

Article

Levee Slope Reliability Based on Cross-Correlated Random Fields

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Featured Application

The proposed NID–Copula–CCRF framework provides a staged tool for screening aging levee sections, identifying dependence-sensitive or spatially continuous weak zones, and prioritizing supplementary investigation, monitoring, and maintenance under limited site data.

Abstract

Aging river levees are heterogeneous flood-defense earthworks whose reliability is affected by limited site data, interparameter dependence, and spatial variability. This study develops an integrated framework combining normal information diffusion (NID), Copula-based dependence identification, cross-correlated random fields (CCRFs), and Monte Carlo reliability analysis. Reliability was evaluated using Monte Carlo simulation based on a simplified infinite-slope benchmark model. Data were obtained from the authors' own site investigations and laboratory tests. NID yields the lowest Kolmogorov–Smirnov statistics for all documented variables and better preserves local fluctuations and lower-strength tails than four classical marginals. Gaussian Copula is preferred for the representative bivariate cases and slightly outperforms the multivariate t Copula for Hongpaihe. For strongly dependent Beiwei data, the correlated model gives a mean factor of safety of 1.235 and a failure probability of 0.153, whereas independence gives a similar mean factor of safety of 1.238 but increases failure probability to 0.200. The weakly dependent main-dike case changes negligibly, while the ϕ – γ pair dominates the trivariate response. Illustrative CCRF realizations show how pointwise dependence can propagate into spatially continuous weak zones. Failure probability is therefore more diagnostic than mean factor of safety when stochastic model form is uncertain. The main novelty lies in the consistent integration and propagation of marginal, dependence, and spatial uncertainties within a unified levee reliability framework.



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Keywords: aging levee; stochastic uncertainty; Copula; cross-correlated random field; failure probability; risk assessment

1. Introduction

River levees are essential flood-defense structures in alluvial plains. Geotechnical performance is governed by material strength and depositional heterogeneity. Construction

history, weathering, and hydraulic loading introduce further structural complexity. Levee failure is seldom an isolated event. Local instability typically triggers a progressive hazard chain. This sequence involves erosion, breaching, and catastrophic downstream flooding. Levee assessment remains fundamental to geohazard diagnosis and infrastructure resilience [1–5]. Major levee systems, such as those along the Yangtze River, represent multi-stage historical structures. Repeated heightening, emergency repairs, and long-term service have produced composite geotechnical systems rather than uniform embankments. The levee body typically comprises materials of varying ages and compaction levels. Foundations consist of complex alluvial sequences involving sands, silts, and clays. From an engineering-geological perspective, localized unfavorable combinations of material properties present the primary challenge. These heterogeneities often trigger slope instability or breach initiation under adverse loading. Limited site investigation data exacerbate these assessment difficulties. Routine levee evaluations frequently rely on deterministic factors of safety based on mean parameter values. While useful for preliminary screening, these metrics fail to capture the uncertainty structure governing engineering risk. Geotechnical reliability research demonstrates that failure probability is highly sensitive to distribution tails, inter-parameter dependence, and the spatial continuity of weak zones [6–12]. Deterministic approaches often mask critical model-form effects. These effects become visible only through explicit uncertainty treatment [13–15].

Spatial variability represents a first-order control in slope reliability. Foundational research on stochastic subsurface characterization demonstrates that correlation length, variance reduction, and weak-zone localization significantly alter failure probability, even under constant mean strength [16–18]. Subsequent studies have extended these concepts to two-dimensional fields, conditional simulations, and system-level assessments [19–28]. Levee stability is frequently governed by the spatial organization of low-strength patches rather than bulk average properties. Interdependence among soil parameters constitutes a second critical uncertainty source. Cohesion (c) and friction angle (ϕ) are inherently correlated in both natural and reworked levee soils. These parameters are conditioned by particle size distribution, density, fabric, and stress history. Unit weight (γ) often shares these geological controls. Neglecting such dependencies results in reconstructed parameter sets that are statistically inconsistent with field measurements. Copula theory addresses this limitation by decoupling marginal distributions from the dependence structure [29–35]. This framework allows site-specific dependence models to be coupled with non-normal marginals. Data scarcity represents a third primary challenge. Levee sampling is frequently constrained by accessibility, budget, and urgent inspection schedules. Classical parametric distributions often smooth over local fluctuations characteristic of small samples from heterogeneous fills. Information-diffusion approaches effectively preserve these fluctuations by diffusing observational information over finite neighborhoods. Unlike rigid single-peak assumptions, this method captures nuanced tail behavior. Refined characterization of these distribution tails significantly shifts the resulting reliability interpretation [25,26,36–43]. Table 1 summarizes the comparison and gap between this study and the existing research.

Table 1. Simplified comparison of representative studies and research gap.

Representative Study	Focus	Limitation	Gap Addressed in This Study
Griffiths et al. (2009) [8]	Spatial variability and slope reliability	Marginal and dependence treated separately	Integrated marginal–dependence–spatial framework
Mainguenaud et al. (2023) [11]	Probabilistic levee reliability	No unified uncertainty propagation	Consistent propagation to F_s and P_f

Table 1. *Cont.*

Representative Study	Focus	Limitation	Gap Addressed in This Study
He et al. (2021) [29]; Wang & Akeju (2016) [34]	Parameter dependence	No spatial propagation	Dependence-consistent CCRF modeling
Tang et al. (2017) [37]; Zhou et al. (2019, 2020) [38,39]	Small-sample & Copula methods	Not linked to full reliability workflow	Unified NID–Copula–CCRF–reliability chain

Existing levee studies frequently treat marginal distributions, random fields, and parameter dependence in isolation. Many investigations focus on individual components, such as Pearson correlation or simplified stochastic fields. Few frameworks integrate small-sample marginal fitting, Copula selection, and cross-correlated random-field (CCRF) simulation into a unified reliability analysis. This comprehensive integration remains essential for establishing statistically consistent and geologically credible slope models. This study expands upon preliminary investigations by strengthening the engineering-geological context of levee hazard evaluation. The research shifts from a purely statistical exercise toward a system-level assessment of aging alluvial structures. The methodology is restructured to emphasize the physical mechanisms driving levee instability [1,13,40,41,44].

The primary objectives are: characterizing marginal distributions of levee soil parameters under small-sample constraints using Normal Information Diffusion (NID); identifying optimal Copula functions for bivariate and trivariate datasets from the Wuhan levee system; quantifying the sensitivity of the factor of safety (F_s) and failure probability (P_f) to marginal and dependence structures; and propagating identified dependencies into CCRF for hazard-oriented levee assessment. By synthesizing these components, the paper demonstrates why refined uncertainty modeling is critical for the engineering geology of aging flood-defense systems. It should be noted that the dataset used in this study was generated through the authors' own site investigation program rather than compiled from earlier publications. The collected datasets were subsequently subjected to data processing methods. Previous studies were used only for theoretical comparison and discussion and did not contribute observations to the dataset analyzed herein. The novelty of this study lies in integrating these components into a unified uncertainty-propagation framework and systematically quantifying the effects of marginal distribution, dependence structure, and spatial variability on the factor of safety (F_s) and probability of failure (P_f). In addition, the framework is evaluated using multiple representative levee datasets and reduced-sample robustness analyses.

2. Materials and Methods

2.1. Geological Setting and Representative Datasets

Wuhan's levee systems, situated in the middle reaches of the Yangtze River, are founded upon highly heterogeneous alluvial deposits. These structures have undergone extensive historical modification through repeated heightening, localized repairs, and recurrent flood-season loading. They function as composite geotechnical systems rather than homogeneous embankments. The levee body typically comprises fill materials of varying ages and compaction levels. Foundations are characterized by complex sequences of interbedded silts, clays, and sandy alluvium. This geological complexity, compounded by prolonged service life and uneven maintenance, dictates that localized weak zones govern stability mechanisms more prominently than section-averaged parameters. Figure 1 illustrates the schematic engineering-geological context of these aging levees.

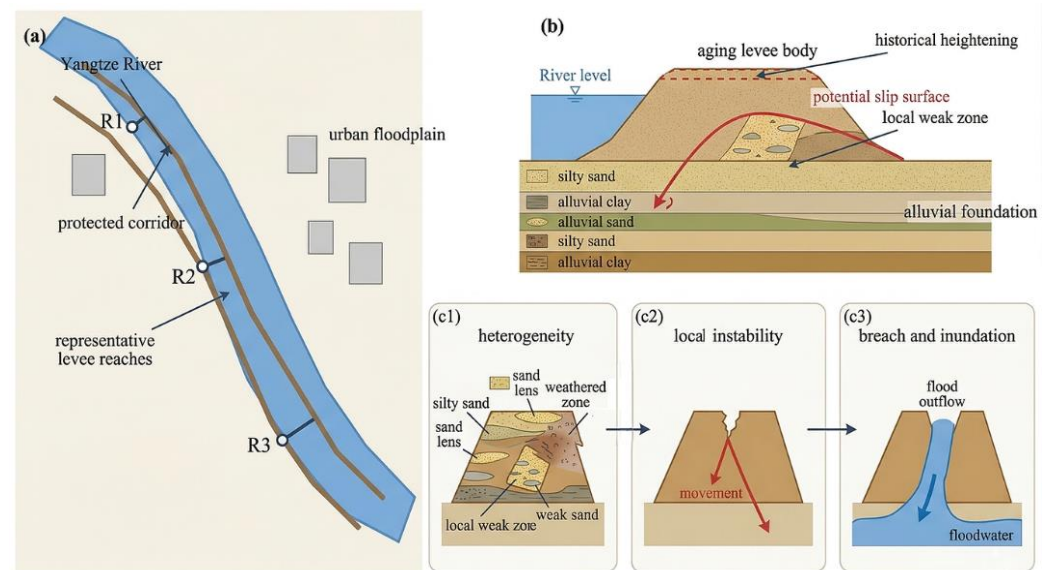


Figure 1. Engineering-geological context and progressive failure process of aging river levees: (a) schematic plan view of representative levee reaches along the Yangtze River and the adjacent protected urban floodplain; (b) typical cross-section of an aging levee showing historical heightening, the layered alluvial foundation, a local weak zone, and a potential slip surface; and (c) conceptual progression from material heterogeneity and local weak-zone formation (c1), through local slope instability (c2), to levee breach and inundation (c3).

Representative datasets originate from five Wuhan levee sections: Beiwei, main-dike, Hanjiang Right, Xiwei, and Hongpaihe. The data configurations encompass bivariate (c , ϕ) and trivariate (c , ϕ , γ) parameter sets. The Beiwei and main-dike datasets focus on cohesion (c) and friction angle (ϕ), providing a basis for evaluating the impacts of varying dependence intensities. Beiwei exhibits Pearson and Kendall coefficients of -0.3960 and -0.2549 , respectively. In contrast, the main-dike dataset shows significantly weaker correlation, with corresponding coefficients of -0.1040 and -0.0350 . The Hongpaihe dataset incorporates unit weight (γ), revealing a more complex multivariate structure: near-neutral c - ϕ correlation, moderately positive c - γ dependence, and strongly positive ϕ - γ coupling. This contrast across sites enables the derivation of generalized methodological conclusions rather than site-specific results. Sample sizes are $n = 91$ for Beiwei, $n = 68$ for the main-dike, $n = 44$ for Hanjiang Right, $n = 80$ for Xiwei, and $n = 39$ for Hongpaihe.

Table 2 summarizes these statistical signatures and their respective distributions. A clear distinction is maintained between empirical datasets and illustrative simulations. Measured statistics, dependence coefficients, and benchmark reliability indices serve as the foundational baseline for this study. By contrast, high-resolution random-field realizations are generated to visualize the methodological propagation into spatial dimensions. These simulations represent transparent illustrations of how identified dependence structures propagate under specified correlation lengths rather than calibrated site-wide field inversions. Consequently, Table 2 constitutes the primary empirical evidence, while subsequent figures clarify the transition from probability modeling to hazard-oriented reliability assessment.

Table 2. Representative levee datasets and statistical descriptors.

Case	Variables	n	Mean and COV	Dependence Signature
Beiwei levee	c - ϕ	91	$c = 17.13 \text{ kPa}$ (0.51); $\phi = 23.39^\circ$ (0.21)	Pearson = -0.3960 ; Kendall = -0.2549

Table 2. Cont.

Case	Variables	<i>n</i>	Mean and COV	Dependence Signature
Main dike	c – φ	68	$c = 24.05$ kPa (0.52); $\varphi = 12.34^\circ$ (0.43)	Pearson = -0.1040 ; Kendall = -0.0350
Hanjiang Right Levee	c – φ	44	$c = 32.15$ kPa (0.38); $\varphi = 18.25^\circ$ (0.25)	Pearson = -0.7323 ; Kendall = -0.5316
Xiwei levee	c – φ	80	$c = 12.96$ kPa (0.40); $\varphi = 18.10^\circ$ (0.24)	Pearson = -0.7615 ; Kendall = -0.5639
Hongpaihe levee	c – φ – γ	39	$c = 18.62$ kPa (0.55); $\varphi = 11.98^\circ$ (0.38); $\gamma = 18.57$ kN/m ³ (0.04)	$\rho(c, \varphi) = -0.0308$; $\rho(c, \gamma) = 0.2828$; $\rho(\varphi, \gamma) = 0.6627$

2.2. Integrated Analytical Framework

Figure 2 delineates the integrated analytical framework adopted in this study. The workflow follows a rigorous sequential progression: (i) marginal distribution identification via Normal Information Diffusion (NID); (ii) multivariate dependence characterization using Copula functions; (iii) spatial variability simulation through Cross-Correlated Random Fields (CCRF); and (iv) probabilistic slope reliability assessment. This hierarchy is functionally mandatory. Precise marginal modeling via NID is required to avoid tail distortion and preserve small-sample fluctuations. These refined marginals provide the essential basis for Copula-based dependence estimation. Only upon establishing a consistent joint probability structure can stochastic properties be propagated into the spatial domain via CCRF. The final reliability analysis translates this multi-scale stochastic model into engineering metrics, specifically the factor of safety (F_S) and failure probability (P_f) [13,33,38,39,41,44].

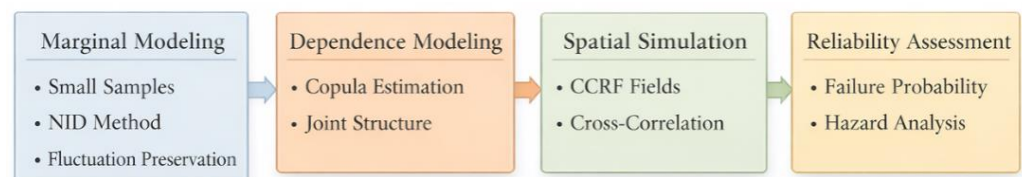


Figure 2. Integrated analytical workflow comprising four sequential stages: NID-based marginal-distribution identification, Copula-based dependence modeling, CCRF-based spatial simulation, and Monte Carlo reliability assessment using F_S and P_f .

This framework is structured to address three critical gaps in the hazard evaluation of aging alluvial levees. Firstly, it examines whether the marginal model preserves the inherent, fluctuation-rich character of limited datasets or induces non-physical oversmoothing. Secondly, it evaluates the fidelity of the dependence model in preserving the measured geometry of the c – φ – γ cloud, particularly near the failure-critical boundaries. Thirdly, it verifies the robust preservation of point-wise dependencies during their propagation into spatial random fields. These three criteria ensure that the resulting stochastic model remains geologically credible and statistically consistent. By linking localized data uncertainty to reach-scale hazard interpretation, this integrated framework provides a robust foundation for the comparative analyses and engineering conclusions presented in the subsequent sections [1,13].

2.3. Marginal Modeling via Normal Information Diffusion (NID)

Marginal distributions for levee soil parameters (c – φ – γ) are identified under small-sample constraints. In this study, four classical parametric models are adopted as reference candidates, namely truncated normal (TN), lognormal (LN), truncated Gumbel (TG),

and Weibull (WB), and their performance is compared against the Normal Information Diffusion distribution (NID). Since soil parameters are physically bounded and positive-valued, comparisons are conducted within a finite support interval rather than an unbounded domain [36–39].

In the NID framework, each observation contributes a local Gaussian diffusion kernel rather than forcing the dataset into a single global unimodal form. For an observation x_i , the diffusion kernel $\mu_i(x)$ is defined as

$$\mu_i(x) = \exp\left[-\frac{(x - x_i)^2}{2h^2}\right] \quad (1)$$

The diffusion bandwidth h is determined by the sample size n and the support interval:

$$h = \zeta_n \frac{(b - a)}{(n - 1)} \quad (2)$$

where ζ_n is the sample-size coefficient, and a and b are the lower and upper bounds of the support interval, respectively.

The resulting probability density function $f(x)$ and cumulative distribution function $F(x)$ are expressed as finite superpositions of local kernels:

$$f(x) = \frac{1}{\sqrt{2\pi}nh} \sum_{i=1}^n \exp\left[-\frac{(x - x_i)^2}{2h^2}\right] \quad (3)$$

$$F(x) = \frac{1}{n} \sum_{i=1}^n \Phi\left(\frac{(x - x_i)}{h}\right) \quad (4)$$

where Φ is the standard normal cumulative distribution function.

The PDF and CDF curve of the four distribution models are shown in Figure 3. The adoption of NID is dictated by the geological characteristics of aging levees. The interbedded alluvial layers and mixed fill provenance often produce empirical distributions with local fluctuations or partial multimodality. Classical one-peak models tend to over-smooth these features, potentially masking critical information in the tail regions. NID preserves these fluctuations, providing a more faithful representation of edge behavior in small datasets [36,38,39].

Model adequacy is quantified via the Kolmogorov–Smirnov (K-S) statistic, which measures the maximum absolute distance between empirical and theoretical cumulative distributions. Beyond statistical fit, the engineering relevance of the marginal model depends on its performance near the low-strength tails. Accurate representation of these lower bounds is essential for geologically credible slope reliability analysis. Figure 4 compares the PDF and CDF fits of the two shear-strength parameters from the Beiwei levee. The empirical distributions show clear local fluctuations, whereas the classical models produce smooth unimodal curves that fail to capture this variability. By contrast, the NID model follows the sample fluctuations more closely in both the PDF and CDF, and thus preserves the key distributional features more effectively [45].

In this paper, the K-S statistic is treated as the first error metric in the model-comparison framework, and the reported values are later compiled in Table 3 and Figure 5. A lower K-S statistic indicates closer agreement between empirical and theoretical cumulative distributions, but its engineering value lies especially in whether the lower-strength tail is represented adequately. This is why the marginal-model comparison is not treated as a purely formal fitting exercise [38,39,45].

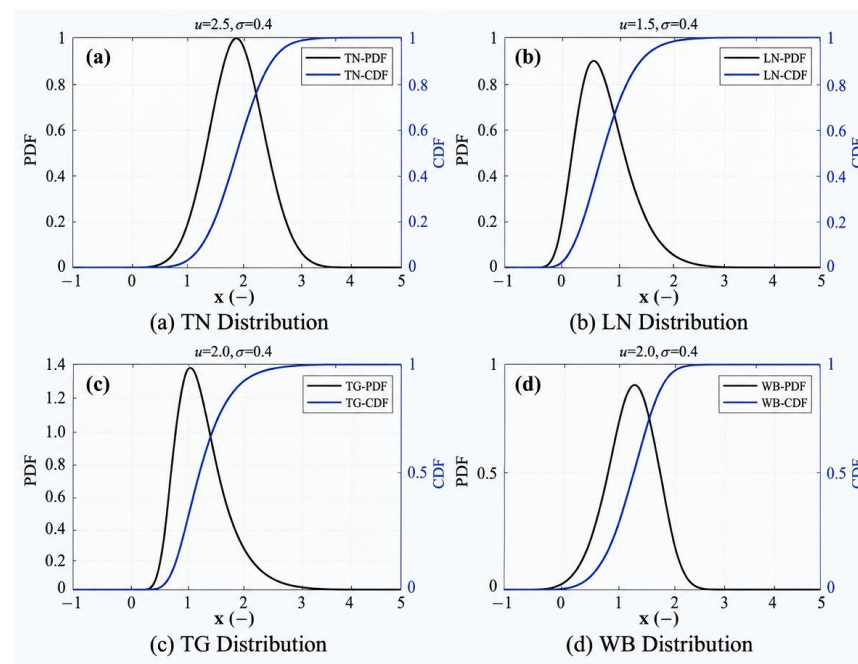


Figure 3. Illustrative probability density functions (PDFs, black curves) and cumulative distribution functions (CDFs, blue curves) of four candidate dimensionless marginal distributions: (a) truncated normal (TN), (b) lognormal (LN), (c) truncated Gumbel (TG), and (d) Weibull (WB). The generic variable x is dimensionless, and the illustrative parameter values are provided in each panel.

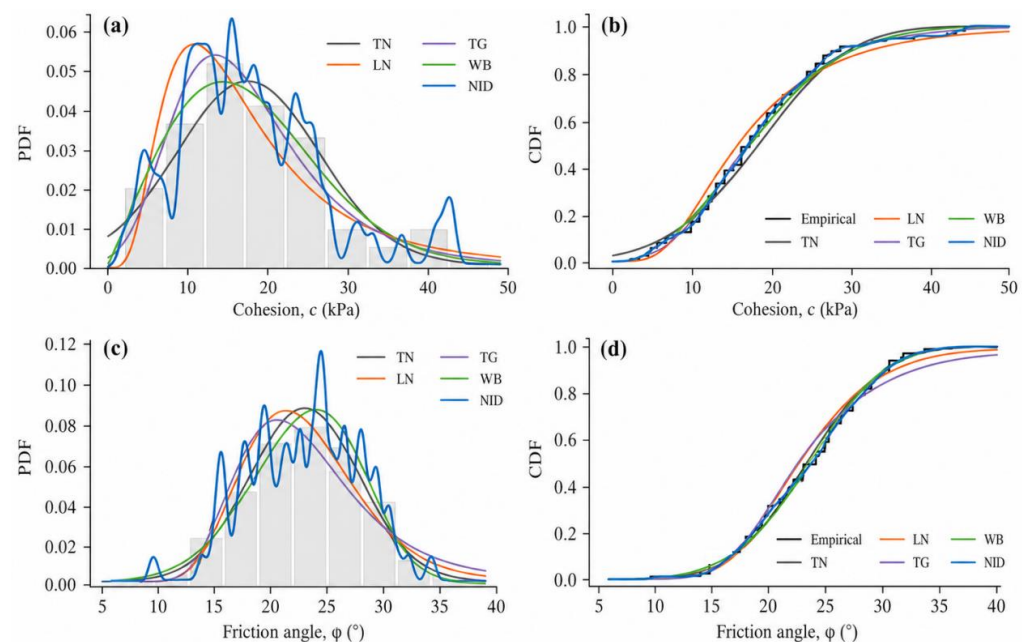


Figure 4. Marginal-distribution fitting for the Beiwei dataset: (a) PDFs of cohesion c ; (b) empirical and fitted CDFs of c ; (c) PDFs of friction angle φ ; and (d) empirical and fitted CDFs of φ . The candidate models are TN, LN, TG, WB, and NID.

Table 3. Reported K–S statistics of candidate marginal models.

Variable	TN	LN	TG	WB	NID	Critical Value
Beiwei c	0.0875	0.0735	0.0525	0.0596	0.0136	0.1404
Beiwei φ	0.0590	0.1006	0.1294	0.0577	0.0140	0.1404

Table 3. Cont.

Variable	TN	LN	TG	WB	NID	Critical Value
Main dike c	0.1313	0.0852	0.0668	0.0691	0.0188	0.1620
Main dike φ	0.0977	0.0837	0.0728	0.0787	0.0209	0.1620
Hongpaihe c	0.1715	0.1036	0.0960	0.1007	0.0105	0.2127
Hongpaihe φ	0.1026	0.1189	0.1170	0.1024	0.0341	0.2127
Hongpaihe γ	0.2229	0.2061	0.1787	0.2715	0.0613	0.2127

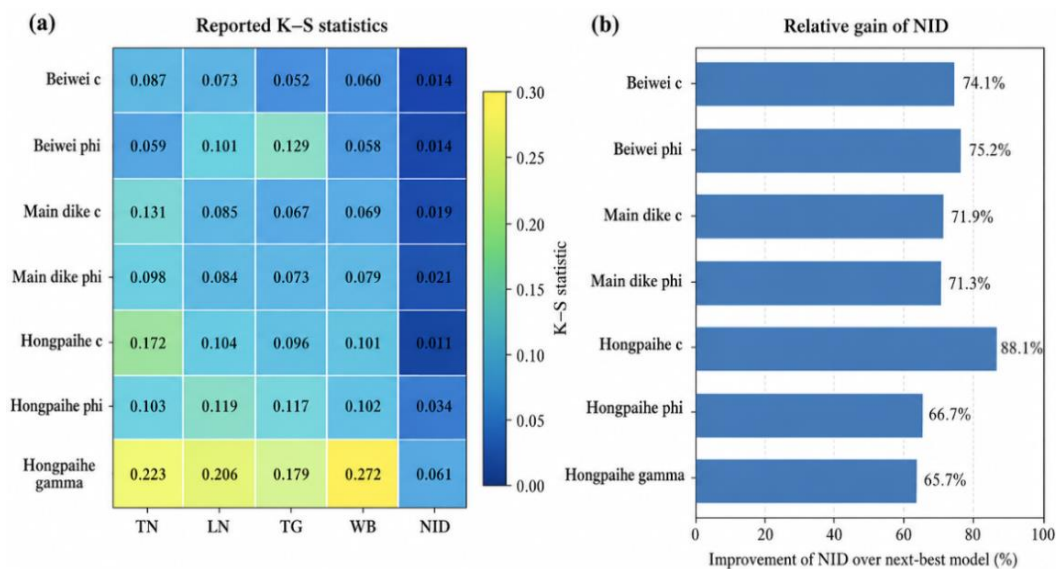


Figure 5. Comparison of marginal-model performance for the representative levee variables: (a) heat map of the reported K–S statistics for TN, LN, TG, WB, and NID, where lower values indicate better agreement with the empirical CDF; (b) percentage reduction in the K–S statistic achieved by NID relative to the next-best classical marginal model.

2.4. Multivariate Dependence Modeling via Copulas

To accurately characterize the stochastic behavior of levee soils, the dependence structure between cohesion (c), friction angle (φ), and unit weight (γ) must be rigorously modeled. The co-variation in these parameters is not random but reflects intrinsic geological controls, including grain size distribution, compaction density, and depositional history. According to Sklar's Theorem, the joint cumulative distribution function $F(X_1, \dots, X_d)$ of a multivariate vector can be uniquely decomposed into its marginal distributions $F_i(x_i)$ and a Copula function C , which governs the coupling logic [30–33,42]:

$$F(X_1, \dots, X_d) = C[F_1(x_1), \dots, F_d(x_d); \theta] \quad (5)$$

where θ denotes the Copula parameter vector identifying the strength and type of dependence. This decomposition allows the non-normal marginal behavior identified by the Normal Information Diffusion model to be retained while the inter-parameter dependence is modeled separately.

For the representative bivariate datasets, four one-parameter Copula families are considered: the Gaussian, Plackett, Frank, and Nelsen's No.16 Copulas. The designation "No.16" refers to Family No.16 in the Archimedean Copula classification presented by Nelsen (2006) [30], rather than to a software-specific model number. For $u, v \in (0, 1]$, Nelsen's No.16 Copula is defined as

$$C_{\theta}^{(16)}(u, v) = \frac{1}{2} \left[S_{\theta}(u, v) + \sqrt{S_{\theta}^2(u, v) + 4\theta} \right], \theta \in [0, \infty) \quad (6)$$

where

$$S_{\theta}(u, v) = u + v - 1 - \theta \left(\frac{1}{u} + \frac{1}{v} - 1 \right) \quad (7)$$

The four candidate Copulas were selected as a parsimonious, rather than exhaustive, comparison set. First, all four are low-dimensional models that can be estimated stably from the relatively limited site-investigation datasets considered in this study. Second, they represent distinct dependence geometries. The Gaussian Copula provides an elliptical and symmetric benchmark; the Plackett Copula provides a flexible non-elliptical but symmetric association structure; the Frank Copula provides a radially symmetric Archimedean model capable of representing both positive and negative association; and Nelsen's No.16 Copula provides an additional non-Gaussian Archimedean alternative particularly relevant to negatively associated variables. The candidate set therefore enables the measured negative dependence between c and φ to be evaluated using several structurally different joint models, rather than assuming Gaussian dependence a priori. Similar candidate sets have been used in previous geotechnical Copula studies to assess the effect of model-form uncertainty on slope reliability [29,31,33,34].

For the trivariate Hongpaihe dataset, the multivariate Gaussian Copula is compared with the multivariate t -Copula. Both models describe symmetric multivariate dependence, whereas the t -Copula additionally permits nonzero symmetric tail dependence. Their comparison therefore examines whether the observed joint behavior of c , φ , and γ requires a heavy-tailed dependence structure or can be represented adequately by a Gaussian dependence model.

Model parsimony and goodness-of-fit are balanced using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) [46,47]:

$$AIC = 2k - 2l \quad (8)$$

$$BIC = k \ln(n) - 2l \quad (9)$$

where k represents the number of fitted parameters, l is the log-likelihood, and n is the sample size. Lower AIC and BIC values indicate a more favorable balance between goodness of fit and model complexity.

To quantify sensitivity to Copula-family selection, the reliability analysis was repeated separately for each fitted bivariate Copula. In these calculations, the NID marginal distributions, infinite-slope parameters ($z = 2.0$ m and $\beta = 18.4^\circ$), and Monte Carlo sample size ($N = 100,000$) were kept unchanged. Therefore, differences in the resulting mean F_S and P_f can be attributed specifically to the selected Copula family. The Gaussian–Copula result was used as the reference, and the relative changes were calculated as

$$\Delta F_{S,i} = \frac{\overline{FS}_i - \overline{FS}_G}{\overline{FS}_G} \times 100\% \quad (10)$$

$$\Delta P_{f,i} = \frac{P_{f,i} - P_{f,G}}{P_{f,G}} \times 100\% \quad (11)$$

where the subscript i denotes the alternative Copula family and G denotes the Gaussian Copula.

The primary objective of the Copula-based approach extends beyond reproducing linear correlation coefficients; it aims to preserve the geometric configuration of unfavorable parameter combinations. This is critical for reliability analysis, as different Copulas with

comparable AIC/BIC values may yield diverging probability densities at the boundaries of the parameter space. Figure 6 summarizes the AIC/BIC-based selection of Copula models for representative bivariate and trivariate levee datasets. In the four bivariate cases, Gaussian Copula consistently yields the lowest AIC, with the strongest advantage observed for the Beiwei levee dataset, where its AIC reaches -13.87 , clearly outperforming Plackett, Frank and Nelsen's No.16 Copulas. For the Main dike dataset, the differences among Gaussian, Plackett and Frank are small because the measured c - ϕ dependence is weak, but Gaussian remains the preferred model. In the Hongpaihe trivariate case, multivariate Gaussian Copula also slightly outperforms the t Copula, with AIC values of -19.67 and -19.58 , respectively. Pairwise results further confirm Gaussian as the most stable model, especially for the sensitive ϕ - γ dependence relevant to reliability prediction. The internal consistency of these models is further examined through equivalent-sample reconstruction, which assesses whether the generated joint distributions preserve the measured cloud orientation, dispersion, and boundary occupancy [33,34,37,42]. This procedure provides a physical-consistency check rather than independent external validation.

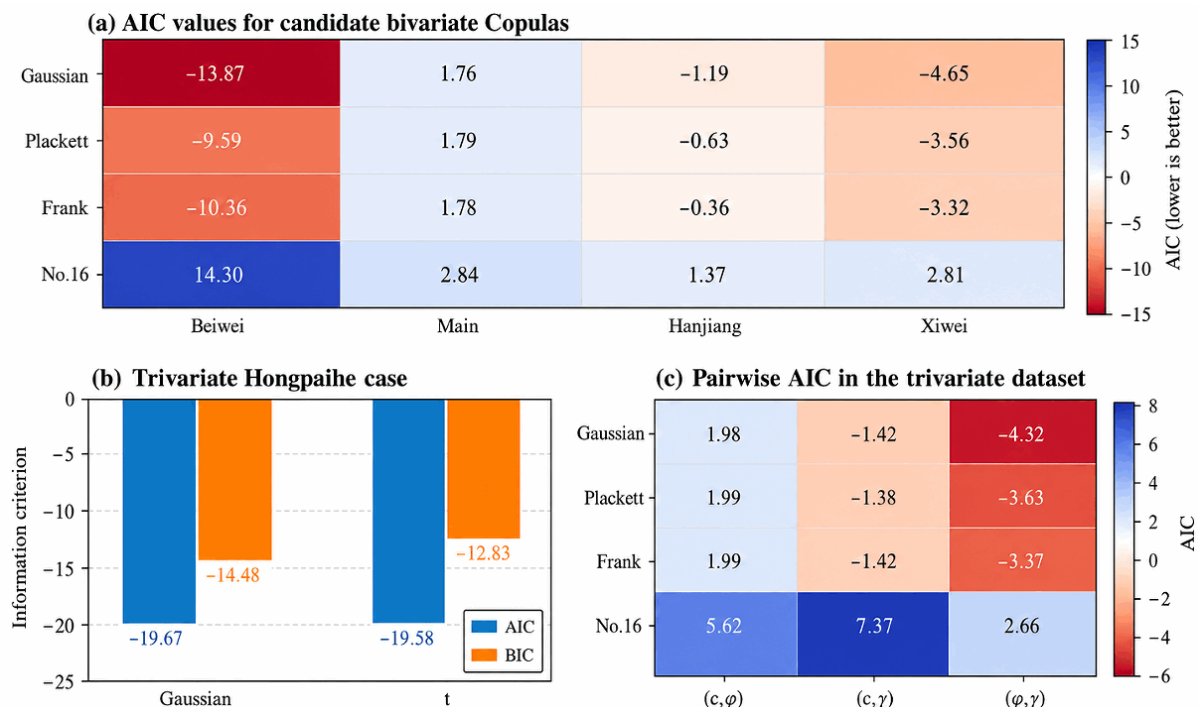


Figure 6. Information-criterion-based Copula comparison: (a) AIC values of the Gaussian, Plackett, Frank, and Nelsen's No.16 Copulas for the Beiwei, main-dike, Hanjiang Right, and Xiwei bivariate datasets; (b) AIC and BIC comparison of the multivariate Gaussian and t -Copulas for the Hongpaihe trivariate dataset; and (c) pairwise AIC values for the c - ϕ , c - γ , and ϕ - γ pairs. Lower values indicate better model performance.

2.5. Spatial Variability Characterization via Cross-Correlated Random Fields (CCRF)

To extend the point-wise Copula dependence structure into the spatial domain, the joint probability distribution is integrated into a Cross-Correlated Random Field (CCRF) framework. This extension simultaneously preserves spatial autocorrelation and inter-parameter cross-correlation, which is essential because levee stability is governed by continuous material zones rather than isolated spatial points. The spatial autocorrelation is characterized by a two-dimensional anisotropic exponential function [8,16–18,35,41,44]:

$$\rho(\Delta x, \Delta y) = \exp\left(-\frac{2|\Delta x|}{l_x} - \frac{2|\Delta y|}{l_y}\right) \quad (12)$$

where Δx and Δy are the horizontal and vertical offsets, l_x and l_y denote the horizontal and vertical correlation lengths, respectively. The cross-correlated Gaussian vector, y , is then generated via Cholesky decomposition:

$$y = L\xi \quad (13)$$

where ξ is a vector of independent standard normal variables, and L is the Cholesky factor of the correlation matrix. Finally, the Gaussian fields are mapped into the target non-Gaussian marginal space using an isoprobabilistic transformation:

$$X = F_X^{-1}[\Phi(y)] \quad (14)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $F_X^{-1}(\cdot)$ signifies the inverse cumulative distribution function of the target marginals.

The illustrative random field simulations are parameterized based on the configurations summarized in Table 4. These settings are strategically selected to reproduce a realistic negative c - φ dependence structure under moderate anisotropic smoothing, providing a representative framework for the investigated levee systems. This spatial extension is physically imperative because slope instability is governed by continuous material zones of finite extent rather than isolated mathematical points [8,35,44].

Table 4. Illustrative CCRF scenarios with prescribed correlation lengths.

Scenario	Purpose	Mean c (kPa)	SD c (kPa)	Mean φ (°)	SD φ (°)	ρ (c, φ)	l_x (m)	l_y (m)
A	Optimal correlated field	17.13	8.71	23.39	4.96	−0.396	18	5
B	Independence field	17.13	8.71	23.39	4.96	0.000	18	5
C	Weakly correlated field	24.05	12.47	12.34	5.31	−0.104	18	5

The horizontal and vertical correlation lengths were prescribed as $l_x = 18$ m and $l_y = 5$ m, respectively, corresponding to an anisotropy ratio of 3.6. These values conceptually represent the stronger horizontal continuity and weaker vertical continuity of layered levee fills and alluvial deposits. The same correlation lengths were retained in all scenarios to isolate the effect of c - φ cross-correlation. They are illustrative parameters rather than values calibrated from dense site-specific observations.

Neglecting either spatial continuity or parameter cross-correlation yields a geologically incomplete representation, compromising the validity of subsequent reliability analyses. Rather than serving as a fully calibrated site-wide inversion, these CCRF realizations function as a transparent mathematical vehicle to propagate point-wise Copula dependence into space, ensuring that marginal fluctuation, multivariate coupling, and spatial variability are seamlessly preserved to maintain geological credibility [27,41,44].

2.6. Hazard-Oriented Reliability Assessment

The ultimate engineering objective of the proposed stochastic framework is a hazard-oriented reliability assessment. Shifting the focus from central deterministic safety-factor estimates, this approach isolates how unfavorable parameter combinations drive the levee slope into a failed state. The performance function Z is defined as [6,9–11,48]:

$$Z = F_S - 1 \quad (15)$$

where F_S is the factor of safety. To isolate the effects of the stochastic structure from complex hydraulic configurations, a simplified infinite-slope benchmark is adopted:

$$F_S = \frac{c + \gamma z \cos^2 \beta \tan \varphi}{\gamma z \sin \beta \cos \beta} \quad (16)$$

where z represents the slip-surface depth and β denotes the slope angle.

In the Monte Carlo benchmark, the slip-surface depth and slope angle were fixed at $z = 2$ m, and $\beta = 18.4^\circ$, respectively. These values were selected to provide a consistent analytical benchmark, while the soil-strength parameters were treated probabilistically. The failure probability (P_f) is subsequently evaluated via Monte Carlo simulation:

$$P_f = \frac{N_f}{N} \quad (17)$$

where N_f is the number of failed realizations and N is the total number of simulations.

In this study, $N = 100,000$ Monte Carlo realizations were performed for each reliability scenario.

The benchmark formulation is deliberately selected to isolate and quantify how marginal misspecification, dependence misspecification, and the loss of spatial structure modify the reliability response, rather than to substitute for a fully coupled seepage-mechanical analysis. Consequently, the interpretation privileges P_f over the mean F_S to facilitate robust hazard diagnosis. The benchmark reliability results establishing these relationships are summarized in Table 5. CCRF should be introduced when spatially continuous weak zones and parameter co-occurrence jointly control failure, and F_S should not be used as a stand-alone index because it may remain nearly unchanged while P_f responds strongly to stochastic model form [6,9,11].

Table 5. Reliability response to stochastic modeling assumptions.

Case/Modeling Factor	Assumption or Model	Mean F_S	P_f	Interpretation
Beiwei levee	Correlated model	1.235	0.153	Dependence lowers failure risk
Beiwei levee	Independence assumption	1.238	0.200	Ignoring dependence increases P_f
Main dike	Correlated model	0.901	0.653	Weak dependence has minor effect
Main dike	Independence assumption	0.898	0.654	Results are nearly unchanged
Hongpaihe levee	Pairwise dependence	—	—	Importance: $\varphi-\gamma > c-\gamma > c-\varphi$
Marginal model	NID vs. classical marginals	Weak–moderate	Strong	Marginal tails mainly affect P_f
Copula type	Alternative Copulas	Weak	Moderate–strong	Joint-cloud geometry controls risk
Cross-correlation	Ignored or preserved	Moderate	Very strong	Cross-correlation should be retained

2.7. Comparative Performance and Sensitivity Metrics

To rigorously evaluate the alternative stochastic models, this study employs a three-tiered assessment framework comprising error measurement, sensitivity analysis, and robustness testing. Marginal-model discrepancy was quantified using the Kolmogorov–Smirnov statistic, whereas Copula model adequacy was assessed using AIC and BIC. These are deterministic goodness-of-fit and model-selection statistics and therefore are not presented with error bars.

2.7.1. Error Measurement

Model fidelity is quantified at multiple conceptual levels. Marginal distribution adequacy is assessed via the Kolmogorov–Smirnov (K-S) statistic, while dependence structures are ranked using AIC and BIC. Beyond scalar statistical metrics, equivalent-sample recon-

struction is utilized to visually verify that the empirical geometry and cloud orientation of the joint parameter space are strictly preserved [37,45–47].

2.7.2. Sensitivity Analysis

This analytical layer isolates how the intensity and configuration of parameter dependence modulate the hazard interpretation. Particular emphasis is placed on the friction angle (ϕ), whose influence is non-linearly amplified by the $\tan\phi$ resistance term in Equation (12). The practical impact of correlation strength is evaluated by contrasting distinct c - ϕ dependence signatures (e.g., strongly negative at Beiwei versus weakly negative at the main dike) and by ranking pairwise contributions in the Hongpaihe trivariate case (Figure 6). Results demonstrate that the ϕ - γ pair dominates the reliability response, followed by c - γ , whereas c - ϕ exerts the least influence. Ultimately, this analysis illustrates how different correlation strengths involving ϕ physically rotate the joint probability cloud relative to the failure domain [29,31,33,34,42].

2.7.3. Robustness Testing and Internal Consistency Assessment

To assess the stability of the stochastic models under limited data, progressive subsampling is used to examine the sensitivity of marginal fitting and Copula ranking to sample reduction. Monte Carlo convergence is also checked to ensure numerical stability of the reliability indices. These procedures evaluate robustness and internal consistency, but they do not constitute independent external validation because the same site datasets are used for model identification and subsequent assessment. Accordingly, the results should be interpreted as internally verified comparative outcomes rather than externally validated field predictions. The multi-criteria framework is shown in Figure 7 [37,42,43].

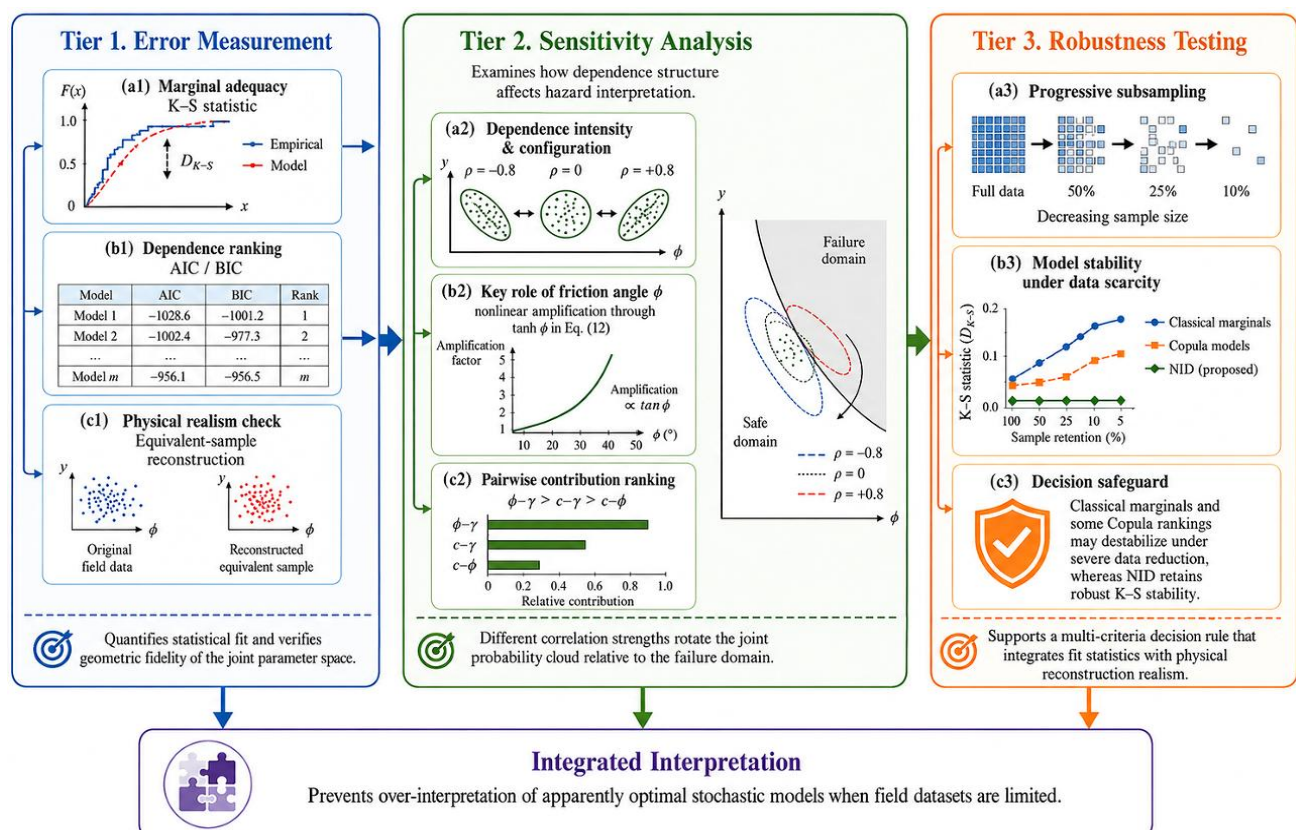


Figure 7. Multi-criteria framework for stochastic-model assessment: Tier 1 evaluates marginal and dependence-model errors; Tier 2 examines dependence sensitivity and parameter influence; and Tier 3 assesses robustness under progressive sample reduction.

Small sample sizes influence the framework at three levels. First, empirical tails, local fluctuations, and possible multimodality become more sensitive to individual observations, increasing marginal-model uncertainty. Second, dependence coefficients and AIC/BIC-based Copula rankings may become unstable because a small number of paired observations can noticeably alter the geometry of the joint sample cloud. Third, these input uncertainties may be amplified in P_f , which depends on relatively sparse probability mass near the failure boundary, whereas the mean F_S is generally more strongly controlled by the central part of the distribution.

3. Results

3.1. Section-Specific Uncertainty Signatures

The representative Wuhan levee datasets show that stochastic uncertainty is strongly section-specific rather than uniform along the levee system. The Beiwei dataset contains 91 paired observations and displays a clear negative c - ϕ dependence, whereas the main-dike dataset contains 68 paired observations and shows only weak negative dependence. The Hongpaihe dataset contains 39 trivariate observations and adds unit weight γ , which exposes a more complex dependence pattern involving c , ϕ and γ . These statistical contrasts are summarized in Figure 8 and constitute the empirical basis for the subsequent marginal, Copula and CCRF analyses.

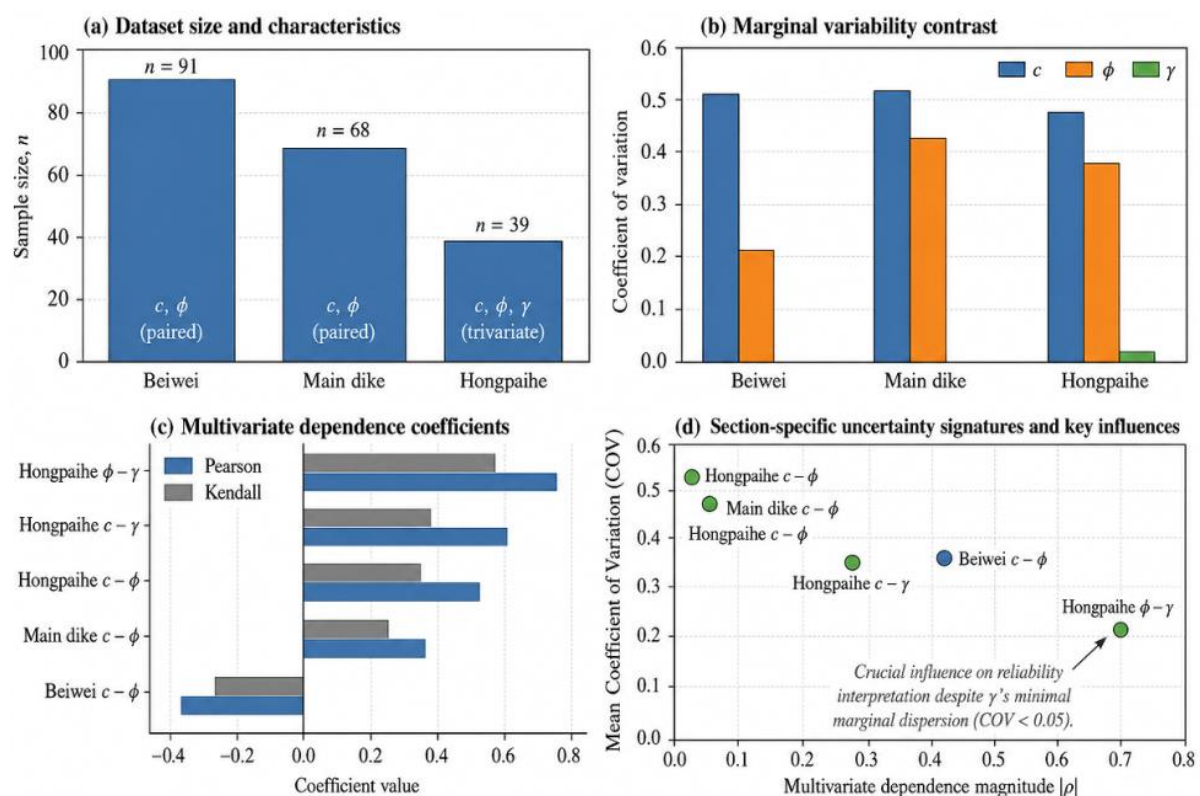


Figure 8. Statistical characteristics of the representative levee datasets: (a) sample sizes and variable configurations; (b) coefficients of variation in c , ϕ , and γ ; (c) measured Pearson and Kendall dependence coefficients; and (d) summary of section-specific marginal variability and dependence magnitude.

A key point emerging from Figure 8 is that parameter importance cannot be judged from marginal variability alone. For example, γ in the Hongpaihe dataset has a much smaller coefficient of variation than c and ϕ , but its strong positive dependence with ϕ later proves to be important for reliability interpretation. This means that a parameter

with limited dispersion can still influence failure probability when it co-varies with a variable that appears directly in the resistance term of the limit-state equation. Therefore, the stochastic model must preserve both the marginal fluctuation and the multivariate geometry of the measured data cloud.

3.2. Marginal Fitting and Small-Sample Robustness

The K-S statistics compiled in Table 3 and visualized in Figure 5, together with the redrawn fitting curves in Figures 4 and 9, demonstrate that NID provides the most stable and robust marginal representation for all fully documented variables. In the Beiwei section, NID yields K-S statistics of 0.0136 for cohesion and 0.0140 for friction angle, both substantially lower than those of the four classical distributions. The same pattern is observed in the main-dike section, where NID gives 0.0188 for c and 0.0209 for ϕ , and it is further reinforced by the Hongpaihe trivariate dataset, in which NID again produces the lowest reported K-S statistics for c , ϕ , and γ , remaining admissible even for unit weight where some classical models become unreliable [38,39].

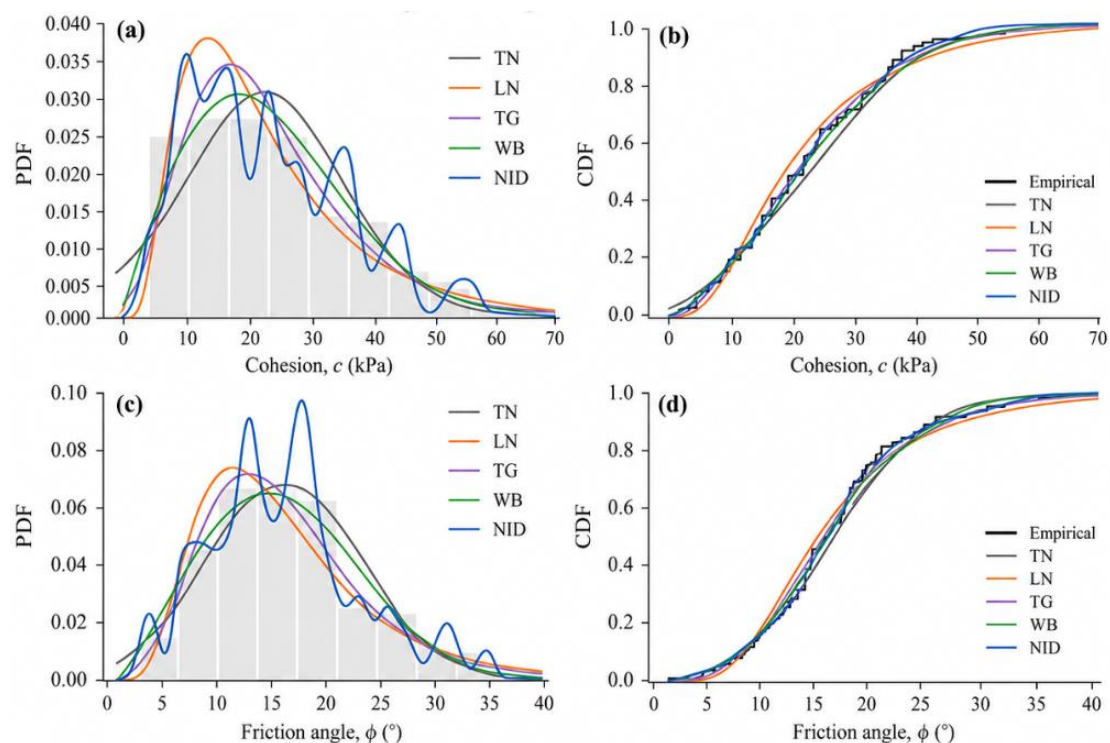


Figure 9. Marginal-distribution fitting for the main-dike dataset: (a) PDFs of cohesion c ; (b) empirical and fitted CDFs of c ; (c) PDFs of friction angle ϕ ; and (d) empirical and fitted CDFs of ϕ . The fitted models are TN, LN, TG, WB, and NID.

The graphical comparisons explain the engineering significance of these numerical differences: measured levee datasets commonly exhibit local fluctuation, partial clustering, edge observations, and occasional multimodal tendencies, reflecting mixed fill provenance, layered alluvial deposition, and non-uniform repair histories. While classical unimodal marginals may still satisfy a formal K-S test, they tend to smooth the low-strength tails that govern the entry of probability mass into the failure domain. By contrast, the NID curves follow these local and tail features more closely, providing not merely a statistically superior fit but also a more defensible stochastic basis for hazard-oriented reliability analysis. Figure 9 extends this visual evidence to the main-dike dataset and complements the Beiwei comparison presented in Figure 4. The sample sizes of the investigated datasets range from 39 to 91, and the resulting model uncertainty is therefore section-dependent.

The reduced-sample results indicate that classical parametric distributions are increasingly influenced by individual observations and may oversmooth local or lower-tail features as the sample size decreases. NID remains comparatively stable in terms of the K–S statistic; however, this relative robustness should not be interpreted as eliminating the epistemic uncertainty associated with limited data. In particular, tail-related reliability estimates should be interpreted more cautiously for the smaller datasets.

3.3. Copula Selection and Equivalent-Sample Reconstruction

The AIC/BIC-based Copula comparison in Figure 6 indicates that Gaussian Copula is the most stable dependence model for the representative bivariate datasets. The advantage is strongest for the Beiwei levee, where Gaussian Copula gives an AIC of -13.8680 , clearly outperforming Plackett, Frank and No.16 Copulas. In the main-dike dataset, the differences among Gaussian, Plackett and Frank are small because the measured c – ϕ dependence is weak. Nevertheless, Gaussian Copula remains the preferred model according to the information criteria [29,33,34].

The reliability calculations further demonstrate the sensitivity to dependence-model selection. For the Beiwei dataset, the fitted dependence models produced mean F_S values ranging approximately from 1.19 to 1.24 and P_f values ranging from 0.153 to 0.225. Relative to the Gaussian Copula, the maximum absolute change was approximately 4.0% for mean F_S and 47.1% for P_f . For the weakly dependent main-dike dataset, the corresponding variations were only approximately 0.3% and 0.2%, respectively. These results indicate that dependence-model selection has a limited effect on the central reliability index but a more evident effect on the failure-tail probability, particularly when the measured dependence is strong.

Equivalent-sample reconstruction clarifies why Copula selection has engineering significance. In the Beiwei section, the Gaussian–Copula reconstruction preserves the negative orientation of the measured c – ϕ cloud, whereas the independence reconstruction distributes samples more diffusely and loses the measured co-variation pattern (Figure 10). In the main-dike section, the difference between correlated and independent reconstruction is less pronounced because the original dependence is weak (Figure 11). This contrast shows that dependence modeling should be section-specific: it is essential where measured dependence is strong, but it may have limited benefit for preliminary screening where dependence is weak [33,34].

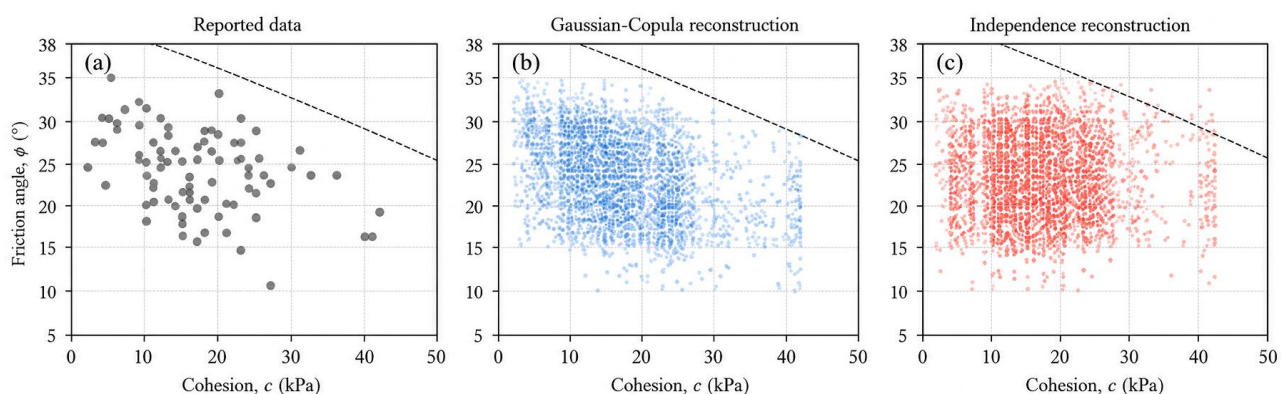


Figure 10. Equivalent-sample reconstruction for the Beiwei dataset: (a) measured c – ϕ observations; (b) samples reconstructed using the fitted Gaussian Copula; and (c) samples generated under the independence assumption. The dashed line denotes the benchmark failure boundary.

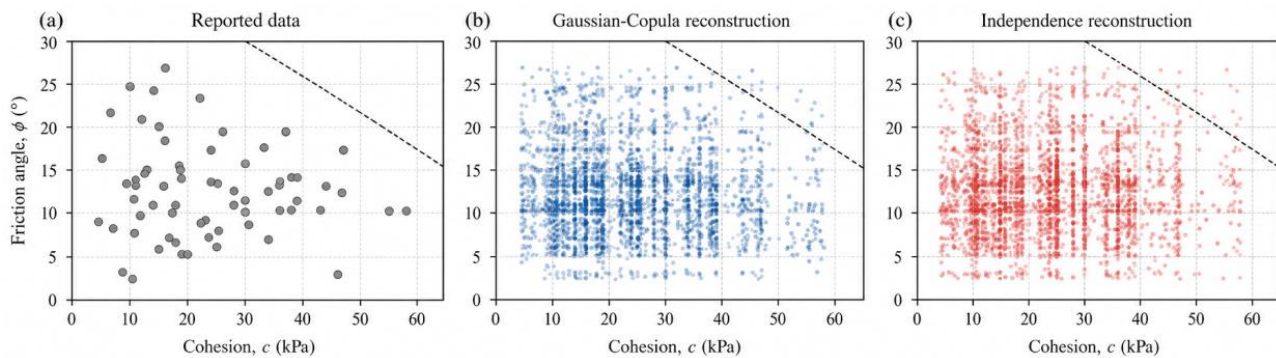


Figure 11. Equivalent-sample reconstruction for the main-dike dataset: (a) measured c – ϕ observations; (b) Gaussian–Copula reconstruction; and (c) independence reconstruction. The dashed line denotes the benchmark failure boundary.

3.4. Reliability Response to Stochastic Assumptions

The benchmark reliability results in Table 5 show that failure probability responds much more strongly than mean FS to stochastic model assumptions. In the Beiwei section, the correlated model gives a mean FS of 1.235 and a failure probability of 0.153, whereas the independence assumption gives a similar mean FS of 1.238 but increases the failure probability to 0.200. The factor of safety therefore suggests almost no difference, while the failure probability indicates a marked change in risk. This is the central reliability result of the bivariate analysis [6,9,10].

The main-dike section behaves differently. The correlated model gives $F_S = 0.901$ and $P_f = 0.653$, whereas the independent model gives $F_S = 0.898$ and $P_f = 0.654$. The negligible difference is consistent with the weak measured c – ϕ dependence in this section. Figure 12 synthesizes the contrast between Beiwei and the main dike. It shows that dependence modeling affects hazard interpretation only when the measured dependence is strong enough to rotate or concentrate the probability cloud relative to the failure domain [29,34].

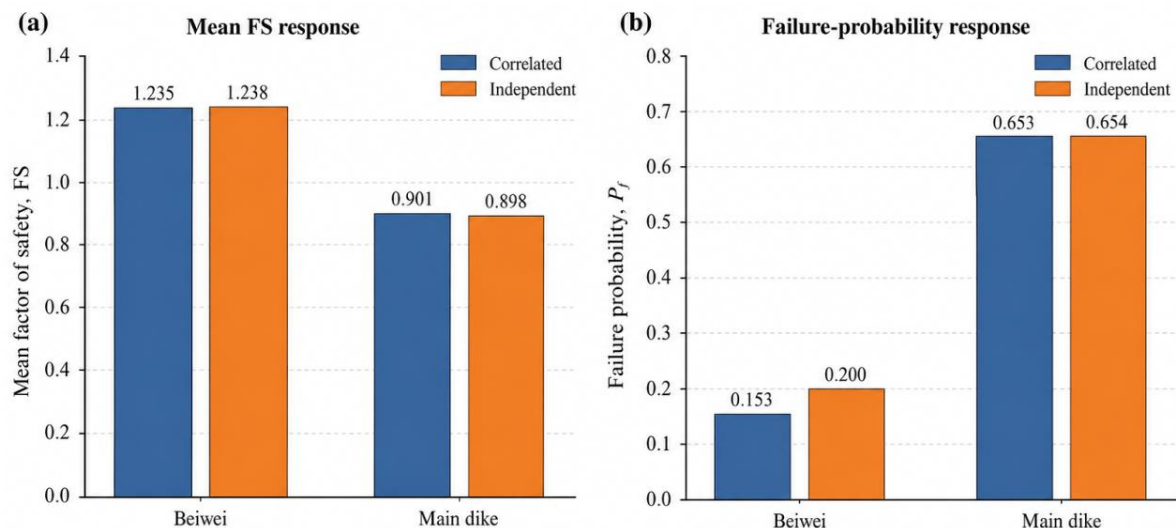


Figure 12. Reliability responses of the correlated and independent bivariate models for the Beiwei and main-dike datasets: (a) mean factor of safety and (b) failure probability. All results were obtained using 100,000 Monte Carlo realizations.

3.5. Trivariate Dependence Effects in the Hongpaihe Case

The Hongpaihe dataset provides a trivariate test of the proposed framework. The reported Pearson coefficients are -0.0308 for c – ϕ , 0.2828 for c – γ , and 0.6627 for ϕ – γ . Thus,

c – ϕ is nearly neutral, c – γ is moderately positive, and ϕ – γ is strongly positive. The trivariate Gaussian Copula slightly outperforms the multivariate t Copula according to AIC and BIC, indicating that a symmetric dependence model is sufficient for this dataset even though pairwise dependence is not uniform [30,31,44].

The pairwise structure shown in Figures 13 and 14 has two implications. Firstly, the most influential dependence pair is not the one most commonly emphasized in conventional slope analysis. The ϕ – γ pair dominates the reliability response, followed by c – γ , whereas c – ϕ has the weakest effect. Secondly, γ cannot be treated as negligible simply because its COV is small. Its strong coupling with ϕ means that it can still affect the distribution of resistance through the geometry of the joint parameter cloud. This finding supports the use of trivariate dependence analysis when unit weight shares geological controls with strength parameters [29,42].

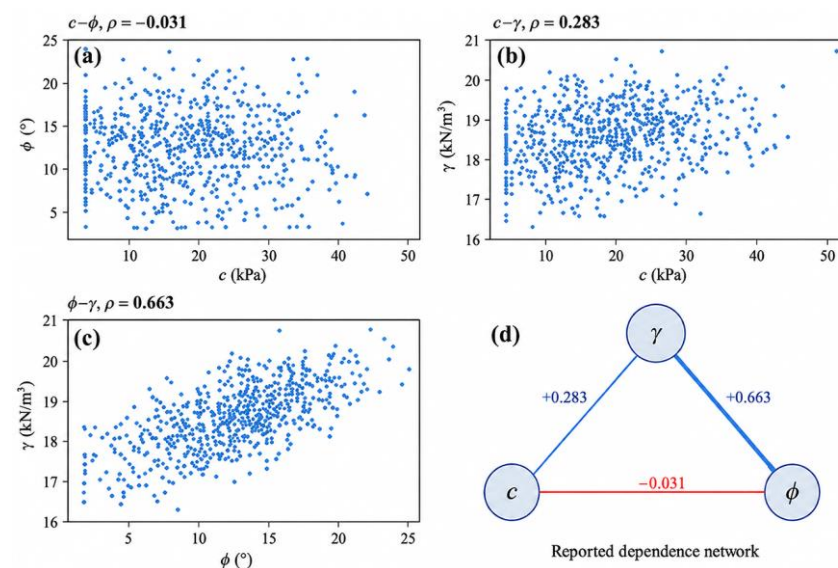


Figure 13. Trivariate dependence structure of the Hongpaihe dataset: scatter plots of (a) c – ϕ , (b) c – γ , and (c) ϕ – γ , together with (d) the corresponding dependence network. The reported values are the measured Pearson coefficients.

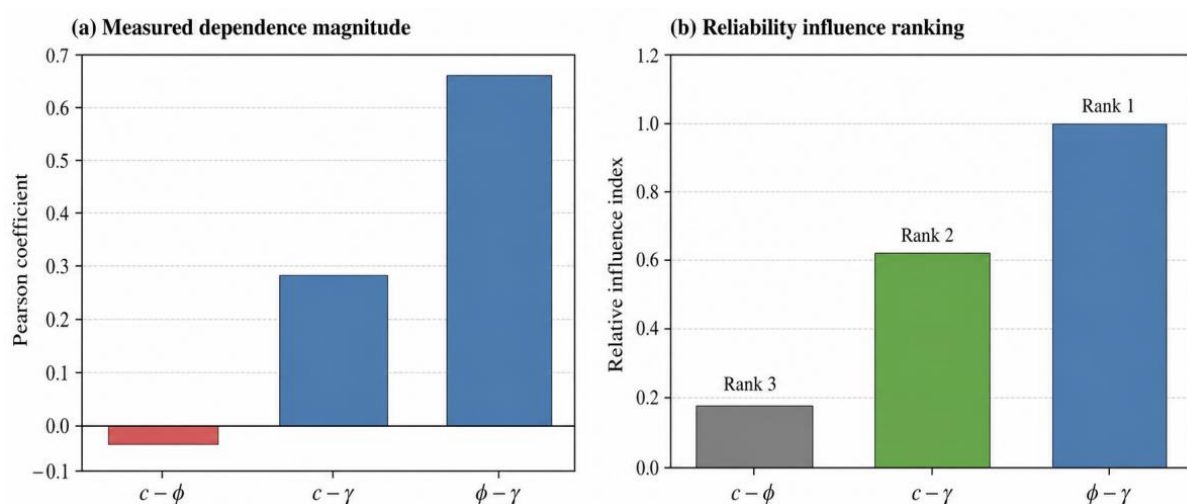


Figure 14. Pairwise dependence and reliability influence in the Hongpaihe trivariate case: (a) measured Pearson coefficients for the three parameter pairs and (b) relative reliability-influence ranking, showing ϕ – γ as the dominant pair, followed by c – γ and c – ϕ .

3.6. Spatial Propagation into Cross-Correlated Random Fields

The CCRF simulations extend the point-wise dependence structure into space. Figure 15 shows representative c and ϕ fields generated under the optimal negatively correlated setting. The fields preserve spatial autocorrelation within each parameter and cross-correlation between parameters. Zones with relatively high c tend, on average, to coincide with lower ϕ , and vice versa. This is the spatial expression of the negative c - ϕ dependence identified at the point scale [8,25–27,35,44].

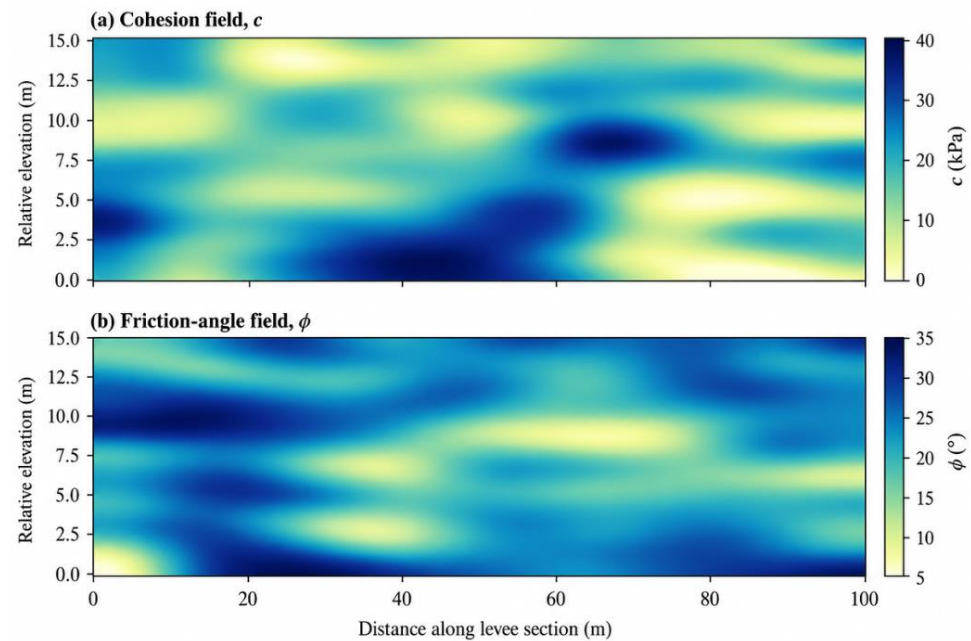


Figure 15. Illustrative CCRF realizations under the optimal negatively correlated scenario: (a) cohesion field c and (b) friction-angle field ϕ . The prescribed parameters are $l_x = 18$ m, $l_y = 5$ m, and $\rho_c, \phi = -0.396$. These fields illustrate dependence propagation and are not calibrated site-wide reconstructions.

Figure 16 compares field-level dependence signatures under optimal correlated, independent and weakly correlated scenarios. The optimal field retains a clear negative c - ϕ structure in cell-wise space; the independent field collapses this structure; and the weakly correlated field occupies an intermediate position similar to the main-dike case. These field-level signatures demonstrate that two models can share similar marginal distributions while producing very different spatial combinations of strength parameters. From an engineering-geological perspective, this distinction is crucial because levee instability is governed by continuous weak zones rather than isolated points [16,41,44].

Quantitatively, Scenarios A and B used identical marginal statistics— $c = 17.13 \pm 8.71$ kPa and $\phi = 23.39 \pm 4.96^\circ$ —and the same correlation lengths, $l_x = 18$ m and $l_y = 5$ m, so that only the prescribed cross-correlation differed. Scenario A had a target correlation of -0.396 and a realization-specific cell-wise correlation of -0.406 , giving an absolute difference of 0.010. Scenario B had a target correlation of zero, whereas the displayed finite realization yielded a cell-wise value of 0.200. This nonzero sample value reflects finite-domain variability and spatial smoothing rather than imposed dependence. Scenario C used $c = 24.05 \pm 12.47$ kPa and $\phi = 12.34 \pm 5.31^\circ$, with target and realized correlations of -0.104 and -0.174 , respectively. These results quantitatively demonstrate how the prescribed point-wise dependence is transferred, with realization-level fluctuations, into the spatial random fields.

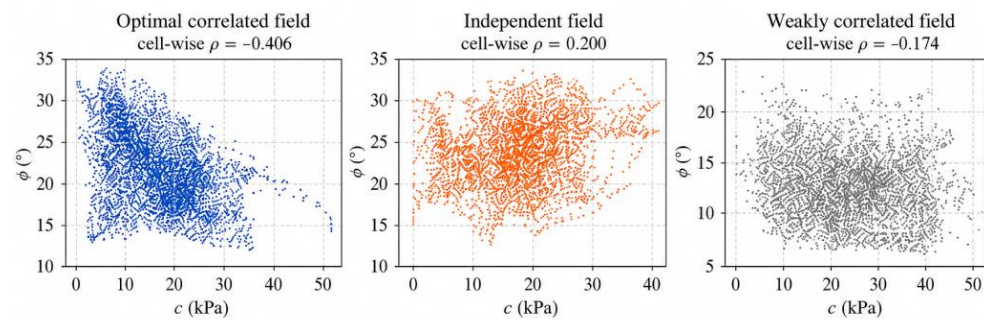


Figure 16. Field-level dependence signatures under alternative CCRF scenarios. The prescribed c - ϕ correlations were -0.396 , 0 , and -0.104 for the optimal correlated, independent, and weakly correlated scenarios, respectively; the panel titles report the corresponding realization-specific cell-wise correlation coefficients.

4. Discussion

4.1. Methodological Innovation Distilled

The preceding results reveal a progressive uncertainty-propagation chain that links marginal variability, dependence structure, spatial random-field representation and reliability response. In this chain, fluctuation-rich marginal behavior is first identified from limited geotechnical data, the dependence geometry between strength parameters is then reconstructed, and the resulting stochastic structure is finally transferred into spatially distributed fields and reliability indices. Figure 17 synthesizes the sensitivity of reliability indices to these modeling choices. A central observation is that their effects are index-dependent rather than uniform. The mean factor of safety (F_S), remains comparatively insensitive to several stochastic assumptions, whereas the failure probability (P_f), responds markedly to marginal tail behavior, dependence geometry, cross-correlation and the spatial continuity of weak zones. This contrast suggests that mean stability indicators may mask risk amplification induced by unfavorable but mechanically coherent combinations of soil strength parameters [13,40,41,43].

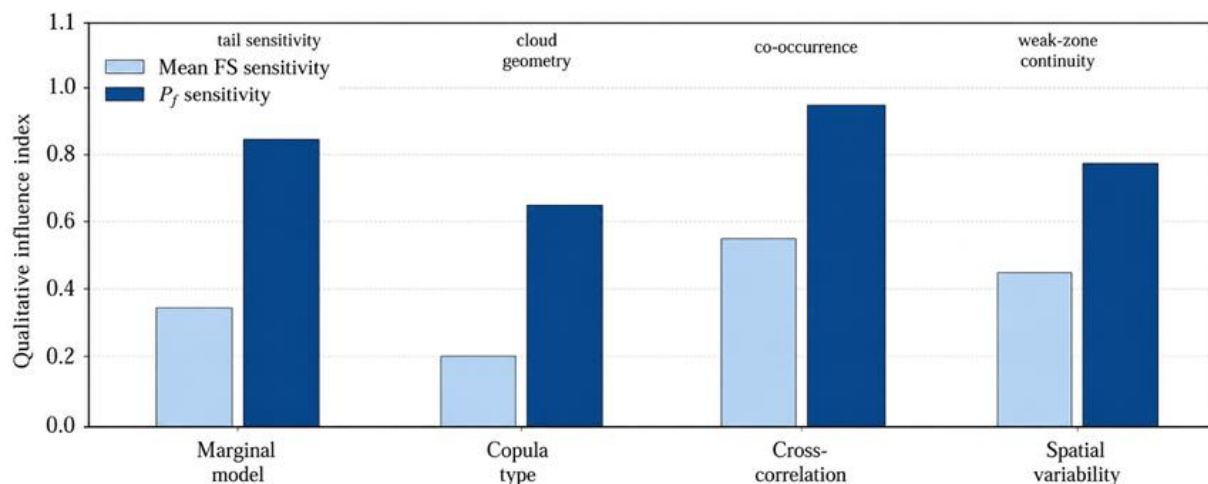


Figure 17. Qualitative synthesis of the relative sensitivity of mean FS and P_f to marginal-model choice, Copula-family choice, interparameter cross-correlation, and spatial weak-zone continuity. The bars summarize the comparative trends observed in the preceding analyses rather than additional statistical estimates.

This interpretation clarifies the methodological contribution of the integrated NID–Copula–CCRF framework. The NID-based marginal characterization improves the representation of limited, heterogeneous and fluctuation-rich datasets; Copula modeling

preserves the directional dependence of unfavorable c - φ combinations; and the CCRF formulation embeds the same dependence structure into the spatial domain. The significance of the framework therefore lies not only in the use of individual stochastic tools, but also in maintaining consistency across successive modeling stages. A reliability assessment is geologically credible only when the stochastic description remains coherent from measured data, through joint probabilistic characterization, to the formation and propagation of spatially continuous weak zones [1,35,44].

4.2. Diagnostic Role of Failure Probability Under Stochastic Model Uncertainty

The results in Sections 3.2–3.6 consistently show that the mean factor of safety and the failure probability respond differently to stochastic model assumptions. The mean factor of safety primarily reflects the central tendency of the simulated resistance state. It therefore remains relatively stable when the main body of the joint parameter cloud is preserved. By contrast, the failure probability is governed by the amount of probability mass crossing the limit-state boundary. Even modest changes in marginal tails, Copula-induced cloud orientation, or field-scale weak-zone continuity can shift the probability mass near the failure domain and produce a clear change in P_f , while the mean F_S changes only slightly [6,9,42,43].

This contrast is evident in the bivariate reliability results, where the correlated and independent models yield almost identical mean F_S values for the Beiwei section but noticeably different P_f values. It is also consistent with the trivariate Hongpaihe results, where the φ - γ dependence exerts a stronger reliability influence than would be inferred from marginal variability alone. At the random-field scale, the same mechanism becomes more spatially explicit: spatial variability controls the continuity of weak zones, whereas cross-correlation controls whether unfavorable parameter combinations occur within those zones [29,33,34].

Figures 18 and 19 synthesize this interpretation. Marginal uncertainty mainly affects the tails and edge regions of the stochastic input, while dependence uncertainty changes the geometry of the joint parameter cloud relative to the failure boundary. When these point-wise effects are propagated into cross-correlated random fields, the resulting field-scale reliability response again shows that FS may conceal model-form sensitivity that remains visible in P_f . Therefore, for hazard-oriented levee assessment, P_f should not be treated as a secondary supplement to F_S . Instead, it should be reported as a diagnostic reliability index that directly reflects the effects of marginal misspecification, dependence misspecification, and loss of spatial structure [13,40].

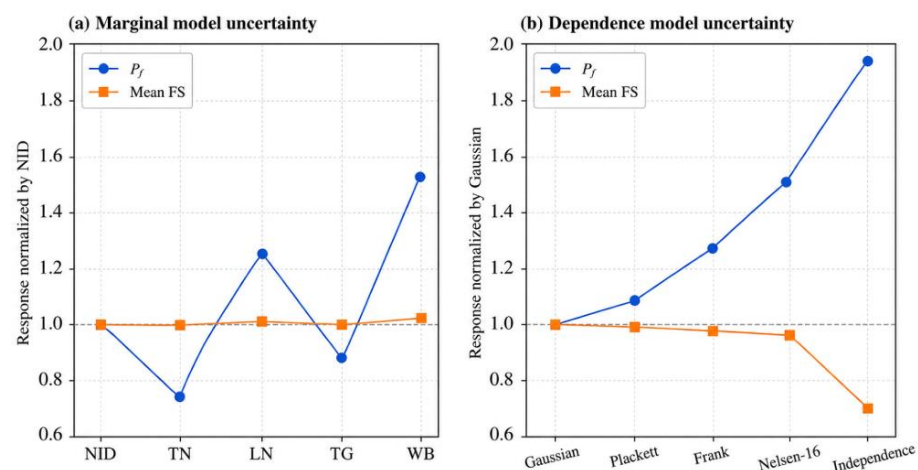


Figure 18. Contrasting responses of mean FS and failure probability to marginal and dependence model uncertainty: (a) sensitivity to marginal-distribution selection; and (b) sensitivity to Gaussian,

Plackett, Frank, Nelsen's No.16, and independence dependence models. All values are normalized by the corresponding reference-model result.

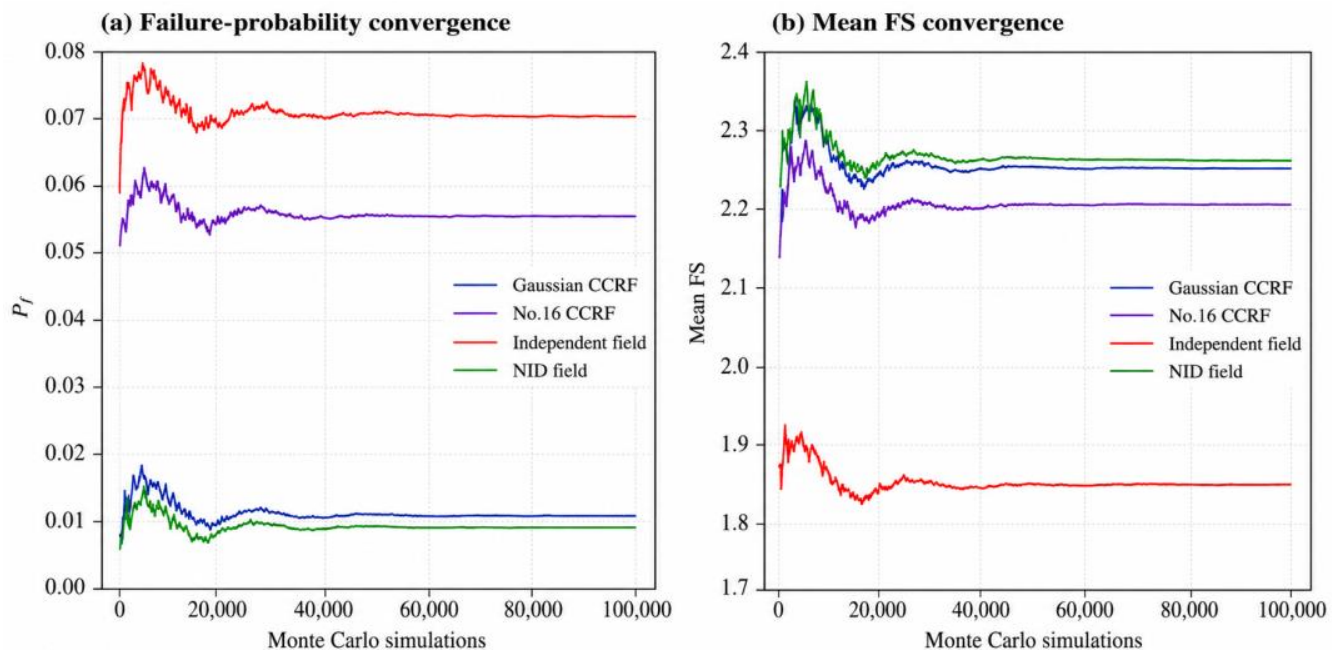


Figure 19. Monte Carlo convergence under alternative random-field assumptions: (a) running estimates of failure probability P_f and (b) running estimates of mean FS as the number of simulations increases to 100,000. The curves compare the selected CCRF and alternative dependence assumptions.

4.3. Engineering-Geological Implications of Cross-Correlated Spatial Variability

The results in Section 3.6 show that the engineering meaning of cross-correlation becomes clearer when the stochastic structure is examined at the field scale. Levee instability is not triggered by a single unfavorable parameter value, but by the spatial organization of unfavorable parameter combinations into a sufficiently continuous weak zone. Spatial autocorrelation and inter-parameter cross-correlation therefore describe two different aspects of geological uncertainty. Spatial autocorrelation controls the continuity, size and persistence of weak or strong patches along the levee section, whereas cross-correlation controls which material parameters co-occur within those patches. A random-field model that preserves spatial continuity but ignores cross-correlation may reproduce heterogeneous-looking fields, but it can still generate geologically inconsistent combinations of c , ϕ and γ [17,18,27,41,44].

Figure 20 summarizes this mechanism in relation to the proposed modeling hierarchy. Small-sample marginal fluctuation determines whether low-strength tails and edge observations are retained. Copula-based dependence modeling determines how these observations are combined in the multivariate parameter space. CCRF then propagates this dependence structure into the spatial domain and determines whether unfavorable combinations remain isolated or evolve into continuous weak zones that can intersect the failure domain. This interpretation is particularly relevant for aging alluvial levees, where repeated heightening, localized repair, mixed fill sources and layered foundations tend to generate spatially persistent zones of elevated instability potential. The CCRF step therefore provides more than a visual random-field representation; it links point-wise statistical dependence to geologically credible weak-zone interpretation and hazard-oriented reliability assessment [1,5,35,44].

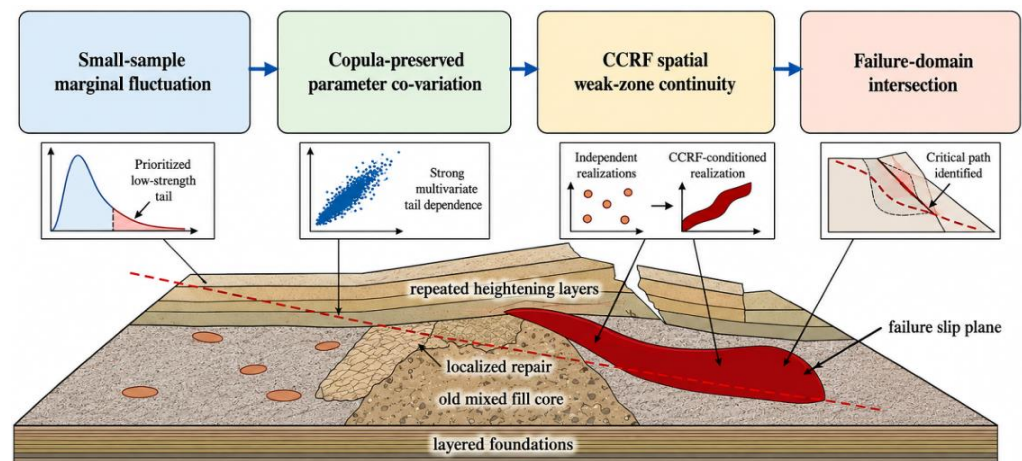


Figure 20. Conceptual mechanism linking stochastic inputs to levee failure: small-sample marginal fluctuations determine the low-strength tails, Copula modeling preserves parameter co-variation, CCRF controls spatial weak-zone continuity, and intersection with the failure domain determines the reliability response.

4.4. Practical Implementation for Levee Screening and Maintenance Prioritization

The practical value of the proposed framework lies in ranking levee sections and allocating investigation effort. The comparison between Beiwei and the main dike shows that advanced stochastic modeling is most valuable where measured dependence is strong or where section-specific heterogeneity is likely to produce unfavorable co-occurrence patterns. In weakly dependent sections, simpler screening may be adequate at an early stage, although such simplification should be revisited if additional data reveal stronger dependence or spatial zoning [2,5,11,15].

Figure 21 translates this logic into a staged workflow. A preliminary survey identifies whether the dataset is small, fluctuation-rich or strongly dependent. NID and Copula analysis then determine whether the point-wise stochastic structure requires dependence-aware reconstruction. CCRF should be introduced for sections in which weak-zone continuity is decision-relevant. The final reliability output should prioritize Pf while retaining FS as a companion index. This staged strategy avoids unnecessary model complexity but places advanced stochastic effort where it can change maintenance or reinforcement decisions [1,13,43].

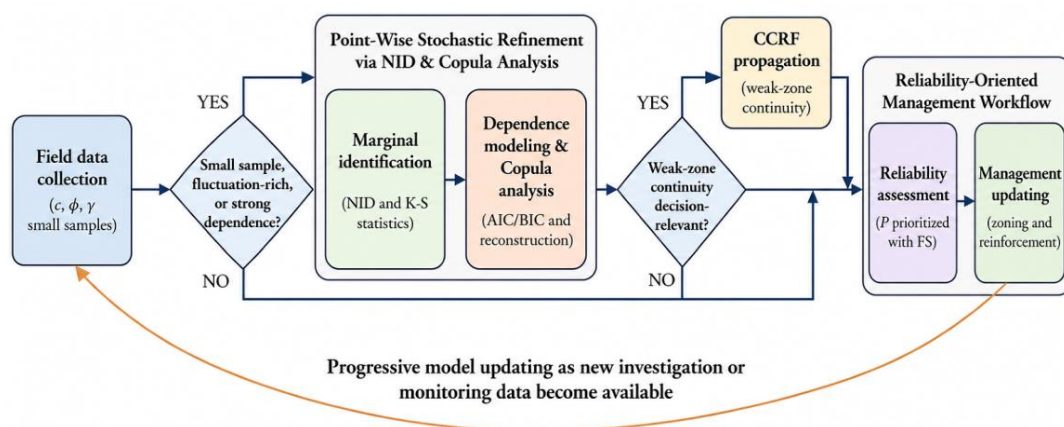


Figure 21. Staged practical implementation of the NID–Copula–CCRF framework, including field-data collection, marginal-distribution identification, dependence assessment, CCRF escalation for priority sections, F_S and P_f evaluation, model updating, and maintenance or supplementary investigation prioritization.

4.5. Scope, Validation Limitations, and Progressive Model Updating

The present validation remains primarily internal. Marginal models are assessed using K–S statistics, Copula models are compared using AIC/BIC and equivalent-sample reconstruction, and reliability results are checked through Monte Carlo convergence and reduced-sample robustness analysis. These procedures confirm statistical fit, numerical stability, and physical consistency within the available datasets. However, they do not provide independent external validation, because no separate levee dataset, long-term monitoring record, or documented failure case was used to test the predictive capability of the full NID–Copula–CCRF framework. Finally, the field simulations are stationary, whereas historical repairs and localized defects can introduce spatial trends [35,44,48,49].

These limitations do not weaken the central conclusion, but they define the next stage of development. The framework is modular: the marginal model, dependence model, spatial correlation length and reliability benchmark can each be updated as new investigation or monitoring data become available. This modularity is useful for long-lived flood-defense systems because data are often accumulated progressively. A practical implementation should therefore begin with point-scale NID and Copula identification, move to CCRF only for priority sections, and finally update the random-field and reliability models as additional stratigraphic or monitoring evidence is obtained [1,25,26,43].

Future validation should proceed in four stages. First, the marginal and Copula models should be evaluated using withheld samples or independent levee sections rather than the same datasets used for calibration. Second, site-specific correlation lengths and nonstationary trends should be estimated from denser borehole, laboratory, geophysical, and monitoring data. Third, the predicted weak zones should be compared with observed pore-pressure anomalies, deformation concentrations, local damage, or documented instability locations. Finally, the reliability results should be benchmarked against rotational limit-equilibrium analyses and fully coupled seepage–stress models under transient flood loading. This staged external-validation strategy would allow the predictive capability and transferability of the framework to be assessed more rigorously.

5. Conclusions

This study evaluates an integrated NID–Copula–CCRF framework using five representative Wuhan levee datasets and a simplified infinite-slope reliability benchmark. Building on the authors' previous component-level studies, the present work advances an integrated framework that consistently propagates marginal, dependence, and spatial uncertainties to F_S and P_f . The main conclusions are as follows:

1. Among the five marginal models examined, NID yielded the lowest K–S statistics for all fully documented variables. For example, the K–S values for c and φ were 0.0136 and 0.0140 for Beiwei and 0.0188 and 0.0209 for the main-dike dataset. These results indicate better fitting performance for the investigated limited-sample datasets, particularly in preserving local fluctuations and lower-tail observations.
2. Among the four bivariate candidate Copulas, the Gaussian Copula was preferred by the AIC/BIC criteria for the investigated datasets. For Beiwei, its AIC was -13.8680 , lower than those of the Plackett, Frank, and Nelsen's No.16 Copulas. For Hongpaihe, the multivariate Gaussian Copula only slightly outperformed the t -Copula, with AIC values of -19.67 and -19.58 , respectively.
3. Within the infinite-slope benchmark analyses, P_f was more sensitive than the mean F_S to the dependence assumption. For Beiwei, the correlated and independent models yielded similar mean F_S values of 1.235 and 1.238, but different P_f values of 0.153 and 0.200. For the weakly dependent main-dike dataset, the corresponding differences were negligible.

4. For the Hongpaihe dataset, the pairwise dependence strength was ranked as $\varphi - \gamma > c - \gamma > c - \varphi$, based on Pearson coefficients of 0.6627, 0.2828, and -0.0308 , respectively. This result indicates that a parameter with a small marginal COV may still influence the benchmark reliability response when it is strongly associated with a resistance-controlling variable.
5. The illustrative CCRF realizations show that changing the prescribed cross-correlation can alter the spatial co-occurrence of c and φ , even when the marginal statistics and correlation lengths are retained. These simulations demonstrate the potential importance of preserving both spatial continuity and interparameter dependence; however, they should not be interpreted as calibrated reconstructions of actual site-wide weak zones.

These conclusions are subject to the scope of the present study. The reliability results are based on a simplified infinite-slope benchmark, the CCRF simulations use prescribed stationary correlation lengths, and external validation using independent levee datasets or monitoring records has not yet been completed. Future studies should evaluate rotational and seepage–stress-coupled failure mechanisms, calibrate nonstationary spatial models using denser field data, and validate the framework across additional levee systems.

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Abbreviations and Symbols

The following abbreviations are used in this manuscript:

AIC	Akaike information criterion
BIC	Bayesian information criterion
CCRF	Cross-correlated random field
CDF	Cumulative distribution function
COV	Coefficient of variation
K–S	Kolmogorov–Smirnov
LN	Lognormal
NID	Normal information diffusion
PDF	Probability density function

TG	Truncated Gumbel
TN	Truncated normal
WB	Weibull
Pf	Failure probability
FS	Factor of safety

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