

Article

Revisiting the Desroziers Diagnostic Across Gaussian and SOAR-Type Spatial Correlation Regimes

Linyi Wang ^{1,2}, Kefeng Zhu ³ and Yuefei Zeng ^{1,2,*}

¹ State Key Laboratory of Climate System Prediction and Risk Management/Key Laboratory of Meteorological Disaster, Ministry of Education/Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science and Technology, Nanjing 210044, China; 202512010039@nuist.edu.cn

² School of Atmospheric Sciences, Nanjing University of Information Science and Technology, Nanjing 210044, China

³ Nanjing Innovation Institute for Atmospheric Sciences, Chinese Academy of Meteorological Sciences-Jiangsu Meteorological Service, Nanjing 210044, China; zhukf@cma.gov.cn

* Correspondence: yuefei.zeng@nuist.edu.cn; Tel.: +86-139-5036-2009

Abstract

The Desroziers diagnostic (D05) is widely used to estimate observation error covariances, but its behavior under different spatial correlation structures remains insufficiently understood. Through twin experiments with a modified shallow water model, this study evaluates D05 across Gaspari-Cohn, uncorrelated, SOAR-type, and mixed Gaspari-Cohn-SOAR spatial correlation regimes. D05 reproduces the prescribed correlation shape well in the Gaspari-Cohn and uncorrelated cases, while small variance biases remain when the assimilation system uses a diagonal observation-error covariance matrix. Under the SOAR-type correlation settings considered here, D05 produces longer diagnosed correlation scales than the prescribed curves, and this positive length-scale bias increases in the mixed Gaspari-Cohn-SOAR experiments. These results indicate that D05 is sensitive to the functional form of the prescribed spatial correlation and to covariance misspecification in the assimilation system. The findings provide guidance for interpreting D05-derived observation-error correlations in atmospheric data assimilation, especially when the true observation-error dependence structure differs from conventional compact-support correlation models.

Keywords: data assimilation; Desroziers diagnostic; observation error covariance; spatial correlation; SOAR correlation; Gaspari-Cohn correlation; shallow water model; twin experiment



Academic Editor: Stefano Federico

Received: 5 June 2026

Revised: 28 July 2026

Accepted: 30 July 2026

Published: 3 August 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

Data assimilation combines model forecasts with observations to produce initial conditions for geophysical prediction. Accuracy depends critically on two error covariance matrices: background **B** and observation **R**. While **B** has received considerable attention, **R** remains challenging to characterize despite its important role in weighting observations [1].

Observation errors encompass instrument error, forward model error, and representation error from unresolved scales [2]. Operational systems traditionally assume **R** is diagonal for efficiency, but observations are frequently correlated. Neglecting these correlations degrades analysis accuracy and forecast skill [3], motivating efforts to estimate correlated observation errors. The Desroziers et al. diagnostic (D05) [4] is the most widely adopted method for estimating **R**, with applications across satellite, radar, and conventional

observations [5–7]. Despite its widespread use, fundamental questions about D05's behavior under realistic conditions remain. The method assumes perfect background covariances, linear operators, and unbiased errors, conditions rarely met [8]. While sensitivity to prescribed error and sampling has been examined [1,9], how D05 performs under different spatial correlation structures is underexplored.

It is important to distinguish between the spatial correlation function of an error field and the probability distribution of the errors. A correlation function such as Gaspari-Cohn or SOAR specifies the second-order spatial dependence between different grid points, whereas marginal and joint distributions describe whether the error values themselves are Gaussian, skewed, heavy-tailed, or otherwise non-Gaussian. A random field may therefore have a SOAR covariance function while still being generated from a multivariate Gaussian distribution. In this study, we focus on the sensitivity of D05 to the prescribed spatial correlation function and avoid interpreting a SOAR-type correlation function alone as proof of distributional non-Gaussianity.

Distributional non-Gaussianity should be assessed using diagnostics of the error samples themselves, not from the correlation function alone. For example, an ideal Gaussian sample has skewness close to zero and Pearson kurtosis close to three; additional checks such as histograms, empirical quantiles, Q-Q plots, and normality tests can be used to diagnose asymmetry, heavy tails, or other departures from Gaussianity. In response to this issue, the present revision explicitly evaluates the prescribed observation-error samples using skewness, Pearson kurtosis, and standardized histograms. These diagnostics are used below to distinguish distributional evidence from evidence associated with the prescribed spatial correlation functions.

This paper addresses this gap through a systematic evaluation of D05 behavior across Gaspari-Cohn, uncorrelated, SOAR-type, and mixed Gaspari-Cohn-SOAR spatial correlation regimes, based on twin experiments using a shallow-water model. Section 2 introduces the numerical model and the mathematical principle of the D05 diagnostic. Section 3 describes the experimental design and presents the main numerical results. Section 4 discusses the interpretation of D05 bias and draws conclusions with future research outlooks.

2. Methodology

2.1. Modified Rotating Shallow Water Model (*modRSW*)

This study employs the modified rotating shallow water model (*modRSW*) developed by Kent et al. [10], which extends classical shallow water dynamics to incorporate simplified moist convective processes. The model predicts three prognostic variables: fluid depth (geopotential height), horizontal fluid velocity (wind), and rainwater content.

The governing equations describe mass and momentum conservation with additional terms for topographic forcing, Coriolis rotation, and a simplified precipitation source-sink scheme. Rainwater is generated under low-level flow convergence and removed via linear relaxation, producing intermittent precipitation features representative of convective-scale atmospheric phenomena. Critical height thresholds ($H_c = 1.02$, $H_r = 1.05$) activate convection and precipitation processes conditionally, while parameters $\beta = 0.2$ and $\alpha = 10 \text{ s}^{-1}$ control rainwater generation and removal rates. A feedback parameter c_0^2 couples rainwater anomalies to the momentum equation. The complete derivation and discretized equations of *modRSW* can be found in [10].

All numerical experiments and diagnostic plotting were implemented using author-developed Python scripts; no commercial reagents, instruments, cell lines, samples, or materials were used.

2.2. The D05 Diagnostic Method

The D05 diagnostic estimates the observation error covariance matrix $\mathbf{R}$ using the expected outer product of the innovation vector $\mathbf{d}_{o-b}$ (first-guess departure) and analysis departure vector $\mathbf{d}_{o-a}$:

$$\mathbf{R}_{\text{est}} = \mathbf{E}[\mathbf{d}_{o-a} \mathbf{d}_{o-b}^T]. \quad (1)$$

Here $\mathbf{y}^o$ denotes the observation vector, $\mathbf{d}_{o-b} = \mathbf{y}^o - \mathbf{H}(\mathbf{x}_b)$ and $\mathbf{d}_{o-a} = \mathbf{y}^o - \mathbf{H}(\mathbf{x}_a)$ represent innovation and analysis departure, respectively, and $\mathbf{H}$ is the nonlinear observation operator.

$\mathbf{R}_{\text{est}}$ provides an unbiased estimate under idealized second-order conditions: linear observation operators, mutually uncorrelated background and observation errors, and correctly specified background and observation error covariances used in the analysis [11]. These assumptions concern the covariance structure entering the analysis equations; Gaussianity is often invoked in optimal-estimation derivations, but the practical D05 estimator in Equation (1) is primarily a covariance diagnostic based on innovation and analysis-departure statistics. Therefore, departures from ideal D05 behavior may arise from several sources, including covariance misspecification, finite ensemble and sample sizes, nonlinear observation operators, or non-Gaussian error distributions. Though these idealized conditions are rarely satisfied in real numerical weather prediction systems, the Desroziers et al. [4] diagnostic remains the dominant practical tool to diagnose correlated observation error statistics. Desroziers et al. [4] originally proposed iterative updates of Equation (1) to approach internally consistent error-covariance estimates, but subsequent studies have shown that a single iteration often produces sufficiently reliable estimates [12]. Given computational constraints for high-dimensional models, almost all operational NWP studies adopt the single-iteration scheme [13–15], which is also followed in this work.

3. Experimental Design and Results

3.1. Experimental Setup

A nature run is generated by integrating the modRSW model forward in time to serve as the “true” atmospheric state. The model domain is discretized with 65 uniform grid points, an integration time step $\Delta t = 0.001$ s, and physical parameters consistent with Section 2. After a spin-up integration, a 20-member time-lagged ensemble is initialized for ensemble data assimilation. The LETKF [16] is adopted as the assimilation algorithm with multiplicative covariance inflation of 1.6 and a horizontal localization radius of 4 grid points. A total of 100 sequential assimilation cycles are performed, and the D05 spatial-correlation diagnostics are evaluated using the post-spin-up cycling statistics. The 65-grid, 100-cycle configuration is the setup used for the revised numerical diagnostics and figures; it was adopted to keep the idealized twin experiments computationally tractable while retaining all three prognostic variables and full-grid observations. The 20-member ensemble size is used consistently in the reported experiments and in the figures below.

Synthetic observations are extracted from the nature run at every model grid point. Each observation value is perturbed by pre-defined observation error fields with prescribed spatial covariance structures:

$$\mathbf{y} = \mathbf{x}_{\text{true}} + \boldsymbol{\varepsilon}, \quad (2)$$

where $\boldsymbol{\varepsilon}$ denotes the observation error vector sampled from the prescribed covariance $\mathbf{R}_{\text{true}}$. Fixed observation error standard deviations are applied across all experiments: 0.05 m for fluid height, 0.05 m s^{−1} for zonal wind, and 0.001 dimensionless units for rainwater mixing ratio. Unless otherwise stated, observation-error correlations are specified within each prognostic variable, and cross-variable error correlations are set to zero, so that the full observation-error covariance matrix is block diagonal by variable. Spatial distance d_{ij} is measured by grid-point separation on the one-dimensional model grid. The prescribed

correlation functions are used to construct positive-definite covariance matrices after scaling by the variable-dependent error variances.

The core experimental variable is the spatial correlation structure of ε . Four major regimes are designed to systematically test D05 performance: Gaspari-Cohn correlation, fully uncorrelated noise, SOAR-type spatial correlation, and mixed Gaspari-Cohn-SOAR spatial correlation with tunable SOAR contribution γ . The terms “Gaspari-Cohn” and “SOAR” in this section refer to the shape of the spatial correlation function, not to the marginal probability distribution of the error values by themselves. Table 1 summarizes the prescribed observation-error spatial correlation regimes.

Table 1. Summary of the prescribed observation-error spatial correlation regimes. The experiment names encode the correlation type and nominal full-length scale for height, wind, and rainwater, respectively; for example, G04G08G12 denotes Gaspari-Cohn correlations with $2L = 4, 8$, and 12 grid points.

Group	Cases	Correlation Function	Nominal Length Scales
Gaspari-Cohn	G04G04G04, G12G12G12, G04G08G12	Gaspari-Cohn	Uniform or variable $2L$
Uncorrelated/mixed	N00N00N00, N00N00G12	Diagonal or diagonal + Gaspari-Cohn	0 or $2L = 12$
SOAR-type	S04S04S04, S04S08S12	SOAR	Uniform or variable $2L$
Gaspari-Cohn-SOAR mixed	H12H12H12_0.25, H12H12H12_0.50, H12H12H12_0.75	Weighted mixture	$2L = 12$

3.1.1. Gaspari-Cohn Spatial Correlation Regime

Observation errors follow the Gaspari-Cohn fifth-order compact-support correlation function $GC(d_{ij}, L)$, where d_{ij} is the spatial distance between grid i and j , and L is half of the nominal correlation length. Three sub-experiments are considered:

- G04G04G04: Height, wind, and rainwater all have correlation length $2L = 4$ grid points
- G12G12G12: All variables with uniform $2L = 12$ grid points
- G04G08G12: Height $2L = 4$, wind $2L = 8$, rain $2L = 12$ grid points

3.1.2. Uncorrelated and Gaspari-Cohn Mixed Regime

True $\mathbf{R}$ is diagonal for uncorrelated cases:

- N00N00N00: All variables fully spatially uncorrelated
- N00N00G12: Height/wind uncorrelated, rain with Gaspari-Cohn correlation $2L = 12$

3.1.3. SOAR-Type Spatial Correlation Regime

Observation error correlation adopts the SOAR function:

$$\text{corr}_{ij} = \left(1 + \frac{d_{ij}}{L}\right) \exp\left(-\frac{d_{ij}}{L}\right), \quad (3)$$

Two sub-experiments are considered:

- S04S04S04: Uniform $2L = 4$ for all three variables
- S04S08S12: Variable-dependent lengths 4, 8, 12 grid points

3.1.4. Mixed Gaspari-Cohn-SOAR Spatial Correlation Regime

Composite error fields are generated by weighting Gaspari-Cohn-correlated and SOAR-correlated components with mixing coefficient $\gamma \in [0, 1]$:

$$\varepsilon_{\text{mix}} = \frac{(1 - \gamma)\varepsilon_{\text{GC}} + \gamma\varepsilon_{\text{SOAR}}}{\sqrt{(1 - \gamma)^2 + \gamma^2}}. \quad (4)$$

The normalization preserves the prescribed marginal standard deviations when the two components are independently sampled with the same variance. The corresponding effective covariance is

$$\mathbf{R}_{\text{mix}} = \sigma^2 \frac{(1 - \gamma)^2 \mathbf{C}_{\text{GC}} + \gamma^2 \mathbf{C}_{\text{SOAR}}}{(1 - \gamma)^2 + \gamma^2}, \quad (5)$$

where $\mathbf{C}_{\text{GC}}$ and $\mathbf{C}_{\text{SOAR}}$ denote the prescribed correlation matrices. Thus, γ changes the effective spatial covariance as well as the SOAR contribution. For the uniform $2L = 12$ mixed cases, the effective prescribed e-folding correlation lengths are 5.17, 7.40, and 11.94 grid points for $\gamma = 0.25, 0.50$, and 0.75 , respectively. $\gamma = 0$ recovers the pure Gaspari-Cohn-correlated component, while $\gamma = 1$ recovers the pure SOAR-correlated component. Three sub-experiments with uniform $2L = 12$ for all variables are considered:

- H12H12H12_0.25: $\gamma = 0.25$ weak SOAR contribution
- H12H12H12_0.50: $\gamma = 0.50$ moderate SOAR contribution
- H12H12H12_0.75: $\gamma = 0.75$ strong SOAR contribution

All graphical results in this section are presented for the 20-member ensemble experiments. Therefore, the experiment description and interpretation below are stated for the 20-member configuration.

3.2. Numerical Results and Analysis

Figure 1 shows temporal RMSE evolution of background and analysis ensemble means across all experiments. Regular sawtooth RMSE patterns are present for height, wind and rainwater variables in every test case, demonstrating the LETKF assimilation cycles run stably without divergence.

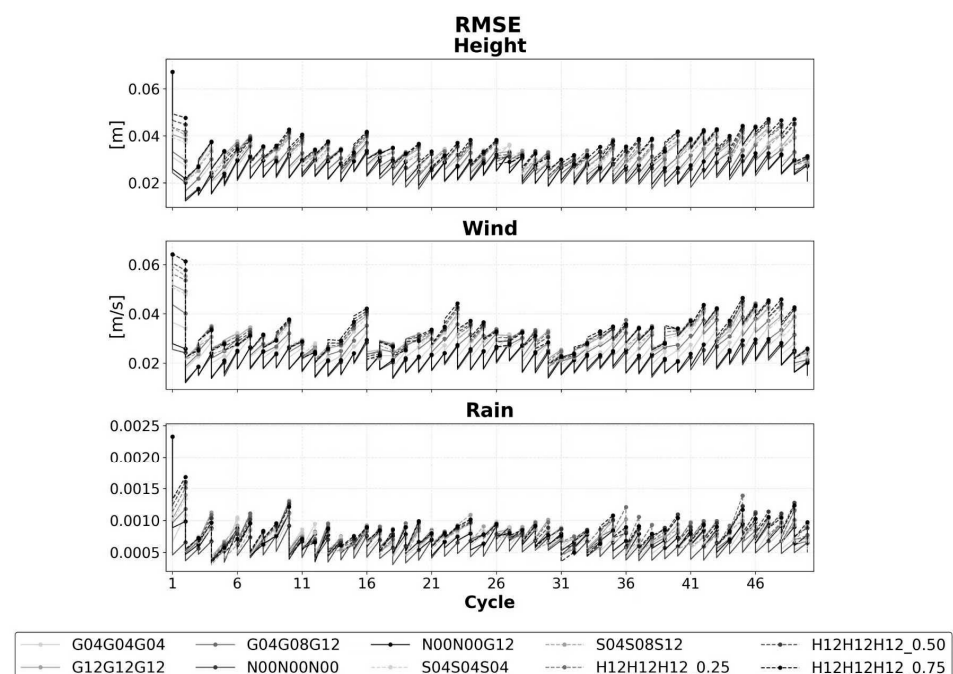


Figure 1. Temporal evolution of RMSE of ensemble mean background fields (bullet markers) and analysis fields across all designed experiments. Periodic sawtooth RMSE curves indicate stable and effective cycling of the LETKF data assimilation system.

Figure 2 reports distributional diagnostics of the prescribed observation-error samples, computed from $\varepsilon = y - x_{\text{true}}$ over all observation locations and assimilation cycles. Across all experiments and variables, skewness remains close to zero, while Pearson kurtosis remains close to the Gaussian reference value of three. The representative standardized histograms also closely follow the standard-normal density. These diagnostics indicate that the generated observation-error samples in the SOAR-type and mixed experiments do not exhibit strong skewness or heavy-tailed behavior. Therefore, the subsequent D05 biases should be interpreted primarily in relation to the prescribed spatial correlation structure, diagonal-**R** cycling, finite ensemble size, and finite sampling, rather than as evidence that the present experiments isolate distributional non-Gaussianity.

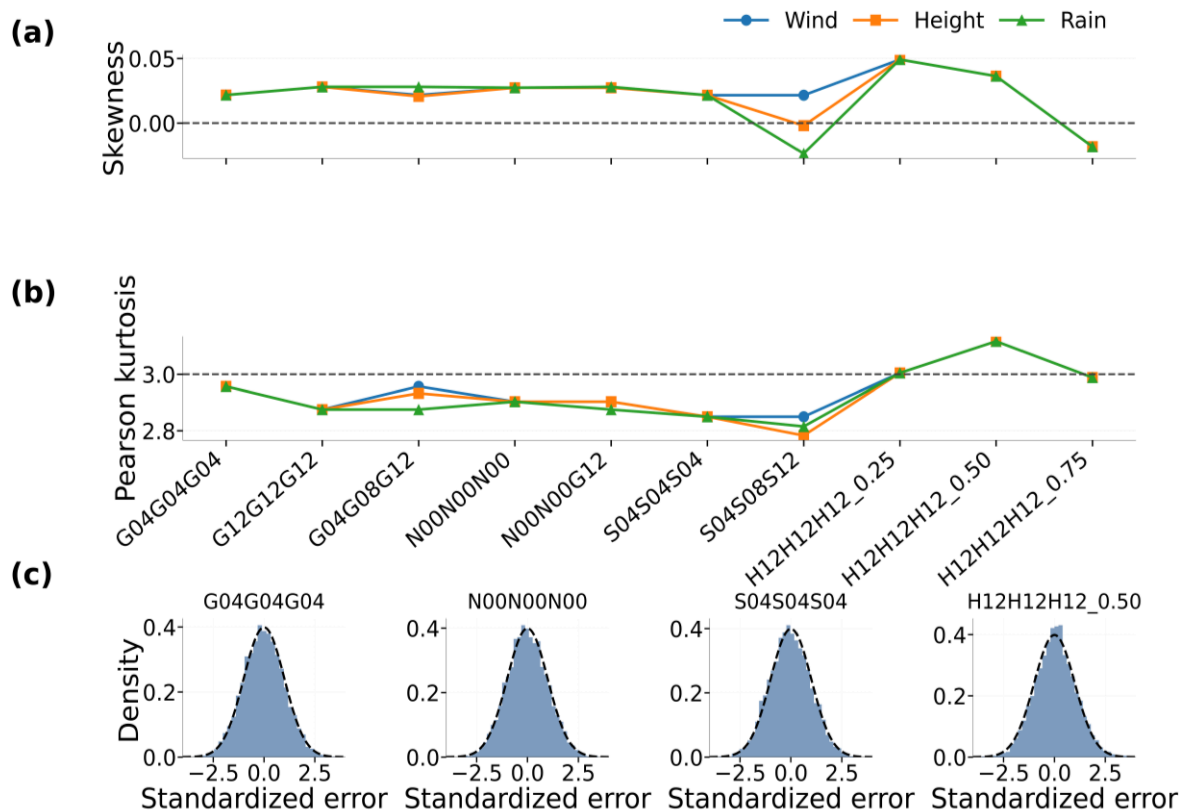


Figure 2. Distributional diagnostics of prescribed observation-error samples in the 20-member experiments. Panels (a–c) show skewness, Pearson kurtosis, and representative standardized histograms. Dashed reference lines/curves denote Gaussian benchmarks (skewness = 0, Pearson kurtosis = 3, and standard-normal density), showing near-Gaussian marginal behavior across the tested spatial-correlation regimes.

Figure 3 reports D05-diagnosed observation error standard deviations versus prescribed values. For Gaspari-Cohn correlation experiments, diagnosed standard deviations are slightly lower than the prescribed values. This negative bias is consistent with the fact that the assimilation system assumes diagonal **R** while the prescribed errors are spatially correlated: the filter can overweight observational information, generating smaller analysis residuals and hence underestimated error variance via D05. For fully uncorrelated error cases, the diagnosed standard deviations match the prescribed values closely. Table 2 provides representative numerical standard-deviation estimates for these diagnostics.

Figure 4 presents correlation diagnosis results under Gaspari-Cohn spatial correlation structures. For uniform correlation lengths ($2L = 4$ or $2L = 12$) and variable lengths across three variables, D05-diagnosed correlation decay curves overlap nearly completely with

the prescribed Gaspari-Cohn functions, confirming that D05 can reproduce these compact-support correlation shapes well in the present experimental setting.

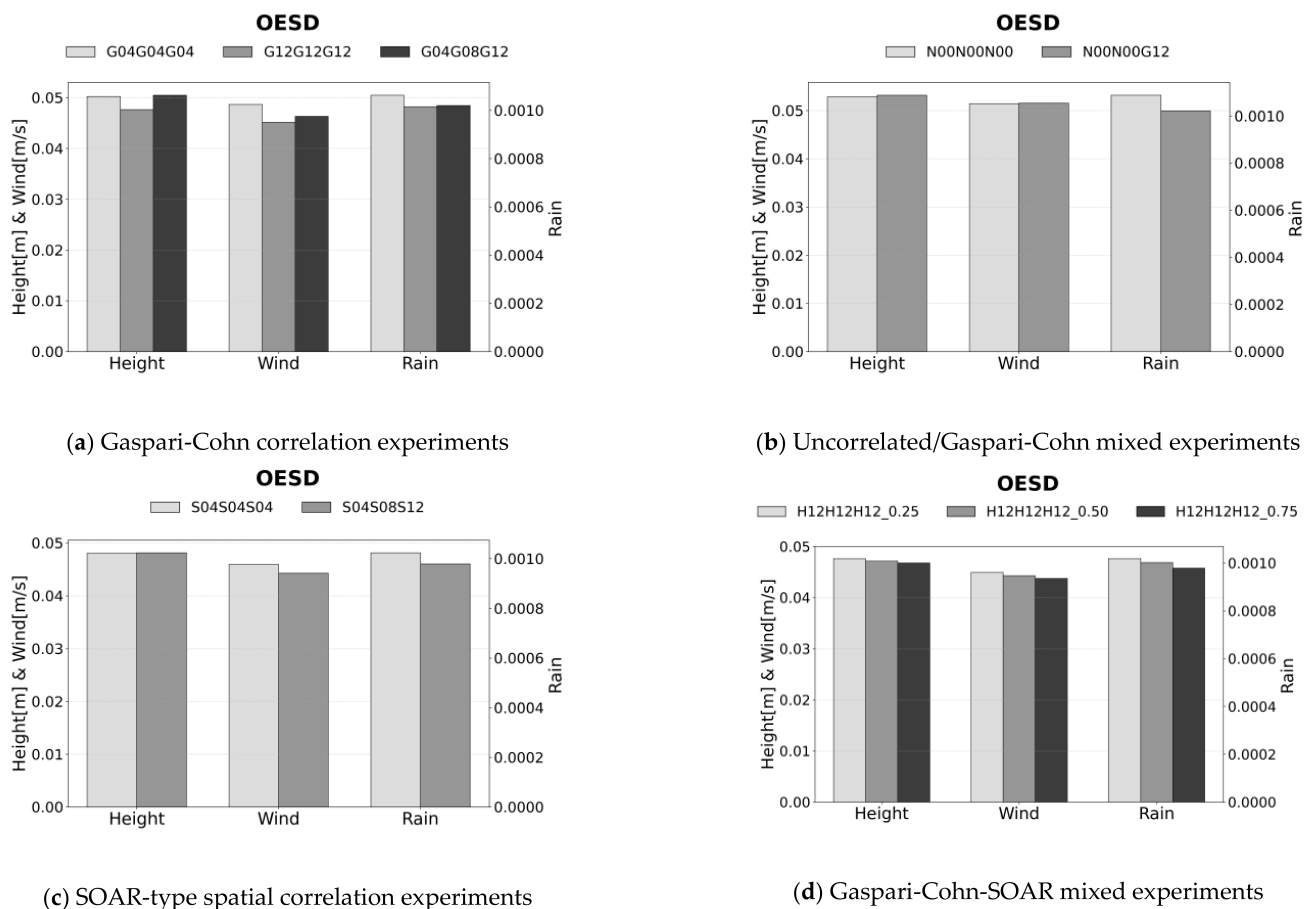


Figure 3. Estimated observation-error standard deviations diagnosed by D05 against prescribed values for four spatial-correlation categories in the 20-member experiments: (a) Gaspari-Cohn correlation experiments, (b) uncorrelated/Gaspari-Cohn mixed experiments, (c) SOAR-type spatial correlation experiments, and (d) Gaspari-Cohn-SOAR mixed experiments. Small departures from the prescribed values reflect the combined effects of diagonal-R cycling, finite ensemble size, and finite diagnostic sampling.

Table 2. Representative prescribed and D05-diagnosed observation-error standard deviations from the public diagnostic-code examples. Bias is computed as diagnosed minus prescribed value. These numerical examples document the magnitude of the variance diagnostic and complement the graphical comparison in Figure 3.

Case	Variable	Prescribed SD	D05 SD	Bias	Bias (%)
N00N00N00	Wind	0.00500	0.00486	−0.00014	−2.80
N00N00N00	Height	1.00000	1.01603	0.01603	1.60
N00N00N00	Rain	0.00005	0.000058	0.000008	16.63
G04G04G04	Wind	0.00500	0.00458	−0.00042	−8.36
G04G04G04	Height	1.00000	1.01222	0.01222	1.22
G04G04G04	Rain	0.00005	0.000057	0.000007	14.88
S04S04S04	Wind	0.00500	0.00472	−0.00028	−5.68
S04S04S04	Height	1.00000	0.99021	−0.00979	−0.98
S04S04S04	Rain	0.00005	0.000056	0.000006	11.01

Figure 5 shows mixed uncorrelated-Gaspari-Cohn experiments. When all variables have fully uncorrelated errors, D05 recovers near-zero spatial correlation accurately. In the

mixed case with uncorrelated height/wind and Gaspari-Cohn-correlated rain errors, D05 separates the near-zero-correlation variables from the correlated rainwater component and reproduces the rainwater correlation length scale. These results indicate that D05 works reliably for the hybrid uncorrelated/Gaspari-Cohn regimes considered here.

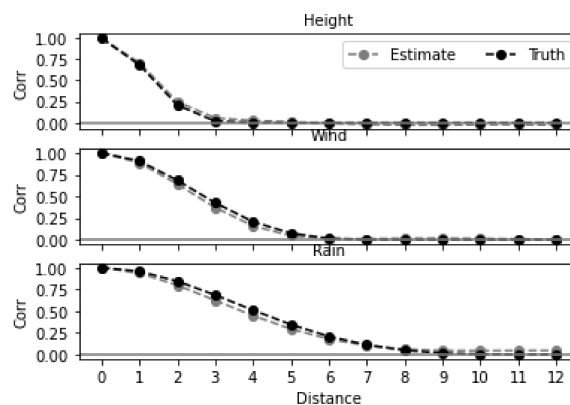
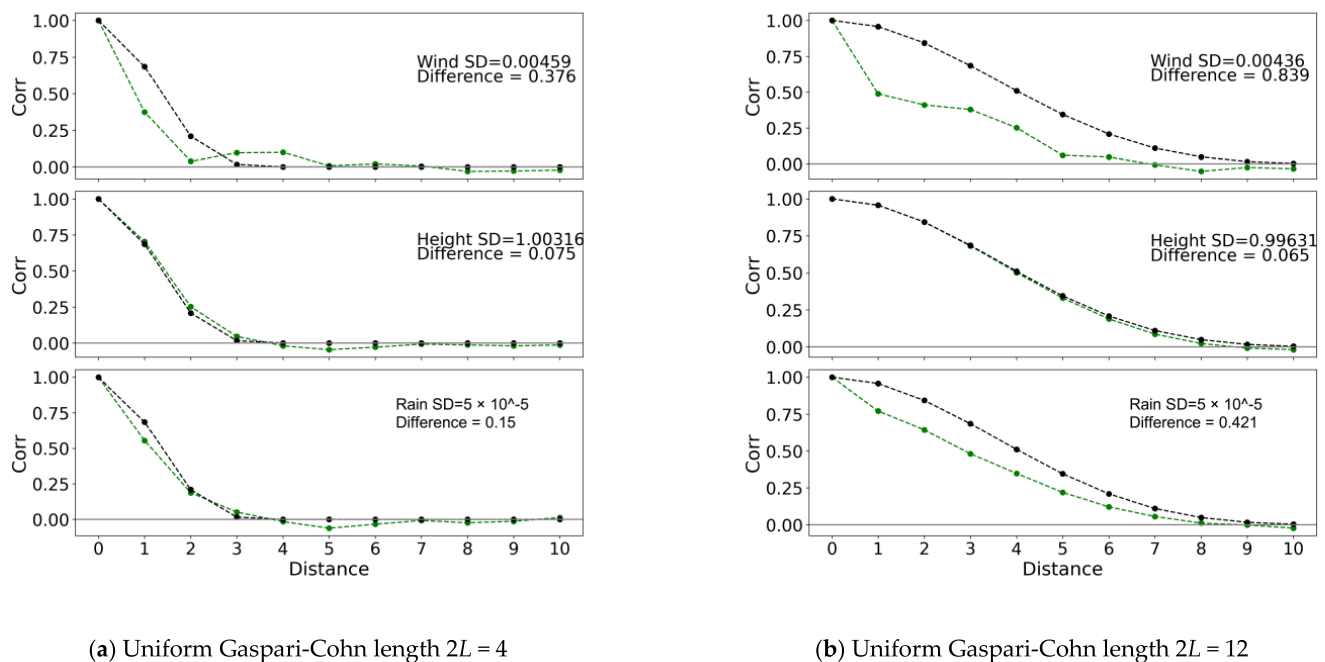


Figure 4. Prescribed versus D05-estimated spatial correlation functions for Gaspari-Cohn observation-error regimes: (a) uniform Gaspari-Cohn length $2L = 4$, (b) uniform Gaspari-Cohn length $2L = 12$, and (c) variable Gaspari-Cohn lengths 4/8/12. Green dashed curves denote D05 estimates, black dashed curves denote prescribed truth, and the gray solid line marks zero correlation. D05 closely reproduces the compact-support correlation decay curves and target length scales in the 20-member experiments.

Figure 6 illustrates D05 performance under SOAR-type spatial correlations. Regardless of uniform or variable prescribed correlation lengths, D05 produces longer diagnosed correlation lengths than the prescribed SOAR curves. This result indicates that D05 is sensitive to the functional form of the spatial correlation used to generate the observation-error covariance. Because the assimilation system still uses a diagonal $\mathbf{R}$ during cycling, the observed bias should be interpreted as the combined effect of SOAR-type dependence, covariance misspecification, the 20-member ensemble size, and finite sampling, rather than as direct evidence of distributional non-Gaussianity alone.

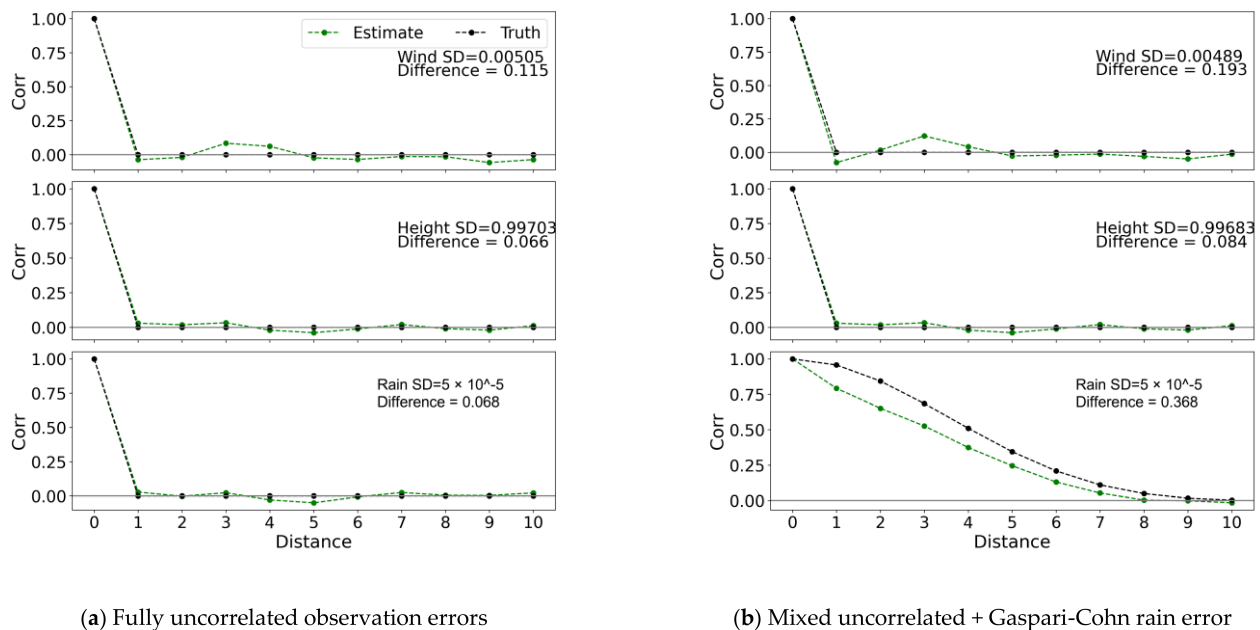


Figure 5. Correlation diagnosis results for uncorrelated and mixed Gaspari-Cohn-uncorrelated error regimes: (a) fully uncorrelated observation errors and (b) mixed uncorrelated plus Gaspari-Cohn rain error. Green dashed curves denote D05 estimates, black dashed curves denote prescribed truth, and the gray solid line marks zero correlation. D05 correctly identifies near-zero spatial correlation for variables with diagonal prescribed R in the 20-member experiments.

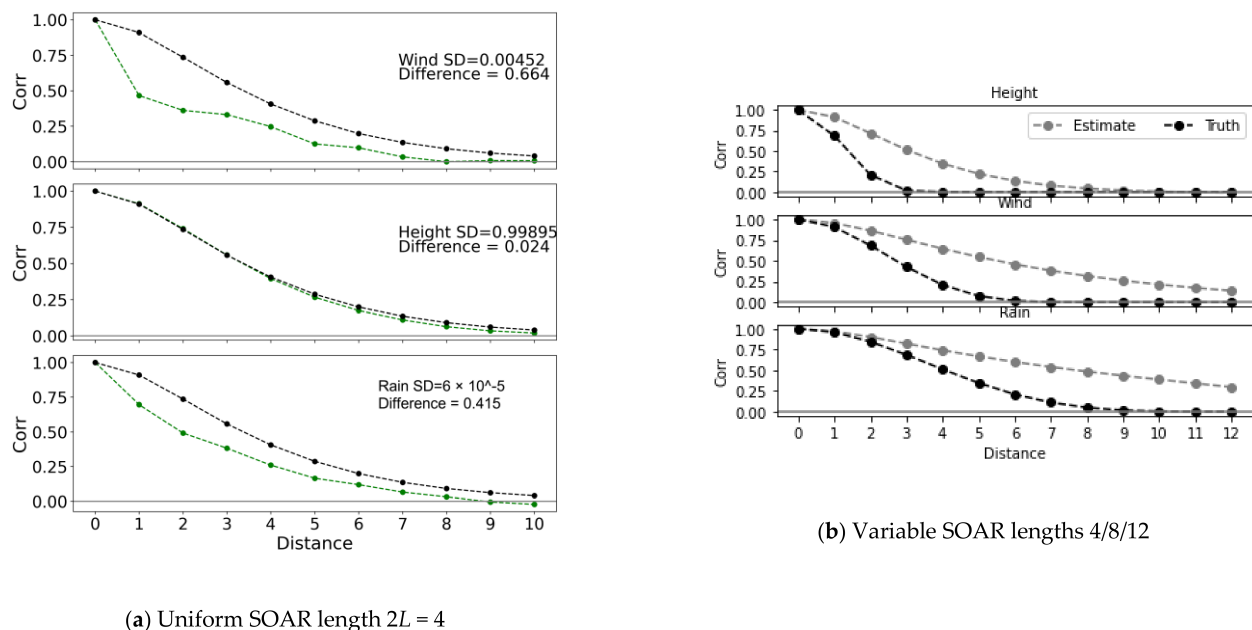


Figure 6. Prescribed SOAR spatial correlation curves and D05-diagnosed correlations: (a) uniform SOAR length $2L = 4$ and (b) variable SOAR lengths 4/8/12. Green dashed curves denote D05 estimates, black dashed curves denote prescribed truth, and the gray solid line marks zero correlation. D05 produces longer diagnosed correlation scales than the prescribed SOAR curves in these 20-member experiments.

To quantify the term “correlation length” used in the interpretation, we define a diagnosed e-folding length ℓ_e as the first grid-point distance at which the normalized correlation curve decreases to $\exp(-1)$, with linear interpolation between neighboring grid distances. The length-scale bias is then

$$\Delta \ell_e = \ell_{e,D05} - \ell_{e,prescribed}.$$

(6)

Table 3 reports representative numerical estimates. The largest negative differences in the SOAR-type wind and rain examples indicate that the D05-estimated curve differs substantially from the prescribed curve shape; this table is therefore used together with the full curves in Figures 4–7, rather than as a replacement for visual curve-shape diagnosis.

Table 3. Representative prescribed and D05-diagnosed e-folding length scales from the public diagnostic-code examples. The curve-difference metric denotes the root-mean-square difference between prescribed and diagnosed correlation curves over the evaluated lags.

Case	Variable	Prescribed ℓ_e	D05 ℓ_e	Bias	Curve Difference
N00N00N00	Wind	0.63	0.61	−0.02	0.443
N00N00N00	Height	0.63	0.64	0.01	0.086
N00N00N00	Rain	0.63	0.67	0.04	0.161
G04G04G04	Wind	1.67	0.69	−0.98	0.684
G04G04G04	Height	1.67	1.72	0.05	0.101
G04G04G04	Rain	1.67	1.44	−0.23	0.208
S04S04S04	Wind	4.32	0.85	−3.47	0.956
S04S04S04	Height	4.32	4.24	−0.08	0.049
S04S04S04	Rain	4.32	3.51	−0.81	0.371

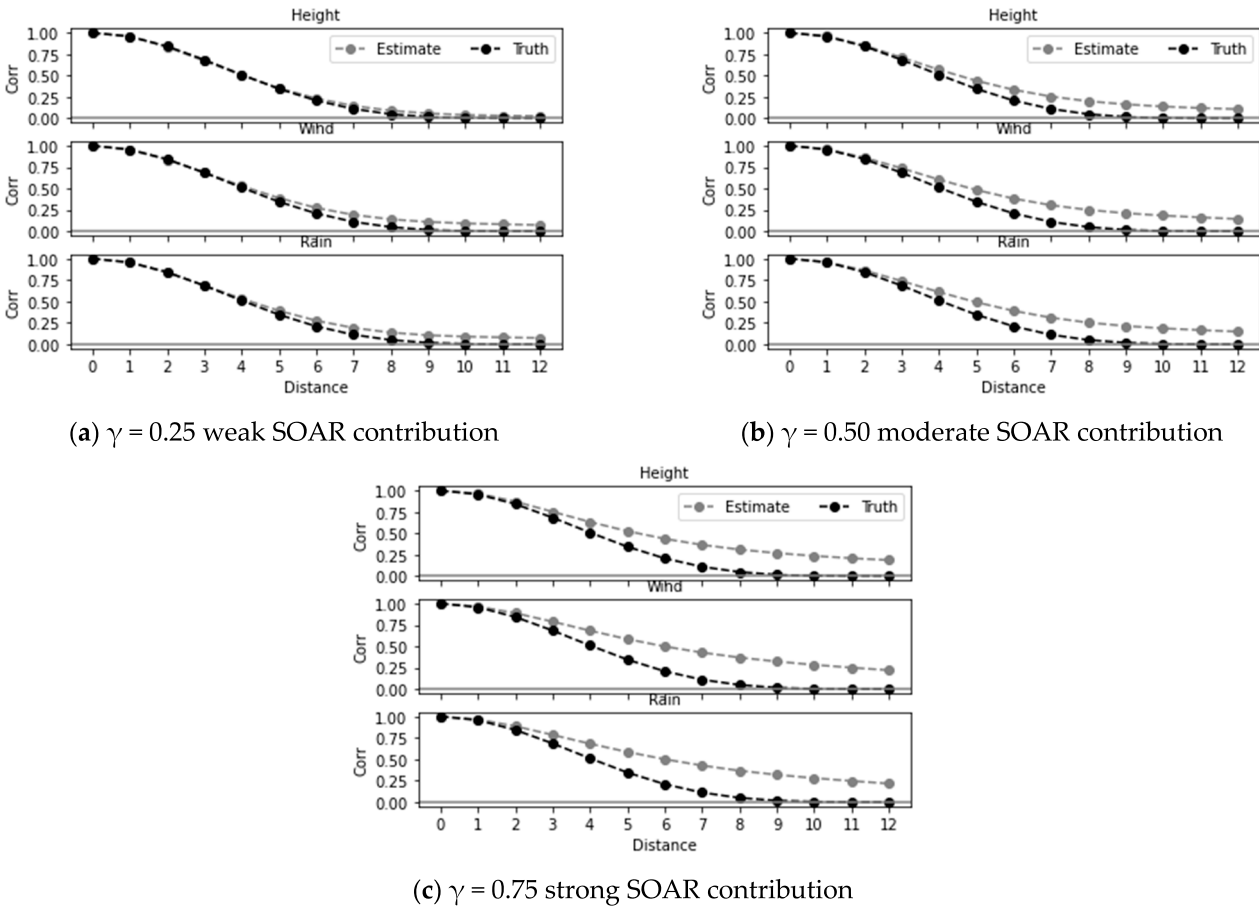


Figure 7. Diagnosed correlation functions for mixed Gaspari-Cohn-SOAR error fields with increasing gamma in the 20-member experiments: (a) gamma = 0.25, (b) gamma = 0.50, and (c) gamma = 0.75. Green dashed curves denote D05 estimates, black dashed curves denote prescribed truth, and the gray solid line marks zero correlation. The nominal prescribed correlation length for all variables is fixed at 2L = 12, while the diagnosed correlation-length overestimation increases with the SOAR contribution.

Figure 7 further quantifies the relationship between SOAR contribution and D05 bias. As mixing coefficient γ increases from 0.25 to 0.75, the gap between diagnosed and prescribed correlation lengths expands continuously. The overestimation bias of D05 therefore increases with the SOAR contribution in the prescribed spatial correlation structure.

The interpretation of this systematic bias requires caution. The distributional diagnostics in Figure 2 show near-zero skewness and near-Gaussian Pearson kurtosis for the generated observation-error samples, including the SOAR-type and mixed experiments. Thus, the present experiments demonstrate sensitivity of D05 to SOAR-type spatial correlation structures under an assimilation setup that uses diagonal $\mathbf{R}$ and a 20-member ensemble, but they do not isolate the effect of marginal or joint distributional non-Gaussianity. A more complete separation would require cross-comparison experiments in which the same correlation function is paired with different error distributions, and the same distribution is paired with different correlation functions. The present results should therefore be read as evidence that SOAR-type spatial dependence and covariance misspecification can lead to positive diagnosed length-scale bias in D05.

4. Conclusions and Outlook

This study systematically evaluates the performance of the widely used Desroziers D05 diagnostic for observation error covariance estimation under controlled twin experiment conditions with a modified shallow water model (modRSW). Four representative observation-error spatial correlation regimes are tested using a 20-member ensemble: Gaspari-Cohn, fully uncorrelated, mixed uncorrelated-Gaspari-Cohn, and SOAR-type correlation with tunable SOAR contribution. The D05-diagnosed error variances and spatial correlation patterns are evaluated against the prescribed synthetic observation-error covariance structures.

Numerical results show that D05 reproduces the prescribed Gaspari-Cohn spatial correlation functions well for uniform correlation lengths, variable correlation lengths across different meteorological variables, and hybrid regimes mixing uncorrelated and Gaspari-Cohn-correlated observational errors. However, small negative biases in diagnosed standard deviations remain when the assimilation system uses diagonal $\mathbf{R}$ while the prescribed true errors are spatially correlated. Thus, D05 performs well for the correlation-shape diagnosis in these cases, but the variance diagnosis is still affected by covariance misspecification in the cycling assimilation system.

In contrast, D05 exhibits systematic positive length-scale bias under the SOAR-type spatial correlation structures considered in this study. The diagnostic consistently produces longer diagnosed correlation scales than the prescribed SOAR curves, and the magnitude of this positive bias increases monotonically with the SOAR contribution in the mixed Gaspari-Cohn-SOAR experiments. Skewness, Pearson kurtosis, and representative standardized histograms of the prescribed observation-error samples show near-Gaussian marginal behavior in these experiments. These findings therefore demonstrate sensitivity of D05 to the functional form of the prescribed spatial correlation and to the use of a diagonal $\mathbf{R}$ in the assimilation step, rather than proving that the diagnosed bias is caused by distributional non-Gaussianity alone.

These findings carry practical implications for operational atmospheric data assimilation. The D05 diagnostic remains a useful and convenient tool for diagnosing observation-error covariance statistics, especially when the true spatial dependence is close to the correlation structure represented in the assimilation system. However, its outputs should be treated with caution when the true observation-error dependence structure differs strongly from conventional compact-support correlation models or when the assimilation system uses a simplified diagonal $\mathbf{R}$. Observation types affected by intermittent precipita-

tion, radar reflectivity, or cloud-contaminated radiances may require additional diagnostics to separate spatial dependence, distributional shape, representation error, and sampling effects before D05-derived correlations are directly adopted into **R**.

Future research should focus on three aspects:

1. Design cross-comparison experiments that further separate correlation-function shape from probability distribution, such as Gaspari-Cohn versus SOAR correlations combined with Gaussian, Student-*t*, lognormal, or skewed error distributions, and report distributional diagnostics including histograms, Q-Q plots, skewness, kurtosis, and normality tests.
2. Conduct sensitivity experiments to explore interactions among prescribed spatial dependence, diagonal versus correlated **R** in the assimilation system, ensemble inflation, horizontal localization, ensemble size, number of assimilation cycles, and random sampling uncertainty.
3. Validate the present idealized twin-experiment conclusions using real-world observational datasets and full-complexity numerical weather prediction models.

Author Contributions: Conceptualization, Y.Z.; methodology, L.W. and Y.Z.; software, L.W.; validation, K.Z. and Y.Z.; formal analysis, L.W. and Y.Z.; investigation, L.W.; resources, Y.Z.; data curation, L.W.; writing—original draft preparation, L.W.; writing—review and editing, Y.Z. and K.Z.; visualization, L.W.; supervision, Y.Z.; project administration, Y.Z.; funding acquisition, Y.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was primarily supported by the Foundation of Beiji Ge (Grant No. BJG202303). The APC was funded by the Foundation of Beiji Ge.

Institutional Review Board Statement: Not applicable; this study only adopts idealized numerical twin experiments without human participants or animal subjects.

Informed Consent Statement: Not applicable; no human subjects were involved in this research work.

Data Availability Statement: Representative scripts and selected diagnostic data illustrating the observation-error generation and Desroziers diagnostic procedures are publicly available at Zenodo: <https://doi.org/10.5281/zenodo.21623540>. Additional materials supporting the full numerical experiments are available from the corresponding author upon reasonable request.

Acknowledgments: The authors appreciate the editor and anonymous reviewers for constructive comments that improved this manuscript. No generative AI tools were used for text drafting, data analysis or figure generation during manuscript preparation.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

D05	Desroziers 2005 diagnostic method
DA	Data Assimilation
LETKF	Local Ensemble Transform Kalman Filter
modRSW	Modified Rotating Shallow Water model
RMSE	Root Mean Square Error
SOAR	Second-Order AutoRegressive correlation function

References

1. Walsworth, A.; Poterjoy, J.; Satterfield, E. Challenges for Inline Observation Error Estimation in the Presence of Misspecified Background Uncertainty. *Mon. Wea. Rev.* **2023**, *151*, 2295–2306. [[CrossRef](#)]
2. Janjic, T.; Bormann, N.; Bocquet, M.; Carton, J.A.; Cohn, S.E.; Dance, S.L.; Losa, S.N.; Nichols, N.K.; Potthast, R.; Waller, J.A.; et al. On the representation error in data assimilation. *Q. J. R. Meteorol. Soc.* **2018**, *144*, 1257–1278.
3. Weston, P.P.; Bell, W.; Eyre, J.R. Accounting for correlated error in the assimilation of high-resolution sounder data. *Q. J. R. Meteorol. Soc.* **2014**, *140*, 2420–2429. [[CrossRef](#)]
4. Desroziers, G.; Berre, L.; Chapnik, B.; Poli, P. Diagnosis of observation, background and analysis-error statistics in observation space. *Q. J. R. Meteorol. Soc.* **2005**, *131*, 3385–3396. [[CrossRef](#)]
5. Bormann, N.; Bonavita, M.; Dragani, R.; Eresmaa, R.; Matricardi, M.; McNally, A. Enhancing the impact of IASI observations through an updated observation-error covariance matrix. *Q. J. R. Meteorol. Soc.* **2016**, *142*, 1767–1780. [[CrossRef](#)]
6. Simonin, D.; Waller, J.A.; Ballard, S.P.; Dance, S.L.; Nichols, N.K. A pragmatic strategy for implementing spatially correlated observation errors in an operational system: An application to Doppler radial winds. *Q. J. R. Meteorol. Soc.* **2019**, *145*, 2772–2790. [[CrossRef](#)]
7. Schraff, C.; Reich, H.; Rhodin, A.; Schomburg, A.; Stephan, K.; Perianez, A.; Potthast, R. Kilometre-Scale Ensemble Data Assimilation for the COSMO Model (KENDA). *Q. J. R. Meteorol. Soc.* **2016**, *142*, 1453–1472. [[CrossRef](#)]
8. Menard, R. Error covariance estimation methods based on analysis residuals: Theoretical foundation and convergence properties derived from simplified observation networks. *Q. J. R. Meteorol. Soc.* **2016**, *142*, 257–273. [[CrossRef](#)]
9. Hu, G.; Dance, S. Sampling and misspecification errors in the estimation of observation-error covariance matrices using observation-minus-background and observation-minus-analysis statistics. *Q. J. R. Meteorol. Soc.* **2024**, *150*, 3052–3077. [[CrossRef](#)]
10. Kent, T.; Bokhove, O.; Tobias, S. Dynamics of an idealized fluid model for investigating convective-scale data assimilation. *Tellus A Dyn. Meteorol. Oceanogr.* **2017**, *69*, 1369332. [[CrossRef](#)]
11. Reichle, R.H.; Walker, R.D.; Koster, R.D.; Houser, P.R. Extended versus ensemble Kalman filtering for land data assimilation. *J. Hydrometeorol.* **2002**, *3*, 728–740. [[CrossRef](#)]
12. Waller, J.A.; Simonin, D.; Dance, S.L.; Nichols, N.K.; Ballard, S.P. Diagnosing Observation Error Correlations for Doppler Radar Radial Winds in the Met Office UKV Model Using Observation-Minus-Background and Observation-Minus-Analysis Statistics. *Mon. Wea. Rev.* **2016**, *144*, 3533–3551. [[CrossRef](#)]
13. Lange, H.; Janjic, T. Assimilation of Mode-S EHS Aircraft Observations in COSMO-KENDA. *Mon. Wea. Rev.* **2016**, *144*, 1697–1711. [[CrossRef](#)]
14. Waller, J.A.; Ballard, S.P.; Dance, S.L.; Kelly, G.; Nichols, N.K.; Simonin, D. Diagnosing horizontal and inter-channel observation error correlations for SEVIRI observations using observation-minus-background and observation-minus-analysis statistics. *Remote Sens.* **2016**, *8*, 581. [[CrossRef](#)]
15. Zeng, Y.; Janjic, T.; Feng, Y.; Blahak, U.; de Lozar, A.; Bauernschubert, E.; Stephan, K.; Min, J. Interpreting estimated Observation Error Statistics of Weather Radar Measurements using the ICON-LAM-KENDA System. *Atmos. Meas. Tech.* **2021**, *14*, 5735–5756. [[CrossRef](#)]
16. Hunt, B.R.; Kostelich, E.J.; Szunyogh, I. Efficient data assimilation for Spatiotemporal Chaos: A Local Ensemble Transform Kalman Filter. *Phys. D Nonlinear Phenom.* **2007**, *230*, 112–126. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.