

Article

Thermomechanical Field Prediction in Subbase Materials Using Physics-Informed Neural Networks

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Abstract

To address the complexities associated with highly nonlinear phase transitions in subbase materials subjected to unidirectional freezing—where direct observation of internal states is challenging—and the limitations of conventional numerical methods in concurrently reconstructing multiphysical fields from sparse experimental data, this study investigates saturated subbase materials from the Dashixia Project. Laboratory freezing experiments were performed, and a two-dimensional axisymmetric physics-informed neural network (PINN) model integrating thermomechanical behavior, phase transition, and deformation was developed. The model incorporated the heat conduction equation (accounting for latent heat), the freezing fraction relaxation equation, and the effective deformation equation into the loss function, employing a continuous freezing fraction to represent phase-change phenomena. Additionally, constraints on the average displacement of the top surface, enhanced sampling near the freezing front, and a dynamic weighting strategy were introduced to improve model performance. This approach enabled the simultaneous prediction of temperature, freezing fraction, and displacement fields. The model achieved a temperature prediction root mean square error (RMSE) of 4.112 °C with an R^2 of 0.600, and a top-surface displacement prediction RMSE of 0.0051 with an R^2 of 0.993. Observations indicated that the freezing front predominantly progressed from the top downward, while the displacement field exhibited a partitioned pattern characterized by negative displacement in the lower region and positive displacement in the upper region. The proposed methodology offers a novel framework for multi-field reconstruction and frost heave prediction in subgrade materials under conditions of limited observational data.

Keywords: physical information neural network; subgrade material; thermo-mechanical coupling; phase change; frost heave deformation prediction



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1. Introduction

In the field of cold-region engineering, subgrade materials serve not only fundamental roles such as load transfer, drainage facilitation, and structural buffering but also actively contribute to temperature regulation and frost heave mitigation within the soil-subgrade system. Consequently, these materials constitute a vital structural layer that significantly influences the long-term performance and durability of infrastructure in cold environments.

As engineering endeavors—including high-altitude railways, highway subgrades, channel linings, and high-fill slopes—progress into high-latitude and high-altitude areas, challenges related to the thermal response, phase-change phenomena, and deformation behavior of subgrade materials under freeze–thaw cycles have become increasingly pronounced. Current studies reveal that engineering failures in permafrost and seasonally frozen soil regions are frequently associated with temperature migration, water–ice phase transitions, unfrozen water dynamics, and the consequent frost heave, thaw settlement, and strength deterioration [1–4]. Moreover, the thermal regime of permafrost and the characteristics of the active layer are subject to continuous regional-scale evolution, thereby heightening the demand for precise prediction of the thermo-mechanical behavior of engineering materials in cold-region contexts [5–7].

In contrast to conventional homogeneous fine-grained soils, engineering subgrade materials exhibit a distinctive multiscale structure composed of “coarse-grained skeleton—fine-grained interstitial matrix—pore water.” Their thermal properties, freezing behavior, and deformation responses are collectively governed by factors including particle size distribution, pore architecture, moisture content, and temperature gradients [8–10]. Variables such as fine-grained content, freezing temperature gradients, boundary heat transfer conditions, and freeze–thaw cycles significantly influence the advancement of the freezing front and the development patterns of frost heave [11–13]. Empirical observations from both field applications and laboratory experiments reveal that frost heave in subbase materials typically manifests phase-dependent behavior, spatial heterogeneity, and path-dependent characteristics [14–16]. Consequently, assessments based solely on compaction parameters at ambient temperature, isolated temperature measurements, or empirical freeze resistance criteria are insufficient to fully capture the complex dynamic interplay among internal temperature variations, phase transitions, and deformation processes during freezing.

In the study of frozen and freezing soil masses, the predominant methodologies currently employed include analytical techniques, finite difference methods, and finite element methods. The theoretical framework underpinning investigations into the freezing front, heat and moisture migration, and frost heave phenomena is grounded in the classical Stefan problem, heat conduction theory, and thermohydraulic coupling models [17–21]. These models have been effectively applied to various scenarios, such as frozen soil slopes, thaw settlement in ice-rich soils, seasonally frozen soil subgrades, and saline frozen soils, thereby advancing the understanding of multi-field coupling mechanisms [22,23]. Nonetheless, when addressing subgrade material specimens characterized by complex boundary conditions, pronounced phase-change zone transitions, and limited observational data, conventional numerical approaches encounter three principal challenges. First, the presence of steep temperature gradients and substantial variability in material properties near the freezing front renders computational outcomes highly sensitive to mesh discretization and parameter selection. Second, the interdependence among the temperature field, phase-change state, and displacement field complicates the simultaneous inversion and calibration of multi-field parameters, resulting in considerable computational demands. Third, laboratory experiments typically provide only sparse measurements of temperature and surface displacement, which impedes the direct reconstruction of continuous spatiotemporal field distributions within the specimen. Consequently, a critical issue in predicting the freezing behavior of subgrade materials lies in effectively leveraging limited experimental data while ensuring adherence to physical principles.

Physics-Informed Neural Networks (PINNs) represent a promising methodology for solving partial differential equations by incorporating sparse data through the integration of governing equations, initial and boundary conditions, as well as observational data

into the neural network's loss function [24]. Karniadakis et al. have extensively detailed the theoretical underpinnings of physics-informed machine learning and explored its potential applications within the domain of scientific computing [25]. Recent advancements in techniques such as adaptive weighting schemes, adaptive activation functions, gradient boosting, and adaptive sampling have enhanced the training stability and computational precision of PINNs, particularly when addressing complex coupled systems and multiscale phenomena [26–28]. In contrast to purely data-driven models, PINNs leverage governing equations to impose constraints on predictions in unobserved regions; compared to conventional numerical methods, they enable the integration of discrete experimental data and the generation of continuous field variable outputs within a unified computational framework [29,30]. Consequently, PINNs offer a novel technical approach to overcoming challenges associated with sparse observational data, pronounced phase-change nonlinearities, and the simultaneous identification of multi-field states in subgrade freezing experiments.

Contemporary machine learning and physics-informed learning methodologies have been progressively employed in the prediction of frost heave, identification of freeze–thaw deformation, and multi-physical field analysis of frozen soils [31–34]. Nonetheless, existing studies predominantly concentrate on natural permafrost or general fine-grained frozen soils. Physics-Informed Neural Networks (PINNs) have been extensively utilized to integrate partial differential equations with phase-change phenomena; thus, the novelty of the present study does not reside in the introduction of a fundamentally new PINN framework, but rather in its specific application and modeling strategy tailored to engineering subgrade material freezing experiments. For subgrade materials exhibiting both coarse- and fine-grained characteristics alongside engineering structural properties, there remains an absence of a unified modeling approach capable of concurrently assimilating finite-temperature and displacement data while reconstructing the temperature field, freezing fraction field, and displacement field. Moreover, given the lack of direct measurements of internal stress, strain, and pore water pressure, further investigation is warranted to elucidate how to characterize the minimal-order coupling relationships among heat conduction, latent heat associated with phase change, and frost heave deformation using a reduced set of state variables.

In light of the aforementioned challenges, this study seeks to evaluate the capability of Physics-Informed Neural Networks (PINNs) to accurately reconstruct and predict the temperature field, freeze fraction field, and vertical displacement field within unidirectionally freezing subgrade materials under conditions of limited experimental observations. To achieve this objective, saturated cylindrical subgrade specimens were selected as the research focus, and a two-dimensional axisymmetric computational domain was utilized. Rather than explicitly modeling the seepage process, the water–ice phase transition was represented through a continuous freezing fraction. The governing equations—including the heat conduction equation accounting for latent heat during phase change, the freezing fraction relaxation equation, and the effective deformation equation—were collectively embedded within the PINN loss function. Model constraints were further reinforced by incorporating temperature measurements at multiple depths alongside average displacement data obtained from the top surface. To enhance training stability, strategies such as intensified sampling near the freezing front, dynamic loss weighting, and staged optimization were implemented. This investigation primarily addresses the following scientific inquiries: (1) the feasibility of simultaneously reconstructing the three state fields under sparse observational data; (2) the spatiotemporal coupling relationships among temperature reduction, freezing fraction augmentation, and displacement evolution; and (3) the model's ability to identify the freezing front and predict the full progression of frost

heave deformation. The findings of this research are expected to offer empirical support and methodological guidance for analyzing the frost sensitivity of subbase materials in cold regions, forecasting frost heave deformation, and developing comprehensive coupled thermal–hydraulic–mechanical models.

2. Theoretical Foundations and Research Methods

2.1. Test Materials

The subgrade raw material was obtained from the Dashixia Project site. Specimens were prepared to achieve a controlled porosity of 16% and a dry density of 2.31 g/cm^3 , molded into saturated cylindrical samples measuring 30 cm in diameter and 30 cm in height. During preparation, water was added to the mixture, which was then thoroughly homogenized and allowed to rest. Subsequently, the mixture was vibrated and compacted in three successive layers, each with a thickness of 10 cm, within the test mold. Following specimen formation, saturation was achieved using the water-head saturation technique. PT100 temperature sensors were installed circumferentially around the specimens at 5 cm intervals to monitor temperature variations throughout the freezing process. Concurrently, the specimens were insulated to ensure thermal stability. A displacement sensor was affixed to the specimen's top plate to measure frost heave deformation, as illustrated in Figure 1. The temperature of the top plate was initially set to -15°C and maintained for 30 min, after which the freezing test commenced with the top plate temperature adjusted to -16.4°C . Both the indoor ambient temperature and the bottom plate temperature were maintained at 1°C . Temperature and deformation data were recorded hourly over a 72 h freezing period.

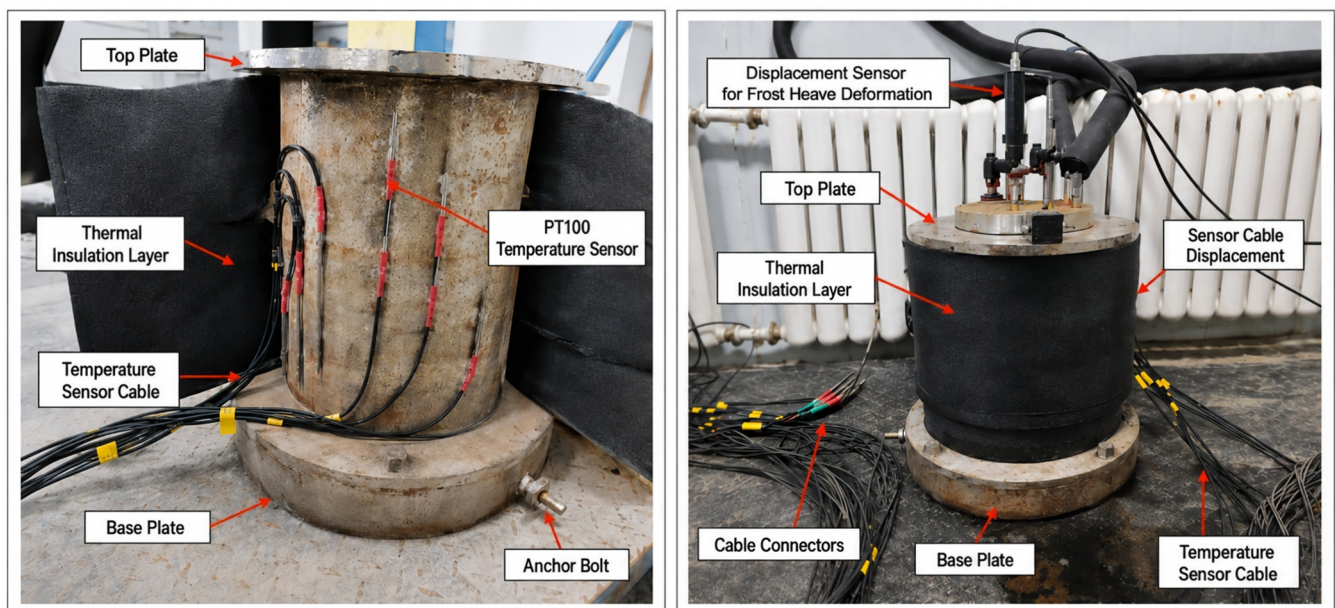


Figure 1. Frost-heave and thaw-settlement gauge with sensor layout.

The specimen is modeled as a cylinder with a radius of $R = 15 \text{ cm}$ and a height of $H = 30 \text{ cm}$. Assuming the test setup and boundaries are approximately axisymmetric, the computational domain is defined as $\Omega = \{(r, z) | 0 \leq r \leq R, 0 \leq z \leq H\}$. In this paper, the coordinate system is uniformly defined as $z = 0$ for the specimen's base and $z = H$ for the top.

Normalized inputs are used to improve the optimization conditions for the neural network, but the control equations must be calculated in physical coordinates.

$$\hat{r} = \frac{r}{R}, \hat{z} = \frac{z}{H}, \hat{t} = \frac{t}{t_{\max}}, t_{\max} = 4320 \text{ min.} \quad (1)$$

2.2. Thermal–Phase-Change Control Equations

In saturated subgrade materials, the freezing process involves concurrent sensible heat variations and latent heat release. Heat transfer within these materials is governed not only by temperature gradients but is also substantially affected by the phase transition between water and ice. Prior research has demonstrated that, in freezing soils or analogous porous media, the latent heat associated with phase change modifies the local equivalent heat capacity and the distribution of temperature gradients. Consequently, this phenomenon significantly impacts both the propagation velocity of the freezing front and the spatial configuration of the temperature field [35,36]. This paper employs an effectively parameterized heat–phase-change equation that accounts for the three dominant mechanisms: sensible heat, latent heat, and axisymmetric heat transfer. The temperature $T(r, z, t)$ and the freezing volume fraction $\phi(r, z, t) \in [0, 1]$ satisfy

$$\rho c \frac{\partial T}{\partial t} + \rho L \frac{\partial \phi}{\partial t} - \lambda \nabla_{axi}^2 T = 0. \quad (2)$$

where the first term represents the change in the material's sensible heat, the second term represents the latent heat of the ice–water phase transition, and the third term represents heat conduction. The axisymmetric Laplace operator is given by

$$\nabla_{axi}^2(\cdot) = \frac{\partial^2(\cdot)}{\partial r^2} + \frac{1}{r} \frac{\partial(\cdot)}{\partial r} + \frac{\partial^2(\cdot)}{\partial z^2}. \quad (3)$$

The equilibrium freezing fraction is approximated using a continuous logical function:

$$\phi_{eq}(T) = \left[1 + \exp\left(k_f(T - T_{ph})\right) \right]^{-1}. \quad (4)$$

This function approaches 1 at $T < T_{ph}$ and approaches 0 at $T > T_{ph}$, with k_f controlling the width of the phase transition region. To avoid the excessive stiffness in the time evolution caused by directly setting $\phi = \phi_{eq}$, a first-order relaxation equation is introduced

$$\frac{\partial \phi}{\partial t} + \tau_{\phi}^{-1}(\phi - \phi_{eq}) = 0. \quad (5)$$

Equations (2)–(5) offer a concise and efficient representation of thermal phase-change phenomena, particularly suited for scenarios with limited experimental data. These equations are advantageous due to their minimal parameter requirements and their compatibility with direct computation through automatic differentiation. However, they do not explicitly model processes such as water migration, salinity variations, or the hysteresis observed in freezing characteristic curves. Consequently, the freezing fraction derived from these equations should be interpreted as a continuous state variable that aligns with temperature and latent heat, rather than as a precise measure of ice volume fraction obtained through direct calibration methods such as nuclear magnetic resonance (NMR) or computed tomography (CT).

2.3. Deformation Prediction Equations and Average Top Surface Area

The third output of the network is the axisymmetric vertical displacement $u(r, z, t)$. In the absence of observations of internal stresses, strains, and pore water pressure, this paper

does not employ elastic-plastic constitutive models with numerous parameters, but instead uses diffusion–source-term-type effective evolution equations [37]:

$$\frac{\partial u}{\partial t} - \eta_u \nabla_{axi}^2 u - [\alpha_T(T - T_0) + \alpha_\phi \phi] = 0. \quad (6)$$

Here, T_0 is the average of the initial temperature profile, $\alpha_T(T - T_0)$ represents the thermal contraction or expansion drive, $\alpha_\phi \phi$ represents the volume expansion drive caused by freezing, and η_u controls spatial consistency. The purpose of this equation is to establish a lowest-order closed-form relationship between temperature, phase change, and displacement, rather than to replace the complete stress balance equations.

The experimental displacement gauge reflects the overall response of the circular top surface. If the values at the mesh points at $r = 0$, $z = H$ are taken directly, this will cause a mismatch between the model's monitored quantities and the experimental measurements. The area-averaged displacement of the axisymmetric top surface should be written as

$$\bar{u}_{top}(t) = \frac{1}{\pi R^2} \int_A u \, dA = \frac{2}{R^2} \int_0^R u(r, H, t) r \, dr. \quad (7)$$

In the calculation, 81 integration points are arranged at $r \in [0, R]$. Trapezoidal integration is used to obtain $\bar{u}_{top}$, which is then subtracted from the initial value and compared with the experimental relative displacement. The radial weighting r ensures that the outer annular region has a larger area than the central region, thereby preventing the central point from unduly dominating the entire top surface. The physical parameters in the model and their roles are shown in Table 1.

Table 1. Physical Parameters and Their Roles in the Model.

Parameter	Symbol	Value	Unit	Role
Specimen Radius	R	15	cm	Axially symmetric geometry
Specimen height	H	30	cm	Vertical computational domain
Density	ρ	2.31	$\text{g}\cdot\text{cm}^{-3}$	Sensible heat term
Equivalent specific heat	c	0.22	$\text{J}\cdot\text{g}^{-1}\cdot^\circ\text{C}^{-1}$	Same effective value is used for both the frozen and unfrozen phases
Latent heat	L	334	$\text{J}\cdot\text{g}^{-1}$	Ice-water phase transition
Equivalent thermal conductivity	λ	1.50	$\text{J}\cdot\text{min}^{-1}\cdot\text{cm}^{-1}\cdot^\circ\text{C}^{-1}$	Effective Parameters for the Frozen/Unfrozen Phases
Thermal deformation coefficient	α_T	1.0×10^{-4}	Model units	Temperature difference source term

Table 1. Cont.

Parameter	Symbol	Value	Unit	Role
Freezing deformation coefficient	$\alpha\varphi$	4.0×10^{-3}	Model unit	Freeze fraction term
Displacement diffusion coefficient	ηu	0.02	$\text{cm}^2 \cdot \text{min}^{-1}$	Smoothing and Propagation of the Deformation Field
Phase transition temperature	T_{ph}	0	$^{\circ}\text{C}$	Freezing Criterion
Phase transition steepness coefficient	k_f	2.5	$^{\circ}\text{C}^{-1}$	Smoothing the freezing curve
Phase transition relaxation coefficient	$\tau\varphi^{-1}$	0.15	min^{-1}	Phase variable approaches equilibrium

2.4. Network Architecture and Multi-Task Output

Physical-Informed Neural Networks (PINNs) achieve a unified solution that integrates data-driven approaches with physical constraints by incorporating control equation residuals, boundary conditions, and observational data into a single loss function [24]. A PINN takes the $(\hat{r}, \hat{z}, \hat{t})$ as input and uses a shared network to simultaneously predict three fields:

$$N_{\theta} : (\hat{r}, \hat{z}, \hat{t}) \mapsto [T_{\theta}, \phi_{\theta}, u_{\theta}]. \quad (8)$$

The main network first maps the three-dimensional input to a 192-dimensional hidden space via a fully connected layer, followed by a series of five residual blocks. Each residual block consists of two layers of fully connected layers with 192 nodes each and hyperbolic tangent activation functions, with an identity mapping added at a ratio of 0.5:

$$\mathbf{h}^{(l+1)} = \mathbf{h}^{(l)} + 0.5 \tanh\left(\mathbf{W}_2^{(l)} \tanh\left(\mathbf{W}_1^{(l)} \mathbf{h}^{(l)} + \mathbf{b}_1^{(l)}\right) + \mathbf{b}_2^{(l)}\right). \quad (9)$$

The output layer generates three scalar values, with the freezing fraction restricted to the interval between 0 and 1 through the application of a sigmoid activation function. The shared backbone architecture facilitates the exchange of information among the three tasks within a unified spatio-temporal representation. Specifically, temperature regulates the equilibrium freezing fraction; the temporal derivative of the freezing fraction contributes to the latent heat component; and both temperature and the freezing fraction collectively influence displacement.

2.5. Initial Conditions, Boundary Conditions, and Configuration Point Sampling

Instead of using a single average value across all measurement points as the initial temperature, we extract the temperature at each elevation along $t = 0$ and linearly interpolate along z to form $T(z, 0)$ at 51 vertical points. The initial freezing fraction and displacement are constrained to 0, respectively. The top and bottom temperature boundaries use the time-varying averages of the sensor channels closest to the corresponding geometric surfaces; adiabatic/symmetric Neumann conditions $\partial T / \partial r = 0$ are applied to the central axis $r = 0$ and the outer edges $r = R$. The lateral boundaries form a complete grid at 31 vertical positions and 41 time positions, rather than being sampled only along the space-time diagonal.

Each Adam iteration randomly samples 6000 uniformly distributed configuration points from the physical domain. Considering that the phase transition gradient is concentrated near the front during the early freezing stage, an additional 2500 enhanced points are sampled from a truncated normal distribution around the identified 60 min freezing onset time. Radial sampling is performed using $r = R\sqrt{\xi}$ to ensure that the configuration points are uniformly distributed in the sense of the axisymmetric surface area. Fixed configuration points are used during the L-BFGS stage to ensure the determinism of the closure function and prevent random sampling during line search from altering the objective function.

2.6. Loss Function and Phased Optimization

The total loss consists of the initial conditions, temperature boundary conditions, lateral boundary conditions, temperature measurements, top surface displacement measurements, and physical residuals [24–26]:

$$L = L_{ic} + L_{bc} + 1.2L_T + w_u L_u + w_f L_{pde}. \quad (10)$$

The temperature and displacement data terms employ scale-normalized Smooth L1 loss to reduce the dominance of outliers on the gradients; the initial displacement and boundary gradients use mean squared error. The scale-normalized Huber loss is given by

$$L_H = \frac{1}{N} \sum_{i=1}^N \text{SmoothL1} \left(\frac{\hat{y}_i - y_i}{s_y}; \beta \right). \quad (11)$$

The physical loss includes the mean square of the energy residual, 0.5 times the phase-change residual, and the deformation residual. If large physical weights are applied immediately in the early stages of training, the network must simultaneously handle data fitting, boundary compliance, and smoothing of higher-order derivatives, which can easily lead to gradient competition. Therefore, during the first 1200 epochs, w_f is linearly increased from 0.10 to 1.00, and w_u is increased from 0.30 to 1.00; From epochs 1200 to 3500, both are set to 1; after epoch 3500, they are increased to 1.6 and 1.3, respectively, to strengthen physical closure once data fitting has largely stabilized.

Optimization was conducted in two stages. In the first stage, Adam was used with an initial learning rate of 3×10^{-4} , weight decay of 10^{-6} , and gradient norm clipping of 1.0. The learning rate was halved based on the plateau phase, up to a maximum of 5000 epochs. In the second stage, strong Wolfe line search L-BFGS was used, with up to 600 iterations and a history length of 50. The random seed was fixed at 12345. All models were implemented using Python 3.10 and PyTorch 2.7, with the main model trained on a CUDA device. Table 2 lists the main training hyperparameters for the network.

Table 2. Key Training Hyperparameters.

Category	Setting	Value
Network	Input/Output	3/3
Network	Width and Residual Blocks	192, 5 blocks
Network	Activation Function	tanh; frozen fractional outputs are Sigmoid

Table 2. *Cont.*

Category	Setting	Value
Adam	Maximum epochs/initial learning rate	5000/3 × 10 ^{−4}
Adam	Weight decay/Gradient clipping	10 ^{−6} /1.0
Scheduler	ReduceLROnPlateau	factor = 0.5, patience = 250, min lr = 10 ^{−6}
L-BFGS	Maximum Iterations/History Length	600/50
Configuration points	Uniform/Front-of-the-Field Reinforcement	6000/2500 (per Adam epoch)
Boundary Grid	Vertical points × time steps	31 × 41
Validation	Time Allocation Ratio	20%
Randomness	Master random seed	12,345

2.7. Evaluation Metrics

The core metric is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}, \quad R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}. \quad (12)$$

Training convergence is not determined by visual inspection of the curve “flattening out,” but rather by a combined criterion of the relative change in the mean of two adjacent 250-epoch windows and the coefficient of variation in the current window:

$$\delta_k = \frac{|\bar{L}_{k-249:k} - \bar{L}_{k-499:k-250}|}{|\bar{L}_{k-499:k-250}| + \varepsilon} \leq 1\%, \quad \text{CV}_k \leq 5\%. \quad (13)$$

The first epoch in which both conditions are simultaneously satisfied is defined as the quantitative convergence point. This criterion constrains both long-term trends and intra-window fluctuations, making it suitable for training processes involving dynamically configured parameters and phased weights.

The meanings of the parameters in the equations of this paper are shown in Table 3.

Table 3. Definitions of Parameters in the Equations.

Parameter	Symbol	Value	Unit
Temperature field (Equations (2)–(6) and (8))	$T(r, z, t)$	Determined jointly by observations and the network	°C
Freezing Fraction (Equations (2), (4)–(6) and (8))	$\phi(r, z, t)$	$0 \leq \phi \leq 1$	Dimensionless
Axisymmetric Laplace operator (Equations (2), (3) and (6))	$\nabla_{\text{axi}}^2(\cdot)$	Calculated according to Equation (3)	cm ^{−2}
Equilibrium freezing fraction (Equations (4) and (5))	$\phi_{\text{eq}}(T)$	$0 \leq \phi_{\text{eq}} \leq 1$	Dimensionless

Table 3. Cont.

Parameter	Symbol	Value	Unit
Vertical displacement field (Equations (6)–(8))	$u(r, z, t)$	Predicted by the network	mm
Average Displacement Across the Top Surface (Equation (7))	$\bar{u}_{\text{top}}(t)$	Calculated by integration according to Equation (7)	Same as u
Top surface area (Equation (7))	A	πR^2	cm ²
Neural network mapping (Equation (8))	N_{θ}	Derived from training	—
Trainable network parameters (Equation (8))	θ	Training Iteration Update	—
Network output (Equation (8))	$T_{\theta}, \phi_{\theta}, u_{\theta}$	From Equation (8), the network output	Expressed in °C, dimensionless units, and displacement units, respectively
Hidden feature vector (Equation (9))	$h^{(l)}$	Internal computations of the network	Dimensionless
Weight matrix (Equation (9))	$W_1^{(l)}, W_2^{(l)}$	Training and learning	—
Bias vector (Equation (9))	$b_1^{(l)}, b_2^{(l)}$	Training	—
Total Loss (Equation (10))	L	$L \geq 0$	Normalized loss
Initial condition loss (Equation (10))	L_{ic}	$L_{\text{ic}} \geq 0$	Same as the total loss
Boundary Condition Loss (Equation (10))	L_{bc}	$L_{\text{bc}} \geq 0$	Same as the total loss
Temperature observation loss (Equation (10))	L_T	$L_T \geq 0$	Same as the total loss
Displacement Observation Loss (Equation (10))	L_u	$L_u \geq 0$	Same as the total loss
Physical Residual Loss (Equation (10))	L_{pde}	$L_{\text{pde}} \geq 0$	Same as the total loss
Dynamic weight of displacement loss (Equation (10))	w_u	$0.30 \rightarrow 1.00 \rightarrow 1.30$	Dimensionless
Dynamic weight for the physical loss (Equation (10))	w_f	$0.10 \rightarrow 1.00 \rightarrow 1.60$	Dimensionless
Huber loss (Equation (11))	L_H	$L_H \geq 0$	Dimensionless
Number of samples (Equations (11) and (12))	N	Positive integer	Units
Observed and predicted values (Equations (11) and (12))	$y_i, \hat{y}_i$	Determined by the experimental data and model output	Same as the target variable
Mean of Observations (Equation (12))	$\bar{y}$	$N^{-1} \sum_{i=1}^N y_i$	Same as y
Root Mean Square Error (Equation (12))	RMSE	$\text{RMSE} \geq 0$	Same as the target variable
Coefficient of determination (Equation (12))	R^2	$R^2 \leq 1$	Dimensionless
Relative Change Rate of Window Mean (Equation (13))	δ_k	At convergence, $\delta_k \leq 1\%$	Dimensionless

Table 3. Cont.

Parameter	Symbol	Value	Unit
Window Average Loss (Equation (13))	$\bar{L}_{a:b}$	Average over the interval from a to b	Same as total loss
Window Coefficient of Variation (Equation (13))	CV_k	Upon convergence, $CV_k \leq 5\%$	Dimensionless
Training Iteration Index (Equation (13))	k	$1 \leq k \leq 5000$	Epoch
Stabilization constant (Equation (13))	ε	A positive number	Dimensionless

3. Experimental Results and Analysis

3.1. Optimization Process

Figure 2 illustrates the progression of total loss, physical loss, temperature loss, and displacement loss over 5000 training epochs, with the vertical axis presented on a logarithmic scale. During the initial phase of training, the total loss exhibited a rapid decline from approximately 10^2 to below 1, demonstrating that the model swiftly captured the fundamental mapping relationships. Thereafter, the total loss decreased in a stepwise manner and stabilized at approximately 0.25 after 3000 epochs. The temperature loss showed an initial sharp reduction to around 0.18, followed by a more gradual decrease, ultimately converging near 0.15. This pattern suggests that the temperature field may constitute the primary constraint on further reductions in total loss. The displacement loss generally decreased from several units to approximately 10^{-2} ; however, it displayed notable periodic fluctuations during the intermediate phase, with particularly pronounced amplitudes observed between 1300 and 1700 iterations. These fluctuations may be attributable to adjustments in loss weighting, variations in sampling batches, or modifications to the learning rate. Overall, all loss metrics achieved reductions on the order of magnitude, and no persistent divergence was observed in the later stages, indicating that the training process ultimately attained a stable convergence state.

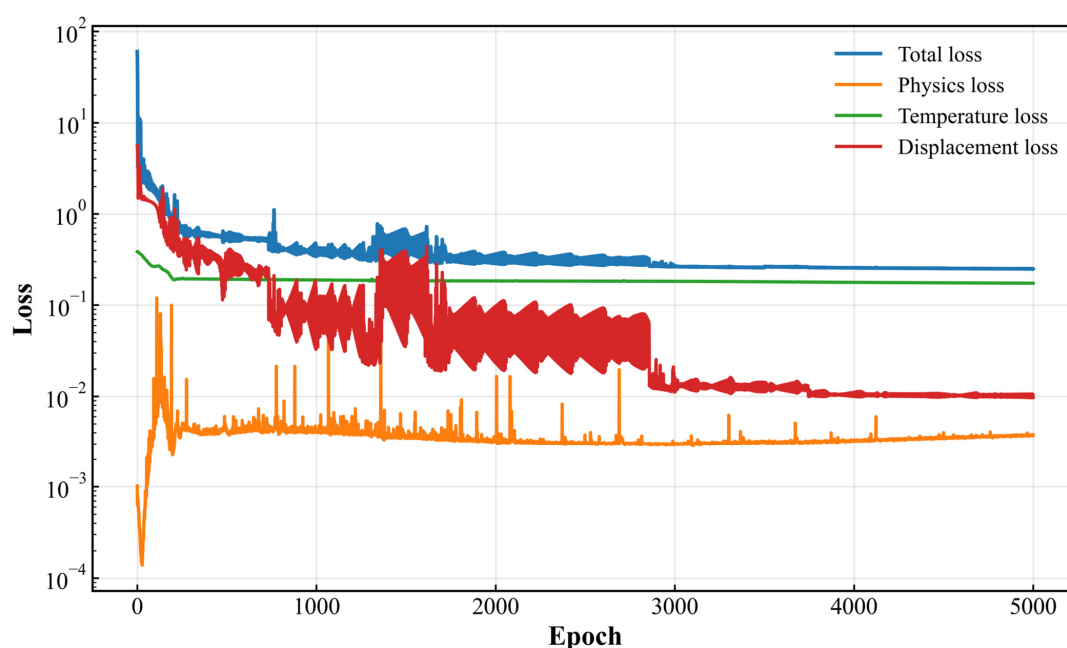


Figure 2. Evolution of total loss, physical loss, temperature loss, and displacement loss during the training of the main model.

Figure 3 depicts the convergence behavior of the total loss at both global and local levels. As shown in Figure 3a, the total loss experienced a rapid decline shortly after the commencement of training, decreasing from values in the tens to below 1 within roughly 500 iterations. Subsequently, the loss entered a phase characterized by gradual optimization. Notable spikes or pronounced oscillations were observed around 800 iterations, between 1300 and 1800 iterations, and between 2500 and 2900 iterations. These fluctuations suggest that the model underwent brief adaptation periods in response to parameter updates, modifications in the learning rate, or adjustments in the weighting of loss components. Beyond 3000 iterations, the loss curve exhibited increased smoothness, with the total loss continuing to decrease slowly and steadily. Figure 3b offers a detailed view of the interval spanning 2500 to 5000 iterations, revealing that the total loss initially oscillated between 0.28 and 0.35 before declining to approximately 0.26 after about 2900 iterations. A minor increase was noted near 3500 iterations, followed by a consistent downward trend, ultimately stabilizing around 0.25.

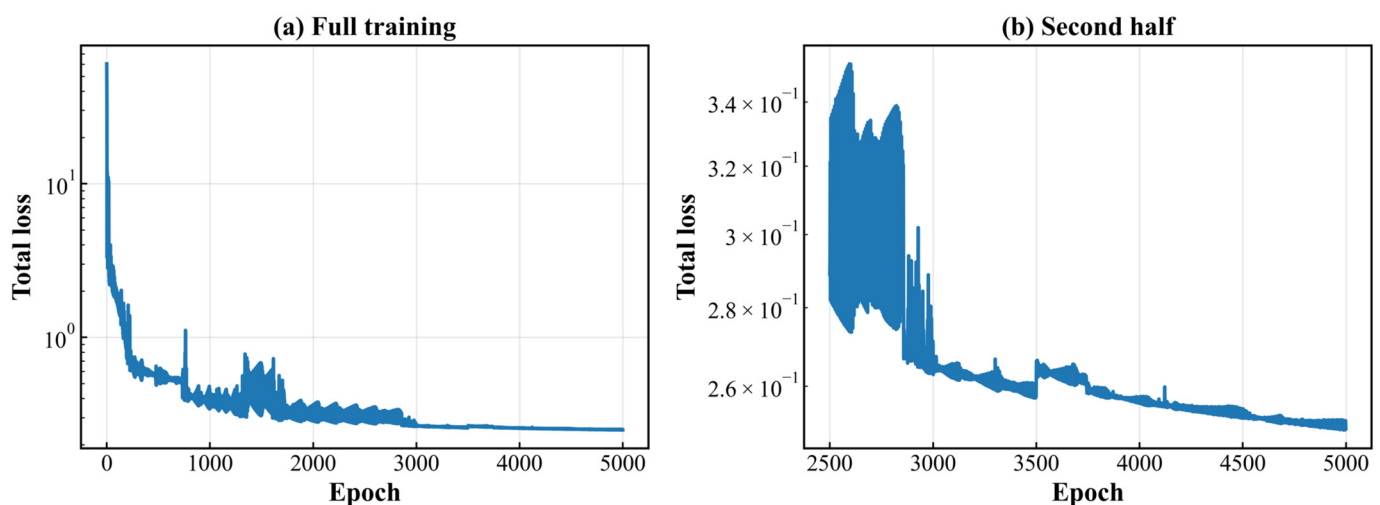


Figure 3. Overall total loss curve and a close-up of the latter half.

Figure 4 employs three quantitative metrics—total loss, the relative change rate of the windowed mean, and the windowed coefficient of variation—to evaluate the convergence status of the model. In Figure 4a, the dashed line marks approximately the 3267th iteration, signifying the point at which the comprehensive convergence criteria were first consistently satisfied. Although the total loss exhibited an overall downward trend prior to this iteration, localized oscillations persisted. Figure 4b utilizes the relative change in the sliding window mean as an indicator of trend stability, applying a threshold of 1%. During the mid-training phase, this metric intermittently approached or briefly fell below the threshold but subsequently rebounded due to fluctuations in loss, suggesting these intervals represented only transient stability. After roughly 3500 iterations, the relative change in the mean predominantly remained below 1%. Figure 4c assesses the magnitude of local fluctuations through the windowed coefficient of variation. Initially, this value exceeded 0.2 at one point; however, following approximately 3000 iterations, it rapidly declined below the 5% threshold and eventually approached zero. Synthesizing these observations of trend behavior and fluctuation intensity indicates that around iteration 3267, the model transitioned from an unstable exploratory phase to a stable convergence phase. Subsequent training primarily served to fine-tune parameters and enhance solution stability. Figure 5 depicts the evolving trade-off between the data-fitting objective and the physical constraint objective throughout training. In the initial phase, the trajectory distribution was notably broad, reflecting considerable uncertainty in parameter estima-

tion. During the intermediate phase, the trajectories exhibited a pronounced inflection, indicative of a shift in optimization direction driven by adjustments to weighting factors and learning rates. In the final phase, the samples progressively concentrated within a region characterized by reduced data loss and relatively stable physical loss.

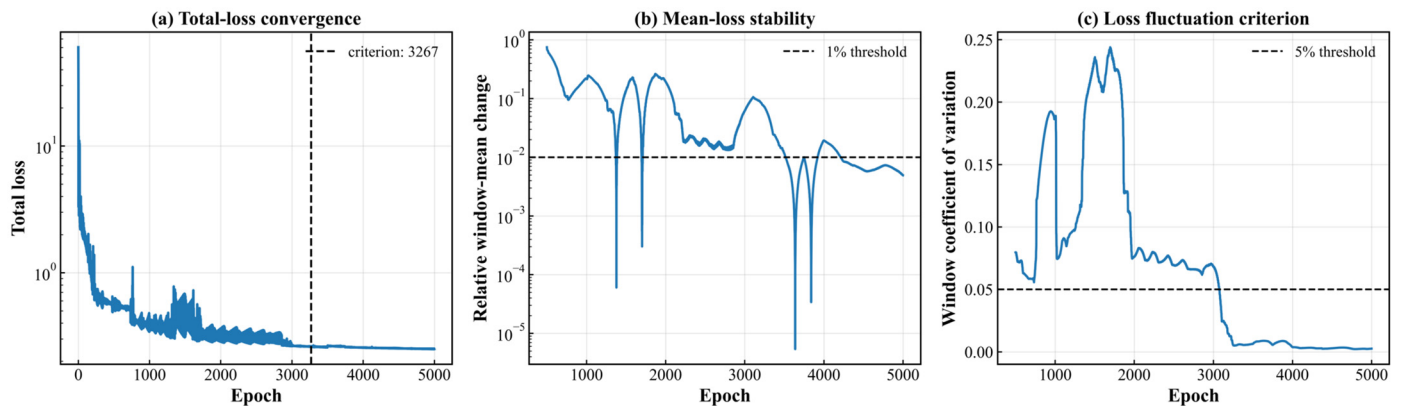


Figure 4. Quantitative convergence criteria based on changes in window mean and coefficient of variation.

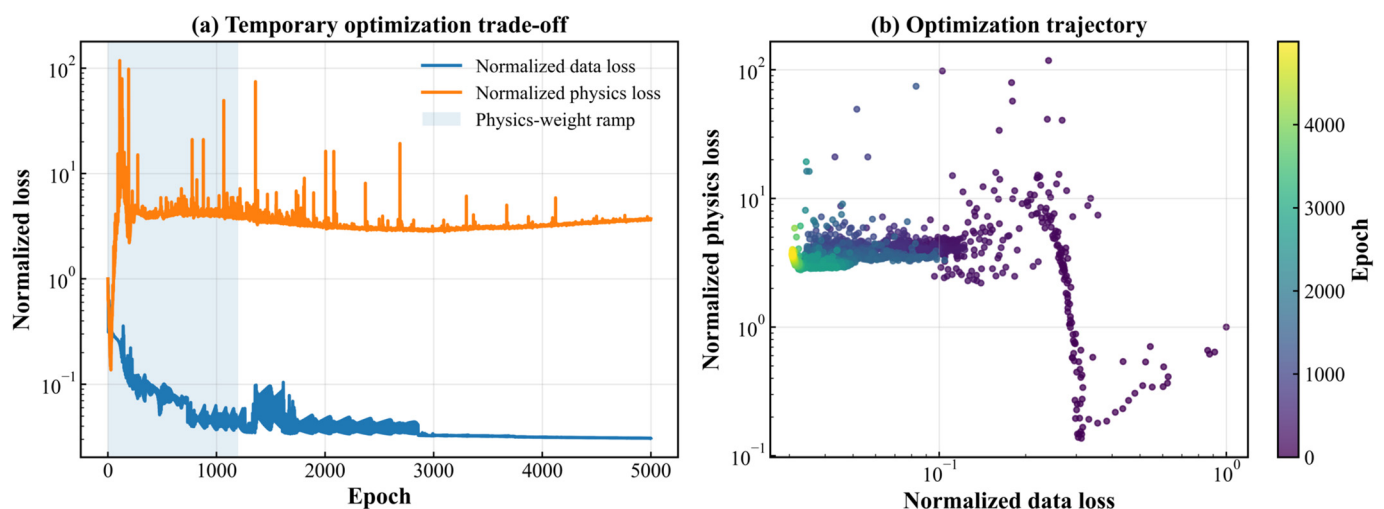


Figure 5. Temporal trade-off between normalized data loss and physical loss, and the optimization trajectory.

Figure 6 presents the residual distributions of the three governing equations within the radial and vertical domains at the final time step. The residuals associated with heat conduction predominantly cluster near the central axis and within the region spanning $z = 0$ to 15 cm. Notably, a pronounced transition from positive to negative residuals is observed at the lower boundary and around $z = 20$ cm, suggesting increased difficulty in satisfying energy conservation in areas characterized by steep temperature gradients, intense boundary heat exchange, or substantial latent heat effects due to phase change. In comparison, the residuals related to phase change exhibit smaller overall magnitudes than those of heat conduction, with the principal discrepancies localized within a thin layer at the bottom, indicating rapid variation in the phase-change variable near the freezing interface. The mechanical residuals are on the order of approximately 10^{-3} and display a generally smooth spatial distribution, transitioning gradually from negative values in the lower-left region to near-zero or positive values in the upper-right region.

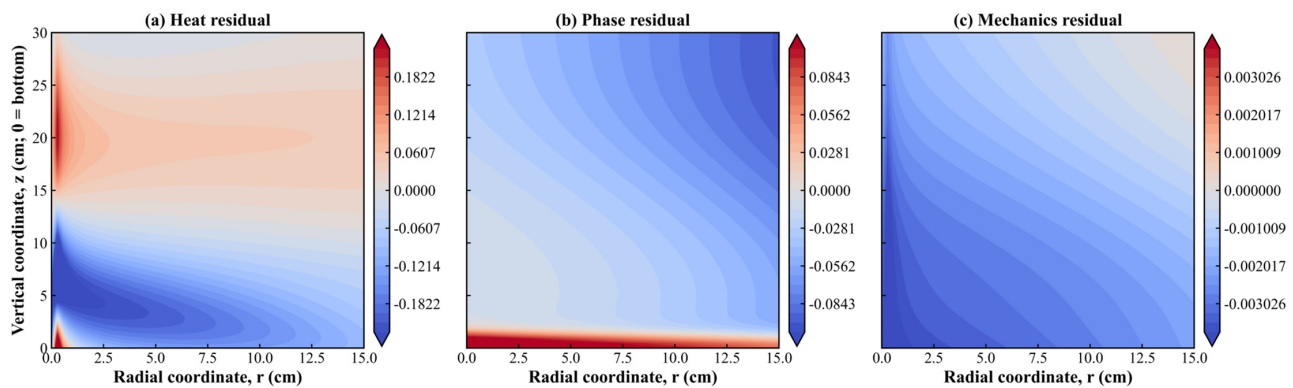


Figure 6. Spatial distribution of spatial residuals for the energy, phase change, and deformation equations at the final time step.

3.2. Fitting of Observed Temperature and Top Surface Displacement

Figure 7 presents a comparison between experimentally measured temperatures and predictions generated by the Physics-Informed Neural Network (PINN) across six representative temperature measurement channels situated at varying depths. In the figure, blue markers denote the experimental data points, while orange curves correspond to the model's predicted values. Overall, temperatures across all channels exhibited a continuous decline over time. The PINN model effectively captured the cooling trend during the intermediate and later stages of the process; notably, beyond approximately 1000 min, the predicted temperature profiles closely aligned with the experimental observations, demonstrating the model's capability to accurately characterize long-term heat conduction dynamics throughout the freezing process. Discrepancies were primarily observed during the initial phase of the experiment, wherein certain channels displayed transient temperature increases, plateaus, or rapid declines. In contrast, the PINN predictions portrayed relatively smooth transitions, thereby inadequately representing these localized transient fluctuations. The most accurate correspondence between model and experiment was found at a depth of 30 cm during the middle and late stages, whereas measurements at depths of 25 cm and 20 cm exhibited some divergence during the early freezing period and the phase transition interval. These deviations may be attributed to variations in boundary conditions, sensor placement differences, and localized nonlinear effects arising from the latent heat associated with phase transitions.

The upper portion of Figure 8 illustrates the experimental measurements of relative top-surface displacement alongside the corresponding Physics-Informed Neural Network (PINN) prediction curves, whereas the lower portion depicts the associated residuals. Prior to and following the initiation of freezing, the top-surface displacement exhibited a rapid increase from near zero, attaining a value of approximately 0.29 at around 700 min. Subsequently, the displacement entered a relatively stable phase characterized by minor periodic fluctuations. This behavior suggests that the specimen experienced rapid frost heave during the initial freezing stage, followed by gradual stabilization in the later stages. The PINN prediction curve effectively captures the rapid displacement increase, the establishment of a plateau, and the slight downward trend observed in the final phase, demonstrating a high degree of concordance with the experimental data. The majority of residuals are confined within the range of -0.01 to 0.01 , oscillating around zero without exhibiting any persistent directional bias, thereby indicating the absence of significant systematic overestimation or underestimation by the model. Notably, residual amplitudes were comparatively larger during the early training phase, reaching values near -0.017 at certain time points, which may be attributed to the pronounced nonlinear response occurring at the onset of freezing.

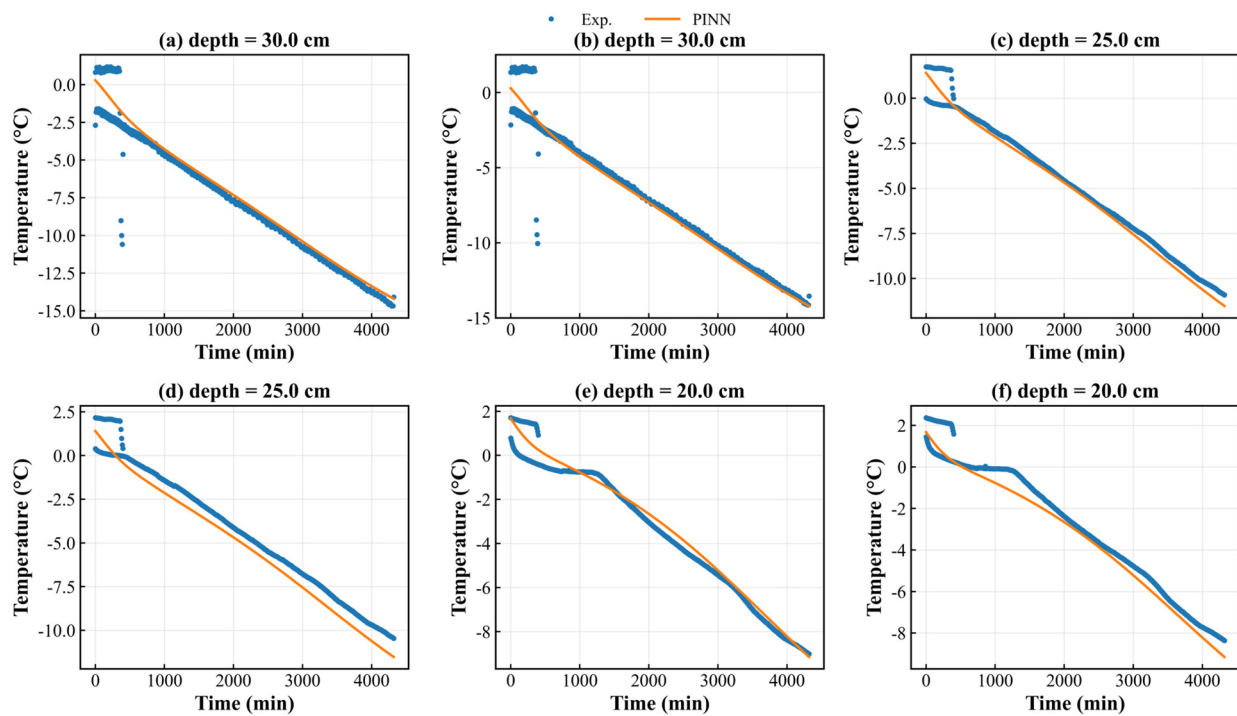


Figure 7. Experimental values and PINN predictions for six representative temperature channels.

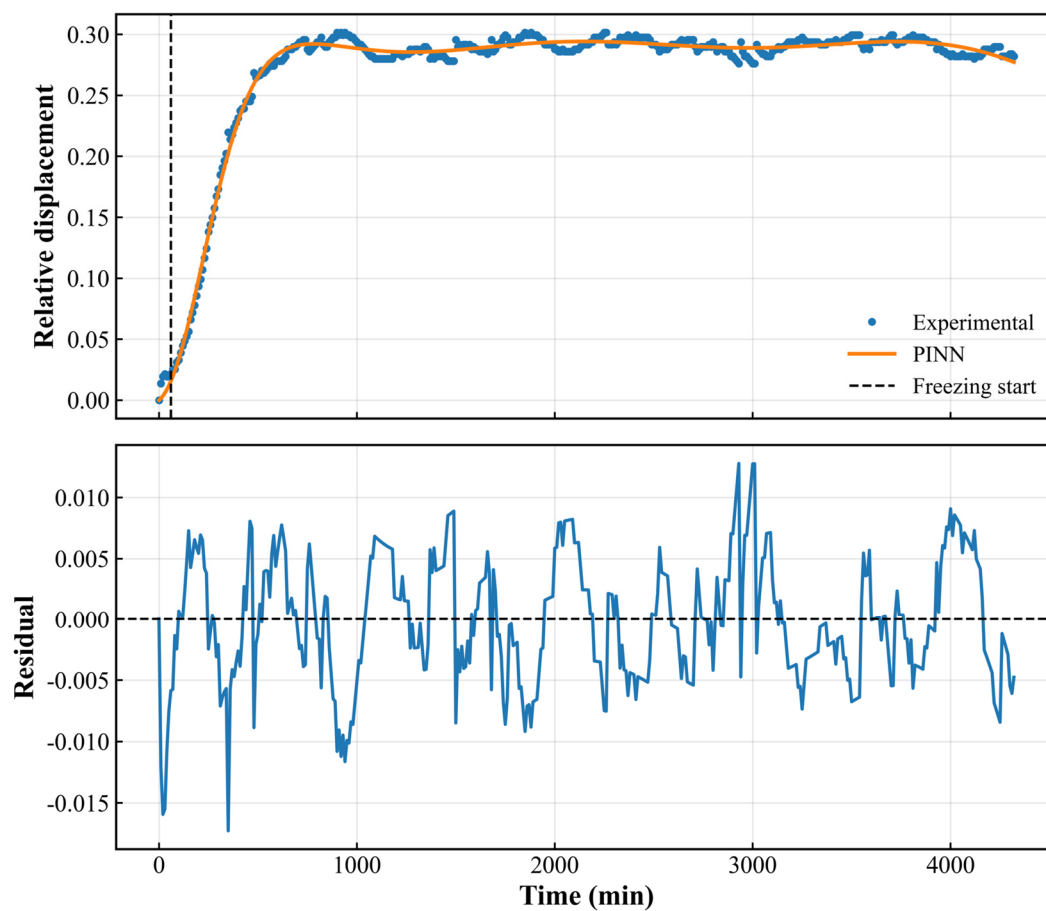


Figure 8. Fitted relative displacement of the top surface and residuals.

Figure 9 presents an assessment of the consistency between temperature predictions generated by the Physics-Informed Neural Network (PINN) model and corresponding ex-

perimental measurements at the overall sample level. The model exhibits strong predictive performance for low-temperature variations during the stable freezing phase. However, within the phase-change temperature range of approximately -5 to 5 °C, there is a marked increase in the dispersion of data points, with certain predicted values surpassing the experimental observations. This suggests that the model inadequately characterizes the temperature plateau near zero degrees Celsius and fails to fully capture the nonlinear dynamics associated with the phase transition. Additionally, samples with experimental temperatures around 18 °C were predicted within a range of 2 to 8 °C, resulting in a cluster of points that significantly deviate from the ideal correlation line; this discrepancy may be attributed to initial elevated temperatures or localized anomalous channels. Figure 10 illustrates the correspondence between predicted and experimental relative displacements measured at the top surface. The majority of data points are densely clustered near the line $y = x$, spanning the entire displacement range from near zero to 0.30 . This distribution indicates that the model effectively captures not only minor displacement changes during the initial stages of frost heave but also accurately predicts the rapid growth phase and the subsequent stabilization plateau. Locally, a slight underestimation is observed within the small displacement range of 0.01 to 0.03 , whereas some points in the range of 0.07 to 0.12 slightly exceed the ideal line. When displacement approaches values between 0.29 and 0.30 , the predicted results exhibit a degree of saturation; nonetheless, the associated error remains minimal. Overall, the PINN model demonstrates excellent fitting capability for surface deformation responses and reliably reconstructs the entire frost heave displacement process. Figure 11 provides a summary of the overall predictive performance of the PINN model concerning both temperature and displacement fields. The root-mean-square error for temperature prediction (T-RMSE) is 4.112 °C, and the coefficient of determination ($T-R^2$) is 0.600 , indicating that the model captures the general temporal trend of temperature variation but inadequately represents local temperature spikes, phase-change plateaus, and initial high-temperature samples. In contrast, the root-mean-square error for displacement prediction (U-RMSE) is only 0.0051 , with a coefficient of determination ($U-R^2$) of 0.993 , demonstrating a high degree of consistency between predicted and experimental displacements. This suggests that the model effectively accounts for nearly all observed displacement changes.

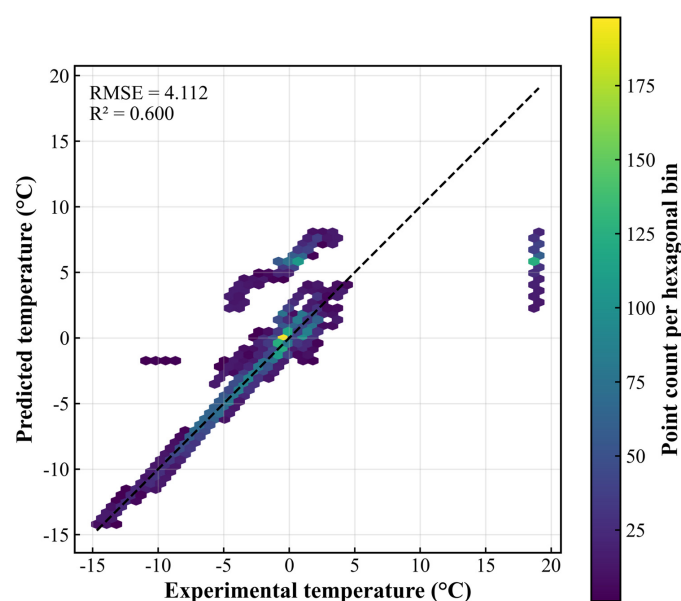


Figure 9. Experimental–predicted density consistency plot for all temperature samples.

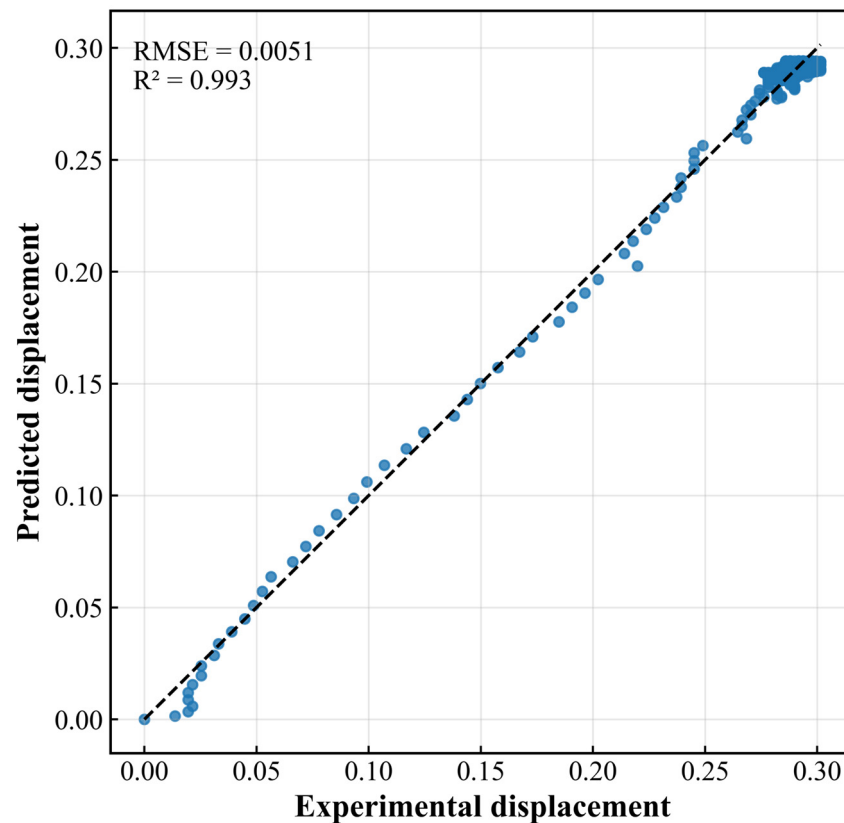


Figure 10. Experimental–predicted agreement plot for relative displacement on the top surface.

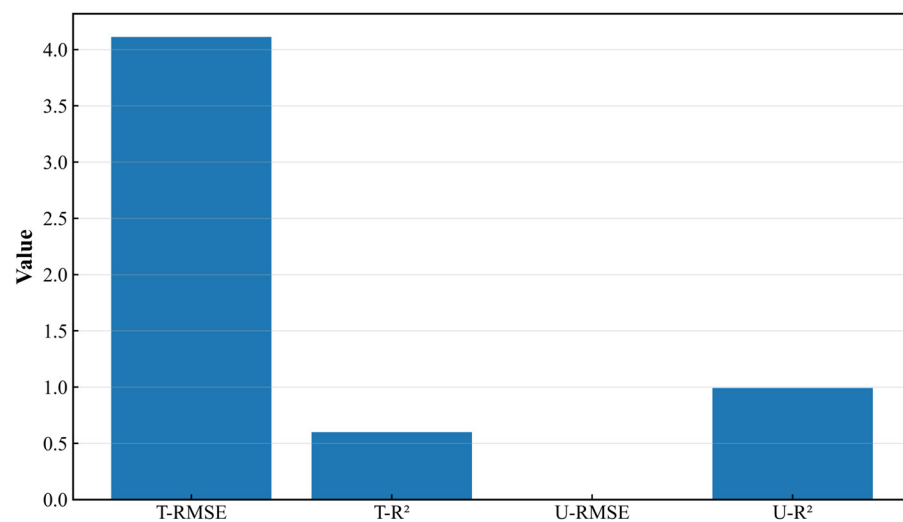


Figure 11. Summary of temperature versus overall RMSE and coefficient of determination for displacement.

3.3. Co-Evolution of Temperature, Freezing Fraction, and Displacement Fields

Figure 12 depicts the temporal progression of the temperature field at 0, 60, 1425.6, and 4320 min. Initially, the temperature exhibited pronounced vertical stratification, decreasing from the relatively warm bottom toward the cooler upper region, while radial variations were minimal. At 60 min, the temperature distribution remained largely consistent, although the low-temperature zone had expanded. By 1425.6 min, an extensive region of subzero temperature had emerged in the upper and middle sections of the specimen, whereas the bottom retained relatively higher temperatures. At 4320 min, nearly the entire specimen was below 0 °C, with the lowest temperatures observed at the top

and the bottom remaining close to 0 °C. The nearly horizontal isotherms suggest that heat transfer was predominantly vertical, with limited radial heterogeneity. Figure 13 illustrates the corresponding changes in the freezing fraction over time. At 0 and 60 min, the freezing fraction was generally low, with slightly elevated values near the central axis and in portions of the upper region. By 1425.6 min, areas exhibiting a high freezing fraction had expanded considerably, particularly near the central axis and within the middle to lower sections of the specimen. At 4320 min, the majority of the specimen displayed a high freezing fraction, with near-complete freezing observed around the central axis between approximately 5 and 15 cm in height, while the peripheral regions and some upper areas remained less frozen. These findings indicate that the freezing process was nonuniform and influenced by the interplay of temperature gradients, radial heat transfer, and phase-change dynamics. Figure 14 presents the evolution of the displacement field. At 0 and 60 min, displacements were minimal and primarily varied along the radial direction. By 1425.6 min, significant changes were evident: negative displacement developed in the lower region, whereas positive displacement appeared in the upper region, resulting in a distinct vertical deformation gradient. At 4320 min, this contrast intensified, with the greatest negative displacement near the bottom and central axis and relatively large positive displacement at the top. The contour patterns indicate that vertical variation predominated the deformation, although some radial heterogeneity persisted. Collectively, the displacement field reflects a differential frost-heave response governed by the coupled evolution of temperature and freezing fraction.

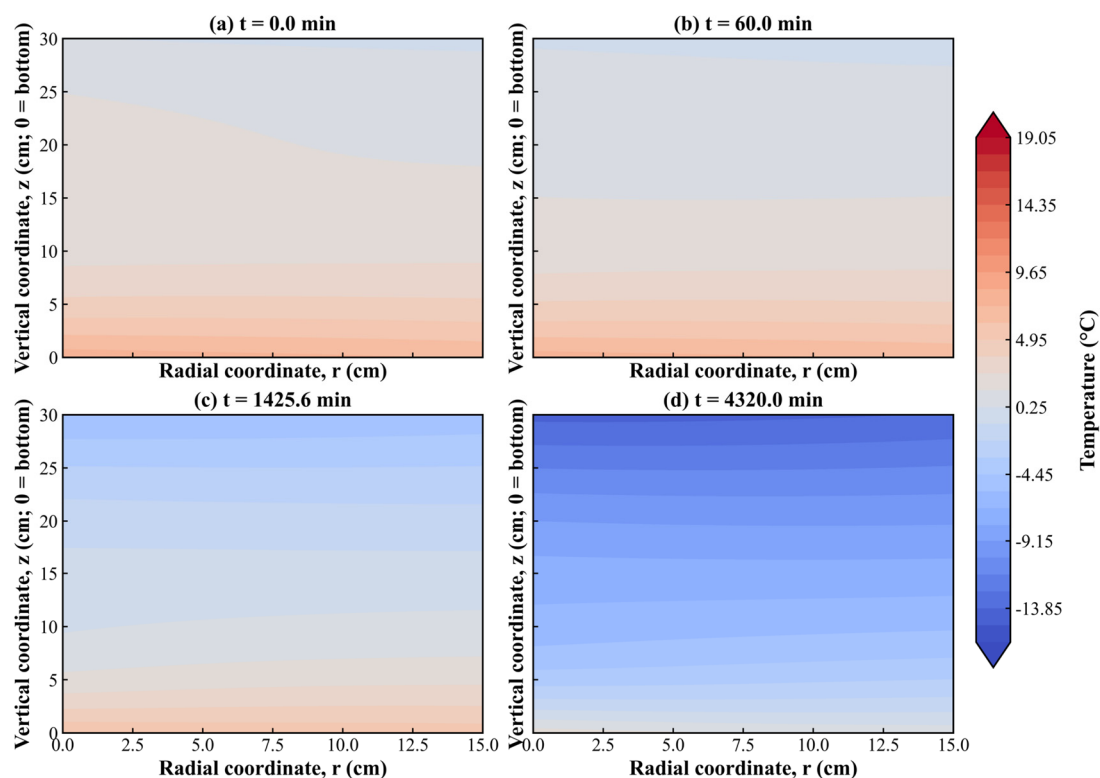


Figure 12. Axially symmetric temperature fields at four representative time points.

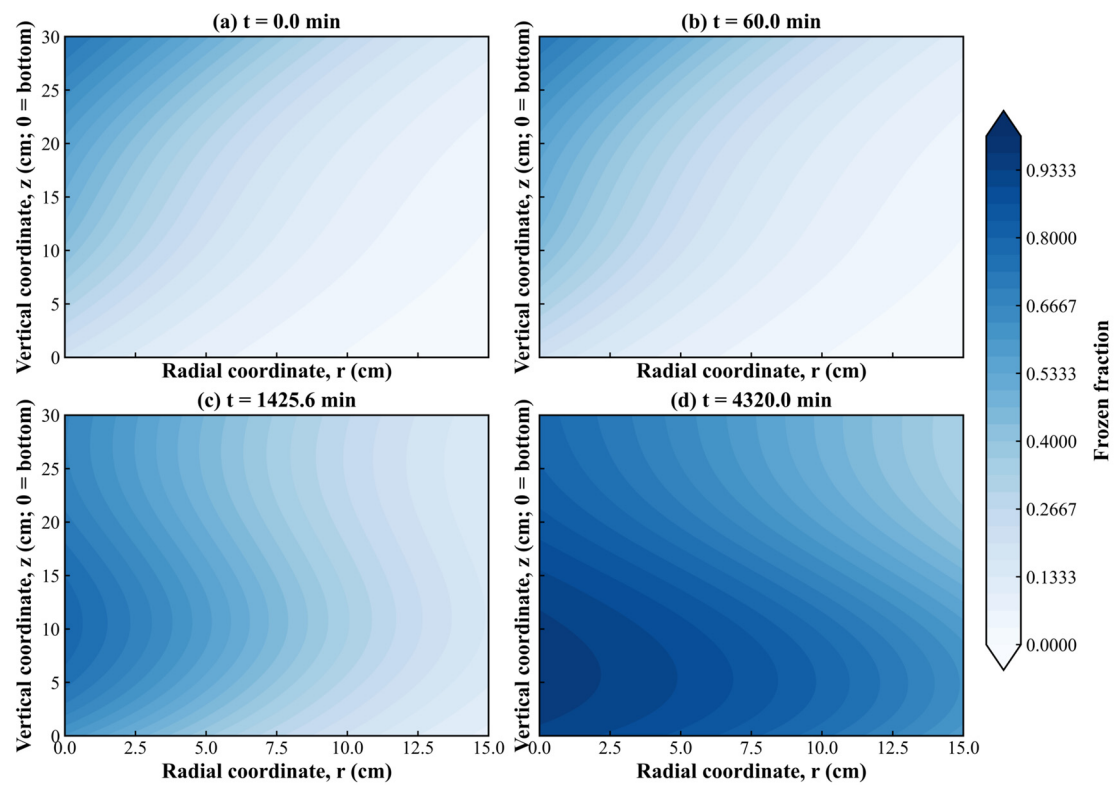


Figure 13. Freezing fraction fields at four representative time points.

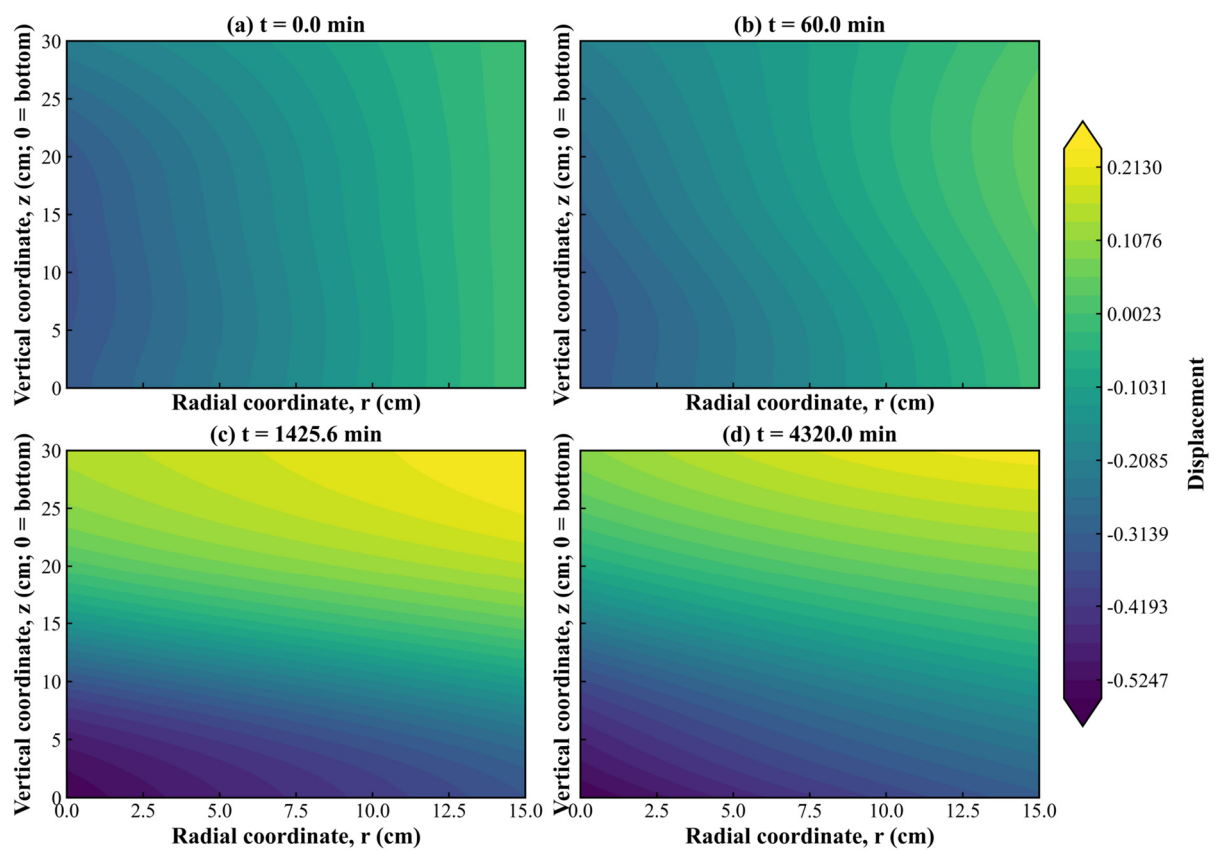


Figure 14. Axisymmetric displacement fields at four representative time points.

3.4. Vertical Profiles, Freezing Front, and Spatiotemporal Propagation

Figures 15–17 collectively depict the interrelated progression of centerline temperature, freezing fraction, and displacement throughout the freezing process. During the initial 0 to 60 min, minimal changes in temperature and displacement profiles suggest that the specimen's internal state remained relatively stable in this early freezing phase. As freezing advanced, the centerline temperature exhibited a general decline; by 1425.6 min, the middle and upper regions had entered subzero temperature ranges, and by 4320 min, the majority of the specimen—excluding areas near the bottom—had cooled below 0 °C. This temperature reduction corresponded with a marked increase in the freezing fraction, which rose from approximately 0.2–0.74 during the early stage to about 0.78–0.95 in the later stage. Notably, the variation in freezing fraction along the specimen's height was non-monotonic, indicating that phase transition dynamics were influenced by boundary conditions and latent heat effects. Concurrently, the displacement field evolved from predominantly negative values in the early stage to a pattern characterized by negative displacement in the lower region, positive displacement in the upper region, and a zero-displacement transition zone centrally located. This spatial variation reflects the impact of non-uniform freezing on deformation behavior. Collectively, these observations underscore that the temperature decrease served as the principal driving force behind the increase in freezing fraction and the redistribution of displacements, demonstrating a distinct coupling among thermal, phase-transition, and mechanical phenomena.

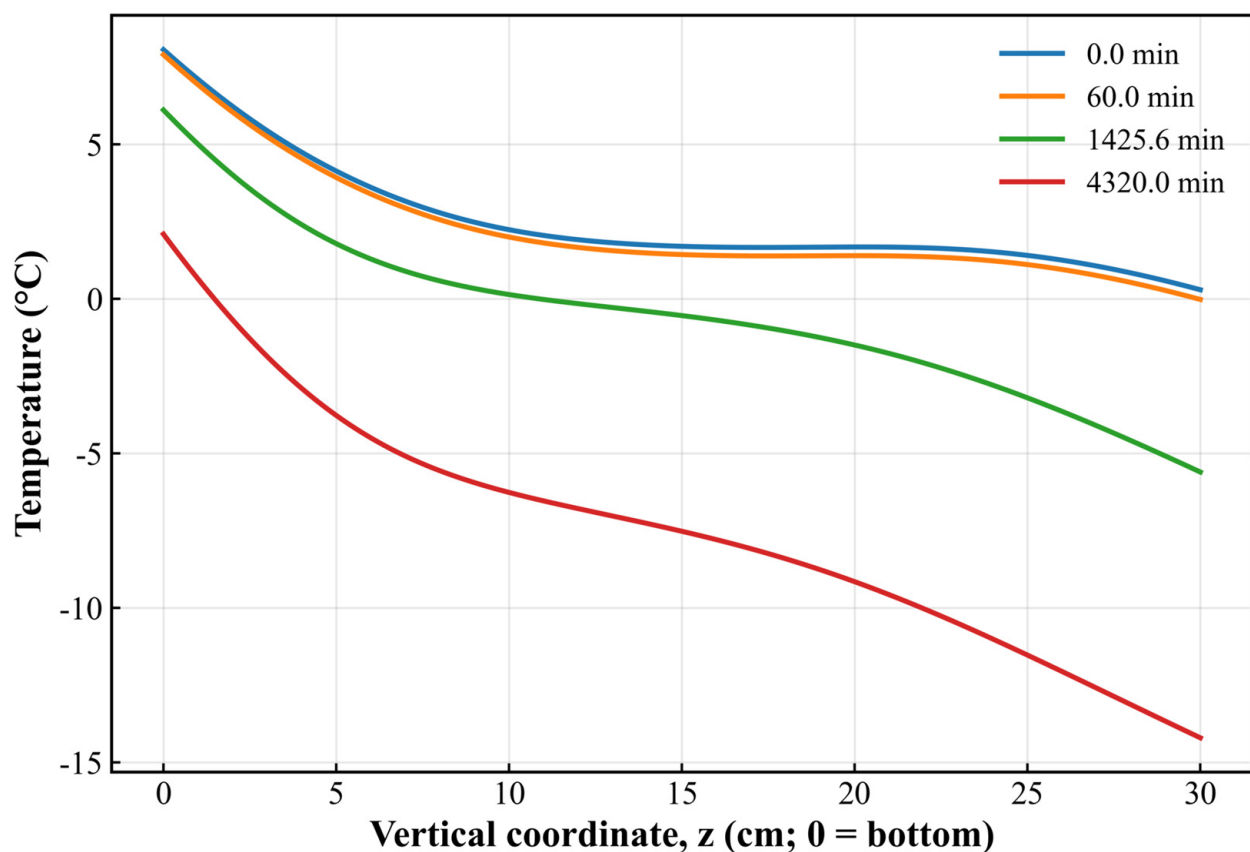


Figure 15. Vertical temperature profiles along the centerline at different representative time points.

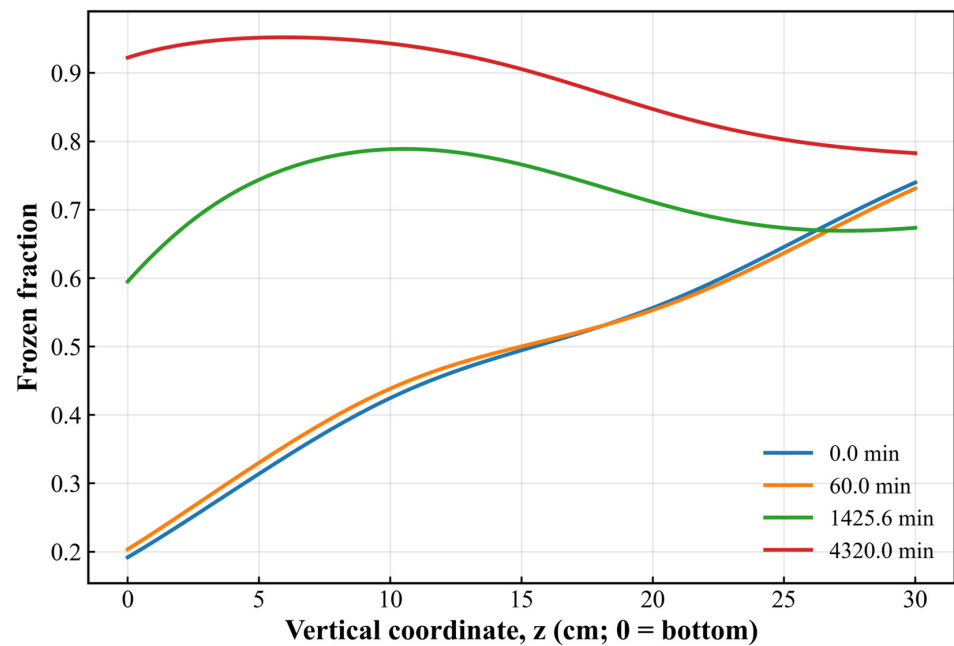


Figure 16. Vertical profiles of the freezing fraction along the centerline at different representative time points.

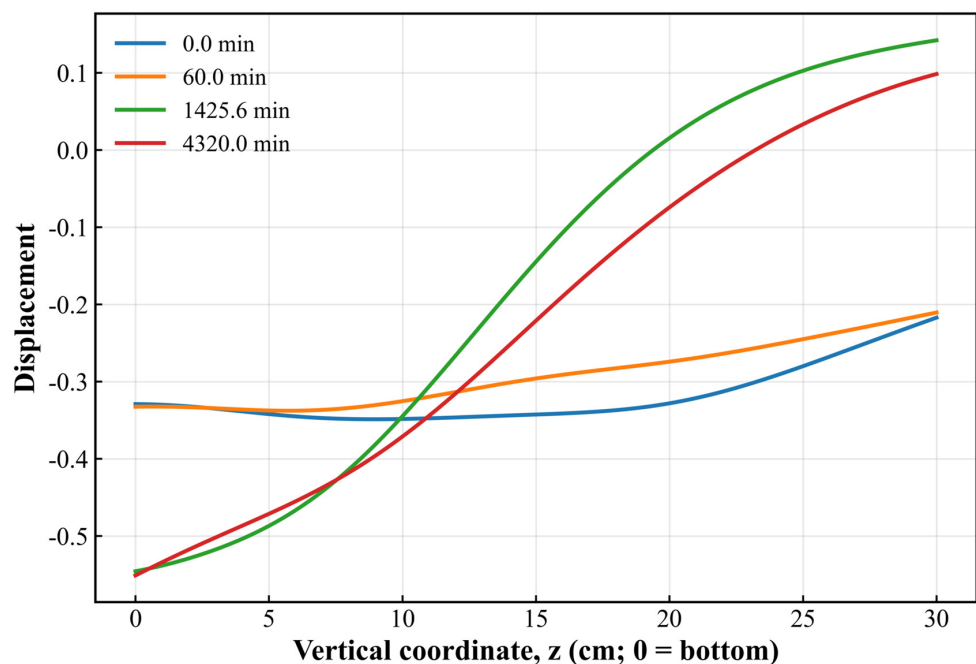


Figure 17. Vertical displacement profiles along the centerline at different representative time points.

Figures 18–21 collectively illustrate the spatiotemporal coupling dynamics among temperature, phase transition, and deformation throughout the specimen's freezing process. As the duration of freezing progresses, the temperature distribution shifts predominantly from warmer to cooler hues, with the 0 °C isotherm gradually descending from the upper to the lower regions. This trend signifies a continuous expansion of the subzero temperature zone, ultimately encompassing the majority of the specimen. Concurrently, the freezing fraction exhibits a steady increase, and the interface corresponding to a freezing fraction of $\phi = 0.5$ migrates over time toward the unfrozen region. The curved shape of this interface suggests that the freezing process is influenced not only by vertical boundary cooling but also by a degree of radial heterogeneity. Initially, the displacement field remains relatively

stable; however, as freezing advances, distinct upper and lower zones emerge. The lower region is characterized predominantly by negative displacement, whereas the upper region transitions to positive displacement, indicative of differential deformation arising from nonuniform freezing. Furthermore, the time-height plot along the specimen's centerline corroborates that both the temperature front and the freezing front propagate downward, and the progression of the freezing fraction demonstrates a pronounced transitional behavior in relation to temperature variations.

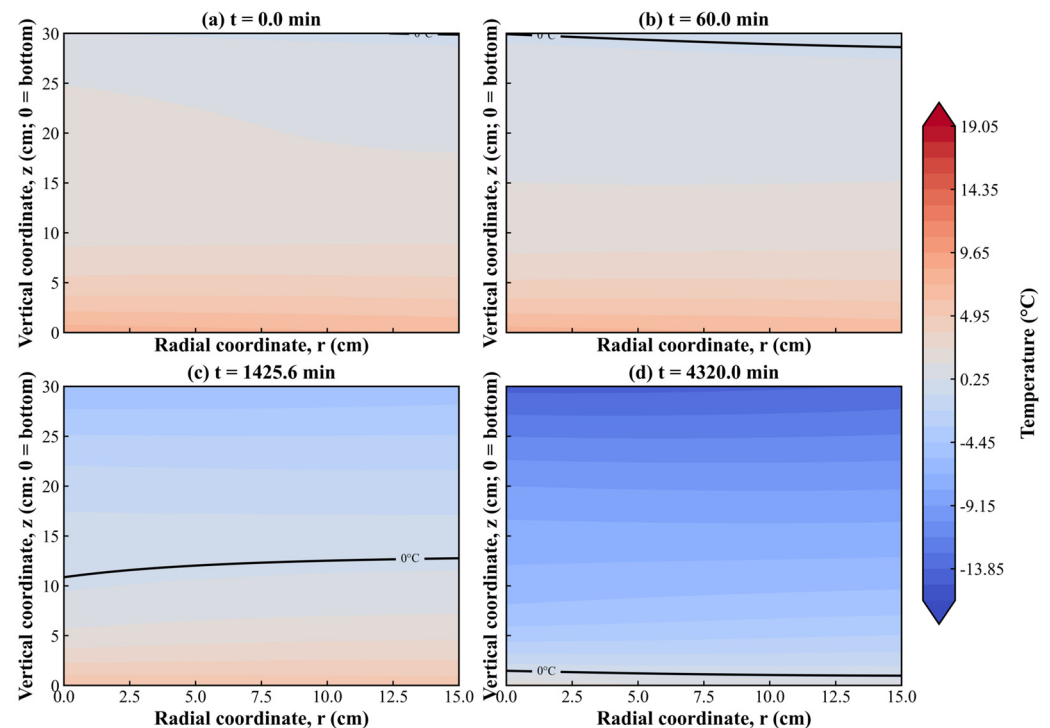


Figure 18. Spatiotemporal progression of the temperature field and the 0 °C isotherm.

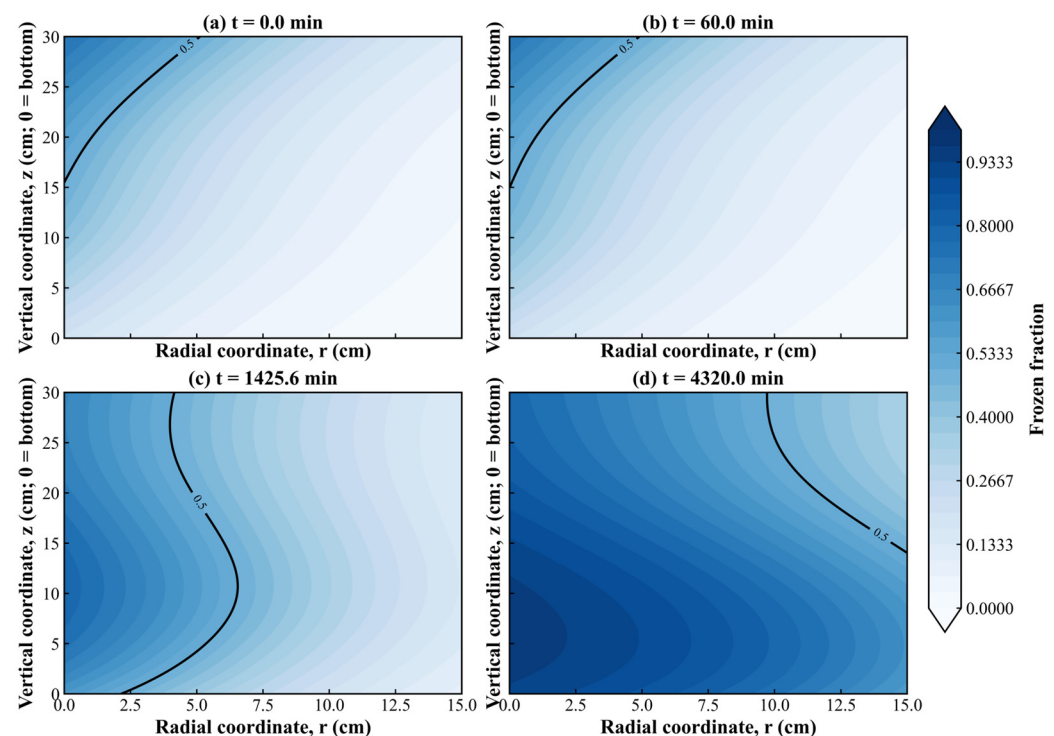


Figure 19. Freezing fraction field and $\phi = 0.5$ front.

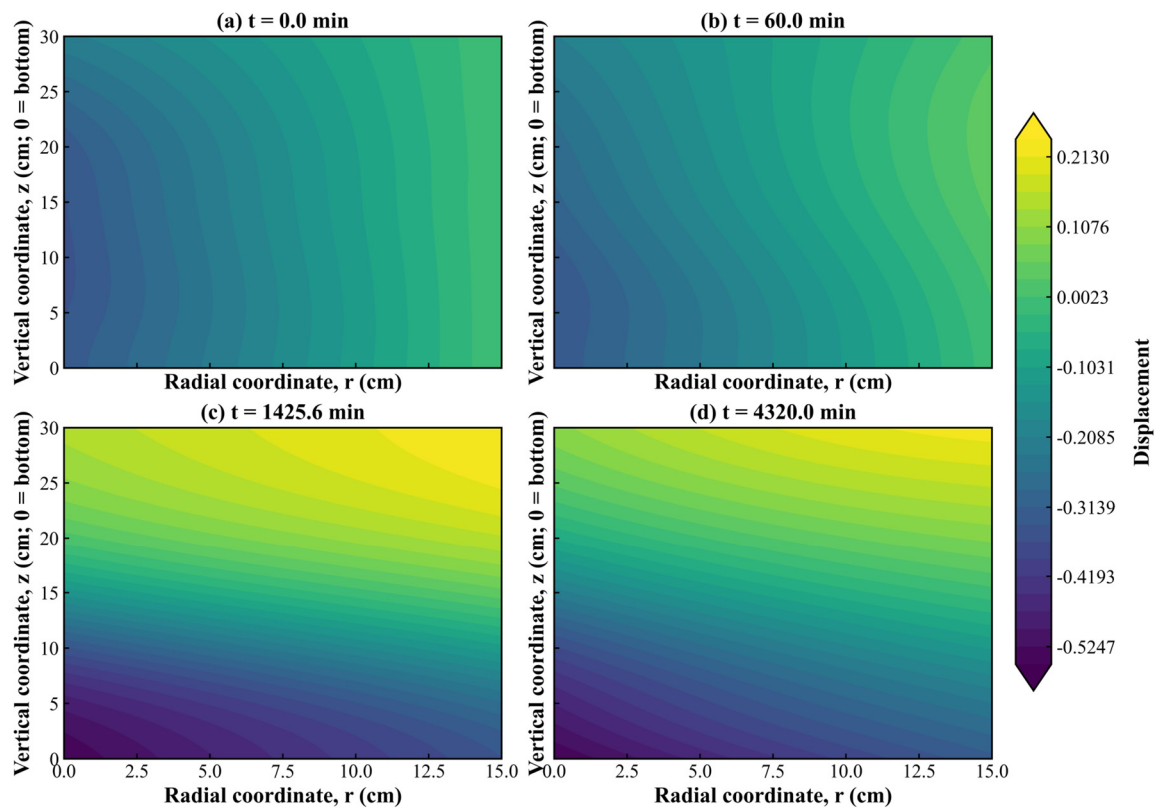


Figure 20. Displacement field and top surface boundary location.

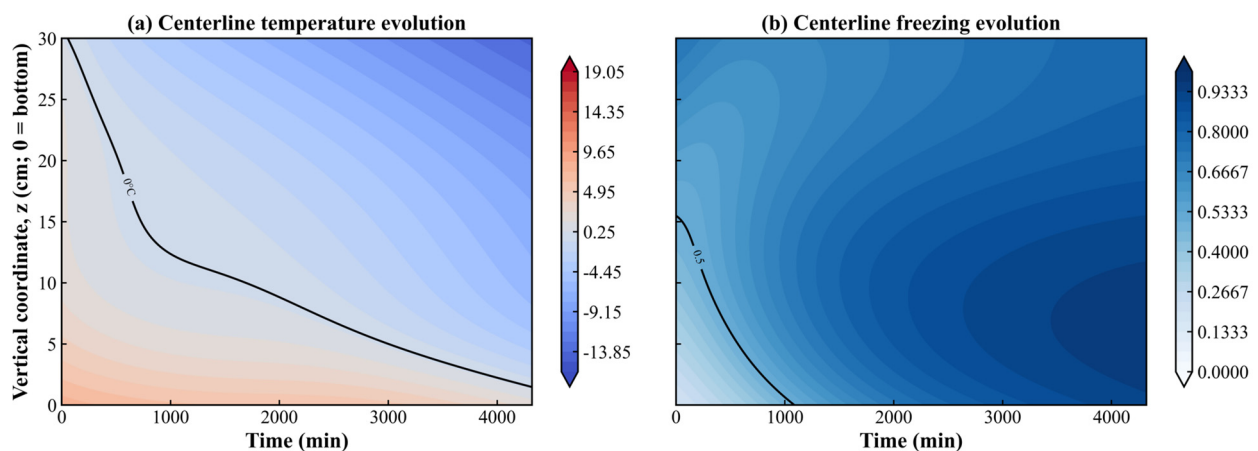


Figure 21. Time–height evolution of the centerline temperature and freezing fraction.

4. Conclusions

To address the challenge of simultaneously monitoring and continuously reconstructing the internal temperature, phase-change state, and frost heave deformation of saturated subgrade materials under unidirectional freezing conditions, this study conducted freezing experiments on cylindrical laboratory specimens and developed a two-dimensional axisymmetric neural network model that integrates thermal, phase-change, and deformation physical information. The model incorporates the heat conduction equation (accounting for the latent heat of phase change), the freezing fraction relaxation equation, and the effective deformation equation into a unified loss function. By combining temperature measurements at multiple depths with displacement observations at the specimen's top surface, the model enables joint prediction of the temperature field, freezing fraction field, and vertical displacement field under conditions of sparse data availability. The primary contributions

of this work lie in the novel application of the physics-informed neural network (PINN) methodology to engineering subgrade freezing tests and the simultaneous modeling of multiple coupled physical fields. The key findings are summarized as follows:

(1) The proposed model is capable of concurrently outputting three state variables—temperature, freeze fraction, and displacement—within a single neural network framework. The temperature field is governed by the equilibrium freeze fraction, while the temporal evolution of the freeze fraction is incorporated into the latent heat term. Together, temperature and freeze fraction jointly drive the displacement evolution, thereby establishing a minimal-order coupling among heat conduction, water–ice phase transition, and frost heave deformation. This integrated framework facilitates continuous reconstruction of fields in unmeasured regions of the specimen based on limited measurement data.

(2) The model training process exhibits distinct phases of optimization. During the initial training stages, competition among data fitting, boundary condition enforcement, and partial differential equation residual minimization resulted in transient peaks in both physical and total loss values. As dynamic weighting strategies and learning rate decay were applied, these fluctuations diminished progressively. Employing quantitative convergence criteria—specifically, a rate of change in the mean loss within adjacent sliding windows below 1% and a coefficient of variation within these windows below 5%—the model achieved stable convergence after approximately 3267 training iterations. This approach demonstrates that quantitative metrics provide a more objective assessment of training progress than relying solely on the visual flattening of loss curves.

(3) The proposed model successfully replicates the general trends observed during the specimen's freezing process, including the temperature decrease, rapid volumetric expansion due to freezing, and subsequent stabilization. The temperature prediction yielded a root mean square error (RMSE) of 4.112 °C and a coefficient of determination (R^2) of 0.600, demonstrating the model's capability to capture the overall heat conduction behavior. Nonetheless, discrepancies remain in accurately predicting the phase transition plateau near 0 °C, transient fluctuations during the initial freezing stages, and certain high-temperature outliers. For the prediction of top surface displacement, the model achieved an RMSE of 0.0051 and an R^2 of 0.993, with residuals predominantly ranging between −0.01 and 0.01, indicating a high degree of accuracy in modeling the entire frost heave displacement process.

(4) The continuous-field predictions reveal that, under unidirectional freezing conditions, the zone of subzero temperature and the freezing front predominantly progress from the top toward the bottom of the specimen. Heat transfer within the specimen is primarily vertical, although some radial heterogeneity is observed. As freezing time advances, the freezing fraction increases progressively, and the displacement field transitions from minor deformation in the early stages to a partitioned pattern characterized by negative displacement in the lower region and positive displacement in the upper region. This behavior reflects the distinctive frost heave phenomena arising from the interplay of non-uniform temperature distribution and phase change dynamics. Furthermore, the temperature front, freezing front, and displacement partition exhibit strong spatiotemporal correlation, indicating the model's effectiveness in capturing the coupled evolution of these state variables.

While this study confirms the feasibility of employing Physics-Informed Neural Networks (PINNs) for multi-field reconstruction in freezing tests of saturated subgrade materials, several limitations persist. First, the model validation was conducted using a single set of saturated cylindrical specimens under one freezing condition, with a relatively limited dataset; thus, its generalizability across varying particle size distributions, moisture con-

tents, freezing temperatures, and boundary conditions remains to be established. Second, systematic comparisons with conventional numerical methods such as finite element and finite difference models, as well as purely data-driven approaches, have not yet been performed. Additionally, the independent effects of components such as the latent heat term, freezing fraction formulation, and dynamic weighting have not been quantified through thawing experiments. Third, the freezing fraction is treated as a latent variable within the model, lacking direct experimental validation via techniques such as nuclear magnetic resonance (NMR) or computed tomography (CT). Moreover, the effective deformation equation does not explicitly incorporate mechanisms related to water migration, pore water pressure, or elastoplastic constitutive behavior. Future research should aim to broaden the experimental conditions and incorporate independent validation datasets, conduct benchmark comparisons and ablation studies, and integrate measurements of ice content, water content, and pore pressure. These efforts will facilitate the development of a fully coupled thermal–hydraulic–mechanical PINN framework, thereby enhancing its applicability in engineering practice.

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References

1. Lai, Y.; Xu, X.; Dong, Y.; Li, S. Present situation and prospect of mechanical research on frozen soils in China. *Cold Reg. Sci. Technol.* **2013**, *87*, 6–18. [[CrossRef](#)]
2. Chen, H.; Gao, X.; Wang, Q. Research progress and prospect of frozen soil engineering disasters. *Cold Reg. Sci. Technol.* **2023**, *212*, 103901. [[CrossRef](#)]
3. Li, K.Q.; Yin, Z.Y.; Yang, Z.H.; Liu, Y. State of the art of mechanical behaviors of frozen soils through experimental investigation. *Cold Reg. Sci. Technol.* **2025**, *236*, 104497. [[CrossRef](#)]
4. Li, K.Q.; Yin, Z.Y.; Qi, J.L.; Liu, Y. State-of-the-art constitutive modelling of frozen soils. *Arch. Comput. Methods Eng.* **2024**, *31*, 3801–3842.
5. Bartsch, A.; Strozzi, T.; Nitze, I. Permafrost monitoring from space. *Surv. Geophys.* **2023**, *44*, 1579–1613. [[CrossRef](#)]
6. Zheng, G.; Yang, Y.; Yang, D.; Dafflon, B.; Yi, Y.; Zhang, S.; Chen, D.; Gao, B.; Wang, T.; Shi, R.; et al. Remote sensing spatiotemporal patterns of frozen soil and the environmental controls over the Tibetan Plateau during 2002–2016. *Remote Sens. Environ.* **2020**, *247*, 111927. [[CrossRef](#)]

7. Jung, Y.T.; Park, S.E.; Kim, H.C. Observation of spatial and temporal patterns of seasonal ground deformation in central Yakutia using time series InSAR data in the freezing season. *Remote Sens. Environ.* **2023**, *293*, 113615. [\[CrossRef\]](#)
8. Cai, X.; Zhang, Q.; Zhang, Y.; Wang, Q.; Luo, B.; Yang, G.; Lau, A. Deformation law and control limit of CRTSIII slab track under subgrade frost heave. *Appl. Sci.* **2021**, *11*, 3520. [\[CrossRef\]](#)
9. Sheng, D.; Zhang, S.; Yu, Z.; Zhang, J. Assessing frost susceptibility of soils using PCHeave. *Cold Reg. Sci. Technol.* **2013**, *95*, 27–38. [\[CrossRef\]](#)
10. Yang, N.; Zheng, H.; Cai, H.; Liu, Y.; Nishimura, S. Study on multidimensional frost heave characteristics and thermal-hydro-mechanical predictive model. *Cold Reg. Sci. Technol.* **2024**, *224*, 104227. [\[CrossRef\]](#)
11. Wu, G.; Xie, Y.; Wei, J.; Yue, X. Freeze-thaw erosion mechanism and preventive actions of highway subgrade soil in an alpine meadow on the Qinghai-Tibet Plateau. *Eng. Fail. Anal.* **2023**, *143*, 106933. [\[CrossRef\]](#)
12. Wu, X.Y.; Niu, F.J.; Lin, Z.J.; Luo, J.; Zheng, H.; Shao, Z. Delamination frost heave in embankment of high speed railway in high altitude and seasonal frozen region. *Cold Reg. Sci. Technol.* **2018**, *153*, 25–32. [\[CrossRef\]](#)
13. Wu, Z.; Chu, C.; Bi, J.; Wang, G.; Cao, W.; Lu, Y.; Sun, T.; Tai, B. Warming performance of a novel high-speed railway embankment in seasonally frozen ground regions. *Constr. Build. Mater.* **2024**, *426*, 136145. [\[CrossRef\]](#)
14. Li, S.M.; Yang, K.S.; Wang, C.C. A semi-empirical model for predicting frost properties. *Processes* **2021**, *9*, 412. [\[CrossRef\]](#)
15. Chen, J.; Li, A.; Bao, C.; Dai, Y.; Liu, M.; Lin, Z.; Niu, F.; Zhou, T. A deep learning forecasting method for frost heave deformation of high-speed railway subgrade. *Cold Reg. Sci. Technol.* **2021**, *185*, 103265. [\[CrossRef\]](#)
16. Guan, W.; Teng, J.; Shan, F.; Liu, J.; Wu, H. A novel transformer-based approach for predicting frost heave of high-speed railway in cold regions. *J. Rock Mech. Geotech. Eng.* **2025**, *17*, 8114–8130. [\[CrossRef\]](#)
17. Li, S.; Zhang, M.; Tian, Y.; Pei, W.; Zhong, H. Experimental and numerical investigations on frost damage mechanism of a canal in cold regions. *Cold Reg. Sci. Technol.* **2015**, *116*, 1–11. [\[CrossRef\]](#)
18. Hai, M.; Wang, M.; Meng, S.; Liu, Y.; Ji, Y.; Zhu, W.; Mu, H. Research on hydro-thermal coupling model of canal foundation soil based on particle grading curve predicting soil-water characteristic curve. *Case Stud. Therm. Eng.* **2024**, *56*, 104270. [\[CrossRef\]](#)
19. Su, A.S.; Hai, M.W.; Wang, M.; Guo, Y.; Wang, F.; Zhang, Q.; Zhou, B. Research on the analytical methods for water-heat coupling in the frost-thaw process of canal slopes under saturated conditions. *Res. Cold Arid. Reg.* **2025**, *17*, 360–370. [\[CrossRef\]](#)
20. Wang, M.; Hai, M.; Su, A.; Xu, J.; Guo, Y.; Yan, H. Study of hydrothermal characteristics of large-scale water conveyance trunk canals in seasonally frozen ground regions under the influence of different initial water contents. *River* **2024**, *3*, 166–180. [\[CrossRef\]](#)
21. Zhou, B.; Wang, M.; Hai, M. A FiPy-DEM Solution Method for Hydrothermal Coupled Modeling of Soil Freezing and Thawing. *J. Cold Reg. Eng.* **2025**, *39*, 04025040. [\[CrossRef\]](#)
22. Hai, M.; Su, A.; Wang, M.; Gao, S.; Lu, C.; Guo, Y.; Xiao, C. Large-Scale Freezing and Thawing Model Experiment and Analysis of Water-Heat Coupling Processes in Agricultural Soils in Cold Regions. *Water* **2023**, *16*, 19. [\[CrossRef\]](#)
23. Liu, Q.; Cai, G.; Zhou, C.; Yang, R.; Li, J. Thermo-hydro-mechanical coupled model of unsaturated frozen soil considering frost heave and thaw settlement. *Cold Reg. Sci. Technol.* **2024**, *217*, 104026. [\[CrossRef\]](#)
24. Raissi, M.; Perdikaris, P.; Karniadakis, G.E. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *J. Comput. Phys.* **2019**, *378*, 686–707. [\[CrossRef\]](#)
25. Karniadakis, G.E.; Kevrekidis, I.G.; Lu, L.; Perdikaris, P.; Wang, S.; Yang, L. Physics-informed machine learning. *Nat. Rev. Phys.* **2021**, *3*, 422–440. [\[CrossRef\]](#)
26. Lu, L.; Meng, X.; Mao, Z.; Karniadakis, G.E. DeepXDE: A deep learning library for solving differential equations. *SIAM Rev.* **2021**, *63*, 208–228. [\[CrossRef\]](#)
27. Wang, S.; Teng, Y.; Perdikaris, P. Understanding and mitigating gradient flow pathologies in physics-informed neural networks. *SIAM J. Sci. Comput.* **2021**, *43*, A3055–A3081. [\[CrossRef\]](#)
28. McClenny, L.D.; Braga-Neto, U.M. Self-adaptive physics-informed neural networks. *J. Comput. Phys.* **2023**, *474*, 111722. [\[CrossRef\]](#)
29. Jagtap, A.D.; Kawaguchi, K.; Karniadakis, G.E. Adaptive activation functions accelerate convergence in deep and physics-informed neural networks. *J. Comput. Phys.* **2020**, *404*, 109136. [\[CrossRef\]](#)
30. Yu, J.; Lu, L.; Meng, X.; Karniadakis, G.E. Gradient-enhanced physics-informed neural networks for forward and inverse PDE problems. *Comput. Methods Appl. Mech. Eng.* **2022**, *393*, 114823. [\[CrossRef\]](#)
31. Yang, Z.; Hai, M.; Wang, M.; Li, X.; Zhou, B.; Guo, Y. A study on optimizing soil frost heave ratio prediction for extreme learning machines using the starfish optimization algorithm. *Eng. Comput.* **2026**, 1–29. [\[CrossRef\]](#)
32. Hai, M.; Wang, M.; Zhou, B.; Zhang, Q. Assessment of frost heave in coarse-grained soil: A novel application of the multi-strategy enhanced dung beetle-optimized KELM model. *Earth Sci. Inform.* **2025**, *18*, 63. [\[CrossRef\]](#)
33. Li, K.Q.; Yin, Z.Y.; Zhang, N.; Liu, H.-C. Physics-informed neural networks for solving the steady-state temperature field in artificial ground freezing. *Can. Geotech. J.* **2025**, *62*, 1–17. [\[CrossRef\]](#)
34. Li, K.Q.; Yin, Z.Y.; Zhang, N.; Li, J. A PINN-based modeling approach for the hydromechanical behavior of unsaturated expansive soils. *Comput. Geotech.* **2024**, *169*, 106174. [\[CrossRef\]](#)
35. Harlan, R.L. Analysis of coupled heat-fluid transport in partially frozen soil. *Water Resour. Res.* **1973**, *9*, 1314–1323. [\[CrossRef\]](#)

36. Hansson, K.; Šimůnek, J.; Mizoguchi, M.; Lundin, L.; van Genuchten, M.T. Water flow and heat transport in frozen soil: Numerical solution and freeze–thaw applications. *Vadose Zone J.* **2004**, *3*, 693–704. [[CrossRef](#)]
37. Rempel, A.W. Formation of ice lenses and frost heave. *J. Geophys. Res. Earth Surf.* **2007**, *112*, F02S21. [[CrossRef](#)]

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