

## Article

# Static Formation Temperature Inversion in Ultra-Deep Wells Based on an IGWO-RBF Surrogate Model

Wenming Li <sup>1</sup>, Feng Lu <sup>1</sup>, Xu Du <sup>1</sup>, Jianfei Xu <sup>1</sup>, Dali Zhang <sup>1</sup>, Wenjie Jia <sup>2</sup> and Zhengming Xu <sup>2,\*</sup>

<sup>1</sup> Sinopec Shengli Petroleum Engineering Co., Ltd., Huanghe Drilling Company, Dongying 257500, China; liwenming337.oss@sinopet.com (W.L.); lufeng4587@163.com (F.L.); goohoot@126.com (X.D.); xujf230.oss@sinopet.com (J.X.); zhangshulizdl@126.com (D.Z.)

<sup>2</sup> School of Energy Resources, China University of Geosciences (Beijing), Beijing 100083, China; 2106240068@email.cugb.edu.cn

\* Correspondence: xuzm@cugb.edu.cn

## Abstract

In ultra-deep well drilling, directly measuring the static formation temperature (SFT) is highly time-consuming, as it requires extended shut-in periods for the wellbore to reach full thermal equilibrium, making it impractical for routine engineering operations. To overcome this challenge, this paper establishes a wellbore–formation transient temperature model (WFTM) and proposes an SFT inversion method based on the Improved Grey Wolf Optimizer (IGWO) and Radial Basis Function (RBF) neural network. The RBF network serves as a surrogate model to replace the WFTM during iterative optimization, avoiding the prohibitive computational cost of repeated WFTM evaluations and enabling rapid prediction of the transient wellbore temperature field. Meanwhile, the IGWO algorithm uses the measured bottomhole circulating temperature (BHCT) as a constraint to optimize the geothermal gradient in SFT inversion. Multi-well validation shows that the RBF surrogate predicts BHCT with relative errors consistently below 1%, demonstrating its effectiveness as a substitute for the WFTM. Compared with the direct iterative approach (IGWO-WFTM), the IGWO-RBF method yields slightly lower SFT inversion accuracy, but this deviation remains within engineering tolerances, and the computational time is reduced by approximately 18 times. Requiring only surface temperature and routinely measured BHCT, the proposed approach offers a practical and efficient pathway for real-time assessment of formation temperature during ultra-deep oil well drilling.

**Keywords:** static formation temperature; geothermal gradient inversion; improved grey wolf optimizer; radial basis function neural network; wellbore–formation transient model

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## 1. Introduction

Ultra-deep well drilling generally faces the challenge of ultra-high temperature formations. High-temperature environments can degrade drilling fluid performance, cause signal loss, or even damage measurement-while-drilling instruments, and simultaneously exacerbate uneven thermal stress on the borehole wall, leading to problems such as borehole instability and failure of the cement sheath. The root cause of the above temperature-related problems lies in excessively high static formation temperature (SFT). SFT refers to the original temperature of the formation before it is penetrated by drilling, serving as the key boundary condition for the wellbore–formation transient temperature field model

and constituting essential data for analyzing geothermal field distribution characteristics, establishing geothermal gradients, and evaluating geothermal resources. However, the cooling effect of drilling fluid circulation causes the measured bottomhole circulating temperature (BHCT) to be far lower than the true SFT during the early shut-in period [1], and full restoration to thermal equilibrium may take days to months, which is an unacceptably long downtime in engineering practice. Therefore, how to reliably invert SFT using BHCT data measured during a short shut-in period has become a critical issue in ultra-deep well drilling engineering.

To estimate static formation temperature, in addition to conventional direct logging methods, researchers have developed various methods based on transient temperature data to indirectly invert for SFT. Dowdle and Cobb [2] first introduced the Horner pressure buildup curve into temperature-recovery analysis, thereby establishing the classical Horner temperature-plot method. Subsequently, Roux et al. [3] proposed an improved Horner method by introducing a dimensionless circulation time correction factor to reduce the underestimation bias of the conventional approach. Kritikos and Kutasov [4] proposed a two-point method that is independent of formation thermal diffusivity and borehole diameter, thereby simplifying the calculation process. Ascencio et al. [5,6], starting from spherical-radial heat flow, proposed a spherical-radial model better suited to downhole conditions, arguing that it better captures the vertical heat-flow component at the bottomhole and generally yields higher SFT estimates than the Horner method. Kutasov and Eppelbaum [7] further generalized the Horner method and proposed a method that requires only three temperature points to estimate SFT. Cao et al. [8], based on a cylindrical-radial heat conduction model, simultaneously estimated multiple parameters, including static formation temperature, thermal invasion distance, formation thermal conductivity, and mud heating efficiency factor, from bottomhole temperature measurements using a nonlinear least-squares inversion method. Chekhonin et al. [9] simultaneously estimated geothermal temperature and its gradient by inverting transient wellbore temperature measurements using a transient heat-transfer model and the damped least-squares method. Wong-Loya et al. [10] employed a rational polynomial least-squares fitting method to describe the temperature recovery process and directly extrapolated SFT from the asymptotic value, which likewise requires no mud or formation thermal properties.

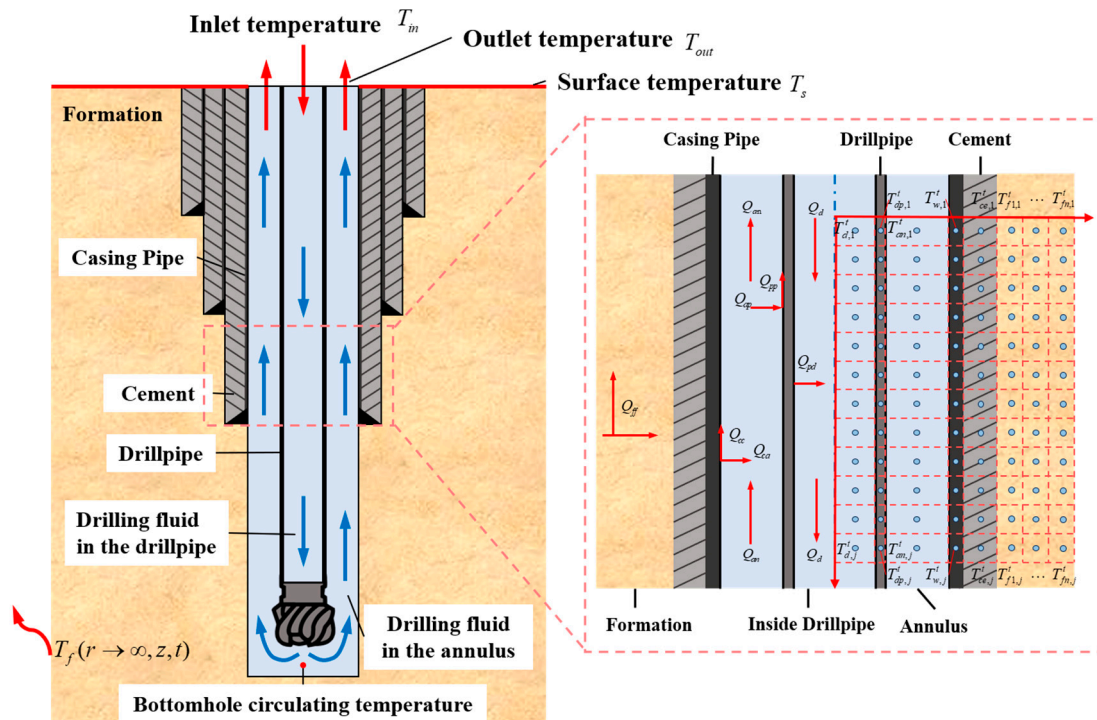
Although the above analytical methods can accurately invert SFT, most of them rely on field logging data such as circulation time, thermophysical parameters, and borehole diameter. Moreover, the model assumptions are overly idealized, making it difficult to balance computational efficiency and accuracy. To address this, researchers have progressively introduced numerical simulations and intelligent algorithms. Espinosa-Paredes et al. proposed inversion strategies based on proportional–integral feedback control [11], cognitive models [12], and the Levenberg–Marquardt nonlinear least-squares method [13], among which the feedback control and cognitive models require only a single temperature measurement to estimate SFT. Subsequently, machine-learning-based methods have attracted considerable attention for SFT prediction. Artificial neural networks, random forests, and nonlinear regression models have been successfully employed to establish the relationship between transient temperature measurements and SFT [14–17]. In addition, several studies investigated the reliability of SFT estimation with respect to uncertainty quantification, input errors, and operating conditions, providing valuable insights into the robustness of prediction results [18–20]. Although these approaches have demonstrated promising predictive capabilities and reduced dependence on detailed thermophysical parameters, purely data-driven models may still suffer from overfitting and limited generalization when training samples are insufficient, whereas high-fidelity numerical simulations are often computationally expensive.

Recently, surrogate models combined with metaheuristic optimization algorithms have emerged as an effective alternative, balancing computational efficiency and prediction accuracy. Among them, the Grey Wolf Optimizer (GWO) has been successfully applied in lithology identification, reservoir evaluation, production forecasting, and well trajectory optimization [21–25]. Meanwhile, the Radial Basis Function (RBF) neural network has been widely adopted in drilling-related applications due to its simple structure, excellent nonlinear approximation capability, and fast training speed [26–30]. Previous studies have demonstrated that GWO- and RBF-based hybrid models can significantly improve prediction accuracy, highlighting their potential for efficient and reliable SFT estimation.

This paper establishes a wellbore–formation transient temperature model (WFTM) and, combined with downhole measured BHCT data, proposes an SFT inversion method based on the Improved Grey Wolf Optimizer and Radial Basis Function neural network (IGWO-RBF). The study by Menad et al. [31] has demonstrated that GWO-RBF outperforms models optimized by PSO, GA, and Differential Evolution (DE) in handling small-sample, nonlinear engineering data. The proposed method takes measured BHCT as input and the geothermal gradient as the key inversion variable, employing the RBF surrogate model to replace the WFTM for temperature calculation, thereby accelerating the optimization iterations. To address the issues of slow convergence and low search efficiency caused by the random initialization strategy in the standard GWO, an initialization strategy based on the proportional prediction center and a dynamic gravitational guidance mechanism is introduced, leading to the Improved Grey Wolf Optimizer (IGWO). This method does not rely on extensive post-drilling measured data; it requires only surface temperature and BHCT to perform SFT inversion during ultra-deep oil well drilling in ultra-high-temperature and high-pressure environments. The present study focuses on ultra-deep oil wells. All field data used for validation are derived from oil exploration wells.

## 2. Wellbore–Formation Transient Temperature Model (WFTM)

During drilling operations, the circulating mud is injected from the wellhead into the drill string and travels downwards to the bit. As it moves along this path, thermal exchange takes place between the relatively cold fluid and the interior surface of the drill pipe. Once the fluid reaches the bottomhole assembly, its flow direction is reversed, and it travels upwards to the surface via the annular passage. Throughout this upward journey, the fluid continuously interacts thermally with the external wall of the drill string, the casing string, and the adjacent formation, effectively carrying heat from the subsurface toward the wellhead. The aforementioned physical mechanisms, along with the associated thermal boundary specifications and the discretization scheme of the computational mesh, are depicted schematically in Figure 1. In accordance with the thermal coupling behavior of the wellbore–formation system, the entire modeling domain is partitioned into five interconnected control volumes: the fluid contained within the drill string, the drill string wall, the annular fluid, the casing–cement sheath assembly, and the surrounding rock. For each of these subdomains, a corresponding energy-conservation equation is formulated to describe the transient thermal response of the coupled system, thereby laying the groundwork for subsequent inversion of the geothermal gradient.



**Figure 1.** Schematic diagram of the transient temperature field and the discretization method in the wellbore and formation during drilling fluid circulation. The red arrows indicate the direction of heat transfer, and the blue arrows indicate the direction of drilling fluid flow.

### 2.1. Mathematical Model

The drillpipe is treated as a one-dimensional thermal conduction medium along the wellbore trajectory, and any heat transfer occurring across the wall thickness is ignored. The drilling fluid circulating in the wellbore is considered a single-phase Newtonian fluid throughout the drilling operation. The surrounding formation is treated as a homogeneous medium whose thermophysical properties remain constant and are not influenced by changes in temperature, pressure, or other environmental factors. It is further assumed that the thermal disturbance induced by drilling diminishes with increasing radial distance from the wellbore, and beyond a sufficiently large distance the formation temperature remains equal to the original geothermal temperature. In addition, the well trajectory is assumed to be smooth and continuous with a regular borehole geometry, and the drill string is considered to remain concentrically aligned with the wellbore without eccentricity or buckling deformation along the entire well section.

Based on the conceptual framework described above, governing equations for each control volume are derived from the principle of energy conservation. The thermal behavior of the coupled wellbore–formation system is represented by considering the dominant heat-transfer mechanisms within each domain, including fluid convection, conductive heat transfer in solid components, interfacial heat exchange between adjacent regions, and internal heat generation where applicable. Accordingly, the temperature governing equations for the drillstring fluid, drillstring wall, annular fluid, wellbore wall, and surrounding formation are established and expressed by Equations (1)–(5), respectively.

(1) Heat transfer model of drilling fluid inside the drillpipe

$$\rho_d c_m \pi \left( \frac{D_{dp,i}}{2} \right)^2 \frac{\partial T_d}{\partial t} = -\rho_d q c_m \frac{\partial T_d}{\partial z} + \pi D_{dp,i} h_{dp,i} (T_{dp} - T_d) \quad (1)$$

Here, the quantity  $\rho_d$  denotes the mass density of the fluid (kg/m<sup>3</sup>), while  $q$  represents the volumetric pumping rate (m<sup>3</sup>/s) and  $c_d$  stands for the specific thermal capacity

of the fluid (J/(kg·°C)). The temperature of the flowing fluid is denoted by  $T_d$  (°C), and the inner diameter of the drillpipe is given by  $D_{dp,i}$  (m). The coefficient  $h_{dp,i}$  characterizes the convective thermal exchange at the internal pipe surface (W/(m<sup>2</sup>·°C)), and  $T_{dp}$  refers to the temperature of the pipe wall (°C). The axial coordinate along the wellbore is represented by  $z$  (m), and  $t$  denotes time (s).

(2) Heat transfer model of the drillpipe wall

$$\lambda_{dp} \frac{\partial^2 T_{dp}}{\partial z^2} + 4 \frac{D_{dp,o} h_{dp,o}}{D_{dp,o}^2 - D_{dp,i}^2} (T_{an} - T_{dp}) + 4 \frac{D_{dp,i} h_{dp,i}}{D_{dp,o}^2 - D_{dp,i}^2} (T_d - T_{dp}) = \rho_{dp} c_{dp} \frac{\partial T_{dp}}{\partial t} \quad (2)$$

In this equation, the parameter  $\lambda_{dp}$  is the thermal conductivity of the pipe (W/(m·°C)), and  $D_{dp,o}$  represents the outer diameter of the drill string (m). The coefficient  $h_{dp,o}$  quantifies the heat transfer between the annulus fluid and the external pipe surface (W/(m<sup>2</sup>·°C)), while  $T_{an}$  denotes the temperature of the fluid occupying the annular space (°C). The density and specific heat of the pipe material are respectively given by  $\rho_{dp}$  (kg/m<sup>3</sup>) and  $c_{dp}$  (J/(kg·°C)).

(3) Heat transfer model of drilling fluid in the annulus

$$\rho_{an} q c_m \frac{\partial T_{an}}{\partial z} + \pi D_{w,i} h_{w,i} (T_{w,i} - T_{an}) + \pi D_{dp,o} h_{dp,o} (T_{dp} - T_{an}) = \rho_{an} c_m \frac{\pi}{4} (D_{w,i}^2 - D_{dp,o}^2) \frac{\partial T_{an}}{\partial t} \quad (3)$$

Here, the subscript *an* denotes the annular domain. The parameter  $\rho_{an}$  is the fluid density in the annulus (kg/m<sup>3</sup>).  $D_{w,i}$  is the inner diameter of the wellbore wall (m), and  $h_{w,i}$  is the convective heat transfer coefficient at the inner wellbore wall (W/(m<sup>2</sup>·°C)). The temperature of the inner wellbore wall is denoted by  $T_{w,i}$  (°C).

(4) Heat transfer model of wellbore wall

$$\rho_f c_f (D_{w,o}^2 - D_{w,i}^2) \frac{\partial T_w}{\partial t} = 4 h_{w,i} D_{w,i} (T_{an} - T_w) + \lambda_f (D_{w,o}^2 - D_{w,i}^2) \frac{\partial^2 T_w}{\partial z^2} + 4 \lambda_f D_{w,o} \frac{T_f - T_w}{r_{w,o} - r_{w,i}} \quad (4)$$

In this equation,  $\rho_f$  is the formation density (kg/m<sup>3</sup>),  $c_f$  is the specific heat capacity of the formation (J/(kg·°C)),  $D_{w,o}$  is the outer diameter of the wellbore wall (m),  $\lambda_f$  is the thermal conductivity of the formation (W/(m·°C)),  $T_w$  is the wellbore wall temperature (°C),  $T_{an}$  is the annular fluid temperature (°C),  $T_f$  is the adjacent formation temperature (°C),  $r_{w,o}$  is the outer radius of the wellbore wall (m),  $r_{w,i}$  is the inner radius of the wellbore wall (m).

(5) Heat transfer model of the formation

$$\frac{\rho_f c_f}{\lambda_f} \frac{\partial T_f}{\partial t} = \frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r} \frac{\partial T_f}{\partial r} + \frac{\partial^2 T_f}{\partial z^2} \quad (5)$$

Here,  $r$  is the radial coordinate measured outward from the wellbore centerline (m).

## 2.2. Initial and Boundary Conditions

### 2.2.1. Initial Condition

Before circulation starts, the wellbore is assumed to be completely filled with drilling fluid and remain static for a sufficiently long period. During this stage, heat exchange between the wellbore and the surrounding formation gradually reaches thermal equilibrium. As a result, the temperatures of the drilling fluid, drillpipe, annular fluid, and formation at the same depth are assumed to be identical and equal to the undisturbed geothermal temperature. The initial temperature distribution is therefore expressed as:

$$T_d(z, t=0) = T_{dp}(z, t=0) = T_{an}(z, t=0) = T_f(z, t=0) = T_s + Gz \quad (6)$$

where  $T_s$  denotes the average surface temperature (°C);  $G$  represents the geothermal gradient (°C/m); and  $z$  is the true vertical depth measured from the ground surface (m).

### 2.2.2. Boundary Conditions

#### (1) Inlet boundary at the wellhead

After circulation begins, drilling fluid is continuously injected into the drillpipe from the surface. The inlet temperature is assumed to remain constant throughout the circulation process and serves as the thermal boundary condition at the wellhead:

$$T_d(z=0, t) = T_{in} \quad (7)$$

where  $T_{in}$  is the inlet temperature of the drilling fluid at the wellhead (°C), and  $t$  denotes circulation time (s).

#### (2) Bottomhole boundary

At the bottomhole location, the downward-flowing drilling fluid exits the drillpipe and enters the annulus. Owing to the intensive heat exchange occurring in this region, the temperatures of the drillpipe fluid, drillpipe wall, and annular fluid are assumed to be equal, indicating local thermal equilibrium:

$$T_d(z=H, t) = T_{dp}(z=H, t) = T_{an}(z=H, t) \quad (8)$$

#### (3) Far-field formation boundary

The formation surrounding the wellbore is assumed to extend infinitely in the radial direction. At a sufficiently large radial distance, the thermal disturbance generated by drilling operations becomes negligible, and the formation temperature remains equal to the original geothermal temperature:

$$T_f(r \rightarrow \infty, z, t) = T_s + Gz \quad (9)$$

#### (4) Upper boundary of the formation

The heat flux across the ground surface is neglected. Therefore, the upper boundary of the formation is treated as an adiabatic boundary, which can be written as:

$$\left. \frac{\partial T_f(r, z, t)}{\partial z} \right|_{z=0} = 0 \quad (10)$$

#### (5) Lower boundary of the formation

The maximum well depth is selected as the lower limit of the computational domain. Since the thermal influence caused by drilling operations does not propagate beyond this depth during the simulation period, an adiabatic condition is also imposed at the lower boundary:

$$\left. \frac{\partial T_f(r, z, t)}{\partial z} \right|_{z=H} = 0 \quad (11)$$

### 2.3. Numerical Solution and Verification of the WFTM

The governing equations presented in Equations (1)–(5) are solved numerically using a finite-difference discretization scheme, with the spatial grid layout following the arrangement shown in Figure 1. In the temporal domain, a fully implicit first-order scheme is applied, while for spatial derivatives, a centered-difference approximation is adopted. All computational codes are written in MATLAB (R2023b) by the authors; the implementation is entirely original and does not rely on any commercial solvers or external libraries. Following the discretization procedure outlined above, algebraic equations are derived for every grid point, and the resulting system is solved using an iterative implicit

marching scheme. The complete solution process is summarized in the flowchart provided in Figure 2. The discretized version of the general energy conservation equation can be expressed in the following compact form:

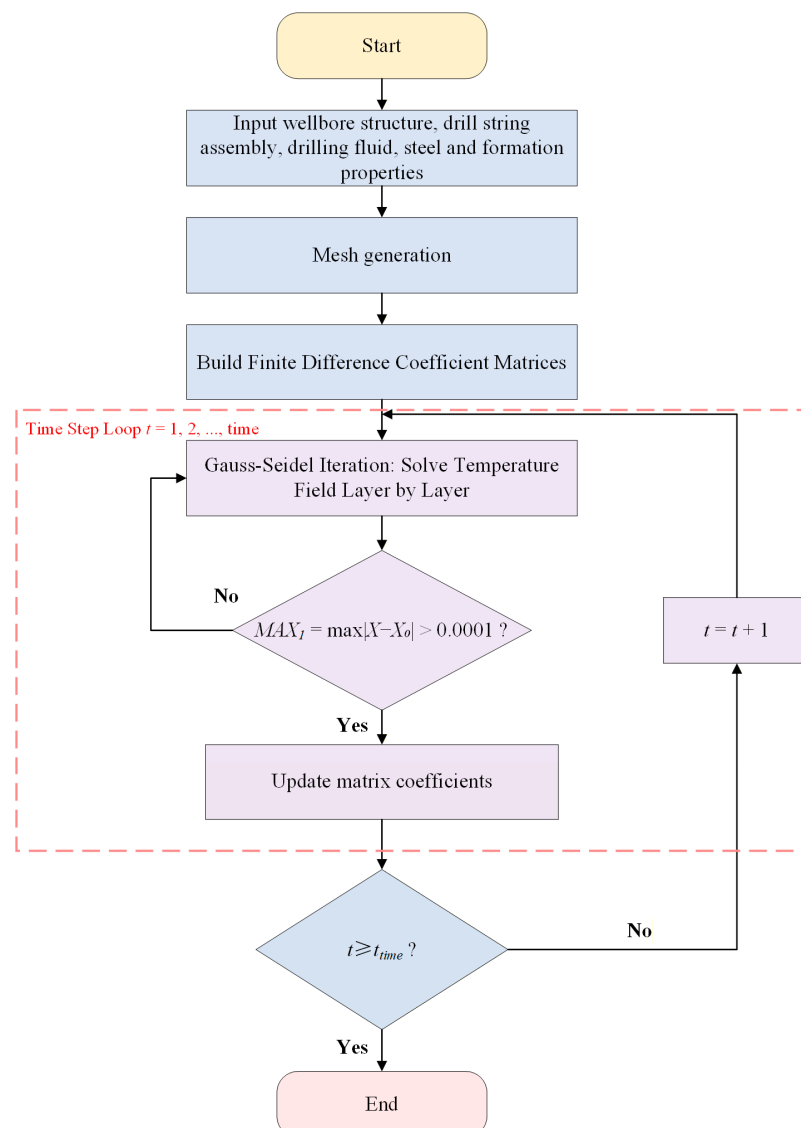
$$A_{i,j}^t T_{i,j}^t + B_{i,j}^t T_{i,j-1}^t + C_{i,j}^t T_{i,j+1}^t + D_{i,j}^t T_{i-1,j}^t + E_{i,j}^t T_{i+1,j}^t = F_{i,j}^t T_{i,j}^{t-1} \quad (12)$$

Taking the heat transfer model of the drilling fluid inside the drill pipe as an example, the discretized equation is:

$$A_1 T_{d,j}^t + B_1 T_{d,j-1}^t + E_1 T_{dp,j}^t = F_1 T_{d,j}^{t-1} \quad (13)$$

where

$$\begin{cases} A_1 = -\frac{\rho_d q c_m}{\Delta z} - \pi D_{d,i} h_{d,i} - \frac{\rho_d c_m \pi D_{d,i}^2}{4\Delta t} \\ B_1 = \frac{\rho_d q c_m}{\Delta z} \\ E_1 = \pi D_{d,i} h_{d,i} \\ F_1 = -\frac{\rho_d c_m \pi D_{d,i}^2}{4\Delta t} \end{cases} \quad (14)$$

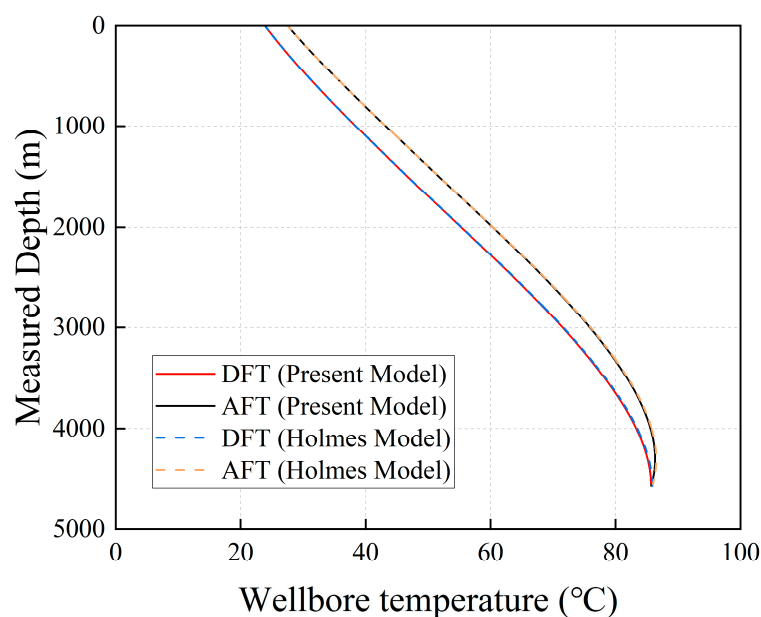


**Figure 2.** Cyclic iterative process of solving wellbore temperature field using finite difference method.

To verify the accuracy of the WFTM established in this paper, the relevant parameters of the test well listed in Table 1 were adopted, and the calculation results of the WFTM were compared with those of the Holmes & Swift model [32]. As shown in Figure 3, at a depth of 4572 m, the bottomhole temperature calculated by the WFTM is 85.6996 °C, while that from the Holmes model is 85.8878 °C, yielding an absolute error of 0.1882 °C and a relative error of only 0.22%. The overall trends of the predicted curves from both models are consistent, both showing that temperature increases with depth, and good agreement is observed over the entire depth range. The above analysis demonstrates that the WFTM presented in this paper achieves high accuracy.

**Table 1.** Parameters of the test well.

| Parameter                | Value   | Parameter                                                        | Value   |
|--------------------------|---------|------------------------------------------------------------------|---------|
| Well depth (m)           | 4572    | Drilling fluid mass flow rate (kg/s)                             | 15.876  |
| Drillpipe radius (m)     | 0.08414 | Drilling fluid specific heat capacity (J/(kg·°C))                | 1674.72 |
| Wellbore radius (m)      | 0.10636 | Drillpipe heat transfer coefficient (W/(m <sup>2</sup> ·°C))     | 170.34  |
| Inlet temperature (°C)   | 23.89   | Borehole wall heat transfer coefficient (W/(m <sup>2</sup> ·°C)) | 5.678   |
| Surface temperature (°C) | 15.28   | Geothermal gradient (°C/m)                                       | 0.0231  |



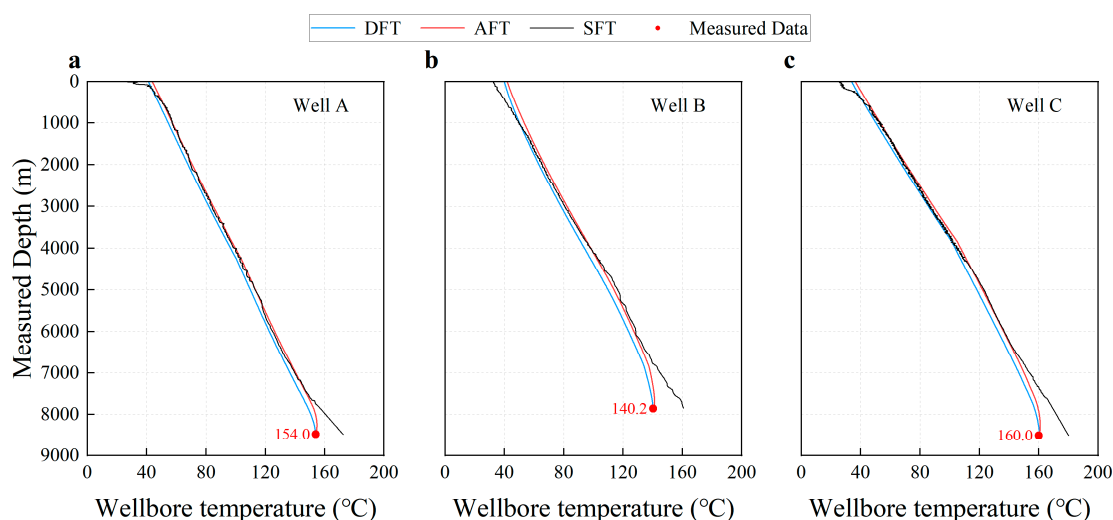
**Figure 3.** Comparison of temperature prediction results among different models.

Furthermore, to fully validate the applicability of the WFTM established in this paper under different field conditions, three ultra-deep oil wells from an oilfield in China were adopted for model validation. The three wells are located in different structural belts, and all exceed 8000 m in depth. Changes in drilling-fluid thermophysical properties may occur as the fluid is exposed to elevated temperatures and pressures in the wellbore. However, these changes are assumed to have a negligible impact on the overall heat-transfer process. As this paper focuses primarily on the geothermal gradient inversion method, the assumption of constant thermophysical properties is adopted. The specific wellbore and thermophysical parameters for the three wells are presented in Tables 2 and 3.

**Table 2.** Wellbore configuration parameters for Well A, Well B, and Well C.

| Parameter                            | Well A  | Well B | Well C | Unit              |
|--------------------------------------|---------|--------|--------|-------------------|
|                                      | Value   | Value  | Value  |                   |
| Well depth                           | 8478.83 | 8132.2 | 8500   | m                 |
| First-section casing setting depth   | 1201    | 1199.7 | 1198.7 | m                 |
| First-section casing outer diameter  | 365.1   | 339.7  | 339.7  | mm                |
| Second-section casing setting depth  | 4834    | 4365   | 4372   | m                 |
| Second-section casing outer diameter | 273.1   | 250.8  | 250.8  | mm                |
| Third-section casing setting depth   | 7821    | 7752   | 7793   | m                 |
| Third-section casing outer diameter  | 193.7   | 177.8  | 177.8  | mm                |
| Drill pipe 1 outer diameter          | 139.7   | 139.7  | 139.7  | mm                |
| Drill pipe 1 inner diameter          | 121.3   | 121.3  | 121.3  | mm                |
| Drill pipe 2 outer diameter          | 114.3   | 114.3  | 114.3  | mm                |
| Drill pipe 2 inner diameter          | 108.6   | 108.6  | 108.6  | mm                |
| Flow rate                            | 50.4    | 43.2   | 54     | m <sup>3</sup> /h |
| Drilling fluid inlet temperature     | 41.3    | 40     | 34     | °C                |
| Circulation time                     | 33      | 21     | 30     | h                 |
| Surface temperature                  | 21.5    | 32.8   | 22     | °C                |

Since the calculation of formation temperature in the WFTM depends on the geothermal gradient, which introduces a degree of arbitrariness due to manual determination, this paper adopts the bottomhole temperature actually measured by the measurement-while-drilling (MWD) instrument as the boundary condition for model computation. Figure 4 compares the temperature prediction results for the three wells. As shown in Figure 4, the predicted results of the proposed model for wells across different structural belts agree well with the measured values. For Well A, at a depth of 8475 m, the predicted temperature is 153.80 °C versus a measured value of 154.00 °C, yielding an absolute error of 0.20 °C. For Well B at a depth of 7840 m, the predicted temperature is 140.22 °C, compared with a measured value of 140.20 °C, yielding an absolute error of 0.02 °C. For Well C at a depth of 8500 m, the predicted temperature is 160.45 °C, compared with a measured value of 160.00 °C, yielding an absolute error of 0.45 °C. The mean absolute error across the three wells is 0.22 °C, and the root mean square error is 0.28 °C, further validating the applicability and reliability of the WFTM established in this paper for different wells.

**Figure 4.** Comparison of temperature prediction results for multiple wells: (a) Well A; (b) Well B; (c) Well C.

**Table 3.** Thermophysical parameters.

| Parameter              | Drilling Fluid | Casing | Cement Sheath | Formation | Unit              |
|------------------------|----------------|--------|---------------|-----------|-------------------|
| Density                | 1110           | 7800   | 2140          | 2600      | kg/m <sup>3</sup> |
| Specific heat capacity | 3000           | 500    | 2000          | 900       | J/(kg · °C)       |
| Thermal conductivity   | 0.6            | 48     | 0.7           | 2.9       | W/(m · °C)        |

### 3. SFT Inversion Based on the IGWO-RBF Model

The WFTM established in this paper takes SFT as the initial and boundary conditions. However, the model requires the geothermal gradient as input to calculate formation temperature, and slight fluctuations in the gradient can lead to significant deviations in formation temperature, thereby affecting the accuracy of the wellbore temperature field. To overcome errors arising from an inappropriate selection of the geothermal gradient and avoid the high computational cost of repeatedly invoking the WFTM, this paper proposes an SFT inversion method based on IGWO-RBF. This method uses an RBF neural network to replace the conventional WFTM forward model as a surrogate model, substantially improving inversion efficiency while maintaining computational accuracy, thereby enabling efficient and accurate inversion of the geothermal gradient.

#### 3.1. Principle of the IGWO-RBF Algorithm

The core concept of the IGWO-RBF method is to take the geothermal gradient as the variable and the measured BHCT as the constraint objective, employ the RBF neural network to replace the WFTM forward model, and achieve global optimization of the geothermal gradient via IGWO. The overall workflow is illustrated in Figure 5, which primarily consists of two stages: RBF surrogate model training and IGWO optimization.

In the formation temperature inversion problem, if only the IGWO algorithm, combined with a finite-difference model, is used for successive iterative calculations, each fitness evaluation requires a complete solution of the temperature field, resulting in a long overall inversion time that cannot meet the demand for rapid processing of field-measured data. To address this issue, this paper trains an RBF neural network based on the established WFTM to construct a rapid mapping relationship between the geothermal gradient and BHCT, thereby enabling fast calculation of the wellbore temperature field, as illustrated in Figure 5.

The proposed inversion framework adopts a point-by-point single-target inversion strategy. For each target depth, we independently invert a single equivalent geothermal gradient  $G$  using the RBF surrogate model coupled with the IGWO. The inverted static formation temperatures at individual depth points are then sequentially connected to construct a continuous temperature profile over the entire well section. This strategy ensures that each depth point is inverted independently, and the profile is assembled by linking pointwise results rather than simultaneously inverting multiple layered gradients in a single optimization run.

First, the approximate geothermal gradient range is determined by consulting the well site's historical logging data, which serves as the input parameter range for the RBF training model. Within the given geothermal gradient search space, Latin hypercube sampling is employed to generate training samples, and the corresponding BHCT is calculated using the established WFTM to construct the training dataset.

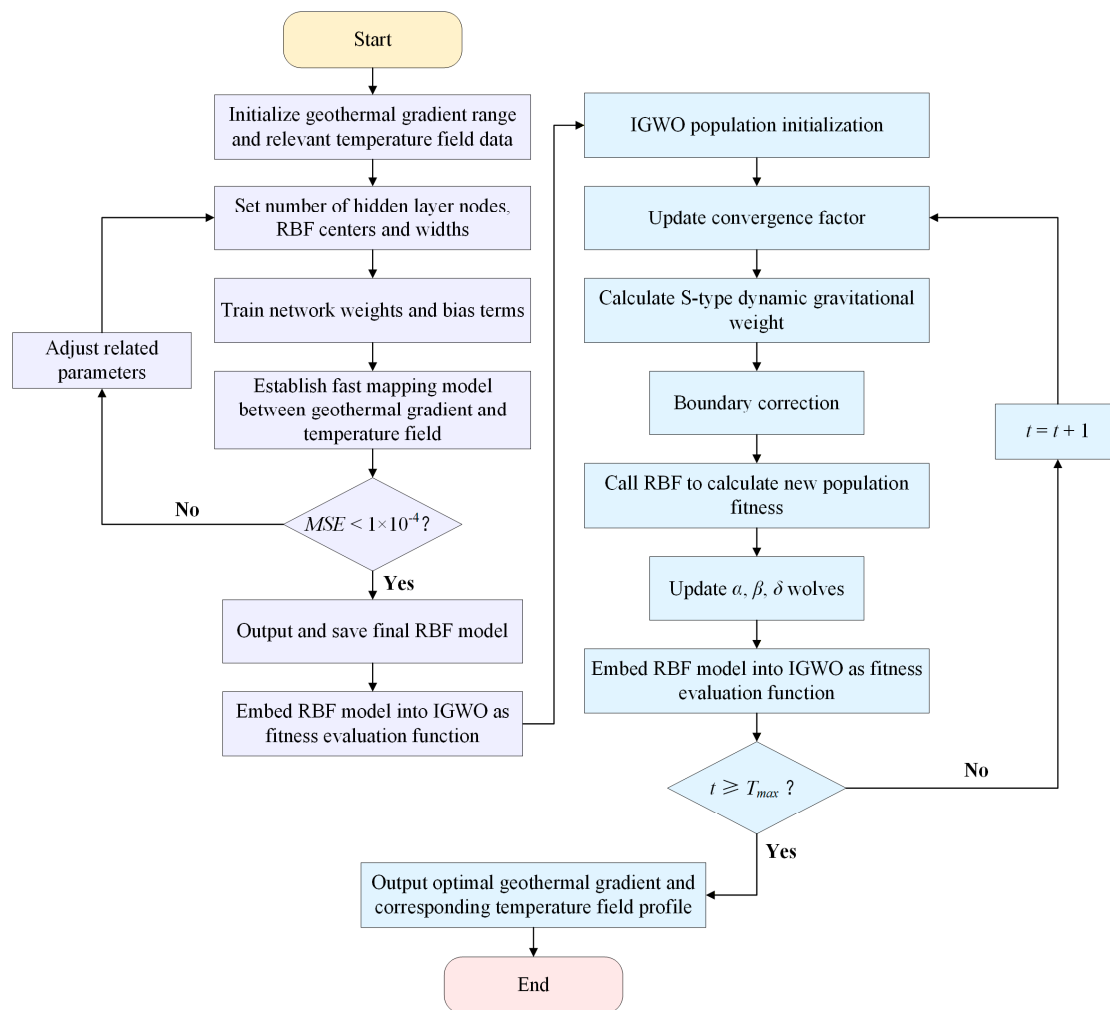


Figure 5. IGWO-RBF model flowchart.

Subsequently, the structure and parameters of the RBF network are determined. The output expression of the RBF network is as follows:

$$T(G) = \sum_{m=1}^M \omega_m \cdot \phi_m(G) + b \quad (15)$$

where  $T(G)$  is the BHCT predicted by the RBF network, °C;  $G$  is the single equivalent geothermal gradient of the deep formation segment (°C/m);  $M$  is the number of hidden layer nodes;  $w_m$  is the connection weight between the  $m$ -th hidden layer node and the output layer;  $\phi_m(G)$  is the radial basis function of the  $m$ -th hidden layer node; and  $b$  is the bias term of the output layer.

This paper selects the Gaussian kernel function as the radial basis function due to its advantages of smoothness and strong local response, which enable effective local approximation of the input space. The form of the Gaussian kernel function is as follows:

$$\phi_m(G) = e^{-\left(\frac{\|G - \mu_m\|^2}{2\sigma_m^2}\right)} \quad (16)$$

where  $\phi_m(G)$  is the output value of the  $m$ -th hidden layer node;  $\mu_m$  is the center vector of the  $m$ -th basis function;  $\|G - \mu_m\|$  denotes the Euclidean distance between the input vector and the center vector; and  $\sigma_m$  is the width parameter of the  $m$ -th basis function.

Finally, the RBF model's prediction accuracy is evaluated using a fitness function. To amplify the influence of large deviations and enhance the search capability of the optimization algorithm in high-error regions, this paper selects the mean square error (MSE) as the fitness evaluation metric, defined as follows:

$$F(G) = (T(G) - T_{bottom}^{meas})^2 \quad (17)$$

where  $F(G)$  is the fitness function value, and  $T_{bottom}^{meas}$  is the measured bottomhole temperature ( $^{\circ}\text{C}$ ).

The trained RBF model is embedded into the grey wolf optimizer as a surrogate model for the WFTM to perform global optimization of the geothermal gradient. The conventional GWO algorithm adopts a random initialization strategy, in which the initial population is uniformly distributed within the search space, lacking prior guidance toward the optimal solution region, resulting in slow convergence and low search efficiency. To address these shortcomings, this paper proposes the IGWO to enhance the efficiency and accuracy of geothermal gradient inversion. In the initialization phase, to concentrate the initial geothermal gradient population near the promising optimal solution region, the initial position of the  $j$ -th dimension of the  $i$ -th individual is formed by superimposing the proportional prediction center with a boundary-constrained random perturbation, calculated as follows:

$$x_{i,j}^{(0)} = X_{center,j} + \delta \cdot (ub_j - lb_j) \cdot \varepsilon \quad (18)$$

where  $x_{i,j}^{(0)}$  denotes the initial position of the  $j$ -th dimension of the  $i$ -th individual;  $X_{center,j}$  is the ideal center value of the  $j$ -th dimension predicted by the proportional method;  $\delta$  is the perturbation amplitude coefficient;  $ub_j$  and  $lb_j$  are the upper and lower bounds of the  $j$ -th dimension search space, respectively; and  $\varepsilon$  is a random number following a uniform distribution in the range  $[-1, 1]$ .

After initialization, the population undergoes iterative updating. Individuals in the population update their positions according to the social hierarchy. The three individuals with the best fitness are designated as the  $\alpha$ ,  $\beta$ , and  $\delta$  wolves, which act as leaders to guide the search direction of the remaining  $\omega$  wolves. The  $\omega$  wolves synthesize the position information of the three leading wolves and determine their candidate new positions through weighted averaging, thereby achieving encirclement and tracking of the optimal solution region. The candidate position calculation formula of standard GWO is as follows:

$$X_{new} = \frac{1}{3} \sum_{k \in \{\alpha, \beta, \delta\}} (X_k - Q_k \cdot |P_k \cdot X_k - X|) \quad (19)$$

$$Q = 2a \cdot r_1 - a, \quad P = 2 \cdot r_2 \quad (20)$$

where  $X_{new}$  is the candidate position vector calculated by the standard GWO algorithm;  $X_k$  is the position vector of the leading wolf  $k$  (i.e., the  $\alpha$ ,  $\beta$ , or  $\delta$  wolf);  $X$  is the position vector of the current grey wolf individual;  $Q_k$  and  $P_k$  are coefficient vectors that control the movement step size and distance weight, respectively;  $r_1$  and  $r_2$  are random vectors uniformly distributed in the interval  $[0, 1]$ ; and  $a$  is the convergence factor.

To further improve the convergence accuracy and stability of the algorithm, this paper introduces a dynamic gravitational guidance mechanism toward the proportional method center  $X_{center}$ . Unlike traditional methods that adopt fixed weights, the gravitational weight designed in this paper varies dynamically with the number of iterations following an S-shaped curve: in the early stage, a higher weight is set to guide the search direction centered on the initial geothermal gradient, enabling the algorithm to search around the initial geothermal gradient; in the middle stage, the weight is gradually reduced to decrease the dependence on the initial value and enhance the algorithm's autonomous exploration capability; in the late stage, the weight is maintained at a low level, allowing the algorithm to focus on local fine search. The final position update formula of IGWO is as follows:

$$X(t_i + 1) = (1 - \omega_{center}(t_i)) \cdot X_{new} + \omega_{center}(t_i) \cdot X_{center} \quad (21)$$

where  $X(t_i + 1)$  is the position vector after the  $t_i + 1$ -th iteration update of the IGWO algorithm;  $X_{new}$  is the candidate position vector obtained by standard GWO;  $X_{center}$  is the center vector predicted by the proportional method, serving as the target point of dynamic gravity; and  $\omega_{center}(t_i)$  is the gravitational weight at the  $t_i$ -th iteration.

The gravitational weight  $\omega_{center}(t_i)$  is dynamically adjusted using an S-shaped curve, calculated as follows:

$$\omega_{center}(t_i) = \omega_{min} + (\omega_{max} - \omega_{min}) \cdot \left( 1 - \frac{1}{1 + e^{-k(p(t_i) - p_0)}} \right) \quad (22)$$

where  $\omega_{max}$  is the upper limit of the gravitational weight, controlling the dependence on the proportional method center in the early stage of the algorithm;  $\omega_{min}$  is the lower limit of the gravitational weight, controlling the weak guidance intensity retained in the late stage;  $k$  is the steepness coefficient of the S-shaped curve;  $p(t_i) = t_i/T_{max}$  is the normalized iteration progress, where  $t_i$  is the current iteration number and  $T_{max}$  is the maximum number of iterations, with  $p(t)$  ranging in  $[0, 1]$ ; and  $p_0$  is the midpoint position of the S-shaped curve.

Finally, the IGWO algorithm is employed to iteratively optimize the assumed geothermal gradient, with MSE minimization as the convergence criterion. When the MSE converges to its minimum, the resulting formation temperature gradient is taken as the static formation temperature gradient, from which the corresponding bottomhole SFT is calculated. The complete procedure of the proposed IGWO-RBF inversion method is summarized in Algorithm 1.

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**Algorithm 1** Pseudocode of IGWO-RBF

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1: Set maximum iterations  $L$ , population size  $N$ , stagnation threshold  $\tau_{max} = 30$ 
2: Generate  $M$  training samples  $\{G_i\}_{i=1}^M$  using Latin hypercube sampling
3: Compute  $T_{BHCT}(G_i)$  using WFTM; train RBF model  $T(G)$  using Equation (15)
4: if RMSE of RBF  $> \varepsilon$  then adjust width  $\sigma$  and retrain
5: for each agent  $i = 1$  to  $N$  do
6:   Initialize  $X_i$  using Equation (18) with proportional center  $X_{center}$ 
7: end for
8: Set  $X_1 = X_{center}$ ,  $t = 1$ ,  $\tau = 0$ , and convergence factor  $a$ 
9: Evaluate fitness  $F(G)$  using Equation (17); determine  $X_\alpha, X_\beta, X_\delta$ 
10: while ( $t \leq L$ ) and stopping criterion not met do
11:   Update convergence factor  $a$  using Equation (8)
12:   if  $\tau > \tau_{max}$  then
13:     Reinitialize all agents except  $X_\alpha$ ; reset  $\tau = 0$ 
14:   end if
15:   for each agent  $i = 1$  to  $N$  do
16:     Compute  $X_{new}$  using standard GWO Equations (19) and (20)
17:     Compute dynamic weight  $\omega_{center}$  using Equation (22)
18:     Update final position using Equation (21) with center  $X_{center}$ 
19:     if  $rand < 0.1(1 - t/L)$  then apply random perturbation
20:     Clip position to  $[lb, ub]$ 
21:   end for
22:   Evaluate  $F(G)$  using Equation (17); update  $X_\alpha, X_\beta, X_\delta$ 
23:   if  $F_\alpha$  unchanged then  $\tau = \tau + 1$  else  $\tau = 0$ 
24:    $t = t + 1$ 
25: end while
26: return  $X_\alpha$  and  $T_{SFT} = T(X_\alpha)$ 

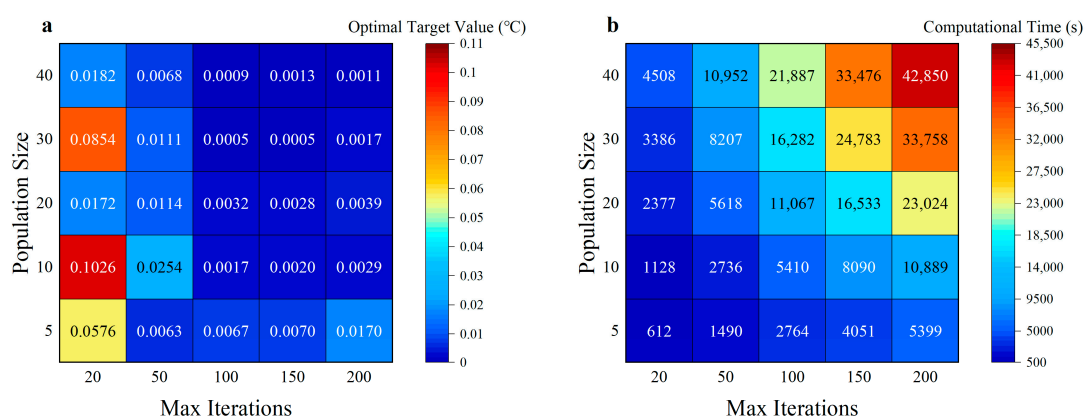
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### 3.2. Parameter Optimization and Performance Evaluation

#### 3.2.1. Accuracy Evaluation of the IGWO Model

To evaluate the optimization performance of the IGWO algorithm proposed in this paper for SFT inversion, Well A is taken as the target well. The IGWO algorithm is employed to optimize the geothermal gradient, after which the BHCT corresponding to different geothermal gradients is calculated using the WFTM. The absolute error between the predicted and measured temperatures is used as the fitness function to analyze the effects of different combinations of population size and maximum number of iterations on the objective function value and computational efficiency. Here, the objective function value is the optimal fitness obtained after convergence of the IGWO algorithm, i.e., the minimum absolute error between the bottomhole temperature calculated by the WFTM and the measured temperature; the smaller this value, the higher the inversion accuracy. The results are shown in Figure 6, where Figure 6a presents the parameter sensitivity heatmap, and Figure 6b illustrates the computational time distribution across different parameter combinations.



**Figure 6.** Optimization performance of the IGWO algorithm under different parameter settings. (a) Objective function sensitivity heatmap; (b) computational time distribution.

As shown in Figure 6a, both the population size and the maximum number of iterations significantly affect the optimization performance and computational efficiency of the IGWO algorithm. When the population size is 5 or 10, the objective function values are generally high and fluctuate considerably across different numbers of iterations, indicating insufficient global search capability and a tendency to become trapped in local optima. As the population size and maximum number of iterations increase, the objective function value decreases markedly, suggesting that the algorithm can search the solution space more thoroughly and improve the accuracy of formation temperature inversion. When the population size is 30, increasing the maximum number of iterations from 100 to 150 yields an objective function value of 0.0005 °C in both cases; however, the runtime increases from 16,282 s to 24,783 s, approximately 1.5 times longer, with no gain in accuracy and a significant decline in efficiency. When the maximum number of iterations is 100, increasing the population size from 30 to 40 reduces the objective function value from 0.0005 °C to 0.0004 °C, a decrease of only 0.0001 °C, while the runtime increases from 16,282 s to 20,365 s, representing an efficiency loss of approximately 25%. Considering both accuracy improvement and computational cost, a population size of 30 and a maximum of 100 iterations constitute the optimal combination. Further increases in population size or the number of iterations yield only marginal improvements in the objective function value, indicating that excessively large parameter settings cannot significantly enhance optimization accuracy.

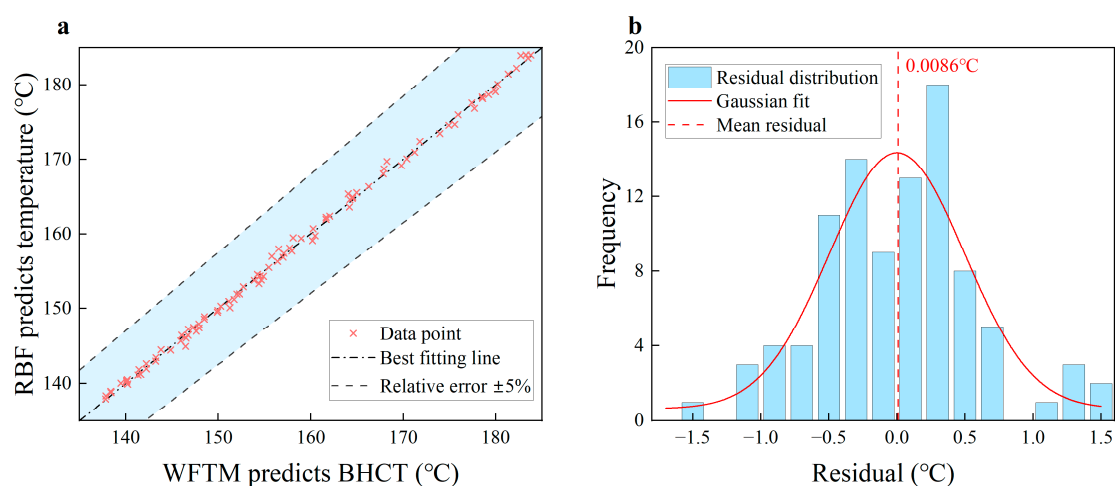
As shown in Figure 6b, the runtime of the IGWO algorithm increases substantially with larger population sizes and maximum iteration numbers. When the population size increases from 5 to 40, and the maximum number of iterations increases from 20 to 200, the computational time rises from 612 s to 42,850 s. This is because larger populations and more iterations increase the number of objective function evaluations, leading to a rapid escalation in the computational cost of optimization. Although larger parameter settings can improve optimization accuracy, the associated computational cost increases considerably, and the improvement in the objective function value gradually levels off.

After comprehensively considering both inversion accuracy and computational efficiency, a population size of 30 and a maximum of 100 iterations are selected as the optimal parameter combination, which achieves a favorable balance between optimization performance and computational cost.

### 3.2.2. Accuracy Evaluation of the RBF Model

The above analysis indicates that the iterative inversion based on the WFTM incurs excessively high computational cost and cannot meet the timeliness requirements of while-drilling operations. It is therefore necessary to introduce the RBF surrogate model to replace the WFTM and substantially improve inversion efficiency. To verify whether the RBF surrogate model can effectively replace the WFTM for BHCT prediction, this paper takes Well A as an example for model training and analysis. The surface temperature of this well is 21.5 °C, and the measured BHCT is 159 °C. Subsequently, the model's predictive performance is verified through error analysis and statistical evaluation, and the effects of the width parameter  $\sigma$  and the number of training samples  $M$  on model performance are analyzed to determine the optimal parameter combination.

Figure 7 presents the error analysis and statistical evaluation of the RBF model prediction results. Figure 7a shows the comparison between the RBF-predicted BHCT and the measured BHCT, and Figure 7b displays the residual distribution of the model and its Gaussian fitting results.



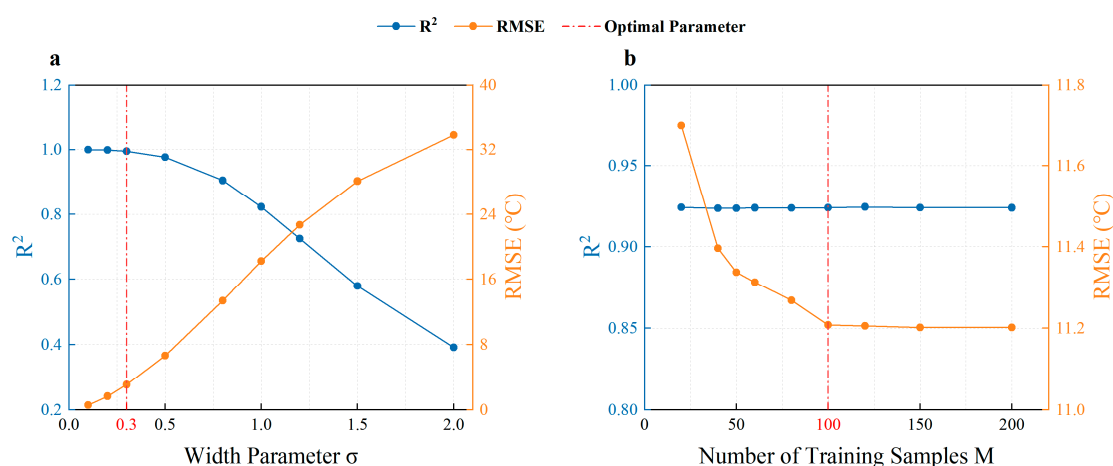
**Figure 7.** Accuracy evaluation of the RBF surrogate model. (a) Comparison between RBF-predicted and measured BHCT values; (b) residual distribution and Gaussian fitting of the RBF model.

Figure 7a demonstrates that the RBF model predictions are highly consistent with the measured BHCT, with an  $R^2$  of 0.9885. Most data points are distributed near the line of best fit and fall within the  $\pm 5\%$  relative error range, indicating that the RBF surrogate model accurately characterizes the predictions of the wellbore transient heat transfer model.

Figure 7b presents the statistical distribution characteristics of the RBF model residuals. The residual distribution exhibits good symmetry about the zero-error line and closely resembles a Gaussian distribution. With a mean residual of merely 0.0086 °C, the model demonstrates excellent unbiasedness and strong predictive reliability. The vast majority of residuals fall within  $\pm 0.5$  °C, with only a few samples exhibiting large errors, demonstrating that the RBF model achieves high prediction accuracy and stability. Moreover, the Gaussian-fit curve agrees well with the residual histogram, further confirming the randomness and reasonableness of the model error distribution.

The effectiveness of the developed RBF model largely depends on the selection of the kernel width  $\sigma$  and the amount of training data available for model fitting, denoted by  $M$ . The width parameter  $\sigma$  controls the response range of the Gaussian kernel function and directly determines the smoothness and generalization capability of the model. An excessively small  $\sigma$  results in an overly narrow kernel function, causing the model to overfit the training data and produce severe oscillations. An excessively large  $\sigma$  makes the kernel function too flat, leading to underfitting and loss of the ability to capture local variations. The number of training samples  $M$  determines the amount of information the model learns, requiring a balance between prediction accuracy and computational cost. In this paper, a sensitivity analysis of  $\sigma$  is conducted under the condition of  $M = 100$ , with five candidate values,  $\sigma = 0.1, 0.2, 0.3, 0.5, 0.8, 1.0, 1.2, 1.5$  and 2, selected for testing to identify the optimal  $\sigma$ . On this basis, the number of training samples  $M$  is further optimized.

As shown in Figure 8a, when  $\sigma = 0.1$ ,  $R^2$  reaches 0.9998, and the RMSE is only 0.5781 °C, suggesting an apparently optimal fit. However, the excessively small  $\sigma$  leads to an overly narrow kernel function, causing the model to exhibit a severe memory effect on the training data, making it highly prone to overfitting during training and resulting in a loss of generalization capability. When  $\sigma = 0.3$ ,  $R^2$  is 0.9949, and the RMSE is 3.0990 °C; the fitting accuracy remains high, and both model smoothness and stability are superior to those at  $\sigma = 0.1$ . When  $\sigma$  is further increased to 1.0,  $R^2$  drops to 0.8229, and the RMSE rises to 18.2325 °C, indicating that the model begins to exhibit an underfitting trend. Considering both fitting accuracy and generalization capability, this paper selects  $\sigma = 0.3$  as the optimal width parameter.



**Figure 8.** Parameter analysis of the RBF model: (a) sensitivity analysis of width parameter  $\sigma$  ( $M = 100$ ); (b) sensitivity analysis of training sample size  $M$  ( $\sigma = 0.3$ ).

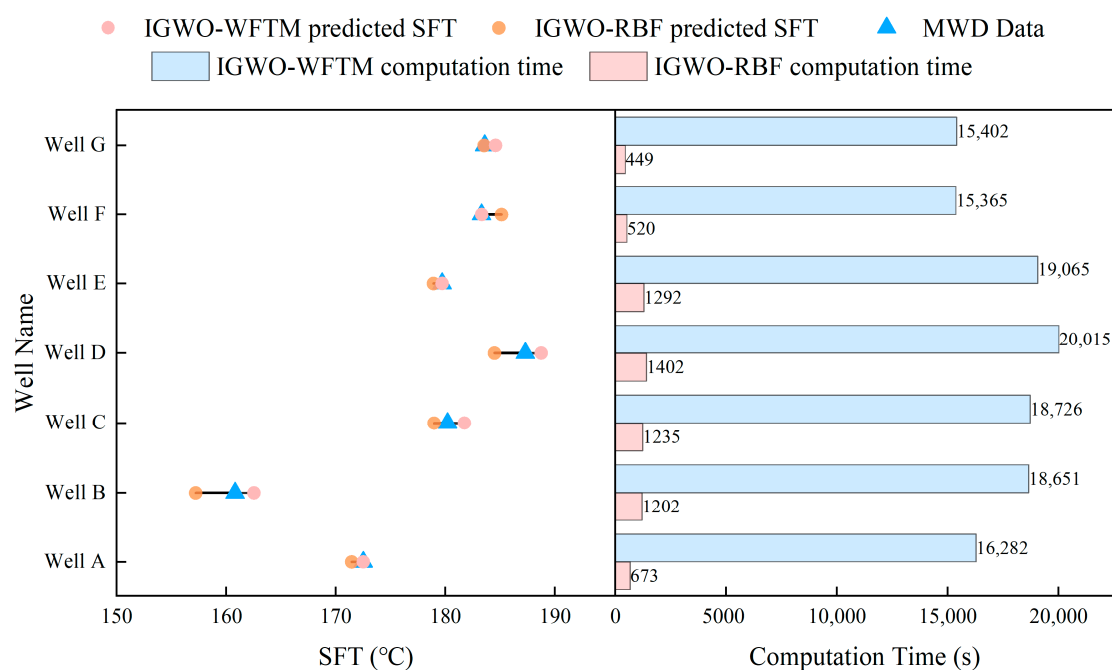
After determining  $\sigma = 0.3$ , a sensitivity analysis of the number of training samples  $M$  is further conducted. The tested  $M$  values include 20, 40, 50, 60, 80, 100, 120, 150, and 200. As shown in Figure 8b, when  $M = 20$ , the RMSE is 11.7 °C and  $R^2$  is 0.9243. As  $M$  increases, the RMSE gradually decreases. When  $M = 100$ , the RMSE drops to 11.2075 °C,

and  $R^2$  improves to 0.9242. When  $M = 150$ , the RMSE is 11.2019 °C and  $R^2$  is 0.9241. When  $M = 200$ , the RMSE is 11.2021 °C and  $R^2$  is 0.9240. From  $M = 100$  to  $M = 200$ , the RMSE decreases by only 0.0055 °C, and the change in  $R^2$  is less than 0.0002, indicating that the improvement in accuracy is extremely limited. It is evident that once  $M$  reaches 100, the model has entered a steady state, and further increases in the number of training samples yield negligible performance gains. Considering both computational efficiency and practical field requirements,  $M = 100$  is selected as the optimal number of training samples.

The predicted values of the RBF surrogate model are in excellent agreement with the measured values, with an average residual of only 0.0086 °C. The error distribution is normal and concentrated around zero, demonstrating the feasibility of replacing the WFTM for temperature prediction. The optimally selected parameter combination of  $\sigma = 0.3$  and  $M = 100$  achieves a favorable balance between fitting accuracy and generalization capability, avoiding both overfitting and underfitting. In terms of computational efficiency, the RBF surrogate model can significantly reduce computational cost while ensuring prediction accuracy, meeting the timeliness requirements of while-drilling operations, and providing a reliable surrogate model foundation for the efficient inversion of the subsequent IGWO algorithm.

### 3.2.3. Accuracy Evaluation of the IGWO-RBF Model

Given the excessively high computational cost of the IGWO inversion method based on iterative WFTM calculations discussed previously, and considering that the RBF surrogate model can efficiently replace the WFTM for BHCT prediction, this section evaluates the inversion accuracy and computational efficiency of the IGWO-RBF model. Seven ultra-deep wells are selected, and both IGWO-WFTM and IGWO-RBF are employed to invert the SFT. The training and computation times for each well are recorded, with the results presented in Figure 9.



**Figure 9.** Comparison of inversion accuracy and computation time between IGWO-RBF and IGWO-WFTM.

The predicted values of the IGWO-WFTM model show a higher degree of agreement with the MWD measured data, achieving a mean absolute error of 0.63 °C and a maximum

absolute error of 1.73 °C across the seven wells. In contrast, the IGWO-RBF model yields a mean absolute error of 1.38 °C and a maximum absolute error of 3.59 °C, with its accuracy slightly lower than that of the former. Although the prediction deviation distribution of IGWO-WFTM is more concentrated across the wells, and IGWO-RBF exhibits noticeable fluctuations in individual wells, the absolute errors of the remaining wells are all within 2.5 °C, and the overall trend is consistent with the measured values. To provide a complete runtime assessment, we selected Well B as a representative case and decomposed its total computational time of 1202.49 s into three stages: offline data generation took approximately 1072 s, RBF surrogate training required about 128 s, and IGWO online inversion took only about 2.49 s. Across the seven wells, the average total training and inversion time per well for IGWO-RBF was 967 s, compared with 17,800 s for IGWO-WFTM, representing an efficiency improvement of approximately 18 times. For online re-training when entering a new section with different bottomhole assembly and drilling fluid systems, the retraining can be performed in advance using the upcoming section's wellbore parameters and thermophysical properties during non-drilling operations such as tripping or equipment maintenance. The entire retraining process takes about 15 to 20 min per section, which does not occupy any drilling time and confirms the practical feasibility of the proposed method.

Despite its marginally superior accuracy, the IGWO-WFTM approach remains impractical for real-time field implementation, as each fitness evaluation entails a full-scale WFTM run, making the overall computational burden unacceptably heavy for on-site applications during drilling. The inversion accuracy of IGWO-RBF exhibits some fluctuations, but its overall mean absolute error of 1.38 °C and mean relative error of approximately 0.8% remain within the acceptable range for engineering applications, while the computational efficiency is substantially improved. By adopting offline training of the RBF surrogate model and online rapid inversion, this model achieves a favorable balance between accuracy and efficiency, making it suitable for real-time geothermal gradient inversion while drilling in ultra-deep wells.

#### 3.2.4. Analysis of Observation Noise Effects

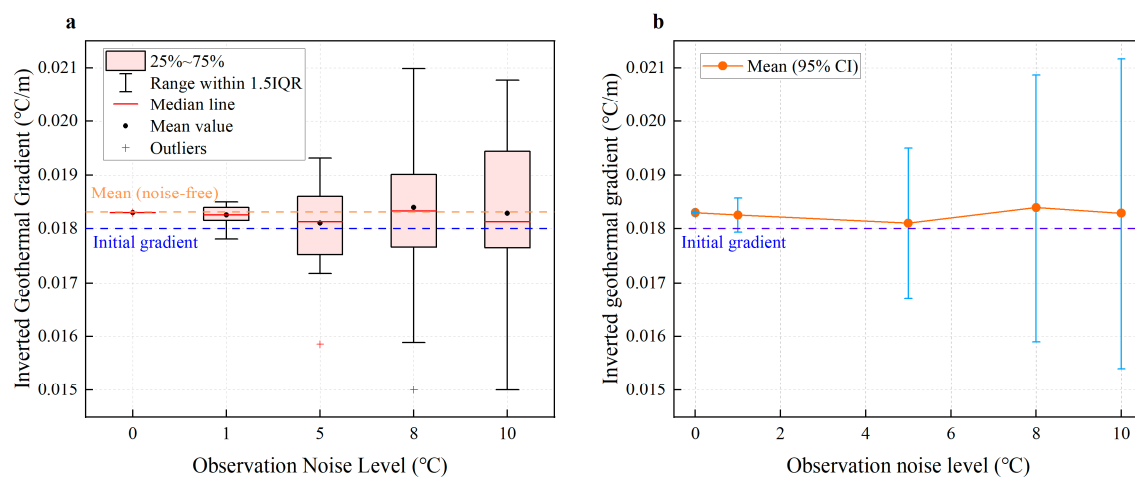
In the practical drilling process of ultra-deep oil wells, the BHCT acquired by MWD tools is inevitably subject to measurement errors and random fluctuations, influenced by the accuracy of the instruments and the high-temperature, high-pressure downhole environment. To comprehensively evaluate the impact of measurement noise and operational variability on the inversion results of the IGWO-RBF method, a Monte Carlo simulation is further introduced in this study to perform propagation analysis of observation noise.

Taking the actual engineering parameters of Well A as a basis, Gaussian white noise with varying standard deviations ( $\sigma$ ) was artificially superimposed on the measured bottom-hole temperature input (159 °C) to simulate sensor errors under different operating conditions. Based on the conventional operating conditions of logging instruments in ultra-deep oil wells, the noise levels  $\sigma$  were set to 0, 1, 5, 8, and 10 °C. For each noise level, the IGWO-RBF inversion was independently repeated 30 times to characterize the combined variability arising from operational randomness and measurement noise.

Figure 10 presents the statistical results of the inverted geothermal gradient at different noise levels. In terms of Figure 10a, the box plots quantitatively illustrate the distribution characteristics. At 0 °C and 1 °C, the results are tightly clustered; for example, the overall range of the 1 °C case is only 0.00068 °C/m, with an extremely narrow interquartile range (IQR). As the noise intensifies to 5 °C, notable outliers begin to appear (spreading from 0.01586 to 0.01931 °C/m). Under the extreme 10 °C condition, the dispersion expands explosively, with the overall range surging to 0.00577 °C/m and the IQR

increasing 15-fold to approximately  $0.0015\text{ }^{\circ}\text{C}/\text{m}$ , indicating significantly elongated distribution tails.

Regarding Figure 10b, it depicts the evolution of the mean inverted gradients and their 95% confidence intervals. Quantitative analysis reveals that while the mean inverted gradient exhibits excellent robustness, fluctuating within the narrow bounds of  $0.01810$  to  $0.01838\text{ }^{\circ}\text{C}/\text{m}$ , with a maximum relative deviation of  $<1.6\%$  from the noise-free baseline, the 95% confidence interval width undergoes a drastic expansion, leaping from a negligible  $2.67 \times 10^{-6}\text{ }^{\circ}\text{C}/\text{m}$  at  $0\text{ }^{\circ}\text{C}$  noise to  $0.00288\text{ }^{\circ}\text{C}/\text{m}$  at  $10\text{ }^{\circ}\text{C}$  noise, an increase of approximately three orders of magnitude. This stark contrast indicates that while the central tendency of the algorithm remains reliable under noise interference, the reliability of individual inversion results severely degrades under extreme conditions.



**Figure 10.** Statistical analysis of SFT inversion results under different noise levels: (a) box plot of the inverted geothermal gradient; (b) mean inverted geothermal gradient with 95% confidence interval.

It is worth emphasizing that such a  $10\text{ }^{\circ}\text{C}$  extreme noise level is physically improbable in actual field operations. In conventional downhole logging environments of ultra-deep oil wells, measurement fluctuations are typically controlled within  $0.5\text{--}5\text{ }^{\circ}\text{C}$  due to the use of industrial-grade digital temperature sensors and rigorous calibration procedures. Therefore, when restricting the evaluation to the realistic operating scenario of  $0\text{--}5\text{ }^{\circ}\text{C}$ , the true uncertainty of the IGWO-RBF inversion method remains well within acceptable engineering tolerance. This confirms that the proposed method possesses excellent anti-noise capability and high engineering reliability for practical ultra-deep oil and gas drilling applications.

## 4. Analysis of Bottomhole Temperature Fitting Performance

### 4.1. Single-Point Inversion of Bottomhole Static Formation Temperature

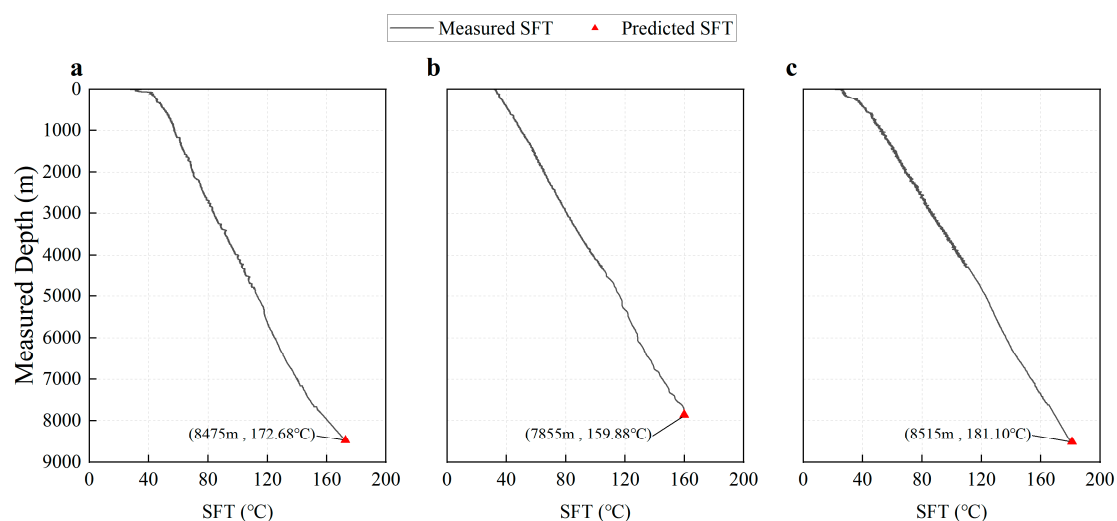
Due to the influence of surface environmental factors, the geothermal temperature in shallow formations varies considerably [33], causing the SFT measured by MWD to exhibit a distinct inflection in the shallow section, and the corresponding geothermal gradient to display significant nonlinear variations. To eliminate interference from shallow geothermal anomalies in the inversion of the deep geothermal gradient, this paper excludes the shallow temperature disturbance section during inversion and uses only the surface temperature as the upper boundary condition to calculate the deep SFT. The exclusion of the shallow disturbed section does not compromise the accuracy of the bottomhole temperature inversion, since the bottomhole temperature is governed primarily by the deep geothermal gradient and the surface temperature boundary condition, whereas the shallow anomalies have negligible influence on the deep thermal field due to the large

distance and the damping of thermal disturbances along the axial direction. To verify the prediction accuracy and applicability of the IGWO-RBF model in single-point SFT inversion, three ultra-deep wells, Well A, Well B, and Well C, located in significantly different structural belts, are selected for inversion testing. The results are shown in Figure 10.

The comparison shown in Figure 10 indicates that the SFT predictions generated by the IGWO-RBF model closely match the measured values. Only minor deviations were observed, confirming the effectiveness of this approach in capturing the underlying temperature patterns. Specifically, the measured SFT of Well A is 172.5 °C, and the inverted value is 172.3 °C, yielding an absolute error of 0.2 °C; the measured SFT of Well B is 160.8 °C, and the inverted value is 160.9 °C, yielding an absolute error of 0.1 °C; the measured SFT of Well C is 180.19 °C, and the inverted value is 180.0 °C, yielding an absolute error of 0.19 °C. The mean absolute error across the three wells is 0.16 °C, and the maximum absolute error does not exceed 0.2 °C, indicating that the proposed model can accurately invert the SFT in ultra-deep wells across different structural belts and is highly applicable.

#### 4.2. SFT Inversion at Different Vertical Depths

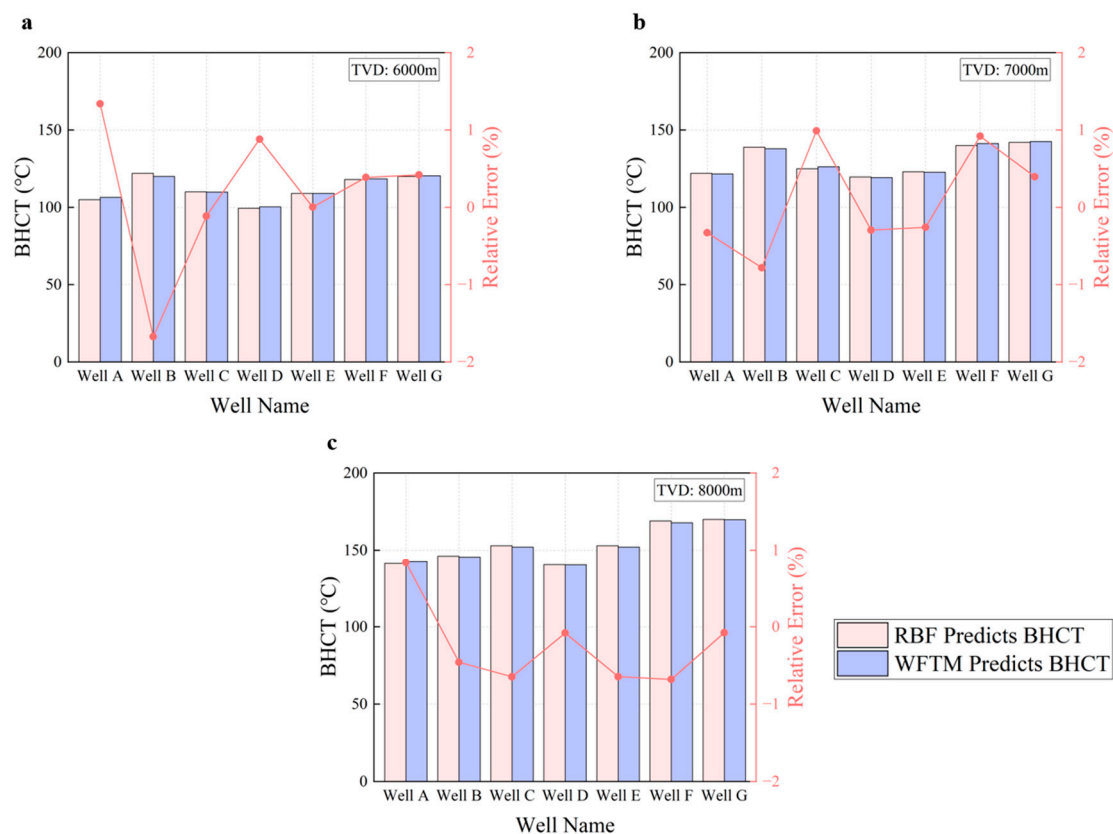
Since the geothermal gradient represents the rate of temperature change with vertical depth, the original formation temperature derived from vertical depth more objectively reflects the distribution characteristics of the geothermal field along the axial direction. To verify the adaptability of the IGWO-RBF model trained in this paper to geothermal gradients at different depths, rather than being limited to inverting a single bottomhole temperature point, seven wells with varying structural parameters, drilling fluid properties, and circulation practices are selected for SFT inversion testing. Assuming a circulation time of 200 h for the temperature recovery to reach a steady state after shut-in, temperature data at vertical depths of 6000 m, 7000 m, and 8000 m are extracted from each well for SFT inversion. For each well, an independent RBF surrogate model is trained on its wellbore configuration and thermophysical parameters, and the same model is then used to invert the SFT at different vertical depths within that well to examine the model's generalization performance and inversion consistency under variable-depth conditions. The inversion results and their comparison with MWD data are shown in Figure 11.



**Figure 11.** Comparison of bottom-hole static formation temperature inversion results for three wells: (a) Well A; (b) Well B; (c) Well C.

Figure 12 presents the relative error distribution of the BHCT predicted by the RBF surrogate model at three vertical depths: 6000 m, 7000 m, and 8000 m. The relative errors

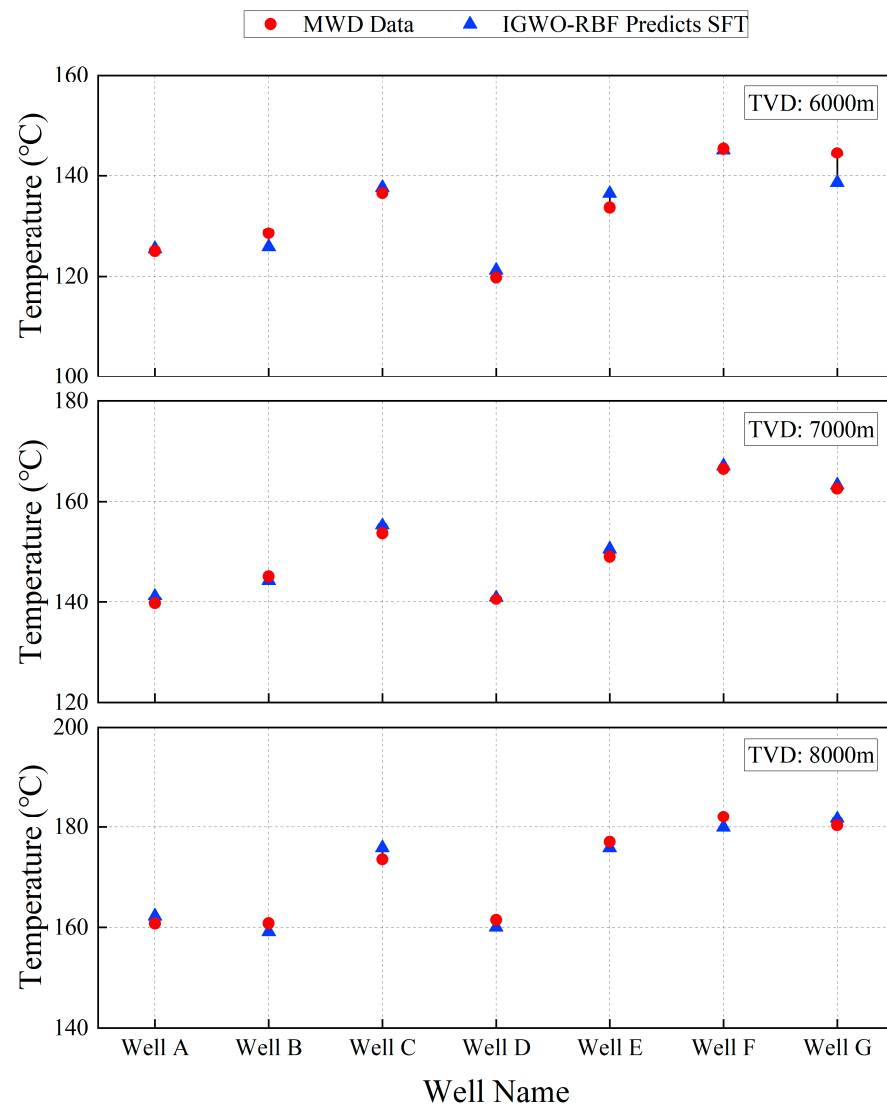
at all three depths are concentrated around the zero line, with positive and negative errors alternating and no obvious systematic overestimation or underestimation trend. Moreover, as the vertical depth increases, the error distribution range gradually narrows, indicating that the RBF model can effectively fit the nonlinear heat-transfer mapping relationship described by the WFTM, with even better fitting performance under deep, high-temperature, high-pressure conditions. The mean absolute relative errors at 6000 m, 7000 m, and 8000 m are 0.68%, 0.57%, and 0.49%, respectively, and the root mean square errors are 0.88%, 0.64%, and 0.58%, respectively, with both metrics decreasing progressively as depth increases. The maximum relative error occurs at Well B at 6000 m (−1.67%), which nevertheless remains within  $\pm 2\%$ , meeting the accuracy requirements of engineering applications. The above results demonstrate that the RBF surrogate model can effectively replace the iteratively computed WFTM while maintaining prediction accuracy, thereby providing a reliable surrogate for the efficient inversion of the subsequent IGWO algorithm.



**Figure 12.** Relative errors of BHCT predicted by RBF and WFTMs at different vertical depths: (a) 6000 m; (b) 7000 m; (c) 8000 m.

Figure 13 compares the SFT inverted by the IGWO-RBF model with the measured MWD values for the seven wells at vertical depths of 6000 m, 7000 m, and 8000 m. The inverted and measured values at all three depths exhibit good consistency, with data points distributed around the ideal fitting line. Among them, the data points at 7000 m are the most concentrated and show the closest agreement with the ideal fitting line, followed by those at 8000 m, which still maintain a good linear trend. The data points at 6000 m are more dispersed, with some wells (e.g., Well G) showing noticeable deviations, attributable to greater disturbance of shallow formations by surface environmental factors and insufficient stabilization of the temperature field. The mean absolute errors at the three depths are 2.08 °C, 1.00 °C, and 1.65 °C, respectively; the mean absolute relative errors are 1.54%,

0.67%, and 0.97%; and the root mean square errors are 2.85 °C, 1.18 °C, and 1.77 °C. At 7000 m, the absolute errors of all seven wells are within  $\pm 1.7$  °C, achieving the highest inversion accuracy. At 8000 m, the maximum absolute error is 2.29 °C (Well F). At 6000 m, except for Well G (error 5.92 °C) and Well E (error  $-2.77$  °C), the errors of the remaining wells are all within  $\pm 1.5$  °C. Overall, the IGWO-RBF model demonstrates good inversion capability across different vertical depths, validating that the proposed method is applicable to inverting the geothermal temperature profile over the entire well section, rather than being limited to a single bottomhole temperature point.



**Figure 13.** Comparison between the SFT inverted by the IGWO-RBF model and the MWD measured data at different vertical depths.

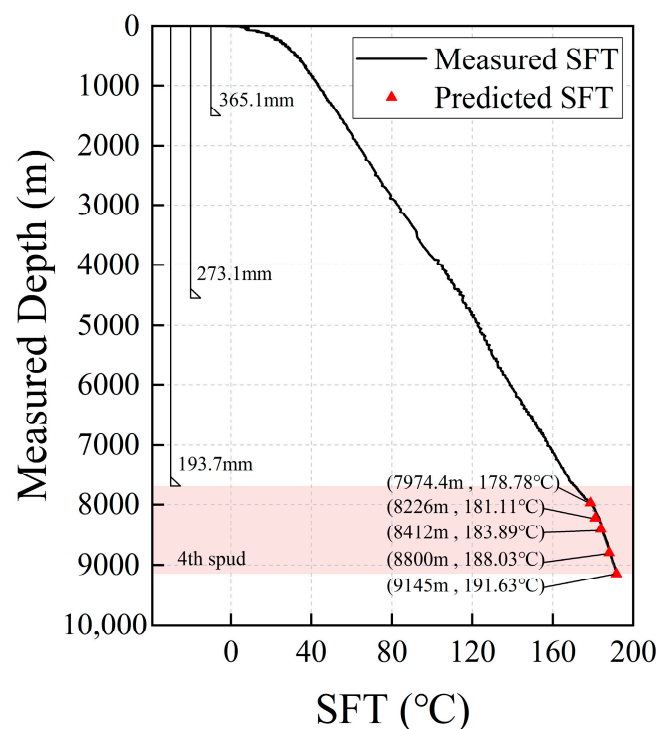
#### 4.3. Prediction of Formation Temperature Profile While Drilling

To further verify the applicability and engineering practicality of the IGWO-RBF model in predicting the formation temperature profile across the entire well section, the fourth-section drilling interval of a directional well is selected for a while-drilling temperature-profile inversion test. The vertical depth range of the fourth section of this well is 7691 m to 9145 m; the BHCT is 172 °C, and the MWD-measured temperature is 192 °C, indicating ultra-deep and ultra-high-temperature drilling conditions. Since different drilling sections correspond to significantly different wellbore configurations (casing levels,

annular dimensions), bottomhole assemblies, and drilling fluid systems, the thermophysical parameters and heat transfer boundary conditions also vary for each section.

Therefore, an independent RBF surrogate model needs to be trained for each section. In this test, the training of the RBF model for the fourth section took 1278 s (approximately 21.3 min), and this training process only needs to be completed once offline. After training, the IGWO optimization algorithm takes an average of only 2.35 s to invert the SFT at different depths, representing a computational efficiency improvement of approximately three orders of magnitude compared with directly invoking the WFTM forward model, which fully meets the engineering timeliness requirement for rapid formation temperature assessment during while-drilling operations.

Figure 14 presents the variation curves of the measured MWD temperature and the SFT predicted by the IGWO-RBF model as a function of vertical depth in the fourth-section interval of this well. In terms of the overall trend, the predicted temperature profile agrees well with the measured profile over the entire well section; both exhibit a monotonic increase with depth, and no obvious systematic deviation is observed. The mean absolute error across the five measurement points is 0.73 °C, and the mean absolute relative error is 0.39%. The maximum absolute error occurs at 8226 m (−1.956 °C), while the absolute errors at the remaining four measurement points are all less than 0.72 °C, demonstrating high inversion accuracy.



**Figure 14.** Comparison of while-drilling formation temperature profiles.

The above results indicate that the IGWO-RBF model can not only accurately invert the bottomhole SFT, but also efficiently construct a continuous geothermal temperature profile over the entire well section. By adopting the strategy of offline training of the RBF surrogate model and online rapid inversion, this method effectively overcomes the bottleneck of excessively high computational cost associated with conventional numerical models under while-drilling conditions. Moreover, the strategy of independently training the RBF model for different drilling sections can adapt to variations in wellbore configuration and thermophysical parameters across sections, demonstrating strong engineering adaptability. This model can provide reliable formation temperature boundary conditions for

real-time prediction of the wellbore temperature field, dynamic adjustment of drilling fluid properties, and borehole stability analysis during ultra-deep well drilling.

## 5. Conclusions

Addressing the engineering challenge of directly measuring SFT in ultra-deep oil well drilling, this paper establishes a WFTM and proposes an SFT inversion method based on the IGWO-RBF. Through theoretical analysis and field data validation, the following main conclusions are drawn:

(1) The established WFTM can accurately describe the wellbore–formation transient heat transfer law during drilling fluid circulation. Comparisons with the Holmes model and measured data from three ultra-deep oil wells show that the mean absolute error of the predicted temperature is 0.22 °C and the root mean square error is 0.28 °C, confirming the model's accuracy and reliability across different well conditions.

(2) To address the issues of slow convergence and low search efficiency caused by the random initialization of the standard GWO algorithm, an IGWO is proposed, incorporating an initialization strategy based on the proportional method prediction center and a dynamic gravitational guidance mechanism. Parameter sensitivity analysis indicates that a population size of 30 and a maximum of 100 iterations constitute the optimal combination, achieving a favorable balance between inversion accuracy and computational efficiency.

(3) The constructed RBF surrogate model can approximate the nonlinear mapping relationship of the WFTM with high accuracy while significantly reducing computational cost, providing a reliable surrogate model for the efficient inversion of the IGWO algorithm. The IGWO-RBF model achieves a mean absolute error of 0.16 °C in single-point SFT inversion, and the mean absolute relative error across different vertical depths is below 1.6%, validating the model's generalization capability for temperature profile inversion over the entire well section. In while-drilling prediction tests, the predicted profile agrees well with MWD measured data, with a mean absolute error of 0.73 °C and a mean absolute relative error of 0.39%. The strategy of independently training the RBF model for different drilling sections can adapt to variations in wellbore configuration and thermo-physical parameters.

(4) The proposed IGWO-RBF method does not require extensive post-drilling measured data; it requires only surface temperature and BHCT to achieve efficient and accurate SFT inversion under while-drilling conditions in ultra-deep oil wells. This provides reliable technical support for wellbore temperature field prediction, dynamic adjustment of drilling fluid properties, and borehole stability analysis in ultra-deep oil well drilling engineering.

### *Limitations and Future Work*

The proposed inversion method currently assumes constant thermophysical properties of the drilling fluid and neglects drillpipe eccentricity and borehole irregularities, which may introduce certain computational errors under ultra-deep, high-temperature, and high-pressure conditions. Moreover, the model validation is currently based on limited field data from ultra-deep oil wells, without comparison with more extensive measured field data. It should be particularly noted that all validation data involved in this study are derived exclusively from ultra-deep oil wells, with no field data from geothermal wells or other well types included; thus, its applicability to different well types requires further verification in the future.

To address these limitations, future research will primarily focus on enhancing the high-fidelity scalability of the model to better accommodate complex well configurations. Subsequent work will establish refined forward models that account for the temperature-

and pressure-dependent properties of drilling fluids, drillpipe eccentricity, and borehole irregularities, so as to effectively reduce the computational errors induced by model simplifications and enhance the applicability and robustness of the method under complex ultra-deep operational conditions.

In addition to the refinement of forward models, bridging the gap toward practical engineering applications constitutes another crucial aspect of our future work. To formulate a technically feasible roadmap, subsequent research will be dedicated to deploying the IGWO-RBF model into the on-site while-drilling data acquisition and monitoring system. By establishing such a while-drilling inversion system, real-time dynamic data processing and continuous inversion can be achieved. Furthermore, a closed-loop mechanism featuring real-time database feedback and model iterative optimization will be constructed, ultimately evolving the proposed method into a practical technical tool capable of effectively supporting engineering operations in ultra-deep and complex wells.

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## Nomenclature

|            |                                                                                                                  |
|------------|------------------------------------------------------------------------------------------------------------------|
| $a$        | convergence factor in GWO, dimensionless                                                                         |
| $b$        | bias term of the RBF network output layer, dimensionless                                                         |
| $c_{dp}$   | specific heat capacity of drillpipe, J/(kg · °C)                                                                 |
| $c_m$      | specific heat capacity of drilling fluid, J/(kg · °C)                                                            |
| $c_f$      | specific heat capacity of formation, J/(kg · °C)                                                                 |
| $D_i$      | hydraulic diameter of the corresponding region, m                                                                |
| $D_{dp,i}$ | inner diameter of drillpipe, m                                                                                   |
| $D_{dp,o}$ | outer diameter of drillpipe, m                                                                                   |
| $D_{w,i}$  | inner diameter of wellbore wall, m                                                                               |
| $D_{w,o}$  | outer diameter of wellbore wall, m                                                                               |
| $F(G)$     | fitness function value, dimensionless                                                                            |
| $G$        | the single equivalent geothermal gradient of the deep formation segment, °C/m                                    |
| $g_j$      | geothermal gradient of the $j$ -th layer segment, °C/m                                                           |
| $h_{dp,i}$ | convective heat transfer coefficient between drilling fluid and inner wall of drillpipe, sW/(m <sup>2</sup> ·°C) |
| $h_{dp,o}$ | convective heat transfer coefficient between annulus fluid and outer wall of drillpipe, W/(m <sup>2</sup> ·°C)   |

|                        |                                                                                                        |
|------------------------|--------------------------------------------------------------------------------------------------------|
| $h_w$                  | convective heat transfer coefficient between annulus fluid and borehole wall, $W/(m^2 \cdot ^\circ C)$ |
| $k$                    | steepness coefficient of the S-type curve, dimensionless                                               |
| $lb_j$                 | lower bound of the j-th dimensional search space, dimensionless                                        |
| $M$                    | number of hidden layer nodes in the RBF network, dimensionless                                         |
| $P_k$                  | coefficient vector controlling distance weight in GWO, dimensionless                                   |
| $p(t_i)$               | normalized iteration progress, dimensionless                                                           |
| $p_0$                  | midpoint position of S-type curve, dimensionless                                                       |
| $Q_k$                  | coefficient vector controlling step size in GWO, dimensionless                                         |
| $q$                    | volumetric flow rate of drilling fluid, $m^3/s$                                                        |
| $R^2$                  | coefficient of determination, dimensionless                                                            |
| $r$                    | radial distance from wellbore center, m                                                                |
| $r_{w,i}$              | inner radius of the wellbore wall, m                                                                   |
| $r_{w,o}$              | outer radius of the wellbore wall, m                                                                   |
| $r_1, r_2$             | uniformly distributed random vectors in $[0, 1]$ , dimensionless                                       |
| $t$                    | circulation time, s                                                                                    |
| $t_i$                  | current iteration number, dimensionless                                                                |
| $T$                    | temperature, $^\circ C$                                                                                |
| $T(G)$                 | the BHCT predicted by the RBF network, $^\circ C$                                                      |
| $T_{bottom}^{meas}$    | measured bottom-hole temperature, $^\circ C$                                                           |
| $T_{max}$              | maximum iteration number                                                                               |
| $T_{surface}$          | surface temperature, $^\circ C$                                                                        |
| $T_d$                  | temperature of drilling fluid inside drillpipe, $^\circ C$                                             |
| $T_{dp}$               | temperature of drillpipe wall, $^\circ C$                                                              |
| $T_{an}$               | the temperature of the annulus fluid, $^\circ C$                                                       |
| $T_{w,i}$              | the temperature of the inner wellbore wall, $^\circ C$                                                 |
| $T_w$                  | the temperature of the wellbore wall, $^\circ C$                                                       |
| $T_f$                  | the temperature of the formation, $^\circ C$                                                           |
| $T_{in}$               | the inlet temperature of the drilling fluid at the wellhead, $^\circ C$                                |
| $ub_j$                 | upper bound of the j-th dimensional search space, dimensionless                                        |
| $w_m$                  | connection weight between the m-th hidden layer node and the output layer, dimensionless               |
| $X$                    | Position vector of the current grey wolf individual, dimensionless                                     |
| $X_k$                  | Position vector of leader wolf k ( $\alpha$ , $\beta$ , or $\delta$ wolf), dimensionless               |
| $X_{center,j}$         | ideal center value of the j-th dimension predicted by the proportional method, dimensionless           |
| $X_{center}$           | center vector predicted by the proportional method, dimensionless                                      |
| $X_{new}$              | candidate position vector computed by standard GWO, dimensionless                                      |
| $x_{i,j}^{(0)}$        | initial position of the j-th dimension of the i-th individual, dimensionless                           |
| $\delta$               | disturbance amplitude coefficient, dimensionless                                                       |
| $\varepsilon$          | uniformly distributed random number in $[-1, 1]$ , dimensionless                                       |
| $\lambda_d$            | the thermal conductivity of drilling fluid, $W/(m \cdot ^\circ C)$                                     |
| $\lambda_{dp}$         | the thermal conductivity of the drillpipe wall, $W/(m \cdot ^\circ C)$                                 |
| $\lambda_f$            | the thermal conductivity of the formation, $W/(m \cdot ^\circ C)$                                      |
| $\mu_m$                | center vector of the m-th basis function, dimensionless                                                |
| $\rho_d$               | the density of drilling fluid inside drillpipe, $kg/m^3$                                               |
| $\rho_{dp}$            | the density of drillpipe wall, $kg/m^3$                                                                |
| $\rho_{an}$            | the density of drilling fluid in annulus, $kg/m^3$                                                     |
| $\rho_f$               | the density of formation, $kg/m^3$                                                                     |
| $\sigma_m$             | width parameter of the m-th basis function, dimensionless                                              |
| $\phi_m(G)$            | output value of the m-th hidden layer node, dimensionless                                              |
| $\omega_{center}(t_i)$ | gravitational weight at the $t_i$ -th iteration, dimensionless                                         |
| $\omega_{max}$         | upper limit of gravitational weight, dimensionless                                                     |
| $\omega_{min}$         | lower limit of gravitational weight, dimensionless                                                     |
| $\ G - \mu_m\ $        | Euclidean distance between the input vector and the center vector, dimensionless                       |
| $\Delta z$             | grid length, m                                                                                         |
| Abbreviations          |                                                                                                        |
| ANN                    | Artificial Neural Network                                                                              |

|          |                                                    |
|----------|----------------------------------------------------|
| BHCT     | Bottomhole Circulating Temperature                 |
| DE       | Differential Evolution                             |
| ELM      | Extreme Learning Machine                           |
| GA       | Genetic Algorithm                                  |
| GSA      | Gravitational Search Algorithm                     |
| GWO      | Grey Wolf Optimizer                                |
| IGWO     | Improved Grey Wolf Optimizer                       |
| IGWO-RBF | Improved Grey Wolf Optimizer–Radial Basis Function |
| IQR      | Interquartile Range                                |
| LSSVM    | Least Squares Support Vector Machine               |
| MSE      | Mean Squared Error                                 |
| MWD      | Measurement While Drilling                         |
| PNN      | Probabilistic Neural Network                       |
| PSO      | Particle Swarm Optimization                        |
| RBF      | Radial Basis Function                              |
| RMSE     | Root Mean Square Error                             |
| RSM      | Response Surface Methodology                       |
| SFT      | Static Formation Temperature                       |
| SVM      | Support Vector Machine                             |
| WFTM     | Wellbore–Formation Transient Heat Transfer Model   |

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