

Article

Risk Assessment of Water Inrush for Underwater Tunnel Based on GT Combination Weighting and Interval Extension Theory

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Abstract

To achieve the accurate prediction of the dynamic risk of water and mud inrush during underwater tunnel construction, a two-stage risk assessment theory and methodology of water and mud inrush based on game theory combination weighting and interval extension theory is proposed, comprising preliminary and secondary assessments. Due to the uncertainty of geological and hydrological conditions and the uncontrollability of construction factors, a ternary fuzzy interval number rather than a fixed value is used to quantify the evaluation index of water inrush. The correlation functions in extension theory are improved. A game-theory-based combination weighting model is constructed by considering the subjective weight of the improved AHP and the objective weight of the FCM. The proposed method is applied to evaluate the water inrush risk of the river-crossing section in the Yuelongmen Tunnel from the Chengdu–Lanzhou Railway. The results show that the risk level of water inrush at the river-crossing section is high, and the evaluation results of the proposed method are consistent with the actual situation. It has been proven that the method is effective and scientific, and it is more suitable for identifying the risk of water inrush in complex geological conditions.

Keywords: tunnel engineering; water and mud inrush; dynamic risk assessment; combination weighting



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1. Introduction

In recent years, China's tunnel engineering has gradually been shifting to the western mountainous areas, where the topography and geological conditions are complex and variable. Geological hazards such as water and mud inrush, large deformations, and rock bursts frequently occur during tunnel construction [1,2]. However, due to the characteristics of strong suddenness, diverse forms, and unpredictability, the occurrence frequency of water and mud inrush and the loss of life and property caused by it are in the forefront [3–5]. Tunnel water and mud inrush is frequently associated with water-bearing and water-conducting geological structures, such as karst caves, underground rivers, and faults. It should be particularly noted that certain fault zones can provide abundant water storage space and preferential flow pathways due to their well-developed voids and fractures.

When these faults are hydraulically connected to surface water bodies or water-rich aquifers, their exposure during tunnel excavation can easily trigger sudden and large-flow water and mud inrush. Therefore, the accurate prediction of water and mud inrush risk during tunnel construction can provide scientific guidance for safe construction management and hazard prevention decision-making. Domestic and foreign scholars have carried out extensive research on the prediction of water and mud inrush risk in tunnels from two aspects: the estimation of water inflow and the identification of construction risk [6–8]. The estimation methods of water inflow include empirical formulas, analytical formulas [9,10], numerical calculation [11], and intelligent prediction [12]. However, due to complex engineering geological and hydrological conditions, the accuracy of these predictions is limited. The methods of construction risk identification include Analytic Hierarchy Process (AHP) [13], attribute recognition model [14–16], fuzzy comprehensive evaluation method [17], ideal point method [18], TOPSIS [19,20], gray correlation theory [21,22], neural network [23,24], and cloud model [8,25]. However, most of the above methods only conduct static risk assessment from a single dimension, ignoring the dynamic changes in construction risk and failing to fully quantify the uncertainty of risk. Furthermore, some methods rely too heavily on the subjective opinions of experts, making the evaluation results susceptible to personal preferences. Therefore, aiming at the risk of water and mud inrush in different stages of tunnel construction, this paper proposes a two-stage assessment theory and methodology for dynamic risk of water and mud inrush in tunnels, namely preliminary assessment and secondary assessment. A novel assessment model based on game theory combination weighting and interval extension theory for accurate identification of water and mud inrush risk is developed. The risk of water and mud inrush in the undercrossing-river section of Yuelongmen Tunnel on the Chengdu–Lanzhou Railway is evaluated.

2. Materials and Methods

2.1. Assessment Theory and Method of Dynamic Risk for Water and Mud Inrush in Tunnels

The risk of water and mud inrush in the whole cycle of tunnel construction is dynamically changing. To accurately predict the risk of water inrush and mud inrush in tunnels and formulate reasonable prevention and control measures of the hazard, a two-stage assessment method for dynamic risk of water and mud inrush in tunnels is presented, which involves a preliminary assessment in the survey and design stage and a secondary assessment in the construction stage, as shown in Figure 1.

2.1.1. The Preliminary Assessment

The preliminary assessment is conducted to evaluate the risk of water inrush in the survey and design stage, with the aim of guiding the rational design of support parameters and construction methods. At this stage, the hazard-pregnant geological environment is the primary factor influencing the occurrence of water and mud inrush. Therefore, through a comprehensive analysis of existing literature, formation lithology I_1 , unfavorable geology I_2 , groundwater level I_3 , topography and geomorphology I_4 , dip angle of rock formation I_5 , layer and interlayer fissures I_6 , and contact zone between dissolvable and insoluble rocks I_7 are selected as the indices of the preliminary assessment [14–16,18]. According to the occurrence probability of geological hazards, the risk of water inrush from high to low is divided into C_1 (Very high risk), C_2 (High risk), C_3 (Medium risk), and C_4 (Low risk). The grading standards of evaluation indices, are in Table 1. The formation lithology I_1 is quantitatively classified based on a comprehensive analysis of rock solubility and the proportion of soluble rocks. The difference between the groundwater level and the tunnel floor elevation is used to quantify the indicator I_3 . The topography and geomorphology I_4 is represented by negative terrain area. The expert scoring method is employed to

quantitatively classify the unfavorable geology I_2 , layer and interlayer fissures I_6 , and contact zone between soluble and insoluble rocks I_7 .

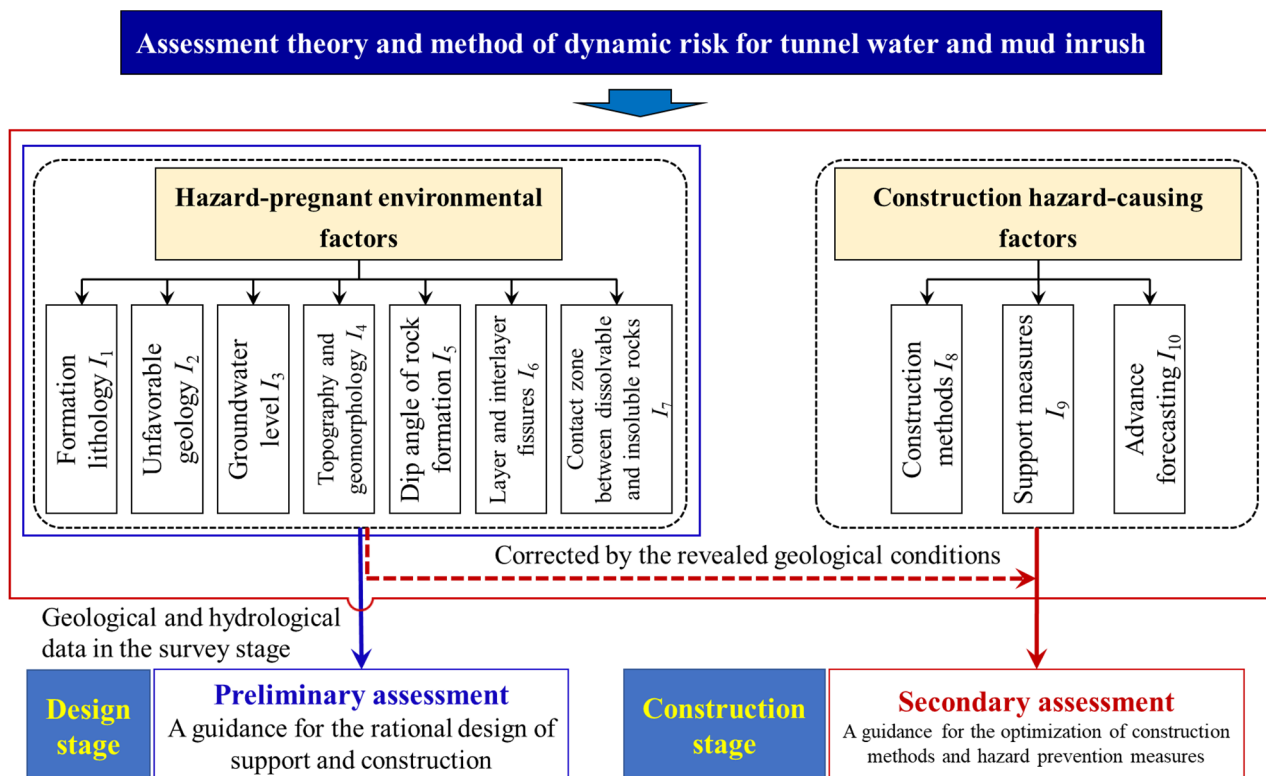


Figure 1. The two-stage assessment method for dynamic risk of water and mud inrush in tunnels.

Table 1. Grading standards of hazard-pregnant environmental indices for water and mud inrush [14,15,18].

Hazard-Pregnant Environmental Indice	Risk Grade of Water Inrush			
	C1	C2	C3	C4
Formation lithology I_1	0.254~0.404	0.104~0.254	0.042~0.104	<0.042
Unfavorable geology I_2	≥ 90	80~90	60~80	<60
Groundwater level I_3 /m	≥ 60 m	30 m~60 m	10 m~30 m	≤ 10 m
Topography and geomorphology I_4	$>60\%$	40~60%	20~40%	0~20%
Dip angle of rock formation $I_5/^\circ$	25~65	10~25/65~80	80~90	0~10
Layer and interlayer fissures I_6	≥ 90	80~90	60~80	<60
Contact zone between dissolvable and insoluble rocks I_7	≥ 90	80~90	60~80	<60

2.1.2. The Secondary Assessment

The secondary assessment is conducted to evaluate the risk of water inrush in an unexcavated section ahead of the tunnel face during the tunnel construction stage. At this stage, the factors influencing the occurrence of water inrush include the hazard-pregnant environmental factors and the construction hazard-causing factors. Therefore, three construction hazard-causing indices, including construction methods I_8 , support measures I_9 , and advance forecasting I_{10} , based on the preliminary assessment of hazard-pregnant environmental indices are introduced as the indices of the secondary assessment. The grading standards of three construction hazard-causing indices are shown in Table 2. In addition, the measured values of the hazard-pregnant environmental indices are corrected according to the engineering and hydrogeological conditions revealed by the tunnel face.

Table 2. Grading standards of construction hazard-causing indices for water and mud inrush [22].

Construction Hazard-Causing Indices	Risk Grade of Water Inrush			
	C ₁ (Very High Risk)	C ₂ (High Risk)	C ₃ (Medium Risk)	C ₄ (Low Risk)
Construction methods I_8	Very unreasonable <60	Unreasonable 60~80	Basically reasonable 80~90	Reasonable ≥90
Support measures I_9	Very unreasonable <60	Unreasonable 60~80	Basically reasonable 80~90	Reasonable ≥90
Advance forecasting I_{10}	Very unreasonable <60	Unreasonable 60~80	Basically reasonable 80~90	Reasonable ≥90

2.2. Assessment Model Based on Combination Weighting and Interval Extension Theory

2.2.1. Improved Interval Extension Theory

Extension theory can transform contradiction problems into compatibility problems with formalized models, and its core is matter element theory and extensible sets. As logical cells, the matter element adopts an ordered triplet $R = (\text{Thing}, \text{Feature}, \text{Value}) = (N, c, v)$ as the basic element for describing things. Usually, the indices describing the risk of an evaluation object are assigned a fixed value. However, the geological and hydrological parameters of tunnels are fuzzy and heterogeneous, and a single value often results in a loss of information. Therefore, a small range interval is used to assign the evaluation index. The steps of the improved interval extension theory are as follows:

- (1) Determining the classical domain, joint domain and matter element of the object

$$R_j = \begin{bmatrix} N_j & c_1 & v_{1j} \\ & c_2 & v_{2j} \\ & \vdots & \vdots \\ & c_n & v_{nj} \end{bmatrix} = \begin{bmatrix} N_j & c_1 & \langle a_{2j}, b_{2j} \rangle \\ & c_2 & \langle a_{2j}, b_{2j} \rangle \\ & \vdots & \vdots \\ & c_n & \langle a_{nj}, b_{nj} \rangle \end{bmatrix} \quad (1)$$

$$R_P = \begin{bmatrix} N_P & c_1 & v_{P1} \\ & c_2 & v_{P2} \\ & \vdots & \vdots \\ & c_n & v_{Pn} \end{bmatrix} = \begin{bmatrix} N_P & c_1 & \langle a_{P1}, b_{P1} \rangle \\ & c_2 & \langle a_{P2}, b_{P2} \rangle \\ & \vdots & \vdots \\ & c_n & \langle a_{Pn}, b_{Pn} \rangle \end{bmatrix} \quad (2)$$

$$R_k = \begin{bmatrix} N_k & c_1 & [v_{k1}^L, v_{k1}^U] \\ & c_2 & [v_{k2}^L, v_{k2}^U] \\ & \vdots & \vdots \\ & c_n & [v_{kn}^L, v_{kn}^U] \end{bmatrix} \quad (3)$$

where R_j is the homologous matter element of the evaluation object corresponding to the j th risk level. N_j is the j th risk level of the evaluation object. c_i is the i th assessment index. $v_{ij} = \langle a_{ij}, b_{ij} \rangle$ is the standardized classical domain of the evaluation index c_i belonging to the risk level N_j . N_P denotes all risk levels. $V_{Pi} = \langle a_{Pi}, b_{Pi} \rangle$ is the joint domain of the evaluation index c_i belonging to all risk levels N_P . N_k is the assessment object. v_{ki}^L and v_{ki}^U are the lower limit and upper limit of the measured values of the evaluation index c_i , respectively.

- (2) Establishing the correlation functions and calculating the correlation degree

The correlation degree of the matter element to be evaluated corresponding to each risk level is as follows:

$$K_j(N_k) = \sum_{i=1}^n w_i K_{ji}(v_{ki}) \quad (4)$$

$$K_{ji}(v_{ki}) = \begin{cases} \frac{-\rho(v_{ki}, v_{0ij})}{|b_{0ij} - a_{0ij}|}, v_{ki} \in v_{0ij} \\ \frac{\rho(v_{ki}, v_{0ij})}{\rho(v_{ki}, v_{pi}) - \rho(v_{ki}, v_{0ij})}, v_{ki} \notin v_{0ij} \end{cases} \quad (5)$$

$$\rho(v_{ki} v_{0ij}) = \left| v_{ki} - \frac{a_{0ij} + b_{0ij}}{2} \right| - \frac{b_{0ij} - a_{0ij}}{2} \quad (6)$$

$$\rho(v_{ki} v_{pi}) = \left| v_{ki} - \frac{a_{pi} + b_{pi}}{2} \right| - \frac{b_{pi} - a_{pi}}{2} \quad (7)$$

where w_i is the weight of the evaluation index c_i . $K_{ji}(v_{ki})$ is the single-index correlation function of the index c_i belonging to the risk level N_{0j} .

Since the measured value of the evaluation index c_i ($i = 1, 2, \dots, n$) is a small range interval $[v_{ki}^L, v_{ki}^U]$, countless values of correlation degree will be obtained by substituting the interval $[v_{ki}^L, v_{ki}^U]$ into Equation (7). It is extremely difficult to solve the multi-index fusion correlation degrees and evaluate the risk level of the object. To simplify the calculation of single-index and multi-index integrated correlation degrees, the assignment-value interval $[v_{ki}^L, v_{ki}^U]$ of the evaluation index c_i is optimized to a ternary fuzzy interval $(v_{ki}^L, v_{ki}^M, v_{ki}^U)$, where v_{ki}^L , v_{ki}^M , and v_{ki}^U are the lower limit, the most likely number, and the upper limit of the measured value of the evaluation index c_i , respectively. Then, the single-index correlation degrees $K_{ji}^M(v_{ki})$, $K_{ji}^L(v_{ki})$, and $K_{ji}^U(v_{ki})$ corresponding to v_{ki}^L , v_{ki}^M , and v_{ki}^U are calculated. The fusion rule of single-index correlation degree is defined as:

$$K_j(N_k) = \sum_{i=1}^n w_i \cdot \frac{2K_{ji}^M(v_{ki}) + K_{ji}^L(v_{ki}) + K_{ji}^U(v_{ki})}{4} \quad (8)$$

(3) Risk level identification

According to the principle of maximum correlation degree $\bar{K}_j(N_k) = \max\{\bar{K}_j(N_k) | j=1,2,\dots,m\}$, the risk level of the matter element to be evaluated is $C_{j'}$.

$$\bar{K}_j(N_k) = \frac{K_j(N_k) - \min_j K_j(N_k)}{\max_j K_j(N_k) - \min_j K_j(N_k)} \quad (9)$$

2.2.2. AHP-FCM Combination Weighting Method Based on Game Theory

(1) Subjective weighting based on AHPv et al. is used to quantify the relative importance between the evaluation indices. However, the 1~9 scale method can easily cause confusion regarding the relative importance between evaluation indices. Therefore, a 1~5 scale method is proposed to construct the judgment matrix $M = (m_{ij})_{n \times n}$ (n is the number of evaluation indices), as shown in Table 3.

Table 3. Improved risk evaluation scores of the 1~5 scale method.

Scale m_{ij}	Linguistic Scale for Importance
1	I_i and I_j are equally importance
3	I_i is obviously more important than I_j
5	I_i is strongly more important than I_j
2, 4	Middle value of upper and lower scales respectively

The weighting values of the evaluation indices are calculated based on the constructed judgment matrix. The specific steps are as follows:

$$W = (w_1, w_2, \dots, w_n) \quad (10)$$

$$w_i = \frac{\overline{w_i}}{\sum_{i=1}^n \overline{w_i}} \quad (11)$$

$$\overline{w_i} = \sqrt[n]{\prod_{j=1}^n m_{ij}} \quad (i = 1, 2, \dots, n) \quad (12)$$

where w_i ($i = 1, 2, \dots, n$) is the weighting value of the evaluation index I_i , s.t. $0 \leq w_i \leq 1$ and $\sum_{i=1}^n w_i = 1$. $\overline{w_i}$ is the geometric mean of the evaluation index I_i .

To verify the rationality of the subjective weights of each evaluation index, a consistency test is needed.

$$CI = \frac{\lambda_{\max} - n}{n - 1}, CR = \frac{CI}{RI} \quad (13)$$

$$\lambda_{\max} = \frac{1}{n} \sum_{i=1}^n \frac{(M \cdot W)_i}{w_i} \quad (14)$$

where CI is the consistency index and RI is the mean random consistency index, which takes its value from the reference [26]. CR is the coincidence coefficient. When CI and CR are less than 0.1, the constructed judgment matrix is scientific.

(2) Objective weighting based on FCM

On the basis of statistical analysis and data collation of typical tunnel water inrush cases, the relationship between various influencing factors and the occurrence of water inrush is analyzed. The frequency counting method (FCM) is employed to determine the objective weights of the evaluation indices. The specific methodology is as follows:

$$w_i^o = \frac{w_i^{o'}}{\sum_{i=1}^N w_i^{o'}}, w_i^{o'} = \frac{f_i}{N} \quad (15)$$

where w_i^o and $w_i^{o'}$ are the objective weights after and before normalization of the evaluation index c_i , respectively. f_i is the frequency of occurrence of the evaluation index c_i . N is the number of statistical water inrush cases. More than 100 cases of tunnel water inrush in China are collected and sorted out [14,23,27], and then the objective weights of the preliminary assessment indices are obtained, as shown in Figure 2.

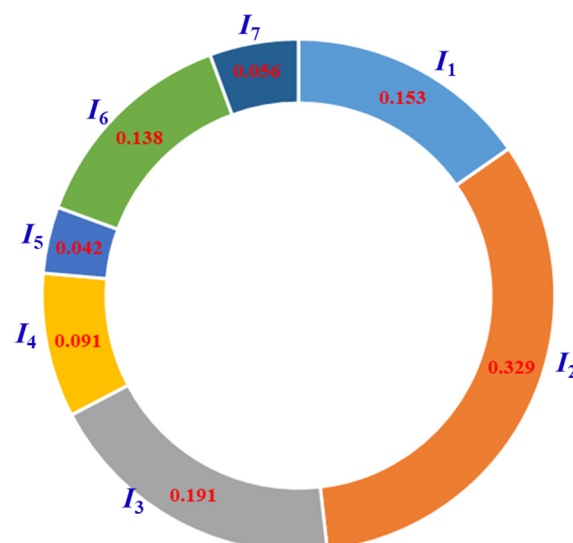


Figure 2. The objective weights of the preliminary assessment indices.

In the secondary assessment, the hazard-causing factors, such as the construction method, support measures and advance forecasting, are introduced. And the objective weights of the hazard-causing factors are analyzed through frequency statistics of numerous tunnel water inrush cases [28], namely $\{w^o(I_8), w^o(I_9), w^o(I_{10})\} = (0.048, 0.048, 0.192)$. According to the proportion of hazard-pregnant environmental factors and construction hazard-causing factors [13], the initial objective weights of the hazard-pregnant indices and the hazard-causing indices are calculated separately and then normalized. The final objective weights of the secondary assessment indices are determined, as shown in Figure 3.

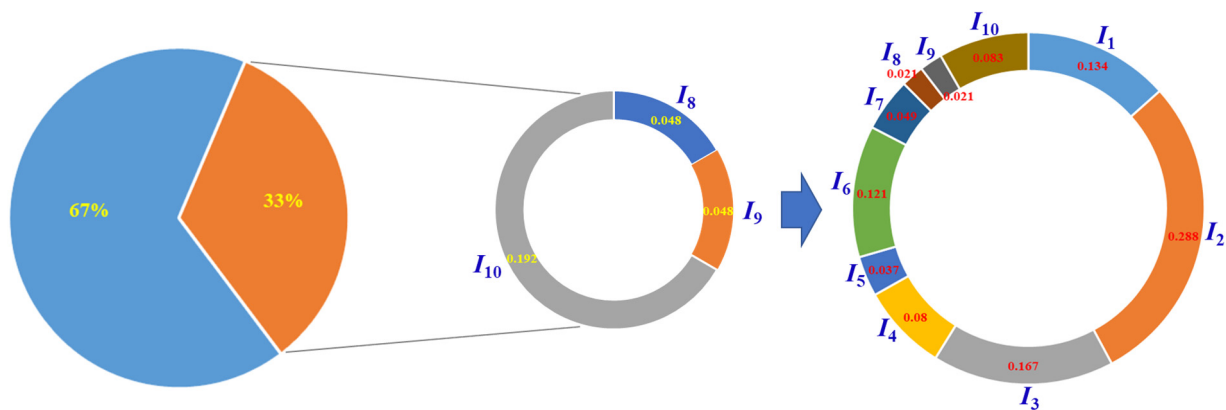


Figure 3. The objective weights of the secondary assessment indices.

(3) Combination weighting based on GT

To solve the best consistent and coordinated relationship between subjective and objective weighting combinations, the concept of minimizing the deviation of linear combination weights from game theory (GT) is applied to comprehensively integrate and assign the weights of the evaluation indices for tunnel water inrush. The calculation formula of combination weight w_i is as follows:

$$w_i = \alpha_1 w_i^S + \alpha_2 w_i^O \quad (16)$$

where w_i^S and w_i^O are the subjective weight and objective weight of the evaluation index c_i , respectively. α_1 and α_2 are respectively the combination coefficients of objective weight and subjective weight, which satisfies $\alpha_1 + \alpha_2 = 1$. The solving process of α_1 and α_2 is as follows:

Firstly, an optimization objective function for the combination coefficients α_1 and α_2 , with the goal of minimizing the deviation between the combination weight vector W and the subjective weight vector W^S and the objective weight vector W^O , is constructed.

$$\min \|\alpha_1 W^S + \alpha_2 W^O - W_k\| \quad (17)$$

where W_k ($k = 1, 2$) is the index weight vector obtained by the i th method.

Secondly, the linear equations equivalent to the first-order derivative conditions for the optimization in Equation (17) are as follows:

$$\begin{bmatrix} W^S \cdot (W^S)^T & W^S \cdot (W^O)^T \\ W^O \cdot (W^S)^T & W^O \cdot (W^O)^T \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} W^S \cdot (W^S)^T \\ W^O \cdot (W^O)^T \end{bmatrix} \quad (18)$$

Next, the optimized combination coefficients α_1^* and α_2^* are calculated and normalized:

$$\alpha_1^* = \frac{|\alpha_1|}{|\alpha_1| + |\alpha_2|}, \alpha_2^* = \frac{|\alpha_2|}{|\alpha_1| + |\alpha_2|} \quad (19)$$

Finally, α_1^* and α_2^* are substituted into Equation (16), and the combined weight w_i is obtained.

3. Results

3.1. Engineering Background

Yuelongmen Tunnel from the Chengdu to Lanzhou Railway in Sichuan Province is a separated tunnel, where the length of the left line is 19,974.3 m and the right line is 20,044.0 m. The maximum burial depth of the tunnel is 1445.5 m. The tunnel passes through multiple geological fault zones and their influence zones, one of which is a Holocene strong active fault zone. The surrounding rock is weak and fractured, which accounts for a high proportion. There are several rivers with perennial running water in deep valleys. The regional precipitation is abundant. The construction difficulty and risk are extremely high [29]. The shallow buried section that underpasses the Gaochuan River, constructed during the rainfall period, is selected as the research area, as shown in Figure 4. The minimum buried depth of the section is only 51 m.

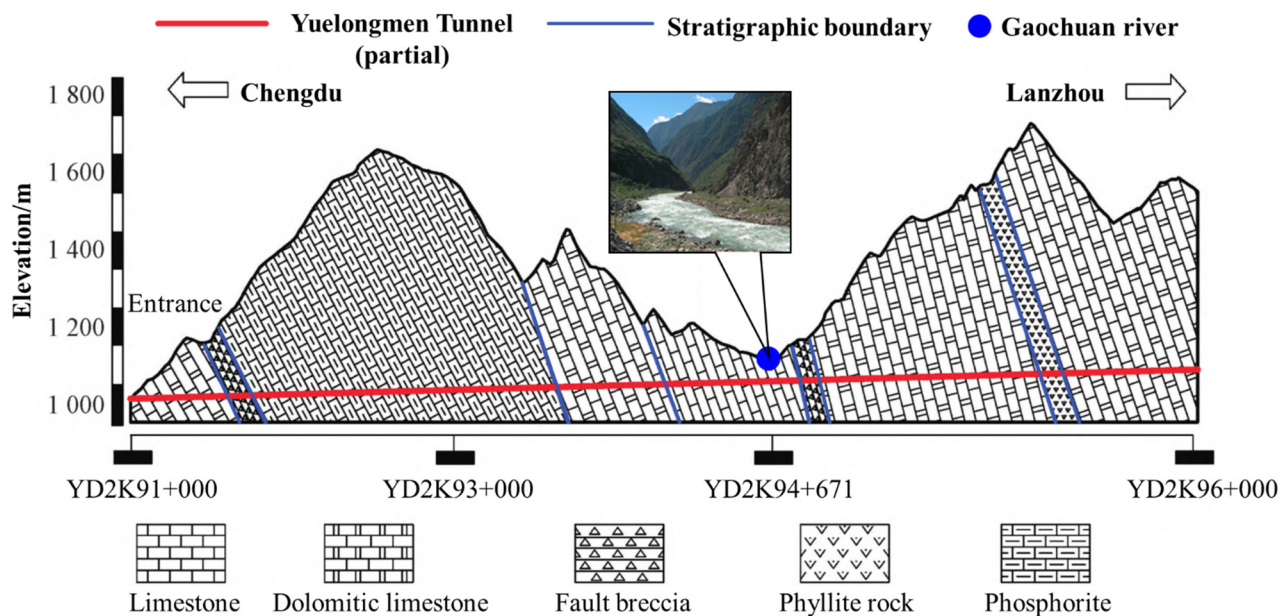


Figure 4. Engineering geological profile of the river-crossing section in the Yuelongmen Tunnel.

3.2. The Result Analysis of Preliminary Assessment

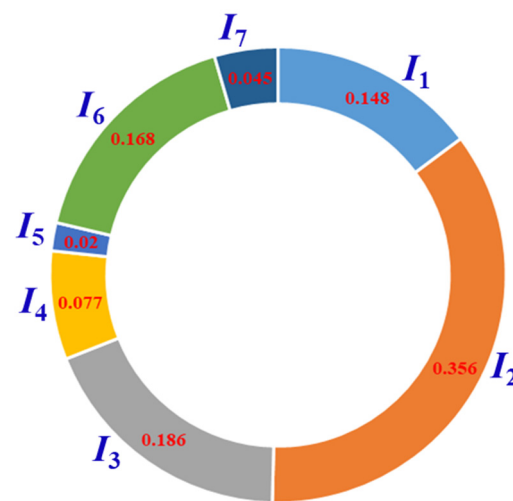
3.2.1. The Weight Determination of the Preliminary Assessment Indices

An industry expert with more than 10 years of tunnel construction experience is invited to construct the judgment matrix between the evaluation indices. The weight values of the hazard-pregnant environmental factors for the preliminary assessment of water inrush are calculated by Equations (10)–(12). And the consistency checking is performed according to Equations (13) and (14), as shown in Table 4.

The subjective weights in Table 4 and the objective weights in Figure 2 are substituted into Equation (18) to calculate the initial α_1 and α_2 . Then they are substituted into Equation (19) for normalization, and the optimized combination coefficients $\alpha_1^* = 0.221$ and $\alpha_2^* = 0.779$ are obtained. The combination weights of the evaluation indices for the preliminary assessment are determined by Equation (16), as shown in Figure 5.

Table 4. Judgment matrix for the weights of hazard-pregnant indexes.

Index	I_1	I_2	I_3	I_4	I_5	I_6	I_7
I_1	1	1/2	1/2	2	2	3	2
I_2	2	1	2	2	3	3	2
I_3	2	1/2	1	2	2	3	2
I_4	1/2	1/2	1/2	1	2	2	2
I_5	1/2	1/3	1/2	1/2	1	2	2
I_6	1/3	1/3	1/3	1/2	1/2	1	1/2
I_7	1/2	1/2	1/2	1/2	1/2	2	1
Weight	0.165	0.260	0.202	0.128	0.099	0.059	0.086
Consistency checking	$\lambda_{max} = 7.272$, $CI = 0.045$, $CR = 0.034$, CI and CR are less than 0.1.						

**Figure 5.** The combination weights of the preliminary assessment indices.

3.2.2. The Correlation Degree Calculation of the Preliminary Assessment

According to the early geological survey data, the formation lithology of the tunnel section beneath the Gaochuan River is dominated by carbonate rocks, such as dolomitic limestone. The tunnel body is located in a deep groundwater circulation zone, with its elevation 10 to 55 m lower than the surface of the river. The karst of this tunnel section is moderately developed. The landform is characterized by dissolution peak clusters, while the interior is mainly presented as dissolution fissures. The measured values of the hazard-pregnant environmental indices are assigned, and the single-index correlation degrees between the risk level and the evaluation indices are solved by Equations (7)–(10), as shown in Figure 6.

And then the multi-index integrated correlation degrees considering the weight of each index are calculated by Equations (4) and (8).

$$K = \{K(C_1), K(C_2), K(C_3), K(C_4)\} = (-0.244, 0.124, -0.172, -0.517)$$

3.2.3. The Risk Identification of the Preliminary Assessment

The $\bar{K}(N_k) = (0.426, 1.00, 0.538, 0)$ is determined by Equation (9). According to the principle of maximum correlation degree, the risk level of water inrush is C_2 (high risk).

3.3. The Result Analysis of Secondary Assessment

3.3.1. The Weight Determination of the Secondary Assessment Indices

The method of constructing the judgment matrix for the subjective weights of the secondary assessment indices is as mentioned above. The judgment matrix for the secondary

assessment indicators, which includes hazard-pregnant environmental factors and construction hazard-causing factors, is constructed by 1 to 5 scale, as shown in Table 5. Then, the weights of the secondary assessment indicators are obtained by Equations (10)–(14), and the consistency checking is satisfied. Finally, the normalization of the index weights is conducted, as shown in Figure 7.

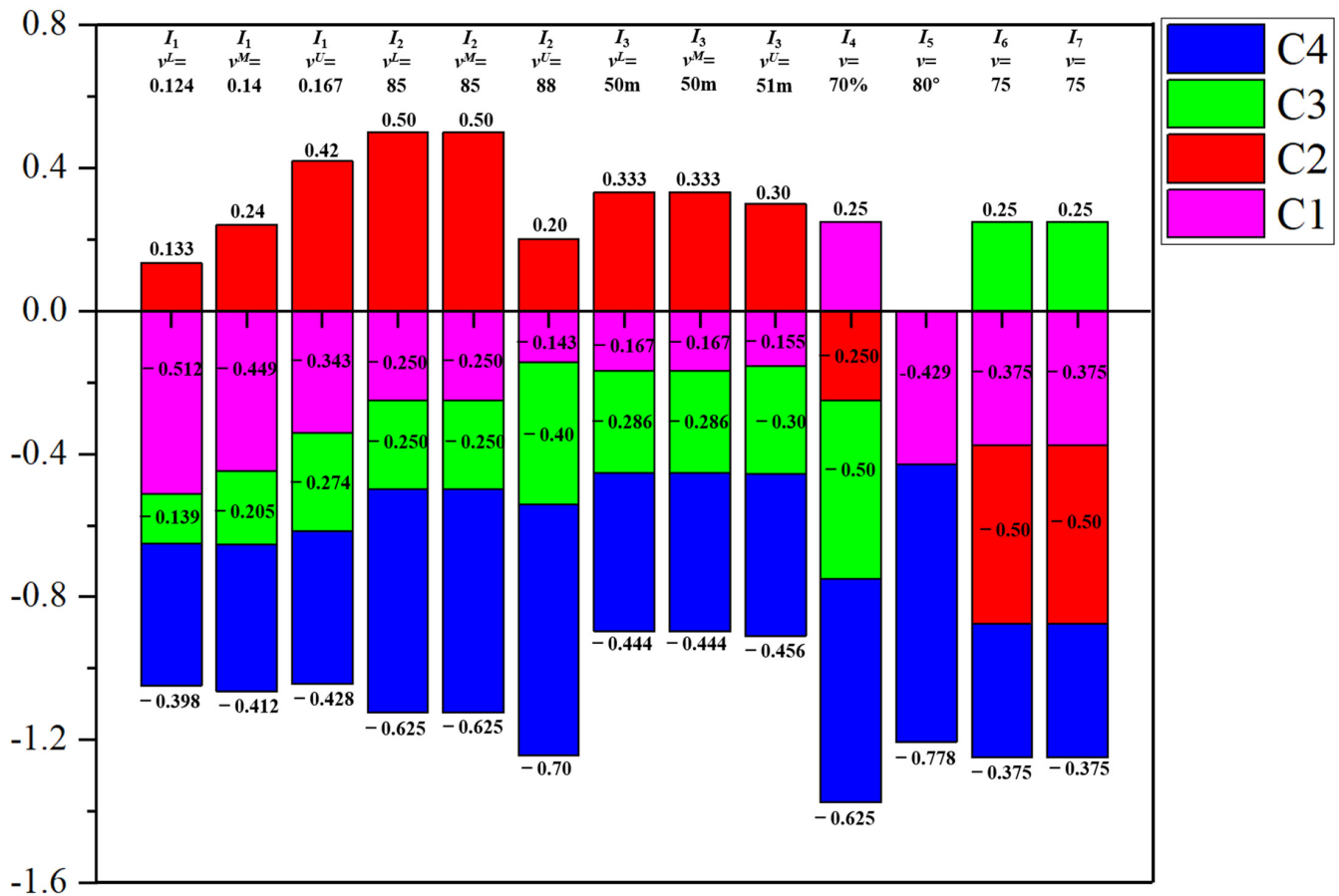


Figure 6. The single-index correlation degrees of the preliminary assessment indices.

Table 5. Judgment matrix for the weights of construction hazard-causing factors.

Index	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
I_1	1	1/2	1/2	2	2	3	2	3	3	2
I_2	2	1	2	2	3	3	2	4	4	3
I_3	2	1/2	1	2	2	3	2	3	3	2
I_4	1/2	1/2	1/2	1	2	2	2	2	2	1
I_5	1/2	1/3	1/2	1/2	1	2	2	1/2	1	1/2
I_6	1/3	1/3	1/3	1/2	1/2	1	1/2	1/2	1	1/2
I_7	1/2	1/2	1/2	1/2	1/2	2	1	1/2	1	1/2
I_8	1/3	1/4	1/3	1/2	2	2	2	1	2	1
I_9	1/3	1/4	1/3	1/2	1	1	1	1/2	1	1/2
I_{10}	1/2	1/3	1/2	1	2	2	2	1	2	1
Weight	0.141	0.213	0.162	0.101	0.064	0.045	0.058	0.076	0.050	0.091
Consistency checking $\lambda_{max} = 10.422$, $CI = 0.047$, $CI = 0.031$, CI and CR are less than 0.1.										

The optimized combination coefficients of the secondary assessment are $\alpha_1^* = 0.301$ and $\alpha_2^* = 0.699$. The combination weights of the evaluation indices for the secondary assessment are determined, as shown in Figure 8.

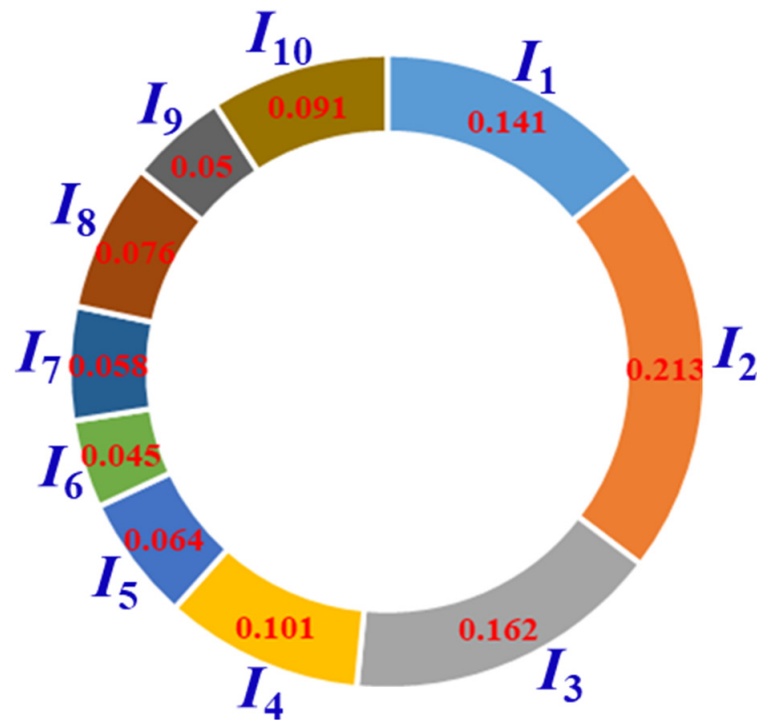


Figure 7. The subjective weights of the secondary assessment indices.

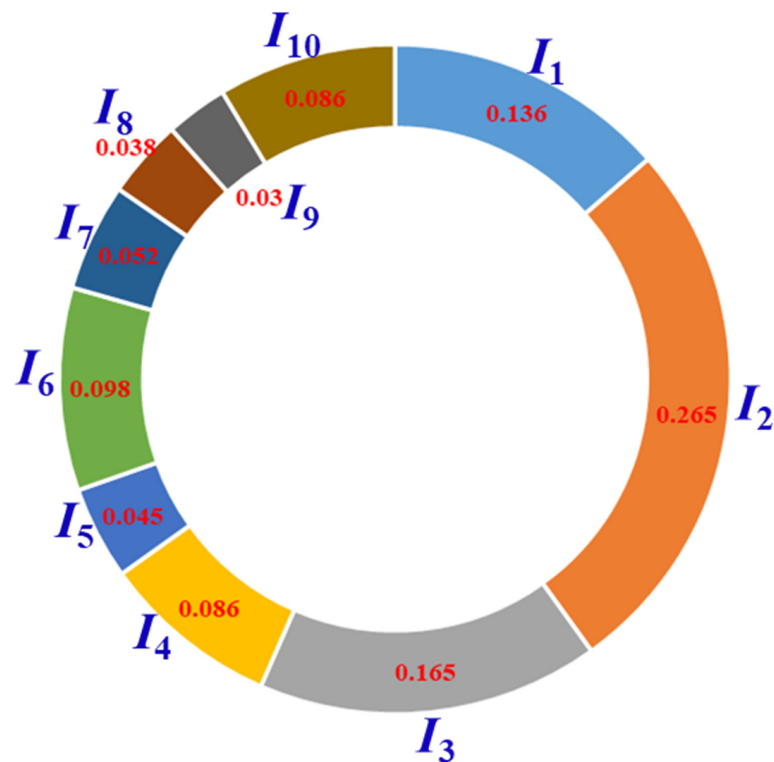


Figure 8. The combination weights of the secondary assessment indices.

3.3.2. The Correlation Degree Calculation of the Secondary Assessment

As the tunnel face reached the YD2K94+671 beneath the Gaochuan River, the lithology is dominated by limestone. The surrounding rock is highly fractured, with well-developed joints and solution fissures. The rock mass is classified as Grade IV, characterized by poor stability and abundant groundwater. A mud-filled weak interlayer is exposed at the upper right portion of the tunnel face. According to the geological conditions revealed by the

excavation of the tunnel face and the advanced geological forecasting data, the assignment values of hazard-pregnant environmental factors of water inrush are modified. The values of the construction hazard-causing factors are assigned based on the construction organization plan and actual construction situation. Subsequently, the single-index correlation degrees of the evaluation indices belonging to each risk level are calculated as shown in Figure 9. The multi-index integrated correlation degrees belonging to each risk level are calculated.

$$K = \{K(C_1), K(C_2), K(C_3), K(C_4)\} = (-0.336, 0.001, -0.069, -0.462)$$

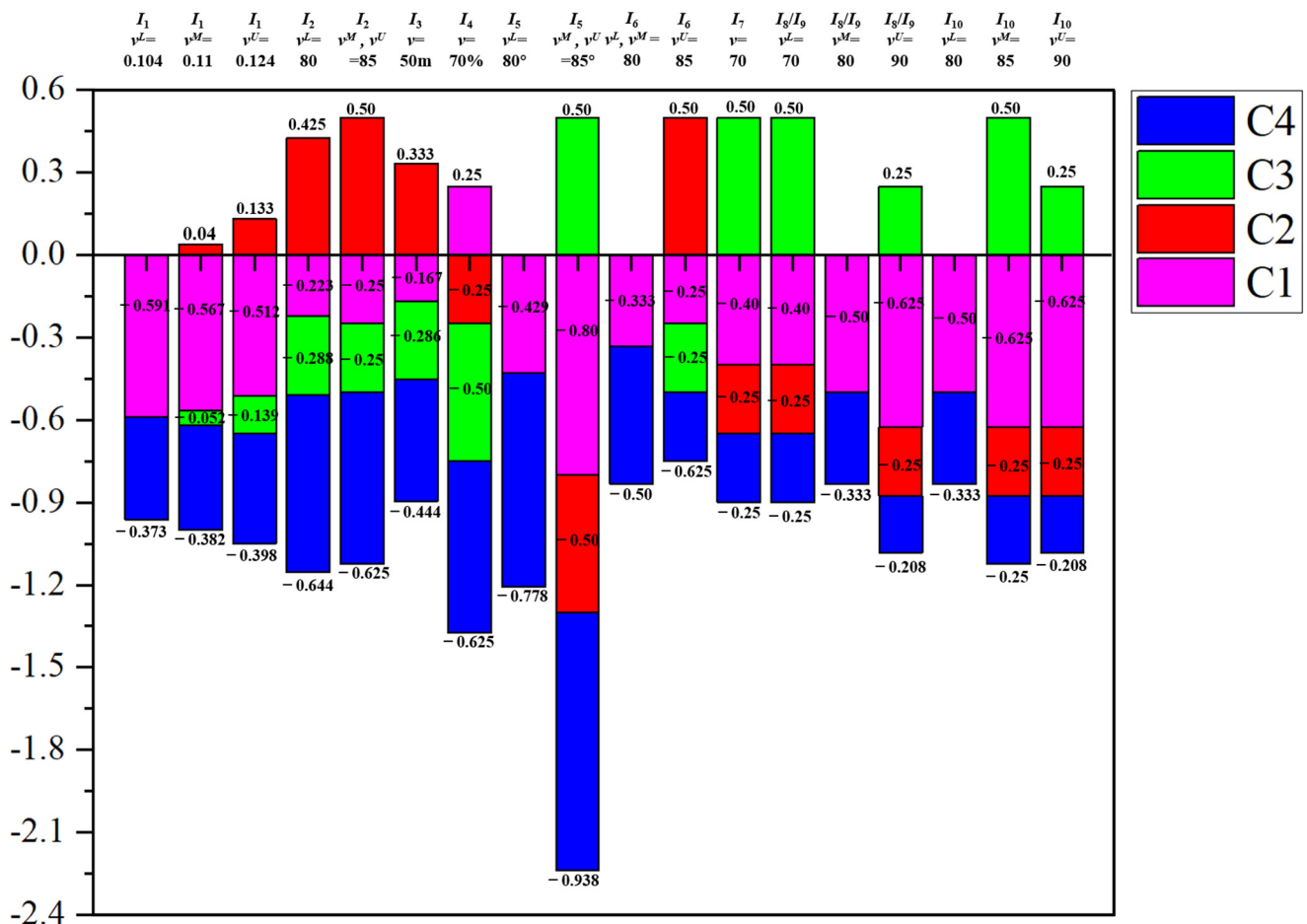


Figure 9. The single-index correlation degrees of the secondary assessment indices.

3.3.3. The Risk Identification of the Secondary Assessment

The $\bar{K}(N_k) = (0.272, 1.00, 0.848, 0)$ is determined by Equation (9). According to the principle of maximum correlation degree, the risk level of water inrush for the tunnel section beneath the Gaochuan River is C_2 (high risk). When the tunnel construction reached the center of the river-crossing section at YD2K94+671, and the tunnel site was in the rainy season, the water inflow at the tunnel face was approximately $7000 \text{ m}^3/\text{d}$ [29]. Li et al. [14] divided the risk level of tunnel water inrush into four grades based on the volume of water inflow; that is, the risk of water inflow more than $10,000 \text{ m}^3/\text{d}$ is Level I, and the risk level of water inflow between $3000 \text{ m}^3/\text{d}$ and $10,000 \text{ m}^3/\text{d}$ is Level II. The results evaluated by the proposed method are compared with those evaluated by other methods [30]. The accuracy of the assessment results and the effectiveness of the proposed method are verified. However, the difference in the correlation degree between Level C_2 and Level C_3 decreased

significantly. This is because advanced drainage and pressure-relief drilling, along with grouting, were implemented for risk control.

4. Discussion

The study proposes a novel two-stage dynamic risk assessment methodology for water and mud inrush in tunnels, integrating an improved assessment model based on game theory combination weighting and interval extension theory. Through validation using the Yuelongmen Tunnel as a case study, the results demonstrate the feasibility and accuracy of the proposed method.

The innovation of this study lies in the two-stage assessment framework for dynamic risk of tunnel water and mud inrush, namely the preliminary assessment in the design stage and the secondary assessment in the construction stage. The preliminary assessment based on seven hazard-pregnant environmental factors provides a risk baseline for optimizing the initial engineering design. The secondary assessment then refines the baseline by incorporating construction hazard-causing factors and original hazard-pregnant environmental factors, which are corrected based on the actual geological conditions exposed during excavation. This methodology effectively mirrors the progressive nature of geological information acquisition in tunneling.

Furthermore, the development of an improved assessment model using game theory combination weighting and ternary fuzzy interval extension theory addresses key limitations in traditional risk assessment. The use of ternary fuzzy interval numbers to quantify evaluation indices, rather than fixed values, offers a more robust way to handle the inherent uncertainty and fuzziness of geological conditions [31]. The construction of single-index and multi-index correlation degree functions for ternary fuzzy intervals allows for a more nuanced calculation of risk levels. Meanwhile, the introduction of game theory to determine the optimal combination weight scheme by calculating subjective weights and objective weights increases the reliability of the final risk classification.

The successful application of this method to the Yuelongmen Tunnel's river-crossing section, with results aligning well with actual water inflow, validates its practical utility. The method's ability to provide clear risk levels and dynamic risk recognition makes it a valuable tool for engineers and project managers. However, it is important to acknowledge certain limitations. The grading criteria of some evaluation indices are qualitative and subjective [14]. The relationship between a single index and water inrush risk under multi-field coupling in tunnels is non-linear, and the existing functions cannot accurately describe it [12,15,24]. Additionally, applying this methodology to a wider range of tunnel projects with diverse geological conditions would further test its generalizability and robustness.

5. Conclusions

(1) A two-stage assessment theory and methodology of dynamic risk of water and mud inrush in tunnels has been proposed, consisting of a preliminary assessment and a secondary assessment. The hazard-pregnant environmental factors, including formation lithology, unfavorable geology, groundwater level, topography and geomorphology, dip angle of rock formation, layer and interlayer fissures, and contact zone between dissolvable and insoluble rocks, are selected as the evaluation indices for the preliminary assessment, which can provide the basis for the optimization of tunnel engineering design. The indicators of the secondary assessment consider the above hazard-pregnant environmental factors and three construction hazard-causing factors, such as construction methods, support measures, and advance forecasting. Meanwhile, the assignment values of the hazard-pregnant environmental factors are corrected according to the geological conditions

exposed by excavation. The results of the secondary assessment can provide guidance for tunnel excavation and support.

(2) An improved assessment model based on game theory combination weighting and interval extension theory for accurate identification of water inrush risk is developed. The ternary fuzzy interval number is used to quantify the measured value of the evaluation index, rather than a fixed value. And the single-index and multi-index correlation degree functions of the extension theory are constructed for the ternary fuzzy interval number. The subjective weights and objective weights of the indices for the risk assessment of water inrush are calculated respectively by using the improved 1–5 scale AHP and frequency counting method. The optimal combination weighting method based on subjective and objective weights is presented by introducing game theory.

(3) The proposed dynamic risk assessment method for tunnel water inrush based on game theory combination weighting and ternary fuzzy interval extension theory is applied to evaluate the risk of water inrush in the crossing-river section of Yuelongmen Tunnel. The assessment results are in good agreement with the field water inflow situation and those of other methods, which verifies the feasibility and accuracy of the method. The proposed method has the advantages of a clear risk level and dynamic risk recognition.

Although the proposed methodology is developed and validated specifically for tunnel water and mud inrush, its methodological framework—comprising preliminary assessment during the design phase, secondary assessment during construction, and progressive updating of evaluation indices—is inherently adaptable to other underground workings.

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