Article

Fiber Lidar for Control of the Ecological State of the Atmosphere

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Abstract: Methods and means of remote control of the ecological state of the atmosphere are constantly improving. Lidar sensing allows obtaining up-to-date information about natural and technogenic sources of atmospheric pollution. There is a wide range of problems in ecological control, where the deployment of an inexpensive mobile lidar network is required. For this purpose, it is suggested to use Q-switch and MOPA fiber lasers in lidars. Q-switch fiber lasers have a simpler design and are more practical to use. However, pulses from Q-switch lasers have long full-pulse durations. In the present work, a lidar signal inversion method (LSIM) is proposed for solving this problem. Verification and outdoor experimentation of the LSIM was carried out with the reference signal method (RSM). The advantage of the proposed RSM is the minimum number of controllable parameters necessary for LSIM verification and approbation. As a result, the accuracy of the obtained results increased. Thus, the possibility of application of the Q-switch fiber lasers for lidar sensing is shown both theoretically and experimentally.

Keywords: atmosphere; Asian dust; ecology; fiber laser; fiber lidar; geometric form factor

1. Introduction

Nowadays, the problem of control of the ecological state of the atmosphere is urgent. Except for natural sources, such as Asian and Saharan dusts, technogenic activity also influences the state of the environment, which requires continuous ecological control [1–4].

As was shown in [5], the methods and means of remote control of the atmospheric state are continuously improving. Lidar sensing allows obtaining up-to-date information on aerosol impurities in the atmosphere and estimating the spatial–temporal dynamics of their spread and transformation of their parameters [6–16].

In this regard, deploying a network of mobile lidars allows a wide range of measures to be implemented. The main lidar component is a laser radiation source. Stability of the laser radiation parameters and the ability of the laser to work autonomously for a long time are the basic requirements for the laser.

Fiber lasers are produced by a number of companies, are structurally simple, and possess increased operational stability and reliability [17–22]. The beam divergence of these lasers is close to that of Gaussian beams. Master oscillator power amplifier (MOPA) fiber lasers are constructed with a two-cascade scheme.

Q-switch fiber lasers have a simpler design and are more practical to use. However, these lasers have complex pulse shapes [23]. The laser pulse duration is indicated by the
manufacturer as a single parameter: the full width at half maximum (FWHM). Its value is typically about 100 ns. The laser pulse is long and decays in a wave-like manner over time. As a whole, the pulse duration can reach several microseconds. Therefore, the problem arises of how to use Q-switch fiber lasers in lidar sensing.

A method for solving this problem is proposed in the present work. The lidar signal inversion method (LSIM) is used to solve the problem of long laser pulses, and the possibility of Q-switch fiber laser application for lidar sensing is shown.

The verification of and outdoor experiment with the LSIM is based on the reference signal method (RSM). An advantage of the RSM is that the result is not affected by the form of the geometric function of the lidar or the form of the lidar response from the atmosphere, depending on the square sensing range.

The block diagram of the fiber lidar, lidar sensing equation, and geometric form factor of the fiber lidar are considered in Section 2, entitled “Fiber Lidar Framework”. The reference signal method for fiber lidar detection of atmospheric impurities, method of fiber lidar signal inversion, and data processing of measurement results are considered in Section 3, entitled “Methods”. Section 4, “Materials”, is devoted to the fiber lidar parameters. Section 5, “Results”, is devoted to the validation of and outdoor experiment with the LSIM. Finally, the method of selecting the criterion for the LSIM is considered in Section 6, entitled “Discussion”.

2. Fiber Lidar Framework

A block diagram of the fiber lidar is shown below. The theoretical background of fiber lidar sensing is briefly described together with the effect of the key lidar parameters on the lidar response in atmospheric sensing.

2.1. Block Diagram of Fiber Lidar

Commercial fiber lasers are delivered with laser radiation output through a light guide connected to a collimator lens. The laser design is simple and requires no additional adjustment during operation. As a rule, the beam divergence at the output of the collimator lens is less than 1 mrad. Thus, a standard fiber laser kit can be immediately included in a lidar.

Figure 1 shows the optical block diagram of a lidar with a fiber laser (referred to as the fiber lidar).

![Figure 1](image.png)

Figure 1. Block diagram of the fiber lidar described in the text in more detail.

Here, *laser control* designates the fiber laser with the control system, and *laser* designates the lens collimator at the laser output. The beam divergence at the light guide output
is close to that of the Gaussian beam. The laser operating modes, including the output pulse energy and pulse repetition frequency, are computer-controlled (PC). Radiation backscattered in the atmosphere is collected by a receiving telescope (telescope) and fed to a photodetector (designated by SPCM) through the light guide. The photodetector is an Si-APD photodiode operating in the single-photon counting module (SPCM). During lidar sensing, a pulse counter (counter control in Figure 1) is triggered synchronously with the laser pulse. The lidar signal waveform is formed with a preset time resolution in photoelectron pulse accumulation mode. The operating pulse counter and data exchange modes of the computer are installed using the Ethernet protocol.

2.2. Lidar Sensing Equation

The optical block diagram of the fiber lidar is shown in Figure 1, with spatial separation of the transmitter and receiver’s optical axes. We designated this separation as $\Delta$ and assumed that the transmitter and receiver optical axes were parallel. We also considered that the lidar return signal was the laser pulse image in the focal plane of the receiving telescope. Figure 2 shows how the image changes with increasing distance from the fiber lidar along the optical axis of the transmitter.

![Figure 2](image)

Figure 2. Laser pulse images in the focal plane of the receiving fiber lidar telescope. The arrow indicates the direction of image displacement, depending on the lidar pulse sensing range.

The laser pulse shape can be neglected and considered to be rectangular if the pulse duration is comparable to or less than the temporal resolution of the recording system. In this case, the lidar sensing equation in the single scattering approximation has the form

$$P(h) = ICG(h)h^2 \beta(h)T^2(h).$$  \hspace{1cm} (1)

Here, $P(h)$ is the power of the lidar return signal backscattered in the atmosphere at the range $h$, $I$ is proportional to the laser pulse energy, $C$ is the instrumental constant, $G(h)$ is the geometric fiber lidar form factor, $\beta(h)$ is the volume backscattering coefficient at the range $h$, and $T(h)$ is the atmospheric transmission:

$$T(h) = \exp\left\{\int_0^h \alpha(h) \, dh\right\},$$  \hspace{1cm} (2)

where $\alpha(h)$ is the volume extinction coefficient.
Geometric Form Factor of Fiber Lidar

The geometric lidar form factor \( G(h) \) can be described in the geometrical optics approximation [24–30]. The direction of motion of the laser pulse is indicated by the arrow in Figure 2. The circles designate cross sections of the pulse image spots with increasing distance from the fiber lidar. In Figure 2, the \( yox \) plane of the coordinate system is located in the focal plane of the telescope, and the \( yoz \) plane is located in the plane passing through the transmitter and receiver’s optical axes. The origin of the coordinates is located on the receiver’s optical axis. Consider that scattered radiation uniformly irradiates the input aperture of the telescope for all sensing ranges. In addition, neglecting the edge effects, we also considered that the intensity of the laser pulse image was uniformly distributed over the focal plane of the telescope. In lidar sensing, the scattering coefficients averaged over the scattering volume are determined experimentally. Therefore, the assumption of the uniform distribution of the laser pulse intensity over the effective area of the laser pulse image is applicable here. Simple geometrical considerations using the lens formula give the following expressions:

\[
\begin{align*}
R_{\text{spot}}(h) &= F \left( \theta + \frac{R_{\text{tel}} + r_{\text{min}}}{h} \right), \\
y(h) &= F \frac{\Delta}{h},
\end{align*}
\]  

(3)

Here, \( R_{\text{spot}}(h) \) is the laser pulse spot radius, depending on the range \( h \), \( F \) is the focal length of the receiving telescope, \( \theta \) is the radial beam divergence, \( R_{\text{tel}} \) is the radius of the receiving aperture of the telescope, \( r_{\text{min}} \) is the radiation’s waist radius at the output of the laser collimator, and \( y(h) \) defines the position of the image spot center on the \( y \) coordinate axis, depending on the range \( h \).

In addition to the lens formula, in the derivation of Equation (3), it was considered that the propagation of fiber laser radiation along the sensing range was described by that of the Gaussian beam. The radiation beam divergence is determined by the formula

\[
\theta = M^2 \frac{\lambda}{\pi r_{\text{min}}},
\]  

(4)

Here, the divergence angle \( \theta \) is in radians, \( \lambda \) is the radiation wavelength, and the minimum radius of the radiation’s beam waist \( r_{\text{min}} \) is defined at the \( 1/e^2 \) level. From Equation (3), it follows that the image spot radius decreases with increasing distance and trends toward the value \( R_{\text{spot}}(\infty) = F \theta \), and the image center approaches the origin of the coordinates.

Let us define the geometric form factor of the fiber lidar as a fraction of the scattered radiation power incident on the receiving aperture of the SPCM light guide with a radius \( r_{\text{fiber}} \) located in the focal plane of the receiving telescope:

\[
G(h) = \frac{S(R_{\text{spot}}(h),d(h),r_{\text{fiber}})}{\pi R_{\text{spot}}^2(h)}.
\]  

(5)

Here, \( S(R,d,r) \) is the intersection area of two circles with radii \( R \) and \( r \), the centers of which are at a distance \( d \) from each other [31]:

\[
S(R,d,r) = \pi d^2 - \pi r^2.
\]  

(6)
\[ S(R,d,r) = r^2 \cos^{-1} \left( \frac{d^2 + r^2 - R^2}{2dr} \right) + R^2 \cos^{-1} \left( \frac{d^2 - r^2 + R^2}{2dr} \right) \]

\[ -\frac{1}{2} \sqrt{(-d + r + R)(d - r + R)(d + r - R)} \]  

If we designate the coordinates of the light guide aperture center in the focal plane as \((x_{\text{FIBER}}, y_{\text{FIBER}})\), then the distance \(d\) is determined with the formula

\[ d(h) = \sqrt{x_{\text{FIBER}}^2 + (y(h) - y_{\text{FIBER}})^2}. \]

Figure 3 shows the estimated geometric fiber lidar form factors, depending on the sensing range.

![Figure 3. Geometric fiber lidar form factor, depending on the sensing range (a), and the same geometric form factor corrected by the squared sensing range as a function of the sensing range (b).](image)

The geometric form factor in Figure 3a was calculated with Equation (5), while the geometric form factor in Figure 3b was corrected by \(h^2\), and it approximately describes the lidar return signal’s waveform. In our estimates, we used the following lidar parameters (in meters): \(\Delta = 0.07\), \(F = 0.3\), \(R_{\text{TEL}} = 0.035\), \(r_{\text{MIN}} = 0.0035\), and \(r_{\text{FIBER}} = 0.0003\) for \(\theta = 1\) mrad. Here, the receiving light guide was placed at the origin of the coordinates of the focal plane of the telescope \((x_{\text{FIBER}} = 0, y_{\text{FIBER}} = 0)\) and \(d(h) = y(h)\). By changing the position of the aperture of the SPCM light guide fiber in the focal plane of the telescope, it is possible to change the dynamic range of the lidar return signal or choose the measurement range gate of the sensing range. Thus, the fiber lidar receiver can be synchronized with the transmitter.

3. Methods

3.1. Reference Signal Method

The method for detecting aerosol impurities in the atmosphere is based on a comparison of backscattered lidar signals received by the fiber lidar. Through scanning, we understand the detection of an aerosol impurity in the chosen sensing direction. The signal of backscattering on atmospheric molecules and aerosol particles from the background environment was selected as the reference signal for a comparison. Note that this can be a signal in the scanning direction without aerosol impurities. The presence of sources of
aerosol pollution is determined from the ratio of the lidar’s return to reference signals (aerosol/background ratio):

\[ R_{\text{SCAN}}(h) = \frac{P_{\text{SCAN}}(h)}{P_{\text{REF}}(h)}. \]  

(8)

Here, \( R_{\text{SCAN}}(h) \) is the ratio of the scanning signals in the sensing range \( h \) to the scanning point, \( P_{\text{SCAN}}(h) \) is the power of the scanning lidar pulse backscattered in the sensing range \( h \), and \( P_{\text{REF}}(h) \) is the power of the reference lidar pulse recorded in the sensing range \( h \). In the first approximation of multiple scattering theory, the lidar return signals \( P_{\text{SCAN}}(h) \) and \( P_{\text{REF}}(h) \) have the forms

\[ P_{\text{SCAN}}(h) = ICG(h)h^2 (\beta_{\text{SOURCE}}(h) + \beta_{\text{REF}}(h)) T_{\text{SOURCE}}^2(h) T_{\text{REF}}^2(h), \]  

(9)

\[ P_{\text{REF}}(h) = ICG(h)h^2 \beta_{\text{REF}}(h) T_{\text{REF}}^2(h). \]  

(10)

Here, \( \beta_{\text{SOURCE}}(h) \) and \( \beta_{\text{REF}}(h) \) are the volume coefficients of backscattering with the aerosol impurities and background environment, respectively, while \( T_{\text{SOURCE}}(h) \) and \( T_{\text{REF}}(h) \) are the atmospheric transmittances for the impurities and background environment:

\[ T_{\text{SOURCE}}(h) = \exp \left\{ -\int_0^h \alpha_{\text{SOURCE}}(h) \, dh \right\}, \]  

(11)

\[ T_{\text{REF}}(h) = \exp \left\{ -\int_0^h \alpha_{\text{REF}}(h) \, dh \right\}, \]  

(12)

where \( \alpha_{\text{SOURCE}}(h) \) and \( \alpha_{\text{REF}}(h) \) are the volume extinction coefficients of the aerosol impurities and the background environment, respectively.

Taking into account Equations (8) and (9), Equation (7) can be transformed into the form

\[ R_{\text{SCAN}}(h) = \frac{\beta_{\text{SOURCE}}(h) + \beta_{\text{REF}}(h)}{\beta_{\text{REF}}(h)} T_{\text{SOURCE}}^2(h). \]  

(13)

For local sources of pollution with low transmission factors, it can be assumed that \( T_{\text{SOURCE}}^2(h) \approx 1 \). Then, Equation (12) assumes the following form:

\[ R_{\text{SCAN}}(h) = \frac{\beta_{\text{SOURCE}}(h)}{\beta_{\text{REF}}(h)} + 1, \]  

(14)

Also, Equation (13) in the transparent atmosphere, in which molecular scattering dominates over aerosol scattering in the background environment, takes the form of the well-known backscattering ratio
\[
R_{\text{SCAN}}(h) = \frac{\beta_{\text{SOURCE}}(h)}{\beta_{\text{MOL}}(h)} + 1. \tag{15}
\]

Here, \( \beta_{\text{MOL}}(h) \) is the volume molecular scattering coefficient in the opposite direction.

Note that, as can be seen in the above relationships, the expressions for the geometric fiber lidar form factor and instrumental constant are abbreviated in the reference signal method, thereby increasing the accuracy of estimation of the impurity parameters in the atmosphere.

### 3.2. Lidar Signal Inversion Method

Figure 4 shows the lidar pulse waveform of the Q-switch series laser.

![Figure 4. Impulse transfer function of the Q-switch series fiber laser as a function of time.](image)

The FWHM parameter specified by the manufacturer defines the pulse duration, being approximately equal to 100 ns. However, in Figure 4, it can be seen that the pulse waveform is a complex function of time. After a fast increase, the pulse decayed in a wave-like manner for a long time. The full pulse duration was about 10 \( \mu \)s. The long laser pulse with the asymmetrical waveform affected the formation of the lidar’s echo signal.

If the laser pulse duration exceeds the time resolution \( \Delta t \) of the registration system, then the laser pulse shape for the lidar in Equation (1) should be taken into account as the impulse transfer function \( I(t) \) of the fiber laser. Here, it is more convenient to proceed to a discrete description of lidar signals because this approximation describes more vividly the lidar sensing process. Owing to the additivity principle, it is possible to represent the laser pulse \( I(t) \) as a sequence of short pulses with durations \( \Delta t \). Then, the radiation scattered in the atmosphere for each short pulse is described by the lidar in Equation (1). Hence, the envelopes of the backscattered lidar return signal will coincide to within a certain factor. In this discrete approximation, a series of short pulses is transmitted into the atmosphere, and the backscattered radiation is recorded with a sensing range resolution \( \Delta h = c\Delta t/2 \). For simplicity, in the discrete approximation, we replaced the designation of the sensing range \( h \) with the serial number of the range gate \( h = n\Delta h \), where \( n = 1, 2, 3, \ldots \) Then, the fiber lidar photodetector recorded the signal \( f(n) \), representing the convolution of the signal \( P(n) \) in Equation (1) with the laser impulse transfer function \( I(n) \):
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\begin{equation}
 f(n) = \sum_{m=1}^{N} I(m)P(n-m), \quad \begin{cases} 
 n = 1,2,3,..,N \\
 m = 1,2,3,..,M \\
 M \leq N
\end{cases}.
\end{equation}

Here, \( P(n) = P(n\Delta h) = P(h) \). The direct solution to the convolution equation is known as the deconvolution operation:

\begin{equation}
 P(n) = \frac{f(n) - \sum_{m=1}^{N} I(m)P(n-m)}{I(1)}.
\end{equation}

Following Equation (17), it is possible to reconstruct the lidar return signal’s waveform \( P(n) \) and estimate the required parameters for the atmospheric state. However, for a complex lidar return signal waveform \( I(t) \) and measurements with errors, it is difficult to put into practice Equation (17). The solution is unstable because successive estimates \( P(n) \) include rounding artifacts and measurement errors from the preceding estimates.

Note that the convolution in Equation (16) is equivalent to obtaining and solving the system of linear equations with the Gauss method. This system of equations in matrix form has the following form:

\begin{equation}
 Ap = f + \varepsilon.
\end{equation}

Here, the \( N \times N \) matrix \( A = \{ a_{ij} \} \) is composed of elements of the laser impulse transfer function \( I(n) \) under the following rule:

\begin{equation}
 a_{ij} = I(i-j), \quad \begin{cases} 
 i,j = 1,2,3,..,N \\
 a_{ij} = 0, \text{ if } 0 > (i-j) > M
\end{cases}.
\end{equation}

Here, \( p = \{ P_{\text{a}} = P(n) \} \), \( f = \{ f_{\text{a}} = f(n) \} \), and \( \varepsilon = \{ \varepsilon_{\text{a}} \} \) is the measurement error vector.

Here, as a consequence of the convolution operation, the system of linear equations in Equation (18) has already been reduced to its lower triangular form. The main diagonal of the matrix \( A \) is composed of the element \( I(1) \), and the lower diagonals comprise elements \( I(2),I(3),..,I(M) \). It is well known that one of the conditions of applicability of all direct methods for solving systems of linear equations is significantly larger values for the elements on the main diagonal compared with all of the others. Following the theorem of uniqueness of matrix reduction to the triangular form, it is possible to conclude that the only condition of a stable solution to the system in Equation (18) is the decaying laser pulse. In addition, the result is influenced by measurement and calculation rounding errors. Therefore, the direct method of solving the system in Equation (18) generally leads to unstable solutions.

The iterative Kaczmarz method [32–36] of solving systems of linear equations belonging to the projective group of methods is the most suitable method for overcoming these difficulties. This method is also known in tomography as the algebraic reconstruction technique (ART) [37,38]. The algorithm of the generalized method is based on the representation of the matrix \( A \) with the rows \( A = \{ A_{k} \} \). In the classical successive approximation method, on each \( k \)th iteration step for the vector \( p \) in the cycle over all rows \( \{ A_{k} \} \) of a matrix \( A \), the discrepancy vector is calculated by the formula
\[ p^k = p^{k-1} + \omega \frac{A^T (f_i - A_i p^{k-1})}{\| A_i \|}, \quad \begin{cases} k = 1, 2, 3, \ldots \\ i = 1, 2, 3, \ldots, N \end{cases} \] (20)

Here, \( \{p^n\} \) is the iterative sequence for the vector \( p \), \( \omega \) is the relaxation parameter, and \( \| A_i \| \) is the Euclidean norm of the \( n \)th row of the matrix \( A \). We note that each addition of the discrepancy vector increases the serial iteration number. The choice of the initial approximation vector \( p^0 = p^1 \) is arbitrary. In the present study, we take advantage of the zero initial approximation, \( p^0 = \bar{p}(h) = 0 \) for all \( h \).

Note that the application of the LSIM transforms the lidar signal as if the laser pulse duration were equal to \( \Delta t \), which is the resolution of the lidar registration system. However, the pulse energy remains the same.

3.3. Data Processing of Measurement Results

The technique of processing measurement results consists of several stages. In the beginning, using tabular data of SPCM testing, the data were corrected for miscalculations. Then, the data were processed using the algorithm of the optimal linear regression method [11,39,40]. It was assumed that the statistics of recording a one-photon pulse obeyed the Poisson distribution. In this case, the noise level could be set equal to estimates of the average accumulated signal or, as was the case with the measurement data, equal to the signal itself. After this, the noise, the level of which was determined by the external background illumination and the intrinsic noise of the avalanche SPCM photodiode, was subtracted from the signals. The intrinsic SPCM noise level was estimated using preliminary laboratory measurements.

4. Materials

4.1. System

A fiber lidar system was developed based on the fiber Q-switch laser, and a series of test measurements was carried out from 2 October 2023 to 1 December 2023. The fiber lidar was deployed in the research laboratory of Hanbat National University in Daejeon, Republic of Korea. Figure 5 shows the external view of the fiber lidar used in our outdoor experiment (left) and the lidar’s position on a local map (right).

![Figure 5](image)

**Figure 5.** Fiber lidar for detecting aerosol pollutants in the atmosphere (left) and the lidar’s position on a local map (right). Here, 1 is a fiber laser radiation collimator, 2 is a fiber lidar receiving telescope, 3 is an SPCM, and 4 is a pulse counter.
Here, 1 is the collimator of the output radiation of the fiber laser [41], 2 is the receiving telescope of the fiber lidar, 3 is the SPCM, and 4 is the pulse counter. Table 1 gives the main fiber lidar parameters. The fiber lidar’s design was such that the optical axis of the telescope was fixed relative to the optical axis of the collimator fiber laser lens.

**Table 1. Fiber lidar parameters.**

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<th>Brand</th>
<th>MFP 20W, Maxphotonics</th>
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<tbody>
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<td>Spectrum width, (FWHM)</td>
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<td></td>
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<tr>
<td>Pulse duration *, (FWHM)</td>
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<tr>
<td>Pulse energy</td>
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</tr>
<tr>
<td>Beam quality (M²)</td>
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* Data of the manufacturer. Full pulse duration is given in the text. ** The module was used in test measurements. *** The sensor was used to measure the laser pulse’s waveform. **** This was used in test measurements and in measurements of the laser pulse’s waveform.

**4.2. Sequence of Operations**

The photodetector light guide of scattered radiation was fixed to the coordinate shifter in the focal plane of the receiving collimator. The bandpass filter was also located here. The input light guide aperture was displaced in the xoy plane. Radiation backscattered from the atmosphere passed through the bandpass filter and the receiving light guide and entered the SPCM module. In the receiving area of the SPCM, being sensitive to the incident radiation, individual photons were detected. Then, the TTL pulses from the SPSM output entered the pulse counter.
The fiber laser operation was controlled by an Arduino Uno R3 programmable controller based on an Atmega 328P microcontroller. The controller was connected to the laser through the DB-25 socket, and signals digitized on the TTL level were fed to the control unit of the fiber laser. After turning on, the external controller generated a sequence of triggering commands and then, at ~100 ms, generated pulses with a frequency equal to that of the laser pulse generation. The logic of operation of the internal fiber laser controller was that it determined the pulse repetition frequency for roughly 100 ms and then independently generated radiation-triggering pulses. After this, the commands of the external controller were used to set and change the laser pulse energy during the course of operation. Thus, the external synchronization of the fiber laser’s launching was absent.

To synchronize the pulse counter with the laser, optical launching through the light guide was employed. For this purpose, a beam-splitting plate was used to deflect a portion of the laser radiation at the triggering light guide and transmit it to the pulse counter as a triggering signal.

The pulse counter was a programmable EBAZ4205 controller based on a system-on-chip (SoC) Zynq 7000 microcontroller. The pulse counter was triggered synchronously with the laser pulse, and pulses arriving at the counter in the memory cells were consistently recorded. The addresses of the memory cells corresponded to the range gates or time intervals of radiation propagation in the atmosphere. The pulse counter operated in accumulation mode, and the lidar return signal profile was formed for a preset number of counter triggering. The operating modes of the counter, including the time period, the number of range gates of pulse accumulation, and the number of accumulation cycles, were set before the beginning of counter operation. The control commands and accumulated data files were transmitted to a computer using the Ethernet protocol. The fiber lidar’s design as a whole was simple and did not need additional adjustments during operation. The noise level created by the control unit of the fiber laser had no significant effect on the lidar’s operation.

5. Results

5.1. Verification

To verify the lidar signal inversion method, we carried out a numerical lidar experiment based on the reference signal method. As was already pointed out above, the advantage of the RSM is that the result is not influenced by the form of the geometric lidar function or the waveform of the lidar return signal from the atmosphere, which depends on the square sensing range.

The verification procedure occurred in the following order:

- Lidar return signals \( P_{\text{REF}}(h) \) from the short-transmitted laser pulse with a duration equal to the resolution of the registration lidar system were formed.
- A square wave signal lasting several strobos was added to one of the signals. It simulated the occurrence of an aerosol impurity within the sensing range. As a result, two lidar signals were formed. A lidar signal without the impurity was designated as the reference signal \( P_{\text{REF}}(h) \). The lidar signal with the impurity was designated as \( P_{\text{SCAN}}(h) \).
- Lidar responses from the long laser pulse \( f(h) \) were formed. Calculations were carried out with Equation (16) as the convolution operation. The signals \( P_{\text{REF}}(h) \) and \( P_{\text{SCAN}}(h) \), and the laser impulse transfer function \( I(t) \) took part in the calculations. As a result, lidar signals \( f_{\text{REF}}(h) \) and \( f_{\text{SCAN}}(h) \) were obtained.

To bring the numerical experiment as close as possible to real conditions, the model’s backscattered lidar return signal \( P_{\text{REF}}(h) \) was generated from the data of the outdoor experiment. The impulse transfer function \( I(t) \) of the laser was also taken from the
measurements shown in Figure 4. Thus, the model’s lidar parameters were kept close to those of the fiber lidar. In turn, the lidar signals \( f(h) \) of the model’s lidar approximately corresponded to the signals of the fiber lidar.

It was accepted that the square wave signal from the aerosol impurity occupied five strobes (37.5 m), starting from strobe number 150 (1125 m), and its level exceeded that of the signal from the surrounding background aerosol twice.

In the next step, using the LSIM, the inverse problem was solved with calculations of the estimates \( \bar{p}_{REF}(h) \) and \( \bar{p}_{SCAN}(h) \). Thus, matrix \( A \) with a size of 600 × 600 counts was composed for the reference signal profile using Equation (19). The lidar signals comprised 600 counts each with a 50 ns time resolution. The laser impulse transfer function comprised 250 counts with the same resolution.

Figure 6a shows the aerosol/background ratios \( f_{SCAN}(h)/f_{REF}(h) \) of the non-reconstructed lidar signals, depending on the sensing range. Figure 6b shows the aerosol/background ratios \( \bar{p}_{SCAN}(h)/\bar{p}_{REF}(h) \) for the reconstructed lidar signals, depending on the sensing range and the LSIM iteration number.

![Figure 6](image.png)

**Figure 6.** Results of the model experiment for detecting impurities in the atmosphere, including the ratio of the non-reconstructed signals, depending on the sensing range (a), and the ratio of the reconstructed signals, depending on the sensing range and the number of LSIM iterations (b).

Here, the ratios of two signals, one of which was taken as a reference, were simulated with the RSM. Figure 6a shows the effect of the laser pulse duration on the signal from the aerosol. It was difficult to distinguish the square wave signal from the impurities; it was blurred and displaced in the sensing range. In Figure 6b, it can be seen that the reconstructed signal from the aerosol impurities monotonously approached the initial signal waveform with an increasing number of iterations of the LSIM. Thus, the dependence of the reconstructed signal on the sensing range also coincided.

The results obtained confirmed the correctness of the main LSIM provisions. In addition, the absence of errors in the computing algorithms was confirmed.

Note here the following property of the simulated results. Since coordinated data without measurement errors were used in the model, we obviated the need for special selection of the relaxation parameter value and set \( \omega = 1 \). Practically, complete absence of calculation artifacts and a sufficiently high rate of convergence for the iterative sequence should also be mentioned here.
5.2. **Outdoor Experiment**

To test the lidar signal inversion method, we performed an outdoor experiment. At this stage of research, test results were obtained with the RSM. This allowed us to qualitatively estimate the results as a whole in the entire sensing range. To calibrate the fiber lidar in absolute units, the sensing results should be compared with the data of independent measurements using methods adequate to the specific aims of the research. The LSIM testing in our outdoor experiment was carried out the following order:

- Control of the temporal stability of the fiber laser;
- Alignment of the fiber lidar and carrying out test measurements in the atmosphere;
- Carrying out the outdoor experiment;
- Experimental data processing;
- Reconstruction of lidar signals using the LSIM;
- Obtaining the measurement results with the RSM.

5.2.1. Control of the Temporal Stability of the Fiber Laser

Figure 7 shows the measurement results for the impulse transfer function of the fiber laser, depending on the time period after the onset of laser pulse generation. To perform these measurements, a portion of the fiber laser radiation was deflected to the silicon photodiode using a beam-splitting plate. The output of the silicon photodiode was connected to the input of the oscilloscope loaded at 50 Ohms. The oscilloscope was synchronized over time with the input signal level. Measurements of the laser pulses were carried out at an interval of 600 s with registration in the oscilloscope memory. The time resolution of the laser pulse profiles was 40 ns. Measurements were carried out at 70% of the maximum laser pulse energy.

![Waveforms of the impulse transfer function of the fiber laser as functions of time after turning on the laser. The profiles of the laser pulses were normalized to the profile measured 1 hour and 10 minutes after switching on the laser.](image)

Our analysis of the measurement results showed that the waveforms of the impulse transfer function and laser pulse energy were stable for about an hour after turning on the fiber laser.

5.2.2. Alignment of the Fiber Lidar and Carrying Out Test Measurements in the Atmosphere

Preliminary adjustment of the optical fiber lidar’s path was carried out in laboratory conditions. At the beginning, parallelism of the optical axes of the fiber laser collimator and receiving telescope was approximately established. The accuracy of alignment of the
axes was determined with the error of machining of lidar parts. Then, a Porro prism was placed on the optical laser radiation axis, and a portion of the laser radiation was deflected in the opposite direction and transmitted to the receiving lidar telescope. The arrangement of the focal plane on the optical axis of the receiving telescope was set through the selection of replaceable rings on the telescope body. The accuracy of the positioning was controlled with the diameter of the spot left by the radiation beam on the absorbing material located in the image plane. Then, by using coordinate displacement, the receiving light guide was accurately positioned in the focal plane of the lens telescope. The position was controlled visually, using an infrared visualization card placed at the light guide output.

At the next stage of this research, a series of test measurements of laser pulses backscattered from the atmosphere was carried out. Laser pulse radiation scattered in the atmosphere was incident at the receiving light guide and entered the avalanche photodiode module (C12703, Hamamatsu). From the output of the module, the lidar signal in analogue mode was fed to the oscilloscope.

Note that the given scheme of signal recording was used only for adjusting and testing the stability of the fiber lidar parameters as well as estimating the dynamic range of lidar signals. The high fiber laser pulse frequency (30 kHz) did not coincide with the low frequency of signal recording with the oscilloscope, thereby leading to a loss of some information.

5.2.3. Carrying Out the Outdoor Experiment

In the course of the outdoor experiment on 6 November 2023, the sensing path of the fiber lidar was oriented at an azimuth angle of 255° and an elevation angle of 15°. Rare low-altitude water droplet cloudiness under stable atmospheric conditions of the background environment with an insignificant amount of aerosol impurities served as a source of aerosol formation. Measurements were carried out for a maximum sensing range of 4500 m in photon counting mode. The measurement cycle comprised lidar signals $f_{\text{SCAN}}(h)$ accumulated for 5 s (150,000 laser pulses).

5.2.4. Experimental Data Processing

The lidar return signal profile averaged over the entire measurement period including measurement cycles without cloudiness served as a reference signal $f_{\text{REF}}(h)$. After preprocessing, files of the $f_{\text{SCAN}}(h)$ and $f_{\text{REF}}(h)$ data comprised 600 counts each, with a spatial resolution of 7.5 m (50 ns time resolution). The profile of the impulse transfer function $f(t)$ of the fiber laser consisted of 250 counts.

5.2.5. Reconstruction of the Lidar Signals Using the LSIM

To invert the lidar signal profile with the LSIM, a matrix $A$ with dimensions of 600 × 600 was constructed. The preset level of convergence was reached after 150–200 iteration cycles. The relaxation parameter in the iterative calculations was set equal to $\omega = 0.03$. The calculation accuracy was controlled by a comparison of the discrepancy norms of the measured signals $f(h)$ and the signal profiles obtained using the convolution operation in Equation (15) for the estimates $\hat{P}_{\text{SCAN}}(h)$ and $\hat{P}_{\text{REF}}(h)$. Note that to obtain the estimates $\hat{P}_{\text{SCAN}}(h)$ and $\hat{P}_{\text{REF}}(h)$, the same number of iterations was used in the inversion procedure.

Figure 8 shows an example of the inversion $\hat{P}(h)$ of one of the experimental lidar profiles $f(h)$.
Figure 8. Example of the reconstructed experimental lidar signal.

In Figure 8, it can be seen that the application of the LSIM eliminated blurring of the experimental lidar signals. The asymmetry of the long laser pulse on the measured lidar signal was also eliminated. As was already indicated above, the reconstructed lidar signal was similar to the signal from the laser pulse, the duration of which was equal to the resolution time of the registration system (i.e., 50 ns). Moreover, the pulse energy remained unchanged and equal to the energy of the long pulse. By analogy, it is possible to compare the effect of the LSIM on the lidar signal with the effect of increasing the sharpness of the blurred image due to image focusing. Artifacts of the inversion procedure can be seen in the lidar return signal profile $P(h)$. These artifacts were caused by regular and random measurement errors. For example, the profile of the impulse transfer function $I(t)$ was not measured during the experiment, thereby introducing systematic errors.

Figure 9 shows the aerosol/background signal ratio profiles $f_{\text{SCAN}}(h)/f_{\text{REF}}(h)$ of the lidar signals, depending on the sensing range.

Figure 9. Aerosol/background signal ratio profiles of the lidar signals, depending on the sensing range.

Figure 9 shows the signals from the aerosol impurities in the atmosphere for a series of 20 subsequent lidar measurements, each of which was accumulated for 5 s. According to the characteristics of rare cloudiness, these signals changed during sensing. The first
region of signal changes was observed from 500 to 1500 m. The second region was observed starting from about 3000 m. It can be seen that the signals from the aerosol impurities in the atmosphere were quite blurred. In the sensing range from 1500 to 2500 m, weak signals were observed. This effect was due to signals in the first region caused by the laser pulse duration.

Figure 10 shows the results of application of the LSIM to the data shown in Figure 9.

![Figure 10](image)

Figure 10. Aerosol/background signal ratio profiles of the lidar signals reconstructed depending on the sensing range obtained after experimental data processing.

Figure 10 shows the profiles of the aerosol/background ratios $R_{\text{SCAN}}(h) = \frac{P_{\text{SCAN}}(h)}{P_{\text{REF}}(h)}$, depending on the sensing range. It can be seen that the signal amplitudes in the regions of signal changes increased by more than four times. The signal blurring was also eliminated. The localization of signals from the impurities in the sensing range was also more pronounced. The LSIM effectively eliminated the effect of the long laser pulse on the results for fiber lidar sensing. At the same time, artifacts can also be seen in Figure 10. They arose due to inaccuracies in measuring the laser impulse transfer function, the effect of noise on the initial signals, and other experimental parameters that were difficult to control. These problems are discussed in the section below.

6. Discussion

One of the problems solved by the LSIM was selection of the calculation end criterion. Experimental data contain random and systematic measurement errors. In addition, the final result was influenced by calculation artifacts caused by rounding errors. The iterative Kaczmarz method is distinguished by monotonic convergence. Therefore, the choice of the relaxation parameter $\omega$ and initial approximation $P^0 = P^1$ determine the strategy for achieving the required convergence level. Assignment of the relaxation parameter is equivalent to choosing the spectral component filtration level for the output signal profile. Since noise is the high-frequency lidar return signal component, a decrease in the relaxation parameter ($\omega \ll 1$) leads to signal smoothing [42,43].

Criterion Selection in the LSIM

The following method was used to select the end criterion of the inversion procedure. The method was based on the assumption of independence of the measurement errors in sensing range gates in the sample $R_{\text{SCAN}} = \{R(h_i)\}$ constructed from all profiles of the ratios $R_{\text{SCAN}}(h)$ for the entire measurement period. We assumed that all $R_{\text{SCAN}}(h) < 1$ in
the sample were caused either by the measurement errors or calculation artifacts, since without measurement errors, we always had $R_{\text{SCAN}}(h) \geq 1$. On the other hand, the values $R_{\text{SCAN}}(h) > 1$ in the sample of ratios $\{R_i(h_i)\}$ consisted of the deterministic signal of the aerosol impurity within the range gate and the measurement errors. Because of the independence of errors, we considered that the contributions of errors and calculation artifacts were the same for $R_{\text{SCAN}}(h) > 1$ and $R_{\text{SCAN}}(h) < 1$.

Figure 11 shows the estimates of the deterministic component in the experimental lidar signal ratios, depending on the number of iterations for the inversion method.

Under these assumptions, for each $k$th iteration step of the projective Kaczmarz method, we obtained the root mean square estimate $s^k$ for the sample $R_{\text{SCAN}}^k = \{R_i(h_i)\}$ of the form

$$s^k = \eta^k_+ - \eta^k_-.$$  \hspace{1cm} (21)

Here, the estimate $\eta^k_+$ comprises all values $R_{\text{SCAN}}(h) > 1$ in the sample:

$$\eta^k_+ = \frac{1}{n} \sum_{i=1}^{E} (1 - R_i(h_i))^2,$$  \hspace{1cm} (22)

where $n$ is the number of corresponding elements in the sample and the estimate $\eta^k_-$ comprises all values $R_{\text{SCAN}}(h) < 1$ in the sample:

$$\eta^k_- = \frac{1}{m} \sum_{i=1}^{E} (1 - R_i(h_i))^2,$$  \hspace{1cm} (23)

where $m$ is the number of the corresponding elements in the sample.

Thus, the estimate $s^k$ is proportional to the contribution of the deterministic signal to the sample of ratios $R_{\text{SCAN}}^k = \{R_i(h_i)\}$ and should monotonically approach a limit with increasing iteration numbers.

To test the method, the sample $\{R_i(h_i)\}$ of measurement results, comprising 11,400 elements, was compiled for each iteration step to solve the system in Equation (20). Figure 11 shows the plot of the estimates $s^k$, depending on the iteration number.

Figure 11. Estimates of the deterministic component in the experimental lidar signal ratios, depending on the number of iterations for the inversion method.
the obtained data demonstrate that the estimate $s^t$ monotonically approached the limit with increasing iteration numbers, which was sufficient for selection of the end criterion for the iterations.

If ecological fiber lidar monitoring is aimed at detecting the presence of aerosol impurities, then a detection threshold should be defined. For this purpose, the discrimination threshold $L_{\text{THRESHOLD}}^t$ can be estimated for the profiles $R_{\text{SCAN}}(h)$, depending on the iteration number:

$$L_{\text{THRESHOLD}}^t = 1 + \alpha \sqrt{\eta^t},$$

where $\alpha$ is the scale factor. Figure 12 shows the dependence of the lower discrimination threshold for $\alpha = 1$ as a function of the iteration number for $R_{\text{SCAN}}(h)$, which was obtained experimentally.

![Figure 12](image)

**Figure 12.** Estimated lower discrimination threshold for the aerosol impurities from the experimental lidar signal ratios, depending on the number of iterations of the inversion method.

These data show that the proposed method for constructing the criteria can be used under experimental conditions in the presence of measurement errors.

### 7. Conclusions

A method for the application of laser pulses with complex waveforms for lidar sensing of the atmosphere was proposed in the present work. The lidar signal inversion method (LSIM) was used to solve this problem. The LSIM was verified and tested. At this stage of our research, the results for verification and an outdoor experiment were obtained within the limits of the reference signal method (RSM), which allowed us to estimate the results for lidar sensing in the entire sensing range as a whole. In addition, application of the RMS allowed us to use only the necessary number of controllable parameters to verify and test the LSIM.

The results of our investigation were presented. Our application of the LSIM was analyzed, the effect of measurement errors on the obtained results was considered, and criteria for selection of the LSIM parameters were proposed. The verification results confirmed the correctness of the main LSIM provisions. In addition, the absence of errors in the computing algorithms was confirmed. The LSIM effectively eliminated the effect of the long laser pulse on the results of fiber lidar sensing. The outdoor experiment’s results show that the signal amplitudes in the regions of signal changes increased by more than four times. Signal blurring was also eliminated. The localization of signals from the impurities in the sensing range was also more pronounced. The results of our work show that
the proposed direction in the development of lidar sensing is promising. Note that to calibrate the fiber lidar in absolute units and develop procedures for comparison of the sensing results with the data of independent measurements, it is necessary to use methods adequate for the specific goals of research. These will be promising directions for the subsequent stages of lidar research.


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**References**


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