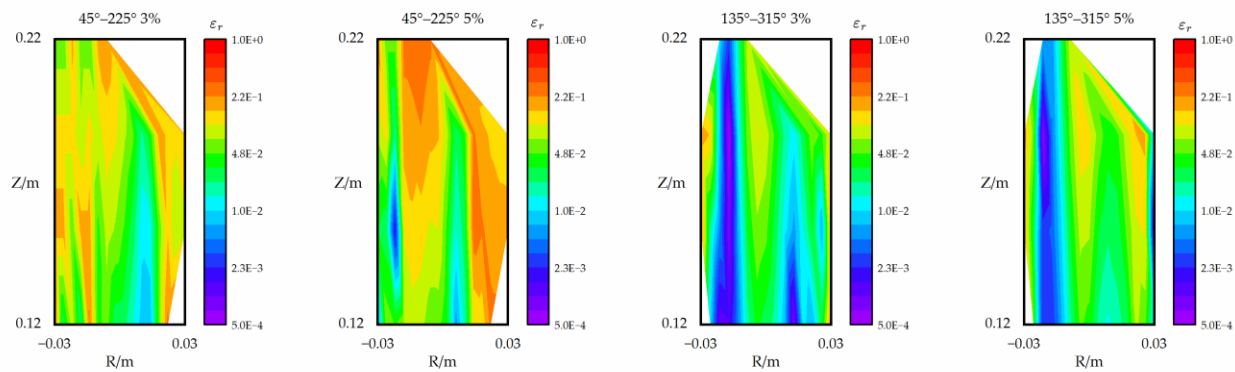


Figure 9. Case 2: Relative error at $\delta = 3\%$ and 5% for both cross-sections.

Table 3 gives the error statistics for Case 2.

Compared to Case 1, the mean errors are higher at each noise level, reflecting the increased complexity of the asymmetric temperature field. The overall reconstruction quality remains sufficient for practical diagnostics, particularly if the region of interest is the flame core.

Table 3. Reconstruction error statistics for Case 2.

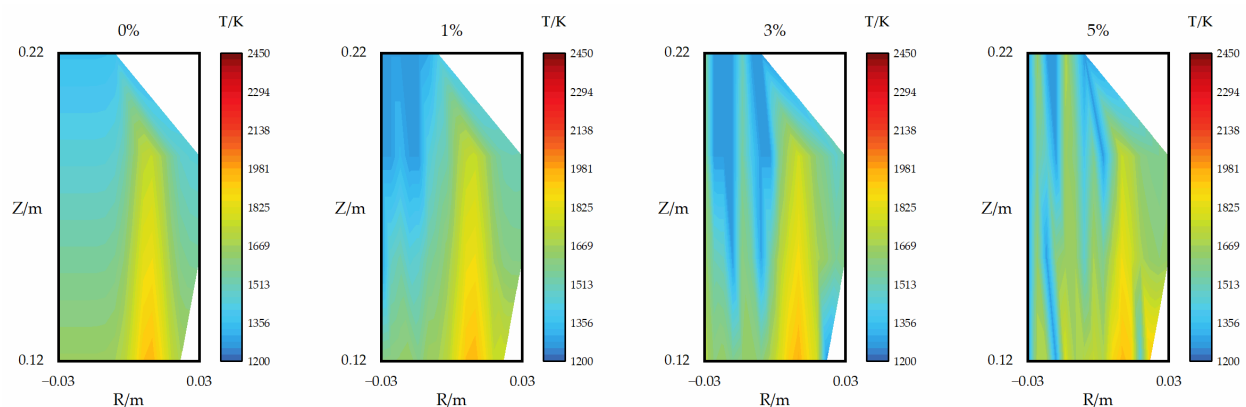
Noise	$\epsilon_{r,min}$	$\epsilon_{r,max}$	$\epsilon_{r,mean}$
0%	6.21×10^{-15}	4.69×10^{-11}	4.95×10^{-12}
1%	1.20×10^{-4}	0.24	0.023
3%	5.26×10^{-4}	0.27	0.071
5%	6.53×10^{-4}	0.29	0.087

3.4. Case 3: Asymmetric Flame with Non-Uniform Absorption Coefficient

Case 3 uses the same asymmetric temperature field as Case 2, but replaces the uniform absorption coefficient with a spatially varying one:

$$\kappa_a(z, r) = -150(0.4 - z)(0.05 - r) + 4, \quad (24)$$

Within the effective reconstruction domain, κ_a ranges from 1.30 m^{-1} to 3.87 m^{-1} , with a mean value of 2.65 m^{-1} . Figure 10 presents the reconstructed temperature field at the two cross-sections (the true temperature distribution is identical to Case 2 and is not repeated). Figure 11 shows the relative error distributions at $\delta = 3\%$ and $\delta = 5\%$ for both cross-sections.



(a)

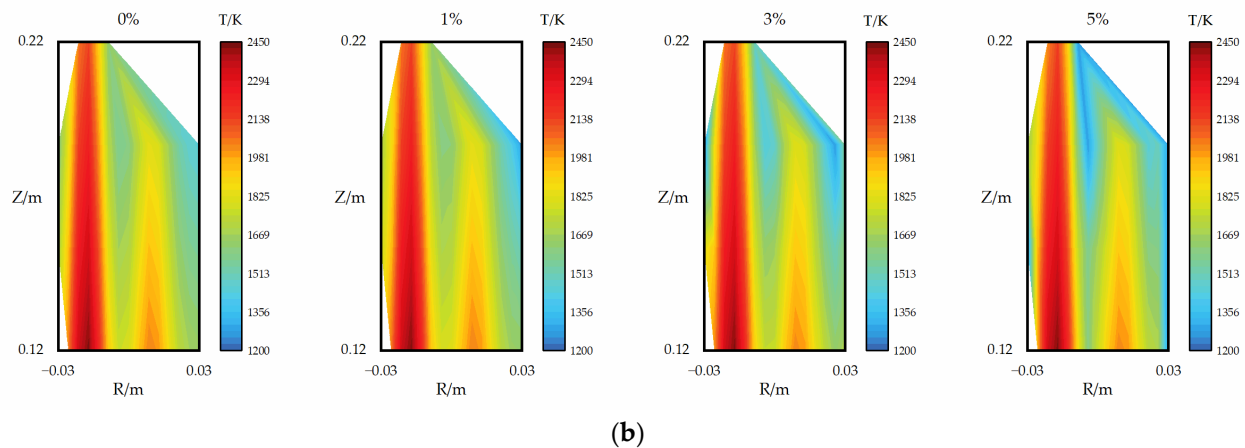


Figure 10. Case 3: Reconstructed temperature at the two cross-sections ($\delta = 0\%$, 1% , 3% , 5%). (a) $45\text{--}225^\circ$ cross-section; (b) $135\text{--}315^\circ$ cross-section.

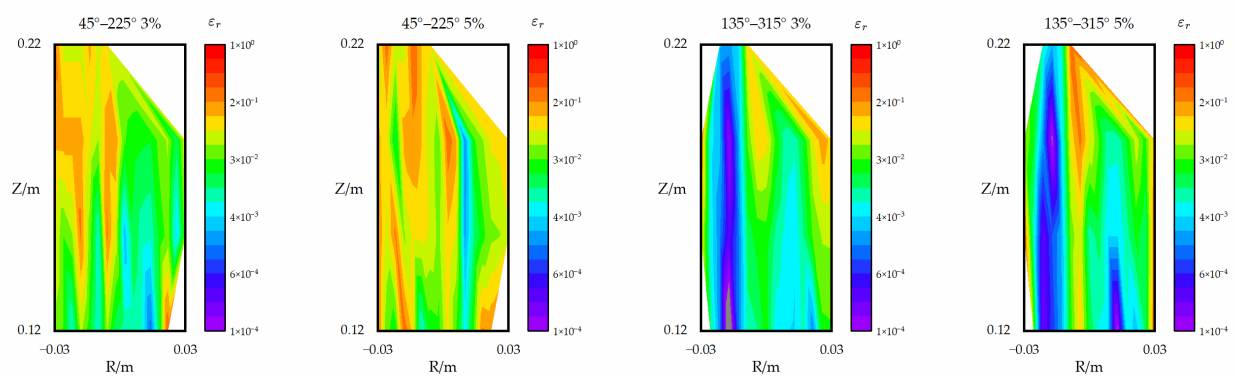


Figure 11. Case 3: Relative error at $\delta = 3\%$ and 5% for both cross-sections.

The overall reconstruction quality is comparable to that of Case 2, with the same spatial pattern: reconstruction errors are consistently lower in the flame core than at the periphery, while boundary-layer oscillations grow with increasing δ . The error distribution maps reveal two characteristic features. First, a clear boundary separates the core from the periphery in the error maps: the low-temperature outer region exhibits more pronounced oscillations. Second, as observed in Case 2, the $135\text{--}315^\circ$ cross-section yields better reconstruction than the $45\text{--}225^\circ$ cross-section. Across both cross-sections, accuracy decreases from the high-temperature center toward the low-temperature boundary.

Table 4 summarizes the error statistics for Case 3. The minimum errors at 1% and 3% noise are both on the order of 10^{-4} – 10^{-5} ; their relative ordering reflects random fluctuation in the noise sample rather than a systematic trend.

Table 4. Reconstruction error statistics for Case 3.

Noise	$\epsilon_{r,\min}$	$\epsilon_{r,\max}$	$\epsilon_{r,\text{mean}}$
0%	4.10×10^{-16}	1.36×10^{-11}	1.17×10^{-12}
1%	1.23×10^{-4}	0.176	0.022
3%	2.98×10^{-5}	0.241	0.041
5%	8.14×10^{-5}	0.270	0.061

3.5. Parameter Sensitivity Analysis

To evaluate the robustness of the reconstruction results with respect to the chosen numerical and geometric parameters, three sets of sensitivity tests were conducted. All tests used the Case 1 configuration

3.5.1. Grid Resolution

The flame domain was discretized at seven grid resolutions, with the total number of cells ranging from 48 to 3072. Table 5 summarizes the reconstruction error, condition number, and computational cost at each resolution.

Table 5. Grid sensitivity.

Total Cells	Effective Cells (Ratio)	Cond (A)	T_lsqr (s)	$\epsilon_{r,mean}$ (%)
$2 \times 4 \times 6 = 48$	16 (33%)	65	0.01	1.38
$3 \times 6 \times 9 = 162$	44 (27%)	345	0.01	3.08
$4 \times 8 \times 12 = 384$	93 (24%)	462	0.03	2.88
$5 \times 10 \times 15 = 750$	161 (21%)	769	0.12	4.30
$6 \times 12 \times 18 = 1296$	270 (21%)	772	0.86	4.92
$7 \times 14 \times 21 = 2058$	415 (20%)	2×10^{18}	1.88	6.23
$8 \times 16 \times 24 = 3072$	585 (19%)	1940	4.56	7.92

The mean error remained stable between 162 and 384 cells, measuring 3.08% and 2.88%, respectively. At 48 cells, the error dropped to 1.38%; however, only 16 effective cells were available at this resolution. A grid this coarse averages out spatial variations and cannot provide meaningful three-dimensional detail. Beyond 750 cells, the error increased steadily, reaching 7.92% at 3072 cells. In this test, the CCD pixel partition is fixed at 40×80 , so the number of detected lights remains unchanged at 2808. With the refinement of the grid, each effective unit will be sampled by fewer rays. The ratio of rays to cells decreased from about 30:1 for 384 cells to 4.8:1 for 3072 cells. The accuracy loss at fine resolutions therefore reflects insufficient ray sampling with respect to the grid.

The condition number of matrix A remains moderate in most resolutions, ranging from 65 to 1940. An exception occurs at $7 \times 14 \times 21$ (2058 cells), in which the number of conditions reaches 2×10^{18} . This anomaly reflects the local coverage defects unique to specific grid subdivisions, in which a few units are intersected by very few detection rays. At the next finer resolution, the condition number returns to 1940.

The $4 \times 8 \times 12$ grid (384 cells) adopted in this study yields a mean error of 2.88%, a condition number of 371, and an LSQR solution time of 0.03 s. These values lie at the favorable end of the trade-off among reconstruction accuracy, matrix conditioning, and computational cost across the tested range.

3.5.2. Rotation Angle ω

The reflector rotation half-angle ω was varied over five values from $\pi/6$ to $5\pi/12$. Table 6 reports the mean reconstruction error, effective cell count, and condition number at each angle.

Table 6. ω sensitivity.

ω	Rays	Effective Cells (Ratio)	Cond (A)	$\epsilon_{r,mean}$ (%)
$\pi/6$	2924	89 (23%)	420	2.99
$\pi/5$	2808	93 (24%)	462	2.84
$\pi/4$	2592	102 (27%)	685	3.26
$\pi/3$	2212	122 (32%)	2660	3.73
$5\pi/12$	1820	133 (35%)	2150	4.82

Two competing factors determine the impact of ω on the reconstruction accuracy. The larger ω expands the angular coverage of the reflector, increasing the number of effective units from 89 at $\omega = \pi/6$ to 133 at $\omega = 5\pi/12$. Larger ω will also cause more pixel positions to be mapped to the light that misses the flame, thus reducing the total number of rays from 2924 to 1820. The average error reflects the trade-off between these two effects. It drops from 2.99% at $\omega = \pi/6$ to a minimum of 2.84% at $\omega = \pi/5$, of which the combination of full effective coverage (93 cells) and high-ray count (2808) is the most favorable. At a wider angle, the error climbed to 4.82% because the loss of light exceeded the increase in coverage. The value $\omega = \pi/5$ used corresponds to the minimum value of the trade-off curve.

3.5.3. Detection Ray Count

The CCD pixel partition $n_x \times n_z$ was varied from 20×40 to 120×240 , producing detection ray counts from 700 to 25,206. Table 7 reports the reconstruction error and computational cost at each setting.

Table 7. Detection ray sensitivity.

$n_x \times n_z$	Rays	Cond (A)	T_lsqr (s)	$\epsilon_{r,mean}$ (%)
20×40	700	1893	0.01	5.34
40×80	2808	462	0.02	2.84
80×160	11,200	1342	0.38	3.11
120×240	25,206	3382	0.78	1.32

Under 700 rays, the information that can be used to constrain 93 effective cells is obviously insufficient. The average error is 5.34%, and the condition number is 1893. Increasing the ray count to 2808, reduce the average error to 2.64%, and reduce the number of conditions to 462, which is the lowest of all four configurations. The error will not decrease monotonically with the addition of further light. Under 11,202 rays, despite the increase in measurement information, the average error rises to 3.11%; the additional rays provided a large amount of redundant sampling of the flame volume and introduced noise without proportionally improving the inversion. Under 25,206 rays, the average error reached a minimum of 1.32%, but the calculation cost increased significantly. The 40×80 configuration provides the most favorable balance in four test settings: competitive accuracy, the lowest condition number and a solution time of only 0.02 s. Table 7 further supports the above explanation. It shows that at a fixed grid resolution of 384 units, increasing the number of rays from 2808 to 25,206 reduces the average error from 2.64% to 1.32%. This trend is consistent with the interpretation that insufficient ray sampling relative to grid resolution is the primary cause of accuracy loss at fine resolutions.

The 40×80 partition (2808 effective rays) provides a favorable balance. The reconstruction accuracy is competitive, the condition number is the lowest among all four tested configurations, and the computational cost remains negligible. This configuration was therefore adopted for the three test cases in Sections 3.2–3.4.

Collectively, these sensitivity tests demonstrate that the parameter values adopted in this study lie at or near the favorable points of their respective trade-off curves. The

reconstruction accuracy is robust to moderate variations in grid resolution, rotation angle, and detection ray count.

3.6. Cross-Case Comparison and Summary

To facilitate cross-case comparison of the noise-dependent reconstruction accuracy, the mean relative errors at $\delta = 1\%$, 3% , and 5% are plotted in Figure 12 as a grouped bar chart. The noise-free results are omitted from the figure for scale clarity.

Several consistent trends emerge from the three cases:

1. The proposed reconstruction method is reasonable in theory. In these three cases, the noise-free reconstruction converges close to machine accuracy, confirming that the combination of the ER-based measurement model and the LSQR solution strategy produces a solvable and accurate linear inversion framework.
2. The noise tolerance follows a predictable pattern. As shown in Figure 12, the average reconstruction error grows with the increase in the noise level. Symmetry improves tolerance; asymmetry and non-uniform absorption increase the error.
3. The error is concentrated in space and is related to temperature. In all cases, the reconstruction error is concentrated at the edge of the flame, because the temperature of the boundary area is lower (the radiation signal is weak) and there are fewer effective detection rays through the peripheral unit. The accuracy is the highest near the high-temperature core and the lowest at the low-temperature boundary.
4. Cross-section dependence under asymmetry. In Cases 2 and 3, the $135^\circ\text{--}315^\circ$ cross-section always produces more accurate reconstruction than the $45^\circ\text{--}225^\circ$ cross-section. Since these two cross-sections correspond to the inherent different regions of the asymmetric temperature field, this difference reflects a geometric factor: the spatial relationship between the detection ray path and the grid direction changes with the variation in the angular position, thus affecting the accuracy of local reconstruction.
5. Effect of non-uniform absorption. Under the condition that all other parameters remain unchanged, Case 3 at 5% noise yields a mean error of 6.1% , compared to 8.7% for Case 2. At lower noise levels, these two cases produce comparable results (2.2% vs. 2.3% at 1%). Since the only difference between Cases 2 and 3 is the spatial variation of κ_a , these results indicate that non-uniform absorption does not necessarily reduce the reconstruction quality under the proposed measurement configuration.

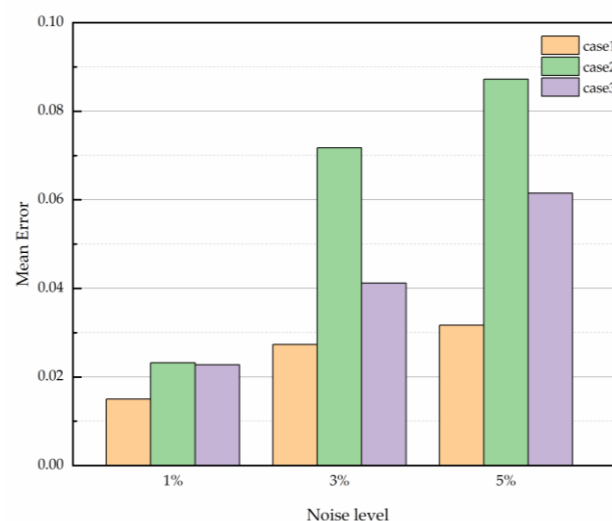


Figure 12. Comparison of mean relative error across three test cases at $\delta = 1\%$, 3% , and 5% .

4. Conclusions

This paper has presented a passive radiation thermometry method that combines an ellipsoidal reflector with CCD-based detection to reconstruct three-dimensional temperature distributions within a flame. The key contributions and findings are summarized as follows.

Theoretical framework. A complete forward model has been established for the ER measurement system. Using the optical geometry of ER, ray tracing is carried out from each detector element through the reflector to the flame volume to obtain the spatial trajectory of each effective detection light. Based on Mie theory, which demonstrates that soot particles in flames have a negligible scattering cross-section, the RTE was simplified to the pure-absorption form and discretized along each ray path. The resulting linear system $I_\lambda = A \cdot I_{b\lambda}$ relates the outgoing spectral intensity measured at the detector to the unknown blackbody intensity distribution within the flame. The inverse problem was solved using the LSQR algorithm, whose iteration count serves as an implicit regularization parameter suitable for large, sparse, and ill-conditioned least-squares systems.

Numerical validation. Three test cases were simulated under identical grid resolution and detection configuration, differing only in the temperature field symmetry and absorption coefficient distribution. Gaussian white noise at levels of 0%, 1%, 3%, and 5% was added to emulate realistic measurement conditions.

- Case 1 (symmetric flame, uniform absorption coefficient) demonstrated that the method recovers the temperature field to near machine precision in the noise-free limit ($\epsilon_{r,\text{mean}} \sim 10^{-12}$), with a mean relative error of 1.5% at 1% noise and 3.1% at 5% noise. The reconstruction quality degrades gracefully, with errors concentrated at the flame periphery.
- Case 2 (asymmetric flame, uniform absorption coefficient) showed that the method extends to three-dimensional temperature distributions without fundamental loss of accuracy, yielding mean errors of 2.3% at 1% noise and 8.7% at 5% noise. A difference between the two observed cross-sections was noted: the 135° – 315° cross-section reconstructs more accurately than the 45° – 225° cross-section, reflecting the geometric relationship between the detection ray paths and the grid orientation in the asymmetric temperature field. The flame core remained accurately recovered under all conditions.
- Case 3 (asymmetric flame, non-uniform absorption coefficient) yielded mean errors of 2.2% at 1% noise and 6.1% at 5% noise. Since all configuration parameters are identical to Case 2, the only variable being the spatial variation of κ_a , these results indicate that non-uniform absorption does not necessarily degrade reconstruction quality under the proposed measurement configuration.

A consistent spatial pattern was observed in all cases: reconstruction errors are concentrated on the periphery of the flame, where temperature and radiative intensity are lower, while the flame core, where the region of primary diagnostic interest is steadily recovered under all tested conditions. Accuracy is highest near the high-temperature core and decreases toward the low-temperature boundary.

Practical significance. The geometry based on ER solves the specific practical limitations of traditional flame temperature measurement. By taking advantage of the two-focus property of the ellipsoid, the system allows a small observation aperture at one focus to access the flame interior, reducing the optical access requirements compared to multi-camera systems. This trade-off, a reconstructed field that only covers the optically accessible sub-volume rather than the entire flame, has clear characteristics and should be evaluated according to the specific diagnostic requirements of each application.

Directions for further work. Several avenues merit further investigation.

First of all, the present study validates the ER thermometry method under a single-position configuration. Due to the limited observation direction, this configuration captures a partial temperature distribution rather than the complete three-dimensional temperature field. Future work may explore a multi-position acquisition strategy, where data from multiple ER positions are jointly inverted, to increase the effective grid coverage and further improve the reconstruction accuracy.

Secondly, in the case of high noise levels, although the boundary layer oscillation is limited to a small part of the effective domain, it may limit the applicability of the method when the peripheral temperature accuracy is crucial. Combined with explicit regularization, these edge artifacts can be suppressed.

Third, this model regards flame as a pure absorption medium. Although according to Mie theory, this is reasonable for soot particles at visible light wavelengths, extending the framework to the absorption scattering medium will expand the applicability of the method to a wider range of combustion conditions and fuel types.

Finally, experimental validation remains the essential next step. The geometric parameters a and b of the ellipsoidal reflector, which were held fixed in the present numerical study, would be determined by the specific reflector used in an experimental setup. The numerical results reported here establish the theoretical feasibility of the method under one representative ER configuration.

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