

Article

Non-Rotating Blackhole Spacetimes with Plasma and Dust: Configurations and Spherically Symmetric Accretion

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Abstract

The passage from generic (non-interacting) plasma to cold plasma with dust around a spherically symmetric black hole is newly analytically studied. The configurations are newly written, and the behaviours of the observer are this way spelt out. The conservation of mass flux and that of the energy flux are used. The velocities of the observer are newly classified in the case of (non-interacting) hot plasma with dust for the transition to a configuration of (non-interacting) cold plasma with dust. The suitable functional dependence of the radial component of the velocity of the observer is now expressed in order to select the configurations which allow for the transition. The passage to a configuration of (non-interacting) cold plasma with dust is proved to be defined after the suitable integration conditions of the mass flux and of the energy flux, plus the suitable functional dependence of the radial component of the velocity of the observer are found. The spherical accretion is newly written. The emissions of the new accretion mechanism are due to (1) the variation in the gravitational potential as from blue further studies of the Author; (2) the radiation due to the change in the gravitational potential as described in the Landau–Lifshitz–Pitaevskii equations; (3) the standard electrodynamics radiation; and (4) the radiation of the positron (quantum mechanism) from the electron–positron pair of the emitted photon (in the Landau–Lifshitz–Lindhard scheme).

Keywords: general relativity; black holes; dust; plasma; accretion

1. Introduction

The cold relativistic (fully ionised) plasma (non-interacting) is an alternative representation of the same fluid macroparticles, which is used in the background description(s). The relativistic limit of plasma to cold plasma is one comprehending vanishing internal energy and vanishing pressure. The cold plasma and the macroparticles ones are coupled to the Maxwell equation. The evolution of the plasma is decided after the ‘Maxwell-fluid’ equations. Up to now, the studies have been performed via numerical simulations [1–3]. The investigation of the azimuthal behaviour is still under design both in the analytical expressions and in the numerical-simulations regime [4].

The role of dust in the accretion of black holes in Early-Universe cosmologies is delineated in reference [5], where radiation hydrodynamics is numerically implemented, while the case of black hole objects surrounded by dust and gas is recapitulated in reference [6]: in the latter item of the bibliography, the accretion process is described as comprehending also the ejection of charged particles after numerical simulation.

The accretion of spherically symmetric black holes surrounded by hot rarefied (non-interacting) plasma is presented in reference [7].



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The relativistic astrophysical implementation of fluid and dust was only very recently analysed in [8]. The role of emitting plasma situated around supermassive black holes of AGNs is involved in the phenomenon of ‘changing-look’ AGNs: it is observed according to the fact that the different types of AGNs exhibit different activities, which are explained as the changing of the properties of plasma and the role of the dust.

The role played by the behaviour of the magnetic field in the interaction with the gravitational field within the framework of quasars and Active Galactic Nuclei (AGNs) is outlined in reference [9]; *ibidem*, the role of the optical fluxes and that of the UV fluxes are studied for the recent phenomena observed in 1ES 1927+654 during December 2017: the radio-to-X-ray ratio was controlled.

The aim of the present paper is the analytical study of the configurations of spherically symmetric black holes surrounded by dust and cold plasma (non-interacting).

The numerical simulation for the study of polytropic-like density profiles at relativistic temperatures is proposed in reference [10].

The present research is motivated by study [10], in which the non-usefulness of the methods of relativistic temperatures is demonstrated at later times (of the plasma system). Furthermore, the protocol of temperature-descending plasma is one apt to be implemented for the requests of reference [11].

The consideration of relativistic dust is in order to accomplish starting analytical expressions of the most recent complete physical characterisation [6] of black hole objects surrounded by gas and of gas.

In the present paper, the study of the characterisation of cold plasma is newly performed after imposing the wished behaviour from a hot plasma (non-interacting model). The configurations of the black hole object surrounded by dust and cold plasma are newly analytically established; in particular, the four velocities are analytically written according to the vanishing-pressure limit: As a new result, not all configurations of hot plasma (non-interacting) are allowed, and the correct ones are newly analytically selected.

Furthermore, some possible accretion paradigms of the black hole objects surrounded by dust and cold plasma (non-interacting) are newly analytically schematised after the characterisation of the thermodynamical functions.

The configurations of the considered spacetimes allow one to newly spell out the emissions analytically and to enumerate them as (1) the radiation after the changes in the gravitational potential defined in reference [12]; (2) the radiation due to the change in gravitational potential due to the Landau–Lifshitz–Pitaevskii scheme; (3) the radiation of standard electrodynamics; and (4) the radiation due to the quantum effect of the passage of the positrons from the positron–electron pairs from the emitted photons.

Indeed, the schematisation of the thermodynamical quantities is newly demonstrated to allow one to select the possible configurations according to the qualities of the fully ionised plasma within the framework of the grand-canonical ensemble.

The novelty of the research is outlined as explaining the study of which kinds of hot plasma may cool down the temperature; furthermore, the approximations of warm dusty plasma and that of the presence of other (i.e., gaseous) matter can be envisaged.

The astrophysical cooling mechanisms of plasma is now characterised. The treatment of the heat transfer by electrons in collisionless magnetised plasma is studied in reference [13] as explaining the thermal conduction as a diffusion process due to radial viscosity. The novelty of this study is the focus of the attention on radial viscosity rather than on azimuthal viscosity.

In the present study, for the chosen relativistic astrophysical conditions, the plasma density is newly found as implied to be constant in the limit to the cooling process.

There results a new model of accretion that is spherically symmetric and compares to axisymmetric plasma inflow onto a compact source as described in the treatments [14–17], i.e., the standard disk [11], the advection-dominated accretion flow [18] with variation, the (hot, luminous) accretion flow [19], the adiabatic ‘inflow–outflow solutions’ [20], the convection-dominated accretion [21,22] flow (CDAF), and Jet-ADAF [23]: their common aim is to study the same process—axisymmetric plasma inflow onto a compact source with effects. The model here presented, differently, can compare the non-rotating spherically symmetric accretion processes described in reference [12].

The paper is organised as follows:

In Section 2, the methodologies are presented; the Darwin Lagrangian and its developments are presented, and the Landau–Lifshits–Pitaevskii plasma models are recalled.

In Section 3, the accretion mechanism of spherically symmetric black holes surrounded by hot plasma is recalled in order to set the comparison with the new wished configuration of the black hole object surrounded by dust and cold plasma (non-interacting).

In Section 4, the new wished-for configuration of the black hole object surrounded by dust and cold plasma (non-interacting) is newly analytically studied, after which the velocities are newly analytically investigated; further, new results prove that not all the physical configurations of hot plasma are apt for the limit to cold plasma: the correct physical configurations are selected. The new study of the corresponding thermodynamic quantities and the implication on the Poisson equation are newly issued.

In Section 4, the accretion paradigms of ‘collapsed objects’ from dust and cold plasma (non-interacting) are newly analytically studied.

In Section 5, the analytical expression of the velocities is written.

In Section 6, the spherically symmetric non-rotating accretion of weakly interacting plasma radiating in a Landau–Lifshitz–Pitaevskii magnetic field is analytically described within the framework outlined in reference [12].

In Section 7, the outlook is presented.

In Appendix A, the analytical expression of the conserved quantities of the general relativistic system considered is spelt out for the sake of introducing the analytical determination of the velocities.

In Appendix B, the tools to be used for numerical simulations within the framework of GRMHD are recalled.

2. Methodologies and Results

2.1. The Darwin Lagrangian

The Darwin Lagrangian, as referred to in reference [24] is a relativistic Lagrangian which describes the charge-charge interaction at any order of approximation v/c of weakly relativistic fluids of the radiation fields.

The action of the radiation fields on the charge is not taken into account because of the rapidity of the emission of fields.

The model is therefore not used in the ultra-relativistic limit.

The Darwin Lagrangian L_D is originally written in reference [25] as

$$L_D = - \sum_{i \neq k} m_i c^2 \sqrt{1 - \beta_i^2} - \frac{1}{2} \sum_{i \neq k} \sum_{k \neq i} \frac{e_i e_k}{r_{ik}} \left[1 - \frac{\beta_i \beta_k}{2} - \frac{(n_{ik} \beta_i)(n_{ik} \beta_k)}{2} \right], \quad (1)$$

where β_i is the relativistic factor $\beta_i = v_i/c$, $\vec{r}_{ik}$ is the distance $\vec{r}_{ik} = \vec{r}_i - \vec{r}_k$, and the unit vector $\vec{n}_{ik}$ is the normalised distance $\vec{r}_{ik}/|\vec{r}_{ik}|$.

The Darwin Lagrangian L_D from Equation (1) is used in thermodynamics theories and kinetic ones of weakly relativistic fluids where the radiation field is neglected.

The Darwin Lagrangian L_D is obtained after the complete Lagrangian of a system of charges [26]. It contains also the Lagrangian functions of the fields.

In the work of Hessén [27], the Darwin formulation of the Darwin Lagrangian in the case of classical-electrodynamics degrees of freedom is reappraised. The remark is *ibidem* expressed that the discrete particle degrees of freedom and the radiation degrees of freedom be isolated.

In reference [28], a relativistic generalisation of the Darwin Lagrangian is proposed; the generalisation is based on a formulation of the vector potential in Equation (11) from *ibidem* and in Equation (12) from *ibidem*. As a result, the ultra-relativistic limit is discussed as that in which it is not admitted to neglect the radiation arising from the newly calculated cross section of Coulomb scattering of the particles; as a result, all the limits are defined. In particular, the kinetics of the slightly relativistic plasma is thoroughly analysed in reference [29], where the interaction retardation is taken at first order.

In the case radiation is neglected, the Lagrangian is rewritten in orders of $\vec{\beta} = \vec{v}/c$ as

$$L = \sum_a \left[-m_a c^2 \sqrt{1 - \beta_a^2} + \frac{e_a}{2} \left(\vec{\beta}_a \cdot \vec{A} - \vec{\phi}(r_a) \right) \right] \quad (2)$$

with the Darwin expansion of the potentials ϕ_a as

$$\phi(r_a) = \sum_{b \neq a} \frac{e_b}{\vec{r}_a - \vec{r}_b} = \sum_{b \neq a} \frac{e_b}{r_{ba}} \quad (3)$$

and the vector potential as

$$\vec{A}(r_a) = \sum_{b \neq a} \frac{e_b}{2r_a} \left[\beta_b + (\vec{\beta}_b \cdot \hat{r}_{ba}) \hat{r}_{ba} \right] \quad (4)$$

A general solution is provided in reference [30], where the schematisation of the vector potential in the Coulomb gauge $\vec{A}_C$ at constant velocity along one of the spatial directions is written; it is provided as follows:

$$\vec{A}_C(\vec{r}) = \frac{e}{\mathcal{R}} \left(\mathcal{G} \xi \beta + \mathcal{H}(\xi^2) (\vec{\beta} \cdot \hat{\mathcal{R}}) \right) \quad (5)$$

with the vector $\vec{R}$ being

$$\vec{R} = \vec{r} - \vec{r}'(t) \quad (6)$$

$\vec{r}'$ is the vector from the source particle and $\vec{r}$ is the point of the field; vector $\vec{\xi}$ is

$$\vec{\xi} = \hat{\mathcal{R}} \wedge \vec{\beta} \quad (7)$$

and $\mathcal{G}$ and $\mathcal{H}$ are the opportune functions. As an example, a constant velocity along the $\hat{x}$ axis allows one to write the model according to $\vec{\beta} = \vec{r}'/c$.

From works [28,31], radiation due to acceleration is studied as negligible, and the expansion in terms of $\vec{v}/c$ is negligible as well at the relevant velocities.

The Darwin Lagrangian is taken from Equation (5) in the case of $\vec{v}/c$ found in reference [27],

$$L_{12} = -\frac{e_1 e_2}{r_{21}} + \mathcal{G} \left(\frac{\vec{v}^2}{c^2} \right) \frac{e_1 e_2}{r_{21}} \frac{\vec{v}^2}{c^2}; \quad (8)$$

in the limit $\vec{v} \rightarrow c$, the second part added in Equation (8) is neglected.

As a result, only the term accounting for the retardation is taken in Equation (8): in the hypothesis of v/c being small, i.e., in the non-ultra-relativistic limit, accelerations are neglected in the Coulomb gauge.

First-order approximation theory in quantum electrodynamics is performed, in which elastic particle-particle scattering is considered, whose kept terms are only those corresponding to equal-energy initial states and final ones.

The Lagrangian is rewritten from reference [24] as

$$L = - \sum_i m_i^2 \sqrt{1 - \beta_i^2} - \frac{1}{2} \sum_i e_i \left[\sum_{k \neq i} \Phi_k(r_i, t) - \beta_i \sum_{k \neq 1} A_k(r_i, t) \right] \quad (9)$$

where Φ is the scalar potential, and the retardation is dropped.

In more detail, from Equation (9), one rewrites the third addend as

$$\frac{1}{8\pi} \int (E^2 - H^2) dV = -\frac{1}{8\pi i} \int \text{div}(\phi \vec{E} + (\vec{A} \vec{H})) dV - \frac{1}{8\pi i} \frac{d}{dt} \int dV \vec{A} \vec{E} + \frac{1}{2} \int (\phi - \frac{1}{c} \vec{j} \vec{A}) \quad (10)$$

The first addend in Equation (10) is transformed into a surface integral, the second addend from Equation (10) is put in the Lagrangian function as it is a total time derivative, and the third addend from Equation (10) equals the second sum of Equation (9).

The Lagrangian Equation (9) is rewritten as

$$L = - \sum_i m_i c^2 \sqrt{1 - \beta_i^2} - \frac{1}{2} \sum_i e_i \left[\sum_{k \neq i} \phi_k(r_i, t) - \beta_i \sum_{j \neq i} A_j(r_i, t) \right] \quad (11)$$

with the Coulomb gauge to write both the equation of the scalar potential ϕ as

$$\Delta \phi = -4\pi \rho, \quad (12)$$

and the equation of the vector potential $\vec{A}$ as

$$\square \vec{A} = -\frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial}{\partial r_i} \frac{\partial}{\partial t} \phi. \quad (13)$$

It is now stressed that the second summand from Equation (9) represents the interaction of the charges (both of electrons and of protons) with the field.

The total energy available for the interaction is written after taking the simple momenta from the Hamiltonian instead of the generalised moments; this way, the second-order terms are neglected. As a result, the radiation of this process is neglected; for the present purposes, all the radiation mechanisms are one described in the following subsection.

The Lagrangian is rewritten as L from ibidem,

$$L = - \sum_i m_i c^2 \sqrt{1 - \beta_i^2} - \frac{1}{2} \sum_{i \neq k} \frac{e_i e_k}{r_{ik}} \left(1 - \frac{\beta_i \beta_k}{1 + x_{ik}^k} - \frac{(n_{ik} \beta_i)(n_{ik} \beta_k)}{x_{ik}^k (1 + x_{ik}^k)} \right), \quad (14)$$

where x_{ik}^k is defined as

$$x_{ik}^k \equiv \left[1 - (\vec{n}_{ik} \beta_k)^2 \right]^{1/2} \quad (15)$$

with $\vec{n}_{ik}$,

$$\vec{n}_{ik} \equiv \frac{\vec{r}_{ik}}{|\vec{r}_{ik}|}, \quad (16)$$

being the direction unit vector.

From Equation (14), no radiation is emitted; the law of conservation of energy follows. The free energy of the system is obtained straightforwardly.

2.2. The Landau–Lifshitz–Lindhard Equations

In the work of Nielsen et al., the Landau–Lifshitz equations for a charged particle in a strong magnetic field in the case the radiation process is not neglected are written; they are here complemented with the Lindhard emission scheme.

The geodesics equation of a charged particle in a strong magnetic field in the case the gravitational potential is neglected is specified from reference [32] (where in the latter reference the equation of the momentum is studied instead),

$$\frac{du^\mu}{d\tau} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2 \left[(\partial_\alpha F^{\mu\nu}u^\alpha u_\nu) + \frac{e^2}{m^2} F^{\mu\nu}F_{\nu\alpha} + \frac{e^2}{m^2} F^\alpha_\nu u^\nu F_{\alpha\lambda} u^\lambda u^\mu \right] \quad (17)$$

with τ being the proper time. Equation (17) is ‘sensitive’ to the magnitude of the ‘strong field’ parameter (as from reference [33])

$$\xi^2 = \frac{F_{\mu\nu}u^\nu}{E_0^2} \quad (18)$$

with

$$E_0^2 = \frac{m^2}{e\hbar} \quad (19)$$

and the initial value $E_0 \simeq 1.32 \cdot 10^{16} \text{ Vcm}^{-1}$. It is my purpose to stress that the verification of the well-posedness of the Lifshitz equation modifies the description of the pure Lorentz equation behaviour in the case of a charged ultra-relativistic particle, e.g., from reference [34].

At $\chi \ll 1$, the radiation spectrum is one following standard electrodynamics.

Moreover, the non-negligible quantum effects are those for the positron (from the positron–electron pair of the emitted photons) with radiation ω^* as

$$\omega^* = \frac{\omega}{1 - \frac{\hbar\omega}{E}} \quad (20)$$

due to the photon recoil [35].

The configurations u^μ to be considered in Equation (17) are those which are specified within the following sections after Appendix A.

2.3. Landau–Lifshitz–Pitaevskii Magneto-Hydrodynamic System of Equations

The main results from the spherically symmetric accretion flows with turbulence in magnetohydrodynamics (MHD) are here recalled from reference [36]. In reference [36], the MHD is treated, where the role of turbulence is separated.

The technique is taken from reference [37], where the viscosity-related addends are kept in the case they are not vanishing with the vanishing behaviour of the velocity. The resulting system is described as time-dependent and coordinate-dependent. The role of the configurations of the observer is therefore crucial, and it will be outlined in the rest of the paper.

The general ‘mass flux equation’ is written as

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{V}) = 0 \quad (21)$$

with ρ being the density and $\vec{V}$ the fluid velocity.

The ‘force balance equation’ is written as

$$\frac{d\vec{V}}{dt} + (\vec{V}\nabla)\vec{V} = -\frac{\nabla p}{\rho} - \nabla\phi_g - \frac{\vec{B} \wedge (\vec{\nabla} \wedge \vec{B})}{4\pi\rho} + \nu\Delta\vec{V} \quad (22)$$

with ϕ_g being the gravitational potential and ν the ‘kinetic viscosity’. The addend $\nu\Delta\vec{V}$ accounts for the energy dissipation via the mechanism of the ‘Kolmogorov cascade’ as taken from reference [38].

The ‘momentum equation’ is worked out after combining Equations (21) and (22), i.e., after the role of the gas internal energy density and that of the gas specific enthalpy have been specified.

The time evolution of the magnetic field $\vec{B}$ is the ‘induction equation’

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}(\vec{V} \wedge \vec{B}) + \nu_M\Delta\vec{B} \quad (23)$$

with ν_M being the magnetic diffusivity.

The magnetic field is therefore found to be solenoidal.

The magnetic field can be written as ‘incompressible random velocity field’.

It is here newly noted that the change in the gravitational potential from Equation (22) is responsible for emission from the accretion object.

3. Materials and Methods

The static spherically symmetric spacetimes with a Schwarzschild solid-angle element are considered with the line element

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2 + r^2(\sin\theta)^2d\phi^2. \quad (24)$$

The relativistic (non-interacting) perfect fluid is briefly discussed.

The General Case

After the analysis of reference [7], the conservation of the mass flux and that of the energy flux are here newly expressed after the most general relations

$$\rho^2 U^r = C_1, \quad (25)$$

and

$$(\rho + p)U_0 r^2 U^r = C_2 \quad (26)$$

The definition of

$$u_\mu u^\mu = -1 \quad (27)$$

implies

$$-\frac{c_2^2}{(p + \rho)(u^r)^2 r^4} + \frac{c_1^2}{\rho^2 r^4} f = -1, \quad (28)$$

from which the constant $C_3 = (C_2/C_1)^2$ is calculated from Equation (25) and from Equation (26).

The new role of c_3 is elucidated after the new requests on ρ in Equation (28), which is developed after f .

4. Spherically Symmetric Black Holes Surrounded of Dust and Plasma

The spherically symmetric spacetimes with a Schwarzschild solid angle element with a black hole object surrounded by relativistic dust and plasma (non-interacting) are here studied.

4.1. The Case of Cold Plasma

In the case of cold plasma with dust (non-interacting), the four vectors are specified for the dust as

$$\rho_d^\mu = (\rho_d, \vec{0}), \quad (29)$$

and for the cold plasma as

$$p_p^\mu = (\rho_p, \vec{p}) \quad (30)$$

in the limit

$$\lim_{p \rightarrow 0} (\rho_p, \vec{p}). \quad (31)$$

The configuration of the four-velocity of the observer can be, in the most general case, chosen according to the spherical symmetry and specified for the considered matter content.

According to the spherical symmetry, the four-velocity vector U^μ is chosen as

$$U^\mu = (U^0, \vec{U}) \quad (32)$$

with

$$\vec{U} = (U^r, 0, 0). \quad (33)$$

After Appendix A, following the analysis outlined in reference [7], the obedience of the four-velocity vector to the conservation of mass flux to that of energy flux is expressed as

$$(\rho_d + \rho_p)r^2U^r = C_1, \quad (34)$$

and

$$(\rho_d + \rho_p)r^2U^rU^r = C_2 \quad (35)$$

with C_1 and C_2 being the opportune constants.

The limit $p \rightarrow 0$ is here newly evaluated for the configuration of the observer. In this case, the following limits are newly verified:

$$\lim_{p \rightarrow 0} \frac{C_1}{C_2} = \frac{1}{u^0}, \quad (36)$$

and

$$\lim_{p \rightarrow 0} \left(\frac{C_1}{C_2} \right)^2 = \lim_{p \rightarrow 0} c_3 \quad (37)$$

It is therefore possible to newly choose the configuration of the observer as

$$U^r = 0. \quad (38)$$

4.2. The Case of the Pressureless Fluid

The limit $p \rightarrow 0$ implies the new condition

$$\lim_{u^r \rightarrow 0} \left[\frac{C_1}{r^2 u^r} + \frac{C_2}{u_0 u^r r^2} \right] = 0. \quad (39)$$

Equation (39) allows one to set the new relation

$$u^r \equiv r^{-2+\epsilon} \quad (40)$$

with the addend in the exponent ϵ being small at leisure.

Therefore, the new limits are calculated

$$\lim_{p \rightarrow 0} \frac{C_1}{r^2 u^r} \equiv \lim_{\epsilon \rightarrow 0} \frac{C_1}{r^2 u^r} = 0 \quad (41)$$

and

$$\lim_{p \rightarrow 0} \frac{C_2}{u_0 r^2 u^r} \equiv \lim_{\epsilon \rightarrow 0} \frac{C_2}{u_0 r^2 u^r} = 0 \quad (42)$$

As a further new result, one newly establishes that, differently, for example, from the case of rarefied hot plasma as from the form explicated after reference [7], no turning points exist.

4.3. Characterisation of the Polytropic Accretion

The study of the conditions characterising the accretion of a black hole object surrounded by cold plasma with dust (non-interacting) is accomplished after the study of the limiting behaviours of the polytropic law,

$$\frac{p}{p_\infty} = \left(\frac{\rho}{\rho_\infty} \right)^\alpha \quad (43)$$

with α being the polytropic exponent, where p is the pressure characterising the hot plasma (non-interacting) from which the transition is modelled; in more detail, p_∞ is the pressure of the hot plasma.

Accordingly, the limit $p \rightarrow 0$ is here newly studied as requested to be finite in

$$\lim_{p \rightarrow 0} \frac{p}{p_\infty} \neq 0 \quad (44)$$

in order to characterise the cold plasma.

The characterisation of the physical quantities at infinity can be performed according to the description of the Darwin Lagrangian as referred to in reference [24] as far as the relativistic corrections to the thermodynamical functions are concerned. In more detail, to obtain the description of the new limit to vanishing pressure description, the conditions on the free energy from a relativistic ideal gas must be imposed on Equation (2.25) from reference [24] in the correct limit from the temperature, whose relativistic law must be that requested from reference [39].

5. Analytical Expression of the Velocities

The calculations newly presented in this section are commented on after Appendix A. The condition Equation (27) is here analysed.

In the case of hot (rarefied) plasma with dust (non-interacting), the possibilities are implied for the choice of U^r ; in particular, as an example, also the choice

$$U^r \equiv V = \text{const} \quad (45)$$

is possible.

Unaccordingly, in the case of the transition to cold plasma here envisaged, the new results are found,

$$U^r \equiv r^{2+\zeta} \quad (46)$$

with ζ being the wished exponent, which must obey the request expressed from Equation (40)

$$\lim_{p \rightarrow 0} \zeta = \epsilon \quad (47)$$

with ϵ as small as the leisure itself, based on Equation (40).

From the definition Equation (27), the values of u^0 are disciplined accordingly as a function of the Schwarzschild radius r_s .

Physical Quantities

After reference [39], the entropy is treated as Lorentz-invariant; the transformation law of the internal energy is requested to be the same as the relativistic one of the temperature.

In the local rest frame, given the internal energy η , from the energy density $\bar{e}$, the rest-energy density is calculated as

$$\bar{e} - \bar{\eta} = \rho \quad (48)$$

with $c \equiv 1$.

In the case of the relativistic dust, the internal energy η_d vanishes.

As far as the relativistic plasma here considered is concerned, the internal energy in the inertial frame η_p and the internal energy in the local rest frame $\bar{\eta}_p$ are related via the Lorentz γ factor as

$$\eta_p = \gamma \bar{\eta}_p \quad (49)$$

In the limit to cold plasma, the internal energy is requested to vanish.

6. Physical Characterisation of the Spherically Symmetric Accretion

The study of 'accretion of matter onto collapsed objects' is due to reference [40] within the framework of investigation of accretion flows produced after 'high-temperature' plasma set by reference [41].

'Steady' accretion flows and their stability are studied analytically in reference [42].

The numerical simulation of the same problem with isothermal gas is demonstrated to produce different results in reference [43].

From reference [44], critical accretion flows are found stable for polytropic indices α as $1 \leq \alpha < 5/3$.

From reference [45], the ratio $\dot{M}/M$ of the accretion rate due to 'accreted matter' is studied for quasars and for massive black holes, where the accretion is taken from interstellar gas.

The aim of the present section is to provide one with analytical schematisations of the accretion paradigms as requested from reference [6].

6.1. Physical Characterisation of the Accretion of the Black Hole Surrounded of Dust and Cold Plasma (Non-Interacting)

The results are easily extended to the case of gaseous-material content.

The accretion rate of a black hole object of mass M is found from reference [14] as

$$\frac{dM}{dt} = 2\pi\alpha(GM)^2 V^{-3} \rho_\infty \quad (50)$$

In the present case of dust with cold plasma (non-interacting), the request that the gas be at rest at infinity is ensured after the Birkhoff theorem in the cases of the Schwarzschild spacetime and in those of the generalised Schwarzschild spacetimes.

From the analysis of reference [46], the accretion rate is studied as

$$\frac{dM}{dt} = 4\pi\lambda(GM)^2 \frac{1}{v_{s\infty}^3} \rho_\infty \quad (51)$$

with $v_{s\infty}$ being the velocity of the sound at infinity.

6.2. Physical Characterisation of the Time-Dependent and Turbulent Accretion of the Black Hole Surrounded by Dust and Cold Plasma

The scenario of the Landau–Lifshitz–Pitaevskii Magneto-Hydrodynamic system of equations as reported in reference [36] is here specified to the presented model.

In the present case, the condition

$$\frac{d\rho}{dt} \equiv 0 \quad (52)$$

is newly implied: the role of the momenta and that of the velocities in the mechanisms of accretion are therefore delineated.

The force balance equation is newly expressed after the Navier–Stokes equation,

$$-\nabla\phi_g - \frac{\vec{B} \wedge (\vec{\nabla} \wedge \vec{B})}{4\pi\rho} = 0, \quad (53)$$

which defines the Poisson equation (obeyed after the gravitational potential ϕ_g) for the total available gravitational mass.

The momentum equation here becomes

$$\frac{\partial}{\partial x_k} \frac{1}{4\pi} \left(\frac{1}{2} B^2 \delta_{ik} - B_i B_k \right) - \frac{\partial}{\partial x_i} \phi_g = 0. \quad (54)$$

The energy equation here becomes

$$\frac{\partial}{\partial t} \left(\rho\eta + \frac{|\vec{B}|^2}{8\pi} \right) = 0 \quad (55)$$

with η being the internal energy.

The physical quantities are here newly further characterised after newly posing the correct limit of η as η_p , and, in particular, the limit of vanishing internal energy for cold plasma must be implemented: the procedures are taken below. In more detail, all the equations stay unvaried, except for Equation (55), whose limit to cold plasma must be imposed as

$$\lim_{\eta \rightarrow 0} \frac{\partial}{\partial t} \left(\rho\eta + \frac{|\vec{B}|^2}{8\pi} \right) = 0 \quad (56)$$

and its implications on the resulting addend $\partial\eta_p/\partial t$ have to be studied again.

7. Conclusions

Within the present paper, the spherically symmetric non-rotating accretion onto a compact object is studied. As a result, the new emissions were scrutinised. They are due to the change in gravitational potential, to the standard electrodynamics and to the quantum effect of the passage of a positron from the electron–positron pair from the emitted photon.

In the present study, spherically symmetric non-rotating black hole spacetimes containing also plasma and dust are studied as far as the understanding of the mechanisms of the polytropic accretion processes as connected with the of cooling of hot plasma.

In more detail, the Lagrangian chosen for the fluid is one from reference [24], in which the weakly relativistic limit is studied: this choice is apt for the description of relativistic–astrophysics-rarefied plasma. The role of dust is further investigated as contributing to the description of the density of the relativistic astrophysical medium. As a new result,

the density is found to be implied to be constant, for which the comparisons with the mechanisms implied after the consideration of viscosity are of interest.

Furthermore, the polytropic accretion is newly studied within the proposed paradigm. The implementation of the Landau–Lifshitz–Pitaevskii Magneto-Hydrodynamic system of equations of the paradigm is implemented: as a result, the force balance equation is written for the potential of the gravitational field.

It is here demonstrated that the paradigm for passing from hot plasma to cold plasma can be one completed after reference [24]. In more detail, the selected model is therefore demonstrated as apt to starting schematising the configurations of the dust with gas as requested from reference [6] after the new characterisation of the thermodynamical functions: the implementation allows one to impose the correct characterisation of the free energy of a relativistic gas and to reconcile the study with the definition of the quantities at infinity ρ_∞ and p_∞ as far as the effects of the gravitational field of the black hole object is concerned.

Furthermore, the study of the limit to hot plasma (as similarly schematised matters) allows one to newly implement the Poisson equation for the definition of the gravitational mass available according to the selected allowed configurations in order to establish an analytical paradigm for reference [6].

The results here presented are pertinent to the characterisation of c_3 elucidated after Equation (28), where the behaviours are dictated by the presence of f .

The complementary problem of the study of heating of high- β plasma is studied in reference [47].

As from the consequences of Equation (42), there are no critical points in the examined cases. The results hold also in the case of a different characterisation of the $r = 0$ singularity, whatever the characterisation of the singularity proposed [48–50], as from the definition of the examined regions of the black hole spacetimes, the behaviour is dictated by the function f of the metric tensor.

As a different perspective, the hydromagnetic interactions of waves and the corresponding effects on magnetised turbulence of high- β , collisionless fluids are studied in reference [51,52], where, in the latter case, turbulence is also taken into account.

The achievements of reference [47,51,52] can therefore be newly applied to the understanding of the accretion of black holes.

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Appendix A. The Relativistic Equations of the Fluid

The relativistic conservation equation is here derived for the fluid in the steady configuration in spherical symmetry.

The stress energy tensor is taken for the fluid as

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}. \quad (\text{A1})$$

The momentum conservation equation is written, in general, as

$$\nabla_\mu J^\mu = 0; \quad (\text{A2})$$

the non-vanishing components of Equation (A2) are specified as the mass flux conservation equation

$$\nabla_i J^i = 0, \quad (\text{A3})$$

which implies, on its turn,

$$\frac{d}{dr} (J^1 r^2) = 0 \quad (\text{A4})$$

and is integrated as

$$\rho r^2 = C_1 \quad (\text{A5})$$

with C_1 being the dimensional constant.

The energy flux conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \quad (\text{A6})$$

is written in its non-vanishing components as

$$\frac{d}{dr} (T^{10} r^2) = 0 \quad (\text{A7})$$

and is integrated as

$$(\rho + p) u^0 u^r r^2 = C_2 \quad (\text{A8})$$

with C_2 being the dimensional constant.

The configuration of the observer (here, one static with the photon) is studied as

$$g_{00} u^0 u^0 + g_{rr} u^r u^r = -1, \quad (\text{A9})$$

from which one obtains

$$\frac{(\rho + p)^2}{\rho^2} (u^0)^2 = \frac{(\rho + p)^2}{\rho^2} \frac{(-1 - g_{rr} u^r u^r)}{f} = C_3, \quad (\text{A10})$$

with C_3 being the dimensional constant.

The pressureless fluid is described after the limit of vanishing pressure.

Appendix B. Uses of the Darwin Lagrangian

The Darwin Lagrangian was used in plasma physics in reference [53] and in reference [53–57].

The role of the use of MHD simulations for weakly ionised plasma is outlined in reference [58].

In the work [59], HD techniques, MHD paradigms and MC methods are compared within the modern developments of numerical investigations in physics of accretion objects onto compact objects. Methods in MHD simulations comprehend the FD methods, the finite-interval methods, the finite-volume methods and the AMR methods [60–65].

In reference [58], the dynamics of weakly ionised plasma are recalled for comparison with the present paper.

In reference [66], the MHD simulations of weakly interacting plasmas are introduced.

In reference [67], the characteristics of weakly interacting plasma in MHD simulations for relativistic plasma objects are presented; within this introduction ibidem, the role of saturated conduction is outlined. As a result, the weakly interacting plasma MHD is stated to be apt to describe ‘radiative-inefficient’ processes.

Within the present paper, the non-radiative plasma interaction processes are accounted for in the Darwin Lagrangian, while the radiative processes are written after the Landau–

Lifshitz–Pitaevskii Magneto-Hydrodynamic paradigm for the first time. The need for the use of the Darwin Lagrangian in MHD studies in relativistic astrophysics is this way newly outlined.

From reference [36] it is recalled that there exist diverse accretion models which need to be further specified.

In the present case, accretion is looked for in a plasma subject to a magnetic field according to the Landau–Lifshitz–Pitaevskii Magneto-Hydrodynamic system of equations. In this case, the magnetic field described by the induction equations.

The resulting magnetic field is solenoidal

$$\nabla \vec{B} = 0 \quad (\text{A11})$$

and the plasma is an incompressible fluid

$$\nabla \vec{V} = 0 \quad (\text{A12})$$

with $\vec{V}$ being the 3-velocity vector of a particle. As the description Equation (A12) is obeyed after the velocities admitted from Appendix A, the magnetic field $\vec{B}$ can be studied accordingly.

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