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Dispersion and Directional Attenuation in Periodic Acoustic-Black-Hole Plates

Ahmed Mehrem ¹, María Campo-Valera ², Luisa Euler ¹, Lutz Staemmler ¹ and Rafael Asorey-Cacheda ^{3,4,*}

¹ Technology Center E-Textiles and Acoustics, German Institutes of Textile and Fiber Research, 73770 Denkendorf, Germany; ahmed.mehrem@ditf.de (A.M.); luisa.euler@ditf.de (L.E.); lutz.staemmler@ditf.de (L.S.)

² School of Engineering and Technology (ESIT), Universidad Internacional de la Rioja, 26006 Logroño, Spain; mariamercedes.campo@unir.net

³ Department of Information and Communication Technologies, Universidad Politécnica de Cartagena, European University of Technology EUt+, 30202 Cartagena, Spain

⁴ People Oriented Smart Technologies Laboratory, European Research Group, European University of Technology, European Union

* Correspondence: rafael.asorey@upct.es

Abstract

This work investigates how unit-cell topology controls flexural-wave dispersion and finite-array attenuation in periodic acoustic-black-hole (ABH) plates. Six configurations are compared under identical material constants, lattice parameter, power-law profile, and residual thickness: a uniform reference plate, two isolated ABH inclusions with circular and y -elongated elliptical shapes, a crossed-strip ABH network, and two cross-connected inclusion-strip networks. The plate is modeled using Kirchhoff–Love theory with spatially varying thickness, and dispersion curves are computed with a finite-difference Bloch (FD–Bloch) formulation along the sampled Γ –X–M– Γ path. The numerical dispersion results are checked against a Gaussian-enriched Ritz method and, for the uniform plate, against the analytical folded dispersion relation. To connect the infinite-periodic predictions with practical attenuation, finite arrays are also analyzed through transmission maps and full-aperture transmission curves. The results show that isolated circular and elliptical ABHs mainly distort dispersion branches, produce partial directional stop behavior, and give only moderate forward-transmission reductions in finite arrays. By contrast, connected low-thickness ligaments strengthen modal coupling and generate wider stop intervals along the sampled path. The crossed-strip and cross-connected circular networks reduce the full-aperture transmission at their selected stop-band frequencies. The cross-connected y -elongated elliptical network gives the strongest response. These results show that ABH topology, and especially the connectivity and anisotropy of the thinned regions, is a decisive design parameter for directional attenuation in lightweight periodic plates.

Keywords: acoustic black hole; Bloch wave; dispersion relation; stop interval; Kirchhoff–Love plate; Gaussian Ritz method; directional attenuation



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1. Introduction

The control of waves in periodic media has its roots in classical lattice theory, where Brillouin showed that periodicity can split wave spectra into propagation and attenuation regions [1]. For thin plates, this idea is especially useful because flexural waves are strongly dispersive and depend on both bending stiffness and areal mass [2,3]. Later developments in phononic materials showed that periodic elastic structures can modify

wave motion through Bragg scattering, modal coupling, and symmetry-dependent band deformation [4–6]. Periodic thickness variation is therefore a natural route for tuning bending-wave dispersion in lightweight plate structures.

Acoustic black holes (ABHs) introduced a complementary attenuation mechanism based on a gradual, usually power-law, thickness reduction. An early flexural-wave description was given by Mironov [7], and its use as a passive vibration treatment was later developed by Krylov and Tilman [8]. In an ABH, the local reduction in thickness lowers the bending stiffness, slows the flexural wave, and concentrates vibration energy near the thin region, where damping can be more effective [9–11]. This local slow-wave behavior made ABHs attractive for lightweight vibration and noise reduction and motivated subsequent studies on how periodic repetition of the graded profile modifies plate dispersion and attenuation.

Early studies on periodic ABH structures showed that the local ABH effect can become a collective dispersion mechanism. Zhu and Semperlotti investigated phononic thin plates with embedded ABHs and demonstrated that graded thickness cells can change flexural-wave band structures [12,13]. Tang and Cheng then showed, in periodic beam structures, that embedded ABH indentations can generate broadband locally resonant band gaps when periodicity, tapering, and damping interact [14]. Zhu and Semperlotti further extended the concept to two-dimensional ABH plate lattices and used periodic ABH cells to form embedded acoustic lenses, showing that graded thickness maps can steer, focus, and redistribute flexural-wave energy without discrete resonators [15]. These works moved the field from isolated ABH damping treatments toward ABH-based phononic architectures.

Subsequent studies focused more directly on how ABH geometry and connectivity affect band gaps, radiation, and directional transmission. Tang and Cheng examined tunneled ABH plates and showed that continuous low-thickness paths can generate directional band-gap behavior [16,17]. Periodic ABH panels reported impaired sound radiation and low-frequency locally resonant attenuation, emphasizing the role of ABH-induced energy localization in reducing the radiating response [18,19]. Other studies merged phononic-crystal concepts with ABH tapers, indicating that Bragg scattering and slow-wave localization can be combined to broaden attenuation ranges in lightweight periodic structures [20–22]. In parallel, ABH lattices were also used to realize topological edge transport in plates, showing that thickness-map design can control modal localization, symmetry breaking, and protected directional propagation [23].

More recently, research has moved toward identifying which ABH layouts produce robust attenuation in practical finite structures. Complete sub-wavelength flexural band gaps have been reported for periodic ABH plates when the unit-cell geometry supports strong coupling between localized and propagating modes [24–26]. Strip-type ABH layouts also open broad band gaps. This result points to the connectivity of the thinned region as an important design variable, not only a geometric detail [27,28]. Metamaterial ABH plates with local resonators further demonstrate that ABH-induced slow waves can be combined with resonant mechanisms for broadband vibration reduction [29–31]. Taken together, these studies show that the same material constants, lattice parameter, residual thickness, and power-law exponent can lead to different dispersion diagrams when the ABH topology changes.

Despite this progress, most studies examine either a single acoustic black hole topology or a narrow set of closely related configurations. Comparisons across the literature therefore remain inconclusive, since reported results are obtained using different material properties, lattice dimensions, residual thicknesses, damping models, numerical schemes, and attenuation measures. These variations obscure the contribution of the unit-cell geometry itself. A systematic comparison is still missing in which isolated, interconnected,

and anisotropic ABH thickness distributions are evaluated under the same mechanical, geometric, and numerical conditions.

The present work addresses this gap through a controlled topology comparison rather than through the introduction of an additional isolated geometry. Six unit-cell configurations are examined using the same material properties, lattice constant, background and residual thicknesses, and power-law exponent. The contributions are three-fold. First, the study isolates the effect of topology by comparing a uniform reference, isolated circular and elliptical ABHs, a crossed-strip network, and two cross-connected inclusion-strip networks within a unified framework. Second, the topology-dependent dispersion curves obtained with the FD–Bloch formulation are independently checked using a Gaussian-enriched Ritz method with a different approximation space. Third, the infinite-periodic dispersion features are related directly to finite-array full-aperture and forward-channel transmission, allowing branch deformation, directional stop behavior, connectivity, and anisotropy to be interpreted together.

Within this study, the ABH thickness map is treated as the primary design variable for redistributing bending stiffness across the unit cell. Isolated tapers, connected ligaments, and elongated inclusions emphasize different physical mechanisms. The comparison therefore provides topology design guidance for selecting between directional filtering, full-aperture transmission reduction, and controlled redistribution of flexural-wave energy.

The remainder of this paper is organized as follows. Section 2 presents the physical model, Bloch formulation, ABH thickness profile, and unit-cell topologies. Section 3 describes the FD–Bloch, GEM, and finite-array simulation methods. Section 4 discusses numerical validation, dispersion behavior, and finite-array attenuation. Section 5 summarizes the main findings and design implications.

2. Physical Model and ABH Topologies

The wave response of a periodic ABH plate is governed jointly by the local thickness profile and by the way thinned regions are arranged within the unit cell. To separate these effects, this section introduces the common Kirchhoff–Love plate model used for all cases, the Bloch-periodic setting, the ABH thickness law, and the six unit-cell topologies studied under identical material and profile parameters.

2.1. Kirchhoff–Love Plate Model

Let $w(x, y, t)$ denote the transverse displacement of a thin plate with spatially varying thickness $h(x, y)$, where x and y are the in-plane coordinates and t is time. Under the Kirchhoff–Love plate hypothesis [2,32], the bending energy of the unit cell Ω is

$$U = \frac{1}{2} \int_{\Omega} D(x, y) \left[w_{,xx}^2 + w_{,yy}^2 + 2\nu w_{,xx} w_{,yy} + 2(1 - \nu) w_{,xy}^2 \right] d\Omega, \quad (1)$$

where E is Young’s modulus, ν is Poisson’s ratio, and the bending stiffness is

$$D(x, y) = \frac{E h^3(x, y)}{12(1 - \nu^2)}. \quad (2)$$

The corresponding kinetic energy is

$$T = \frac{1}{2} \int_{\Omega} \rho h(x, y) \dot{w}^2 d\Omega, \quad (3)$$

where ρ is the material density. Because $D \propto h^3$ (Equation (2)), even a modest reduction in thickness produces a large local decrease in bending stiffness, which is the physical origin of the ABH slow-down effect [7,9]. Kirchhoff–Love theory is retained for the present

thin-plate comparison. However, transverse shear deformation and rotary inertia may affect the highest-frequency results, which is acknowledged as a limitation of the model.

2.2. Bloch Periodicity and Eigenvalue Problem

For harmonic motion, $w(\mathbf{x}, t) = W(\mathbf{x})e^{i\omega t}$, where $\mathbf{x} = (x, y)$ and ω is the angular frequency. The finite-dimensional form of the Kirchhoff–Love weak formulation gives the generalized eigenvalue problem

$$\mathbf{K}(\boldsymbol{\kappa}) \mathbf{q} = \omega^2 \mathbf{M} \mathbf{q}, \quad (4)$$

where $\boldsymbol{\kappa} = (k_x, k_y)$ is the Bloch wave vector, $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, and $\mathbf{q}$ collects the discrete displacement degrees of freedom. Bloch periodicity on a square lattice with period a imposes:

$$W(x + a, y) = W(x, y) e^{ik_x a}, \quad W(x, y + a) = W(x, y) e^{ik_y a}. \quad (5)$$

The dispersion relation $\omega(\boldsymbol{\kappa})$ is evaluated along the boundary of the irreducible Brillouin zone,

$$\Gamma = (0, 0), \quad X = \left(\frac{\pi}{a}, 0\right), \quad M = \left(\frac{\pi}{a}, \frac{\pi}{a}\right), \quad (6)$$

following the standard Γ – X – M – Γ path [1].

For the uniform reference plate, $h(x, y) = h_0$, the analytical folded dispersion relation becomes

$$\omega(\boldsymbol{\kappa}) = \sqrt{\frac{D_0}{\rho h_0}} |\boldsymbol{\kappa} + \mathbf{G}|^2, \quad D_0 = \frac{E h_0^3}{12(1 - \nu^2)}, \quad \mathbf{G} = \frac{2\pi}{a} (n_x, n_y), \quad (7)$$

where D_0 is the bending stiffness of the uniform reference plate, $\mathbf{G}$ is a reciprocal-lattice vector, and $(n_x, n_y) \in \mathbb{Z}^2$. Equation (7) serves as the primary validation target for both numerical solvers.

2.3. ABH Topologies

Six unit-cell configurations are studied as shown in Figure 1. All share the same material, lattice, and ABH-profile parameters $(E, \rho, \nu, a, h_0, h_c, m)$, so differences in the undamped dispersion can be attributed to the topology of the thickness field. The finite-array attenuation additionally reflects the geometry-following damping distribution described in Section 3. The topology-specific geometric parameters are the circular ABH radius R , the elliptical semi-axes R_x and R_y , and the strip half-width r . The cases are ordered from the uniform reference to isolated ABH inclusions and then to connected networks, which makes it possible to separate local ABH effects from inter-cell connectivity. The ABH region in each non-uniform topology is described by a normalized local coordinate $s \in [0, 1]$ and a truncated power-law profile,

$$h(s) = h_c + (h_0 - h_c) s^m, \quad 0 \leq s \leq 1, \quad (8)$$

where h_c is the residual thickness, h_0 is the background thickness, m is the power-law exponent, and $s = 0$ corresponds to the thinnest ABH point. Outside the thinned ABH region, $h = h_0$. This profile follows the power-law ABH concept introduced by Mironov [7] and is consistent with practical truncated ABH realizations reviewed by Pelat et al. [9].

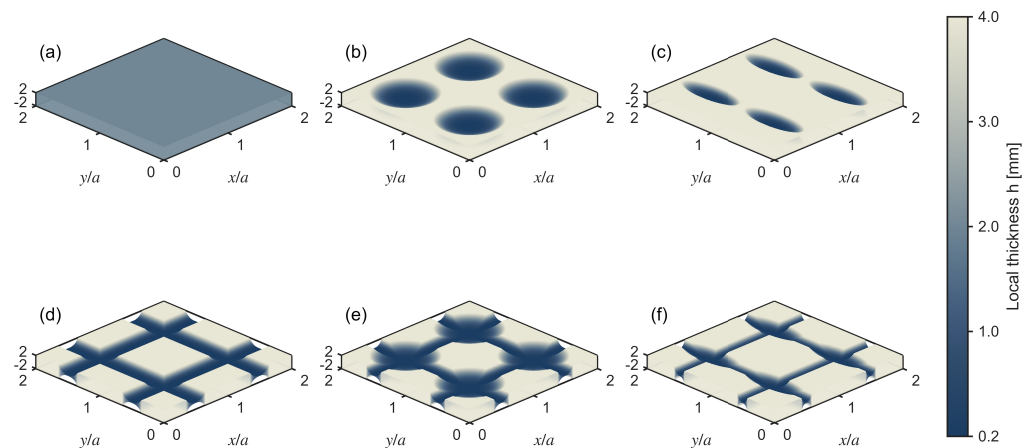


Figure 1. ABH topology cases: (a) uniform reference plate, (b) isolated circular ABH inclusion, (c) isolated y -elongated elliptical ABH inclusion, (d) crossed-strip ABH network, (e) cross-connected circular ABH network, and (f) cross-connected y -elongated elliptical ABH network. The color map denotes local plate thickness.

The uniform plate, for which $h(x, y) = h_0$ everywhere, serves as the reference case and admits the analytical solution in Equation (7). Building on this baseline, the isolated circular ABH is defined by the normalized radius $s = \sqrt{(x - a/2)^2 + (y - a/2)^2} / R$, with h given by Equation (8) inside $s \leq 1$ and $h = h_0$ elsewhere. The isolated y -elongated elliptical ABH uses the normalized coordinate $s = \sqrt{[(x - a/2)/R_x]^2 + [(y - a/2)/R_y]^2}$, where R_x and R_y are the ellipse semi-axes and $R_y > R_x$; this elongation breaks the square-lattice symmetry and introduces in-plane anisotropy. Connectivity is then introduced through the crossed-strip network, in which two orthogonal strips of half-width r pass through the cell center and have the thickness fields

$$h_x = h_c + (h_0 - h_c) \left(\frac{|x - a/2|}{r} \right)^m, \quad h_y = h_c + (h_0 - h_c) \left(\frac{|y - a/2|}{r} \right)^m, \quad (9)$$

so that the cell thickness is $h = \min(h_0, h_x, h_y)$ and the resulting low-stiffness path is continuous in both lattice directions. The two remaining hybrid topologies combine this connectivity with local ABH inclusions: the cross-connected circular network uses $h = \min(h_0, h_{\text{circle}}, h_x, h_y)$, whereas the cross-connected y -elongated elliptical network uses $h = \min(h_0, h_{\text{ellipse}}, h_x, h_y)$. Here, h_{circle} and h_{ellipse} denote the circular and elliptical ABH thickness fields obtained from Equation (8). These hybrid fields combine directional anisotropy with continuous inter-cell connectivity.

3. Numerical Methodology

The numerical analysis combines reciprocal-space and real-space calculations to study the wave behavior of each ABH topology. Dispersion curves are first computed with the FD-Bloch solver, and the results are independently checked with the GEM using a different approximation space. Finite-array simulations are then performed with the same material, ABH-profile, and lattice parameters, so that the Bloch predictions can be compared directly with attenuation in physical space.

3.1. FD-Bloch Dispersion Solver

For each unit-cell topology, the dispersion relation is first computed with a finite-difference Bloch formulation. The unit cell is discretized on an $N_x \times N_y$ Cartesian grid,

where N_x and N_y are the numbers of grid points in the two in-plane directions, and the central finite-difference curvature operators $\mathbf{L}_{xx}$, $\mathbf{L}_{yy}$, and $\mathbf{L}_{xy}$ are modified at the periodic boundaries by the Bloch phase factors from Equation (5). The discrete stiffness matrix follows directly from the bending-energy form in Equation (1):

$$\mathbf{K}_{\text{FD}} = \mathbf{L}_{xx}^H \mathbf{W}_D \mathbf{L}_{xx} + \mathbf{L}_{yy}^H \mathbf{W}_D \mathbf{L}_{yy} + \nu \left(\mathbf{L}_{xx}^H \mathbf{W}_D \mathbf{L}_{yy} + \mathbf{L}_{yy}^H \mathbf{W}_D \mathbf{L}_{xx} \right) + 2(1 - \nu) \mathbf{L}_{xy}^H \mathbf{W}_D \mathbf{L}_{xy}, \quad (10)$$

where $\mathbf{W}_D = \text{diag}[D(x_i, y_j) \Delta x \Delta y]$ contains the quadrature-weighted stiffness at each grid point, (x_i, y_j) are grid coordinates, Δx and Δy are the grid spacings, and the superscript H denotes the Hermitian transpose. Since the kinetic energy in Equation (3) is local and mass-lumped, the FD mass matrix is diagonal:

$$\mathbf{M}_{\text{FD}} = \text{diag}[\rho h(x_i, y_j) \Delta x \Delta y]. \quad (11)$$

Equations (10) and (11) are substituted into Equation (4), which is solved at each κ point using a standard sparse eigensolver. The calculation is repeated along the same Γ -X-M- Γ path for every topology, providing a consistent basis for comparing branch deformation and stop intervals. A stop interval is identified when the maximum frequency of one branch remains below the minimum frequency of the following branch along the sampled path. The shaded regions therefore denote stop intervals along Γ -X-M- Γ .

3.2. GEM Validation

The FD-Bloch results are checked with an independent Gaussian-enriched Ritz method, which uses a weak-form projection rather than finite-difference curvature operators. For the ABH topologies, the Gaussian centers are placed using a geometry-aware strategy so that the thinned regions and high-gradient thickness zones are better represented. In this formulation, the displacement amplitude is approximated by a hybrid Bloch basis,

$$W(x, y) \approx \sum_{j=1}^{N_g} q_j^{(g)} \psi_j(x, y; \kappa) + \sum_{\ell=1}^{N_p} q_\ell^{(p)} \varphi_\ell(x, y; \kappa), \quad (12)$$

where ψ_j denotes a Bloch-periodized Gaussian basis function, φ_ℓ denotes a Bloch plane-wave enrichment function, $q_j^{(g)}$ and $q_\ell^{(p)}$ are Ritz coefficients, and N_g and N_p are the corresponding numbers of basis functions. The Gaussian basis is written as

$$\psi_j(x, y; \kappa) = \sum_{m_x=-N_I}^{N_I} \sum_{m_y=-N_I}^{N_I} e^{i(k_x m_x a + k_y m_y a)} g_j(x - x_j - m_x a, y - y_j - m_y a), \quad (13)$$

with the local Gaussian envelope

$$g_j(\xi, \eta) = \exp\left(-\frac{\xi^2 + \eta^2}{2\sigma_j^2}\right), \quad (14)$$

where $\xi = x - x_j - m_x a$ and $\eta = y - y_j - m_y a$, m_x and m_y are image-cell indices, σ_j is the Gaussian width, and N_I is the number of image cells retained in each lattice direction. Thus, the Gaussian envelope is centered at (x_j, y_j) in the reference unit cell and repeated over neighboring cells with the Bloch phase factor. The plane-wave enrichment functions are

$$\varphi_{n_x n_y}(\mathbf{x}; \kappa) = e^{i[(k_x + G_x)x + (k_y + G_y)y]}, \quad (15)$$

where (G_x, G_y) are the Cartesian components of a reciprocal-lattice vector. These enrichment functions improve the representation of the uniform folded dispersion in Equation (7). The use of Gaussian basis functions follows established ABH plate formulations [27,33], while the geometry-aware center placement provides additional resolution in the regions that control the topology-dependent dispersion response.

Substituting Equation (12) into the weak form associated with Equations (1) and (3) gives the Ritz stiffness and mass matrices

$$\begin{aligned} \mathbf{K}_{\text{GEM}} = & \Phi_{xx}^H \mathbf{W}_D \Phi_{xx} + \Phi_{yy}^H \mathbf{W}_D \Phi_{yy} \\ & + \nu \left(\Phi_{xx}^H \mathbf{W}_D \Phi_{yy} + \Phi_{yy}^H \mathbf{W}_D \Phi_{xx} \right) \\ & + 2(1 - \nu) \Phi_{xy}^H \mathbf{W}_D \Phi_{xy}. \end{aligned} \quad (16)$$

$$\mathbf{M}_{\text{GEM}} = \Phi^H \mathbf{W}_M \Phi, \quad (17)$$

where $\mathbf{W}_D = \text{diag}[D(x_i, y_i)\Delta A]$, $\mathbf{W}_M = \text{diag}[\rho h(x_i, y_i)\Delta A]$, ΔA is the quadrature area weight, Φ contains the hybrid basis functions evaluated at the quadrature points, and Φ_{xx} , Φ_{yy} , and Φ_{xy} contain their second derivatives. This Ritz projection follows the standard Rayleigh–Ritz method [32]. The same eigenvalue problem in Equation (4) is then solved using $(\mathbf{K}_{\text{GEM}}, \mathbf{M}_{\text{GEM}})$, and agreement between the GEM and FD–Bloch spectra is used as the numerical consistency check before interpreting the topology-dependent dispersion features.

3.3. Finite-Array Simulation and Attenuation Metric

After the infinite-periodic dispersion analysis, finite 10×10 unit-cell arrays are simulated in the time domain to assess how the predicted pass- and stop-band behavior appears in a bounded structure. The governing equation is the variable-coefficient Kirchhoff–Love plate equation with effective viscous damping,

$$\rho h(x, y) w_{,tt} + \gamma(x, y; \omega_d) w_{,t} + \mathcal{L}_{\text{KL}}[w, D] = q(x, y, t), \quad (18)$$

where $q(x, y, t)$ is the harmonic point-force density and $\omega_d = 2\pi f_0$ is the angular driving frequency. The total damping coefficient is

$$\gamma(x, y; \omega_d) = \rho h(x, y) [\eta(x, y) \omega_d + \sigma_{\text{sp}}(x, y)], \quad (19)$$

where $\eta(x, y)$ is the dimensionless structural loss-factor map and $\sigma_{\text{sp}}(x, y)$ is the absorbing-boundary sponge coefficient. This is a spatially varying, mass-scaled viscous damping model evaluated at the driving frequency and is not classical Rayleigh damping of the form $\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}$.

The structural loss factor is increased within the ABH regions according to

$$\eta(s) = \begin{cases} \eta_{\text{core}}, & 0 \leq s \leq s_c, \\ \eta_{\text{bg}} + (\eta_{\text{ABH}} - \eta_{\text{bg}})(1 - s^m), & s_c < s \leq 1, \\ \eta_{\text{bg}}, & s > 1, \end{cases} \quad (20)$$

where $\eta_{\text{bg}} = 0.001$, $\eta_{\text{ABH}} = 0.015$, $\eta_{\text{core}} = 0.030$, and $s_c = 0.25$. For strip ABHs, s is the normalized perpendicular distance from the strip centerline, and the largest local value is retained where different ABH components overlap. This distribution provides an effective representation of localized damping treatment; a separate viscoelastic layer is not modeled

explicitly. The sponge damping is applied only near the computational boundaries and is defined as

$$\sigma_{\text{sp}}(x, y) = \begin{cases} \sigma_{\text{max}} \left(\frac{L_{\text{sp}} - d_{\text{min}}(x, y)}{L_{\text{sp}}} \right)^p, & d_{\text{min}} < L_{\text{sp}}, \\ 0, & d_{\text{min}} \geq L_{\text{sp}}, \end{cases} \quad (21)$$

where d_{min} is the distance to the nearest external boundary, $L_{\text{sp}} = 44\Delta x$, $\sigma_{\text{max}} = 8.0 \times 10^5 \text{ s}^{-1}$, and $p = 2$.

The variable-coefficient plate operator is expanded in moment-divergence form [2]:

$$\mathcal{L}_{\text{KL}}[w, D] = -(M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy}), \quad (22)$$

with the bending and twisting moment resultants

$$M_{xx} = -D(w_{,xx} + \nu w_{,yy}), \quad M_{yy} = -D(w_{,yy} + \nu w_{,xx}), \quad M_{xy} = -D(1 - \nu) w_{,xy}. \quad (23)$$

The finite-array response is calculated using an explicit finite-difference time-domain scheme with 24 grid points per lattice constant. A harmonic point source is placed in the left margin between the two central rows of the array. Graded sponge layers are applied along all four sides, while zero transverse displacement is imposed over the outermost two grid rows and columns to define the external boundary. The same finite-array model is post-processed using a kinetic-energy transmission metric,

$$T_{\text{dB}} = 10 \log_{10} \left(\frac{E_{\text{out}}}{E_{\text{in}}} \right), \quad (24)$$

where E_{out} and E_{in} are the spatially integrated kinetic-energy measures over the exit and entrance windows of the crystal, respectively. Both full-aperture and forward-channel metrics are computed. For connected topologies exhibiting stop intervals across the sampled high-symmetry path, the full-aperture metric is used. For isolated topologies with partial directional attenuation, the forward-channel metric is used to avoid including laterally scattered energy in the transmitted response.

All Bloch and finite-array calculations use a steel-like isotropic plate with Young's modulus $E = 210 \text{ GPa}$, density $\rho = 7800 \text{ kg m}^{-3}$, and Poisson's ratio $\nu = 0.30$. The lattice constant is $a = 50 \text{ mm}$, the background thickness is $h_0 = 4 \text{ mm}$, the residual ABH thickness is $h_c = 0.267 \text{ mm}$ ($h_c/h_0 \approx 1/15$), and the power-law exponent is $m = 3$ [27]. These material and profile parameters are fixed across all configurations. The same loss-factor values and damping rule are also applied, while their spatial distribution follows each ABH topology. The finite-array attenuation therefore reflects the combined effects of topology-dependent wave behavior, energy localization, and localized dissipation. The numerical implementation and post-processing were carried out in MATLAB R2025a (The MathWorks, Inc., Natick, MA, USA).

4. Results and Discussion

The six unit-cell topologies are studied in terms of their Bloch spectra and finite-array attenuation. The uniform plate is used first to verify the FD–Bloch and GEM formulations against the analytical folded dispersion. The topology-dependent dispersion features from Figure 2 are then examined, followed by finite 10×10 simulations at selected pass-band and stop-band frequencies to connect the infinite-periodic results with real-space attenuation.

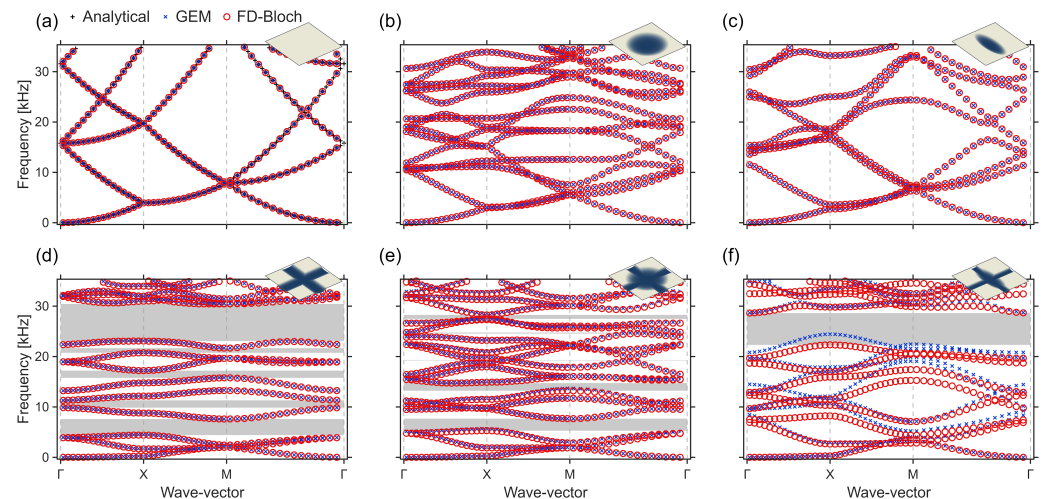


Figure 2. Dispersion relations for the six unit-cell topologies: (a) uniform plate, (b) isolated circular ABH, (c) isolated y -elongated elliptical ABH, (d) crossed-strip network, (e) cross-connected circular network, and (f) cross-connected y -elongated elliptical network. Blue markers: GEM; red circles: FD-Bloch; gray bands: stop intervals along the sampled path. Insets show the corresponding unit-cell thickness profiles in perspective view.

4.1. Topology Effects in Bloch Dispersion

The uniform plate provides the reference case because its folded Bloch dispersion is known analytically from Equation (7). In Figure 2a, the FD-Bloch and GEM results coincide with the analytical folded solution, confirming that both numerical solvers reproduce the expected Kirchhoff–Love plate dispersion before ABH thickness variation is introduced. For the isolated circular and y -elongated elliptical ABHs in Figure 2b,c, the main effect is a distortion of the uniform-plate branches, with branch splitting and several avoided crossings. The close overlap between the FD-Bloch and GEM curves indicates that these changes are captured consistently by the two independent formulations. Since the thinned regions remain confined within each unit cell and do not connect to neighboring cells, the associated stop behavior is mostly partial and direction-dependent, rather than forming a broad interval along the full sampled Γ –X–M– Γ path. The circular inclusion preserves the in-plane symmetry of the square unit cell; therefore, the branch shifts remain relatively balanced, and the branch interactions remain localized. The y -elongated ellipse, however, introduces a preferred direction into the cell. As a result, the dispersion becomes more direction-dependent, with stronger branch splitting and clearer changes in the branch slopes, especially along the diagonal M– Γ segment.

The connected cases show a different dispersion behavior from the isolated inclusions. In the crossed-strip network, Figure 2d, the thin regions form continuous low-stiffness paths in both lattice directions. This increases inter-cell coupling, flattens several branches, and opens wider stop intervals along the sampled Γ –X–M– Γ path. The effect is physically expected from the cubic thickness dependence of the bending stiffness: for $h_c/h_0 \approx 1/15$, the stiffness ratio of the thinnest ligaments is $D_c/D_0 \approx (h_c/h_0)^3 \approx 3 \times 10^{-4}$. Therefore, even narrow connected ligaments can dominate the flexural response of the cell. For Figure 2e,f, adding circular or elliptical inclusions to the strip network further rearranges the branches by combining local ABH trapping with inter-cell connectivity. Among the connected topologies, the cross-connected elliptical network gives the widest stop intervals. This is attributed to the combined effect of continuous low-stiffness ligaments and the anisotropy introduced by the elongated inclusion, which together promote stronger modal coupling and branch separation. For this most complex geometry, the FD-Bloch and GEM results show larger discrepancies than in the other cases. The larger

discrepancy for the cross-connected elliptical topology is attributed to its steeper and more complex thickness gradients, which are represented differently by the FD–Bloch and GEM discretizations. Nevertheless, both methods capture the same general trend: connected anisotropic topologies produce stronger branch rearrangement and wider stop intervals than isolated ABH inclusions.

4.2. Finite-Array Attenuation and Interpretation

Time-domain simulations of 10×10 arrays quantify how the Bloch band structure translates into real-space transmission loss, using the metric defined in Equation (24).

The isolated-inclusion arrays show partial and direction-dependent attenuation rather than full-aperture isolation. This behavior is consistent with the dispersion diagrams in Figure 2b,c, where the isolated circular and elliptical ABHs mainly produce branch distortion and partial stop behavior, rather than wide complete stop intervals along the full sampled path. Because the displacement maps are normalized by the maximum amplitude in each panel, the quantitative comparison is taken from the forward-channel transmission curves. For the circular ABH array in Figure 3, the uniform reference at 22.5 kHz gives $T_{dB} \approx -11.0$ dB. The ABH pass-band case at 13.0 kHz gives -12.5 dB, while the selected directional stop-band candidate at 22.5 kHz gives -15.3 dB. The finite-array field still shows residual propagation through the crystal, which confirms that this frequency should not be interpreted as a full-aperture attenuation interval. Instead, it corresponds to a partial directional attenuation regime. The source position between rows may also affect the modal content injected into the array and can contribute to the remaining transmission. The isolated elliptical array in Figure 4 shows a similar but slightly clearer trend. The uniform reference at 12.0 kHz gives $T_{dB} \approx -8.7$ dB, the ABH pass-band case at 10.0 kHz gives -10.0 dB, and the selected directional stop-band candidate at 12.0 kHz gives -13.8 dB. Thus, isolated ABH inclusions reduce the forward-channel transmission at the selected directional attenuation frequencies, but the reduction remains moderate. This is expected because the thinned regions are local, do not form continuous low-stiffness paths across the lattice, and mainly scatter or redistribute the flexural field rather than producing full-aperture blocking.

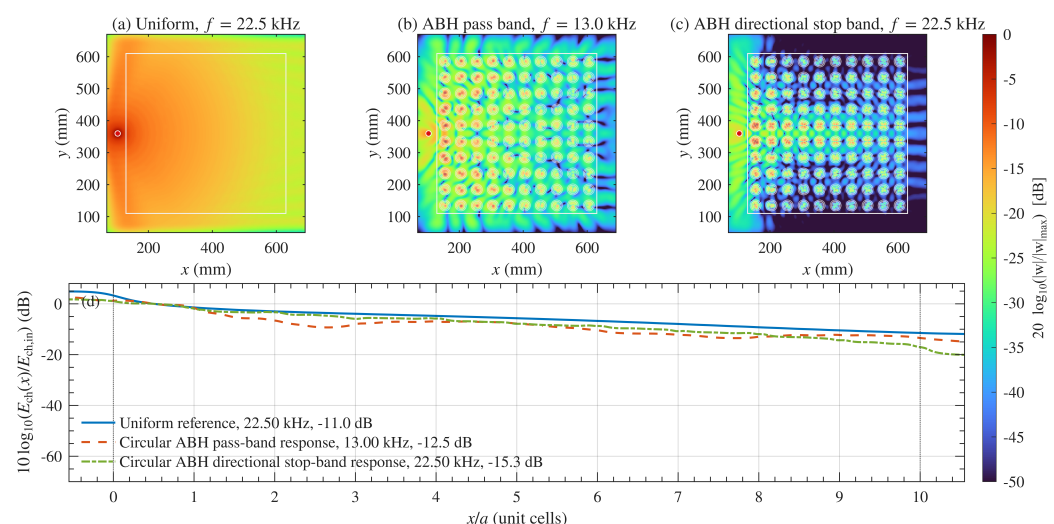


Figure 3. Finite-array response of the isolated circular ABH array: (a) uniform reference at 22.5 kHz, (b) pass-band case at 13.0 kHz, (c) directional stop-band case at 22.5 kHz, and (d) forward-channel transmission T_{dB} from Equation (24) versus propagation distance in unit cells.

Continuous ligaments substantially increase full-aperture attenuation at the selected stop-band frequencies. In the crossed-strip network in Figure 5, the uniform reference gives

$T_{dB} \approx -6.0$ dB, while the ABH pass-band case at 4.0 kHz remains comparable at -4.7 dB. In contrast, the stop-band case at 6.0 kHz drops sharply to -35.6 dB. The cross-connected circular network in Figure 6 shows the same trend: the reference and pass-band cases reach -6.3 dB and -4.5 dB, respectively, whereas the stop-band case reaches -33.4 dB. These large pass-to-stop contrasts indicate that the connected low-stiffness network produces strong spatial decay across the full aperture, consistent with the stop intervals observed in the Bloch dispersion diagrams.

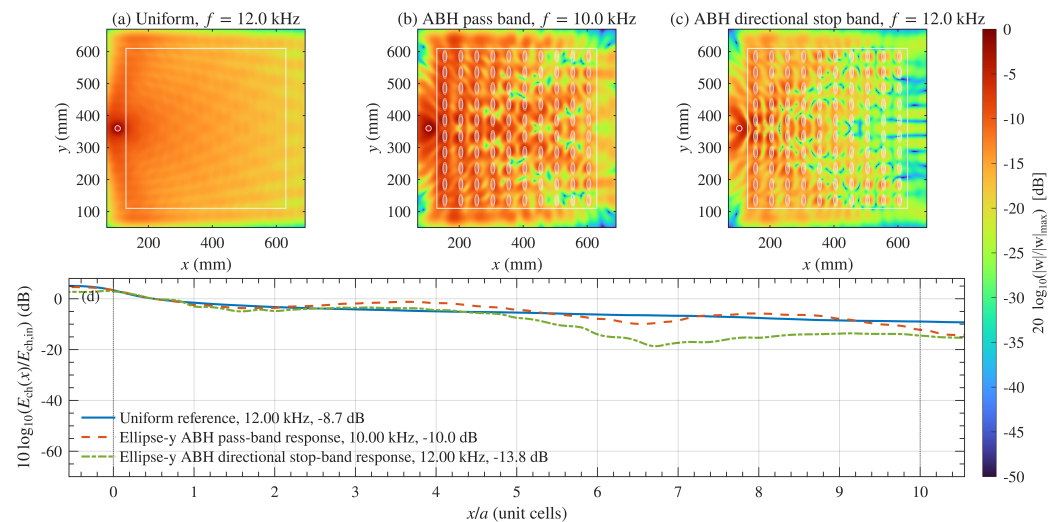


Figure 4. Finite-array response of the isolated y -elongated elliptical ABH array: (a) uniform reference at 12.0 kHz, (b) pass-band case at 10.0 kHz, (c) directional stop-band case at 12.0 kHz, and (d) forward-channel transmission T_{dB} .

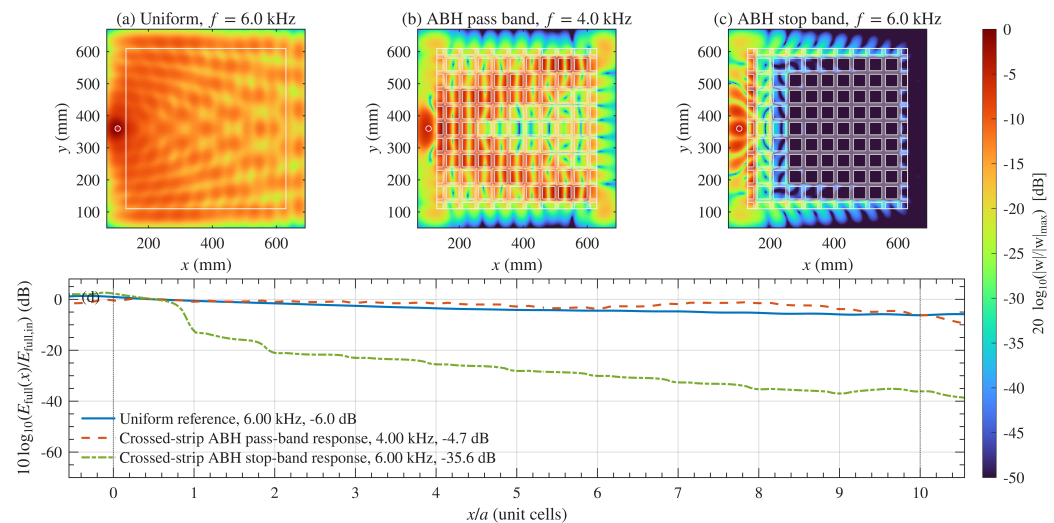


Figure 5. Finite-array response of the crossed-strip ABH network: (a) uniform reference at 6.0 kHz, (b) pass-band case at 4.0 kHz, (c) stop-band case at 6.0 kHz, and (d) full-aperture transmission T_{dB} .

The cross-connected elliptical network gives the strongest suppression among the tested configurations. As shown in Figure 7, the uniform reference at 25.0 kHz gives $T_{dB} \approx -9.0$ dB. The ABH pass-branch case at 15.0 kHz already shows a substantial reduction to -21.4 dB, while the stop-band case at 25.0 kHz reaches -60.4 dB. The attenuation observed at 15.0 kHz indicates that a pass branch in the infinite-periodic dispersion diagram does not necessarily lead to high transmission through a finite, damped array. In this case, finite-size scattering, source-mode mismatch, energy localization within the regions assigned higher loss factors, and energy trapping may all contribute to the reduced transmitted response. The much stronger decay at 25.0 kHz then confirms the stop-band character

of this frequency. These results show that the combination of connected low-stiffness ligaments and geometric anisotropy provides the most effective attenuation, and they underline the importance of combining Bloch analysis with finite-array simulations for practical design.

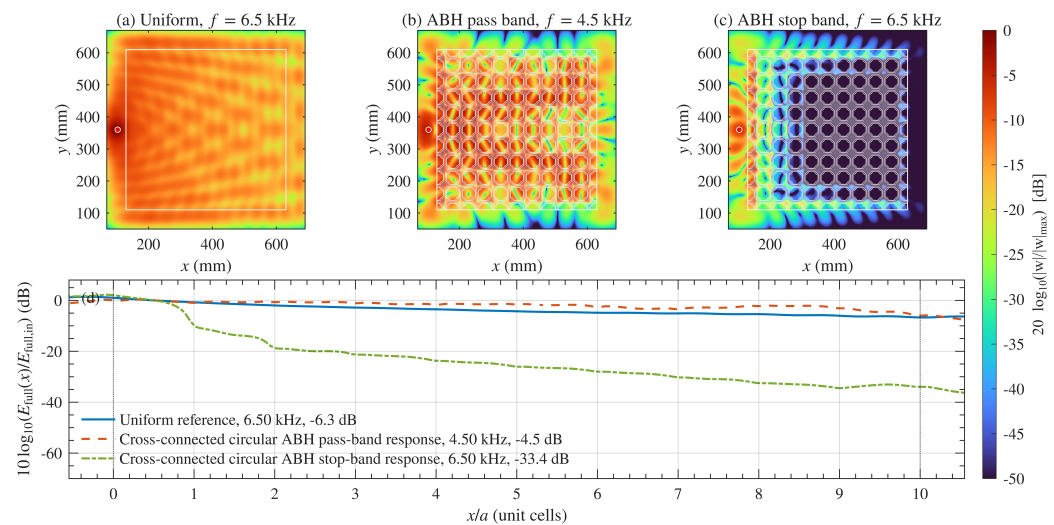


Figure 6. Finite-array response of the cross-connected circular ABH network: (a) uniform reference at 6.5 kHz, (b) pass-band case at 4.5 kHz, (c) stop-band case at 6.5 kHz. (d) full-aperture transmission T_{dB} .

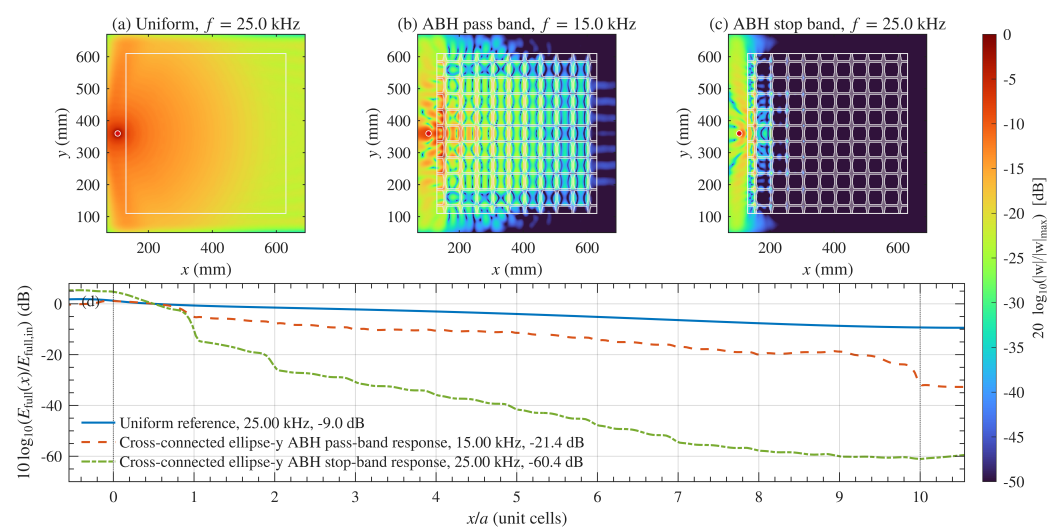


Figure 7. Finite-array response of the cross-connected y -elongated elliptical ABH network: (a) uniform reference at 25 kHz, (b) off-gap/pass-branch reference at 15 kHz, (c) stop-band case at 25 kHz, and (d) full-aperture transmission T_{dB} .

4.3. Experimental Implementation and Model Limitations

The numerical predictions could be assessed experimentally using a finite steel plate containing one of the investigated ABH arrays. The thickness profiles could be manufactured by CNC milling, while the plate could be excited harmonically using an electrodynamic shaker or a bonded piezoelectric actuator positioned upstream of the periodic region. A scanning laser Doppler vibrometer could then measure the transverse-velocity field over the plate. Spatially integrated velocity or kinetic-energy indicators evaluated over entrance and exit windows would provide an experimental counterpart to the transmission metrics used in the present simulations. Related experimental studies of periodic ABH plates indicate that such an approach is feasible [27,30].

Manufacturing tolerances are expected to influence the precise locations and widths of the predicted attenuation intervals. The residual thickness is particularly important because the local bending stiffness varies as $D \propto h^3$. Consequently, a small absolute deviation in the minimum thickness can produce a substantially larger relative change in local stiffness. The finite-array simulations use an effective spatial damping distribution in which the loss factor is higher in the thinned ABH regions than in the background plate. This represents the dissipative action of a localized damping treatment, but a separate viscoelastic layer, including its added mass and stiffness, is not modeled explicitly. In an experimental realization, the assumed loss factors would therefore need to be calibrated from measurements of the manufactured plate. Additional uncertainties may arise from deviations in the strip widths, inclusion dimensions, power-law profile, surface roughness, and cell-to-cell repeatability. These effects may change the local modal frequencies and reduce the ideal periodicity of the manufactured plate. Further differences between the numerical and experimental responses may arise from uncertainty in the material properties, non-ideal support conditions, and reflections from the finite plate boundaries, which may shift the attenuation frequencies and reduce the measured transmission contrast. Nevertheless, the relative trends associated with isolated and connected ABH topologies are expected to remain experimentally observable.

5. Conclusions

This study investigated how ABH unit-cell topology controls the dispersion and finite-array attenuation of periodic thin plates. A Kirchhoff–Love model with spatially varying thickness was combined with Bloch-periodic boundary conditions. The FD–Bloch solver was independently checked using a Gaussian–Ritz formulation, and the uniform plate was validated against the analytical folded dispersion relation. The six-topology comparison shows that isolated circular and y -elongated elliptical ABHs mainly produce directional branch deformation and partial stop behavior. In contrast, the crossed-strip and cross-connected networks generate wider stop intervals along the sampled Γ –X–M– Γ path because their low-thickness regions are continuous across the unit cell. The cross-connected elliptical network combines ligament connectivity with geometric anisotropy and gives the strongest finite-array attenuation among the cases studied. Finite 10×10 simulations show how the dispersion features appear in real-space transmission. The crossed-strip and cross-connected circular networks exhibit strong full-aperture suppression at their selected stop-band frequencies, whereas the isolated circular and elliptical inclusions mainly reduce the forward-channel transmission. The cross-connected elliptical network also shows substantial attenuation at the selected pass-branch frequency, where finite-size scattering, damping, and source–mode coupling may reduce transmission beyond the infinite-periodic prediction. Combining Bloch analysis with finite-array simulations provides a practical basis for ABH-plate design. The results should, however, be interpreted as numerical trends, since manufacturing tolerances, boundary conditions, damping uncertainty, and higher-order plate effects may shift the predicted attenuation frequencies and levels. Future work will address experimental validation, damping calibration, and comparison with a Mindlin–Reissner model.

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